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# Mathematics and Decision Science

Highlights

Ground Handling Operations

General Class of Polynomials

Management: An Agent-Based

An Unified Study of Some Multiple

Discovering Thoughts, Inventing Future

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# Certain Fractional Derivative Formulae Involving the Product of a General Class of Polynomials and the Multivariable Gimel-Function

By Frédéric Ayant

*Abstract-* In the present paper, we obtain three unified fractional derivative formulae. The first involves the product of a general class of polynomials and the multivariable Gimel-function. The second involves the product of a general class of polynomials and two multivariable Gimel-functions and has been obtained with the help of the generalized Leibniz rule for fractional derivatives. The last fractional derivative formulae also involves the product of a general class of polynomials and the multivariable Gimel-function but it is obtained by the application of the first fractional derivative formulae twice and, it involve two independents variables instead of one. The polynomials and the functions involved in all our fractional derivative formulae as well as their arguments which are of the type $x^{\rho}\prod_{i=1}^{s}(x^{t_j}+\alpha_j)^{\sigma_j}$ The formulae are the very general character and thus making them useful in applications. In the end, we shall give a particular case.

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CERTAINFRACTIONALDERIVATIVEFORMULAEINVOLVINGTHEPRODUCTOFAGENERALCLASSOFPOLYNOMIALSANDTHEMULTIVARIABLEGIMELFUNCTION

*Strictly as per the compliance and regulations of:*

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## I. Introduction And Preliminaries

Throughout this paper, let $\mathbb{C}, \mathbb{R}$ and $\mathbb{N}$ be set of complex numbers, real numbers and positive integers respectively. Also $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. We define a generalized transcendental function of several complex variables.

$$\gimel(z_1,\cdots,z_r) = \gimel^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)}}_{p_{i_2},q_{i_2},\tau_{i_2};R_2;p_{i_3},q_{i_3},\tau_{i_3};R_3;\cdots;p_{i_r},q_{i_r},\tau_{i_r}:R_r:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r \end{array}\right|$$

$$[(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}};[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$
$$[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}};$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)};A_{rj})_{1,n_r}],$$
$$[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots; \quad \ldots$$

$$[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)};A_{rji_r})_{n_r+1,p_r}]: \ [(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}]$$
$$[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)};B_{rji_r})_{1,q_r}]: \ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)};D_j^{(1)})_{1,m^{(1)}}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})_{m^{(1)}+1,q_i^{(1)}}]$$

$$\left.\begin{array}{l};\cdots;[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})_{m^{(r)}+1,p_i^{(r)}}] \\ ;\cdots;[(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,n^{(r)}}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_{ji^{(r)}}^{(r)})_{n^{(r)}+1,q_i^{(r)}}]\end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega = \sqrt{-1}$

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$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{n_2}\Gamma^{A_{2j}}(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k)}{\sum_{i_2=1}^{R_2}[\tau_{i_2}\prod_{j=n_2+1}^{p_{i_2}}\Gamma^{A_{2ji_2}}(a_{2ji_2}-\sum_{k=1}^{2}\alpha_{2ji_2}^{(k)}s_k)\prod_{j=1}^{q_{i_2}}\Gamma^{B_{2ji_2}}(1-b_{2ji2}+\sum_{k=1}^{2}\beta_{2ji2}^{(k)}s_k)]}$$

$$\frac{\prod_{j=1}^{n_3}\Gamma^{A_{3j}}(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k)}{\sum_{i_3=1}^{R_3}[\tau_{i_3}\prod_{j=n_3+1}^{p_{i_3}}\Gamma^{A_{3ji_3}}(a_{3ji_3}-\sum_{k=1}^{3}\alpha_{3ji_3}^{(k)}s_k)\prod_{j=1}^{q_{i_3}}\Gamma^{B_{3ji_3}}(1-b_{3ji3}+\sum_{k=1}^{3}\beta_{3ji3}^{(k)}s_k)]}$$

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$$\frac{\prod_{j=1}^{n_r}\Gamma^{A_{rj}}(1-a_{rj}+\sum_{k=1}^{r}\alpha_{rj}^{(k)}s_k)}{\sum_{i_r=1}^{R_r}[\tau_{i_r}\prod_{j=n_r+1}^{p_{i_r}}\Gamma^{A_{rji_r}}(a_{rji_r}-\sum_{k=1}^{r}\alpha_{rji_r}^{(k)}s_k)\prod_{j=1}^{q_{i_r}}\Gamma^{B_{rji_r}}(1-b_{rjir}+\sum_{k=1}^{r}\beta_{rjir}^{(k)}s_k)]} \tag{1.2}$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m^{(k)}}\Gamma^{D_j^{(k)}}(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n^{(k)}}\Gamma^{C_j^{(k)}}(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m^{(k)}+1}^{q_{i^{(k)}}}\Gamma^{D_{ji^{(k)}}^{(k)}}(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n^{(k)}+1}^{p_{i^{(k)}}}\Gamma^{C_{ji^{(k)}}^{(k)}}(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

For more details, see Ayant [2].

The contour $L_k$ is in the $s_k(k=1,\cdots,r)$- plane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where $\sigma$ if is a real number with loop, if necessary to ensure that the poles of $\Gamma^{A_{2j}}\left(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k\right)(j=1,\cdots,n_2),\Gamma^{A_{3j}}\left(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k\right)$ $(j=1,\cdots,n_3),\cdots,\Gamma^{A_{rj}}\left(1-a_{rj}+\sum_{i=1}^{r}\alpha_{rj}^{(i)}\right)(j=1,\cdots,n_r),\quad \Gamma^{C_j^{(k)}}\left(1-c_j^{(k)}+\gamma_j^{(k)}s_k\right)(j=1,\cdots,n^{(k)})(k=1,\cdots,r)$to the right of the contour $L_k$ and the poles of $\Gamma^{D_j^{(k)}}\left(d_j^{(k)}-\delta_j^{(k)}s_k\right)(j=1,\cdots,m^{(k)})(k=1,\cdots,r)$ lie to the left of the contour $L_k$. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained of the corresponding conditions for multivariable H-function given by as :

$|arg(z_k)|<\frac{1}{2}A_i^{(k)}\pi$ where

$$A_i^{(k)}=\sum_{j=1}^{m^{(k)}}D_j^{(k)}\delta_j^{(k)}+\sum_{j=1}^{n^{(k)}}C_j^{(k)}\gamma_j^{(k)}-\tau_{i^{(k)}}\left(\sum_{j=m^{(k)}+1}^{q_i^{(k)}}D_{ji^{(k)}}^{(k)}\delta_{ji^{(k)}}^{(k)}+\sum_{j=n^{(k)}+1}^{p_i^{(k)}}C_{ji^{(k)}}^{(k)}\gamma_{ji^{(k)}}^{(k)}\right)$$

$$-\tau_{i_2}\left(\sum_{j=n_2+1}^{p_{i_2}}A_{2ji_2}\alpha_{2ji_2}^{(k)}+\sum_{j=1}^{q_{i_2}}B_{2ji_2}\beta_{2ji_2}^{(k)}\right)-\cdots-\tau_{i_r}\left(\sum_{j=n_r+1}^{p_{i_r}}A_{rji_r}\alpha_{rji_r}^{(k)}+\sum_{j=1}^{q_{i_r}}B_{rji_r}\beta_{rji_r}^{(k)}\right) \tag{1.4}$$

Following the lines of Braaksma ([3] p. 278), we may establish the asymptotic expansion in the following convenient form :

$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,),max(\,|z_1|,\cdots,|z_r|\,)\to 0$

$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,),min(\,|z_1|,\cdots,|z_r|\,)\to\infty$ where $i=1,\cdots,r:$

$$\alpha_i=\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]\text{ and }\beta_i=\max_{1\leqslant j\leqslant n^{(i)}}Re\left[C_j^{(i)}\left(\frac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right)\right]$$

Ref

2. F. Ayant, An expansion formula for multivariable Gimel-function involving generalized Legendre Associated function, International Journal of Mathematics Trends and Technology (IJMTT), 56(4) (2018), 223-228.

**Remark 1.**

If $n_2 = \cdots = n_{r-1} = p_{i_2} = q_{i_2} = \cdots = p_{i_{r-1}} = q_{i_{r-1}} = 0$ and $A_{2j} = A_{2ji_2} = B_{2ji_2} = \cdots = A_{rj} = A_{rji_r} = B_{rji_r} = 1$ $A_{rj} = A_{rji_r} = B_{rji_r} = 1$, then the multivariable Gimel-function reduces in the multivariable Aleph- function defined by Ayant [1].

**Remark 2.**

If $n_2 = \cdots = n_r = p_{i_2} = q_{i_2} = \cdots = p_{i_r} = q_{i_r} = 0$ and $\tau_{i_2} = \cdots = \tau_{i_r} = \tau_{i^{(1)}} = \cdots = \tau_{i^{(r)}} = R_2 = \cdots = R_r = R^{(1)} = \cdots = R^{(r)} = 1$, then the multivariable Gimel-function reduces in a multivariable I-function defined by Prathima et al. [9].

**Remark 3.**

If $A_{2j} = A_{2ji_2} = B_{2ji_2} = \cdots = A_{rj} = A_{rji_r} = B_{rji_r} = 1$ and $\tau_{i_2} = \cdots = \tau_{i_r} = \tau_{i^{(1)}} = \cdots = \tau_{i^{(r)}} = R_2 = \cdots = R_r = R^{(1)} = \cdots = R^{(r)} = 1$, then the generalized multivariable Gimel-function reduces in multivariable I-function defined by Prasad [8].

**Remark 4.**

If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the multivariable H-function defined by Srivastava and Panda [15,16].

Srivastava ([14],p. 1, Eq. 1) has defined the general class of polynomials

$$S_N^M(x) = \sum_{K=0}^{[N/M]} \frac{(-N)_{MK}}{K!} A_{N,K} x^K \tag{1.5}$$

On suitably specializing the coefficients $A_{N,K}$, $S_N^M(x)$ yields some of known polynomials, these include the Jacobi polynomials, Laguerre polynomials, and others polynomials ([17],p. 158-161).

We shall define the fractional integrals and derivatives of a function $f(x)$ ([11], p. 528-529), see also [5-7] as follows :

Let $\alpha, \beta$ and $\gamma$ be complex numbers. The fractional integral $(Re(\alpha) > 0)$ and derivative $(Re(\alpha) < 0)$ of a function $f(x)$ defined on $(0, \infty)$ is given by

$$I_{0,x}^{\alpha,\beta,\gamma} f(x) = \left[ \begin{array}{c} \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} F\left(\alpha+\beta, -\gamma; \alpha; 1-\frac{t}{x}\right) f(t) \mathrm{d}t (Re(\alpha) > 0). \\ . \\ \frac{d^q}{dx^q} \mathrm{I}_{0,x}^{\alpha+q,\beta-q,\gamma-q} f(x), (Re(\alpha) \leqslant 0, 0 < Re(\alpha) + q \leqslant 1, (q = 1, 2, 3, \cdots.), \end{array} \right. \tag{1.6}$$

where $F$ is the Gauss hypergeometric serie.

The operator $I$ includes both the Riemann-Liouville and the Erdélyi-Kober fractional operators as follows :

The Riemann-Liouville operator

$$R_{0,x}^{\alpha} f(x) = \left[ \begin{array}{c} \mathrm{I}_{0,x}^{\alpha,-\alpha,\gamma} f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t) \mathrm{d}t (Re(\alpha) > 0). \\ . \\ \frac{d^q}{dx^q} \mathrm{R}_{0,x}^{\alpha+q} f(x), (Re(\alpha) \leqslant 0, 0 < Re(\alpha) + q \leqslant 1, (q = 1, 2, 3, \cdots.), \end{array} \right. \tag{1.7}$$

The Erdélyi-Kober operators

$$E_{0,x}^{\alpha,\gamma} f(x) = I_{0,x}^{\alpha,0,\gamma} f(x) = \frac{x^{-\alpha-\gamma}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} t^{\gamma} f(t) \mathrm{d}t, (Re(\alpha) > 0). \tag{1.8}$$

## Main formulae.

**Theorem 1.**

$$I_{0,x}^{\alpha,\beta,\gamma} \left\{ x^{\rho} \prod_{i=1}^{s} (x^{t_i} + \alpha_i)^{\sigma_i} \prod_{j=1}^{t} S_{N_j}^{M_j} \left[ e_j x^{\lambda_j} \prod_{i=1}^{s} (x^{t_i} + \alpha_i)^{\eta_i^{(j)}} \right] \gimel \left( z_1 x^{u_1} \prod_{i=1}^{s} (x^{t_i} + \alpha_i)^{-v_i'}, \cdots, z_r x^{u_r} \prod_{i=1}^{s} (x^{t_i} + \alpha_i)^{-v_i^{(r)}} \right) \right\}$$

$$= \alpha_1^{\sigma_1} \cdots \alpha_r^{\sigma_r} x^{\rho-\beta} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_t=0}^{[N_t/M_t]} \frac{(-N_1)_{M_1 K_1} \cdots (-N_t)_{M_t K_t}}{K_1! \cdots K_t!} A_{N_1,K_1} \cdots A_{N_t,K_t}$$

Ref

1. F. Ayant, An integral associated with the Aleph-functions of several variables. International Journal of Mathematics Trends and Technology (IJMTT), 31(3) (2016), 142-154.

$$e_1^{K_1}\cdots e_t^{K_t}\alpha_1^{\sum_{j=1}^t\eta_1^{(j)}K_j}\cdots\alpha_s^{\sum_{j=1}^t\eta_s^{(j)}K_j}x^{\sum_{j=1}^t\lambda_jK_j}$$

$$\gimel^{U;0,s+2+n_r:V}_{X;s+2+p_{i_r},s+2+q_{i_r},\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c} \mathrm{z}_1\alpha_1^{-v_1'}\cdots\alpha_s^{-v_s'}x^{u_1} & \mathbb{A};\mathrm{A}_1,\mathbf{A}:A \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r\alpha_1^{-v_1^{(r)}}\cdots\alpha_s^{-v_s^{(r)}}x^{u_r} & \cdot \\ \alpha_1^{-1}x^{t_1} & \cdot \\ \cdot & \cdot \\ \cdot & \mathbb{B};\mathbf{B},\mathrm{B}_1:B;\underbrace{(0,1;1);\cdots;(0,1;1)}_{s} \\ \alpha_s^{-1}x^{t_s} & \end{array}\right) \tag{2.1}$$

Notes

where

$$\mathbb{A}=[(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}},[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{(r-1)j};\alpha_{(r-1)j}^{(1)},\cdots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})_{1,n_{r-1}}],$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}};\alpha_{(r-1)ji_{r-1}}^{(1)},\cdots,\alpha_{(r-1)ji_{r-1}}^{(r-1)};A_{(r-1)ji_{r-1}})_{n_{r-1}+1,p_{i_{r-1}}}] \tag{2.2}$$

$$A_1=\left(-\rho-\sum_{j=1}^t\lambda_jK_j;u_1,\cdots,u_r,t_1,\cdots,t_s;1\right),\left(-\beta-\gamma-\rho-\sum_{j=1}^t\lambda_jK_j;u_1,\cdots,u_r,t_1,\cdots,t_s;1\right),$$

$$\left(1+\sigma_1+\sum_{j=1}^t\eta_1^{(i)}K_j;v_1',\cdots,v_1^{(r)},1,\underbrace{0,\cdots,0}_{s-1};1\right),\cdots,\left(1+\sigma_s+\sum_{j=1}^t\eta_s^{(i)}K_j;v_s',\cdots,v_s^{(r)},\underbrace{0,\cdots,0}_{s-1},1;1\right)$$

$$\mathbf{A}=[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)},\underbrace{0,\cdots,0}_{s};A_{rj})_{1,n_r}],[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{s};A_{rji_r})_{\mathfrak{n}+1,p_{i_r}}] \tag{2.4}$$

$$A=[(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}];\cdots;$$

$$[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})_{m^{(r)}+1,p_i^{(r)}}] \tag{2.5}$$

$$\mathbb{B}=[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}},[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}};\beta_{(r-1)ji_{r-1}}^{(1)},\cdots,\beta_{(r-1)ji_{r-1}}^{(r-1)};B_{(r-1)ji_{r-1}})_{1,q_{i_{r-1}}}] \tag{2.6}$$

$$\mathbf{B}=[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{s};B_{rji_r})_{1,q_{i_r}}] \tag{2.7}$$

$$B_1=\left(\beta-\rho-\sum_{j=1}^t\lambda_jK_j;u_1,\cdots,u_r,t_1,\cdots,t_s;1\right),\left(-\alpha-\gamma-\rho-\sum_{j=1}^t\lambda_jK_j;u_1,\cdots,u_r,t_1,\cdots,t_s;1\right),$$

$$\left(1+\sigma_1+\sum_{j=1}^t\eta_1^{(i)}K_j;v_1',\cdots,v_1^{(r)},\underbrace{0,\cdots,0}_{s};1\right),\cdots,\left(1+\sigma_s+\sum_{j=1}^t\eta_s^{(i)}K_j;v_s',\cdots,v_s^{(r)},\underbrace{0,\cdots,0}_{s};1\right) \tag{2.8}$$

$$\mathrm{B} = [(\mathrm{d}_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,m^{(1)}}], [\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}, \delta_{ji^{(1)}}^{(1)}; D_{ji^{(1)}}^{(1)})_{m^{(1)}+1,q_i^{(1)}}]; \cdots ;$$

$$[(\mathrm{d}_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,m^{(r)}}], [\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}, \delta_{ji^{(r)}}^{(r)}; D_{ji^{(r)}}^{(r)})_{m^{(r)}+1,q_i^{(r)}}] \tag{2.9}$$

Notes

$$U = 0, n_2; 0, n_3; \cdots ; 0, n_{r-1}; V = m^{(1)}, n^{(1)}; m^{(2)}, n^{(2)}; \cdots ; m^{(r)}, n^{(r)}; \underbrace{(1,0); \cdots ; (1,0)}_{s} \tag{2.10}$$

$$X = p_{i_2}, q_{i_2}, \tau_{i_2}; R_2; \cdots ; p_{i_{r-1}}, q_{i_{r-1}}, \tau_{i_{r-1}} : R_{r-1};$$

$$Y = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}; \cdots ; p_{i^{(r)}}, q_{i^{(r)}}; \tau_{i^{(r)}}; R^{(r)}; \underbrace{(0,1); \cdots ; (0,1)}_{s} \tag{2.11}$$

Provided

$$Re(\alpha) > 0, t_i, \lambda_j, \eta_i^{(j)}, u_k, v_i^{(k)} > 0; (i = 1, \cdots , s); (j = 1, \cdots , t); (k = 1, \cdots , r)$$

$$Re(\rho) + \sum_{i=1}^{r} u_i \min_{1 \leqslant j \leqslant m^{(i)}} Re \left[ D_j^{(i)} \left( \frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right] + 1 > 0$$

$$\left| arg \left( z_k x^{u_k} \prod_{i=1}^{s} (x^{t_i} + \alpha_i)^{-v_i'} \right) \right| < \frac{1}{2} \pi A_k^{(l)}$$

Proof

To prove the fractional derivative formula, we first express the product of a general class of polynomials occurring on. Its left-hand side in the series form with the help of (1.5), next we express the Gimel-function regarding Mellin-Barnes multiple integrals contour with the help of (1.1). Interchange the order of summations and $(s_1, \cdots , s_r)$- integrals and taking the fractional derivative operator inside (which is permissible under the conditions stated above ). And make simplifications. Next, we express the terms $(x^{t_1} + \alpha_1)^{\sigma_1 + \sum_{j=1}^{t} \eta_1^{(j)} k_j - \sum_{k=1}^{r} v_1^{(k)} s_k}, \cdots , (x^{t_s} + \alpha_1)^{\sigma_s + \sum_{j=1}^{t} \eta_s^{(j)} k_j - \sum_{k=1}^{r} v_s^{(k)} s_k}$ so obtained in terms of Mellin-Barnes multiple integrals contour, see ([14], p.18, Eq.(2.6.4) and p.10 Eq. (2.1.1)). Now interchanging the order of $(s_{r+1}, \cdots , s_{r+t})$ and $(s_1, \cdots , s_r)$- integrals (which is permissible under the conditions stated above ). And evaluating the $x$-integral thus obtained by using the following formula with the help of ([10], p.16, Lemma 1)

$$I_{0x}^{\alpha,\beta,\gamma}(x^{\lambda}) = \frac{\Gamma(1+\lambda)\Gamma(1-\beta+\gamma+\lambda)}{\Gamma(1-\beta+\lambda)\Gamma(1+\alpha+\gamma+\lambda)} x^{\lambda-\beta} \tag{2.12}$$

provided $Re(\lambda) > max[0, Re(\beta - \gamma)] - 1$

and we interpret the resulting multiple integrals contour with the help of (1.1) in term of the gimel-function of $(r+s)$-variables, after algebraic manipulations, we obtain the theorem.

