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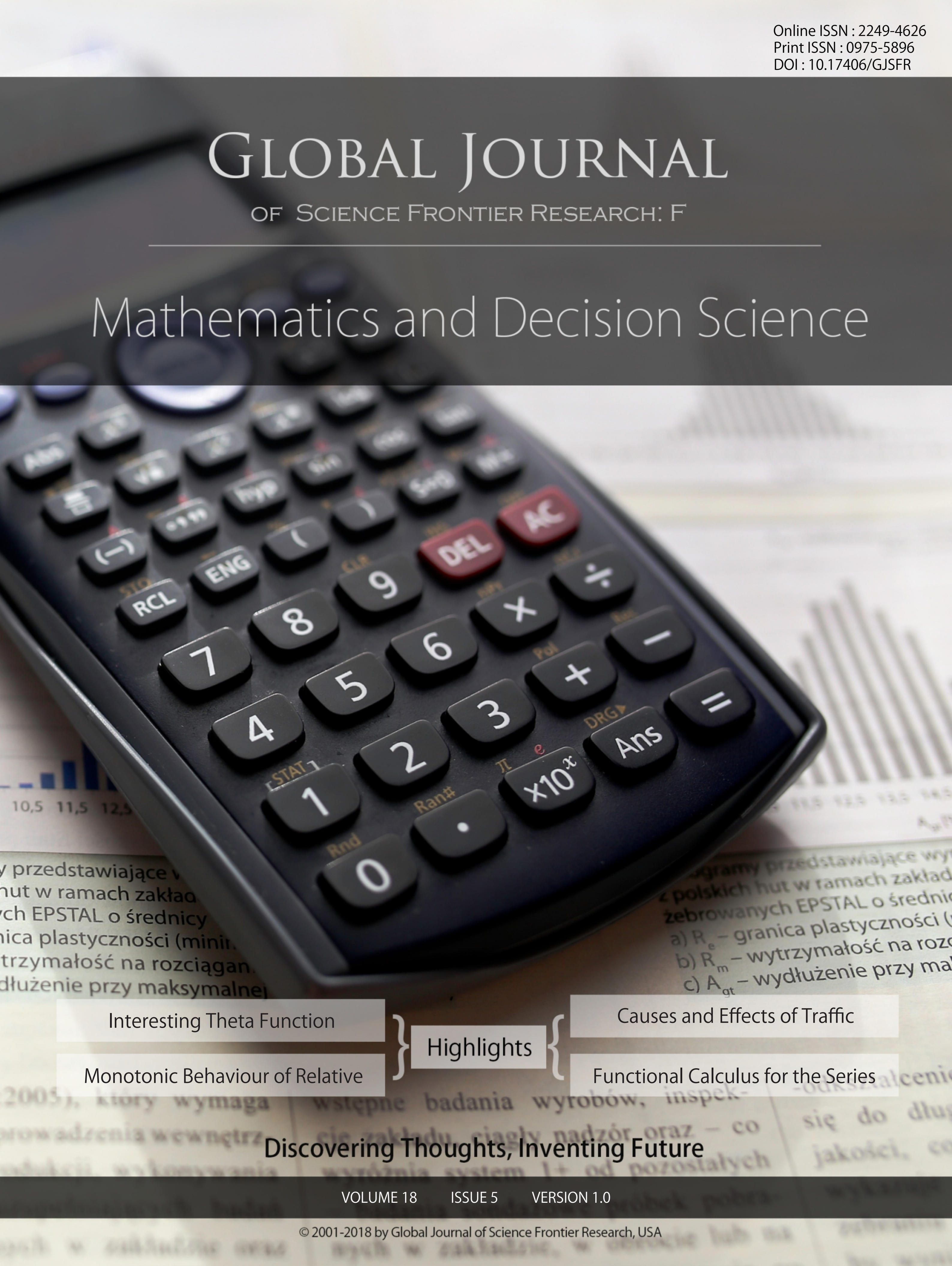

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Monotonic Behaviour of Relative

Highlights

Causes and Effects of Traffic

Functional Calculus for the Series

Discovering Thoughts, Inventing Future

VOLUME 18 ISSUE 5 VERSION 1.0

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Monotonic Behaviour of Relative Increments of Pearson Distributions

By Sereko Kooepile-Reikeletseng
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Abstract- Theory has been developed in order to classify distributions ac-cording to monotonic behaviour of their relative increment functions. We apply the results to Pearson distributions.

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Monotonic Behaviour of Relative Increments of Pearson Distributions

Sereko Kooepile-Reikeletseng

Abstract- Theory has been developed in order to classify distributions ac-cording to monotonic behaviour of their relative increment functions. We apply the results to Pearson distributions.

I. Introduction

Let $f(x)$ be the probability density function of a continuous random variable X with open support I. The corresponding distribution function of X, $X(x)$, is defined by

$$F(x) = \int_{-\infty}^{x} f(t)dt$$

The relative increment function, h, of a distribution function, F, is defined by

$$h(x) = \frac{F(x+c) - F(x)}{1 - F(x)}$$

where c is a positive constant.

Lemma 1. Let F be a twice differentiable distribution function with

$$F(x) < 1, \; F'(x) = f(x) > 0$$

for all x in I. We define the function Ψ as follows

$$\Psi(x) = \frac{(F(x) - 1).f'(x)}{f^2(x)}$$

If $\Psi < 1 (\Psi > 1)$, then it has been proven that the function h strictly increases (strictly decreases)[2]

A probability distribution with probability density function $f(x)$ is said to be a Pearson distribution if

$$\frac{f'(x)}{f(x)} = \frac{Q(x)}{q(x)}$$

where $Q(x) = Ax + B$ and $q(x) = ax^2 + bx + c$ and A, B, a, b, c are real constants with

$$a^2 + b^2 + c^2 > 0, \; A^2 + B^2 > 0 \qquad [1]$$

The following four theorems have been proven before and we will use them to formulate results about Pearson distributions.

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Ref

2. Szabo, Z. : Investigation of Relative Increments of Distribution Functions. Publ. Math. De-brecen 49 (1996), pp. 99-112

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Theorem 1.
If the probability density function f has the following properties.

(1.1) $I = (r, s) \subseteq \mathbb{R}$ is the largest finite or infinite open interval in which $f > 0$.
(1.2) There exists m in I at which f' is continuous and $f'(m) = 0$
(1.3) $f' > 0$ in (r, m) and $f' < 0$ in (m, s)
(1.4) f is twice differentiable in (m, s)
(1.5) $\left(\frac{f}{f'}\right)' = \frac{d}{dx}\left(\frac{f}{f'}\right) > 0$ in (m, s),

Then the corresponding continuous relative increment function, h, behaves as follows: If $\Psi(s^-) = \lim_{x\to s^-} \Psi(x)$ exists, then

- h strictly increases in I if $\Psi(s^-) \leq 1$
- h strictly increases in (r, y) and strictly decreases in (y, s) for some y in I if $\Psi(s^-) > 1$.[2]

Theorem 2.
If the probability density function f has the following properties.

(2.1) $I = (r, s) \subseteq \mathbb{R}$ is the largest finite or infinite open interval in which $f > 0$.
(2.2) $m = r$
(2.3) $f' < 0$ in (r, s)
(2.4) f is twice differentiable in (r, s)
(2.5) $\left(\frac{f}{f'}\right)' = \frac{d}{dx}\left(\frac{f}{f'}\right) < 0$ in (r, s), then r is finite.

(i) If $\Psi(r^+) < 1$ or ($\Psi(r^+) = 1$ and $\Psi < 1$ in some right neighbourhood of r), then $\Psi < 1$ in I and the corresponding relative increment function strictly increases in I.
(ii) If $\Psi(r^+) > 1$, then
- If $\Psi(s^-) \geq 1$, then $\Psi > 1$ and the relative increment function strictly decreases in I.
- If $\Psi(s^-) < 1$, then $\Psi > 1$ in (r, y) and $\Psi < 1$ in (y, s) for some $y \in I$, so the relative increment function strictly decreases first and then strictly increases.[2]

III. Monotonic Behaviour of Relative Increments of Pearson Distributions

Theorem 3.
If the probability density function f has the following properties.

(3.1) $I = (r, s) \subseteq \mathbb{R}$ is the largest finite or infinite open interval in which $f > 0$.
(3.2) $m = r$
(3.3) $f' < 0$ in (r, s)
(3.4) f is twice differentiable in (r, s)
(3.5) $\left(\frac{f}{f'}\right)' = \frac{d}{dx}\left(\frac{f}{f'}\right) > 0$ in (r, s), then r is finite

(i) If $\Psi(m^+) > 1$ or ($\Psi(m^+) = 1$ and $\Psi > 1$ in some right neighbourhood of m), then $\Psi > 1$ in I and the corresponding relative increment function strictly decreases in I.
(ii) If $\Psi(m^+) < 1$, then
- If $\Psi(s^-) < 1$, then $\Psi < 1$ and the relative increment function strictly increases in I.
- If $\Psi(s^-) > 1$, then $\Psi < 1$ in (m, x) and $\Psi > 1$ in (x, s) for some $x \in I$, so the relative increment function strictly increases first and then strictly decreases.[4]

Theorem 4.
Let $f(x)$ be a probability density function of U type distribution where (1.1) – (1.4) are fulfilled. Suppose $(\frac{f}{f'})' > 0$ in (m, y) and $(\frac{f}{f'})' < 0$ in (y, s) for some $y \in (m, s)$. Then the corresponding continuous relative increment function, h, behaves as follows:

- If $\Psi(y) < 1$ and $\Psi(s^-) < 1$ then strictly increases in I or

Ref

2. Szabo, Z. : Investigation of Relative Increments of Distribution Functions. Publ. Math. De-breen 49 (1996), pp. 99-112

- If $\Psi(y) \geq 1$ then
  - if $\Psi(s^-) > 1$ h strictly increases in (r, x_0) for some $x_0 \in (m, y)$ and strictly decreases in (x_0, s).
  - if $\Psi(s^-) <$ then h strictly increases in (r, x_1) for some $x_1 \in (m, y)$ and strictly decreases in (x_1, x_2) for some $x_2 \in (y, s)$ then finally strictly increases in (x_2, s).[4]

Theorem 5.

Let $f(x)$ be a probability density function of J type distribution. Assume $m = r$, (1.1), (1.3), (1.4) are fulfilled. Suppose $(\frac{f}{f'})' < 0$ in (m, Y) and $(\frac{f}{f'})' > 0$ in (Y, s) for some $Y \in (m, s)$. Then the corresponding continuous relative increment function, h, behaves as follows: if $\Psi(m^+) = \lim_{x \to m^+} \Psi(x)$ exists, then

- If $\Psi(m^+) < 1$ and
  - $\Psi(s^-) < 1$ then h strictly increases in I or
  - $\Psi(s^-) > 1$ then h strictly increases in (m, x_3) for some $x_3 \in (y, s)$ and strictly decreases in (x_3, s)
- If $\Psi(m^+) \geq 1$ and
  - $\Psi(y) > 1$ then h strictly decreases in I
  - $\Psi(y) < 1$ then if
    - ∗ if $\Psi(s^-) < 1$ then h strictly decreases in (m, x_4) for some $x_4 \in (m, y)$ and strictly increases in (x_4, s)

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∗ if $\Psi(s^-) > 1$ then h strictly decreases in (m, x_5) for some $x_5 \in (m, y)$ and strictly increases in (x_5, x_6) for some $x_6 \in (y, s)$ then finally strictly decreases in (x_6, s) [4]

V. Main Results

For the next two theorems, we use the notation used in [3] **Theorem 6.** Let $f(x)$ be a density function of a Pearson distribution where (1.1) – (1.4) are fulfilled Let $M = b.B - A.c$, $L = a.B^2 - A.M$, $D = a.L$, $D_1 = \sqrt{D}$ and $b_1 = \frac{b}{2a}$ for $a \neq 0$.

(1) If $A = 0$ and $a < 0$ then $s + b_1 \geq 0$.

(2) If $a.A > 0$ and $q(\frac{-B}{A}) \neq 0$ and $D \geq 0$ then $Y(m) \leq 0$ and $0 \leq Y(s) \leq 2D_1$

(3) If $a.A < 0$ and $q(\frac{-B}{A}) \neq 0$ and $D \geq 0$ then $0 \leq Y(m) \leq 2D_1$ and $Y(s) \leq 0$

where $Y(v) = a(Av + B) + D_1$.

Then all the assertions of Theorem 4 hold.

Proof. By theorem 4, it is sufficient to prove that

$$(\frac{f}{f'})' > 0 \text{ in } (m, y) \text{ and } (\frac{f}{f'})' < 0 \text{ in } (y, s)$$

for some y in (m, s) We have

$$\left(\frac{f}{f'}\right)' = \left(\frac{q}{Q}\right)' = \frac{p(x)}{Q^2}$$

where

$$p(x) = aAx^2 + 2aBx + M$$

and

$$\left(\frac{f}{f'}\right)' > 0 \text{ if } p(x) > 0 \text{ and } \left(\frac{f}{f'}\right)' < 0 \text{ if } p(x) < 0$$

If $a \neq 0$ and $D \geq 0$ then the roots of $p(x)$ are

$$x_1 = \frac{(-aB - D_1)}{(aA)} \text{ and } x_2 = \frac{(-aB + D_1)}{(aA)}$$

where $D_1 = D^{\frac{1}{2}}$.

Ref

4. Szabo Z. : On Monotonic Behaviour of Relative Increments of Unimodal Distributions.

Case 1: $A \neq 0$, $a \neq 0$ and $q(\frac{-B}{A}) \neq 0$, so q and Q have no common zero.

1.1. If $a.A > 0$, then $p(x)$ is convex. If $D \geq 0$ then there will be two real roots x_1 and x_2 where $x_1 \leq x_2$.

$$p(x) > 0 \text{ in } (m, y) \text{ and } p(x) < 0 \text{ in } (y, s)$$

if and only if

$$m \leq x_1, \ x_1 = y \text{ and } x_1 < s < x_2$$

which means that

$$Y(m) < 0 \text{ and } 0 \leq Y(s) \leq 2D_1$$

1.2. If $a.A < 0$, then $p(x)$ is concave.

$$p(x) > 0 \text{ in } (m, y) \text{ and } p(x) < 0 \text{ in } (y, s)$$

if and only if

$$x_1 \leq m \leq x_2 \text{ and } s \geq x_2 = y$$

which means that

$$0 \leq Y(m) \leq 2D_1 \text{ and } Y(s) \leq 0$$

Case 2 $A = 0$ and $a \neq 0$ therefore

$$\frac{q}{Q} = \frac{ax^2 + bx + c}{B}$$

and

$$(\frac{q}{Q})' = \frac{2ax + b}{B}$$

If $B > 0$ then $(\frac{q}{Q})' > 0$ if $x > \frac{-b}{a}$ and $(\frac{q}{Q})' < 0$ if $x < \frac{-b}{a}$. If $a < 0$ then $s > \frac{-b}{2a}$

If $B < 0$ then $(\frac{q}{Q})' > 0$ if $x < \frac{-b}{a}$ and $(\frac{q}{Q})' < 0$ if $x > \frac{-b}{a}$. If $a > 0$ then $s > \frac{-b}{2a}$

Case 3 If $a = 0$, then $p(x) = \frac{b}{B}$. If $\frac{b}{B} > 0$ then Theorem 1 applies. If $\frac{b}{B} < 0$ then there is no case like it as seen in remark 2.1 in [3].

