Global Journal of Science Frontier Research: A
Physics and Space Science
Volume 18 Issue 1 Version 1.0 Year 2018
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Chemical in Homogeneities, Electric Currents, and Diffusion Waves in Stars

By Vadim URPIN
A.F.Ioffe Institute of Physics and Technology

Abstract- The stars of the middle main sequence often have spot-like chemical structures at their surfaces. We consider diffusion caused by electric currents and argue that such current-driven diffusion can form chemical in homogeneities in plasma. Diffusion was considered using partial momentum equations derived by the Chapman-Enskog method. We argue that diffusion caused by electric currents can substantially change the surface chemistry of stars and form spotted chemical structures even in a relatively weak magnetic field. The considered mechanism can be responsible for a formation of element spots in Hg-Mn and Ap-stars. Diffusion in the presence of electric currents can be accompanied by propagation of a particular type of magneto hydrodynamic modes in which only the impurity number density oscillate. Such modes exist if the magnetic pressure is much greater than the gas pressure and can be the reason of variations of the abundance peculiarities in stars.

Keywords: *stars: magnetic fields - stars: abundances - stars: chemically peculiar- stars: spots.*

GJSFR-A Classification : FOR Code: 020105

CHEMICALINHOMOGENEITIESELECTRICCURRENTSANDDIFFUSIONWAVESINSTARS

Strictly as per the compliance and regulations of:

Chemical in Homogeneities, Electric Currents, and Diffusion Waves in Stars

Vadim URPIN

Abstract- The stars of the middle main sequence often have spot-like chemical structures at their surfaces. We consider diffusion caused by electric currents and argue that such current-driven diffusion can form chemical in homogeneities in plasma. Diffusion was considered using partial momentum equations derived by the Chapman-Enskog method. We argue that diffusion caused by electric currents can substantially change the surface chemistry of stars and form spotted chemical structures even in a relatively weak magnetic field. The considered mechanism can be responsible for a formation of element spots in Hg-Mn and Ap-stars. Diffusion in the presence of electric currents can be accompanied by propagation of a particular type of magneto hydrodynamic modes in which only the impurity number density oscillate. Such modes exist if the magnetic pressure is much greater than the gas pressure and can be the reason of variations of the abundance peculiarities in stars.

Keywords: stars: magnetic fields - stars: abundances - stars: chemically peculiar- stars: spots.

I. Introduction

The stars of the middle main sequence often have relatively quiescent surface layers, and the abundance peculiarities can develop in their atmospheres since, in general, there are physical processes that lead to evolution of atmospheric chemistry during the main sequence lifetime. Chemical composition can evolve in the atmospheres of such stars, for example, because of loss of heavy ions caused by gravitational settling. Also, the atmosphere can acquires ions driven upwards by radiative acceleration due to the radiative energy flux (see Michaud 1970, Michaud et. al. 1976, Vauclair et al. 1979, Alecian & Stift 2006). Many stars with peculiar chemical abundances show line-profile variations caused by element spots on their surface (see, e.g., Pyper 1969, Khokhlova 1985, Silvester et al. 2012). The exact reasons of inhomogeneous surface distributions on stars are unknown. It was thought that chemical spots can only occur in the presence of a strong organized magnetic field. Indeed, some stars exhibit the presence of such magnetic fields. For example, Ap stars show variations of both spectral lines and magnetic field strength that can be caused by rotation of chemical and magnetic spots. Often such stars have the strongest concentration of heavy elements around the magnetic poles (see, e.g., Havnes 1975). A reconstruction of the stellar magnetic geometry from observations is a very complex problem for decade. The magnetic Doppler imaging code developed by Piskunov & Kochukhov (2002) makes it possible to derive the magnetic map of a star self-consistently with the distribution of chemical elements. The reconstructions show that the magnetic and chemical maps can be extremely complex (Kochukhov et al. 2004a). For instance, Kochukhov et al. (2004b) have found that almost all elements (except, may be, Li and O) of the Ap-star HR 3831 do not follow the symmetry of the dipolar magnetic field but are distributed in a rather complex manner. The calculated distributions demonstrate the complexity of diffusion in Ap-stars and discard a point of view that diffusion leads to a formation of the chemical spots symmetric with respect to the londitudinal magnetic field (Kochukhov 2004). Likely, chemical distributions are affected by a number of poorly understood phenomena in the surface layers of stars.