**Theorem 2.**

$$I_{0,x}^{\alpha,\beta,\gamma} \left\{ x^{\rho} \prod_{i=1}^{s} (x^{t_i} + \alpha_i)^{\sigma_i} \prod_{j=1}^{t} S_{N_j}^{M_j} \left[ e_j x^{\lambda_j} \prod_{i=1}^{s} (x^{t_i} + \alpha_i)^{\eta_i^{(j)}} \right] \gimel \left( z_1 x^{u_1} \prod_{i=1}^{s} (x^{t_i} + \alpha_i)^{-v_i'}, \cdots , z_r x^{u_r} \prod_{i=1}^{s} (x^{t_i} + \alpha_i)^{-v_i^{(r)}} \right) \right.$$

$$\left. \gimel \left( z_{r+1} x^{u_{r+1}} \prod_{i=1}^{s-1} (x^{t_i} + \alpha_i)^{-v_i^{(r+1)}}, \cdots , z_{r+\tau} x^{u_{r+\tau}} \prod_{i=1}^{s-1} (x^{t_i} + \alpha_i)^{-v_i^{(r+\tau)}} \right) \right\}$$

$$= \alpha_1^{\sigma_1}\cdots\alpha_s^{\sigma_s}x^{\rho-\beta}\sum_{l=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\binom{-\beta}{l}\frac{(-N_1)_{M_1K_1}\cdots(-N_t)_{M_tK_t}}{K_1!\cdots K_s!}A_{N_1,K_1}\cdots A_{N_t,K_t}$$

$$e_1^{K_1}\cdots e_t^{K_t}\alpha_1^{\sum_{j=1}^t\eta_1^{(j)}K_j}\cdots\alpha_s^{\sum_{j=1}^t\eta_s^{(j)}K_j}x^{\sum_{j=1}^t\lambda_jK_j}e_1^{K_1}\cdots e_t^{K_t}$$

$$\gimel^{U,U';0,2s+3+n_r+n_{r+\tau},V,V'}_{X,X';p_{i_r}+p_{i_{r+\tau}}+2s+3,q_{i_r}+q_{i_{r+\tau}}+2s+3,\tau_{i_r},\tau_{i_{r+\tau}}:R_r,R_{r+\tau}:Y,Y'}\left(\begin{array}{c|c} \mathrm{z}_1\prod_{i=1}^s\alpha_i^{-v_i'}x^{u_1} & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r\prod_{i=1}^s\alpha_i^{-v_i^{(r)}}x^{u_r} & \\ \alpha_1^{-1}x^{t_1} & \mathbb{A},\mathbb{A}';\,\mathrm{A}_2,\mathbf{A},\mathbf{A}':A;A' \\ \cdot & \cdot \\ \cdot & \cdot \\ \alpha_s^{-1}x^{t_s} & \cdot \\ \mathrm{z}_{r+1}\prod_{i=1}^{s-1}\alpha_i^{-v_i^{(r+1)}}x^{u_{r+1}} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_{r+\tau}\prod_{i=1}^{s-1}\alpha_i^{-v_i^{(r+\tau)}}x^{u_{r+\tau}} & \mathbb{B},\mathbb{B}';\,\mathbf{B},\mathbf{B}',\mathrm{B}_2:B;B' \\ \cdot & \\ \cdot & \\ \alpha_1^{-1}x^{t_1} & \\ \cdot & \\ \cdot & \\ \alpha_{s-1}^{-1}x^{t_{s-1}} & \end{array}\right) \tag{2.13}$$

where

$\mathbb{A}$ $=[(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}},[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$

$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{(r-1)j};\alpha_{(r-1)j}^{(1)},\cdots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})_{1,n_{r-1}}],$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}};\alpha_{(r-1)ji_{r-1}}^{(1)},\cdots,\alpha_{(r-1)ji_{r-1}}^{(r-1)};A_{(r-1)ji_{r-1}})_{n_{r-1}+1,p_{i_{r-1}}}] \tag{2.14}$$

$\mathbb{A}$' $=[(\mathrm{a}_{(r+2)j};\alpha_{(r+2)j}^{(r+1)},\alpha_{(r+2)j}^{(r+2)};A_{j(r+2)})]_{1,n_{r+2}},[\tau_{i_r+2}(a_{(r+2)ji_{r+2}};\alpha_{(r+2)ji_{r+2}}^{(1)},\alpha_{(r+2)ji_{r+2}}^{(r+2)};A_{(r+2)ji_{r+2}})]_{n_{r+2},p_{i_{r+2}}},$

$[(\mathrm{a}_{(r+3)j};\alpha_{(r+3)j}^{(r+1)},\alpha_{(r+3)j}^{(r+2)},\alpha_{(r+3)j}^{(r+3)};A_{(r+3)j})]_{1,n_{r+3}},$

$[\tau_{i_{r+3}}(a_{(r+3)ji_{r+3}};\alpha_{(r+3ji_{r+3}}^{(r+1)},\alpha_{(r+3)ji_{r+3}}^{(r+2)},\alpha_{(r+3)ji_{r+3}}^{(r+3)};A_{(r+3)ji_{r+3}})]_{n_{r+3}+1,p_{i_{r+3}}};\cdots;$

$[(\mathrm{a}_{(r+\tau-1)j};\alpha_{(r+\tau-1)j}^{(r+1)},\cdots,\alpha_{(r+\tau-1)j}^{(r+\tau-1)};A_{(r+\tau-1)j})_{1,n_{r+\tau-1}}],$

$$[\tau_{i_{\tau+r-1}}(a_{(\tau+r-1)ji_{\tau+r-1}};\alpha_{(r-1)ji_{\tau+r-1}}^{(r+1)},\cdots,\alpha_{(r+\tau-1)ji_{r+\tau-1}}^{(r++\tau-1)};A_{(r+\tau-1)ji_{r+\tau-1}})_{n_{r+\tau-1}+1,p_{i_{r+\tau-1}}}] \tag{2.15}$$

$$A_2=\left(-\sum_{j=1}^t\lambda_jK_j;u_1,\cdots,u_r,t_1,\cdots,t_s,\underbrace{0,\cdots,0}_{\tau+s-1};1\right),\left(1-\gamma-\sum_{j=1}^t\lambda_jK_j;u_1,\cdots,u_r,t_1,\cdots,t_s,\underbrace{0,\cdots,0}_{\tau+s-1};1\right),$$

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$$\left(1-\sum_{j=1}^{t}\lambda_j K_j; u_1,\cdots,u_r,t_1,\cdots,t_s,\underbrace{0,\cdots,0}_{\tau+s-1};1\right),\left(-\alpha-\gamma-\sum_{j=1}^{t}\lambda_j K_j; u_1,\cdots,u_r,t_1,\cdots,t_s,\underbrace{0,\cdots,0}_{\tau+s-1};1\right),$$

$$\left(1+\sigma_1,\underbrace{0,\cdots,0}_{r+s};v_1^{(r+1)},\cdots,v_1^{(r+\tau)},1,\underbrace{0,\cdots,0}_{s-2};1\right),\left(1+\sigma_{s-1},\underbrace{0,\cdots,0}_{r+s};v_{s-1}^{(r+s)},\cdots,v_{s-1}^{(r+\tau)},1,\underbrace{0,\cdots,0}_{s-2};1\right),$$

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$$\left(1+\sum_{i=0}^{t}\eta_i K_i; v_1',\cdots,v_1^{(r)},1,\underbrace{0,\cdots,0}_{\tau+2s-2};1\right),\left(1+\sigma_s+\sum_{i=0}^{t}\eta_i K_i; v_1',\cdots,v_1^{(r)},\underbrace{0,\cdots,0}_{s-1},1,\underbrace{0,\cdots,0}_{\tau+s-1};1\right) \tag{2.16}$$

$$\mathbf{A}=[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)},\underbrace{0,\cdots,0}_{\tau+2s-1};A_{rj})_{1,n_r}],[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{\tau+2s-1};A_{rji_r})_{\mathfrak{n}+1,p_{i_r}}] \tag{2.17}$$

$$\mathbf{A}'=[(\mathrm{a}_{(r+\tau)j};\underbrace{0,\cdots,0}_{r+s},\alpha_{(r+\tau)j}^{(1)},\cdots,\alpha_{(r+\tau)j}^{(r+\tau)},\underbrace{0,\cdots,0}_{s-1};A_{(r+\tau)j})_{1,n_r}],$$

$$[\tau_{i_{r+\tau}}(a_{(r+\tau)ji_{r+\tau}};\underbrace{0,\cdots,0}_{r+s},\alpha_{(r+\tau)ji_{r+\tau}}^{(1)},\cdots,\alpha_{(r+\tau)ji_{r+\tau}}^{(r+\tau)},\underbrace{0,\cdots,0}_{s-1};A_{(r+\tau)ji_{r+\tau}})_{\mathfrak{n}+1,p_{i_{r+\tau}}}] \tag{2.18}$$

$$A=[(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}];\cdots;$$

$$[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})_{m^{(r)}+1,p_i^{(r)}}] \tag{2.19}$$

$$A'=[(\mathrm{c}_j^{(r+1)},\gamma_j^{(r+1)};C_j^{(r+1)})_{1,n^{(r+1)}}],[\tau_{i^{(r+1)}}(c_{ji^{(r+1)}}^{(r+1)},\gamma_{ji^{(r+1)}}^{(r+1)};C_{ji^{(r+1)}}^{(r+1)})_{n^{(r+1)}+1,p_i^{(r+1)}}];\cdots;$$

$$[(c_j^{(r+\tau)},\gamma_j^{(r+\tau)};C_j^{(r+\tau)})_{1,m^{(r+\tau)}}],[\tau_{i^{(r+\tau)}}(c_{ji^{(r+\tau)}}^{(r+\tau)},\gamma_{ji^{(r+\tau)}}^{(r+\tau)};C_{ji^{(r+\tau)}}^{(r+\tau)})_{m^{(r+\tau)}+1,p_i^{(r+\tau)}}] \tag{2.20}$$

$$\mathbb{B}=[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}},[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}};\beta_{(r-1)ji_{r-1}}^{(1)},\cdots,\beta_{(r-1)ji_{r-1}}^{(r-1)};B_{(r-1)ji_{r-1}})_{1,q_{i_{r-1}}}] \tag{2.21}$$

$$\mathbb{B}'=[\tau_{i_{r+2}}(b_{(r+2)ji_{r+2}};\beta_{(r+2)ji_{r+2}}^{(r+1)},\beta_{(r+2)jir+2}^{(r+2)};B_{(r+2)ji_{r+2}})]_{1,q_{i_{r+2}}},[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;$$

$$[\tau_{i_{r+3}}(b_{(r+3)ji_{r+3}};\beta_{(r+3)ji_{r+3}}^{(r+1)},\beta_{(r+3)jir+3}^{(r+2)},\beta_{(r+3)ji_{r+3}}^{(r+3)};B_{(r+3)ji_{r+3}})]_{1,q_{i_{r+3}}};\cdots;$$

$$[\tau_{i_{r+\tau-1}}(b_{(r+\tau-1)ji_{r+\tau-1}};\beta_{(r+\tau-1)ji_{r+\tau-1}}^{(r+1)},\cdots,\beta_{(r+\tau-1)ji_{r+\tau-1}}^{(r+\tau-1)};B_{(r+\tau-1)ji_{r+\tau-1}})_{1,q_{i_{r+\tau-1}}}] \tag{2.22}$$

$$\mathbf{B}=[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{\tau+2s-1};B_{rji_r})_{\mathfrak{n}+1,q_{i_r}}] \tag{2.23}$$

$$\mathbf{B'} = [\tau_{i_{r+\tau}}(b_{(r+\tau)ji_{r+\tau}}; \underbrace{0,\cdots,0}_{r+s}, \beta^{(1)}_{(r+\tau)ji_{r+\tau}}, \cdots, \beta^{(r+\tau)}_{(r+\tau)ji_{r+\tau}}, \underbrace{0,\cdots,0}_{s-1}; B_{(r+\tau)ji_{r+\tau}})_{\mathfrak{n}+1,q_{i_{r+\tau}}}] \tag{2.24}$$

$$B_2 = \left(-\rho; \underbrace{0,\cdots,0}_{r+s}, u_{r+1},\cdots,u_{r+\tau}, t_1,\cdots,t_{s-1}; 1\right), \left(\beta-l-\rho; \underbrace{0,\cdots,0}_{r+s}, u_{r+1},\cdots,u_{r+\tau}, t_1,\cdots,t_{s-1}; 1\right),$$

$$\left(\beta-l-\gamma; \underbrace{0,\cdots,0}_{r+s}, u_{r+1},\cdots,u_{r+\tau}, t_1,\cdots,t_{s-1}; 1\right), \left(1+\sigma_s+\sum_{i=0}^{t}\eta_i K_i; v_1',\cdots,v_1^{(r)}, \underbrace{0,\cdots,0}_{\tau+2s-1}; 1\right)$$

$$\left(-\rho-\alpha-\gamma; \underbrace{0,\cdots,0}_{r+s}, u_{r+1},\cdots,u_{r+\tau}, t_1,\cdots,t_{s-1}; 1\right), \left(1+\sigma_{s-1}; \underbrace{0,\cdots,0}_{r+s}, v_{s-1}^{(r+1)},\cdots,v_{s-1}^{(r+\tau)}, \underbrace{0,\cdots,0}_{s-1}; 1\right) \tag{2.25}$$

$$\left(1+\sigma_1; \underbrace{0,\cdots,0}_{r+s}; v_1^{(r+1)},\cdots,v_1^{(r+\tau)}, \underbrace{0,\cdots,0}_{s-1}; 1\right), \left(1+\sum_{i=0}^{t}\eta_i K_i; v_1',\cdots,v_1^{(r)}, \underbrace{0,\cdots,0}_{\tau+2s-1}; 1\right) \tag{2.26}$$

$$\mathrm{B} = [(\mathrm{d}_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,m^{(1)}}], [\tau_{i^{(1)}}(d^{(1)}_{ji^{(1)}}, \delta^{(1)}_{ji^{(1)}}; D^{(1)}_{ji^{(1)}})_{m^{(1)}+1,q_i^{(1)}}]; \cdots;$$

$$[(\mathrm{d}_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,m^{(r)}}], [\tau_{i^{(r)}}(d^{(r)}_{ji^{(r)}}, \delta^{(r)}_{ji^{(r)}}; D^{(r)}_{ji^{(r)}})_{m^{(r)}+1,q_i^{(r)}}]; \underbrace{(0,1;1),\cdots,(0,1;1)}_{s} \tag{2.27}$$

$$\mathrm{B'} = [(\mathrm{d}_j^{(r+1)}, \delta_j^{(r+1)}; D_j^{(r+1)})_{r+1,m^{(r+1)}}], [\tau_{i^{(r+1)}}(d^{(r+1)}_{ji^{(r+1)}}, \delta^{(r+1)}_{ji^{(r+1)}}; D^{(r+1)}_{ji^{(r+1)}})_{m^{(r+1)}+1,q_i^{(r+1)}}]; \cdots;$$

$$[(\mathrm{d}_j^{(r+\tau)}, \delta_j^{(r+\tau)}; D_j^{(r+\tau)})_{1,m^{(r+\tau)}}], [\tau_{i^{(r+\tau)}}(d^{(r+\tau)}_{ji^{(r+\tau)}}, \delta^{(r+\tau)}_{ji^{(r+\tau)}}; D^{(r+\tau)}_{ji^{(r+\tau)}})_{m^{(r+\tau)}+1,q_i^{(r+\tau)}}]; \underbrace{(0,1;1),\cdots,(0,1;1)}_{s-1} \tag{2.28}$$

$$U = 0, n_2; 0, n_3; \cdots; 0, n_{r-1} \quad U' \text{ is similar to } U$$

$$V = m^{(1)}, n^{(1)}; m^{(2)}, n^{(2)}; \cdots; m^{(r)}, n^{(r)}; \underbrace{(1,0);\cdots;(1,0)}_{s} \tag{2.29}$$

$$V' = m^{(1)}, n^{(r+1)}; m^{(r+2)}, n^{(r+2)}; \cdots; m^{(r+\tau)}, n^{(r+\tau)}; \underbrace{(1,0);\cdots;(1,0)}_{s-1} \tag{2.30}$$

$$Y = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}; \cdots; p_{i^{(r)}}, q_{i^{(r)}}; \tau_{i^{(r)}}; R^{(r)}; \underbrace{(0,1);\cdots;(0,1)}_{s} \tag{2.31}$$

$$Y' = p_{i^{(r+1)}}, q_{i^{(r+1)}}, \tau_{i^{(r+1)}}; R^{(r+1)}; \cdots; p_{i^{(r+\tau)}}, q_{i^{(r+\tau)}}; \tau_{i^{(r+\tau)}}; R^{(r+\tau)}; \underbrace{(0,1);\cdots;(0,1)}_{s-1} \tag{2.32}$$

Provided

$$Re(\alpha) > 0, t_i, \lambda_j, \eta_i^{(j)}, u_k, v_i^{(k)} > 0; (i=1,\cdots,s); (j=1,\cdots,t); (k=1,\cdots,r), u_{r+l}, v_i^{(r+l)} > 0; (l=1,\cdots,\tau)$$

$$Re(\rho) + \sum_{i=1}^{r+\tau} u_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] + 1 > 0$$

Notes

$$\left|arg\left(z_k x^{u_k}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{-v'_i}\right)\right| < \frac{1}{2}\pi A_k^{(l)} \quad \text{and} \quad \left|arg\left(z_{r+\tau} x^{u_{r+\tau}}\prod_{i=1}^{s-1}(x^{t_i}+\alpha_i)^{-v_i^{(r+\tau)}}\right)\right| < \frac{1}{2}\pi A_k'^{(l)}$$