Theorem 7

Let $f(x)$ be the density function of a Pearson distribution with $m = r$, (1.1), (1.3), (1.4) are fulfilled and M, L, D defined as in theorem 5.

(1) If $A = 0$ and $a < 0$ then $m + b_1 \leq 0$.
(2) If $a.A > 0$ and $q(\frac{-B}{A}) \neq 0$ and $D \geq 0$ and $Y(m) \leq 0$ and $0 \leq Y(s) \leq 2D_1$
(3) If $a.A < 0$ and $q(\frac{-B}{A}) \neq 0$ and $D \geq 0$ and $0 \leq Y_1(m) \leq 2D_1$ and $Y_1(s) \leq 0$

where $Y_1(v) = a(Av + B) - D_1$.

Then all the assertions of Theorem 5 hold.

Proof. By theorem 5, it is sufficient to prove that

$$(\tfrac{f}{f'})' < 0 \text{ in } (m, y) \text{ and } (\tfrac{f}{f'})' > 0 \text{ in } (y, s)$$

for some y in (m, s).

Case 1: $A \neq 0$ and $q(\frac{-B}{A}) \neq 0$, so q and Q have no common zero.

1.1. If $a.A > 0$, then $p(x)$ is convex. If $D \geq 0$ then there will be two real roots x_1 and x_2 where $x_1 < x_2$.

$$p(x) < 0 \in (m, y) \text{ and } p(x) > 0 \in (y, s)$$

if and only if

$$m = x_1, \ x_2 = y \text{ and } s > x_2$$

Ref

3. Szabo, Z. : Relative Increments of Pearson Distributions. Acta Math. Aca. Paed. Nyiregyh. 15 (1999), pp. 45-54. [Electronic Journal, websites www.bgytf.hu/amapn, www.emis.de/journals]

which means that

$$Y_1(s) > 0$$

VI. Monotonic Behaviour of Relative Increments of Pearson Distributions

1.2. If $a.A < 0$, then $p(x)$ is concave.

$$p(x) < 0 \in (m, y) \text{ and } p(x) > 0 \in (y, s)$$

if and only if

$$m < x_1 \text{ which means that } Y_1(m) > 0,\ y = x_1 \text{ and } s = x_2$$

Case 2 $A = 0$ therefore

$$\frac{q}{Q} = \frac{ax^2 + bx + c}{B}$$

and

$$(\frac{q}{Q})' = \frac{2ax + b}{B}$$

If $B > 0$ then $(\frac{q}{Q})' < 0$ if $x < \frac{-b}{a}$ and $(\frac{q}{Q})' > 0$ if $x > \frac{-b}{a}$. If $a > 0$ then $m < \frac{-b}{2a}$

If $B < 0$ then $(\frac{q}{Q})' < 0$ if $x > \frac{-b}{a}$ and $(\frac{q}{Q})' > 0$ if $x < \frac{-b}{a}$. If $a < 0$ then $m < \frac{-b}{2a}$

Case 3 If $a = 0$, then $p(x) = \frac{b}{B}$. If $\frac{b}{B} < 0$ then Theorem 2 applies.

If $\frac{b}{B} > 0$ then Theorem 3 applies.

We state the following Lemma as it helps in classifying distributions. These are outlined and proven in [3].

Lemma 2. Let $f(x)$ be the density function of a Pearson distribution, then then:

- I Let $s = \infty$ and $f_\infty = lim_{x\to\infty} x.f(x)$
  - I.1 If $A = a = f_\infty = 0$ and $b + B \neq 0$ then $\Psi(\infty) = \frac{B}{(b+B)}$
  - I.2 If $A \neq 0$ and $a = 0$ then $\Psi(\infty) = 1$
  - I.3 If $A.a.(a + A) \neq 0$ and $f_\infty = 0$ then $\Psi(\infty) = \frac{A}{(a+A)}$
  - I.4 If $(a = A = 0\ ,\ b.f_\infty = 0)$ or $(A = 0\ ,\ a \neq 0)$ or $(a.A.f_\infty \neq 0)$ then $\Psi(\infty) = 0$
- II Let s be a finite number and let $lim_{x\to s^-} f(x) = 0$
  - II.1 If $[A.Q(s).q(s) \neq 0]$ or $[A.q'(s) \neq 0, Q(s) = q(s) = 0]$ or $[A = 0, q(s) \neq 0]$ or $[AA.Q(s) \neq 0, q(s) = q'(s) = 0]$, then $\Psi(s^-) = 1$
  - II.2 If $A \neq 0, q(-\frac{B}{A}).Q(s).[Q(s) + q'(s)] \neq 0\ ,\ q(s) = 0$ then $\Psi(s^-) = \frac{Q(s)}{[Q(s)+q'(s)]}$
  - II.3 If $A \neq 0\ ,\ q(-\frac{B}{A}).q(s) \neq 0\ ,\ Q(s) = 0$ then $\Psi(s^- = 0)$
  - II.4 If $A \neq 0\ ,\ q(-\frac{B}{A}) = q(s) = 0\ ,\ a + A \neq 0$ and either $Q(s) = q'(s) = 0$ or $Q(s) \neq 0$ then $\Psi(s^-) = \frac{A}{(a+A)}$
  - II.5 If $A = a = q(s) = 0\ ,\ q'(s).(b + B) \neq 0$ then $\Psi(s^-) = \frac{B}{b+B}$

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Example 1

$$f(x) = \frac{1}{\Gamma(n-1)} x^{-n} \exp(\frac{-k}{x})\ ,\ n = 2, 3, 4, \ldots\ ,\ I = (0, \infty)$$

We have

$$\frac{f'}{f} = \frac{-nx+k}{x^2} \text{ so } A = -n\ ,\ B = k\ ,\ a = 1\ ,\ b = 0\ ,\ c = 0$$

Ref

3. Szabo, Z. : Relative Increments of Pearson Distributions. Acta Math. Aca. Paed. Nyiregyh. 15 (1999), pp. 45-54. [Electronic Journal, websites www.bgytf.hu/amapn, www.emis.de/journals]

Theorem 6 part 3 applies since $q(\frac{-B}{A}) = \frac{k^2}{n^2} \neq 0$ and $a.A = -n < 0$.
$mode = \frac{k}{n} < y = \frac{2k}{n}$, so it's a U-type distribution with
$p(x) = x(2k - nx) > 0$ for $x < \frac{2k}{n}$ and $p(x) < 0$ for $x > \frac{2k}{n}$

$$\lim_{x\to\infty} xf(x) = \lim_{x\to\infty} \frac{1}{\Gamma(n-1)} x^{(-n+1)} \exp(-k/x) = 0$$

so $I.3$ in Lemma 2 applies giving

$$\psi(\infty) = \frac{A}{a+A} = \frac{-n}{-n+1} > 1$$

This means that $\psi(y) > 1$ since if it was less than 1 then according to theory $\psi(x)$ would decrease and remain less than 1 in (y, s). So the relative increment function increases and then decreases.

Notes

Example 2

$f(x) = K(1+x^2)^{-n} \exp(-\arctan(x))$, $n > \frac{1}{2}$, K is a constant and $I = (-\infty, \infty)$

We have

$$\frac{f'}{f} = \frac{2nx \quad 1}{x^2+1} \text{ so } A = -2n \text{ , } B = -1 \text{ , } a = 1 \text{ , } b = 0 \text{ , } c = 1$$

Theorem 6 part 3 applies since $q(\frac{-B}{A}) \neq 0$ and $a.A = -2n < 0$.

$mode = \frac{-1}{2n} < y = \frac{-1+\sqrt{4n^2+1}}{2n}$, so it's a U-type distribution with
$p(x) = -2(-nx^2 + x - n) > 0$ for

$$x < \frac{-1+\sqrt{4n^2+1}}{2n}$$

and

$$p(x) < 0 \text{ for } x > \frac{-1+\sqrt{4n^2+1}}{2n}$$

$$\lim_{x\to\infty} xf(x) = 0$$

so I.3 in Lemma 2 applies giving

$$\psi(\infty) = \frac{A}{a+A} = \frac{-2n}{-2n+1} > 1$$

This means that $\psi(y) > 1$ since if it was less than 1 then according to theory $\psi(x)$ would decrease and remain less than 1 in (y, s). So the relative increment function increases and then decreases.

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References Références Referencias

1. Johnson N. and Kotz S. : Distributions in Statistics. Continuous Univariate Distributions, vol. 1,2, Houghton Mi¨in, Boston - New York, 1970.
2. Szabo, Z. : Investigation of Relative Increments of Distribution Functions. Publ. Math. De-brecen 49 (1996), pp. 99-112
3. Szabo, Z. : Relative Increments of Pearson Distributions. Acta Math. Aca. Paed. Nyiregyh. 15 (1999), pp. 45-54. [Electronic Journal, websites www.bgytf.hu/amapn, www.emis.de/journals]
4. Szabo Z. : On Monotonic Behaviour of Relative Increments of Unimodal Distributions.
5. Acta Math. Acad. Paed. Nyiregyh. 22 (2006), pp. 113-119. [Electronic Journal, website www.emis.de/journals]

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Causes and Effects of Traffic Congestions in Nigeria

By Casmir. C. Onyeneke
University of Calabar

Abstract- Nigera, the most populated country in Africa is faced with series of traffic congestions. This indisputable issue of traffic hold is occasioned by the growing population concentration, rapid urbanization, and increasing commercial and economic activities. High population generates heavy vehicular traffic, leading to vehicular conflict and congestion as well as other mobility related challenges, which adversely affect the ultimate goal of people's mobility. The country is expected to offer high accessibility and mobility advantages, coupled with the provision of transportation infrastructures which are being threatened by mobility challenges resulting in low productivity and loss of man-hours within Nigeria, thus adversely affecting the overall wellbeing of residents.

Keywords: *traffic congestions, road network, coefficient of variation, standard deviation, road accidents, standard of living, population, infrastructure, drivers' license, number plate, economy.*

GJSFR-F Classification: *FOR Code: MSC 2010: 47N30*

CAUSESANDEFFECTSOFTRAFFICCONGESTIONSINNIGERIA

Strictly as per the compliance and regulations of:

Causes and Effects of Traffic Congestions in Nigeria

Casmir. C. Onyeneke

Abstract- Nigera, the most populated country in Africa is faced with series of traffic congestions. This indisputable issue of traffic hold is occasioned by the growing population concentration, rapid urbanization, and increasing commercial and economic activities. High population generates heavy vehicular traffic, leading to vehicular conflict and congestion as well as other mobility related challenges, which adversely affect the ultimate goal of people's mobility. The country is expected to offer high accessibility and mobility advantages, coupled with the provision of transportation infrastructures which are being threatened by mobility challenges resulting in low productivity and loss of man-hours within Nigeria, thus adversely affecting the overall wellbeing of residents. This research examines the causes of traffic congestion as well as its effects on the citizens of Nigeria through questionnaire administration, based on a physical characteristics survey and the analysis of drivers' license and number plates recorded across the 36 states of country plus the federal capital territory. Desk study of relevant documents, and interviews were conducted. Inferential and non-inferential statistics were employed for data analysis. Findings revealed a chaotic acquisition of drivers' license, vehicles number plates and land use pattern, resulting into traffic and transportation blockage, vehicular conflict and avoidable traffic congestion, longer travel time and low productivity among others.

Keywords: traffic congestions, road network, coefficient of variation, standard deviation, road accidents, standard of living, population, infrastructure, drivers' license, number plate, economy.

I. Introduction

The examination of the background of the causes and effects of traffic congestions in Nigeria revealed huge existence of the negative effects of the scenario. In this research work, efforts were made to bring related information from various sources of literature with key issues put across to ascertain what is exactly happening, the likely challenges that are to be faced if measures are not taken to curb traffic congestion. The goal of this study, objectives and questions, positioned the necessary actions required to ensure straightforward and easy flow of traffic in Nigeria and other related countries. Nigeria, as the case study was chosen to represent other countries and societies that experience similar conditions. In describing the scope, location and limits of the study in terms of coverage, the period in which the study was carried out and the relevance of its findings, Nigeria was specifically chosen as the largest country in Africa and one of the most populated countries of the world. Nigeria faces extreme traffic congestion where cars in the country travel at speeds of about 3-5km/hr. On most of the busy days of the week such as the workdays and during rush hours like the morning and evening hours, there exist extremely significant congestions popularly known as hold-ups or goes-slow.

The 2017 record of the National Bureau of Statistics (NBS) estimated the vehicle population of 11458370 as at the first quarter of the year with 58.8 percent of commercial vehicles [1]. The number of private vehicles amounted to 44.5 percent of the

Ref

1. Iloani Francis Arinze (2017), Nigeria has 11.5m Vehicles at Q1 2017 NBS; Daily Trust.

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total vehicle population in Nigera for the same year. Also, the government and diplomatic vehicles contributed to respective figures of 1.65 and 0.1 percents of the total vehicle population in the country. The report published by NBS reviewed that the Lagos and FCT produced the highest number of national drivers licenses while Yobe and Kebbi States had the least out of the total number of 220736. In the same year, a total number of 113358 vehicle number plates were produced in the first quarter alone. Lagos and Sokoto states produced the least while Ebonyi and Anambra produced the highest. In the third quarter, the total number of registered vehicles is 11547236. In 2013, an abstract statistical study estimated that more than 1004469 motor vehicle number plates 395539 drivers' licenses and 254667 motor cycles number plates were produced across the 36 states of Nigeria and Federal Capital Territory (FCT).

Comparing the vehicle population in Nigeria to the overall population of people in the country as at 2016, the vehicle population ratio is 0.06. Rampant population growth could be a contributing factor to traffic congestion in Nigeria's capital cities Nigeria hence it needs to be checked [2]. Traffic congestion in Nigeria is also likely to be caused by poor conditioned roads that cannot contain the vehicle population, ongoing road constructions that usually take long, population and economic growth hence increased purchasing power and demand for vehicles and stubborn drivers that do not follow road rules and regulations which need to be investigated.

Traffic congestion leads to increased pollution of the air with fumes from cars, increased travel times leading to delays hence loss of investment opportunities [3]. It could also bring about increased pedestrian risks and road accidents. The objectives of this research include the identification of the degree and depth of traffic congestion and the evaluation of the causes in Nigeria. Also, maximum efforts were made to identify and establish relationship between the causes and effects of the congestion. Critically, the review of existing theories in available literature as related to the research developments were examined which helped to develop this unique and meaningful study. The definitions of key terms that were used throughout the study compared and contrasted the information from different sources to come up with summary of the indices that causes traffic congestions and their effects to growth and development of Nigeria's economy. Also, findings made at the cause of this study thoroughly identified main issues relating to the causes and effects of traffic congestions and proffers possible solutions.

