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First Order Reactant of Dusty Fluid MHD Turbulence Prior to the Ultimate Phase of Decay for Four-Point Correlation in A Rotating System

By M. Rahman & M. A. Bkar Pk
Rajshahi University

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First Order Reactant of Dusty Fluid MHD Turbulence Prior to the Ultimate Phase of Decay for Four-Point Correlation in a Rotating System

M. Rahman [α] & M. A. Bkar Pk [σ]

Abstract- In this paper, first order chemical reactant will be analyzed to study the effect of the fluctuation of MHD turbulence before the ultimate phase of decay in present of dust particles in a rotating system at four point correlation. Following Deissler's approach two, three and four point correlation equations have been obtained, and the set of correlation equations is made determinate by neglecting the quintuple correlations in comparison to the third and fourth order correlation terms. By taking Fourier-transforms, the correlation equations are converted to spectral form. Finally, integrating the energy spectrum over all wave numbers. The energy decay of MHD turbulence for first-order reactant in the presence of dust particles in a rotating system is obtained, and the result is shown graphically in the text.

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I. Introduction

Chemical reactions occur in the gas phase, in solution in a variety of solvents, at gas-solid and other interfaces, in the liquid state, and in the solid state. It is sometimes convenient to work with amounts of substances instead of with concentrations. The essential characteristic of turbulent flows is that chaotic fluctuations are random. Chemical reaction as used in chemistry, chemical engineering, physics, fluid mechanics, heat, and mass transport. The behavior of dust particles in a turbulent flow depends on the concentrations of the particles and the size of the particles concerning the scale of the disordered fluid.

Deissler (Deissler 1958) developed a theory on the decay of homogeneous turbulence before the final period. Deissler (Deissler 1960) further described a assumption of decaying homogeneous turbulence. By considering Deissler's hypothesis, Sarker and Kishore (Sarker and Kishore 1991) studied the decay of MHD insecurity before the final period. Ahmed (Ahmed 2013) discussed the turbulent energy of dusty fluid in a rotating system. Sarker and Ahmed (Sarker and Ahmed 2011) studied the fiber motion in unclean fluid turbulent flow with two-point correlation. Kulandaivel *et* al. (Kulandaivel *et al.*2009) considered the chemical reaction on moving vertical plate with constant mass flux in the presence of thermal radiation.

Author α: Lecturer, Department of Natural Science, Varendra University, Rajshahi-6204, Bangladesh. e-mail: mizan.ru.am@gmail.com
Author σ: Associate Professor, Department of Applied Mathematics University of Rajshahi, Rajshahi-6205, Bangladesh.
e-mail: abubakarpk_ru@yahoo.com

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Islam and Sarker (Islam and Sarker 2001) calculated the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Loeffler and Deissler (Loeffler and Deissler 1961) studied the decompose of temperature fluctuations in homogeneous turbulence before the final period. Chandrasekhar (Chandrasekhar 1951a) studied the invariant theory of isotropic turbulence in magneto-hydrodynamics. Corrsin (Corrsin 1951b) considered the spectrum of isotropic temperature fluctuations in isotropic turbulence. Sarker *et al.* (Sarker *et al.* 2010) studied the effect of Coriolis force on dusty viscous fluid between two parallel plates in the MHD flow. Azad *et al.* (Azad *et al.* 2011) also studied the statistical theory of some distribution functions in MHD turbulent flow for velocity and concentration undergoing a first order reaction in a rotating system. Azad *et al.* (Azad *et al.* 2015) also discussed 3-point distribution functions in the statistical theory in MHD turbulent flow for velocity magnetic temperature and concentration undergoing a first- order reaction. Bkar Pk *et al.*(Bkar Pk *et al.* 2012) discussed the decay of energy of MHD turbulence for four-point correlation. Bkar Pk *et al.* (Bkar Pk *et al.* 2013) also calculated the decay of MHD turbulence before the ultimate phase in the presence of dust particle for four-point correlation. Bkar Pk (Bkar Pk *et al.* 2016) further studied the effect of first-order chemical reaction for Coriolis force and dust particles for small Reynolds number in the atmospheric over territory.

In this paper, we have studied the first order reactant of MHD turbulence before the ultimate phase of decay in the presence of dust particles for the case of four-point correlation in a rotating system. The energy decay of MHD turbulence depends on the variation of the magnitude of dust particle parameters in the magnetic field and causes a vital role between pure and non-pure system. The effects due to dust particle in magnetic field fluctuation of MHD turbulence have been graphically discussed. It is observed that energy decays increases with the increases of the values in M (dust particle) and minimums at the point where the dust particle is absence. The energy decay of MHD fluid turbulence for four-point correlation in the presence of dust particle is more rapidly than the energy decay of MHD turbulence for clean fluid.

II. Four Point Correlation and Equation

For first order reactant in four-point correlation in the present of dust particle, we take the momentum equation of MHD turbulence at the point p and the induction equation of magnetic field fluctuation at p', p'' and p''' as

$$\frac{\partial u_l}{\partial t}+u_k\frac{\partial u_l}{\partial x_k}-h_k\frac{\partial h_l}{\partial x_k}=-\frac{\partial w}{\partial x_l}+\nu\frac{\partial^2 u_l}{\partial x_k\partial x_k}-2\varepsilon_{pkl}\Omega_p u_i+f(u_l-v_l)-Ru_l \tag{1}$$

$$\frac{\partial h_i'}{\partial t}+u_k'\frac{\partial h_i'}{\partial x_k'}-h_k'\frac{\partial u_i'}{\partial x_k'}=\frac{\nu}{p_M}\frac{\partial^2 h_i'}{\partial x_k'\partial x_k'} \tag{2}$$

$$\frac{\partial h_j''}{\partial t}+u''\frac{\partial h_j''}{\partial x_k''}-h''\frac{\partial u_j''}{\partial x_k''{}_k}=\frac{\nu}{p_M}\frac{\partial^2 h_j''}{\partial x_k''\partial x_k''} \tag{3}$$

$$\frac{\partial h_m'''}{\partial t}+u_k'''\frac{\partial h_m'''}{\partial x_k'''}-h_k'''\frac{\partial u_m'''}{\partial x_k'''}=\frac{\nu}{p_M}\frac{\partial^2 h_m'''}{\partial x_k'''} \tag{4}$$