Notes

Proof

We take $f(x) = x^{\rho}\prod_{i=1}^{s-1}(x^{t_i}+\alpha_i)^{\sigma_i}\gimel\left(z_{r+1}x^{u_{r+1}}\prod_{i=1}^{s-1}(x^{t_i}+\alpha_i)^{-v_i^{(r+1)}},\cdots,z_{r+\tau}x^{u_{r+\tau}}\prod_{i=1}^{s-1}(x^{t_i}+\alpha_i)^{-v_i^{(r+\tau)}}\right)$ and

$$g(x) = (x^{t_s}+\alpha_s)^{\sigma_s}\prod_{j=1}^{t}S_{N_j}^{M_j}\left[e_j x^{\lambda_j}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{\eta_i^{(j)}}\right]\gimel\left(z_1 x^{u_1}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{-v'_i},\cdots,z_1 x^{u_1}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{-v_i^{(r)}}\right)$$

in the left-hand side of (2.13) and apply the following Leibniz rule for the fractional integrals

$$I_{0,x}^{\alpha,\beta,\gamma}\left[f(x)g(x)\right] = \sum_{l=0}^{\infty} I_{0,x}^{\alpha,\beta-l,\gamma}[f(x)]I_{0,x}^{\alpha,l,\gamma}[g(x)] \tag{2.33}$$

we obtain the fractional derivative formula (2) after simplification, using the theorem 1 and the result ({4], p. Eq. (6))

**Theorem 3.**

$$I_{0,x}^{\alpha,\beta,\gamma}I_{0,y}^{\alpha',\beta',\gamma'}\Bigg\{x^{\rho}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{\sigma_i}(y^{t'_i}+\beta_i)^{\sigma'_i}\prod_{j=1}^{t}S_{N_j}^{M_j}\left[e_j x^{\lambda_j}y^{\zeta_j}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{\eta_i^{(j)}}(y^{t'_i}+\beta_i)^{\tau_i^{(j)}}\right.$$

$$\left.\gimel\left(z_1 x^{u_1}y^{u'_1}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{-v'_i}(y^{t'_i}+\beta_i)^{-w'_i},\cdots,z_r x^{u_r}y^{u'_r}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{-v_i'^{(r)}}(y^{t_i}+\beta_i)^{-w_i^{(r)}}\right)\right\}$$

$$= \alpha_1^{\sigma_1}\cdots\alpha_s^{\sigma_s}\beta_1^{\sigma'_1}\cdots\beta_s^{\sigma'_s}x^{\rho-\beta}y^{\rho'-\beta'}\sum_{K_1=0}^{[M_1/N_1]}\cdots\sum_{K_t=0}^{[M_t/N_t]}\frac{(-N_1)_{M_1K_1}\cdots(-N_t)_{M_tK_t}}{K_1!\cdots K_t!}A'_{N_1,K_1}\cdots A_{N_t,K_t}^{(t)}$$

$$e_1^{k_1}\cdots e_t^{k_t}\alpha_1^{\sum_{j=1}^{t}\eta_1^{(j)}K_j}\cdots\alpha_s^{\sum_{j=1}^{t}\eta_s^{(j)}K_j}\beta_1^{\sum_{j=1}^{t}\tau_1^{(j)}K_i}\cdots\beta_s^{\sum_{j=1}^{t}\tau_s^{(j)}K_i}x^{\sum_{j=1}^{t}\lambda_jK_j}y^{\sum_{j=1}^{t}\zeta_jK_j}$$

$$\gimel_{X;p_{i_r}+2s+4,q_{i_r}+s+2,\tau_{i_r}:R_r:Y}^{U;0,2s+4+n_r:V}\left(\begin{array}{c|c} \mathrm{z}_1\prod_{i=1}^{s}\alpha_i^{-v'_i}\beta_i^{-w'_i}x^{u_1}y^{u'_1} & \\ \cdot & \mathbb{A},;\ \mathrm{A}_3,\mathbf{A}:A \\ \cdot & \cdot \\ \mathrm{z}_r\prod_{i=1}^{s}\alpha_i^{-v_i^{(r)}}\beta_i^{-w_i^{(r)}}x^{u_r}y^{u'_r} & \cdot \\ \alpha_1^{-1}x^{t_1} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \alpha_s^{-1}x^{t_s} & \\ \beta_1^{-1}y^{t'_1} & \mathbb{B};\ \mathbf{B},\mathrm{B}_3:B \\ \cdot & \\ \cdot & \\ \beta_s^{-1}x^{t_s} & \end{array}\right) \tag{2.34}$$

where

$$\mathbb{A} = [(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}},[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{(r-1)j};\alpha_{(r-1)j}^{(1)},\cdots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})_{1,n_{r-1}}],$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}};\alpha^{(1)}_{(r-1)ji_{r-1}},\cdots,\alpha^{(r-1)}_{(r-1)ji_{r-1}};A_{(r-1)ji_{r-1}})_{n_{r-1}+1,p_{i_{r-1}}}] \tag{2.35}$$

$$A_3=\left(-\rho-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,\underbrace{0,\cdots,0}_{s},t_1,\cdots,t_s;1\right),\left(1+\sigma_1+\sum_{j=1}^{t}\eta_1^{(j)}K_j,v_1',\cdots,v_1^{(r)},\underbrace{0,\cdots,0}_{s},1\underbrace{0,\cdots,0}_{s-1};1\right),$$

$$\left(1+\sigma_s'+\sum_{j=1}^{t}\tau_1'^{(j)}K_j,w_1',\cdots,w_1^{(r)},1,\underbrace{0,\cdots,0}_{2s-1};1\right),\left(\beta-\gamma-\rho-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,\underbrace{0,\cdots,0}_{s},t_1,\cdots,t_s;1\right),$$

$$\left(\beta-\rho-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,\underbrace{0,\cdots,0}_{s},t_1,\cdots,t_s;1\right),\left(-\alpha-\gamma-\rho-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,\underbrace{0,\cdots,0}_{s},t_1,\cdots,t_s;1\right),$$

$$\left(1+\sigma_1'+\sum_{j=1}^{t}\tau_1'^{(j)}K_j,w_1',\cdots,w_1^{(r)},\underbrace{0,\cdots,0}_{s},1,\underbrace{0,\cdots,0}_{s-1};1\right),$$

$$\left(1+\sigma_1'+\sum_{j=1}^{t}\tau_1'^{(j)}K_j,w_1',\cdots,w_1^{(r)},1,\underbrace{0,\cdots,0}_{2s-1};1\right),\left(1+\sigma_s+\sum_{j=1}^{t}\eta_s^{(j)}K_j,v_1',\cdots,v_1^{(r)},1,\underbrace{0,\cdots,0}_{2s-1};1\right),$$

$$\left(1+\sigma_1;\underbrace{0,\cdots,0}_{r+s};v_1^{(r+1)},\cdots,v_1^{(r+\tau)},\underbrace{0,\cdots,0}_{s-1};1\right),\left(1+\sigma_s'+\sum_{j=1}^{t}\tau_s'^{(j)}K_j,w_1',\cdots,w_1^{(r)},1,\underbrace{0,\cdots,0}_{s-1};1,\underbrace{0,\cdots,0}_{s};1\right) \tag{2.36}$$

$$\mathbf{A}=[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)},\underbrace{0,\cdots,0}_{2s};A_{rj})_{1,n_r}],[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{2s};A_{rji_r})_{\mathfrak{n}+1,p_{i_r}}] \tag{2.37}$$

$$A=[(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}];\cdots;$$

$$[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})_{m^{(r)}+1,p_i^{(r)}}] \tag{2.39}$$

$$\mathbb{B}=[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}},[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}};\beta^{(1)}_{(r-1)ji_{r-1}},\cdots,\beta^{(r-1)}_{(r-1)ji_{r-1}};B_{(r-1)ji_{r-1}})_{1,q_{i_{r-1}}}] \tag{2.40}$$

$$\mathbf{B}=[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{2s};B_{rji_r})_{\mathfrak{n}+1,q_{i_r}}] \tag{2.41}$$

$$B_3=\left(\beta'-\gamma'-\rho'-\sum_{j=1}^{t}\lambda_jK_j;u_1,\cdots,u_r,\underbrace{0,\cdots,0}_{s},t_1,\cdots,t_s;1\right),$$

Notes

$$\left(\beta'-\rho'-\sum_{j=1}^{t}\lambda_j K_j; u_1,\cdots,u_r,\underbrace{0,\cdots,0}_{s},t_1,\cdots,t_s;1\right),$$

$$\left(-\alpha'-\gamma'-\rho'-\sum_{j=1}^{t}\zeta_j K_j; u'_1,\cdots,u'_r,\underbrace{0,\cdots,0}_{s},t_1,\cdots,t_s;1\right),$$

$$\left(-\rho'-\sum_{j=1}^{t}\zeta_j K_j; u'_1,\cdots,u'_r,\underbrace{0,\cdots,0}_{s},t_1,\cdots,t_s;1\right),$$

$$\left(1+\sigma_1+\sum_{j=1}^{t}\eta_1^{(j)}K_j,v'_1,\cdots,v_1^{(r)},\underbrace{0,\cdots,0}_{2s};1\right),\left(1+\sigma_s+\sum_{j=1}^{t}\eta_s^{(j)}K_j,v'_1,\cdots,v_1^{(r)},\underbrace{0,\cdots,0}_{2s-1};1\right),$$

$$\left(1+\sigma_s+\sum_{j=1}^{t}\eta_s^{(j)}K_j,v'_1,\cdots,v_1^{(r)},\underbrace{0,\cdots,0}_{2s};1\right),\left(1+\sigma_1+\sum_{j=1}^{t}\eta_1^{(j)}K_j,v'_1,\cdots,v_1^{(r)},\underbrace{0,\cdots,0}_{s},0,\underbrace{0,\cdots,0}_{2s-1}\right),$$

$$\left(1+\sigma'_1+\sum_{j=1}^{t}\tau_1'^{(j)}K_j,w'_1,\cdots,w_1^{(r)},\underbrace{0,\cdots,0}_{2s};1\right),\cdots,\left(1+\sigma'_s+\sum_{j=1}^{t}\tau_1'^{(j)}K_j,w'_1,\cdots,w_1^{(r)},\underbrace{0,\cdots,0}_{2s};1\right) \quad (2.42)$$

$$\mathrm{B}=[(\mathrm{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,m^{(1)}}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})_{m^{(1)}+1,q_i^{(1)}}];\cdots;$$

$$[(\mathrm{d}_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_{ji^{(r)}}^{(r)})_{m^{(r)}+1,q_i^{(r)}}];\underbrace{(0,1;1),\cdots,(0,1;1)}_{2s} \quad (2.43)$$

$$U=0,n_2;0,n_3;\cdots;0,n_{r-1}:V=m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)};\underbrace{(1,0);\cdots;(1,0)}_{2s} \quad (2.44)$$

$$Y=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)};\underbrace{(0,1);\cdots;(0,1)}_{2s} \quad (2.45)$$

Provided

$$Re(\alpha),Re(\alpha')>0,\lambda_j,\eta_i^{(j)}>0,t_i,t'_i,\tau_i^{(j)},u_k,u'_k,v_i^{(k)},w_i^{(k)}>0(i=1,\cdots,s);(j=1,\cdots,k);(k=1,\cdots,r)$$

$$Re(\rho)+\sum_{i=1}^{r}u_i\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]+1>0,Re(\rho')+\sum_{i=1}^{r}u'_i\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]+1>0,$$

$$\left|\bar{a}rg\left(z_i x^{u_i}y^{u_i}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{-v_i^{(r)}}(y^{t_i}+\beta_i)^{-w_i^{(r)}}\right)\right|<\frac{1}{2}\pi A_k^{(l)}$$

Proof

To prove the theorem 3, we use the theorem 1 twice, first with respect to the variable $y$ and then with respect to the variable $x$, in this situation,the variables $x$ and $y$ are independent variables.

## II. Special Case

In this section, we shall see the particular case studied by Soni and Singh ([12],p. 561, Eq. (14))

Consider the theorem 1, if we take $t=2$ and reduce the polynomial $S_{N_1}^{M_1}$ to the Hermite polynomial ([17], p. 158. Eq.

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(1.4)), the polynomial $S_{N_2}^{M_2}$ to the Laguerre polynomial ([17], p. 159. Eq.(1.8)), the multivariable Gimel-function to the product of r different Whittaker functions ([14], p. 18, Eq. (2.6.7)), we obtain the following result

**Corollary.**

$$I_{0,x}^{\alpha,\beta,x}\left\{x^{+\sum_{l=1}^{r}b_l+\frac{n_1}{2}}\prod_{i=1}^{s}(x^{t_i}+\alpha_i)^{\sigma_i}H_{n_1}\left(\frac{1}{2\sqrt{x}}\right)\prod_{l=1}^{r}e^{-\frac{z_lx}{2}}W_{\mu_l v_l}(z_lx)\right\}$$

$$\frac{\prod_{l=1}^{r}z_l^{-b_l}\alpha_1^{\sigma_1}\cdots\alpha_s^{\sigma_s}x^{\rho-\beta}}{\Gamma(-\sigma_1)\cdots\Gamma(-\sigma_s)}\sum_{K_1=0}^{[N_1/2]}\sum_{K_2=0}^{N_2}\frac{(-N_1)_{2K_1}(-N_2)_{K_2}}{K_1!K_2!}(-)^{K_1}\binom{N_2+\theta}{N_2}\frac{x^{K_1+K_2}}{(\theta+1)_{K_2}}$$

$$H_{2,2:1,2,\cdots,1,1;1,2,\cdots,1,1}^{0,2:2,0;\cdots;2,0;1,1;\cdots;1,1}\left(\begin{array}{c|c} \mathrm{z}_1x & (-\rho-K_1-K_2;1,\cdots,1,t_1,\cdots,t_s),(\beta-\gamma-\rho-K_1-K_2;1,\cdots,1,t_1,\cdots,t_s); \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_rx & \cdot \\ \alpha_1^{-1}x^{t_1} & \cdot \\ \cdot & \cdot \\ \cdot & \\ \alpha_s^{-1}x^{t_s} & (\beta-\rho-K_1-K_2;1,\cdots,1,t_1,\cdots,t_s),(-\alpha-\gamma-\rho-K_1-K_2;1,\cdots,1,t_1,\cdots,t_s): \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{b}_1-u_1+1,1);\cdots;(b_r-u_r+1,1);(1+\sigma_1,1);\cdots;(1+\sigma_s,1) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \left(b_1\pm v_1+\frac{1}{2},1\right);\cdots;\left(b_r\pm v_r+\frac{1}{2},1\right);\underbrace{(0,1),\cdots,(0,1)}_{s} \end{array}\right) \tag{3.1}$$

The validity conditions mentioned above are verified.

**Remarks :**

We obtain easily the same relations with the functions defined in section 1.
Soni and Singh [12] have obtained the same relations with the multivariable H-function.

## III. Conclusion

The fractional derivative formulae evaluated in this study are unified in nature and act as key formulae. Thus the general class of polynomials involved here reduce to a large variety of polynomials and so from theorems 1, 2 and 3; we can further obtain various fractional derivatives formulae involving a number of simpler polynomials. Secondly by specializing the various parameters as well as variables in the generalized multivariable Gimel-function, we get several formulae involving a remarkably wide variety of useful functions ( or product of such functions) which are expressible in terms of E, F, G, H, I, Aleph-function of one and several variables and simpler special functions of one and several variables. Hence the formulae derived in this paper are most general forms and may prove to be useful in several interesting cases appearing in the literature of Pure and Applied Mathematics and Mathematical Physics.

## References Références Referencias


[1] F. Ayant, An integral associated with the Aleph-functions of several variables. International Journal of Mathematics Trends and Technology (IJMTT), 31(3) (2016), 142-154.

[2] F. Ayant, An expansion formula for multivariable Gimel-function involving generalized Legendre Associated function, International Journal of Mathematics Trends and Technology (IJMTT), 56(4) (2018), 223-228.

[3] B.L.J. Braaksma, Asymptotics expansions and analytic continuations for a class of Barnes-integrals, Compositio Math. 15 (1962-1964), 239-341.

[4] K.C. Gupta and R.C. Soni, A study of H-functions of one and several variables, J. Rajasthan Acad. Phys. Sci. 1(2002), 89-94.

[5] K.S. Miller and B. Ross, An introduction, to the fractional calculus and fractional differential equation (New York: John Wiley and Sos), 1993.

[6] K.B. Oldham and J. Spanier, The fractional calculus (New York : Academic press), 1974.


Ref


12. R.C. Soni and D. Singh, certain fractional derivative formulae involving the product of a general class of polynomials and the multivariable H-function, Proc. Indian. Acad. Sci. 112(4) (2002), 551-562.

Notes

[7] M. Saigo, A remark on integral operators involving the Gauss hypergeometric functions, Math. Rep. Kyushu Univ. 11(1978), 135-143.

[8] Y.N. Prasad, Multivariable I-function , Vijnana Parisha Anusandhan Patrika 29 (1986), 231-237.

[9] J. Prathima, V. Nambisan and S.K. Kurumujji, A Study of I-function of Several Complex Variables, International Journal of Engineering Mathematics Vol (2014), 1-12.

[10] M. Saigo and R.K. Raina, Fractional calculus operators associated with a general class of polynomials, Fukuoka Univ. Sci. Report 18 (1988), 15-22.

[11] M. Saigo and R.K. Raina, Fractional calculus operators associated with the H-function of several variables: Analysis, Geometry and Groups: Hadronic Press (1993), 527-538.

[12] R.C. Soni and D. Singh, certain fractional derivative formulae involving the product of a general class of polynomials and the multivariable H-function, Proc. Indian. Acad. Sci. 112(4) (2002), 551-562.

[13] H.M. Srivastava, A contour integral involving Fox's H-function, Indian. J. Math. 14(1972), 1-6.

[14] H.M. Srivastava, KC. Gupta and S.P. Goyal, The H-function of one and two variables with applications, South Asian Publisher, New Delhi and Madras, 1982.

[15] H.M. Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24 (1975),119-137.

[16] H.M. Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of several complex variables II. Comment. Math. Univ. St. Paul. 25 (1976), 167-197.

[17] H.M. Srivastava and N.P. Singh, The integration of certains products of the multivariable H-function with a general class of polynomials, Rend. CirC. Mat. Palermo. 32(2)(1983), 157-187.

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# Ground Handling Operations Management: An Agent-Based Modelling Approach

By Hayat El Asri, Abderrahim Agnaou, Ali Fakhrulddin & Ali Al-Humairi
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*Abstract-* This work addresses the resource allocation-routing problem in the aircraft Ground Handling (GH) context. This problem is classified as one of the challenges that most of the ground-handling operators face nowadays. It consists of allocating a number of resources including workers and vehicles to aircraft of different service requirements and specifications. The allocation deals with the assignment of *m* teams of *n* skilled workers to *k* special purpose vehicles to serve *i* aircraft on the air-side area, while the routing is to determine the optimal route used by a group of workers integrated with vehicles when serving one or more aircraft. Both the allocation and routing tasks are affected by a number of constraints including different aircraft types and sizes, different aircraft service times, and various GH operation details based on flight types.