Lagos, Abuja, Kano, Port-Harcourt, Aba and other capital cities are business districts with city centre converses, where development is compact. The cost of land is considered relatively high, in comparison to suburban areas. The high cost of land with high rise structures, justifies the dense development pattern in the capital cities. All the thirty-six state capitals including the Federal Capital Territory and other major cities are jam-packed with commercials, offices, retails, and cultural centre of the cities and usually are the centre points for transportation. Some of the theories and concepts usually adopted in the management of capital cities include central place theory that consider cities as a system; compact city theory, which according to [4], points to initiatives of promoting compact cities that emphasizes higher level of mixing and integration of functions; concept of accessibility that centre on proximity of two or more places, emphasizing availability of opportunity in geographical region and freedom of individuals to participate [5].

The traffic congestion is drive by too many people working in the capital city areas, coupled with narrow streets and shortage of off-street parking facilities. This makes on-street parking inevitable and reduces road design capacity resulting in traffic

Ref

5. Jende Hsu, A.M. and Surti, V.H.M. (1976). Demand Model for Bus Network Design. Transportation Journal of ASCE Vol. 102, No. TE3.

jam. Solutions tried so far have not yielded the desired result in alleviating traffic congestion in cities, these include construction of ring-roads or by-passes, introduction of congestion charges or outright banning of motor cycles from entering the capital cities, park and ride schemes, car-pooling, traffic calming, public transport subsidies, toll road, regulatory changes to increase density, parking levies and restrictions in area well served by public transport; among other [6].Traffics are the automobiles that are on a road at a particular time. They are the automobiles that are utilized for the purpose of transportation [7]. Traffic on road consists of pedestrians, ridden or herded animals, vehicles, streetcars, buses and other conveyances, either singly or together, while using the public way for purposes of travel [8]. The abstract statistical study estimated that more than 1004469, 560987, 303274 vehicles were registered in 2013, 2014 and 2015.

The other half of congestion is caused by random events such as accidents and unusual weather conditions (rain, storms, etc.), which are unexpected and unplanned. Non-recurrent congestion is linked to the presence and effectiveness of incident response strategies. As far as accidents are concerned, their randomness is influenced by the level of traffic as the higher the traffic on specific road segments the higher the probability of accidents [9]. Traffic Congestion occurs when there is an imbalance between transport demand and supply at a specific point in time and in a specific section of the transport system. It implies that the available transport infrastructure at that given time is being overstretched. Congestion can be perceived as an unavoidable consequence of the usage of scarce transport resources, particularly if they are not priced. The last decades have seen the extension of roads in rural but particularly in urban areas, most of them free of access. Those infrastructures were designed for speed and high capacity, but the growth of urban circulation occurred at a rate higher than often expected [10].

Congestion is one of the most prevalent transport problems in large urban states, usually above one million inhabitants [11]. It is particularly linked with motorization and the diffusion of the automobile, which has increased the demand for transport infrastructures. However, the supply of infrastructures has often not been able to keep up with the growth of mobility. Since vehicles spend the majority of the time parked, motorization has exceeded the parking lot available which has created space consumption problems particularly in capital cities. Some state capitals are mono centric structure as single centered and poorly planned cities; Nigeria cannot exist without traffic Jam. According to [12], there is an ever increasing importation of used cars. Currently there are over 11880000 registered cars in Nigeria, poorly planned roads and building, reckless driving and motor cycling and an ever growing population in Nigeria due to rural-urban migration are the common factors.

One of the major negative effects of traffic congestion in Nigeria is the cause of high difficulties for non-motorized transport. These difficulties are either the outcome of intense traffic, where the mobility of pedestrians, bicycles and vehicles are impaired, but also because of a deliberate lack of consideration for pedestrians and bicycles in the physical design of infrastructures and facilities. This is a common sight in most urban centers in where vehicle owners are at loggerhead with motorbike riders. It causes incessant loss of public spaces especially in Lagos. The majority of roads are publicly owned and free of access. According to the state of environment report, the national environment watchdog, the increase in motor vehicles has resulted in an increase in pollution, traffic congestion and a reduction in the quality of life for urban dwellers. It says, adding that motorbikes are hazardous and add to the traffic congestion, air pollution and disorganization in urban centres.

Ref

6. Andrew Downie (2008-04-21). "The World's Worst Traffic Jams". Time. Retrieved 2008-06-20.

Burning fossil fuels like oil and petrol from vehicles emit carbon dioxide, a main greenhouse gas which contributes to global warming. Vehicle fuel has a sulphur gas component which causes respiratory diseases of which cancer is one of them [13]. Alongside burning tyres on roads and release of burnt oil by vehicles is not only hazardous to human health, but also roads as they shorten their lifespan. Increased traffic has adverse impacts on public activities which once crowded the streets such as markets, shops/cubicle, parades and processions, games, and community interaction [14]. Traffic flows influence the life and interactions of residents and their usage of street space. More traffic impedes social interactions and street activities. The Nigerian state capitals have to deal with thousands of cars running through their streets each day. Traffic congestion is a big problem for everyone within the cities. The main causes of this problem are lack of regulatory rules on purchase of more cars, poor road management, and poor practices on behalf of employers.

One of the main reasons why there exists more congestion is due to more cars on the road. The adult population is increasing and therefore more people want their own personal transport to get around with. As the number of cars increase the chance of congestion also increases. This is coupled with a lack of proper infrastructure. Though in smaller towns and villages congestion is almost unheard of, they constitute little or no significant impact to the country's economy. Councils and national governments fail to act on the looming threat of heavy congestion until it happens. The city roads do not expand along with an increasingly car reliant population. A single street with a lane on each side before might not suffice in ten years after the population has increased. Authorities often fail to convert this into a dual carriageway.

II. Research Methodology

The study explored the problem in a positive view, using descriptive research strategy because it aims to know more about the phenomena that are responsible for traffic congestion and its effects in Nigeria. This research examined the problem in both descriptive and exploratory manner. It also looked into the problem by exploring the views of different set of respondents, as well as exploring different literatures related with the study. The population included various riders, passengers, pedestrians and traffic police officers found in the randomly selected major cities in Nigeria. Primary data was collected from the respondents that were chosen in Nigeria cities while the secondary data was obtained from the published works of the National Bureau of Statistics. The summary and analysis of the data was done using R program and SPSS.

The research analysis was divided into descriptive and inferential analysis. Descriptive analysis was adopted in the study of distribution of one variable. This study provided us with the information about the various types of traffic jam, their intensity and their span. After that, the use of inferential analysis was basically adopted to analyze the data. In other words, inferential analysis simultaneously analyzes the variables. The interdependence between the variables, their correlation, and variance analysis were employed to draw the inferences. Comparing the distribution of the drivers' license in the respective years, it is quite clear that 2015 has least variation relative to the mean. The coefficient of variation was particularly useful when comparing results for the three years of produced drivers license, number plates of motor vehicles and motor cycles. The different measures or values were examined to ascertain the increase or decrease in traffic congestions. The formula for the coefficient of variation was used as the ratio of the mean and standard deviation. In symbols: CV

Ref

13. Link, Heike (2004). "PEP-A Yield-Management Scheme for Rail Passenger Fares in Germany". Japan Railway & Transport Review. 38: 54.

= (SD/) × 100. Multiplying the coefficient by 100 is an optional step to get a percentage, as opposed to a decimal.

Researches on city structure, especially dealing with the problems of congestion and traffic jam adopted mathematical theories such as [15], 'three phase traffic theory', which likened traffic flows to the rule of fluid dynamics, noting pipe suddenly freezing representing traffic jam triggered 'butterfly effect' with spontaneous driver's maneuvering or traffic light control, distorting traffic flow. When the capital cities were observed in terms of problems and challenges, two major issues were identified. The first one is mainly urban decay or run-down, creating undesirable condition to reside; and secondly, traffic congestion within the city centres. Congestion is said to occur when transport demand exceeds transport supply at a specific point in time and in a specific section of the transport system [16]. Analytical Summary of the Annual Vehicle Statistics in Nigeria was assessed during the study by the examination of the drivers' licenses and number plates processed per state, and from the year 2013 to 2015. One of the best ways used to assess the level of variability of the data set was through the analysis and examination of variation about the mean.

III. Results and Discussions

In most statistical analysis, it is not common to see a situation where the value of standard deviation is greater than the value of the mean. However, it is often seen in real life situations and during practical researches. A sample's standard deviation that is of greater magnitude than its mean can indicate different things depending on the data being analyzed. Statically, the mean is seen as the center of location while the standard deviation measures how dispersed the individual observations occur around the mean. Since the mean is seen as the midpoint of a statistical data and the standard deviation is described the data's spread, it is easier to find the distribution of the license, number plates of motor vehicles and motor cycles across the states in the respective years. It helps to know the cities that are over congested with the number of vehicles. All the standard deviations in the case study are larger than the mean except motor cycle number plates produced in 2015. This was as a result of the limitation of the use of motor vehicles in some major cities in Nigeria. It helped to harmonize the distribution and use of motor vehicles. It is also evidence that if a similar measure is taken in the use of motor vehicles, there is no doubt the fact that traffic congestions will be minimal in Nigeria.

Table 1: Summary of the drivers licenses processed (2013 – 2015)

States	X2013	X2014	X2015
ABIA: 1	Min.: 1274	Min.: 1349	Min.: 2375
ADAMAWA: 1	1st Qu.: 2588	1st Qu.: 3464	1st Qu.: 8916
AKWA-IBOM: 1	Median: 4576	Median: 7954	Median: 17623
ANAMBRA: 1	Mean: 10690	Mean: 17650	Mean: 31133
BAUCHI: 1	3rd Qu.: 8726	3rd Qu.: 13253	3rd Qu.: 31303
BAYELSA: 1	Max.: 127261	Max.:225284	Max.:298319
BENUE: 1	Sum: 395539	Sum:653046	Sum:1151906
BORNO: 1	Var.:460959583	Var.:1402510511	Var.:2520309750
CROSSRIVER: 1	CV.: 2.008416	CV.: 2.121819	CV.: 1.612523
(Other): 28	Sd.: 21469.97	Sd.: 37450.11	Sd.: 50202.69

Ref

15. Adrian May (2007). "The philosophy and practice of Taktfahrplan: a case-study of the East Coast Main Line" (Working Paper). Working Paper 579. Institute of Transport Studies, University of Leeds, Leeds, UK. Retrieved 2008-04-27.

The wide and uneven distribution of the data was shown in the nature of the standard deviation. When the histogram of the data was observed, there is indication of some cities from Kwara to Nasarawa being widely spread. For example, say your data represent distances measured above and below sea level. Using this information, some inferences were made about the causes and effects of traffic congestions in Nigeria. Some states in Nigeria were significantly higher than the mean level. Also comparing the population distribution in Nigeria, there was a serious indication that the rapid cluster of the vehicles in some particular cities was the indication of incessant traffic congestion in Nigeria. For example, in the first dataset, you could determine whether a particular point was significantly higher above mean level than all others. It represented a statistical variance that was worth investigating based on the amount of standard deviations measured around the mean it was located. One point to clarify is that the concept of standard deviation is not limited to normally distributed data. It is a general concept that applies to data that arise from any distribution.

The coefficient of variation (CV) was used to measure relativity variability [17]. The ratio of the standard deviation to the mean for the drivers licenses produced per state in 2013, 2014 and 2015 respectively were 2.008416, 2.121819 and 1.612523. It was the ratio of the standard deviation to the mean (average). In other words, the standard deviations were 200.84, 212.18 and 161.25 percents of the mean.

Table 2: Summary of Motor Vehicle Number Plate Produced per State and Type (2013-2015)

States	X2013	X2014	X2015
ABIA: 1	Min.: 8	Min.: 0	Min.: 0
ADAMAWA: 1	1st Qu. : 6607	1st Qu.: 4716	1st Qu.: 1821
AKWA-IBOM: 1	Median: 9950	Median: 10478	Median: 3915
ANAMBRA: 1	Mean: 25112	Mean: 14025	Mean: 7397
BAUCHI: 1	3rd Qu.: 17216	3rd Qu.: 16805	3rd Qu.: 6913
BAYELSA: 1	Max.: 36907	Max.: 85155	Max.: 76399
BENUE: 1	Sum: 1004469	Sum: 560987	Sum: 303274
BORNO: 1	Var.: 3453009363	Var.: 251043305	Var.: 157850340
CROSS RIVER: 1	Sd.: 58762.31	Sd.: 15844.35	Sd.: 12563.85
DELTA: 1	CV.: 2.340009	CV.: 1.129722	CV.: 1.698506
(Other): 31	NA's: 1	NA's:1	

Observing the motor vehicle plate number produced per state from 2013 to 2015 in Nigeria, the mean average production decreased across the years. The total of 1004469 was produced in 2013 while the least was 2015 with a figure of 303274. It was observed that the year with the least coefficient of variation was 2014. It was also observed that the minimum values of zero were observed in 2015 and 2014. In other words, there were indices such Osun, Lagos and Oyo states which had no plate number production. This is as a result of accumulated ones that had to be sold. The complementary Types were not available in the years that preceded 2015.

Ref

17. Transport Research Board (TRB) (2000). Highway Capacity manual (HCM 2000): Transport Research Board, National Research Council, Washington D. C. USA.

Table 3: Summary of Motor Cycle Number Plate Produced per State and Type (2013-2015)

States	X2013	X2014	X2015
ABIA: 1	Min.: 0	Min.: 0	Min.: 0
ADAMAWA: 1	1st Qu.: 1543	1st Qu.: 2401	1st Qu.: 2000
AKWA-IBOM: 1	Median: 4860	Median: 6871	Median: 4233
ANAMBRA: 1	Mean: 6367	Mean: 10460	Mean: 6580
BAUCHI: 1	3rd Qu.: 7197	3rd Qu. : 13327	3rd Qu.: 11114
BAYELSA: 1	Max.: 45521	Max.:52460	Max.: 18061
BENUE: 1	Sum: 254667	Sum:418417	Sum: 269795
BORNO: 1	Var.: 65955825	Var.: 152938318	Var.: 33235757
CROSS RIVER: 1	Sd.: 8121.319	Sd.: 12366.82	Sd.: 5765.046
DELTA: 1	CV.: 1.275533	CV.: 1.182296	CV.: 0.876147
(Other): 31	NA's: 1	NA's: 1	

The Federal government restriction of the movement of motor cycles in some major cities in Nigeria aided the evenly distributed plate numbers of motor cycles produced across the states and types in Nigeria. The coefficient of variation in 2015 was o0.8761 which simples means that the ration of the standard deviation with respect to the mean could year a value of 87.61 percent. In the cause of the studies, it was discovered that the more dispersed the drivers' license and plate numbers across the states, the less traffic congestions experienced across the years. In order words, rural empowerment is very necessary. The government should not focus attention only in some major states, thereby abandoning others. It causes urban migration, congestions and overcrowded vehicles [18]. It was obvious that one of the causes of traffic congestion is when the population of vehicles outflow the available infrastructures such as good roads, availability of traffic lights and other road regulators.