Often, a formation of the chemical spots is related to anisotropic diffusion in a strong magnetic field. Indeed, the magnetic field of Ap-stars (~10^3–10^4G) can magnetize electrons in plasma that, generally, leads to anisotropic transport. Anisotropy of diffusion in a magnetized plasma is characterized by the the Hall parameter, $x_e = \omega_{Be}\tau_e$, where $\omega_{Be} = eB/m_e c$ is the gyrofrequency of electrons and τ_e is their relaxation time; B is the magnetic field. If the base ground plasma is presumably hydrogen, then $\tau_e = 3\sqrt{m_e}(k_b T)^{3/2}/4\sqrt{2\pi}e^4 n\ \Lambda$ (see, e.g., Spitzer 1998) where n and T are the number density of electrons and their temperature, Λ is the Coulomb logarithm. At $x_e \geq 1$, the rates of diffusion along and across the magnetic field become different. The condition $x_e \geq 1$ yields the following estimate of the magnetic field that magnetizes plasma

$$B \geq B_e = 2.1 \times 10^3 \Lambda_{10} n_{15} T_4^{-3/2}\ \text{G}, \qquad (1)$$

where $\Lambda_{10} = \Lambda/10$, $n_{15} = n/10^{15}$, and $T_4 = T/10^4 K$. Some Ap-stars that exhibit spot-like chemical structures have a sufficiently strong magnetic field that satisfies this condition. Note, however, that the magnetic field (1) magnetizes only electrons and, as a result, its effect on diffusion of heavy ions is relatively weak. Perhaps, a much stronger field that magnetizes protons is required in order to produce strong chemical in homogeneities in stars. In this case, one requires $y > 1$ where $y = eB\tau_p/m_p c$ is the Hall parameter for protons and

Author: A. F. Ioffe Institute of Physics and Technology, 194021 St. Petersburg, Russia. e-mail: vadim.urpin@uv.es

$\tau_p = 3\sqrt{m_p}(k_B T)^{3/2}/4\sqrt{2\pi}e^4 n \Lambda$ is the relaxation time for protons (see, e.g., Spitzer 1998). The condition $y > 1$ yields

$$B > B_p = 10^5 n_{15} T_4^{-3/2} \Lambda_{10} \ \text{G}. \quad (2)$$

Such field is substantially stronger than the field detected at the surface of Ap-stars.

In recent years, the discovery of chemical inhomogeneities in the so-called Hg-Mn stars has rised additional doubts regarding the magnetic origin of these inhomogeneities. The aspect of inhomogeneous distribution of some chemical elements over the surface of HgMn stars was discussed first by Hubrig & Mathys (1995). In contrast to Ap-stars, no strong large-scale madnetic field of kG order has ever been detected in HgMn stars. For instance, Wade et al. (2004) find no longitudinal field above 50 G in the brightest Hg-Mn star αAnd with distributed inhomogeneously chemical elements. The authors also establish an upper limit of the global field at $\approx$ 300 G that is obviously not sufficient to magnetize plasma. Weak magnetic fields in the atmospheres of Hg-Mn stars have been detected also by a number of authors (see, e.g., Hubrig & Castelli 2001, Hubrig et al. 2006, Makaganiuk et al. 2011, 2012). In a recent study by Hubrig et al. (2012), the previous measuments of the magnetic field have been re-analysed and the presence of a weak longitudinal magnetic field up to 60-80 G has been revealed in several HgMn stars. On the other hand, magnetic fields up to a few hundred Gauss have been detected in several HgMn stars (see, e.g., Mathys & Hubrig 1995). Measurements by Hubrig at al. (2010) reveal a longitudinal magnetic field of the order of a few hundred Gauss in the spotted star AR Aur. The complex interrelations between the magnetic field and the chemical structures clearly indicate how incomplete is our understanding of diffusion in stars.

In this paper, we consider one more diffusion process that can be responsible for a formation of chemical inhomogeneities in stars. This process is relevant to electric currents and well studied in a laboratory plasma (see, e.g., Vekshtein et al. 1975) but have not been considered in detail in stellar conditions. By making use of a simple model, we show in this paper that interaction of the electric currents with different sorts of ions leads to their diffusion in the direction perpendicular to both the electric current and magnetic field. This type of diffusion can alter the surface chemical distributions even if the magnetic field is substantially weaker than B_e.

II. Basic Equations

Consider a cylindrical plasma configuration with the magnetic field parallel to the axis z, $\vec{B} = B(s)\vec{e}_z$; (s, φ, z) and $(\vec{e}_s, \vec{e}_\varphi, \vec{e}_z)$ are cylindrical coordinates and the corresponding unit vectors. The electric current in such configuration is

$$j_\varphi = -(c/4\pi)(dB/ds). \quad (3)$$

We suppose that $j_\varphi \to 0$ at large s and, hence, $B \to B_0$=const at $s \to \infty$. Note that $B(s)$ can not be arbitrary function of s because, generally, the magnetic configurations can be unstable for some dependences *B(s)* (see, e.g., Tayler 1973, Bonanno & Urpin 2008a,b). The characteristic timescale of this instability is usually of the order of the time taken for an Alvén wave to travel around the star that is much shorter than the diffision timescale. Therefore, a formation of chemical structures in such magnetic configurations is impossible. Note that, in some cases, the considered configuration can mimic real magnetic fields with a high accuracy. This is valid, for example, for the magnetic field near the magnetic pole where the fild lines are very close to a cylindrical geometry (see, e.g., Urpin & Van Riper 1993).