Multiplying equations (1) by $h_i'h_j''h_m'''$ (2) by $u_l h_j'' h_m'''$ (3) by and $u_l h_i' h_m'''$ (4) by and $u_l h_i' h_j''$ and adding the four equations, we then taking the space or time averages or ensemble average, both of the values give the same representation. Space or time averages denoted by $\overline{(.......)}$ and $\langle .... \rangle$ respectively. We get,

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$$\frac{\partial}{\partial t}(\overline{u_l h_i' h_j'' h_m'''}) + \frac{\partial}{\partial x_k}(\overline{u_l u_k h_i' h_j'' h_m'''}) - \frac{\partial}{\partial x_k}(\overline{h_k h_l h_i' h_j'' h_m'''}) + \frac{\partial}{\partial x_k'}(\overline{u_l u_k h_i' h_j'' h_m'''}) -$$

$$\frac{\partial}{\partial x_k'}(\overline{u_l u_i' h_k' h_j'' h_m'''}) + \frac{\partial}{\partial x_k''}(\overline{u_l u_k'' h_i' h_j'' h_m'''}) - \frac{\partial}{\partial x_k''}(\overline{u_l u_j'' h_i' h_k'' h_m'''}) + \frac{\partial}{\partial x_k'''}(\overline{u_l u_k''' h_i' h_j'' h_m'''}) -$$

$$\frac{\partial}{\partial x_k'''}(\overline{u_l u_j''' h_i' h_j'' h_m'''}) = -\frac{\partial}{\partial x_l}(\overline{w h_i' h_j'' h_m'''}) + \frac{\partial^2}{\partial x_k \partial x_k}(\overline{u_l h_i' h_j'' h_m'''}) + \frac{\nu}{p_M}[\frac{\partial^2}{\partial x_k' \partial x_k'}(\overline{u_l h_i' h_j'' h_m'''}) +$$

$$\frac{\partial^2}{\partial x_k'' \partial x_k''}(\overline{u_l h_i' h_j'' h_m'''}) + \frac{\partial^2}{\partial x_k''' \partial x_k'''}(\overline{u_l u_k h_i' h_j'' h_m'''})]$$

$$-2\varepsilon_{\mathrm{pkl}}\Omega_p\left(\overline{u_l h_i' h_j'' h_m'''}\right) + f\left[\left(\overline{u_l h_i' h_j'' h_m'''}\right) - \left(\overline{v_l h_i' h_j'' h_m'''}\right)\right] - \left(\overline{R u_l h_i' h_j'' h_m'''}\right) \tag{5}$$

By using $\frac{\partial}{\partial x_k''} = \frac{\partial}{\partial r_k'}$, $\frac{\partial}{\partial x_k'} = \frac{\partial}{\partial r_k'}$, $\frac{\partial}{\partial x_k} = -(\frac{\partial}{\partial r_k'} + \frac{\partial}{\partial r_k'} + \frac{\partial}{\partial r_k''})$ into equation (5) and then following nine dimensional Fourier transforms as [Equations (6)-(13) in [Bkar Pk *et al.* 2013]] and interchange of point's p' and p etc. in the subscripts with the facts

$$\overline{u_l u_k''' h_i' h_j'' h_m'''} = \overline{u_l u_k' h_i' h_j'' h_m'''};\ \overline{u_l u_k''' h_i' h_j'' h_m'''} = \overline{u_l u_k' h_i'' h_j'' h_m'''};$$

$$\overline{u_l u_m''' h_i' h_j'' h_m'''} = \overline{u_l u_i' h_i' h_k' h_j'' h_m'''};\ \overline{u_l u_j'' h_i' h_k'' h_m'''} = \overline{u_l u_i' h_i' h_k' h_j'' h_m'''};$$

then taking contraction for indices i and j of the resulting equation we obtain,

$$\frac{\partial}{\partial t}(\overline{\phi_l \gamma_i' \gamma_i'' \gamma_m'''}) + \frac{\nu}{p_M}[(1+p_M)(k^2 + k'^2 + k''^2) + 2p_M(kk' + k'k'' + kk'') - 2\varepsilon_{\mathrm{pkl}}\Omega_p p_M/\nu + Rp_M/\nu](\overline{\phi_l \gamma_i' \gamma_i'' \gamma_m'''})$$

$$-f\left(\overline{\phi_l \gamma_i' \gamma_i'' \gamma_m'''}\right) + f\left(\overline{\eta_l \gamma_i' \gamma_i'' \gamma_m'''}\right) = i(k_k + k_k' + k_k'')(\overline{\phi_l \phi_k \gamma_i' \gamma_i'' \gamma_m'''})$$

$$-i(k_k + k_k' + k_k'')(\overline{\phi_l \gamma_k \gamma_i' \gamma_i'' \gamma_m'''}) - i(k_k + k_k' + k_k'')(\overline{\phi_l \gamma_k' \gamma_i' \gamma_i'' \gamma_m'''})$$

$$+i(k_k + k_k' + k_k'')(\overline{\phi_l \phi_i' \gamma_k' \gamma_i'' \gamma_m'''}) + i(k_k + k_k' + k_k'')(\overline{\delta \gamma_i' \gamma_i'' \gamma_m'''}) \tag{6}$$