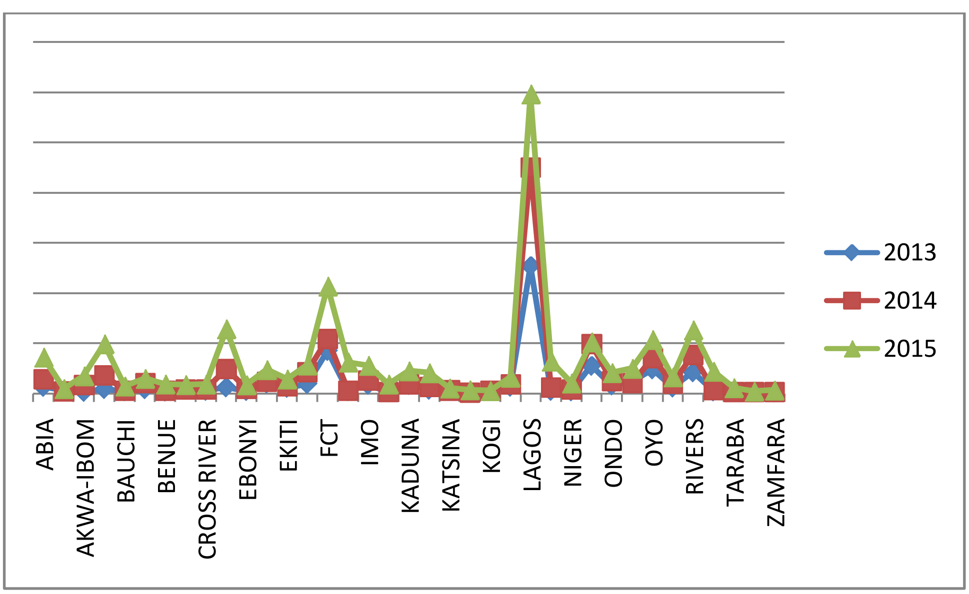

Fig. 1: Drivers licenses processed across the states (2013 -2015)

The above graph shows the increase in vehicle population in Nigeria over the years and it clearly shows a gradual increase in vehicles purchased. Nigeria revenue records showed that more than 1100246 vehicles enter the country every year compared

Ref

18. Kadiyali, L.R. (2008). Traffic Engineering and Transport Planning Seventh Edition, Second Reprint Khanna Publishers, NaiSarak, Delhi.

to the figure for the same period in 1962. Another single cause of traffic congestion in Nigeria is road accidents. Great number of patients admitted in Nigeria hospitals in 2015, 1988 were victims of road accidents. The major cities that connect Lagos – Ibadan express way and other highway road of southeast to the southwestern part of Nigeria experienced a huge increase of road accident from year 2015 to 2017 as a result of congestions. Lagos to Ibadan express road is one of the busiest in the country with the highest traffic count; the number plate of vehicles and licenses issued in Lagos is the largest in the country which accounts for the highest number of accidents being recorded in the road.

Notes

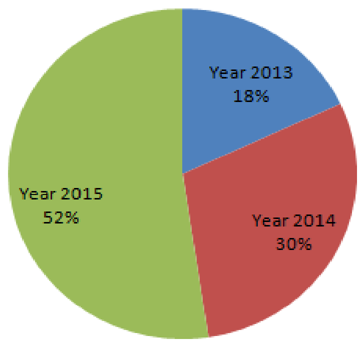

Fig. 2: Drivers licenses processed (2013 -2015)

The chat shows that there is a rapid increase in the number of license issued in Nigeria. It is obvious that measures should be put in place to ensure more roads are created to meet up with the increase number of vehicles that enter the country. Figure 2 shows the percentage of plate numbers issued from 2013 to 2015. The increased number of vehicles has attributed to the increased number of delays which has greatly increased traffic congestion in Nigeria.

IV. Conclusion

The analysis showed high level of traffic congestion in the cities of Nigeria which affect negatively the socioeconomic well-being of the citizens in the country. Also, there was significant indication that the number of vehicles in some States like Lagos, Edo and Osun states was higher than expected. It indicates that the western part of Nigeria experience huge congestion than other areas.

In the demographic analysis of the respondents, Respondents were asked to indicate their age bracket and below were their responses. Findings show that the number of respondents whose age lie between 20-30 years was 22 percent, those who lie between 31-40 years was 34.1 percent and those at 41years and above was 43.9 percent. There is respondent below 20 years recorded. Most of the respondents were above 40 years. This implies that most respondents that were interviewed were actually adults. Respondents were asked to indicate their profession and or Academic qualifications and below were their responses. The findings revealed that 39 percent of the respondents are diploma holders; 48.8 percent are degree holders while 12.2 percent are certificate holders. The findings therefore imply that the majority of the respondents were literate, since most of them attained qualifications at a higher education level.

It is largely the pain and agony of a failed public transport service, mainly offered by 14-seater owned mini buses that many Nigerians invest in, personal cars, resulting in traffic jams that cost the economy a whooping 500 million naira daily on burnt fuel. The jam also accounts for 30 percent of productive time lost daily; a situation worsened by most motor cycles, a popular alternative Nigerians use to fend off jam in most situations. The Federal Road Safety Commission (FRSC) notes that second hand cars pose grave health and environment challenges than the new ones. That is to say; a second hand car's combustion efficiency is not as good as a new car's yet about 80 percent of the cars found in Nigeria are second hand cars.

References Références Referencias

Notes

1. Iloani Francis Arinze (2017), Nigeria has 11.5m Vehicles at Q1 2017 NBS; Daily Trust.
2. Helaakoshi, R. and A. Merilainen,(2001). Transport, Poverty and Disability in Developing Countries. Linca Consultants Ltd.
3. George, E.G. (1979). Perception of Public Transportation, in Public Transportation Planning, Operations, and Management George E. G. and Lester J. (eds) Prentice-Hall INC. Englewood Cliff, New Jersey.
4. Mfinanga, D.A. and Ochieng, M.O.A. (2006). Development of a Model for Assessing Urban Public Transport Level of Service in Cities of Developing Nations, African Journal of Science and Technology (AJST) Science and Engineering series. Vol. 7, No. 2. pp. 35-53.
5. Jende Hsu, A.M. and Surti, V.H.M. (1976). Demand Model for Bus Network Design. Transportation Journal of ASCE Vol. 102, No. TE3.
6. Andrew Downie (2008-04-21). "The World's Worst Traffic Jams". Time. Retrieved 2008-06-20.
7. Duranton, Gilles; Turner, Matthew A. (2011). "The Fundamental Law of Road Congestion: Evidence from U.S. Cities". American Economic Review. 101 (6): 2616–52. doi:10.1257/aer.101.6.2616.
8. Isla, Rouselle (2016). "Find Out What's The Cost of Traffic In Metro Manila". iMoney.ph. Retrieved 2018-03-28.
9. Hermann Knoflacher (2006). "A new way to organize parking: the key to a successful sustainable transport system for the future". Environment and Urbanization. International Institute for Environment and Development. 18 (2): 387–400. doi:10.1177/0956247806069621.
10. Shoup, Donald C. (2005). The High Cost of Free Parking. American Planning Association. ISBN 1-884829-98-8.
11. Knoflacher, Hermann (January 2001). Stehzeuge. Der Stau ist kein Verkehrsproblem (in German). Vienna: Böhlau. ISBN 978-3-205-98988-2.
12. Goddard, Haynes (1997). "Using Tradeable Permits to Achieve Sustainability in the World's Large Cities". Environmental and Resource Economics. Springer Netherlands. 10 (1): 63–99. doi:10.1023/A:1026444113237.
13. Link, Heike (2004). "PEP-A Yield-Management Scheme for Rail Passenger Fares in Germany". Japan Railway & Transport Review. 38: 54.
14. Andersen, Bjørn (1993). "A survey of the Swiss public transport system and policy". Transport Reviews. 13 (1): 61–81. doi:10.1080/01441649308716835.
15. Adrian May (2007). "The philosophy and practice of Taktfahrplan: a case-study of the East Coast Main Line" (Working Paper). Working Paper 579. Institute of Transport Studies, University of Leeds, Leeds, UK. Retrieved 2008-04-27.
16. Ogilvie, David; Matt Egan; Val Hamilton; Mark Petticrew (2004). "Promoting walking and cycling as an alternative to using cars: systematic review". British Medical Journal. BMJ Publishing Group. 329 (7469): 763. doi:10.1136/bmj.38216.714560.55. PMC 520994 . PMID 15385407.

17. Transport Research Board (TRB) (2000). Highway Capacity manual (HCM 2000): Transport Research Board, National Research Council, Washington D. C. USA.
18. Kadiyali, L.R. (2008). Traffic Engineering and Transport Planning Seventh Edition, Second Reprint Khanna Publishers, NaiSarak, Delhi.

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Deformation in a Three-Phase-Lag Model of Orthotropic Thermoviscoelastic Material

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Abstract- The present paper is aimed at studying the effect of viscosity on thermoelastic interactions in three-phase-lag model of a homogeneous thermally conducting orthotropic material whose surface is subjected to thermal excitations. The governing equations are solved by applying Laplace and Fourier transforms technique. Eigen value approach is used to obtain the expressions for the variables considered. Numerical computations are performed for a specific material and result obtained are represented graphically for temperature gradient boundary. Comparsions are made with in the theory in the presence and absence of viscosity effect.

Keywords: generalized thermo elasticity, orthotropic material, relaxation time, laplace and fourier transforms, instantaneous loading, continuous loading.

GJSFR-F Classification: FOR Code: MSC 2010: 35D40

DEFORMATIONINATHREEPHASELAGMODELOFORTHOTROPICTHERMOVISCOELASTICMATERIAL

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Leena Rani

Abstract- The present paper is aimed at studying the effect of viscosity on thermoelastic interactions in three-phase-lag model of a homogeneous thermally conducting orthotropic material whose surface is subjected to thermal excitations. The governing equations are solved by applying Laplace and Fourier transforms technique. Eigen value approach is used to obtain the expressions for the variables considered. Numerical computations are performed for a specific material and result obtained are represented graphically for temperature gradient boundary. Comparsions are made with in the theory in the presence and absence of viscosity effect.

Keywords: generalized thermo elasticity, orthotropic material, relaxation time, laplace and fourier transforms, instantaneous loading, continuous loading.

I. Introduction

Most materials experience volumetric variations when are subjected to temperature variations and the consequent thermal stresses developed due to temperature gradient in the surface vicinity results in micro-crack and others imperfection development at the surface of anisotropic materials. Thus owing to anisotropic material's applications in aeronautics, astronautics, plasma physics, nuclear reactors and high-energy particle and in various others engineering sciences, theory of thermoelasticity has aroused intense attention in our challenge to understand the nature of the interaction between temperature and strain fields.

Thermo elasticity theory, Chadwick (1960, 1979) and Nowacki (1962, 1975), of thermal disturbances has aroused considerable interest in the last century, but systematic research started only after thermal waves – called second sound – were first measured in materials like solid helium, bismuth and sodium fluoride. Thus, the thermoelasticity theories, which admit a finite speed for thermal signals, have been receiving a lot of attention for the past thirty years. In contrast to the conventional coupled thermoelasticity theory basedon a parabolic heat equation, Biot (1956), which predicts an infinite speed for the propagation of heat, these theories involve a hyperbolic heat equation and are referred to as generalized thermoelasticity theories.

The Lord and Shulman (1967) theory introduces a single time constant todictate the relaxation of thermal propagation as well as the rate of change ofstrain rate and the rate of change of heat generation, and obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier law for isotropic bodies. Green and Lindsay (1972) developed a temperature rate dependent

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thermoelasticity that includes two thermal relaxation times and does not violate the classical Fourier law of heat conduction, when body under consideration has center of symmetry.

Dhaliwal and Sherief (1980) derived the governing field equations of generalized thermoelasticity for anisotropic media and also developed a variational principle for these equations. Dhaliwal and Rokne (1989) investigated the one dimensional thermal shock problem with two relaxation times. Simionescu (1992) studied the effect of concentrated loads in quasi-static coupled thermoelasticity.

Green and Naghdi (1993) proposed a new theory of thermoelasticity without energy dissipation and presented the derivation of a complete set of governing equations of the linearized version of the theory for homogenous and isotropic materials in terms of displacement and temperature fields and proved the uniqueness of the solutions of the corresponding initial mixed boundary value problem. An important feature of this theory, which is not present in other theories, is that this theory does not accommodate dissipation of thermal energy.

Tzou(1995) and Chandrasekharaiah(1998) developed dual-phase-lags thermoelastic model, In these models two different phase-lags, i.e.,one for the heat flux vector and other for the temperature gradient have been introduced in theFourier's law. Cimmelli(1998) studied thermodynamics of anisotropie solids near absolute zero.

Das and Lahiri (2001) employed the eigen value approach to determine the thermal stress in an orthotropic elastic slab due to prescribed surface temperatures.

Kumar and Rani(2004)investigated the disturbance due to mechanical and thermal sources in generalized orthorhombic thermoelastic material.

Kumar and Rani(2007) considered a two-diamensional problem of thermoelasticity and discussed the effects of mechanical and thermal sources in generalized orthorhombic thermoelastic material. Ieşan and Quintanilla(2009) studied inner structure and microtemperatures of thermoelastic bodies. Dolotov and Kill (2012) considered a dynamic problem for an elastic half-space with asymmetric normal loading on its boundary and obtained expressions for the components of the stress tensor in the form of series, possessing asymptotic properties, which converge for short values of the time. Liu, Lin and Li(2013) discussed convergence result for the *thermoelasticity* of type III.

Sherief and El-Latief (2014) applied the fractional order theory of thermoelasticity to a 2D problem for a half-space, solved it with Laplace and exponential Fourier transform techniques and study the effect of the fractional derivative parameter on the behavior of the solution. Marotti de Sciarra and Salerno (2014) discussed thermodynamic functions in *thermoelasticity* without energy dissipation. Yermolenko and Ivanov(2014) developed the principles of correspondence between static boundary value problems of thermoviscoelasticity and *thermoelasticity* Abbas, Kumar and Rani (2015)studied ramp-type heating in a thermally conducting cubic crystal. Zhyhailo and Bajkowski (2016) discussed axisymmetrical problem of *thermoelasticity* for halfspace with gradient coating. El-Karamany and Ezzat (2016)proposed three models of generalized thermoelasticity: a single –phase - lag Green–Naghdi theory of type III, a dual-phase–lag Green–Naghdi theory of type II and of type III for linear anisotropic inhomogeneous material. *Fernández and Masid(2017) studied mixture of thermoelastic solids with two temperatures*

The present work aims to determine the distributions of the displacement component, stresses and temperature distribution in a three-phase-lag model of

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homogeneous, thermally conducting, orthotropic materialdue to thermal loading in presence and absence of the viscosity for two values of time. Expressions for the physical quantities are obtained using eigen value approach and are presented graphically. The results of the problem may be applied to a wide class of geophysical problems involving temperature change. The physical applications are encountered in the context of problems such as ground explosions and oil industries. This problem is also useful in the field of geomechanics, where the interest is in various phenomenon occurring in earthquakes and measurement of displacements, stresses and temperature field due to the presence of certain sources.