We assume that plasma is fully ionized and consists of electrons e, protons p, and a small admixture of heavy ions *i*. The number density of species *i* is small and does not influence the dynamics of plasma. Therefore, these ions can be treated as trace particles interacting only with a background hydrogen plasma. The hydrostatic equilibrium in such plasma is given by

$$-\nabla p + \vec{F} + \frac{1}{c}\vec{j} \times \vec{B} = 0, \quad (4)$$

where p is the gas pressure, ρ is the density, and $\vec{F}$ is an external force acting on plasma. Since the baseground plasma is hydrogen and fully ionized, $p \approx 2nk_B T$ where k_B is the Boltzmann constant. In stellar conditions, $\vec{F}$ is usually the sum of two forces, $\vec{F} = \vec{F}_g + \vec{F}_{rad}$, where $\vec{F}_g = \rho\vec{g}$ is the gravity force and $\vec{F}_{rad}$ is caused by radiative acceleration due to the radiative energy flux from the interior. We assume that the both external forces $\vec{F}_g$ and $\vec{F}_{rad}$ act in the vertical direction. Then, the z-component of Eq. (4) determines the vertical distribution of a background plasma and reads $\partial p/\partial z = F_z$. The s-component of these equation describes the transverse structure of a magnetic atmosphere. For the sake of simplicity, we consider the case T=const and neglect the contribution of thermodiffusion. Integrating the scomponent of Eq. (4), we obtain

$$n = n_0 \left(1 + \beta_0^{-1} - \beta^{-1}\right), \quad (5)$$

where $\beta = 8\pi p_0/B^2$; $(p_0, n_0, T_0, \beta_0$ are the values of (p, n, T, β) at $s \to \infty$.

The partial momentum equations in fully ionized multi component plasma have been considered by a number of authores (see, e.g., Urpin 1981). This study deals mainly with the hydrogen-helium plasma. However, the derived equations can be applied for hydrogen plasma with a small admixture of any other ions if their number density is small. If the mean hydrodynamic velocity of plasma is zero and only small

Global Journal of Science Frontier Research (A) Volume XVIII Issue I Version I 48 Year 2018

diffusive velocities are non-vanishing, the partial momentum equation for the species *i* reads

$$-\nabla p_i + Z_i e n_i \left(\vec{E} + \frac{\vec{V}_i}{c} \times \vec{B} \right) + \vec{R}_{ie} + \vec{R}_{ip} + \vec{F}_i = 0, \quad (6)$$

where Z_i is the charge number of the species i, p_i and n_i are its partial pressure and number density, $\vec{E}$ is the electric field in plasma, and $\vec{V}_i$ is the diffusion velocity. Since diffusive velocities are usually very small, we neglect the terms proportional $(\vec{V}_i \cdot \nabla)\vec{V}_i$ in the momentum equation (6). The force $\vec{F}_i$ is the external force on species i; in stellar conditions, $\vec{F}_i$ is the sum of gravitational and radiative forces. The forces $\vec{R}_{ie}$ and $\vec{R}_{iH}$ are caused by the interaction of ions i with electrons and protons, respectively. Note that forces $\vec{R}_{ie}$ and $\vec{R}_{iH}$ are internal, but the sum of internal forces over all plasma components is zero in accordance with Newton's third law. If n_i is small compared to the number density of protons, $\vec{R}_{ie}$ is given by

$$\vec{R}_{ie} = -(Z_i^2 n_i / n) \vec{R}_e \quad (7)$$

where $\vec{R}_e$ is the force acting on the electron gas (see, e.g., Urpin 1981). Since $n_i \ll n, \vec{R}_e$ is determined mainly by scattering of electrons on protons but scattering on ions i gives a small contribution to $\vec{R}_e$. Therefore, we can use for $\vec{R}_e$ the expression for hydrogen plasma calculated by Braginskii (1965). In our model of isothermal plasma, the expression for $\vec{R}_e$ reads

$$\vec{R}_e = -\alpha_{\parallel} \vec{u}_{\parallel} - \alpha_{\perp} \vec{u}_{\perp} + \alpha_{\wedge} \vec{b} \times \vec{u}, \quad (8)$$

where $\vec{u} = -\vec{j}/en$ is the current velocity of electrons; $\vec{b} = \vec{B}/B$; the subscripts $\parallel, \perp$, and $\wedge$ denote the parallel, perpendicular, and the so called Hall components of the corresponding vector; $\alpha_{\parallel}, \alpha_{\perp}$ and $\alpha_{\wedge}$ are the coefficients calculated by Braginskii (1965). Taking into account Eq.(3), we have

$$\vec{u} = (c/4\pi en)(dB/ds)\vec{e}_{\varphi}. \quad (9)$$

Since $\vec{B} \perp \vec{u}$ in our model, we have $\vec{u}_{\parallel} = 0$. In this paper, we consider diffusion only in a relatively weak magnetic field that does not magnetize electrons, $x_e \ll 1$. Substituting Eq.(8) into Eq.(7) and using coefficients $\alpha_{\perp}$ and $\alpha_{\wedge}$ calculated by Braginskii (1965) with the accuracy in linear terms in x_e, we obtain

$$R_{ie\varphi} = Z_i^2 n_i \left(0.51 \frac{m_e}{\tau_e} u \right), \quad R_{ies} = Z_i^2 n_i \left(0.21 x \frac{m_e}{\tau_e} u \right). \quad (10)$$

If T =const, the friction force $\vec{R}_{ip}$ is proportional to the relative velocity of ions *i* and protons. Like $\vec{R}_e$ (see Eq.(8)), this force also has a tensor character and, generally, depends on the magnetic field.