If we take the derivative concerning x_l of equation (1) at p, we have,

$$-\frac{\partial^2 w}{\partial x_l \partial x_l} = \frac{\partial^2}{\partial x_l \partial x_l}(u_l u_k - h_l h_k) \tag{7}$$

Multiplying (7) by $h_i' h_j'' h_m'''$, using time averages and writing the equation regarding the independent variables, $\vec{r}$, $\vec{r'}$, $\vec{r''}$ we have,

$$-(\overline{\delta\gamma_i'\gamma_j''\gamma_m'''}) = \frac{(K_l K_k + K_l K_k' + K_l K_k'' + K_l' K_k + K_l' K_k' + K_l' K_k'' + K_l'' K_k + K_l'' K_k + K_l'' K_k' + K_l'' K_k'')}{K_l K_l + K_l' K_l' + K_l' K_l'' + 2K_l K_l' + 2K_l' K_l'' + 2K_l K_l''}$$

$$(\overline{\phi_l \phi_k \gamma_i' \gamma_j'' \gamma_m'''} - \overline{\gamma_l \gamma_k \gamma_i' \gamma_j'' \gamma_m'''}) \tag{8}$$

Equation (8) can be used to eliminate from $(\overline{\delta\gamma_i'\gamma_j''\gamma_m'''})$ equation (6). Equation (6) and (8) are the spectral equations corresponding to the four-point correlation equation.

where $\omega = \frac{P}{\rho} + \frac{1}{2}\left|\overline{h}\right|^2$ is the total MHD pressure,

$R =$ First order chemical reactant

$p(x,t) =$ Hydrodynamic pressure,

$\rho =$ Fluid density,

$P_M = \frac{\nu}{\lambda}$ Magnetic Prandtl number,

$\Omega_s =$ angular velocity components,

ε_{skm} is the alternating tensor,

$\nu =$ Kinematics viscosity,

$\lambda =$ Magnetic diffusivity,

$h_i(x,t) =$ Magnetic field fluctuation,

$u_k(x,t) =$ Turbulent velocity,

$m_i = \frac{4}{3}\pi R^3{}_i \rho_i$, is the mass of a single spherical dust particle of the radius and R_i and ρ_i a constant density of the material in the dust particles.

$f = \frac{KN}{\rho}$, is the dimensions of frequency, K is the Stock's drug resistance, N is a constant number density of dust particle,

$v_l =$ Dust velocity component, t is the time,

$x_k =$ Space co-ordinate and repeated subscripts are summed, from 1 to 3.

III. Three Point Correlation and Equation

The spectral equations corresponding to the three-point correlation equations by contraction of the indices i and j are

$$\frac{\partial}{\partial t}\overline{(\phi_l \beta_i' \beta_i'')} + \frac{\nu}{p_M}[(1+P_M)(K^2 + K'^2) + 2p_M KK']\,\overline{(\phi_l \beta_i' \beta_i'')} = i\,(K_k + K_k')\,\overline{(\phi_l \phi_k \beta_i' \beta_i'')} -$$

$$i\,(K_k + K_k')\,\overline{(\beta_l \beta_k \beta_i' \beta_i'')} - i(K_k + K_k')\,\overline{(\phi_l \phi_k' \beta_i' \beta_i'')} + i(K_k + K_k')\,\overline{(\phi_l \phi_i' \beta_i' \beta_i'')} + i(k_l + k_l')\overline{\gamma \beta_i' \beta_i''} \tag{9}$$

and

$$-(\gamma\,\overline{\beta_i' \beta_j''}) = \frac{(K_l K_k + K_l' K_k + K_l k_k' + K_l' K_k')}{(K_l{}^2 + K_l'^2 + 2K_l K_l')}\,(\overline{\phi_l \phi_k \beta_i' \beta_i''} - \overline{\beta_l \beta_k \beta_i' \beta_j''}) \tag{10}$$

where the spectral tensors are defined by [equations (19)-(24) in [Bkar Pk *et al.* 2013]]

A relation between $\phi_l\phi_k'\beta_i'\beta_j''$ *and* $\phi_l\gamma_i'\gamma_j''\gamma_m'''$ can be obtained, by letting $\vec{r''}=0$ in equation.

$$\left\langle u_l h_i'(\bar{r}) h_j''(\bar{r}') h_m'''(\bar{r}'')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_l\gamma_i'(\bar{k})\gamma_j''(\bar{k}')\gamma_m'''(\bar{k}'')\right\rangle \exp\left[i(\bar{k}.\bar{r}+\bar{k}'.\bar{r}'+\bar{k}''.\bar{r}''\right]d\bar{k}d\bar{k}'d\bar{k}''$$

and comparing the result with the equation

$$\left\langle u_l u_k'(\bar{r}) h_i'(\bar{r}) h_j''(\bar{r}')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_l\phi_k'(\bar{k})\beta_i'(\bar{k})\beta_j''(\bar{k}')\right\rangle \exp\left[i(\bar{k}.\bar{r}+\bar{k}'.\bar{r}'\right]d\bar{k}d\bar{k}',$$

we get

$$\left\langle \phi_l\phi_k'(\bar{k})\beta_i'(\bar{k})\beta_j''(\bar{k}')\right\rangle = \int_{-\infty}^{\infty}\left\langle \phi_l\gamma_i'(\bar{k})\gamma_j''(\bar{k}')\gamma_m'''(\bar{k}'')\right\rangle \exp\left[i(\bar{k}.\bar{r}+\bar{k}'.\bar{r}'+\bar{k}''.\bar{r}''\right]d\bar{k}d\bar{k}'d\bar{k}'' \quad (11)$$