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II. Basic Equations

The constitutive relations for orthotropicthermoelastic medium following Dhaliwal and Sherief (1980) and Green and Lindsay (1972) are given by

$$\mathrm{t}_{ij} = c_{ijkl}e_{kl} - \beta_{ij}\left(1+\tau_a \frac{\partial}{\partial t}\right)T_{,i}, \qquad \beta_{ij} = c_{ijkl}\alpha_{kl} \quad (\mathrm{i,j,k,l}=1,2,3) \tag{1}$$

Equation of motion for anorthotropic thermoelastic medium in the absence of body force is given by

$$t_{ij,j} = \rho \ddot{u}_i, \tag{2}$$

The heat conduction equation following Green and Nadhdi (1993) and Choudhuri (2007) is

$$K_{ij}\left(1+\tau_T \frac{\partial}{\partial t}\right)\dot{T}_{,\mathrm{ij}} + K^*{}_{ij}\left(1+\tau_v \frac{\partial}{\partial t}\right)T_{,\mathrm{ij}} = \left(1+\tau_q \frac{\partial}{\partial t}+\tau_q^2 \frac{\partial^2}{\partial t^2}\right)(\rho c_e \ddot{T} + \mathrm{T_o}\beta_{ij}\ddot{u}_{i,j}) \tag{3}$$

where list of symbols has been given at the end of the paper. The comma notation is used for spatial derivatives and dot notation represents time differentiation.

c_{ijkl} *satisfies the (Green) symmetry conditions:*

$$c_{ijkl} = c_{klij} = c_{ijlk} = c_{jikl}.$$

III. Formulation And Solution Of The Problem

We consider a homogenous, orthotropicthermoelastic half-space in the undeformed state at uniform temperature T_0. The rectangular Cartesian co-ordinate system (x,y,z) having origin on the plane surface z=0 with z-axis pointing vertically into medium is introduced. The boundary of the half-space is affected by thermal loading, which depends on time t and spatial coordinate z (- $\infty < z < \infty$).

In equation (2), we have used the contracted Voigt notation for the stiffness $\mathrm{c}_{\mathrm{ijkl}}$ to c_{ij} according to the scheme $11 \to 1,\quad 22 \to 2,\quad 33 \to 3,\quad 23 \to 4,\quad 13 \to 5,\quad 12 \to 6$.

In order to account for the material damping behavior the material coefficients c_{ij} are assumed to be function of the time operator $\mathrm{D}= \frac{\partial}{\partial \mathrm{t}}$, i.e.

$$\mathrm{c}_{\mathrm{ij}} = \mathrm{c}_{\mathrm{ij}}^* \tag{4}$$

where $\mathrm{c}_{\mathrm{ij}}^* = \mathrm{c}_{\mathrm{ij}}(D)$

Assuming that the viscoelastic nature of the material is described by the Voigt model of linear viscoelasticity (1963) , we write

$$c_{ij}(D) = c_{ij}\left(1 + \tau_0 \frac{\partial}{\partial t}\right), \tag{5}$$

where τ_0 is the relaxation time assumed to be identical for each c_{ij}.

Making use of the c_{pq} from equation (2) in equation (1) then the field equations and constitutive relations for such a medium in the absence of body forces and heat sources in non-dimensional form after suppressing the primes can be rewritten as

$$u_{,xx} + c_1^* u_{,zz} + c_2^* w_{,xz} - T_{,x} = \ddot{u}, \tag{6}$$

$$c_1^* w_{,xx} + c_3^* w_{,zz} + c_2^* u_{,xz} - \bar{\beta} T_{,z} = \ddot{w}, \tag{7}$$

$$(1+\tau_T \frac{\partial}{\partial t})(\dot{T}_{,xx} + \bar{K}\dot{T}_{,zz}) + (1/\omega^*)(1+\tau_v \frac{\partial}{\partial t})(\bar{K}_2 T_{,xx} + \bar{K}_1 T_{,zz})$$
$$= \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\{\ddot{T} + \epsilon_1 (\ddot{u}_{,x} + \ddot{w}_{,z})\}, \tag{8}$$

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where comma notation is used for spatial derivatives, we have defined the quantities

$$x' = \frac{\omega_1^* x}{v_1}, \quad z' = \frac{\omega_1^* z}{v_1}, \quad u' = \frac{\rho v_1 \omega_1^*}{\beta_1 T_0} u, \quad t' = \omega_1^* t, \quad w' = \frac{\rho v_1 \omega_1^*}{\beta_1 T_0} w,$$

$$T' = \frac{T}{T_0}, \quad c_1^* = \frac{c_{55}}{c_{11}}, \quad c_2^* = \frac{c_{13}+c_{55}}{c_{11}}, \quad c_3^* = \frac{c_{33}}{c_{11}}, \quad \bar{K}_1 = \frac{K_3^*}{K_1},$$

$$\omega' = \frac{\omega}{\omega_1^*}, \quad \tau_T' = \omega_1^* \tau_T, \quad \tau_v' = \omega_1^* \tau_v, \quad t_1' = \omega_1^* t_1, \quad \tau_q' = \omega_1^* \tau_q,$$

$$\bar{\beta} = \frac{\beta_3}{\beta_1}, \quad \varepsilon_1 = \frac{\beta_1^2 T_o}{\rho k_1 \omega_1^*}, \quad \bar{K}_2 = \frac{K_1^*}{K_1}, \quad a' = \frac{\omega_1^*}{c_2} a, \quad \bar{K} = \frac{K_3}{K_1}. \tag{9}$$

$$t_{zz}' = \frac{t_{zz}}{\beta_1 T_0}, \quad t_{zx}' = \frac{t_{zx}}{\beta_1 T_0}, \quad h' = \frac{h v_1}{\omega_1^*}, \tag{10}$$

and $v_1 = \left(\frac{c_{11}}{\rho}\right)^{\frac{1}{2}}$ and $\omega_1^* = \frac{c_e c_{11}}{K_1}$ are, respectively, the velocity of compressional waves in x-direction and characteristic frequency of the medium.

The initial and regularity conditions are given by

$$u(x,z,0) = 0 = \dot{u}(x,z,0),$$

$$w(x,z,0) = 0 = \dot{w}(x,z,0),$$

$$T(x,z,0) = 0 = \dot{T}(x,z,0) \quad \text{for } z \geq 0, \quad -\infty < x < \infty \tag{11}$$

$$\text{and } u(x,z,t) = w(x,z,t) = T(x,z,t) = 0 \quad \text{for } t > 0 \quad \text{when} \quad z \to \infty \tag{12}$$

Applying the Laplace and Fourier transforms

$$\hat{f}(x,z,p)=\int_0^\infty f(x,z,t)e^{-pt}dt \quad \text{and}$$

on

$$\tilde{f}(\xi,\ z,\ p)=\int_{-\infty}^{\infty}\hat{f}(x,\ z,\ p)e^{i\xi x}dx \tag{13}$$

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the resulting expressions, we obtain

$$\frac{d^2\tilde{u}}{dz^2}=R_{11}\tilde{u}+R_{13}\tilde{T}+R_{15}\frac{d\tilde{w}}{dz}, \tag{14}$$

$$\frac{d^2\tilde{w}}{dz^2}=R_{22}\tilde{w}+R_{24}\frac{d\tilde{u}}{dz}+R_{26}\frac{d\tilde{T}}{dz}, \tag{15}$$

$$\frac{d^2\tilde{T}}{dz^2}=R_{31}\tilde{u}+R_{33}\tilde{T}+R_{35}\frac{d\tilde{w}}{dz}. \tag{16}$$

where

$$R_{11}=\frac{\xi^2+p^2}{c_1},\qquad R_{13}=\frac{-i\xi}{c_1},\qquad R_{15}=\frac{i\xi c_2}{c_1},$$

$$R_{22}=\frac{c_1\xi^2+p^2}{c_3},\qquad R_{26}=\frac{\overline{\beta}}{c_3},\qquad R_{24}=\frac{i\xi c_2}{c_3},$$

$$R_{31}=-\frac{i\xi\varepsilon_1 N_3 p^2}{(N_1+N_2)\overline{K}_1},\quad R_{33}=\frac{N_1\xi^2+N_2\overline{K}_2+N_3p^2}{(N_1+N_2)\overline{K}_1},\quad R_{35}=-\frac{\overline{\beta}\varepsilon_1 N_3 p^2}{(N_1+N_2)\overline{K}_1},$$

$$N_1=p+\tau_T p^2,\qquad N_2=\frac{1+\tau_v p}{\omega_1^*}\qquad N_3=1+\tau_q p+\tau_q^2 p^2.$$

The equations (14)-(16) can be written as

$$\frac{d}{dz}W(\xi,z,p)=A(\xi,p)W(\xi,z,p), \tag{17}$$

where

$$\mathrm{W}=\begin{bmatrix}\mathrm{U}\\ \mathrm{U}'\end{bmatrix},\qquad \mathrm{A}=\begin{bmatrix}\mathrm{O} & \mathrm{I}\\ \mathrm{A}_1 & \mathrm{A}_2\end{bmatrix},\qquad \mathrm{U}=\begin{bmatrix}\tilde{\mathrm{u}}\\ \tilde{\mathrm{w}}\\ \tilde{\mathrm{T}}\end{bmatrix},\qquad \mathrm{U}'=\begin{bmatrix}\tilde{\mathrm{u}}'\\ \tilde{\mathrm{w}}'\\ \tilde{\mathrm{T}}'\end{bmatrix},$$

$$\mathrm{O}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix},\qquad \mathrm{I}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\qquad \mathrm{A}_1=\begin{bmatrix}0&\mathrm{R}_{15}&0\\ \mathrm{R}_{24}&0&\mathrm{R}_{26}\\ 0&\mathrm{R}_{35}&0\end{bmatrix},$$

$$\mathrm{A}_2=\begin{bmatrix}\mathrm{R}_{11}&0&\mathrm{R}_{13}\\ 0&\mathrm{R}_{22}&0\\ \mathrm{R}_{31}&0&\mathrm{R}_{33}\end{bmatrix},$$

To solve the equation (17), we take

$$W(\xi,z,p)=X(\xi,p)\,e^{qz} \tag{18}$$

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so that

$A(\xi, p)W(\xi, z, p) = qW(\xi, z, p)$

which leads to an eigenvalue problem. The characterstic equation corresponding to the matrix A is given by

$$\det[A-qI]=0 \tag{19}$$

which on expansion leads to

$$q^6 - \lambda_1 q^4 + \lambda_2 q^2 - \lambda_3 = 0 \tag{20}$$

where

$\lambda_1 = R_{15} R_{24} + R_{33} + R_{22} + R_{11} + R_{26}R_{35}$

$\lambda_2 = R_{15}R_{24}R_{33} - R_{13}R_{24}R_{35} + R_{22}R_{33} + R_{11}R_{26}R_{35}$
$\quad - R_{31}R_{15}R_{26} + R_{11}R_{33} - R_{31}R_{13} + R_{11}R_{22}$

$\lambda_3 = R_{22}(R_{11}R_{33} - R_{31}R_{13}),$

The roots of equation (20) are $\pm q_\ell (\ell = 1, 2, 3)$.

The eigenvalues of the matrix A are roots of equation (20). The eigenvector X (ξ,p) corresponding to the eigenvalues q_ℓ can be determined by solving the homogeneous equation

$$[A-qI]\, X(\xi, p)=0 \tag{21}$$

The set of eigenvectors $X_\ell(\xi, p)$, $(\ell=1,2,3,4,5,6)$ may be obtained as

$$X_\ell(\xi, p) = \begin{bmatrix} X_{\ell 1}(\xi, p) \\ X_{\ell 2}(\xi, p) \end{bmatrix}$$

where

$$X_{\ell 1}(\xi, p) = \begin{bmatrix} -\xi \\ a_\ell q_\ell \\ b_\ell \end{bmatrix}, \quad X_{\ell 2}(\xi, p) = \begin{bmatrix} -\xi q_\ell \\ a_\ell q_\ell^2 \\ b_\ell q_\ell \end{bmatrix}, \quad q = q_\ell, \ell = 1,2,3.$$

$$X_{\ell_a 1}(\xi, p) = \begin{bmatrix} -\xi \\ -a_\ell q_\ell \\ b_\ell \end{bmatrix}, \quad X_{\ell_a 2}(\xi, p) = \begin{bmatrix} \xi q_\ell \\ a_\ell q_\ell^2 \\ -b_\ell q_\ell \end{bmatrix}, \quad \ell_a = \ell + 3, \quad q = -q_\ell, \ell = 1,2,3.$$

and

$$a_\ell = \frac{\{(\bar{\beta} - c_2)\xi^2 + p^2\bar{\beta} - c_1\bar{\beta}q_\ell^2\}}{\Delta_\ell},$$

$$b_\ell = \frac{\{c_1 q_\ell^2 \xi - (\xi^2 + p^2)\xi\}\{(c_1\xi^2 + p^2) - q_\ell^2(c_3 - c_2\bar{\beta})\} - q_\ell^2 c_2 \xi \ \{(\xi^2 + p^2) - c_1 q_\ell^2\}\bar{\beta} - c_2\xi^2\}}{\xi\,\Delta_\ell},$$

$$\Delta_\ell = i\{(c_1\xi^2 + p^2) - (c_3 - c_2\overline{\beta})q_\ell^2\}, \qquad \ell = 1,2,3.$$

The solution of equation (21) is given by

$$W(\xi,z,p) = \sum_{\ell=1}^{3}[B_\ell X_\ell(\xi,p)\exp(q_\ell z) + B_{\ell+3}X_{\ell+3}(\xi,p)\exp(-q_\ell z)], \tag{22}$$

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where $B_\ell(\ell = 1,2,3,4,5,6)$ are arbitrary constants.