*Keywords:* *resource allocation, routing, aircraft, ground handling operations, autonomous system, algorithms, simulation.*

*GJSFR-F Classification:* *FOR Code: MSC 2010: 11Y16*

*Strictly as per the compliance and regulations of:*

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# Ground Handling Operations Management: An Agent-Based Modelling Approach

Hayat El Asri [α], Abderrahim Agnaou [σ], Ali Fakhrulddin [ρ] & Ali Al-Humairi [Ѡ]

*Abstract-* This work addresses the resource allocation-routing problem in the aircraft Ground Handling (GH) context. This problem is classified as one of the challenges that most of the ground-handling operators face nowadays. It consists of allocating a number of resources including workers and vehicles to aircraft of different service requirements and specifications. The allocation deals with the assignment of *m* teams of *n* skilled workers to *k* special purpose vehicles to serve *i* aircraft on the air-side area, while the routing is to determine the optimal route used by a group of workers integrated with vehicles when serving one or more aircraft. Both the allocation and routing tasks are affected by a number of constraints including different aircraft types and sizes, different aircraft service times, and various GH operation details based on flight types. Therefore, the main aim of this research is to develop an autonomous system to deal with the allocation, planning, and scheduling of resources. The methodology followed involves a number of algorithms that act and interact with one another towards achieving the best allocation and routing process. The simulation results have shown a net decrease in the service time and an overall team utilisation of 92%.

*Keywords: resource allocation, routing, aircraft, ground handling operations, autonomous system, algorithms, simulation.*

## I. Introduction

With over 3 billion passengers and millions of flights every year, aviation is one of the busiest sectors in the world. Air transport is an economic engine that has globalised the world's economy and facilitated tourism and business between different countries and continents (ATAG, 2014).GH operations embody the airside activities at airports (Fitouri-Trabelsi et al., 2015). They comprise all the services required by aircraft between landings and take-offs (CAPA, 2014). Most of these operations require the use of special vehicles that are specific to each type of operation.They are usually categorised as: ramp, on-ramp, or on-board services. Each of these categories include a number of services. However, airline companies may opt for another schema to follow depending on a number of factors such as the total budget of GH operations and the number of available workers.

In this research paper, the issue of aircraft GH resource allocation, scheduling and routing is investigated. Because GH operations consist of many distinct tasks that not all ground handlers can perform, workers of different skills are required. Moreover, the different operations to be completed depend on two parameters the aircraft size and the flight type. There are three aircraft sizes: large, medium, and small, and two flight types: international and domestic. Furthermore, a number of vehicles are required to perform all the tasks. These vehicles are categorized based on their specialisation and may be needed for one or more tasks. Likewise, workers have different skills; however, some of them can perform more than one operation and therefore can be used

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accordingly. Workers will be assigned not only based on their skills, but also depending on their idle time. The worker with the least utilisation will be called in first. Following the same logic, the aircraft that is taking off first will be given priority.

This paper is organised as follow: section II presents the work context followed the multi-agent based modelling section where the algorithms used are explained. The simulation and results are explained in section IV, followed by the conclusion and future works sections.

## II. Work Context

GH operations refer to all the operations needed to service an aircraft between the period of landing and take-off. Figure describes the main GH usually needed:

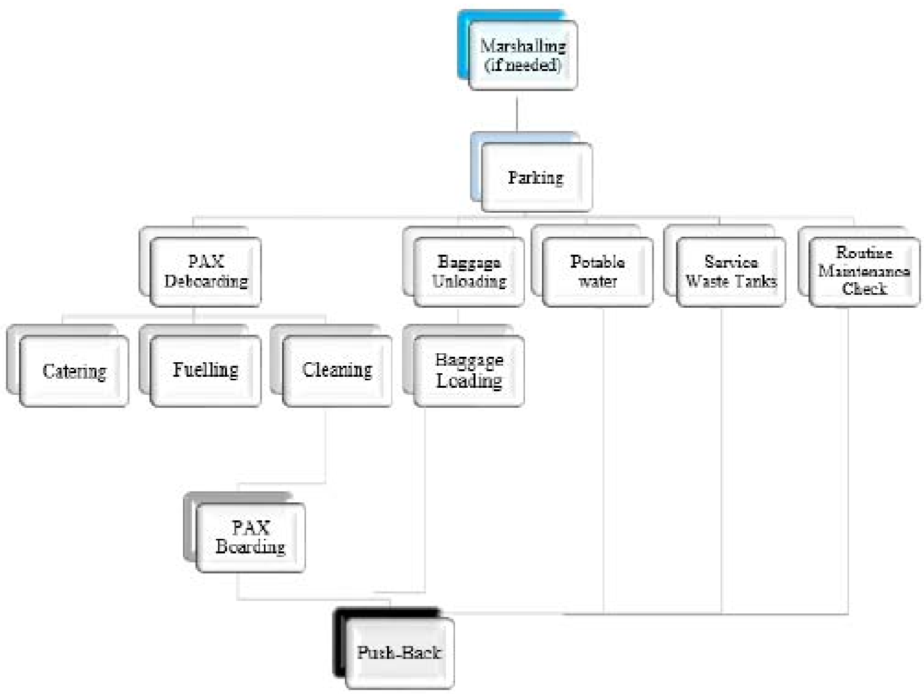


*Figure 1:* GH Operations

Several vehicles are needed to complete GH operations. Some of the vehicles are represented in the diagram below:

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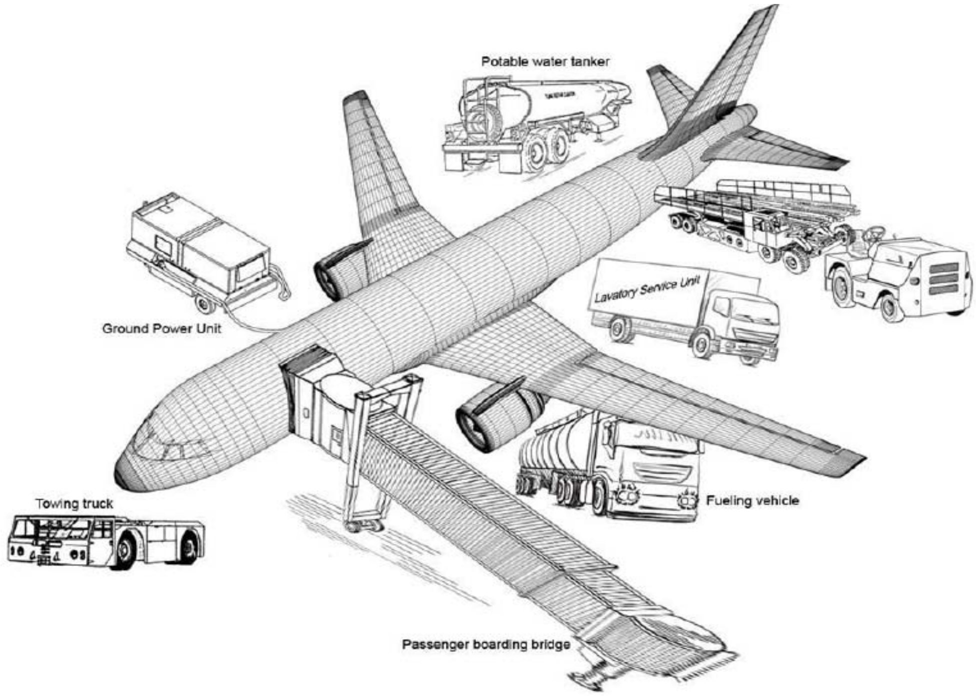


*Figure 2:* Vehicles needed in GH

In this research, the issue of aircraft GH resource allocation, scheduling and routing is investigated. Because GH operations consist of many distinct tasks that not all ground handlers can perform, workers of different skills are required. Moreover, the different operations to be completed depend on two parameters the aircraft size and the flight type. As mentioned above, there are three aircraft sizes: large, medium, and small, and two flight types: international and domestic. Furthermore, a number of vehicles are required to perform all the tasks. These vehicles are categorized based on their specialisation and may be needed for one or more tasks.

GH operations have been studied for decades and great advances in the GH processes were made. Nonetheless, some aspects of GH operations have not been improved, and on-going research is still being carried out. One aspect of the turnaround processes, however, was not investigated to this date. It is contended that the benefits of combining vehicle routing with the workers' availability and skills should be exploited by managers to optimally allocate crews to aircraft.

*In the light of the above, the question driving this research is:*

Can GH resource allocation, scheduling and routing be fully optimised? The following parameters are taken into account: workers' availability and skills, vehicles' availability and specialisation, aircraft inter-arrival, and due times.

## III. Multi-Agent Based Modelling

The system architecture is a conceptual diagram that demonstrates how the system operates. The purpose behind this system is to allocate, plan, and optimise ground-handling resources efficiently by calculating the shortest paths for vehicles and implementing intelligent search and optimisation algorithms.

The core of this system are Multi-Agent based model along with a number of embedded algorithms including Hamiltonian Cycle, Fibonacci Heap, and Genetic Algorithms.

These algorithms have been developed to optimise allocation of resources including both workers and vehicles to different GH operations. These algorithms are implemented in the back-end and are not accessible through the GUI.

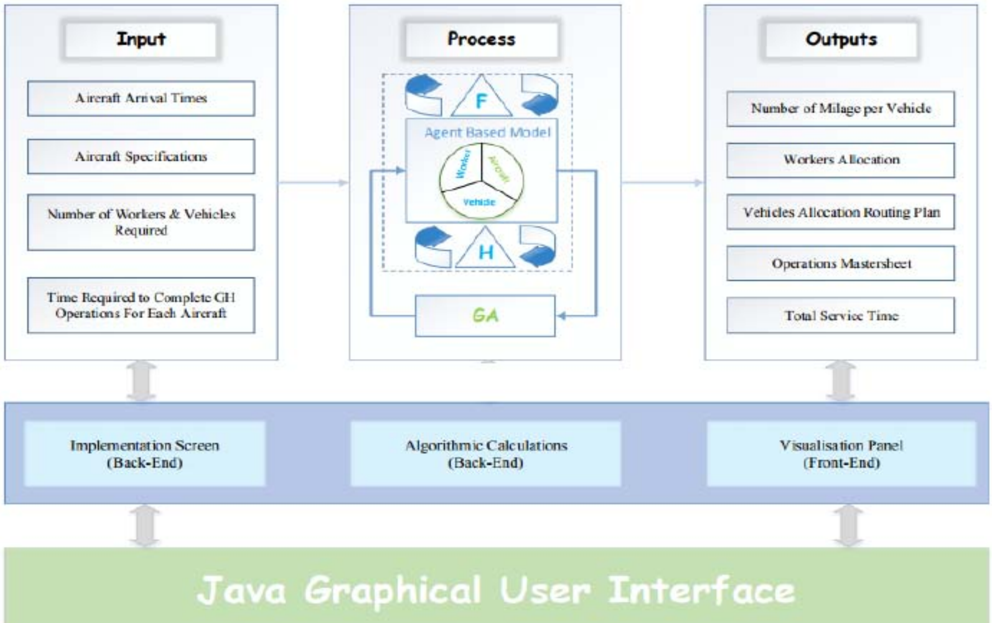


*Figure 3:* Conceptual Model

A number of inputs are used and fed into the system. These inputs include aircraft inter-arrival times, aircraft specifications, number of workers and vehicles required, and finally the time required to complete all GH operations. Several algorithms embedded within the ABM are developed to optimise resources allocation of GH operations. Hamiltonian Cycle with Fibonacci Heap algorithms are used to optimise the vehicles routing plan, while a Genetic Algorithm is used to optimise the required number of workers and vehicles.

Every time a random number of aircraft is generated, the Hamiltonian graph changes. The nodes and edges of the graph are directly linked, and therefore dependent, of the number of aircraft generated. Furthermore, the KPIs for GH operations include optimised routes, optimised GH operations, optimised number of vehicles and workers, and an optimised GH operations planning and schedules. These are the outputs of the system.

An Agent-Based approach along with a number of algorithms as components are used to process inputs and turn them into the required KPIs. The Agent-Based approach is used to mimic the GH operations and a number of algorithms are integrated with the developed ABM to optimise the performance of the allocated resources including workers and vehicles.

## IV. Simulation & Results

A 7-day simulation was run to test the system and get the results that are presented within this section.

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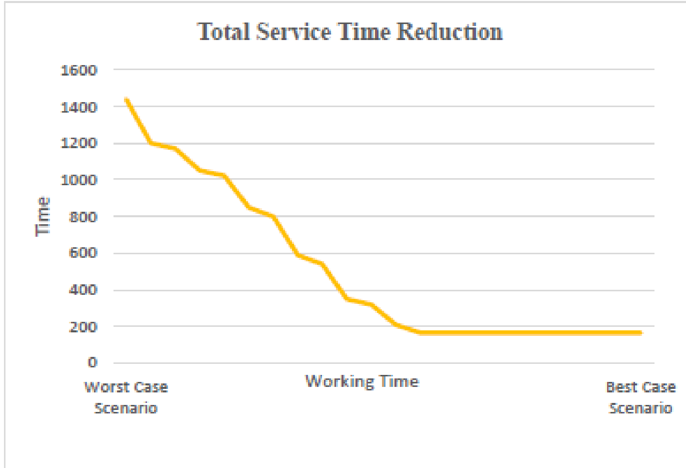


*Figure 4:* Total Service Time Reduction

The total service time reduction shows the worst case versus the best case scenarios. The Genetic Algorithm (GA) used was run for about 5 hours to get these results.

Concerning the vehicles working hours for vehicles serving international aircraft, it is higher than the ones serving local flights. This is due to the fact that more time is allocated to international flights as a number of things must be checked thoroughly before take-off. For instance, routine maintenance check for local aircraft may take 10 to 30 minutes, while the same operation takes between 30 and 50 minutes for international flights.

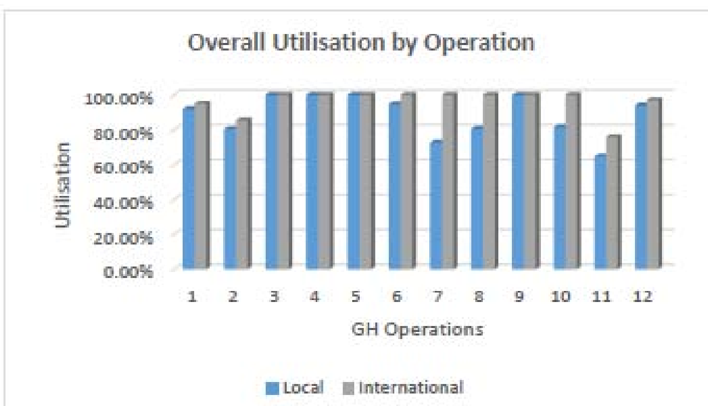


*Figure 5:* Overall Utilisation by Operation

An overall utilisation of over 92.26% was achieved, as shown in the below graph.

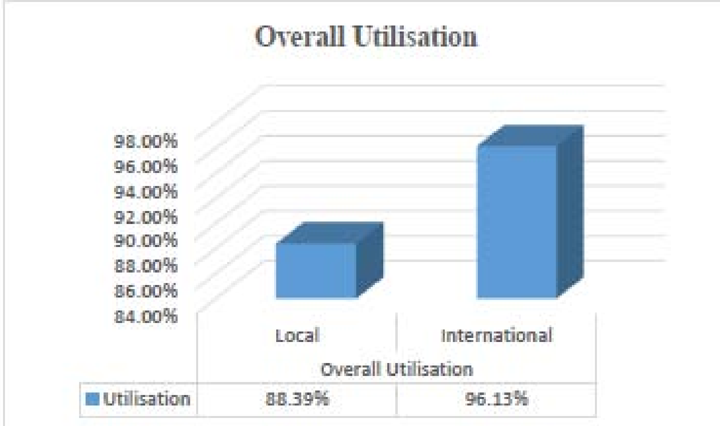


*Figure 6:* Overall Utilisation

The graph below shows both the total service time for vehicles serving both local and international flight types.


*Figure 7:* Total Service Time

We notice from the graph that international flight destinations are given more time. Nonetheless, one should keep in mind that the total number of aircraft of international destinations and those of local destinations are randomly generated. This may have impacted the total service time for some days. For instance, 52% of the aircraft generated on day 1, and 81% of the aircraft generated on day 4 are of international flight type.

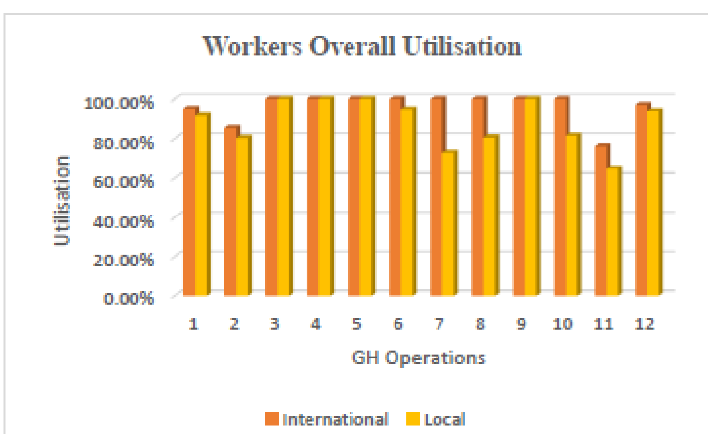


*Figure 8:* Workers Utilisation

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The overall utilisation for workers serving local and international aircraft is high. The overall utilisation of the workers serving international aircraft is higher than the one of workers serving local aircraft.

To determine the resource allocation plan, a 7-day simulation was run from Monday to Sunday to randomly generate workers, vehicles, and aircraft. Most of the workers and vehicles are used every day. This is what makes the utilisation very high. Both workers and vehicles have very little idle time and work all day long serving different aircraft.

## V. Conclusion

Optimally allocating workers to vehicles and assigning them to one or more aircraft while taking into account the availability of resources and aircraft arrival and due times is the problem addressed within the scope of this research. An attempt to solve this problem was made through implementing a multi-agent based simulation model.

A thorough analysis of the results, routing tables, and allocation tables of 103 aircraft was carried out. A number of graphs were generated to show the total working hours, the vehicles utilisation, the total service time, the workers utilisation, and the total service time reduction.

The simulation results are promising as the overall vehicles utilisation is 92% and the total working hours are very high. This implies that all resources are well allocated, optimised, and used.

## VI. Future Works

Finding the optimal resource allocation and routes was the main purpose behind this study. Different aircraft models and types were taken into account. Two aircraft models were studied: Boeing & Airbus; different sizes were considered as well: large, medium, and small aircraft. The simulation study consisted of generating a random number of aircraft of different destination types: local or international where 103 aircraft total were studied.

For future works, adapting this system to other problems in other areas of study, such as supply chain where using the vehicle routing section of this research to solve the dynamic allocation of vehicles in logistics, is being considered. Furthermore, adding more rules to achieve better agents' interaction is considered as well.

## References Références Referencias

1. Air Transport Action Border (2014). Aviation Benefits. AVIATION BENEFITS BEYOND BORDERS. [online] Available at: http://aviationbenefits.org/media/26786/ATAG__AviationBenefits2014_FULL_LowRes.pdf [Accessed 4 Aug. 2017].
2. Fitouri-Trabelsi, S., Mora-Camino, F., Nunes-Cosenza, C. and Weigang, L. (2015). Global Journal of Science Frontier Research: F Mathematics and Decision Sciences. Integrated Decision Making for Ground Handling Management. [online] 15(1). Available at: https://hal-enac.archives-ouvertes.fr/hal-01117103/document [Accessed Mar. 2017].
3. Cabinet Office Department of Transport (2013). 2010 to 2015 government policy: transport security - GOV.UK. [online] Available at: https://www.gov.uk/government/publications/2010-to-2015-government-policy-transport-security/2010-to-2015-government-policy-transport-security#background [Accessed 3 Aug. 2017].