The force $\vec{R}_{ip}$ has an especially simple shape if $A_i = m_i/m_p \gg 1$ (see Urpin 1981) and we consider only this case. We neglect the influence of the magnetic field on $\vec{R}_{ip}$ since this influence becomes important only in a strong magnetic field $\geq B_p$. Taking into account that the velocity of the background plasma is zero, $\vec{V}_p = 0$, the friction force $\vec{R}_{ip}$ can be written as

$$\vec{R}_{ip} = (0.42 m_i n_i Z_i^2 / \tau_i)(-\vec{V}_i), \quad (11)$$

where $\tau_i = 3\sqrt{m_i}(k_B T)^{3/2} / 4\sqrt{2\pi} e^4 n \Lambda; \tau_i / Z_i^2$ is the timescale of ion-proton scattering; we assume that Λ is the same for all types of scattering (see, e.g., Urpin 1981).

III. Diffusion Velocity

The cylindrical components of Eq.(6) yield

$$-\frac{d}{ds}(n_i k_B T) + Z_i e n_i \left(E_s + \frac{V_{i\varphi}}{c} B \right) + R_{ies} + R_{ips} = 0, \quad (12)$$

$$Z_i e n_i \left(E_{\varphi} - \frac{V_{is}}{c} B \right) + R_{ie\varphi} + R_{ip\varphi} = 0, \quad (13)$$

$$-\frac{d}{dz}(n_i k_B T) + Z_i e n_i E_z + R_{iez} + R_{ipz} + F_{iz} = 0. \quad (14)$$

In our simplified magnetic configuration, we have $R_{iez} = 0$. Eqs.(12)-(14) depend on cylindrical components of the electric field, E_s, E_{φ}, and E_z. These components can be determined from the momentum equations for electrons and protons

$$-\nabla(n k_B T) - en \left(\vec{E} + \frac{\vec{u}}{c} \times \vec{B} \right) + \vec{R}_e + \vec{F}_e = 0, \quad (15)$$

$$-\nabla(n k_B T) + en\vec{E} - \vec{R}_e + \vec{F}_p = 0. \quad (16)$$

In these equations, we neglect collisions of electrons and protons with the ions i since these ions are considered as the test particles and their number density is assumed to be small. The sum of Eqs.(15) and (16) yield the equation of hydrostatic equilibrium (4). The difference of Eqs.(16) and (15) yields the following expression fo the electric field

$$\vec{E} = -\frac{1}{2}\frac{\vec{u}}{c} \times \vec{B} + \frac{\vec{R}_e}{en} - \frac{1}{2en}(\vec{F}_p - \vec{F}_e). \quad (17)$$

Taking into account the friction force $\vec{R}_e$ (Eq.(8)) and the coefficients $\alpha_{\perp}$ and $\alpha_{\wedge}$, calculated by Braginskii (1965), we obtain with accuracy in linear terms in x_e

$$E_s = -\frac{uB}{2c} - \frac{1}{e}\left(0.21 \frac{m_e u}{\tau_e} x_e \right), E_{\varphi} = -\frac{1}{e}\left(0.51 \frac{m_e u}{\tau_e} \right),$$

$$E_z = -\frac{1}{2en}(F_{pz} - F_{ez}). \qquad (18)$$

Substituting Eqs.(7) and (19) into vertical component of the momentum equation (15), we obtain the following expression for the velocity of vertical diffusion

$$V_{iz} = -D\frac{d\ln n_i}{dz} + \frac{D}{n_i k_B T}F_z^{(i)}, \qquad (19)$$

where $D = 2.4c_i^2\tau_i/Z_i^2$ is the diffusion coefficient, $c_i^2 = k_B T/m_i$, and

$$F_z^{(i)} = F_{iz} - \frac{Z_i n_i}{2n}(F_{pz} - F_{ez}). \qquad (20)$$

Often, radiative acceleration due to the radiative energy flux and gravitational settling give the main contribution to the external force $F_z^{(i)}$ (Michaud et al. 1976). The diffusion velocity caused by these forces can be relatively large and, therefore, the vertical diffusion often is faster than diffusion in the tangential direction parallel to the surface. As a result, the vertical distribution of chemical elements reaches a quasi-steady equilibrium on a relatively short timescale. We will show, however, that the horizontal diffusion can form spots faster than the vertical diffusion if the magnetic field is weak.