Thus equation (22) represents the solution of the general problem in the plane strain case of generalized homogeneous thermoelasticity by employing the eigenvalue approach and therefore can be applied to a broad class of problems in the Laplace and Fourier transforms. Displacements and temperature distribution that satisfy the regularity conditions (12) are given by

$$\tilde{u}(\xi,z,p) = -\xi(B_4 e^{-q_1 z} + B_5 e^{-q_2 z} + B_6 e^{-q_3 z}), \tag{23}$$

$$\tilde{w}(\xi,z,p) = -(a_1 q_1 B_4 e^{-q_1 z} + a_2 q_2 B_5 e^{-q_2 z} + a_3 q_3 B_6 e^{-q_3 z}), \tag{24}$$

$$\tilde{T}(\xi,z,p) = (b_1 B_4 e^{-q_1 z} + b_2 B_5 e^{-q_2 z} + b_3 B_6 e^{-q_3 z}), \tag{25}$$

IV. Application

a) Dynamic thermoelasticcase

i. Thermoelastic Interactions due to Thermal Source

The boundary conditions at the plane surface are

$$t_{zz} = 0, \quad t_{zx} = 0, \qquad \text{at } z = 0$$

$$\frac{\partial T}{\partial z}(x, z=0) = r(x,t), \qquad \text{for the temperature gradient boundary,}$$

or

$$T(x, z=0) = r(x,t), \qquad \text{for the temperature input boundary,.} \tag{26}$$

where $r(x,t) = \eta(x)F(t)$

Applying the Laplace and Fourier transforms defined by (12),we get

$r(\xi,p) = \tilde{\eta}(\xi)\tilde{F}(p)$

Making use of Eqs. (1), (9)-(13) in the boundary conditions given by Eq. (26) and with the help of Eqs. (23) –(25), we obtain the expressions for displacement components, stresses and temperature distribution as

$$\tilde{u} = -r(\xi,p)\frac{\xi\sum_{m=1}^{3}\Delta''_m e^{-q_m z}}{T_0\Delta_2^*}, \ \tilde{w} = -r(\xi,p)\frac{\sum_{m=1}^{3}a_m q_m\Delta''_m e^{-q_m z}}{T_0\Delta_2^*}, \ \tilde{t}_{zx} = r(\xi,p)\frac{\sum_{m=1}^{3}s_m\Delta''_m e^{-q_m z}}{T_0\Delta_2^*},$$

$$\tilde{t}_{zz} = r(\xi,p)\frac{\sum_{m=1}^{3}p_m\Delta''_m e^{-q_m z}}{T_0\Delta_2^*}, \ \tilde{T} = -r(\xi,p)\frac{\sum_{m=1}^{3}b_m\Delta''_m e^{-q_m z}}{T_0\Delta_2^*}. \tag{27}$$

where

$$\Delta''_1 = p_2 s_3 - s_2 p_3, \quad \Delta''_2 = p_2 s_3 - s_1 p_3, \quad \Delta''_3 = p_1 s_2 - s_1 p_2,$$

$$s_m = \frac{\xi k_\ell (1 + i a_\ell)}{c_{11}}, \qquad p_\ell = \frac{i\xi c_{13} + a_\ell q_\ell^2 c_{33} - \overline{\beta} b_\ell}{c_{11}}, \quad \ell = 1,2,3.$$

On replacing Δ *by* $(\frac{\omega_1^* T_0}{V_1})\Delta_1^*$ and $T_0 \Delta_2^*$ respectively, we obtain the expressions for temperature gradient boundary and temperature input boundary.

we set a triangular pulse $\eta(x) = \begin{cases} a + x, & -a \le x \le 0 \\ a - x, & 0 < x \le a \\ 0, & |x| > a \end{cases}$

in equation (26). Using equations (9)-(10) and applying the Laplace and Fourier transform defined by equation (13), we get

$$\tilde{\eta}(\xi) = \left[2\left\{1 - \cos(\frac{\xi c_2 a}{\omega_1^*})\right\} \Big/ \xi^2 \right], \ \xi \neq 0.$$

which leads to an eigenvalue problem. The characterstic equation corresponding to the

Case 1: Instantaneousloading:

The plane boundary z=0 is assumed to be traction free and is subjected to an instantaneous input in temperature, i.e.

$$F(t) = F_0 \delta(t)$$

with

$$\tilde{F}(p) = F_0, \tag{28}$$

where F_0 is a constant representing the magnitude of constant temperature and $\delta(t)$ is the Dirac delta function.

Case 2: Continuous loading:

The plane boundary z=0 is subjected to a continuousinput in temperature, i.e. $F(t) = F_0 H(t)$,

with

$$\tilde{F}(p) = \frac{F_0}{p} \tag{29}$$

where F_0 is a constant representing the magnitude of constant temperature and $H(t)$ is the Heaviside unit step function.

V. Special Cases

Transformed solutions of equation (27) reducesto various models of thermoelasticity as:

(1) Classical thermoelastic model - K^*_{ij} =0.

(2) Dual phase-lag-model of thermoelasticity- K_{ij} ¿¿ K^*_{ij} , K^*_{ij} =0.

(3) Lord shulman (L-S) model- K^*_{ij} =0, $\tau_T = 0$, $\tau_v = 0$, $\tau_q = \tau$.

(4) Coupled thermoelasticity(CT) model- K^*_{ij} =0, $\tau_T = \tau_q = \tau_v = 0$.

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(5) Uncoupled thermoelasticity(UCT) model - $\varepsilon_1 = 0, K^*_{ij} = 0,\ \tau_T = \tau_q = \tau_v = 0.$

(6) Green-Nighdi(G-N)model(Type-1)- $K^*_{ij} = 0.$

(7) Green-Nighdi(G-N)model(Type-II)- $K_{ij} \,¡¡\, K^*_{ij}, \tau_T = \tau_q = \tau_v = 0.$

(8) Green-Nighdi(G-N)model(Type-III)- $\tau_T = \tau_q = \tau_v = 0.$

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VI. Particular Cases

a) Transversely isotropic materials

This type of medium has only one axis of thermal and elastic symmetry. We take the z axis along the axis of symmetry. Then the non-vanishing elastic and thermal parameters are

$$c_{11} = c_{33},\quad K_1 = K_3,\quad \alpha_1 = \alpha_3,\ K_1^* = K_3^*.$$

b) Cubic crystal

For cubic crystals, the nonvanishing elastic and thermal parameters are

$$c_{11} = c_{33},\quad K_1 = K_3 = K_1^* = K_3^* = K,\quad \beta_1 = \beta_3 = \beta,\quad \alpha_1 = \alpha_3 = \alpha_t$$

c) Isotropic media

For isotropic material every direction is a direction of elastic as well as thermal symmetry and the nonvanishing elastic and thermal parameters are

$$c_{11} = c_{33} = \lambda + 2\mu,\quad c_{13} = \lambda,\quad c_{55} = \mu,\quad K_1 = K_3 = K_1^* = K_3^* = K,$$

$$\alpha_1 = \alpha_3 = \alpha_t,\qquad \beta_1 = \beta_3 = \beta = (3\lambda + 2\mu)\alpha_t$$

VII. Inversion Of The Transforms

To obtain the solution of the problem in the physical domain, we must invert the transforms in equation (26) for three phase lag theory of thermoelasticity. These expressions are functions of z, the parameters of Laplace and Fourier transforms p and ξ, respectively, and hence are of the form $\tilde{f}(\xi, z, p)$. To get the function f(x,z,t) in the physical domain, first we invert the Fourier transform using

$$\hat{f}(x,z,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x}\,\tilde{f}(\xi,z,p)\,d\xi = \frac{1}{\pi}\int_0^{\infty}(\cos(\xi x)\,f_e - i\sin(\xi x) f_0)\,d\xi, \tag{30}$$

where f_e and f_0 are, respectively, even and odd parts of the function $\tilde{f}(\xi, z, p)$.

Thus, expression (30) gives us the Laplace transform $\hat{f}(x, z, p)$ of the function f(x,z,t). Following Honig and Hirdes(1984), the Laplace transform function $\hat{f}(x, z, p)$ can be inverted to f(x,z,t).

The last step is to calculate the integral in equation (30). The method for evaluating this integral is described by Press et al. (1986), which involves the use of Romberg's integration with adaptive step size. This, also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

VIII. Numerical Result And Discussion

Following Dhaliwal and Singh (1980), we take the case of magnesium crystal-like material for numerical calculations. The physical constants used are:

$\varepsilon = 0.0202$, $c_{11} = 5.974 \times 10^{10}\ \mathrm{Nm^{-2}}$, $c_{12} = 2.624 \times 10^{10}\ \mathrm{Nm^{-2}}$,

$\rho = 1.74 \times 10^{3}\ \mathrm{kgm^{-3}}$, $c_{44} = 3.278 \times 10^{10}\ \mathrm{Nm^{-2}}$, $c_e = 1.04 \times 10^{3}\ \mathrm{J\ kg^{-1}degree^{-1}}$

$\omega_1^* = 3.58 x 10^{11}\ \mathrm{s^{-1}}$, $K_1 = K_3 = 1.7 \times 10^{2}\ \mathrm{Wm^{-1}degree^{-1}}$, $\beta_1 = \beta_3 = 2.68 \times 10^{6}\ \mathrm{Nm^{-2}degree^{-1}}$,

$F_0 = 1$, a=1, $T_0 = 298^0$K.

The comparison of normal boundary displacement w and boundary temperature field T, and normal stress t_{zz} for instantaneous thermoviscoelastic material (ITVM) and instantaneous thermoelastic material (ITM) are depicted in Figures 1-3 and continuous thermoviscoelastic material (CTVM) and continuous thermoelastic material (CTM) are depicted in Figures 4-6.for three-phase-lag theory of thermoelasticity. The computations were carried out for two values of time t=1.0 and t=2.0, non-dimensional relaxation times $\tau_v = 0.02$, $\tau_a = 0.05$, $\tau_T = 0.04$, $\tau_q = 0.06$ at z=1.0 in the range $0 \le x \le 10$.

a) Thermoelastic Interactions due to Thermal Source (Temperature gradient boundary) Dynamic thermoelastic case:

Figure 1. depictsthe variation of normal displacement 'w' with distance x for ITM and ITVM for different values of time in context of three phase-lag-model. At t=1.0 the viscosity effect on 'w' is prominent in the ranges $0 \le x \le 2.5,\ 3.5 \le x \le 4.5, 7 \le x \le 8.5$ and less in rest of ranges where asat t=2.0 the viscosity effect on 'w' is more in the ranges $0.5 \le x \le 3,\ 7 \le x \le 9.5$ and less in rest of ranges.

Figure 2. determinethe variation of temperature distribution T with distance x. At t=1.0 the t=2.0 the ITVM and ITM show opposite oscillatory behavior in the whole range $0 \le x \le 10$.

Figure 3. displays the variation of normal stress $t_{zz}$with distance x. Near the point of application of source, the magnitude of normal stress for ITM is more ITVM and then become oscillatory in the whole range about zero for the time t=1.0 and t=2.0,respectively.

Figure 4. showsthe variation of normal displacement 'w' with distance x for CTM and CTVM. The viscosity effect is more prominent than thermal effect in the range $0 \le x \le 2.5$ for time t=1.0 and t=2.0, respectively, and less in rest of the range.

Figure 5. depictsthe variation of temperature distribution T with distance x. The values of temperature shows same oscillatory pattern in the whole range for CTVM and CTM for time t=1.0 and t=2.0,respectively.

Figure 6. displaysthe variation of normal stress $t_{zz}$with distance x.The thermal effect is more prominent than viscosity effect in the range $0 \le x \le 7$ and reverse in rest of the range $7 \le x \le 10$ for both values of time t=1.0 and t=2.0,respectively.

IX. Conclusion

The problem of investigating displacement component, temperature and stress components in anhomogeneous anisotropic thermoelastic half-space is studied in the purview of viscothermoelasticity. Eigen value technique is employed to express the

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results mathematically. Theoretically obtained field variables are also shown through a specific model to present the results in graphical form.
The results of the present work can be summarized as

1. The Laplace and Fourier techniqueis used to derive expressions for displacement components, stresses and temperature distribution for dynamic thermo-elastic case.
2. The values of all physical quantities are observed to follow oscillatory pattern about zero in the whole range with increase in distance x.
3. The viscosity effect has a significant role in the considered physical quantities.
4. The time effect has significant influence on the distribution of the considered physical quantities.

Notes

Nomenclature

$\vec{u}=(u,v,w)$ - displacement vector

T(x,y,z,t) -temperature change

c_{ijkl} - isothermal elastic parameters,

t- time

t_{ij}-stress tensor

e_{ij}-strain tensor

T_0 -uniform temperature

ρ _density

τ_T, τ_a, τ_v and τ_q-thermal relaxation times

α_{kl} -linear thermal expansion tensor.

$K^*_{ij}=\frac{c_e c_{11}}{4}$ -the material characteristic constant of the theory.

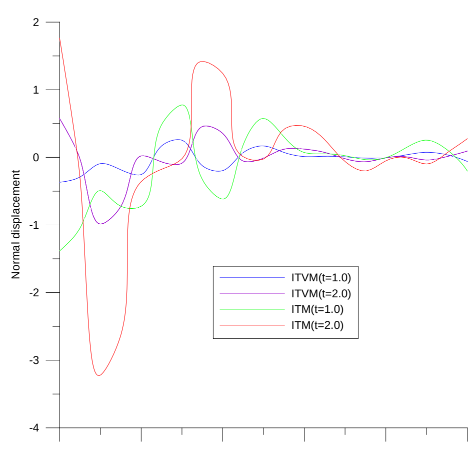

Fig. 1: Variation of Normal Displacement with Distance X.

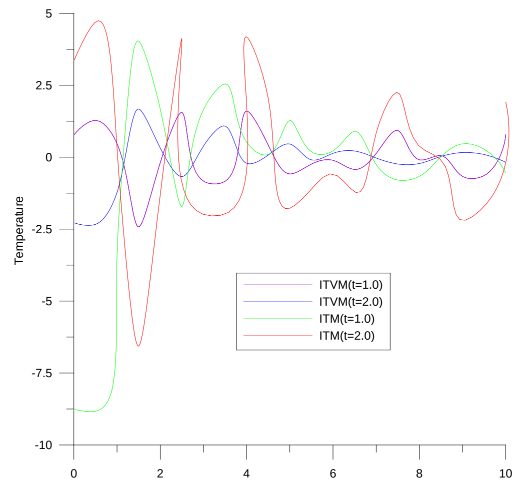

Fig. 2: Variation of Temperature with Distance X.

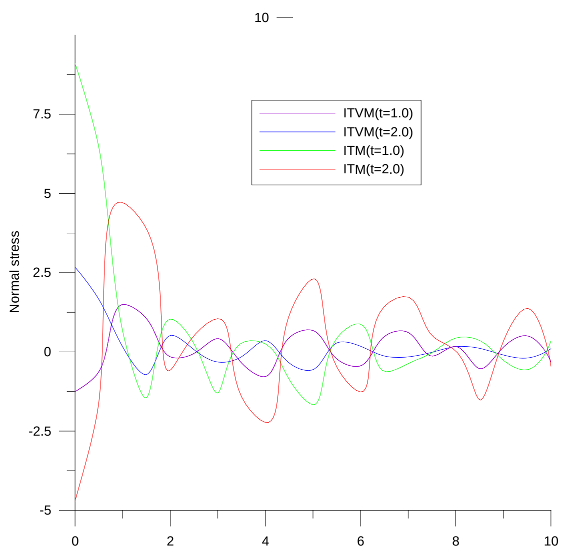

Fig. 3: Variation of Normal stress with Distance X.

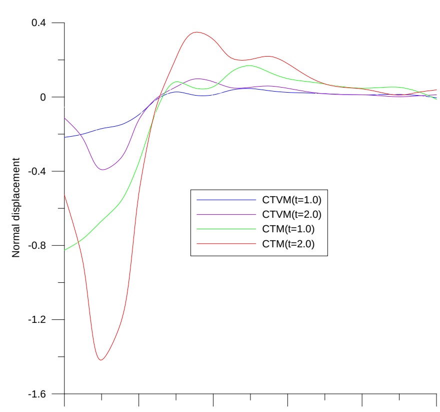

Fig. 4: Variation of Normal Displacement with Distance X.

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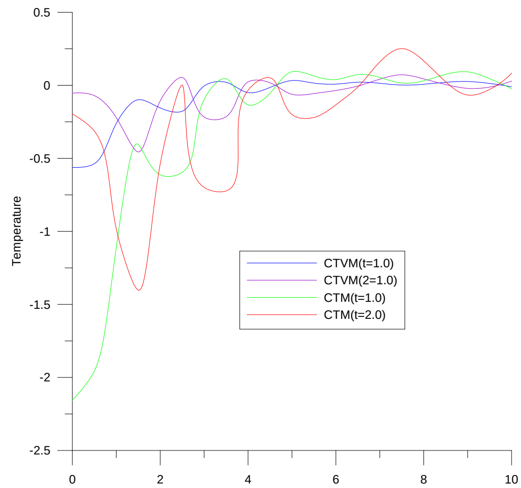

Fig. 5: Variation of Temperature with Distance X.