4. Garcia Ansola, P., Garcia Higuera, A., Otamendi, F. and de las Morenas, J. (2014). Agent-Based Distributed Control for Improving Complex Resource Scheduling: Application to Airport Ground Handling Operations. IEEE Systems Journal, 8(4), pp.1145-1157.
5. Kabongo, P. (2015). A Multi Agent Based Model for Ground Handling Management at Airports. [online] Available at: http://www.translab.unb.br/images/slides/14-ground-handling-management.pdf [Accessed Mar. 2017].
6. Wu, C. and Caves, R. (2007). Modelling and optimization of aircraft turnaround time at an airport. [online] Available at: http://www.aviation.unsw.edu.au/downloads/papersWu/ 1_TP&T_V27N1_2004_2.pdf [Accessed 19 Mar. 2017].


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# An Unified Study of Some Multiple Integrals with Multivariable Gimel-Function

By Frédéric Ayant

*Abstract-* In this paper, we first evaluate a unified and general finite multiple integral whose integrand involves the product of the functions $_PF_Q, S_N^M$ and the multivariable Gimel-function occurring in the integrand involve the product of factors of the form $x^{\rho-1}(a-x)^{\sigma}\left[1+(bx)^{l}\right]^{-\lambda}$ while that of $_PF_Q$ occurring herein involves a finite series of such finite series of such factors. On account of the most general nature of the functions occurring in the integrand of our main integral, a large number of new and known integrals can easily be obtained from ot merely by specializing the functions and parameters involved therein. At the end of this study, we illustrate a new integral whose integrand involves a product of the Jacobi polynomial, the Appell's function $F_3$ and the Bessel function $J_v$

*Keywords: multivariable gimel-function, multiple integral contours, a general class of polynomials, general finite multiple integral, generalized hypergeometric function.*

*GJSFR-F Classification: FOR Code: 33C99, 33C60, 44A20*

*Strictly as per the compliance and regulations of:*

Notes

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*Keywords: multivariable gimel-function, multiple integral contours, a general class of polynomials, general finite multiple integral, generalized hypergeometric function.*

## I. Introduction And Preliminaries

Throughout this paper, let $\mathbb{C}, \mathbb{R}$ and $\mathbb{N}$ be set of complex numbers, real numbers, and positive integers respectively. Also $\mathbb{N}_0 = \mathbb{N}\cup\{0\}$. We define a generalized transcendental function of several complex variables.

$$\gimel(z_1,\cdots,z_r) = \gimel^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)}}_{p_{i_2},q_{i_2},\tau_{i_2};R_2;p_{i_3},q_{i_3},\tau_{i_3};R_3;\cdots;p_{i_r},q_{i_r},\tau_{i_r}:R_r:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{array}\right.$$

$$[(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}};[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$
$$[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}};$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)};A_{rj})_{1,n_r}],$$
$$[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;\quad\cdots$$

$$[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)};A_{rji_r})_{n_r+1,p_r}]:\ [(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}]$$
$$[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)};B_{rji_r})_{1,q_r}]:\ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)};D_j^{(1)})_{1,m^{(1)}}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})_{m^{(1)}+1,q_i^{(1)}}]$$

$$\left.\begin{array}{l};\cdots;[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})_{m^{(r)}+1,p_i^{(r)}}] \\ ;\cdots;[(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,n^{(r)}}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_{ji^{(r)}}^{(r)})_{n^{(r)}+1,q_i^{(r)}}]\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

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$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{n_2}\Gamma^{A_{2j}}(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k)}{\sum_{i_2=1}^{R_2}[\tau_{i_2}\prod_{j=n_2+1}^{p_{i_2}}\Gamma^{A_{2ji_2}}(a_{2ji_2}-\sum_{k=1}^{2}\alpha_{2ji_2}^{(k)}s_k)\prod_{j=1}^{q_{i_2}}\Gamma^{B_{2ji_2}}(1-b_{2ji2}+\sum_{k=1}^{2}\beta_{2ji2}^{(k)}s_k)]}$$

$$\frac{\prod_{j=1}^{n_3}\Gamma^{A_{3j}}(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k)}{\sum_{i_3=1}^{R_3}[\tau_{i_3}\prod_{j=n_3+1}^{p_{i_3}}\Gamma^{A_{3ji_3}}(a_{3ji_3}-\sum_{k=1}^{3}\alpha_{3ji_3}^{(k)}s_k)\prod_{j=1}^{q_{i_3}}\Gamma^{B_{3ji_3}}(1-b_{3ji3}+\sum_{k=1}^{3}\beta_{3ji3}^{(k)}s_k)]}$$

$$\cdot \qquad \cdot \qquad \cdot \qquad \cdot$$

$$\cdot \qquad \cdot \qquad \cdot \qquad \cdot$$

$$\cdot \qquad \cdot \qquad \cdot \qquad \cdot$$

$$\frac{\prod_{j=1}^{n_r}\Gamma^{A_{rj}}(1-a_{rj}+\sum_{k=1}^{r}\alpha_{rj}^{(k)}s_k)}{\sum_{i_r=1}^{R_r}[\tau_{i_r}\prod_{j=n_r+1}^{p_{i_r}}\Gamma^{A_{rji_r}}(a_{rji_r}-\sum_{k=1}^{r}\alpha_{rji_r}^{(k)}s_k)\prod_{j=1}^{q_{i_r}}\Gamma^{B_{rji_r}}(1-b_{rjir}+\sum_{k=1}^{r}\beta_{rjir}^{(k)}s_k)]} \tag{1.2}$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m^{(k)}}\Gamma^{D_j^{(k)}}(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n^{(k)}}\Gamma^{C_j^{(k)}}(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m^{(k)}+1}^{q_{i^{(k)}}}\Gamma^{D_{ji^{(k)}}^{(k)}}(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n^{(k)}+1}^{p_{i^{(k)}}}\Gamma^{C_{ji^{(k)}}^{(k)}}(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

For more details, see Ayant [2].

The contour $L_k$ is in the $s_k(k=1,\cdots,r)$- plane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where $\sigma$ if is a real number with loop, if necessary to ensure that the poles of $\Gamma^{A_{2j}}\left(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k\right)(j=1,\cdots,n_2),\Gamma^{A_{3j}}\left(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k\right)$ $(j=1,\cdots,n_3),\cdots,\Gamma^{A_{rj}}\left(1-a_{rj}+\sum_{i=1}^{r}\alpha_{rj}^{(i)}\right)(j=1,\cdots,n_r),\quad \Gamma^{C_j^{(k)}}\left(1-c_j^{(k)}+\gamma_j^{(k)}s_k\right)(j=1,\cdots,n^{(k)})(k=1,\cdots,r)$to the right of the contour $L_k$, and the poles of $\Gamma^{D_j^{(k)}}\left(d_j^{(k)}-\delta_j^{(k)}s_k\right)(j=1,\cdots,m^{(k)})(k=1,\cdots,r)$ lie to the left of the contour $L_k$. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained of the corresponding conditions for multivariable H-function given by as :

$|arg(z_k)|<\frac{1}{2}A_i^{(k)}\pi$ where

$$A_i^{(k)}=\sum_{j=1}^{m^{(k)}}D_j^{(k)}\delta_j^{(k)}+\sum_{j=1}^{n^{(k)}}C_j^{(k)}\gamma_j^{(k)}-\tau_{i^{(k)}}\left(\sum_{j=m^{(k)}+1}^{q_i^{(k)}}D_{ji^{(k)}}^{(k)}\delta_{ji^{(k)}}^{(k)}+\sum_{j=n^{(k)}+1}^{p_i^{(k)}}C_{ji^{(k)}}^{(k)}\gamma_{ji^{(k)}}^{(k)}\right)$$

$$-\tau_{i_2}\left(\sum_{j=n_2+1}^{p_{i_2}}A_{2ji_2}\alpha_{2ji_2}^{(k)}+\sum_{j=1}^{q_{i_2}}B_{2ji_2}\beta_{2ji_2}^{(k)}\right)-\cdots-\tau_{i_r}\left(\sum_{j=n_r+1}^{p_{i_r}}A_{rji_r}\alpha_{rji_r}^{(k)}+\sum_{j=1}^{q_{i_r}}B_{rji_r}\beta_{rji_r}^{(k)}\right) \tag{1.4}$$

Following the lines of Braaksma ([3] p. 278), we may establish the asymptotic expansion in the following convenient form :

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,),max(\,|z_1|,\cdots,|z_r|\,)\rightarrow 0$$

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,),min(\,|z_1|,\cdots,|z_r|\,)\rightarrow\infty \text{ where } i=1,\cdots,r:$$

Ref

2. F. Ayant, An expansion formula for multivariable Gimel-function involving generalized Legendre Associated function, International Journal of Mathematics Trends and Technology (IJMTT), 56(4) (2018), 223-228.

$$\alpha_i = \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] \text{ and } \beta_i = \max_{1\leqslant j\leqslant n^{(i)}} Re\left[C_j^{(i)}\left(\frac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right)\right]$$

**Remark 1.**
If $n_2 = \cdots = n_{r-1} = p_{i_2} = q_{i_2} = \cdots = p_{i_{r-1}} = q_{i_{r-1}} = 0$ and $A_{2j} = A_{2ji_2} = B_{2ji_2} = \cdots = A_{rj} = A_{rji_r} = B_{rji_r} = 1$ $A_{rj} = A_{rji_r} = B_{rji_r} = 1$, then the multivariable Gimel-function reduces in the multivariable Aleph- function defined by Ayant [1].

**Remark 2.**
If $n_2 = \cdots = n_r = p_{i_2} = q_{i_2} = \cdots = p_{i_r} = q_{i_r} = 0$ and $\tau_{i_2} = \cdots = \tau_{i_r} = \tau_{i^{(1)}} = \cdots = \tau_{i^{(r)}} = R_2 = \cdots = R_r = R^{(1)} = \cdots = R^{(r)} = 1$, then the multivariable Gimel-function reduces in a multivariable I-function defined by Prathima et al. [6].

**Remark 3.**
If $A_{2j} = A_{2ji_2} = B_{2ji_2} = \cdots = A_{rj} = A_{rji_r} = B_{rji_r} = 1$ and $\tau_{i_2} = \cdots = \tau_{i_r} = \tau_{i^{(1)}} = \cdots = \tau_{i^{(r)}} = R_2 = \cdots = R_r = R^{(1)} = \cdots = R^{(r)} = 1$, then the generalized multivariable Gimel-function reduces in multivariable I-function defined by Prasad [5].

**Remark 4.**
If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the multivariable H-function defined by Srivastava and Panda [11,12].

Srivastava ([7],p. 1, Eq. 1) has defined the general class of polynomials

$$S_N^M(x) = \sum_{K=0}^{[N/M]} \frac{(-N)_{MK}}{K!} A_{N,K} x^K \tag{1.5}$$

On suitably specializing the coefficients $A_{N,K}$, $S_N^M(x)$ yields some known polynomials, these include the Jacobi polynomials, Laguerre polynomials, and others polynomials ([12],p. 158-161).

## II. Main Integral

In this section, we evaluate a unified multiple finite integrals involving the multivariable Gimel-function with general arguments.

**Theorem.**

$$\int_0^{a_1}\cdots\int_0^{a_s}\prod_{j=1}^{s}\left[x_j^{\rho_j-1}(a_j-x_j)^{\sigma_j}\left[1+(b_jx_j)^{l_j}\right]^{-\lambda_j}\right]\prod_{k=1}^{t}S_{N_k}^{M_k}\left[Y_k\prod_{j=1}^{s}\left[x_j^{e_j^{(k)}}(a_j-x_j)^{f_j^{(k)}}\left[1+(b_jx_j)^{l_j}\right]^{-g_j^{(k)}}\right]\right]$$

$$\beth\left[\begin{array}{c} \mathrm{z}_1\prod_{j=1}^{s}\left[x_j^{u_j^{(1)}}(a_j-x_j)^{v_j^{(1)}}\left[1+(b_jx_j)^{l_j}\right]^{-w_j^{(1)}}\right] \\ \cdot \\ \cdot \\ \mathrm{z}_r\prod_{j=1}^{s}\left[x_j^{u_j^{(r)}}(a_j-x_j)^{v_j^{(r)}}\left[1+(b_jx_j)^{l_j}\right]^{-w_j^{(r)}}\right] \end{array}\right]$$

$${}_PF_Q\left[(E_P);(F_Q);\sum_{j=1}^{s}B_j\left[x_j^{\mu_i}(a_i-x_j)^{\upsilon_j}\left[1+(b_jx_j)^{l_j}\right]^{-\omega_j}\right]\right]\mathrm{d}x_1\cdots\mathrm{d}x_r = \frac{\prod_{j=1}^{Q}\Gamma(F_j)}{\prod_{j=1}^{P}\Gamma(E_j)}\prod_{j=1}^{s}a_j^{\rho_j+\sigma_j}$$

Ref

1. F. Ayant, An integral associated with the Aleph-functions of several variables. International Journal of Mathematics Trends and Technology (IJMTT), 31(3) (2016), 142-154.

$$\prod_{k=1}^{t}\sum_{K_k=0}^{[N_k/M_k]}\frac{(-N_k)_{M_kK_k}A_{N_k,K_k}\left[Y_k\prod_{j=1}^{s}a_j^{e_j^{(k)}+f_j^{(k)}}\right]^{K_k}}{K_k!}$$

$$\gimel^{U;0,P+3s+n+n_r:V}_{X;P+3s+n+p_{i_r},Q+2s+q+q_{i_r},\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c} \mathrm{z}_1\prod_{j=1}^{s}a_j^{u_j^{(1)}+v_j^{(1)}} & \mathbb{A};\mathrm{A}_s,\mathbf{A}:A \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r\prod_{j=1}^{s}a_j^{u_j^{(r)}+v_j^{(r)}} & \cdot \\ \text{-}\mathrm{B}_1a_1^{\mu_1+\upsilon_1} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \text{-}\mathrm{B}_ra_r^{\mu_r+\upsilon_r} & \cdot \\ (\mathrm{a}_1b_1)^{l_1} & \cdot \\ \cdot & \mathbb{B};\mathbf{B},\,\mathrm{B}_s:B;\underbrace{(0,1;1);\cdots;(0,1;1)}_{2s} \\ \cdot & \\ (\mathrm{a}_sb_s)^{l_s} & \end{array}\right) \tag{2.1}$$

where

$$\mathbb{A}=[(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}},[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{(r-1)j};\alpha_{(r-1)j}^{(1)},\cdots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})_{1,n_{r-1}}],$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}};\alpha_{(r-1)ji_{r-1}}^{(1)},\cdots,\alpha_{(r-1)ji_{r-1}}^{(r-1)};A_{(r-1)ji_{r-1}})_{n_{r-1}+1,p_{i_{r-1}}}] \tag{2.2}$$

$$A_s=(1-E_j;\underbrace{0,\cdots,0}_{r},\underbrace{1,\cdots,1}_{s},\underbrace{0,\cdots,0}_{s};1),\left(1-\rho_1-\sum_{k=1}^{t}e_1^{(k)}K_k;u_1^{(1)},\cdots,u_1^{(r)},u_1,\underbrace{0,\cdots,0}_{s-1},l_1,\underbrace{0,\cdots,0}_{s-1};1\right)$$

$$,\cdots,\left(1-\rho_s-\sum_{k=1}^{t}e_s^{(k)}K_k;u_s^{(1)},\cdots,u_s^{(r)},\underbrace{0,\cdots,0}_{s-1},\mu_s,\underbrace{0,\cdots,0}_{s-1},l_s;1\right),$$

$$\left(-\sigma_1-\sum_{k=1}^{t}f_1^{(k)}K_k;v_1^{(1)},\cdots,v_1^{(r)},\upsilon_1,\underbrace{0,\cdots,0}_{2s-1};1\right),\cdots,\left(-\sigma_s-\sum_{k=1}^{t}f_s^{(k)}K_k;v_s^{(1)},\cdots,v_s^{(r)},\underbrace{0,\cdots,0}_{s-1},\upsilon_s,\underbrace{0,\cdots,0}_{s-1};1\right)$$

$$\left(1-\lambda_1-\sum_{k=1}^{t}g_1^{(k)}K_k;w_1^{(1)},\cdots,w_1^{(r)},\omega_1,\underbrace{0,\cdots,0}_{s-1},1,\underbrace{0,\cdots,0}_{s-1};1\right),\cdots,$$

$$\left(1-\lambda_s-\sum_{k=1}^{t}g_s^{(k)}K_k;w_s^{(1)},\cdots,w_s^{(r)},\underbrace{0,\cdots,0}_{s-1},\omega_s,\underbrace{0,\cdots,0}_{s-1},1;1\right) \tag{2.3}$$

$$\mathbf{A}=\;[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)},\underbrace{0,\cdots,0}_{2s};A_{rj})_{1,n_r}],[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{2s};A_{rji_r})_{\mathfrak{n}+1,p_{i_r}}] \tag{2.4}$$

$$A=[(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}];\cdots;$$

Notes

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$$[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})_{m^{(r)}+1,p_i^{(r)}}] \tag{2.5}$$

$$\mathbb{B}=[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}},[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}};\beta_{(r-1)ji_{r-1}}^{(1)},\cdots,\beta_{(r-1)ji_{r-1}}^{(r-1)};B_{(r-1)ji_{r-1}})_{1,q_{i_{r-1}}}] \tag{2.6}$$

$$\mathbf{B}=\ [\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{2s};B_{rji_r})_{1,q_{i_r}}] \tag{2.7}$$

$$B_s=(1-F_j;\underbrace{0,\cdots,0}_{r},\underbrace{1,\cdots,1}_{s},\underbrace{0,\cdots,0}_{s};1)_{1,p},$$

$$\left(-\rho_1-\sigma_1-\sum_{k=1}^{t}(e_1^{(k)}+f_1^{(k)})K_k;u_1^{(1)}+v_1^{(1)},\cdots,u_1^{(r)}+v_1^{(r)},;\mu_1+\upsilon_1,\underbrace{0,\cdots,0}_{s-1},l_1,\underbrace{0,\cdots,0}_{s-1};1\right),\cdots,$$

$$\left(-\rho_s-\sigma_s-\sum_{k=1}^{t}(e_s^{(k)}+f_s^{(k)})K_k;u_s^{(1)}+v_s^{(1)},\cdots,u_s^{(r)}+v_s^{(r)},\underbrace{0,\cdots,0}_{s-1},\mu_s+\upsilon_s,\underbrace{0,\cdots,0}_{s-1},l_s;1\right)$$

$$\left(1-\lambda_1-\sum_{k=1}^{t}g_1^{(k)}K_k;w_1^{(1)},\cdots,w_1^{(r)},\omega_1,\underbrace{0,\cdots,0}_{2s-1};1\right),\cdots,$$

$$\left(1-\lambda_s-\sum_{k=1}^{t}g_s^{(k)}K_k;w_s^{(1)},\cdots,w_s^{(r)},\underbrace{0,\cdots,0}_{s-1},\omega_s,\underbrace{0,\cdots,0}_{s};1\right) \tag{2.8}$$

$$\mathrm{B}=[(\mathrm{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,m^{(1)}}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})_{m^{(1)}+1,q_i^{(1)}}];\cdots;$$

$$[(\mathrm{d}_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_{ji^{(r)}}^{(r)})_{m^{(r)}+1,q_i^{(r)}}] \tag{2.9}$$

$$U=0,n_2;0,n_3;\cdots;0,n_{r-1};V=m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)};\underbrace{(1,0);\cdots;(1,0)}_{2s} \tag{2.10}$$

$$X=p_{i_2},q_{i_2},\tau_{i_2};R_2;\cdots;p_{i_{r-1}},q_{i_{r-1}},\tau_{i_{r-1}}:R_{r-1};$$

$$Y=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)};\underbrace{(0,1);\cdots;(0,1)}_{2s} \tag{2.11}$$

Provided

$$\lambda_j,e_j^{(k)},f_j^{(k)},g_j^{(k)},u_j^{(i)},v_j^{(i)},w_j^{(i)},\mu_j,\upsilon_i,\omega_j>0;(j=1,\cdots,s);(k=1,\cdots,t);(i=1,\cdots,r)$$

$$Re(\rho_k)+\sum_{i=1}^{r}u_k^{(i)}\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0\,;Re(\sigma_k+1)+\sum_{i=1}^{r}v_k^{(i)}\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0$$

$$\left|arg\left(z_i\prod_{j=1}^{s}\left[x_j^{u_j^{(i)}}(a_j-x_j)^{v_j^{(i)}}\left[1+(b_jx_j)^{l_j}\right]^{-w_j^{(i)}}\right]\right)\right|<\frac{1}{2}\left(A_i^{(k)}-\sum_{j=1}^{s}(u_j^{(i)}+v_j^{(i)}+w_j^{(i)})\right)\pi$$

where $A_i^{(k)}$ is defined by (1.4).