The tangential components of the difusion velocity can be obtained from Eqs. (13) and (14). Taking into acount Eq. (12) for $\vec{R}_{ip}$, one can transform Eqs. (13)-(14) into

$$V_{is} - qV_{i\varphi} = A, \quad V_{i\varphi} + qV_{is} = G, \qquad (21)$$

where

$$A = \frac{D}{n_i k_B T}\left(-\frac{dp_i}{ds} + Z_i e n_i E_s + R_{ies}\right), \qquad (22)$$

$$G = \frac{D}{n_i k_B T}(Z_i e n_i E_\varphi + R_{ie\varphi}), \quad q = 2.4\frac{eB}{Z_i m_i c}\tau_i. \qquad (23)$$

Then, the diffusion velocities in the s- and φ-directions are

$$V_{is} = \frac{A + qG}{1+q^2}, \quad V_{i\varphi} = \frac{G - qA}{1+q^2}. \qquad (24)$$

The parameter q is of the order of $\omega_{Bi}\tau_i$ and is small even for magnetic fields typical for Ap-stars. Then, we have for $q \ll 1$

$$V_{is} \approx A, \quad V_{i\varphi} \approx G. \qquad (25)$$

Substituting Eqs. (10) and (19) into expressions (23)-(24) for A and G, we obtain the following expressions for the diffusion velocities

$$V_{is} = V_{n_i} + V_B, \quad V_{n_i} = -D\frac{d\ln n_i}{ds}, \quad V_B = D_B\frac{d\ln B}{ds} \qquad (26)$$

$$V_{i\varphi} = D_{B\varphi}\frac{dB}{ds} \qquad (27)$$

V_{ni} is the velocities of ordinary diffusion and V_B is the diffusion velocity caused by the electric current. The corresponding diffusion coefficients are

$$D = \frac{2.4c_i^2\tau_i}{Z_i^2}, \quad D_B = \frac{2.4c_A^2\tau_i}{Z_i A_i}(0.21Z_i - 0.71), \qquad (28)$$

$$D_{B\varphi} = 1.22\sqrt{\frac{m_e}{m_i}}\frac{c(Z_i-1)}{4\pi e n Z_i}. \qquad (29)$$

where $c_i^2 = k_B T/m_i$ and $c_A^2 = B^2/(4\pi n m_p)$. Eqs. (27)-(28) describe the drift of ions i under the combined influence of ∇n_i and $\vec{j}$.

IV. Distribution of Ions Caused by Electric Currents

Consider the equilibrium distribution of heavy ions in our model. In equilibrium, we have $V_{is} = 0$ and Eq.(27) yields

$$D\frac{d\ln n_i}{ds} = D_B\frac{d\ln B}{ds}. \qquad (30)$$

The term on the r.h.s. describes the effect of electric currents on the distribution of impurities. Note that this type of diffusion is driven by the electric current rather than an inhomogeneity of the magnetic field. The conditions $dB/ds \neq 0$ and $j \neq 0$ are equivalent in our simplified magnetic configuration. Eq. (4) yields

$$\frac{d}{ds}(nk_B T) = -\frac{B}{8\pi}\frac{dB}{ds}. \qquad (31)$$

Substituting Eq. (32) into Eq.(31) and integrating, we obtain

$$\frac{n_i}{n_{i0}} = \left(\frac{n}{n_0}\right)^{\mu}, \qquad (32)$$

where

$$\mu = -2Z_i(0.21Z_i - 0.71) \qquad (33)$$

and n_{i0} is the value of n_i at $s \to \infty$. Denoting the local abundance of the element i as $\gamma_i = n_i/n$ and taking into account Eq. (5), we have

$$\frac{\gamma_i}{\gamma_{i0}} = \left(\frac{n}{n_0}\right)^{\mu-1} = \left(1 + \frac{1}{\beta_0} - \frac{1}{\beta}\right)^{\mu-1}, \qquad (34)$$

where $\gamma_{i0} = n_{i0}/n_0$. Local abundances turn out to be flexible to the field strength and, particularly, this concerns ions with large charge numbers. The exponent (μ−1) can reach large negative values for elements with large Z_i and, hence, produce strong abundance anormalies. For instance, (μ−1) is equal 1.16, -0.52, and -2.04 if Z_i =2, 3, and 4, respectively. Note that (μ−1) changes its sign as Z_i increases: (μ−1) > 0 if Z_i= 2 but (μ− 1) < 0 for $Z_i \geq 3$. Therefore, elements with $Z_i \geq 3$ are in deficit ($\gamma_i < \gamma_{i0}$) in the region with a weak magnetic field ($B < B_0$) but, on the contrary, these elements should be overabundant in the spot where the magnetic field is stronger than the external field B_0.

Note that the dependence of the exponent (μ−1) on Z_i can be responsible for the increase in He abundance in magnetic stars with stellar age. This increase was first discovered by Bailey et al. (2014) and is very unexpected within the frame of the standard theory because radiative levitation of He is very weak and becomes weaker as the star evolves. However, the increase in He abundance seems to be rather natural if one takes into account the current-driven diffusion. Indeed, observations indicate that the magnetic field decreases with the stellar age (see, e.g., Bailey et al. (2014)) because of ohmic dissipation and, hence, a contrast between the magnetic spots and ambient plasma becomes weaker. As it follows from Eq. (34), a weaker contrast of the magnetic field leads to a higher local abundance of He in a spot.