IV. Two Point Correlation and Equation

The spectral equation corresponding to the two-point correlation equation taking contraction of the indices is

$$\frac{\partial}{\partial t}\left\langle \varphi_i\varphi_i'(\bar{k})\right\rangle + \frac{2\nu}{p_M}k^2\left\langle \varphi_i\varphi_i'(\bar{k})\right\rangle = 2ik_k[\left\langle \alpha_i\varphi_k\varphi_i'(\bar{k})\right\rangle - \left\langle \alpha_k\varphi_i\varphi_i'(-\bar{k})\right\rangle] \quad (12)$$

where $\varphi_i\varphi_i'$ and $\alpha_i\phi_k\phi_i'$ are defined by

$$\left\langle h_i h_i'(\bar{r})\right\rangle = \int_{-\infty}^{\infty}\left\langle \varphi_i\varphi_i'(\bar{k})\right\rangle \exp\left(i\bar{k}.\bar{r}\right)dk \quad (13)$$

and

$$\left\langle h_i h_k h_i'(\bar{r})\right\rangle = \int_{-\infty}^{\infty}\left\langle \alpha_i\varphi_k\varphi_i'(\bar{k})\right\rangle \exp\left(i\bar{k}.\bar{r}\right)dk \quad (14)$$

The relation between $\alpha_i\varphi_k\varphi_j'(\vec{k})$ and $\varphi_l\beta_i'\beta_j''$ is obtained by letting $\vec{r'}=0$ in equation

$$\left\langle u_l h_i'(\bar{r}) h_j''(\bar{r}')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_i\beta_i'(\bar{k})\beta_j''(\bar{k}')\right\rangle \exp\left[i(\bar{k}.\bar{r}+\bar{k}'.\bar{r}'\right]d\bar{k}d\bar{k}'$$

and comparing the result with equation

$$\left\langle w h_i'(\bar{r}) h_j''(\bar{r}')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \gamma\beta_i'(\bar{k})\beta_j''(\bar{k}')\right\rangle \exp\left[i(\bar{k}.\bar{r}+\bar{k}'.\bar{r}'\right]d\bar{k}d\bar{k}'$$

then

$$\left\langle \alpha_i\varphi_k\varphi_i'(\bar{k})\right\rangle = \int_{-\infty}^{\infty}\phi_l\beta_i'(\bar{k})\beta_i''(\bar{k}')dk' \quad (15)$$

V. Solutions Neglecting Quintuple Correlation

Using $f\left(\overline{\eta_l\gamma_i'\gamma_i''\gamma_m'''}\right)=L\left(\overline{\phi_l\gamma_i'\gamma_i''\gamma_m'''}\right)$ and 1- $L=s$ in equation (6) and after simplifying, we neglect all the terms on the right hand side of the resulting equation and then integrating between t_1 to t, we obtained

$$\left\langle\phi_l\gamma_i'\gamma_j''\gamma_m'''\right\rangle=\left\langle\phi_l\gamma_i'\gamma_j''\gamma_m'''\right\rangle_1$$

$$\exp[\left\{-\frac{\nu}{p_M}(1+p_M)\left(k^2+k'^2+k''^2+2kk'+2k'k''+2kk'\right)-2\varepsilon_{\mathrm{pkl}}\Omega_p u_l+fs-R\right\}(t-t_1)] \quad (16)$$

Where is $\left\langle\phi_l\gamma_i'\gamma_j''\gamma_m'''\right\rangle_1$ the value of at $\left\langle\phi_l\gamma_i'\gamma_j''\gamma_m'''\right\rangle$ at $t=t_1$ that is stationary value for small value of k, k' and k'' when the quintuple correlations are negligible? Substituting of equations (10), (11), (16) in equation (9) we get,

$$\frac{\partial}{\partial t}(\overline{k_k\phi_l\beta_i'\beta_i''})+\frac{\nu}{p_M}[(1+p_M)(k^2+k'^2)+2p_M kk'](\overline{k_k\phi_l\beta_i'\beta_i''})$$

$$[a]_1\int_{-\infty}^{\infty}\exp[\frac{-\nu}{p_M}(t-t_1)\{(1+p_M)\left(k^2+k'^2+k''^2\right)+2p_M(kk'+k'k''+kk')\}]$$

$$\exp[(fs-2\varepsilon_{\mathrm{pkl}}\Omega_p-R)(t-t_1)]\,dk''$$

$$+[b]_1\int_{-\infty}^{\infty}\exp[\frac{-\nu}{p_M}(t-t_1)\{(1+p_M)\left(k^2+k'^2+k''^2\right)+2p_M(kk'-kk'')\}]\exp[(fs-2\varepsilon_{\mathrm{pkl}}\Omega_p-R)(t-t_1)]\,dk''$$

$$+[c]_1\int_{-\infty}^{\infty}\exp[\frac{-\nu}{p_M}(t-t_1)\{(1+p_M)\left(k^2+k'^2+k''^2\right)+2p_M(kk'-k'k'')\}]\exp[(fs-2\varepsilon_{\mathrm{pkl}}\Omega_p-R)(t-t_1)]dk'' \quad (17)$$