Proof

First, we replace the polynomials $S_N^M(x)$ in series with the help of (1.5). We interchange the orders of series and the $(x_1, \cdots, x_s)$-integrals. Next, we express the generalized hypergeometric function of one variable regarding of generalized Kampé de Fériet function of s variables with the help of ([9], p.39, Eq. 30), and express this Kampé de Fériet function regarding of H-function of s variables with the help of ([10], p.272, Eq. 4.7). Next, we express the H-function of s-variables and the Gimel-function regarding of their respective Mellin-Barnes multiple integrals contour. Now we change the order of the $\zeta_i$, $\eta_j$ and $x_j$-integrals $(i = 1, \cdots, r; j = 1, \cdots, s)$ which is permissible under the stated conditions. Finally, on evaluating the $x_j$-integrals , after algebraic manipulations; we obtain ( say I)

Notes

$$I = \frac{\prod_{j=1}^{Q}\Gamma(F_j)}{\prod_{j=1}^{P}\Gamma(E_j)}\prod_{k=1}^{t}\sum_{K_k=0}^{[N_k/M_k]}\frac{(-N_k)_{M_kK_k}A_{N_k,K_k}Y_k^{K_k}}{K_k!}\frac{1}{(2\pi\omega)^{r+s}}\int_{L_1}\cdots\int_{L_r}\int_{L_{r+1}}\cdots\int_{L_{r+s}}$$

$$\psi(s_1, \cdots, s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{\zeta_i}\frac{\prod_{j=1}^{P}\Gamma\left((E_j + \sum_{j=1}^{s}\eta_j\right)}{\prod_{j=1}^{Q}\Gamma\left((F_j + \sum_{j=1}^{s}\eta_j\right)}\prod_{j=1}^{s}[\Gamma(\eta_j)(-B_j)^{\eta_j}$$

$$a_j^{\left(\rho_j+\sigma_j+\sum_{k=1}^{t}(e_j^{(k)}+f_j^{(k)}K_k+\sum_{i=1}^{r}[u_j^{(i)}+v_j^{(i)}\zeta_i+(\mu_i+\upsilon_i)\eta_i\right)}\frac{\Gamma\left(1+\sigma_j+\sum_{k=1}^{t}f_j^{(k)}K_k+\sum_{i=1}^{r}v_j^{(i)}\zeta_i+\upsilon_i\eta_j\right)}{\Gamma\left(\lambda_j++\sum_{k=1}^{t}g_j^{(k)}K_k+\sum_{i=1}^{r}vw_j^{(i)}\zeta_i+\omega_i\eta_j\right)}$$

$$H_2^1\left[(\mathrm{b}_ja_j)^l\left|\begin{matrix}(1\text{-}\lambda_j-\sum_{k=1}^{t}g_j^{(k)}k_k+\sum_{i=1}^{r}v_j^{(i)}\zeta_i-\omega_j\eta_j,1;1),(1-\rho_j-\sum_{k=1}^{t}e_j^{(k)}K_k+\sum_{i=1}^{r}u_j^{(i)}\zeta_i-\mu_j\eta_j,1;1)\\ \cdot \\ (0,1;1),(\text{-}\rho_j-\sigma_j-\sum_{k=1}^{t}(e_j^{(k)}+f_j^{(k)})K_k-\sum_{i=1}^{r}(u_j^{i)}+v_j^{(i)})\zeta_i-(\mu_j-\upsilon_j)\eta_j,l_j;1)\end{matrix}\right.\right] \quad (2.12)$$

Now we express the s-product of the H-functions of one variable occurring in the above expression regarding of their respectives Mellin-Barnes integral contour and we interpret the resulting multiple integrals contour with the help of (1.1) in term of the Gimel-function of $(r + 2s)$-variables, after algebraic manipulations, we obtain the theorem.

## III. Special Cases

We consider the particular case studied by Gupta and Jain ([4], 9. 79-80, Eq. (3.1)) Taking $s = r = 2, l_1 = l_2 = t = 1$ in the result (2.1) and further reduce the general polynomial $S_N^M$ into Jacobi polynomials $P_n^{(\alpha,\beta)}(1-2x)$, the Gimel-function of two variables into Appell's function $F_3$ and the generalized hypergeometric function ${}_PF_Q$ into the Bessel's function $J_\upsilon$ with the help of following results ([13], p. 159, Eq. (1.6)), ([8] p.18, Eq. (2.6.3), (2.6.5)), we obtain the following double integral after algebraic manipulations :

$$\int_0^{a_1}\int_0^{a_2}\prod_{j=1}^{2}\left[x_j^{\rho_j-1}(a_j-x_j)^{\sigma_j}(1+b_jx_j)^{-\lambda_j}\right]P_n^{(\alpha,\beta)}\left(1-2yx_1^{e_1}x_2^{e_2}\right)F_3(k_1,k_2,h_1,h_2;L;z_1x_1^{u_1},z_2x_2^{u_2})$$

$$[B_1x_1^{\mu_1}+B_2x_2^{m}{}^{u_2}]^{-\frac{\upsilon}{2}}J_\upsilon\left[2\sqrt{B_1x_1^{\mu_1}+B_2x_2^{\mu_2}}\right]\mathrm{d}x_1\mathrm{d}x_2 = \frac{\Gamma(L)\Gamma(1+\sigma_1)\Gamma(1+\sigma_2)a_1^{\rho_1+\sigma_1}a_2^{\rho_2+\sigma_2}}{\Gamma(k_1)\Gamma(k_2)\Gamma(h_1)\Gamma_(h_2)\Gamma(\lambda_1)\Gamma(\lambda_2)}$$

$$\sum_{R=0}^{n}\frac{(-n)_R\binom{\alpha+n}{n}(\alpha+\beta+n+1)_R(ya_1^{e_1}a_2^{e_2})^R}{R!(\alpha+1)_R}H_{2,4:2,1;2,1;0,1;0,1;1,1;1,1}^{0,2:1,2;1,2;1,0;1,0;1,1;1,1}$$

$$\left(\begin{array}{c|c} \text{-z}_1 a_1^{u_1} & \\ \text{-z}_2 a_2^{u_2} & (1\text{-}\rho_1 - e_1 R; u_1, 0, \mu_1, 0, 1, 0), (1-\rho_2 - e_2 R; 0, u_2, 0, \mu_2, 0, 1) : (1-k_1, 1), (1-k_2, 1); \\ \text{B}_1 a_1^{\mu_1} & \cdot \\ \text{B}_2 a_2^{\mu_2} & \cdot \\ \text{a}_1 b_1 & (\text{-}\upsilon; 0,0,1,1,0,0), (-\rho_1 - \sigma_1 - e_1 R; u_1, 0, \mu_1, 0, 1, 0), (-\rho_2 - \sigma_2 - e_2 R; 0, u_2, 0, \mu_2, 0, 1), \\ \text{a}_2 b_2 & \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-h}_1,), (1-h_2, 1); -; -; (1-\lambda_1, 1); (1-\lambda_2, 1) \\ \cdot \\ \cdot \\ (1\text{-L};1,1,0,0,0,0):(0,1)\ ;\ (0,1);\ (0,1),(0,1);(0,1);(0,1) \end{array}\right) \tag{3.1}$$

The validity conditions mentioned above are verified.

**Remarks :**

We obtain easily the same relations with the functions defined in section 1.

Gupta and Jain. [4] have obtained the same relations about the multivariable H-function.

## IV. Conclusion

The importance of our results lies in their manifold generality. Firstly, given of unified multiple integrals with general classes of polynomials, generalized hypergeometric function with general arguments utilized in this study, we can obtain a large variety of single, double or multiple simpler integrals specializing the coefficients and the parameters in these functions. Secondly by specializing the various parameters as well as variables in the generalized multivariable Gimel-function, we get a several formulae involving a remarkably wide variety of useful functions ( or product of such functions) which are expressible in terms of E, F, G, H, I, Aleph-function of one and several variables and simpler special functions of one and several variables. Hence the formulae derived in this paper are most general in character and may prove to be useful several interesting cases appearing in literature of Pure and Applied Mathematics and Mathematical Physics.

## References Références Referencias


1. F. Ayant, An integral associated with the Aleph-functions of several variables. International Journal of Mathematics Trends and Technology (IJMTT), 31(3) (2016), 142-154.
2. F. Ayant, An expansion formula for multivariable Gimel-function involving generalized Legendre Associated function, International Journal of Mathematics Trends and Technology (IJMTT), 56(4) (2018), 223-228.
3. B.L.J. Braaksma, Asymptotics expansions and analytic continuations for a class of Barnes-integrals, Compositio Math. 15 (1962-1964), 239-341.
4. K. C Gupta and R Jain, A unified study of some multiple integrals, Soochow Journal of Mathematics, 19(1) (1993), 73-81.
5. Y.N. Prasad, Multivariable I-function, Vijnana Parisha Anusandhan Patrika 29 (1986), 231-237.
6. J. Prathima, V. Nambisan and S.K. Kurumujji, A Study of I-function of Several Complex Variables, International Journal of Engineering Mathematics Vol (2014), 1-12.
7. H.M. Srivastava, A contour integral involving Fox's H-function, Indian. J. Math. 14(1972), 1-6.
8. H.M. Srivastava, KC. Gupta and S.P. Goyal, The H-function of one and two variables with applications, South Asian Publisher, New Delhi and Madras, 1982.
9. H.M. Srivastava and Pee W. Karlsson, Multiple Gaussian hypergeometric serie, John Wiley and Sons (Ellis Horwood Ltd.), New York, 1985.
10. H.M. Srivastava and R. Panda, Some bilateral generating functions for a class of generalized hypergeometric polynomials, J. Reine Angew. Math. 283/284 (1976), 265-274.


Ref

4. K. C Gupta and R Jain, A unified study of some multiple integrals, Soochow Journal of Mathematics, 19(1) (1993), 73-81.

11. H.M. Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24 (1975), 119-137.

12. H.M. Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of several complex variables II. Comment. Math. Univ. St. Paul. 25 (1976), 167-197.

13. H.M. Srivastava and N.P. Singh, The integration of certains products of the multivariable H-function with a general class of polynomials, Rend. CirC. Mat. Palermo. 32(2)(1983), 157-187.

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# New Approaches to the Solution of the Problem of the Propagation of Electrical Energy Fluxes in the Material Media and the Long Lines

By F. F. Mende

*Abstract-* In the article new approaches to the solution of the problem of the propagation of electrical energy fluxes in the material media and the long lines are examined. The electrodynamics of plasma is examined and it is shown that the absolute value of the vector of poytinga can be obtained with the examination of the motion of specific electric field energy and kinetic kinetic energy of the charges, concentrated in the single volumes of plasma. Is obtained wave of equation for the plasma. The electrodynamics of dielectrics is examined and is obtained wave equation for them. Are examined processes occurring in the long lines, filled with plasma or dielectrics and predicted new phenomenon transverse plasma resonance in the limited nonmagnetized plasma. The use of transverse plasma resonance opens the possibility of designing of the lasers of large power.

*Keywords: maxwell's equation, wave equation, poynting's vector, plasma, dielectric, plasma resonance, transverse plasma resonance, long line, laser.*

*GJSFR-F Classification: FOR Code: MSC 2010: 35Q61*

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*Strictly as per the compliance and regulations of:*

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## I. Introduction

If we as the direction of propagation of electromagnetic (EM) wave in the free space select axis $Z$, and the vector of electric field to direct along the axis $x$, the for this component of field wave equation, obtained from Maxwell's equations, will be written down:

$$\frac{\partial^2 E_x}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2},$$

where $c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$ - speed of light. where $\varepsilon_0$ and $\mu_0$ - dielectric and magnetic constant of vacuum.

The function satisfies this equation

$$E_x=E_{0x}\sin(\omega t-kz),$$

where $k=\frac{\omega}{c}$ - wave number.

Eletromagnitnaya wave with its propagation transfers energy. The quantity of energy, transferred by wave in one second, through the single area, normal to the direction of propagation, is determined by the Poynting's vector. This vector is formally introduced as follows for the free space in the system of SI:

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$$\mathbf{\Pi} = [\mathbf{E}\times\mathbf{H}].$$

The relation of the absolute values of electrical and magnetic field in EM to wave in the free space is determined by the wave drag of the free space $Z_0$

$$\frac{E}{H} = Z_0 ,$$

where

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} .$$

Consequently, the absolute value of the Poynting's vector is equal

$$\Pi = \frac{1}{Z_0} E^2 = \frac{1}{\sqrt{\frac{\mu_0}{\varepsilon_0}}} E^2 .$$

The average value of the acting function $E_x = E_{0x}\sin(\omega t - kz)$ is equal $\frac{E_{0x}}{\sqrt{2}}$. The average value of the absolute value of the Poynting's vector will comprise for this case

$$\Pi = \frac{1}{Z_0} E^2 = \frac{1}{2Z_0} E_{0x}^2 = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\; E_{0x}^2 . \qquad (1.1)$$

The transfer of energy EM by wave can be calculated by energy method.

The specific electric field energy, stored up per unit of volume, is determined by the relationship

$$W_0 = \frac{1}{2}\varepsilon_0 E_{0x}^2 .$$

If EM wave moves with the speed of light $c$ , that it will per unit time through the single area, located normal to the propagation of wave, transfer the energy

$$cW_0 = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\; E_{0x}^2 ,$$

The obtained value coincides with the value of the absolute value of the Poynting's vector(1.1).

## II. Electrodynamics Of Plasmo-Like Media

By plasma media we will understand such, in which the charges can move without the losses. To such media in the first approximation, can be related the superconductors, free electrons or ions in the vacuum (subsequently conductors). In the absence magnetic field in the media indicated equation of motion for the electrons takes the form:

$$m\frac{d\vec{v}}{dt} = e\vec{E} , \qquad (2.1)$$

Notes

where $m$ - electron mass, $e$ - electron charge, $\vec{E}$ - the tension of electric field, $\vec{v}$ - speed of the motion of charge.

In the work [9] it is shown that this equation can be used also for describing the electron motion in the hot plasma. Therefore it can be disseminated also to this case. Using an expression for the current density

$$\vec{j} = ne\vec{v} \tag{2.2}$$

from (2.1) we obtain the current density of the conductivity

$$\vec{j}_L = \frac{ne^2}{m} \int \vec{E}\ dt \tag{2.3}$$

in relationship (2.2) and (2.3) the value of $n$ represents electron density. After introducing the designation of

$$L_k = \frac{m}{ne^2} \tag{2.4}$$

we find

$$\vec{j}_L = \frac{1}{L_k} \int \vec{E}\ dt\ . \tag{2.5}$$

In this case the value $L_k$ presents the specific kinetic inductance of charge carriers [2-6]. Its existence connected with the fact that charge, having a mass, possesses inertia properties. Pour on $\vec{E} = \vec{E}_0 \sin \omega t$ relationship (2.5) it will be written down for the case of harmonics:

$$\vec{j}_L = -\frac{1}{\omega L_k} \vec{E}_0 \cos \omega t \tag{2.6}$$

For the mathematical description of electro dynamic processes the trigonometric functions will be here and throughout, instead of the complex quantities, used so that would be well visible the phase relationships between the vectors, which represent electric fields and current densities.

from relationship (2.5) and (2.6) is evident that $\vec{j}_L$ presents inductive current, since. its phase is late with respect to the tension of electric field to the angle $\frac{\pi}{2}$.

If charges are located in the vacuum, then during the presence of summed current it is necessary to consider bias current

$$\vec{j}_\varepsilon = \varepsilon_0 \frac{\partial\ \vec{E}}{\partial\ t} = \varepsilon_0 \vec{E}_0 \cos \omega t\ .$$

Is evident that this current bears capacitive nature, since. its phase anticipates the phase of the tension of electrical to the angle $\frac{\pi}{2}$. Thus, summary current density will compose [3-5]

$$\vec{j}_\Sigma = \varepsilon_0 \frac{\partial\ \vec{E}}{\partial\ t} + \frac{1}{L_k} \int \vec{E}\ dt\ ,$$

or

$$\vec{j}_\Sigma = \left( \omega\varepsilon_0 - \frac{1}{\omega L_k} \right) \vec{E}_0 \cos\omega t \tag{2.7}$$

If electrons are located in the material medium, then should be considered the presence of the positively charged ions. However, with the examination of the properties of such media in the rapidly changing fields, in connection with the fact that the mass of ions is considerably more than the mass of electrons, their presence usually is not considered.

In relationship (2.7) the value, which stands in the brackets, presents summary susceptance of this medium $\sigma_\Sigma$ and it consists it, in turn, of the the capacitive $\sigma_C$ and by the inductive $\sigma_L$ the conductivity

$$\sigma_\Sigma = \sigma_C + \sigma_L = \omega\varepsilon_0 - \frac{1}{\omega L_k}.$$

Relationship (2.7) can be rewritten and differently:

$$\vec{j}_\Sigma = \omega\varepsilon_0 \left( 1 - \frac{\omega_0^2}{\omega^2} \right) \vec{E}_0 \cos\omega t,$$

where $\omega_0 = \sqrt{\frac{1}{L_k \varepsilon_0}}$ - plasma frequency.