It is generally believed that the standard diffusion smoothes chemical in homogeneities on a timescale of the order of L^2/D where L is the length scale of a nonuniformity. However, this is not the case for a chemical distribution given by Eq. (34) which can exist during a much longer time than $\sim L^2/D$. In our model, distribution (34) is reached due to balance of two diffusion processes, standard ($\propto \nabla n_i$) and current-driven ($\propto dB/ds$) diffusion which push heavy ions in the opposite directions. As a result, $V_{is} = 0$ in the equilibrium state and this state can be maintained as long as the electric currents exist. Therefore, the characteristic lifetime of chemical structures is of the order of the decay time of electric currents that is determined by ohmic dissipation and is $\sim 4\pi\sigma L^2/c^2$ where σ is the electrical conductivity. Decay of the magnetic field is very slow in stellar conditions and the decay timescale can be longer than the diffusion timescale if $D > c^2/4\pi\sigma$. Under such conditions, the lifetime of a spot is entirely determined by the ohmic decay time.

Note tha V_{is} = 0 in the equilibrium state but the φ-component of the diffusion velocity is non-zero. It turns out that impurities rotate around the magnetic exis even if equlibrium is reached, $V_{i\varphi} \neq 0$. The direction of rotation depends on the sign of dB/ds and is opposite to the electric current. Since electrons move in the same direction, heavy ions turn out to be carried along electrons. Different ions move with different velocities around the axis, and the difference between different sorts of ions, $\Delta V_{i\varphi}$, is of the order of

$$\Delta V_{i\varphi} \sim \frac{c}{4\pi e n}\sqrt{\frac{m_e}{m_i}}\frac{dB}{ds} \sim 3\times 10^{-3}\frac{B_4}{n_{14}L_{10}A_i^{1/2}}\ \frac{\mathrm{cm}}{\mathrm{s}}, \tag{35}$$

where $B_4 = B/10^4$ G, $n_{14} = n/10^{14}\mathrm{cm}^{-3}$, and $L_{10} = L/10^{10}$. Since different impurities rotate around the magbetic axis with different velocities, periods of such rotation also are different for different ions. The difference in periods can be estimated as

$$\Delta P = \frac{2\pi L}{\Delta V} \sim 10^6 \frac{L_{10}^2 n_{14} A_i^{1/2}}{B_4}\ \mathrm{yrs}. \tag{36}$$

If the distribution of impurities is non-axisymmetric then such diffusion in the azimuthal direction should lead to slow variations in the abundance peculiarities.

V. Diffusion Waves

In our model of plasma with a cylindrical symmetry, the continuity equation for ions i reads

$$\frac{\partial n_i}{\partial t} - \frac{1}{s}\frac{\partial}{\partial s}\left(sD\frac{\partial n_i}{\partial s} - s n_i \frac{D_B}{B}\frac{dB}{ds}\right) = 0. \tag{37}$$

Together with Eqs. (29)-(30), this equation describes diffusion of ions i in the presence of electric currents.

Let us assume that plasma is in a diffusion equilibrium (Eq. (31)) and, hence, the distribution of elements in such a basic state is given by Eqs.(33)-(35). Consider the behaviour of small disturbances of the number density of impurity from this equilibrium by making use of a linear analysis of Eq. (36). Since the number density of impurity *i* is small, its influence on parameters of the basic state is negligible. For the sake of simplicity, we assume that small disturbances are axisymmetric and do no depend on the vertical coordinate, z. Such disturbances have a shape of cylindrical waves. Denoting disturbances of the impurity number density by δn_i and linearizing Eq. (36), we obtain the equation governing the evolution of such small disturbances,

$$\frac{\partial \delta n_i}{\partial t} - \frac{1}{s}\frac{\partial}{\partial s}\left(sD\frac{\partial \delta n_i}{\partial s} - s \delta n_i \frac{D_B}{B}\frac{dB}{ds}\right) = 0. \tag{38}$$

We consider disturbances with the wavelength shorter than the lengthscale of B. In this case, we can use the so called local approximation and assume that disturbances are $\propto \exp(-iks - M\varphi)$ where k is the wavevector, $ks \gg 1$, and *M* is the azimuthal wavenumber. Since the basic state does not depend on t, δn_i can be represented as $\delta n_i \propto \exp(i\omega t - iks - iM\varphi)$ where ω should be calculated from the dispersion equation. We consider two particular cases of the compositional waves, $M = 0$ and $M \gg ks$.