At, t_1 $\gamma's$ have been assumed independent of $\vec{k''}$; that assumption is not, made for other times. This is one of several ideas made concerning the initial conditions, although continuity equation satisfied the circumstances. The complete specification of initial turbulence is complicated; the ideas for the initial situation made here in are partially by simplicity. Substituting $dk''=dk_1''dk_2''dk_3''$ and integrating concerning k_1'', k_2'' and k_3'', we get,

$$\frac{\partial}{\partial t}(\overline{k_k\phi_l\beta_i'\beta_i''})+\frac{\nu}{p_M}\left[(1+p_M)\left(k^2+k'^2\right)+2p_M kk'\right](\overline{k_k\phi_l\beta_i'\beta_i''})$$

$$=\frac{\sqrt{\pi.p_M}}{\sqrt{[\nu(t-t_1)(1+p_M)}}[a_1]\exp\left[-\frac{\nu(t-t_1)(1+p_M)}{p_M}\left\{\frac{(1+2p_M)\left(k^2+k'^2\right)}{(1+p_M)^2}+\frac{2p_M kk'}{(1+p_M)^2}\right\}\right]\exp[(fs-2\varepsilon_{\mathrm{pkl}}\Omega_p-R)(t-t_1)]$$

$$+\frac{\sqrt{\pi.p_M}}{\sqrt{[\nu(t-t_1)(1+p_M)}}[b_1]\exp\left[-\frac{\nu(t-t_1)(1+p_M)}{p_M}\left\{\frac{(1+2p_M)\left(k^2\right)}{(1+p_M)^2}+\frac{2p_M kk'}{(1+p_M)}+k'^2\right\}\right]\exp[(fs-2\varepsilon_{\mathrm{pkl}}\Omega_p-R)(t-t_1)]$$

$$+\frac{\sqrt{\pi.p_M}}{\sqrt{[\nu(t-t_1)(1+p_M)}}[c_1]\exp\left[-\frac{\nu(t-t_1)(1+p_M)}{p_M}\left\{k^2+\frac{(1+2p_M)\left(k'^2\right)}{(1+p_M)^2}+\frac{2p_M kk'}{(1+p_M)}\right\}\right]\exp[(fs-2\varepsilon_{\mathrm{pkl}}\Omega_p-R)(t-t_1)] \quad (18)$$

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Integration of equation (18) with respect to time, and in order to simplify calculations, we will assume that $[a]_1 = 0$;That is we assume that a function sufficiently general to represent the initial conditions can obtain by considering only the terms involving $[b]_1$ and $[c]_1$,then substituting of equation (15) in equation (12) and setting $H = 2\pi k^2 \varphi_i \varphi_i'$ result in

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$$\frac{\partial H}{\partial t} + \frac{2\nu k^2}{p_M} H = G \tag{19}$$

Where,

$$G = k^2 \int_{-\infty}^{\infty} 2\pi i [\ \overline{k_k \phi_l \beta_i' \beta_i''(\vec{k},\vec{k'})}\ \ \overline{k_k \phi_i \beta_i' \beta_i''(-\vec{k},-\vec{k'})}\ ]_0 .$$

$$\exp[-\frac{\nu}{p_M}(t-t_0)\{(1+p_M)(k^2+k'^2)+2p_M kk'\}]dk'$$

$$+ k^2 \int_{-\infty}^{\infty} \frac{2\pi^{\frac{5}{2}} i}{\nu} [b(\vec{k}.\vec{k'}) - b(-\vec{k}.-\vec{k'})]_1 \exp[(fs - 2\varepsilon_{pkl}\Omega_p - R)(t-t_1)] .$$

$$-\!-\ \omega^{-1} \exp[-\omega^2 \left(\frac{(1+2p_M)k^2}{(1+p_M)^2} + \frac{2P_m kk'}{1+p_M} + k'^2 \right)]$$

$$+ k \exp\ [-\omega^2 (\ \ (1+p_M)(k^2+k'^2)+2p_M kk'\ ] \int_0^{\frac{\omega k}{2}} \exp(x^2)dx\}dk'$$

$$+ k^2 \int_{-\infty}^{\infty} \frac{2\pi^{\frac{5}{2}} i}{\nu} [c(\vec{k}.\vec{k'}) - c(-\vec{k}.-\vec{k'})]_1 \exp[(fs - 2\varepsilon_{pkl}\Omega_p - R)(t-t_1)]$$

$$-\!-\ \omega^{-1} \exp[-\omega^2 \left(k^2 + \frac{2P_m kk'}{1+p_M} + \frac{(1+2p_M)k'^2}{(1+p_M)^2} \right)]$$

$$+ k' \exp[-\omega^2 (\ (1+p_M)(k^2+k'^2)+2p_M kk')]. \int_0^{\frac{\omega k'}{2}} \exp(x^2)dx\}dk' \tag{20}$$

where H is the magnetic energy spectrum function, which represents contributions from various wave number (or eddy sizes) to the energy and G is the energy transfer function, which is responsible for the transfer of energy between wave number. In order to make further calculations, an assumption must be made for the forms of the bracketed quantities with the subscripts 0 and 1 in equation (20) which depends on the initial conditions.

$$(2\pi)^2 [\ \overline{k_k \phi_l \beta_i' \beta_i''(\vec{k},\vec{k'})} \text{ - } \overline{k_k \phi_i \beta_i' \beta_i''(-\vec{k},-\vec{k'})}\ ]_0 = -\xi_0 (k^2 k'^4 - k^4 k'^2) \tag{21}$$

Where ξ_0 is a constant depending on the initial conditions for the other bracketed quantities in equation (20), we get,

$$\frac{4\pi^{\frac{7}{2}}i}{\nu}[b(\vec{k}.\vec{k'})-b(-\vec{k}.-\vec{k'})]_1=\frac{4\pi^{\frac{7}{2}}i}{\nu}[c(\vec{k}.\vec{k'})-c(-\vec{k}.-\vec{k'})]_1=-2\,\xi_1(k^4k'^6-k^6k'^4) \qquad (22)$$