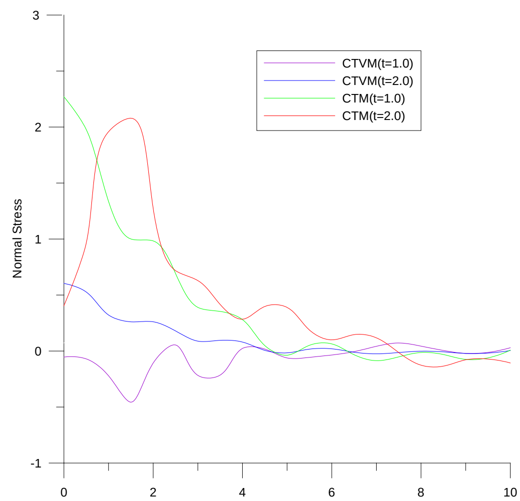

Fig. 6: Variation of Normal stress with Distance X.

References Références Referencias

1. Abbas, I.A., Kumar, R. and Rani, L. (2015) "Thermoelastic interaction in a thermally conducting cubic crystal subjected to ramp-type heating", Applied Mathematics and Computation, Volume 254, pp. 360–369.
2. Biot M.A., 1956, Thermoelasticity and irreversible thermodynamics, *Journal of Applied Physics*, 27, 240-253
3. Chadwick P., 1960, *Progress in Solid Mechanics* , R. Hill and I.N. Sneddon (Edit.), North Holland Publishing Co.
4. Chadwick P., 1979, Basic properties of plane harmonic waves in a presented heat conducting elastic material, *J. of Thermal Stresses*, 2, 193-214
5. Choudhuri, S.S.R (2007): On a thermoelastic three-phase-lag Model, J. Thermal Stresses, vol. 30, pp.231-238.
6. Chandrasekharaiah D.S.(1998) Hyperbolic Thermoelasticity: A Review of Recent Literature, *Appl. Mech. Rev.*, vol. 51, pp. 705–729.
7. Cimmelli, V.A (1998): Thermodynamics of anisotropie solids near absolute zero. Mathematical and Computer Modelling, vol. 28, pp.79-89.

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Das, N.C. and Lahiri, A. (2001), Dynamic thermal stresses in an orthotroic elastic slab due to prescribed surface temperatures, J. Indian Acad. Math., vol.23, pp 1-16.
8. Dhaliwal, R.S. and Sherief, H.H. (1980); Generalized thermoelasticity for anisotropic media; Quart. of Applied Mathematics, vol.38,pp.1-8.
9. Dhaliwal, R.S. and Singh, A., Dynamic coupled thermoelasticity; Hindustan Pub. Corp.(India), Delhi, 1980.
10. Dolotov, M.V. ans Kill, I.D.(2012): The dynamic problem for an elastic half-space with asymmetric normal loading of its boundary, Journal of Applied Mathematics and Mechanics, Volume 76, Issue 6, Pages 722-731.
11. Dhaliwal, R.S. and Rokne, J.G. (1989), One dimensional thermal shock problem with two relaxation times, J. Thermal Stresses, vol.12, pp 259-279.
12. El-Karamany, A.S, Ezzat, M.A. (2016): On the phase- lag Green–Naghdi thermoelasticity theories, Applied Mathematical Modelling, vol. 40, pp. 5643-5659
13. Fernández, J.R. and Masid, M.(2017): A mixture of *thermoelastic* solids with two temperatures, Computers & Mathematics with Applications, vol. 73,pp.1886-1899.
14. Green A.E. and Lindsay K.A.(1972): *Thermoelasticity.* - J.Elasticity, vol.2, pp. 1-7.
15. Green, A.E. and Naghdi, P.M. (1993), Thermoelasticity without energy dissipation; J. Elasticity, vol.31, pp189-208.
16. Honig, G and Hirdes, U.(1984): *A method for the numerical inversion of Laplace transform,* Journal of Computational and Applied Mathematics,vol.10, pp.113-132.
17. Han, Jian-Jun and Hasebe, N. (2002), Green's functions of point heat source in various thermoelastic boundary value problems, J. Thermal Stresses, vol.25,pp.153-167.
18. Hany H. Sherief, A.M. Abd El-Latief(2014): Application of fractional order theory of thermoelasticity to a 2D problem for a half-space, Applied Mathematics and Computation, vol. 248, pp. 584–592.
19. Ieşan,D and Quintanilla, R(2009): On *thermoelastic* bodies with inner structure and microtemperatures, vol. 354, pp 12-23.
20. Kaliski, S. (1963): Absorption of magneto-viscoelastic surface waves in a real conductor in magnetic field, *Proc. Vib.Prob.* vol.4: pp319- 329.
21. Kumar, R. and Rani, L. (2004): Deformation due to mechanical and thermal sources in generalized orthorhombic thermoelastic material, Sadhana, 29, 429-447.
22. Kumar, R. and Rani, L. (2007): Disturbance due to mechanical and thermal sources in orthorhombicthermoelastic material, IJAME, 12, 677-692.
23. Liu, Y., Lin, Y. and Li, Y. (2013): Convergence result for the *thermoelasticity* of type III, Applied Mathematics Letters, vol.26, pp. 97-102.
24. Lord H.W.and Shulman Y. (1967): A generalized dynamical theory of thermoelasticity. - J. Mech. Phys. Solids, vol. 15, pp. 299-309.
25. Marotti de Sciarra, F and Salerno, M (2014): On thermodynamic functions in *thermoelasticity* without energy dissipation, European Journal of Mechanics - A/Solids, vol.46, pp. 84-95.
26. NowackiW., (1962), *Thermoelasticity*, Int. Ser. Monographs in Aeronautics and Astronautics, PWN Warsaw.
27. NowackiW., (1975), *Dynamic Problems of Thermoelasticity*, Leyden: Noordhoff. Press W.H., Teukolshy S.A., Vellerling W.T. and Flannery B.P., Numerical Recipes, p.134, Cambridge University Press, Cambridge, 1986.

Notes

28. Simionescu, O., The effect of concentrated loads in quasi-static coupled thermoelasticity; Stud. Cerc. Mat., 44(1992) 543-548.
29. Tzou D. Y.(1995): A Unified Field Approach for Heat Conduction from Macro to Micro-Scales, ASME J. Heat Transf., vol. 117, pp. 8–16.
30. Yermolenko, G.Y and Ivanov, Y.Y(2014): The principles of correspondence between static boundary value problems of thermoviscoelasticity and thermoelasticity, vol. 78, pp. 613-616.
31. Zhyhailo, R.K and Bajkowski, A.S.(2016): Axisymmetrical problem of *thermoelasticity* for halfspace with gradient coating, International Journal of Mechanical Sciences,vol.106, pp. 62-71.

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Interesting Theta Function Identities Related to Jacobi Triple-Product

By Getachew Abiye Salilew
Madda Walabu University

Abstract- In this article, the author provide elementry proof for two (pre-sumably new) theta function identities.

Keywords and Phrases: *infinite series, theta function identities.*

GJSFR-F Classification: *FOR Code: MSC 2010: 03G12*

Strictly as per the compliance and regulations of:

Interesting Theta Function Identities Related to Jacobi Triple-Product

Getachew Abiye Salilew

[4]. G. J. Jacobi, *Fundamental Nova Theoriae Functionum Ellipticarum*, Sumptibus Fratrum Borntrger, Regiomonti, 1829.

Abstract- In this article, the author provide elementry proof for two (pre-sumably new) theta function identities.
Keywords and Phrases: infinite series, theta function identities.

I. Introduction

Since Jacobi introduce the so-called theta functions, they have been extensively investigated and have many applications in diverse areas for example number theory, quantum physics, quadratic forms and elliptic functions (see the references here and references cited therein) . We recall the well known Jacobi triple product (see, e.g., [4]; see also 1, 2, 3, 5, 6, 7, 8, 9, 10] as follows:

$$\sum_{i=-\infty}^{\infty} y^i x^{i^2} = \prod_{i=1}^{\infty}(1-x^{2i})(1+yx^{2i-1})(1+y^{-1}x^{2i-1}) \qquad (|x|<1, y\neq 0) \tag{1}$$

We define the following three fundamental theta functions (see, e.g., [2, 3, 7]):

$$f(-x) = \sum_{n=-\infty}^{\infty} (-1)^n x^{\frac{n(3n-1)}{2}} = \prod_{i=1}^{\infty}(1-x^i) \tag{2}$$

$$\phi(x) = \sum_{n=-\infty}^{\infty} x^{n^2} = \prod_{i=1}^{\infty}(1+x^{2i+1})(1+x^{2i-1})(1-x^{2i}) \tag{3}$$

$$\psi(x) = \sum_{n=0}^{\infty} x^{\frac{n(n+1)}{2}} = \prod_{i=1}^{\infty} \frac{(1-x^{2i})}{(1-x^{2i-1})} \tag{4}$$

The main object of the present article is to prove two (presumably new) identities involving the three functions given in (2), (3) and (4) in elementary way.

II. The Main Results

In this section, we begin by expressing the functions $f(-x)$, $\phi(x)$ and $\psi(x)$ in rising powers of x as follows:

$$f(-x) = 1 + \sum_{n=1}^{\infty}(-1)^n\left(x^{\frac{n(3n-1)}{2}} + x^{\frac{n(3n+1)}{2}}\right)$$

$$= 1 - x - x^2 + x^5 + x^7 - x^{12} - x^{15} + x^{22} + x^{26} - \cdots ; \tag{5}$$

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$$\phi(x) = 1 + 2\sum_{n=1}^{\infty} x^{n^2} = 1 + 2x + 2x^4 + 2x^9 + 2x^{16} + \cdots ; \tag{6}$$

and

$$\psi(x) = 1 + \sum_{n=1}^{\infty} x^{\frac{n(n+1)}{2}} = 1 + x + x^3 + x^6 + x^{10} + x^{15} + \cdots . \tag{7}$$

Now the main results state in the following Theorem.

Theorem. *Each of the following relationships holds true:*

$$f(-x^3)[\psi(x) - x\psi(x^9)] = f(-x^6)\phi(-x^9) \tag{8}$$

and

$$[\phi(x^2) - \phi(x^{18})][\phi(x^6) - \phi(x^{54})][\psi(x^4) - x^4 \quad (x^{36})][\psi(x^{12}) - x^{12} \quad (x^{108})]$$
$$\psi$$
$$= x^5[\psi^2(x^{27})\{\psi(x) - x\psi(x^9)\}^2 - \psi^2(-x^{27})\{\psi(-x) + x\psi(-x^9)\}^2] \tag{9}$$

where the functions $f(-x)$, $\phi(x)$ and $\psi(x)$ are given by (2), (3), (4), respectively.

Proof. First of all, we shall prove our first identity (8). Let η_1 and ξ_1 denote the left-hand and the right-hand sides of the identity (8), respectively. Then, in order to compute the value for η_1, by using (5) [for $x \mapsto x^3$] and (7) [for $x \mapsto x^9$] respectively, we have:

$$\eta_1 = (1 - x^3 - x^6 + x^{15} + x^{21} - x^{36} - \psi \cdot)[(1 + x\psi + +x^3 + x^6 + +x^{10}$$
$$+x^{15} + \cdots) - x(1 + x^9 + x^{27} + x^{54} + x^{90} + x^{135} + \cdots)]$$

which, after multiplication and further simplification, yields

$$\eta_1 = 1 - x^6 - 2x^9 - x^{12} + 2x^{15} + 2x^{21} + x^{30} + 2x^{36} - -2x^{39} - x^{42}$$
$$-2x^{48} - 2x^{51} + 2x^{66} - x^{72} + 2x^{78} + 2x^{87} - x^{90} + +2x^{93} + 2x^{99}$$
$$-2x^{108} - 2x^{111} - 2x^{123} - \cdots \tag{10}$$

In a similar way, we can compute the value for ξ_1, by applying (5) [for $x \mapsto x^6$] and (6) [for $x \mapsto -x^9$] as follows:

$$\xi_1 = (1 - x^6 - x^{12} + x^{30} + x^{42} - x^{72} - x^{90} + x^{132} + \cdots)\times$$
$$\times(1 - 2x^9 + 2x^{36} - 2x^{81} + 2x^{144} - 2x^{225} + 2x^{324} - \cdots)$$

after simplifications and using little algebra, we obtain:

$$\xi_1 = 1 - x^6 - 2x^9 - x^{12} + 2x^{15} + 2x^{21} + x^{30} + 2x^{36} - 2x^{39} - x^{42}$$
$$-2x^{48} - 2x^{51} + 2x^{66} - x^{72} + 2x^{78} + 2x^{87} - x^{90} + 2x^{93} + 2x^{99} -$$
$$-2x^{108} - 2x^{111} - 2x^{123} - 2x^{126} + \cdots \tag{11}$$

By comparing equations (10) and (11), we readily arrive at the identity (8).

We next prove the second identity (9). Let η_2 and ξ_2 denote the left-hand

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and the right-hand sides of the identity (9), respectively. Then, in order to compute the value for η_2, by using (6) [for $x \mapsto x^2$, $x \mapsto x^6$, $x \mapsto x^{18}$ and $x \mapsto x^{54}$] and (7) [for $x \mapsto x^4$, $x \mapsto x^{12}$, $x \mapsto x^{36}$ and $x \mapsto x^{108}$], as follows:

$$\eta_2 = [2x^2 + 2x^8 + 2x^{32} + 2x^{50} + 2x^{98} + 2x^{128} + 2x^{200} + \cdots][2x^6 + 2x^{24} + \\ +2x^{96} + 2x^{150} + 2x^{294} + \cdots][1 + x^{12} + x^{24} + x^{60} + x^{84} + x^{144} + \\ +x^{180} + \cdots][1 + x^{36} + x^{72} + x^{108} + x^{252} + \cdots]$$

which, after simplification and by using algebraic manipulation, yields

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$$\eta_2 = 4x^8[1 + x^6 + x^{12} + 2x^{18} + 2x^{24} + 3x^{30} + 2x^{36} + 3x^{42} + 4x^{48} + 3x^{54} + \\ +5x^{60} + 5x^{66} + 4x^{72} + 5x^{78} + 6x^{84} + 6x^{90} + 5x^{96} + 5x^{102} + x^{104} + \cdots] \quad (12)$$

Now we have to compute the value ξ_2, using (7) [for $x \mapsto -x^{27}$, $x \mapsto -x^9$, $x \mapsto -x$, $x \mapsto x^9$ and $x \mapsto x^{27}$] as follows:

$$\xi_2 = x^5[(1 + 2x^3 + 3x^6 + 2x^9 + x^{12} + 2x^{15} + 2x^{18} + 4x^{21} + 2x^{24} + 4x^{27} + \\ +5x^{30} + 6x^{33} + 8x^{36} + 4x^{39} + 7x^{42} + 6x^{45} + 10x^{48} + 8x^{51} + x^{54} + \\ +6x^{57} + \cdots) - (1 - 2x^3 + 3x^6 - 2x^9 + x^{12} - 2x^{15} + 2x^{18} - 4x^{21} + \\ +2x^{24} - 4x^{27} + 5x^{30} - 6x^{33} + 8x^{36} - 4x^{39} + 7x^{42} - 6x^{45} + 10x^{48} - \\ -8x^{51} + x^{54} - 6x^{57} + \cdots)]$$

after simplifications, we have:

$$\xi_2 = 4x^8[1 + x^6 + x^{12} + 2x^{18} + 2x^{24} + 3x^{30} + 2x^{36} + 3x^{42} + 4x^{48} + 3x^{54} + \\ +5x^{60} + 5x^{66} + 4x^{72} + 5x^{78} + 6x^{84} + 6x^{90} + 5x^{96} + 5x^{102} + x^{104} + \cdots] \quad (13)$$

The identity (9) now follows upon comparing the equations (12) and (13).