Cylindrical waves with $M = 0$. Substituting δn_i into Eq. (38), we obtain the dispersion equation for $M = 0$

$$i\omega = -\omega_R + i\omega_B, \; \omega_R = Dk^2, \; \omega_B = kD_B(d\ln B/ds). \quad (39)$$

This dispersion equation describes cylindrical waves in which only the number density of impurity oscillates. The quantity ω_R characterizes decay of waves with the characteristic timescale $\sim (Dk^2)^{-1}$ typical for a standard diffusion. The frequency ω_B describes oscillations of impurities caused by the combined action of electric current and the Hall effect. Note that the frequency can be of any sign but ω_R is always positive. The compositional waves are aperiodic if $\omega_R > |\omega_B|$ and oscillatory if $|\omega_B| > \omega_R$. This condition is equivalent to

$$c_A^2/c_s^2 > Z_i^{-1}|0.21Z_i - 0.71|^{-1}kL, \quad (40)$$

where c_s is the sound speed, $c_s^2 = k_B T/m_p$. Compositional waves become oscillatory if the field is strong and the magnetic pressure is substantially greater than the gas pressure. The frequency of diffusion waves is higher in the region where the magnetic field has strong gradients. The order of magnitude estimate of ω_I is

$$\omega_I \sim kc_A(1/Z_iA_i)(c_A/c_i)(l_i/L), \quad (41)$$

where $l_i = c_i\tau_i$ is the mean free-path of ions i. Note that different impurities oscillate with different frequences.

Non-axisymmetric waves with $M \gg ks$. In this case, the dispersion equation reads

$$i\omega = -\omega_R + i\omega_{B\varphi}, \quad \omega_{B\varphi} = (M/s)BD_{B\varphi}(d\ln B/ds). \quad (42)$$

Non-axisymmetric waves rotate around the cylindric axis with the frequency $\omega_{B\varphi}$ and decay slowly on the diffusion timescale $\sim \omega_R^{-1}$. The frequency of such waves is typically higher than that of cylindrical waves. One can estimate the ratio of these frequencies as

$$(\omega_{B\varphi}/\omega_B) \sim (BD_{B\varphi}/D_B) \sim (1/A_i x_e)(M/ks). \quad (43)$$

Since we consider only weak magnetic fields ($x_e \gg 1$), the period of non-axisymmetric waves is shorter for waves with $M > A_i x_e (ks)$. The ratio of the diffusion timescale and period of non-axisymmetric waves is

$$(\omega_{B\varphi}/\omega_R) \sim (1/x_e)(c_A^2/c_s^2)(Z_i/A_i)(1/kL) \quad (44)$$

and can be large. Hence, these waves can be oscillatory.

VI. Conclusion

We have considered diffusion of elements under a combined influence of standard and current-driven diffusion mechanisms. A diffusion velocity caused by the electric current can be estimated as

$$V_B \sim c_A(c_A/c_i)(1/Z_iA_i)(l_i/L \quad (45)$$

if the magnetic field is relatively weak and electrons are not magnetized. Generally, this velocity can be comparable to velocities caused by other diffusion mechanisms. The current-driven mechanism can form chemical inhomogeneities in plasma even if the magnetic field is weak ($\sim 10 - 100$ G) whereas other diffusion processes require a substantially stronger magnetic field (see Eqs. (1) and (2)). Using Eq. (48), the velocity of current-driven diffusion can be estimated as

$$V_B \sim 1.1 \times 10^{-4} A_i^{-1/2} B_4^2 n_{15}^{-2} T_4^{3/2} \Lambda_{10} L_{10}^{-1} \;\; \text{cm/s}, \quad (46)$$

where $\Lambda_{10} = \Lambda/10$, $B_4 = B/10^4$ G, and $L_{10} = L/10^{10}$ cm The velocity V_B turns out to be sensitive to the field ($\propto B^2$) and, therefore, diffusion in a weak magnetic field requires a longer time to reach equilibrium abundances (34).

The current-driven mechanism leads to a drift of ions in the direction perpendicular to both the magnetic field and electric current. Therefore, a distribution of chemical elements in plasma depends essentially on the geometry of fields and currents. The mechanism considered can operate both in laboratory plasma and in various astrophysical bodies where the electric currents are non-vanishing.

The considered mechanism does not depend on the nature of electric currents and can operate if the current is maintained by some mechanism or if it is of the fossil origin. In the latter case, the decay time of Ohmic dissipation in the spot must be longer than the diffusion time scale. If the length scale of the field is L, the decay time scale is $t_d \sim 4\pi\sigma L^2/c^2$ where σ is the conductivity. In subphotospheric layers, we can estimate $\sigma \sim 3 \cdot 10^{14}\ \text{s}^{-1}$ and $t_d \sim 10^7 L_{10}^2$ yrs. The time scale of diffusion from subphotospheric layers is $t_B \sim H/V_B$ where H is the heigh scale. Using Eq. (34) and assuming $B \sim 100$ G, we obtain $t_B \sim 3 \cdot 10^6 H_8 L_{10}$ yrs where H8 = H/108 cm. Hence, the current-driven diffusion is faster than the Ohmic dissipation if $L_{10} > 1$ and it can form the observed chemical inhomogeneities.