Remembering, $d\vec{k'}=-2\pi \overrightarrow{k'^2}d(\cos\theta)dk'$, $kk'=kk'\cos\theta$, and carrying out the integration with respect to θ we get,

$$G=\int_0^\infty[\frac{\xi_0(k^2k'^4-k^4k'^2)kk'}{\nu(t-t_0)}\{\exp[-\frac{\nu}{p_M}(t-t_0)\{(1+p_M)(k^2+k'^2)-2p_Mkk'\}]$$

$$-\exp[-\frac{\nu}{p_M}(t-t_0)\{(1+p_M)(k^2+k'^2)+2p_Mkk'\}]$$

$$+\frac{\xi_1(k^4k'^6-k^6k'^4)kk'}{\nu(t-t_0)}\exp[(fs-2\varepsilon_{\mathrm{pkl}}\Omega_p-R)(t-t_1)](\omega^{-1}\exp[-\omega^2\left(\frac{(1+2p_M)k^2}{(1+p_M)^2}-\frac{2p_Mkk'}{(1+p_M)}+k'^2\right)$$

$$-\omega^{-1}\exp[-\omega^2\left(\frac{(1+2p_M)k^2}{(1+p_M)^2}+\frac{2p_Mkk'}{(1+p_M)}+k'^2\right)]$$

$$+\omega^{-1}\exp[-\omega^2\left(k^2-\frac{2p_Mkk'}{(1+p_M)}+\frac{(1+2p_M)k'^2}{(1+p_M)^2}\right)]$$

$$-\omega^{-1}\exp[-\omega^2\left(k^2+\frac{2p_Mkk'}{(1+p_M)}+\frac{(1+2p_M)k'^2}{(1+p_M)^2}\right)]$$

$$+\{k\exp[-\omega^2((1+p_M)(k^2+k'^2)-2p_Mkk')]$$

$$-k\exp[-\omega^2((1+p_M)(k^2+k'^2)+2p_Mkk')]\}\int_0^{\frac{\omega k}{2}}\exp(x^2)dx$$

$$+\{k'\exp[-\omega^2((1+p_M)(k^2+k'^2)-2p_Mkk')]$$

$$-k'\exp[-\omega^2((1+p_M)(k^2+k'^2)+2p_Mkk')]\}\int_0^{\frac{\omega k'}{2}}\exp(x^2)dx)]dk' \qquad (23)$$

where, $\omega=\left(\frac{\nu(t-t_1)(1+p_M)}{p_M}\right)^{\frac{1}{2}}$.

Integrating equation (23) with respect to k' we have,

$$\mathrm{G}=G_\beta+G_\gamma\exp[(fs-2\varepsilon_{\mathrm{pkl}}\Omega_p-R)(t-t_1)] \qquad (24)$$

The integral expression in equation (24), the quantity G_β represents the transfer function arising due to consideration of magnetic field at three-point correlation

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equation; G_γ arises from consideration of the four-point equation. Integration of equation (24) over all wave numbers shows that

$$\int_0^\infty G.d\vec{k}=0 \tag{25}$$

Indicating that the expression for G satisfies the conditions of continuity and homogeneity, physically, it was to be expected, since G is a measure of transfer of energy and the numbers must be zero. From equation (19),

we get, $H=\exp\left[-\frac{2\nu k^2(t-t_0)}{p_M}\right]\int G\exp\left[-\frac{2\nu k^2(t-t_0)}{p_M}\right]dt+J(k)\exp\left[-\frac{2\nu k^2(t-t_0)}{p_M}\right]$,

$J(k)=N_0k^2/\pi$, is a constant of integration and can be obtained as by Corrsion [Corrsion 1951b]. Therefore we obtained,

$$H=\frac{N_0k^2}{\pi}\exp\left[\frac{-2\nu k^2(t-t_0)}{p_M}\right]+\exp\left[\frac{-2\nu k^2(t-t_0)}{p_M}\right]\int(G_\beta+G_{\gamma_1}+G_{\gamma_2}+G_{\gamma_3}+G_{\gamma_4})$$

$$\exp\left[\frac{-2\nu k^2(t-t_0)}{p_M}\right]dt \tag{26}$$

where, $$G=G_\beta+G_{\gamma_1}+G_{\gamma_2}+G_{\gamma_3}+G_{\gamma_4} \tag{27}$$

After integration equation (26) becomes

$$H=\frac{N_0k^2}{\pi}\exp\left[-\frac{2\nu k^2(t-t_0)}{p_M}\right]+H_\beta+[H_{\gamma_1}+H_{\gamma_2}+H_{\gamma_3}+H_{\gamma_4}]\exp[(fs-2\varepsilon_{\text{pkl}}\Omega_p-R)(t-t_1)] \tag{28}$$

From equation (28) we get,

$$H=H_1+H_2\exp[(fs-2\varepsilon_{\text{pkl}}\Omega_p-R)(t-t_1)] \tag{29}$$

$$H_1=\frac{N_0k^2}{\pi}\exp\left[-\frac{2\nu k^2(t-t_0)}{p_M}\right]+H_\beta,\ H_2=\left(H_{\gamma_1}+H_{\gamma_2}+H_{\gamma_3}+H_{\gamma_4}\right);$$

In equation (29) H_1 and H_2 magnetic energy spectrum arising from consideration of the three and four-point correlation equations respectively. Equation (29) can be integrated over all wave numbers to give the total magnetic turbulent energy. That is

$$\frac{\overline{h_ih_i'}}{2}=\int_0^\infty Hdk \tag{30}$$

Now,

$$\int_0^\infty H_1dk=\frac{N_0p^{3/2}{}_M\nu^{-3/2}(t-t_0)^{-3/2}}{8\sqrt{2\pi}}+\xi_0Q\nu^{-6}(t-t_0)^{-5}$$

$$\int_0^\infty H_2 dk = \xi_1 [L_1 \nu^{-17/2}(t-t_1)^{-15/2} + L_2 \nu^{-19/2}(t-t_1)^{-17/2}]. \exp[(fs - 2\varepsilon_{\text{pkl}}\Omega_p - R)(t-t_1)]$$

where

$$L_1 = Q_2 + Q_4 + Q_6 + Q_7,\ L_2 = Q_1 + Q_3 + Q_5$$

and $G_\beta,\ G_\gamma\ \ H_\beta, H_{\gamma_1}, H_{\gamma_2}, H_{\gamma_3}, H_{\gamma_4}, Q, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7$ are defined in [Bkar Pk *et al.* 2013].