And large temptation here appears to name the value

$$\varepsilon^*(\omega) = \varepsilon_0 \left( 1 - \frac{\omega_0^2}{\omega^2} \right) = \varepsilon_0 - \frac{1}{\omega^2 L_k},$$

By the depending on the frequency dielectric constant of plasma, that also is made in all existing works on physics of plasma. But this is incorrect, since. this mathematical symbol is the composite parameter, into which simultaneously enters the dielectric constant of vacuum and the specific kinetic inductance of charges. It is clear from the previous examination that the parameter $\varepsilon^*(\omega)$ gives the possibility in one coefficient to combine derivative and the integral of harmonic function, since they are characterized by only signs and thus impression is created, that the dielectric constant of plasma depends on frequency. It should be noted that a similar error is perfected by such well-known physicists as Akhiezer, Tamm, Ginsburg [7-12].

This happened still and because, beginning to examine this question, Landau introduced the determinations of dielectric constant only for the static pour on, but he did not introduce this lopredeleniya for the variables pour on. Let us introduce this determination.

If we examine any medium, including plasma, then current density (subsequently we will in abbreviated form speak simply current) it will be determined by three components, which depend on the electric field. The current of resistance losses there will be synchronizedto electric field. The permittance current, determined by first-order derivative of electric field from the time, will anticipate the tension of electric field on

Notes

the phase to $\frac{\pi}{2}$. This current is called bias current. The conduction current, determined by integral of the electric field from the time, will lag behind the electric field on the phase to $\frac{\pi}{2}$. All three components of current indicated will enter into the second Maxwell's equation and others components of currents be it cannot. Moreover all these three components of currents will be present in any nonmagnetic regions, in which there are losses. Therefore it is completely natural, the dielectric constant of any medium to define as the coefficient, confronting that term, which is determined by the derivative of electric field by the time in the second Maxwell's equation. In this case one should consider that the dielectric constant cannot be negative value. This connected with the fact that through this parameter is determined energy of electrical pour on, which can be only positive.

Notes

Without having introduced this clear determination of dielectric constant, Landau begins the examination of the behavior of plasma in the ac fields. In this case is not separated separately the bias current and conduction current, one of which is defined by derivative, but by another integral, is written as united bias current. It makes this error for that reason, that in the case of harmonic oscillations the form of the function, which determine and derivative and integral, is identical, and they are characterized by only sign. Performing this operation, Landau does not understand, that in the case of harmonic electrical pour on in the plasma there exist two different currents, one of which is bias current, and it is determined by the dielectric constant of vacuum and derivative of electric field. Another current is conduction current and is determined by integral of the electric field. these two currents are antiphase. But since both currents depend on frequency, moreover one of them depends on frequency linearly, and another it is inversely proportional to frequency, between them competition occurs. The conduction current predominates with the low frequencies, the bias current, on the contrary, predominates with the high. However, in the case of the equality of these currents, which occurs at the plasma frequency, occurs current resonance.

Let us emphasize that from a mathematical point of view to reach in the manner that it entered to Landau, it is possible, but in this case is lost the integration constant, which is necessary to account for initial conditions during the solution of the equation, which determines current density in the material medium.

The obviousness of the committed error is visible based on other example.

Relationship (2.7) can be rewritten and differently:

$$\vec{j}_{\Sigma}=-\frac{\left(\frac{\omega^2}{\omega_0^2}-1\right)}{\omega L}\ \vec{E}_0\cos\omega t$$

and to introduce another mathematical symbol

$$L^*(\omega)=\frac{L_k}{\left(\frac{\omega^2}{\omega_0^2}-1\right)}=\frac{L_k}{\omega^2 L_k\varepsilon_0-1}\quad .$$

In this case also appears temptation to name this bending coefficient on the frequency kinetic inductance.

Thus, it is possible to write down:

$$\vec{j}_\Sigma = \omega\varepsilon^*(\omega)\ \vec{E}_0 \cos\omega t\,,$$

or

$$\vec{j}_\Sigma = -\frac{1}{\omega L^*(\omega)}\ \vec{E}_0 \cos\omega t\ .$$

But this altogether only the symbolic mathematical record of one and the same relationship (2.7). Both equations are equivalent. But view neither $\varepsilon^*(\omega)$ nor $L^*(\omega)$ by dielectric constant or inductance are from a physical point. The physical sense of their names consists of the following:

$$\varepsilon^*(\omega) = \frac{\sigma_X}{\omega}\ ,$$

i.e. $\varepsilon^*(\omega)$ presents summary susceptance of medium, divided into the frequency, and

$$L_k{}^*(\omega) = \frac{1}{\omega\sigma_X}$$

it represents the reciprocal value of the work of frequency and susceptance of medium.

As it is necessary to enter, if at our disposal are values $\varepsilon^*(\omega)$ and $L^*(\omega)$, and we should calculate total specific energy. Natural to substitute these values in the formulas, which determine energy of electrical pour on

$$W_E = \frac{1}{2}\varepsilon_0 E_0^2$$

and kinetic energy of charge carriers

$$W_j = \frac{1}{2} L_k\, j_0^2\,. \tag{2.8}$$

Hence it follows that the summary specific energy, accumulated per unit of volume of plasma is equal

$$W_\Sigma = \frac{1}{2}\varepsilon_0 E_0^2 + \frac{1}{2} L_k\, j_0^2\,.$$

It is not difficult to show that in this case the total specific energy can be obtained from the relationship of

$$W_\Sigma = \frac{1}{2}\cdot\frac{d\big(\omega\varepsilon^*(\omega)\big)}{d\omega} E_0^2\ , \tag{2.9}$$

from where we obtain

$$W_\Sigma = \frac{1}{2}\varepsilon_0 E_0^2 + \frac{1}{2}\ \frac{1}{\omega^2 L_k} E_0^2 = \frac{1}{2}\varepsilon_0 E_0^2 + \frac{1}{2} L_k\, j_0^2\ .$$

We will obtain the same result, after using the formula

$$W_\Sigma = \frac{1}{2}\ \frac{d\left[\dfrac{1}{\omega L_k{}^*(\omega)}\right]}{d\omega}\ E_0^2\,.$$

Notes

The given relationships indicate, that the specific energy EM of wave consists not only of potential energy of electrical pour on, as we counted earlier, but into it it enters still and kinetic energy of charge carriers.

With the examination of any media by our final task appears the presence of wave equation. In this case this problem is already practically solved. Maxwell's equations for this case take the form:

$$rot\ \vec{E} = -\mu_0 \frac{\partial\ \vec{H}}{\partial\ t},$$
$$rot\ \vec{H} = \varepsilon_0 \frac{\partial\ \vec{E}}{\partial\ t} + \frac{1}{L_k}\int \vec{E}\ dt, \qquad (2.10)$$

system of equations (2.10) completely describes all properties of nondissipative conductors. From it we obtain

$$rot\ rot\ \vec{H}+\mu_0\varepsilon_0\frac{\partial^2\vec{H}}{\partial\ t^2}+\frac{\mu_0}{L_k}\vec{H}=0 \qquad (2.11)$$

For the case pour on, time-independent, equation (2.11) passes into the London equation

$$rot\ rot\ \vec{H}+\frac{\mu_0}{L_k}\vec{H}=0\ ,$$

where $\lambda_L{}^2=\frac{L_k}{\mu_0}$ - London depth of penetration.

Thus, it is possible to conclude that the equations of London being a special case of equation (2.11), and do not consider bias currents on Wednesday. Therefore they do not give the possibility to obtain the wave equations, which describe the processes of the propagation of electromagnetic waves in the superconductors.

The wave equation in this case it appears as follows for the electrical field:

$$rot\ rot\ \vec{E}+\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial\ t^2}+\frac{\mu_0}{L_k}\vec{E}=0 \qquad (2.12)$$

The carried out examination showed that the dielectric constant of this medium was equal to the dielectric constant of vacuum and this permeability on frequency does not depend. The accumulation of potential energy is obliged to this parameter. Furthermore, this medium is characterized still and the kinetic inductance of charge carriers and this parameter determines the kinetic energy, accumulated on Wednesday.

Taking into account the fact that $div\vec{E}=0$ from (2.12) we obtain wave equation with the right side

$$\frac{\partial^2\vec{E}}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}=\frac{\mu_0}{L_k}\vec{E}\,.$$

Solution of this equation will give the value of the unknown function.

## III. Physical Processes in the Parallel Resonant Circuit

In order to show that the single volume of conductor or plasma according to its electrodynamic characteristics is equivalent to parallel resonant circuit with the lumped parameters, let us examine parallel resonant circuit. The connection between the voltage $U$, applied to the outline, and the summed current $I_\Sigma$, which flows through this chain, takes the form of

$$I_\Sigma = I_C + I_L = C\frac{dU}{dt} + \frac{1}{L}\int U\ dt,$$

where $I_C = C\frac{dU}{dt}$ - current, which flows through the capacity, and $I_L = \frac{1}{L}\int U\ dt$ - current, which flows through the inductance.

For the case of the harmonic stress $U = U_0 \sin\omega t$ we obtain

$$\text{of } I_\Sigma = \left(\omega C - \frac{1}{\omega L}\right) U_0 \cos\omega t, \qquad (3.1)$$

In relationship (3.1) the value, which stands in the brackets, presents summary susceptance $\sigma_\Sigma$ this medium and it consists it, in turn, of the capacitive $\sigma_C$ and by the inductive $\sigma_L$ the conductivity

$$\sigma_\Sigma = \sigma_C + \sigma_L = \omega C - \frac{1}{\omega L}.$$

In this case relationship (3.1) can be rewritten as follows:

$$I_\Sigma = \omega C\left(1 - \frac{\omega_0^2}{\omega^2}\right) U_0 \cos\omega t,$$

where $\omega_0{}^2 = \frac{1}{LC}$ - the resonance frequency of parallel circuit.

And here, just as in the case of conductors, appears temptation, to name the value

$$C^*(\omega) = C\left(1 - \frac{\omega_0^2}{\omega^2}\right) = C - \frac{1}{\omega^2 L} \qquad (3.2)$$

by the depending on the frequency capacity. Conducting this symbol it is permissible from a mathematical point of view; however, inadmissible is awarding to it the proposed name, since. this parameter of no relation to the true capacity has and includes in itself simultaneously and capacity and the inductance of outline, which do not depend on frequency.

is accurate another point of view. Relationship (3.1) can be rewritten and differently:

$$I_\Sigma = -\frac{\left(\frac{\omega^2}{\omega_0^2} - 1\right)}{\omega L} U_0 \cos\omega t,$$

Notes

Notes

and to consider that the chain in question not at all has capacities, and consists only of the inductance depending on the frequency

$$L^*(\omega)=\frac{L}{\left(\frac{\omega^2}{\omega_0^2}-1\right)}=\frac{L}{\omega^2 LC-1} \ . \tag{3.3}$$

But, just as $C^*(\omega)$, the value of $L^*(\omega)$ cannot be called inductance, since this is the also composite parameter, which includes simultaneously capacity and inductance, which do not depend on frequency.

Using expressions (3.2) and (3.3), let us write down:

$$I_\Sigma=\omega C^*(\omega)\ U_0 \cos\omega t\,, \tag{3.4}$$

or

$$I_\Sigma=-\frac{1}{\omega L^*(\omega)}\ U_0 \cos\omega t\,. \tag{3.5}$$

The relationship (3.4) and (3.5) are equivalent, and separately mathematically completely is characterized the chain examined. But view neither $C^*(\omega)$ nor $L^*(\omega)$ by capacity and inductance are from a physical point, although they have the same dimensionality. The physical sense of their names consists of the following:

$$C^*(\omega)=\frac{\sigma_X}{\omega}\,,$$

i.e. $C^*(\omega)$ presents the relation of susceptance of this chain and frequency, and

$$L^*(\omega)=\frac{1}{\omega\sigma_X}\,,$$

it is the reciprocal value of the work of summary susceptance and frequency.

Accumulated in the capacity and the inductance energy, is determined from the relationships

$$W_C=\frac{1}{2}CU_0{}^2\,, \tag{3.6}$$

$$W_L=\frac{1}{2}LI_0{}^2\,. \tag{3.7}$$

How one should enter for enumerating the energy, which was accumulated in the outline, if at our disposal are $C^*(\omega)$ and $L^*(\omega)$? Certainly, to put these relationships in formulas (3.6) and (3.7) cannot for that reason, that these values can be both the positive and negative, and the energy, accumulated in the capacity and the inductance, is always positive. But if we for these purposes use ourselves the parameters indicated, then it is not difficult to show that the summary energy, accumulated in the outline, is determined by the expressions:

$$W_\Sigma=\frac{1}{2}\ \frac{d\sigma_X}{d\omega}\ U_0{}^2\,, \tag{3.8}$$

$$W_{\Sigma}=\frac{1}{2}\ \frac{d\sigma_{X}}{d\omega}\ U_{0}^{2}, \tag{3.8}$$

or

$$W_{\Sigma}=\frac{1}{2}\ \frac{d\left[\omega C^{*}(\omega)\right]}{d\omega}\ U_{0}^{2}, \tag{3.9}$$

or

$$W_{\Sigma}=\frac{1}{2}\ \frac{d\left(\dfrac{1}{\omega L^{*}(\omega)}\right)}{d\omega}\ U_{0}^{2}. \tag{3.10}$$

If we paint equations (3.8) or (3.9) and (3.10), then we will obtain identical result, namely:

$$W_{\Sigma}=\frac{1}{2}CU_{0}^{2}+\frac{1}{2}LI_{0}^{2}$$

where $U_0$ - amplitude of stress on the capacity, and $I_0$- amplitude of the current, which flows through the inductance.

If we compare the relationships, obtained for the parallel resonant circuit and for the conductors, then it is possible to see that they are identical, if we make of $E_0 \to U_0$, $j_0 \to I_0$, $\varepsilon_0 \to C$ and $L_k \to L$. Thus, the single volume of conductor, with the uniform distribution of electrical pour on and current densities in it, it is equivalent to parallel resonant circuit with the lumped parameters indicated. In this case the capacity of this outline is numerically equal to the dielectric constant of vacuum, and inductance is equal to the specific kinetic inductance of charges.

A now let us visualize this situation. In the audience, where are located specialists, who know radio engineering and of mathematics, comes instructor and he begins to prove, that there are in nature of no capacities and inductances, and there is only depending on the frequency capacity and that just she presents parallel resonant circuit. Or, on the contrary, that parallel resonant circuit this is the depending on the frequency inductance. View of mathematics will agree from this point. However, radio engineering they will calculate lecturer by man with the very limited knowledge. Specifically, in this position proved to be now those scientists and the specialists, who introduced into physics the frequency dispersion of dielectric constant.

## IV. Transverse Plasma Resonance

Longitudinal mechanical Langmuir resonance is observed during the imposition on the plasma of ac field in it. Physical understanding of the processes, proceeding in the plasma made possible to open the previously unknown phenomenon, which taught name transverse plasma resonance, for the first time described in the article [14].
Let us examine in more detail this phenomenon.

Is known that the plasma resonance is longitudinal. But longitudinal resonance cannot emit transverse electromagnetic waves. However, with the explosions of nuclear charges, as a result of which is formed very hot plasma, occurs electromagnetic radiation in the very wide frequency band, up to the long-wave radio-frequency band. Today are not known those of the physical mechanisms, which could explain the

Ref

14. Никольский В. В., Никольская Т. И. Электродинамика и распространение радиоволн М: Наука, 1989.- 543 с.

appearance of this emission. On existence in the nonmagnetized plasma of any other resonances, except Langmuir, earlier known it was not, but it occurs that in the confined plasma the transverse resonance can exist, and the frequency of this resonance coincides with the frequency of Langmuir resonance, i.e., these razonansy are degenerate. Specifically, this resonance can be the reason for the emission of electromagnetic waves with the explosions of nuclear charges.

Notes

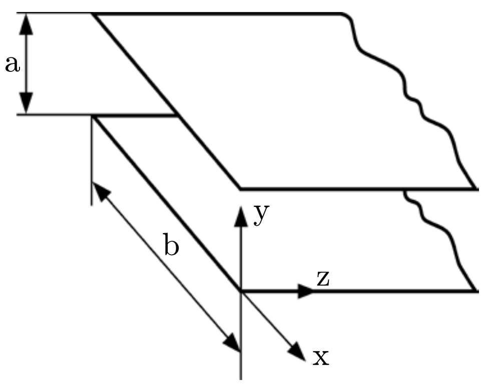


*Fig. 1:* The two-wire circuit, which consists of two ideally conducting planes

For explaining the conditions for the excitation of this resonance let us examine the long line, which consists of two ideally conducting planes, as shown in Fig 1.

Linear (falling per unit of length) capacity and inductance of this line without taking into account edge effects they are determined by the relationships:

$$C_0 = \varepsilon_0 \frac{b}{a}$$

$$L_0 = \mu_0 \frac{a}{b}.$$

Therefore with an increase in the length of line its total capacitance $C_\Sigma = \varepsilon_0 \frac{b}{a} z$ and summary inductance $L_\Sigma = \mu_0 \frac{a}{b} z$ increase proportional to its length.

If we into the extended line place the plasma, charge carriers in which can move without the losses, and in the transverse direction pass through the plasma the current $I$, then charges, moving with the definite speed, will accumulate kinetic energy. Let us note that here are not examined technical questions, as and it is possible confined plasma between the planes of line how. In this case only fundamental questions, which are concerned transverse plasma resonance in the nonmagnetic plasma, are examined. Since the transverse current density in this line is determined by the relationship

$$j = \frac{I}{bz} = nev$$

that summary kinetic energy of the moving charges can be written down

$$W_{k\Sigma} = \frac{1}{2} \frac{m}{ne^2} abzj^2 = \frac{1}{2} \frac{m}{ne^2} \frac{a}{bz} I^2. \tag{4.1}$$

Relationship (4.1) connects the kinetic energy, accumulated in the line, with the square of current; therefore the coefficient, which stands in the right side of this relationship before the square of current, is the summary kinetic inductance of line.

$$L_{k\Sigma} = \frac{m}{ne^2} \cdot \frac{a}{bz} . \qquad (4.2)$$

Thus, The value

$$L_k = \frac{m}{ne^2} \qquad (4.3)$$

presents the specific kinetic inductance of charges. Relationship (4.3) is obtained for the case of the direct current, when current distribution is uniform.

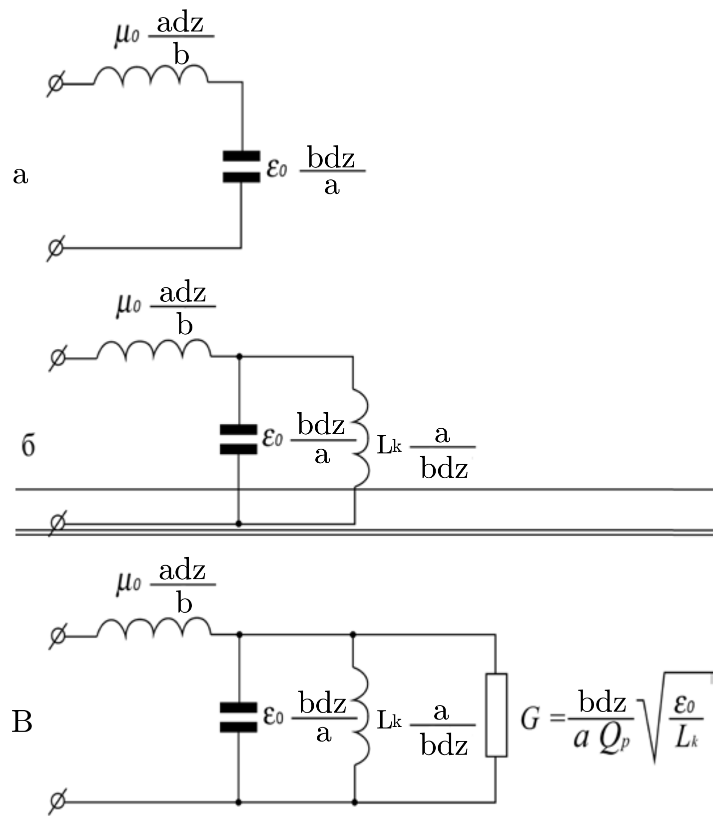


*Fig. 2:* а - the equivalent the schematic of the section of the two-wire circuit

б - the equivalent the schematic of the section of the two-wire circuit, filled with nondissipative plasma;

в - the equivalent the schematic of the section of the two-wire circuit, filled with dissipative plasma.