References Références Referencias

1. Jorge Luis Cimadevilla Villacorto and M. P. Chaudhary, *On Certain theta function identities,* submitted to Ramanujan J.
2. H. Schroter, De Aequationibus Modularibus, Disserrtatio Inaugurolis, Al bertino, Litterarum University, Regiomonti, 1854.
3. M. D. Hirschorn, *A simple proof of an identity of Ramanujan,* J. Austral. Math. Soc. Ser. A, 44(1)(1988), 42-45.
4. G. J. Jacobi, *Fundamental Nova Theoriae Functionum Ellipticarum,* Sumpti bus Fratrum Borntrger, Regiomonti, 1829.
5. M. P. Chaudhary, G. A. Salilew and J. Choi, *Two theta function identities,* Far East J. Math. Sci. 101 (2017), 1833-1837.
6. M. P. Chaudhary, G. A. Salilew and J. Choi, *Two identities involving theta functions,* East Asian Math. J. 33 (2017), 291-293.
7. H. M. Srivastava, M. P. Chaudhary and Sangeeta Chaudhary, *Some theta-function identities related to Jacobis triple-product identity,* Eur. J. Pure Appl. Math, 11 (1) (2018), 1-9.
8. S. Cooper, *The quinntuble product identity,* Int. J. Number Theory, (2)(1)(2006), 115-161.
9. Z. Cao, *Integer matrix exact covering sytems and product identities for theta functions,* Inter. Math. Res. Notices. 19(2011), 4471-4514.
10. M. P. Chaudhary and Junesang Choi, *Note on modular relations for Roger Ramanujan type identities and representation for Jacobi identities,* East Asian Math. J. (31)(5)(2015), 659-665.

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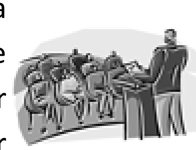

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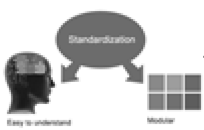

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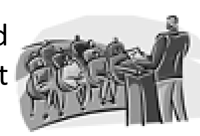

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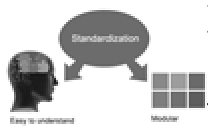

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- The professional accredited with Fellow honor, is entitled to various benefits viz. name, fame, honor, regular flow of income, secured bright future, social status etc.

- In addition to above, if one is single author, then entitled to 40% discount on publishing research paper and can get 10%discount if one is co-author or main author among group of authors.
- The Fellow can organize symposium/seminar/conference on behalf of Global Journals Incorporation (USA) and he/she can also attend the same organized by other institutes on behalf of Global Journals.
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- • This individual has learned the basic methods of applying those concepts and techniques to common challenging situations. This individual has further demonstrated an in–depth understanding of the application of suitable techniques to a particular area of research practice.

Note :

"

- In future, if the board feels the necessity to change any board member, the same can be done with the consent of the chairperson along with anyone board member without our approval.
- In case, the chairperson needs to be replaced then consent of 2/3rd board members are required and they are also required to jointly pass the resolution copy of which should be sent to us. In such case, it will be compulsory to obtain our approval before replacement.
- In case of "Difference of Opinion [if any]" among the Board members, our decision will be final and binding to everyone.

"

Preferred Author Guidelines

We accept the manuscript submissions in any standard (generic) format.

We typeset manuscripts using advanced typesetting tools like Adobe In Design, CorelDraw, TeXnicCenter, and TeXStudio. We usually recommend authors submit their research using any standard format they are comfortable with, and let Global Journals do the rest.

Alternatively, you can download our basic template from https://globaljournals.org/Template.zip

Authors should submit their complete paper/article, including text illustrations, graphics, conclusions, artwork, and tables. Authors who are not able to submit manuscript using the form above can email the manuscript department at submit@globaljournals.org or get in touch with chiefeditor@globaljournals.org if they wish to send the abstract before submission.

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2. Authors must accept the privacy policy, terms, and conditions of Global Journals.
3. Ensure corresponding author's email address and postal address are accurate and reachable.
4. Manuscript to be submitted must include keywords, an abstract, a paper title, co-author(s') names and details (email address, name, phone number, and institution), figures and illustrations in vector format including appropriate captions, tables, including titles and footnotes, a conclusion, results, acknowledgments and references.
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6. Proper permissions must be acquired for the use of any copyrighted material.
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- Findings
- Writings
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- Printed material
- Graphic representations
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The corresponding author should mention the name and complete details of all co-authors during submission and in manuscript. We support addition, rearrangement, manipulation, and deletions in authors list till the early view publication of the journal. We expect that corresponding author will notify all co-authors of submission. We follow COPE guidelines for changes in authorship.

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Unless specified in the notification, the Editorial Board's decision on publication of the paper is final and cannot be appealed before making the major change in the manuscript.

Acknowledgments

Contributors to the research other than authors credited should be mentioned in Acknowledgments. The source of funding for the research can be included. Suppliers of resources may be mentioned along with their addresses.

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Preparing your Manuscript

Authors can submit papers and articles in an acceptable file format: MS Word (doc, docx), LaTeX (.tex, .zip or .rar including all of your files), Adobe PDF (.pdf), rich text format (.rtf), simple text document (.txt), Open Document Text (.odt), and Apple Pages (.pages). Our professional layout editors will format the entire paper according to our official guidelines. This is one of the highlights of publishing with Global Journals—authors should not be concerned about the formatting of their paper. Global Journals accepts articles and manuscripts in every major language, be it Spanish, Chinese, Japanese, Portuguese, Russian, French, German, Dutch, Italian, Greek, or any other national language, but the title, subtitle, and abstract should be in English. This will facilitate indexing and the pre-peer review process.

The following is the official style and template developed for publication of a research paper. Authors are not required to follow this style during the submission of the paper. It is just for reference purposes.

Manuscript Style Instruction (Optional)

- Microsoft Word Document Setting Instructions.
- Font type of all text should be Swis721 Lt BT.
- Page size: 8.27" x 11'", left margin: 0.65, right margin: 0.65, bottom margin: 0.75.
- Paper title should be in one column of font size 24.
- Author name in font size of 11 in one column.
- Abstract: font size 9 with the word "Abstract" in bold italics.
- Main text: font size 10 with two justified columns.
- Two columns with equal column width of 3.38 and spacing of 0.2.
- First character must be three lines drop-capped.
- The paragraph before spacing of 1 pt and after of 0 pt.
- Line spacing of 1 pt.
- Large images must be in one column.
- The names of first main headings (Heading 1) must be in Roman font, capital letters, and font size of 10.
- The names of second main headings (Heading 2) must not include numbers and must be in italics with a font size of 10.

Structure and Format of Manuscript

The recommended size of an original research paper is under 15,000 words and review papers under 7,000 words. Research articles should be less than 10,000 words. Research papers are usually longer than review papers. Review papers are reports of significant research (typically less than 7,000 words, including tables, figures, and references)

A research paper must include:

a) A title which should be relevant to the theme of the paper.
b) A summary, known as an abstract (less than 150 words), containing the major results and conclusions.
c) Up to 10 keywords that precisely identify the paper's subject, purpose, and focus.
d) An introduction, giving fundamental background objectives.
e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition, sources of information must be given, and numerical methods must be specified by reference.
f) Results which should be presented concisely by well-designed tables and figures.
g) Suitable statistical data should also be given.
h) All data must have been gathered with attention to numerical detail in the planning stage.

Design has been recognized to be essential to experiments for a considerable time, and the editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned unrefereed.

i) Discussion should cover implications and consequences and not just recapitulate the results; conclusions should also be summarized.
j) There should be brief acknowledgments.
k) There ought to be references in the conventional format. Global Journals recommends APA format.

Authors should carefully consider the preparation of papers to ensure that they communicate effectively. Papers are much more likely to be accepted if they are carefully designed and laid out, contain few or no errors, are summarizing, and follow instructions. They will also be published with much fewer delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and suggestions to improve brevity.

Format Structure

It is necessary that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

All manuscripts submitted to Global Journals should include:

Title

The title page must carry an informative title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) where the work was carried out.

Author details

The full postal address of any related author(s) must be specified.

Abstract

The abstract is the foundation of the research paper. It should be clear and concise and must contain the objective of the paper and inferences drawn. It is advised to not include big mathematical equations or complicated jargon.

Many researchers searching for information online will use search engines such as Google, Yahoo or others. By optimizing your paper for search engines, you will amplify the chance of someone finding it. In turn, this will make it more likely to be viewed and cited in further works. Global Journals has compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Keywords

A major lynchpin of research work for the writing of research papers is the keyword search, which one will employ to find both library and internet resources. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining, and indexing.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy: planning of a list of possible keywords and phrases to try.

Choice of the main keywords is the first tool of writing a research paper. Research paper writing is an art. Keyword search should be as strategic as possible.

One should start brainstorming lists of potential keywords before even beginning searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in a research paper?" Then consider synonyms for the important words.

It may take the discovery of only one important paper to steer in the right keyword direction because, in most databases, the keywords under which a research paper is abstracted are listed with the paper.

Numerical Methods

Numerical methods used should be transparent and, where appropriate, supported by references.

Abbreviations

Authors must list all the abbreviations used in the paper at the end of the paper or in a separate table before using them.

Formulas and equations

Authors are advised to submit any mathematical equation using either MathJax, KaTeX, or LaTeX, or in a very high-quality image.

Tables, Figures, and Figure Legends

Tables: Tables should be cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g., Table 4, a self-explanatory caption, and be on a separate sheet. Authors must submit tables in an editable format and not as images. References to these tables (if any) must be mentioned accurately.

Figures

Figures are supposed to be submitted as separate files. Always include a citation in the text for each figure using Arabic numbers, e.g., Fig. 4. Artwork must be submitted online in vector electronic form or by emailing it.

Preparation of Eletronic Figures for Publication

Although low-quality images are sufficient for review purposes, print publication requires high-quality images to prevent the final product being blurred or fuzzy. Submit (possibly by e-mail) EPS (line art) or TIFF (halftone/ photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Avoid using pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings). Please give the data for figures in black and white or submit a Color Work Agreement form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution at final image size ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs): >350 dpi; figures containing both halftone and line images: >650 dpi.

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Tips for Writing a Good Quality Science Frontier Research Paper

Techniques for writing a good quality Science Frontier Research paper:

1. Choosing the topic: In most cases, the topic is selected by the interests of the author, but it can also be suggested by the guides. You can have several topics, and then judge which you are most comfortable with. This may be done by asking several questions of yourself, like "Will I be able to carry out a search in this area? Will I find all necessary resources to accomplish the search? Will I be able to find all information in this field area?" If the answer to this type of question is "yes," then you ought to choose that topic. In most cases, you may have to conduct surveys and visit several places. Also, you might have to do a lot of work to find all the rises and falls of the various data on that subject. Sometimes, detailed information plays a vital role, instead of short information. Evaluators are human: The first thing to remember is that evaluators are also human beings. They are not only meant for rejecting a paper. They are here to evaluate your paper. So present your best aspect.

2. Think like evaluators: If you are in confusion or getting demotivated because your paper may not be accepted by the evaluators, then think, and try to evaluate your paper like an evaluator. Try to understand what an evaluator wants in your research paper, and you will automatically have your answer. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

3. Ask your guides: If you are having any difficulty with your research, then do not hesitate to share your difficulty with your guide (if you have one). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work, then ask your supervisor to help you with an alternative. He or she might also provide you with a list of essential readings.

4. Use of computer is recommended: As you are doing research in the field of science frontier then this point is quite obvious. Use right software: Always use good quality software packages. If you are not capable of judging good software, then you can lose the quality of your paper unknowingly. There are various programs available to help you which you can get through the internet.

5. Use the internet for help: An excellent start for your paper is using Google. It is a wondrous search engine, where you can have your doubts resolved. You may also read some answers for the frequent question of how to write your research paper or find a model research paper. You can download books from the internet. If you have all the required books, place importance on reading, selecting, and analyzing the specified information. Then sketch out your research paper. Use big pictures: You may use encyclopedias like Wikipedia to get pictures with the best resolution. At Global Journals, you should strictly follow here.

6. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right? It is a good habit which helps to not lose your continuity. You should always use bookmarks while searching on the internet also, which will make your search easier.

7. Revise what you wrote: When you write anything, always read it, summarize it, and then finalize it.

8. Make every effort: Make every effort to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in the introduction—what is the need for a particular research paper. Polish your work with good writing skills and always give an evaluator what he wants. Make backups: When you are going to do any important thing like making a research paper, you should always have backup copies of it either on your computer or on paper. This protects you from losing any portion of your important data.

9. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several unnecessary diagrams will degrade the quality of your paper by creating a hodgepodge. So always try to include diagrams which were made by you to improve the readability of your paper. Use of direct quotes: When you do research relevant to literature, history, or current affairs, then use of quotes becomes essential, but if the study is relevant to science, use of quotes is not preferable.

10. Use proper verb tense: Use proper verb tenses in your paper. Use past tense to present those events that have happened. Use present tense to indicate events that are going on. Use future tense to indicate events that will happen in the future. Use of wrong tenses will confuse the evaluator. Avoid sentences that are incomplete.

11. Pick a good study spot: Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

12. Know what you know: Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

13. Use good grammar: Always use good grammar and words that will have a positive impact on the evaluator; use of good vocabulary does not mean using tough words which the evaluator has to find in a dictionary. Do not fragment sentences. Eliminate one-word sentences. Do not ever use a big word when a smaller one would suffice.

Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

14. Arrangement of information: Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

15. Never start at the last minute: Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

16. Multitasking in research is not good: Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

17. Never copy others' work: Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

18. Go to seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

19. Refresh your mind after intervals: Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.

20. Think technically: Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

21. Adding unnecessary information: Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

22. Report concluded results: Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

23. Upon conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

The introduction: This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear: Adhere to recommended page limits.

Mistakes to avoid:

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

Title page:

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

Abstract: This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

Approach:

- Single section and succinct.
- An outline of the job done is always written in past tense.
- Concentrate on shortening results—limit background information to a verdict or two.
- Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

Introduction:

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.

The following approach can create a valuable beginning:

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

Approach:

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

Procedures (methods and materials):

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

Materials may be reported in part of a section or else they may be recognized along with your measures.

Methods:

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

What to stay away from:

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

Figures and tables:

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

Discussion:

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- Give details of all of your remarks as much as possible, focusing on mechanisms.
- Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

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