Therefore, from equation (30)

$$\frac{\overline{h_i h_i'}}{2} = \frac{N_0 p^{3/2}{}_M \nu^{-3/2}(t-t_0)^{-3/2}}{8\sqrt{2\pi}} + \xi_0 Q \nu^{-6}(t-t_0)^{-5}$$

$$+ \xi_1 [L_1 \nu^{-17/2}(t-t_1)^{-15/2} + L_2 \nu^{-19/2}(t-t_1)^{-17/2}] \exp[(fs - 2\varepsilon_{\text{pkl}}\Omega_p - R)(t-t_1)] \quad (31)$$

This is the first order reactant of MHD turbulence in presence of dust particles for four-point correlation. It can be written as

$$\langle h^2 \rangle = A(t-t_0)^{-3/2} + B(t-t_0)^{-5} + [C(t-t_1)^{-15/2} + D(t-t_1)^{-17/2}]\exp[(M - 2\varepsilon_{\text{pkl}}\Omega_p - R)(t-t_1)] \quad (32)$$

If R=0 in equation (32) then it becomes

$$\langle h^2 \rangle = A(t-t_0)^{-3/2} + B(t-t_0)^{-5} + [C(t-t_1)^{-15/2} + D(t-t_1)^{-17/2}]\exp[(M - 2\varepsilon_{\text{pkl}}\Omega_p)(t-t_1)] \quad (33)$$

Where, $M=fs$ is the dust particle parameter. This was obtained earlier by Bkar Pk et al. [Bkar PK et al. 2013].

In the absent of M, $2\varepsilon_{\text{pkl}}\Omega_p$ and R equation (32) reduces to the form

$$\langle h^2 \rangle = A(t-t_0)^{-3/2} + B(t-t_0)^{-5} + C(t-t_1)^{-15/2} + D(t-t_1)^{-17/2}] \quad (34)$$

This is the decay of energy of MHD turbulence for four-point correlation. It is fully same with Bkar PK et al. [Bkar PK et al. 2012].

If ξ_1=0, then equation (34) becomes

$$\langle h^2 \rangle = A(t-t_0)^{-3/2} + B(t-t_0)^{-5}$$

This is the decay of MHD turbulence for three-point correlation. This is totally same with the result obtained by Sarker and Kishore [Sarker and Kishore 1991].

VI. Results and Discussion

This study shows that the terms associated with the higher-order correlations die out faster than those associated with the lower-order ones. If the quadruple and quintuple correlations were not neglected, then more conditions the negative higher power of $(t-t_1)$ would be added to the equation(32), and for large times the last terms in the equations (32), becomes negligible, leaving the -3/2 power decay law for the final period.

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*h*1, *h*2, *h*3, *h*4 and *h*5 are energy decay curves of equation (32) at 0.5, 1, 1.5, 2 and 2.5 respectively. For different values of *M,* $2\varepsilon_{\mathrm{pkl}}\Omega_p$ and *R* we have seen the energy decay curves in the figures bellow. In the presence of dust particles energy decay increases, and more increases either rotating force or chemical reaction is absent. In the absence of dust particles and Coriolis force (or chemical reaction) energy decay curves increases due to decreases of the chemical reaction (or Coriolis force) and maximum at the point where chemical reaction (or Coriolis force) is equal to zero. Energy decay increases with the decreases of chemical reaction and maximum at zero.

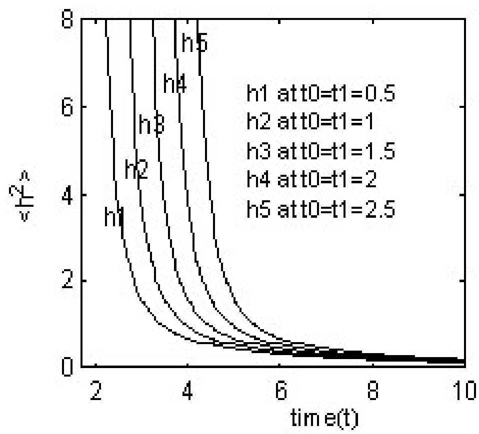

Figure 1: Energy curves for *M*=3, $2\varepsilon_{\mathrm{pkl}}\Omega_p = 0.50$ *R*=0.50

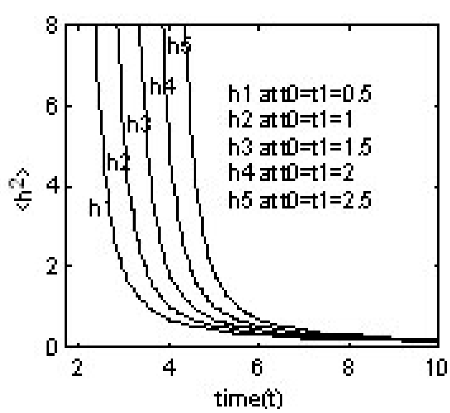

Figure 2: Energy decay curves of equation (32) if *M*=3, $2\varepsilon_{\mathrm{pkl}}\Omega_p = 0$, *R*=0.50