Like other diffusion processes, the current-driven diffusion can lead to a formation of chemical spots if the star has relatively quiescent surface layers. This condition is fulfilled in various type of stars and, therefore, the current-driven diffusion can manifest itself in different astrophysical bodies. For example, this mechanism can contribute to formation of element spots in Ap-stars. The magnetic fields have been detected in many of such spotted stars and, likely, these magnetic fields are maintained by electric currents located in the surface layers. Quiescent surface layers may exist in other types of stars as well, for example, in white dwarfs and neutron stars. Many neutron stars have strong magnetic fields and, most likely, topology of these fields is very complex with spot-like structures at the surface. As it was shown, such magnetic configurations can be responsible for the formation of a spotlike element distribution at the surface. Such chemical structures can be important, for instance, for the emission spectra, diffusive nuclear burning (Brown et al. 2002, Chang & Bildsten 2004), etc. Evolution of neutron stars is very complicated, particularly, in binary systems (see, e.g., Urpin et al. 1998a,b) and, as a result, a surface chemistry can be complicated as well. Diffusion processes may play an important role in this chemistry.

Our study reveals that a particular type of magnetohydrodynamic waves exist in multicomponent plasma in the presence of electric currents. These waves are characterized by oscillations of the impurity number density and exist only if the magnetic pressure exceeds essentially the gas pressure. The frequency of such waves is given by Eq.(39) and turns out to be relatively small. Note that different impurities oscillate with different frequences. Therefore, the local abandances of different elements can exhibit variations with the time. The characteristic timescale of these variations is shorter in plasma with a stronger magnetic field.

Acknowledgements

The author thanks the Russian Academy of Sciences for financial support under the programme OFN-15.

References Références Referencias

1. Alecian, G., Stift, M.J. 2006. A&A, 454, 571
2. Bailey, J.D., Landstreet, J.D., & Bagnulo, S. 2014. A&A, 561, A147
3. Bonanno A., Urpin, V. 2008a. A&A, 477, 35
4. Bonanno, A., Urpin, V. 2008b. A&A, 488, 1
5. Braginskii, S. In “Reviews of Plasma Physics” (Ed. M.Leontovich), vol. 1, p.205, Consultants Bureau, New York (1965)
6. Brown, E., Bildsten, L., & Chang, P. 2002. ApJ, 574, 920
7. Chang, P., Bildsten, L. 2004. ApJ, 605, 830
8. Havnes, O. 1975. A&A, 38, 105
9. Hubrig, S., Mathys, G. 1995. Com. Ap., 18, 167
10. Hubrig, S., Castelli, F. 2001. A&A, 375, 963
11. Hubrig, S., North, P., Sch¨oller, M., & Mathys, G. 2006. Astron. Nachr., 327, 289
12. Hubrig, S., Savanov, I., Ilyin, et al. 2010. MNRAS, 408, L61
13. Hubrig, S., Gonsales, J., Ilyin, et al. 2012. A&A, 547, A90
14. Khokhlova, V. 1985. Sov. Sci. Rev. (Sec. E: Astrophysics and Space Phys. Reviews), v.4, pp. 99-159
15. Kochukhov, O., Bagmulo, S., Wide, G., et al. 2004a. A&A, 414, 613
16. Kochukhov, O., Drake, N., Piskunov, N., de la Reza, R. 2004b. A&A, 424, 935
17. Makaganiuk, V., Kochukhov, O., Piskunov, N., Jeffers, S., Johns-Krull, C., Keller, C., Rodenhuis, M., Snik, F., Stempels, H., & Valenti, J. 2011. A&A, 529, A160.
18. Makaganiuk, V., Kochukhov, O., Piskunov, N., Jeffers, S., Johns-Krull, C., Keller, C., Rodenhuis, M., Snik, F., Stempels, H., & Valenti, J. 2012. A&A, 539, A142
19. Mathys, G., Hubrig, S. 1995. A&A, 293, 810
20. Michaud, G. 1970. ApJ, 160, 641
21. Michaud, G., Charland, Y., Vauclair, S., & Vauclair, G. 1976. ApJ, 210, 447
22. Piskunov, N. & Kochukhov, O. 2002. A&A, 381, 736
23. Pyper, D. 1969. ApJS, 164, 347
24. Silvester, J., Wade, G.A., Kochukhov, O., Bagnulo, S., Folsom, C.P., Hanes, D. 2012. MNRAS, 426, 1003
25. Spitzer, L. Physical Processes in the Interstellar Medium. 1978. New York: Wiley
26. Tayler, R.J. 1973. MNRAS, 161, 365
27. Urpin, V. 1981. Ap&SS, 79, 11
28. Urpin, V., & Van Riper, K. 1993. ApJ, 411, L87
29. Urpin, V. 2015. Astr. Nach., 336, 266
30. Urpin, V., Geppert, U., & Konenkov, D. 1998a. MNRAS, 295, 907
31. Urpin, V., Konenkov, D., & Geppert, U. 1998b. MNRAS, 299, 73
32. Vauclair, S., Hardorp, J., & Peterson, D. 1979, ApJ, 227, 526
33. Vekshtein, G., Riutov, D., & Chebotaev, P. 1975. SvJPP, 1, 220
34. Wade, G., Abecassis, M., Auriere, M., et al. In “Magnetic Stars” (eds. Yu. Glagolevskij, D. Kudryavtsev, and I. Romanyuk), 2004, p.108-113.

Global Journals Guidelines Handbook 2018

www.GlobalJournals.org