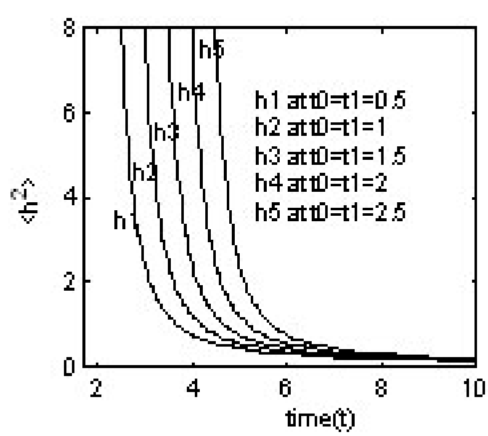

Figure 3: Energy decay curves of equation (32) if *M*=3, $2\varepsilon_{\mathrm{pkl}}\Omega_p = 0$, *R*=0

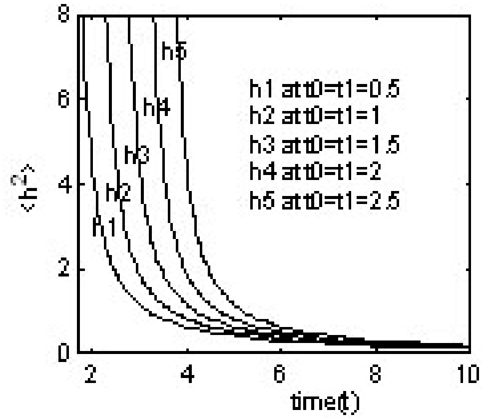

Figure 4: Energy decay curves of equation (32) if M=0, $2\varepsilon_{\text{pkl}}\Omega_p = 0.50$, R=0.50

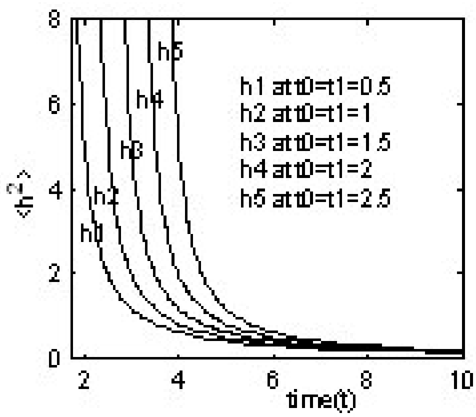

Figure 5: Energy decay curves of equation (32) if M=0, $2\varepsilon_{\text{pkl}}\Omega_p = 0$, R=0.50

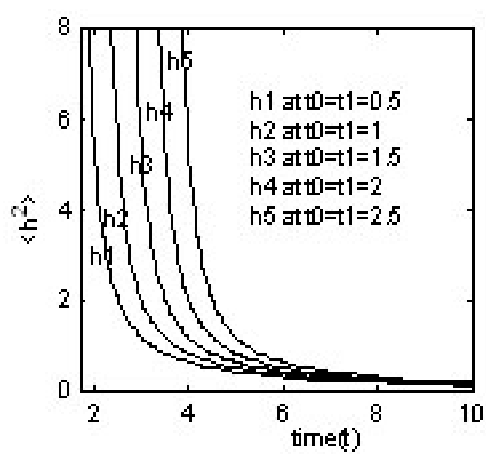

Figure 6: Energy decay curves of equation (32) if M=0, $2\varepsilon_{\text{pkl}}\Omega_p = 0$, R=0.25

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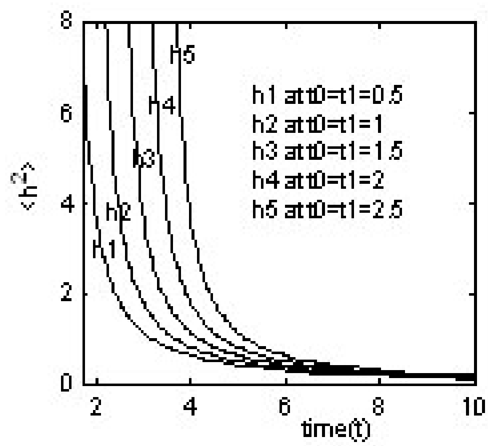

Figure 7: Energy decay curves of equation (32) if $M=0, 2\varepsilon_{pkl}\Omega_p = 0,\ R=5$

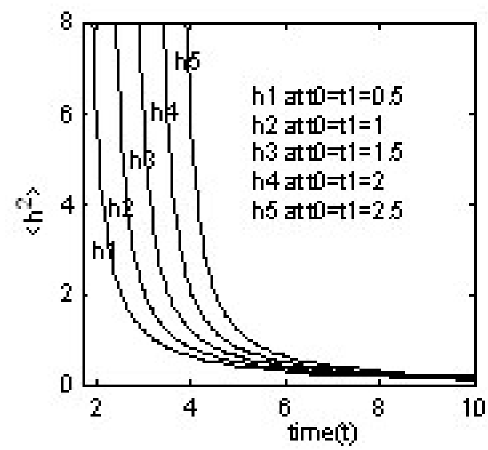

Figure 8: Energy decay curves of equation (34) if $M=0, 2\varepsilon_{pkl}\Omega_p = 0, R=0$

From the above figures and discussion, we conclude that if $(M - 2\varepsilon_{pkl}\Omega_p - R) \ge 0$ then energy decay increases rapidly and if $(M - 2\varepsilon_{pkl}\Omega_p - R) < 0$ in the chemical reaction energy decay of MHD fluid turbulence for four-point correlation energy decay increases more slowly.

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