Subsequently for the larger clarity of the obtained results, together with their mathematical idea, we will use the method of equivalent diagrams. The section, the lines examined, long $dz$ can be represented in the form the equivalent diagram, shown in Fig. 2 (a).

From relationship (4.2) is evident that in contrast to $C_\Sigma$ and $L_\Sigma$ the value $L_{k\Sigma}$ with an increase in $z$ does not increase, but it decreases. Connected this with the fact that with an increase in $z$ a quantity of parallel-connected inductive elements grows.

Notes

The equivalent the schematic of the section of the line, filled with non dissipative plasma, it is shown in Fig. 2(б). Line itself in this case will be equivalent to parallel circuit with the lumped parameters:

Notes

$$C = \frac{\varepsilon_0 bz}{a},$$

$$L = \frac{L_k a}{bz}$$

in series with which is connected the inductance

$$\mu_0 \frac{adz}{b}.$$

But if we calculate the resonance frequency of this outline, then it will seem that this frequency generally not on what sizes depends, actually:

$$\omega_\rho^2 = \frac{1}{CL} = \frac{1}{\varepsilon_0 L_k} = \frac{ne^2}{\varepsilon_0 m}.$$

Is obtained the very interesting result, which speaks, that the resonance frequency macroscopic of the resonator examined does not depend on its sizes. Impression can be created, that this is plasma resonance, since. the obtained value of resonance frequency exactly corresponds to the value of this resonance. But it is known that the plasma resonance characterizes longitudinal waves in the long line they, while occur transverse waves. In the case examined the value of the phase speed in the direction $z$ is equal to infinity and the wave vector $\vec{k}=0$.

In this case the wave number is determined by the relationship:

$$k_z^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_\rho^2}{\omega^2}\right) \quad (4.4)$$

and the group and phase speeds

$$v_g^2 = c^2\left(1 - \frac{\omega_\rho^2}{\omega^2}\right) \quad (4.5)$$

$$v_F^2 = \frac{c^2}{\left(1 - \frac{\omega_\rho^2}{\omega^2}\right)} \quad (4.6)$$

where $c = \left(\frac{1}{\mu_0 \varepsilon_0}\right)^{1/2}$ - speed of light in the vacuum.

For the present instance the phase speed of electromagnetic wave is equal to infinity, which corresponds to transverse resonance at the plasma frequency. Consequently, at each moment of time pour on distribution and currents in this line uniform and it does not depend on the coordinate $z$, but current in the planes of line in the direction $z$ is absent. This, from one side, it means that the inductance $L_\Sigma$ will not have effects on electrodynamic processes in this line, but instead of the conducting planes can be used any planes or devices, which limit plasma on top and from below.

From relationships (4.4), (4.5) and (4.6) is evident that at the point $\omega = \omega_p$ occurs the transverse resonance with the infinite quality. With the presence of losses in the resonator will occur the damping, and in the long line in this case $k_z \neq 0$, and in the line will be extended the damped transverse wave, the direction of propagation of which will be normal to the direction of the motion of charges. It should be noted that the fact of existence of this resonance is not described by other authors.

Before to pass to the more detailed study of this problem, let us pause at the energy processes, which occur in the line in the case of the absence of losses examined.

Pour on the characteristic impedance of plasma, which gives the relation of the transverse components of electrical and magnetic, let us determine from the relationship:

$$Z = \frac{E_y}{H_x} = \frac{\mu_0 \omega}{k_z} = Z_0 \left(1 - \frac{\omega_\rho^2}{\omega^2}\right)^{-1/2},$$

where $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ - characteristic (wave) resistance of vacuum.

The obtained value $Z$ is characteristic for the transverse electrical waves in the waveguides. It is evident that when $\omega \to \omega_p$, then $Z \to \infty$, and $H_x \to 0$. When $\omega$ ¿ $\omega_p$ of in the plasma there is electrical and magnetic component of field. The specific energy of these pour on it will be written down:

$$W_{E,H} = \frac{1}{2}\varepsilon_0 E_{0y}^2 + \frac{1}{2}\mu_0 H_{0x}^2$$

Thus, the energy, concluded in the magnetic field, in $\left(1 - \frac{\omega_\rho^2}{\omega^2}\right)$ of times is less than the energy, concluded in the electric field. Let us note that this examination, which is traditional in the electrodynamics, is not complete, since. in this case is not taken into account one additional form of energy, namely kinetic energy of charge carriers. Occurs that pour on besides the waves of electrical and magnetic, that carry electrical and magnetic energy, in the plasma there exists even and the third - kinetic wave, which carries kinetic energy of current carriers. The specific energy of this wave is written:

$$W_k = \frac{1}{2} L_k j_0^2 = \frac{1}{2} \cdot \frac{1}{\omega^2 L_k} E_0^2 = \frac{1}{2}\varepsilon_0 \frac{\omega_\rho^2}{\omega^2} E_0^2 .$$

Thus, total specific energy is written as

$$W_{E,H,j} = \frac{1}{2}\varepsilon_0 E_{0y}^2 + \frac{1}{2}\mu_0 H_{0x}^2 + \frac{1}{2} L_k j_0^2 .$$

Thus, for finding the total energy, by the prisoner per unit of volume of plasma, calculation only pour on $E$ and $H$ it is insufficient.

Notes

At the point of $\omega = \omega_p$ are carried out the relationship:

$$W_H = 0$$

$$W_E = W_k$$

Notes

i.e. magnetic field in the plasma is absent, and plasma presents macroscopic electromechanical resonator with the infinite quality, resounding at the frequency $\omega_p$.

Since with the frequencies of ¿ of the wave, which is extended in the plasma, it bears on itself three forms of the energy: electrical, magnetic and kinetic, then this wave can be named [elektromagnitokineticheskoy]. Kinetic wave is the wave of the current density $\vec{j} = \frac{1}{L_k} \int \vec{E}\ dt$. This wave is moved with respect to the electrical wave the angle $\frac{\pi}{2}$.

Until now, the physically unrealizable case has been considered, when there are no losses in the plasma, which corresponds to an infinite Q-factor of the plasma resonator.If losses are located, moreover completely it does not have value, by what physical processes such losses are caused, then the quality of plasma resonator will be finite quantity. For this case Maxwell's equation they will take the form:

$$\begin{aligned} rot\ \vec{E} &= -\mu_0 \frac{\partial\ \vec{H}}{\partial\ t} \\ rot\ \vec{H} &= \sigma_{p.ef}\ \vec{E} + \varepsilon_0 \frac{\partial\ \vec{E}}{\partial\ t} + \frac{1}{L_k} \int \vec{E}\ dt \end{aligned} \tag{4.7}$$

The presence of losses is considered by the term $\sigma_{p.ef} \vec{E}$, and, using near the conductivity of the index $ef$, it is thus emphasized that us does not interest very mechanism of losses, but only very fact of their existence interests. The value $\sigma_{ef}$ determines the quality of plasma resonator. For measuring $\sigma_{ef}$ should be selected the section of line by the length $z_0$, whose value is considerably lower than the wavelength in the plasma. This section will be equivalent to outline with the lumped parameters:

$$C = \varepsilon_0 \frac{bz_0}{a} \tag{4.8}$$

$$L = L_k \frac{a}{bz_0} \tag{4.9}$$

$$G = \sigma_{\rho.ef} \frac{bz_0}{a} \tag{4.10}$$

where $G$ - conductivity, connected in parallel $C$ and $L$.

Conductivity and quality in this outline enter into the relationship:

$$G = \frac{1}{Q_\rho} \sqrt{\frac{C}{L}},$$

from where, taking into account (4.8 - 4.10), we obtain

$$\sigma_{\rho.ef} = \frac{1}{Q_\rho}\sqrt{\frac{\varepsilon_0}{L_k}} \tag{4.11}$$

Thus, measuring its own quality plasma of the resonator examined, it is possible to determine $\sigma_{p.ef}$ . Using (4.2) and (4.11) we will obtain

$$\begin{aligned} rot\ \vec{E} &= -\mu_0 \frac{\partial\ \vec{H}}{\partial\ t} \\ rot\ \vec{H} &= \frac{1}{Q_\rho}\sqrt{\frac{\varepsilon_0}{L_k}}\ \vec{E} + \varepsilon_0 \frac{\partial\ \vec{E}}{\partial\ t} + \frac{1}{L_k}\int \vec{E}\ dt \end{aligned} \tag{4.12}$$

The equivalent the schematic of this line, filled with dissipative plasma, is represented in 3 (в).

Let us examine the solution of system of equations (4.12) at the point $\omega=\omega_p$, in this case, since

$$\varepsilon_0 \frac{\partial\ \vec{E}}{\partial\ t} + \frac{1}{L_k}\int \vec{E}\ dt = 0,$$

we obtain

$$rot\ \vec{E} = -\mu_0 \frac{\partial\ \vec{H}}{\partial\ t},$$

$$rot\ \vec{H} = \frac{1}{Q_P}\sqrt{\frac{\varepsilon_0}{L_k}}\ \vec{E}.$$

These relationships determine wave processes at the point of resonance.

If losses in the plasma, which fills line are small, and strange current source is connected to the line, then it is possible to assume:

$$\begin{aligned} & rot\ \vec{E} = 0 \\ & \frac{1}{Q_p}\sqrt{\frac{\varepsilon_0}{L_k}}\ \vec{E} + \varepsilon_0 \frac{\partial\ \vec{E}}{\partial\ t} + \frac{1}{L_k}\int \vec{E}\ dt = \vec{j}_{CT} \end{aligned} \tag{4.13}$$

where $\vec{j}_{CT}$ - density of strange currents.

After integrating (7.13) with respect to the time and after dividing both parts to, we will obtain

$$\omega_p^2\ \vec{E} + \frac{\omega_p}{Q_p}\cdot\frac{\partial\ \vec{E}}{\partial\ t} + \frac{\partial^2 \vec{E}}{\partial\ t^2} = \frac{1}{\varepsilon_0}\cdot\frac{\partial\ \vec{j}_{CT}}{\partial\ t} \tag{4.14}$$

if we (4.2) integrate over the surface of normal to the vector of and to designate

Notes

$$\omega_p^2 P_E + \frac{\omega_p}{Q_p} \cdot \frac{\partial P_E}{\partial t} + \frac{\partial^2 P_E}{\partial t^2} = \frac{1}{\varepsilon_0} \cdot \frac{\partial I_{CT}}{\partial t} \tag{4.15}$$

where $I_{CT}$ - strange current.

Notes

Equation (4.15) is the equation of harmonic oscillator with the right side, characteristic for the two-level lasers, which opens the possibility of designing of the lasers of large power.

## V. Electrodynamics Of The Dielectrics

Let us examine the simplest case, when oscillating processes in atoms or molecules of dielectric obey the law of mechanical oscillator [15].

$$\left(\frac{\beta}{m} - \omega^2\right)\mathbf{r}_m = \frac{e}{m}\mathbf{E}, \tag{5.1}$$

where $\mathbf{r}_m$ - deviation of charges from the position of equilibrium, $\beta$ - coefficient of elasticity, which characterizes the elastic electrical binding forces of charges in the atoms and the molecules. Introducing the resonance frequency of the bound charges

$$\omega_0 = \frac{\beta}{m},$$

we obtain from (5.1):

$$\mathbf{r}_m = -\frac{e\,\mathbf{E}}{m(\omega^2 - \omega_o^2)}. \tag{5.2}$$

Is evident that in relationship (5.2) as the parameter is present the natural vibration frequency, into which enters the mass of charge. This speaks, that the inertia properties of the being varied charges will influence oscillating processes in the atoms and the molecules.

Since the general current density on medium consists of the bias current and conduction current

$$\operatorname{rot}\mathbf{H} = \mathbf{j}_\Sigma = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + ne\mathbf{v},$$

that, finding the speed of charge carriers in the dielectric as the derivative of their displacement through the coordinate

$$\mathbf{v} = \frac{\partial \mathbf{r}_m}{\partial t} = -\frac{e}{m(\omega^2 - \omega_o^2)} \frac{\partial \mathbf{E}}{\partial t},$$

from relationship (5.2) we find

$$\operatorname{rot}\mathbf{H} = \mathbf{j}_\Sigma = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} - \frac{1}{L_{kd}(\omega^2 - \omega_0^2)} \frac{\partial \mathbf{E}}{\partial t}. \tag{5.3}$$

Let us note that the value

$$L_{kd} = \frac{m}{ne^2}$$

presents the kinetic inductance of the charges, entering the constitution of atom or molecules of dielectrics, when to consider charges free. Therefore (5.3) let us rewrite in the form:

$$\text{rot}\,\mathbf{H} = \mathbf{j}_\Sigma = \varepsilon_0\left(1 - \frac{1}{\varepsilon_0 L_{kd}(\omega^2 - \omega_0{}^2)}\right)\frac{\partial \mathbf{E}}{\partial t}. \tag{5.4}$$

Since the value

$$\frac{1}{(\varepsilon_0 L_{kd})} = \omega_{pd}{}^2$$

it represents the plasma frequency of charges in atoms and molecules of dielectric, if we consider these charges free, then relationship (9.4) takes the form:

$$\text{rot}\,\mathbf{H} = \mathbf{j}_\Sigma = \varepsilon_0\left(1 - \frac{\omega^2{}_{pd}}{(\omega^2 - \omega_0{}^2)}\right)\frac{\partial \mathbf{E}}{\partial t}. \tag{5.5}$$

It is possible to name the value

$$\varepsilon^*(\omega) = \varepsilon_0\left(1 - \frac{\omega^2{}_{pd}}{(\omega^2 - \omega_0{}^2)}\right) \tag{5.6}$$

by the effective dielectric constant of dielectric and it depends on frequency. But this mathematical parameter is not the physical dielectric constant of dielectric, but has composite nature. It includes three those not depending on the frequency of the value: electrical constant, natural frequency of atoms or molecules and plasma frequency for the charge carriers, entering their composition, if we consider charges free.

Let us examine two limiting cases:

*a) If $\omega << \omega_0$ then from (5.6) we obtain*

$$\text{rot}\,\mathbf{H} = \mathbf{j}_\Sigma = \varepsilon_0\left(1 + \frac{\omega_{pd}{}^2}{\omega_0{}^2}\right)\frac{\partial \mathbf{E}}{\partial t}. \tag{5.7}$$

In this case the coefficient, confronting the derivative, does not depend on frequency, and it presents the static dielectric constant of dielectric. As we see, it depends on the natural frequency of oscillation of atoms or molecules and on plasma frequency. This result is intelligible. Frequency in this case proves to be such low that the charges manage to follow the field and their inertia properties do not influence electrodynamic processes. In this case the bracketed expression in the right side of

Notes

relationship (5.7) presents the static dielectric constant of dielectric. As we see, it depends on the natural frequency of oscillation of atoms or molecules and on plasma frequency. Hence immediately we have a prescription for creating the dielectrics with the high dielectric constant. In order to reach this, should be in the assigned volume of space packed a maximum quantity of molecules with maximally soft connections between the charges inside molecule itself.

Notes

*b) The case is exponential* $\omega >> \omega_0$, *in this case*

$$\mathrm{rot}\,\mathbf{H} = \mathbf{j}_\Sigma = \varepsilon_0\left(1 - \frac{\omega_{pd}{}^2}{\omega^2}\right)\frac{\partial \mathbf{E}}{\partial t}$$

and dielectric became conductor (plasma) since. the obtained relationship exactly coincides with the equation, which describes plasma.

One cannot fail to note the circumstance that in this case again nowhere was used this concept as polarization vector, but examination is carried out by the way of finding the real currents in the dielectrics on the basis of the equation of motion of charges in these media. In this case in this mathematical model as the initial electrical characteristics of medium are used the values, which do not depend on frequency.

From relationship (5.5) is evident that in the case of fulfilling the equality of $\omega = \omega_0$, the amplitude of fluctuations is equal to infinity. This indicates the presence of resonance at this point. The infinite amplitude of fluctuations occurs because of the fact that they were not considered losses in the resonance system, in this case its quality was equal to infinity. In a certain approximation it is possible to consider that lower than the point indicated we deal concerning the dielectric, whose dielectric constant is equal to its static value. Higher than this point we deal already actually concerning the metal, whose density of current carriers is equal to the density of atoms or molecules in the dielectric.

Now it is possible to examine the question of why dielectric prism decomposes polychromatic light into monochromatic components or why rainbow is formed. For this the phase speed of electromagnetic waves on Medium must depend on frequency (frequency wave dispersion). Let us add to (5.5) the first Maxwell equation:

$$\mathrm{rot}\,\mathbf{E} = -\mu_0\frac{\partial \mathbf{H}}{\partial t}; \quad \mathrm{rot}\,\mathbf{H} = \varepsilon_0\left(1 - \frac{\omega^2{}_{pd}}{(\omega^2 - \omega_0{}^2)}\right)\frac{\partial \mathbf{E}}{\partial t},$$

from where we immediately find the wave equation:

$$\nabla^2\mathbf{E} = \mu_0\varepsilon_0\left(1 - \frac{\omega_{pd}{}^2}{\omega^2 - \omega_0{}^2}\right)\frac{\partial^2 \mathbf{E}}{\partial t^2}. \tag{5.8}$$

If one considers that

$$\mu_0\varepsilon_0 = \frac{1}{c^2},$$

where $c$ - the speed of light, then is easy to see the presence in dielectrics of frequency dispersion. But the dependence of phase speed on the frequency is connected not with

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the dependence on it of physical dielectric constant. In the formation of this dispersion it will participate immediately three, which do not depend on the frequency, physical quantities: the self-resonant frequency of atoms themselves or molecules, the plasma frequency of charges, if we consider it their free, and the dielectric constant of vacuum.

Let us accept that $div\mathbf{E}=0$ , then from (5.8) we obtain

$$\frac{\partial^2\mathbf{E}}{\partial z^2}=\frac{1}{c^2}\left(1-\frac{{\omega_{pd}}^2}{\omega^2-{\omega_0}^2}\right)\frac{\partial^2\mathbf{E}}{\partial t^2}.$$

This equation is reduced to the standard form

$$\frac{\partial^2\mathbf{E}}{\partial z^2}=\frac{1}{{v_d}^2}\frac{\partial^2\mathbf{E}}{\partial t^2}$$

if we place the velocity of propagation EM of wave in the dielectric

$$v_d=\frac{c}{\sqrt{\left(1-\frac{{\omega_{pd}}^2}{\omega^2-{\omega_0}^2}\right)}}.$$

Let us show that the equivalent the schematic of dielectric presents the sequential resonant circuit, whose inductance is the kinetic inductance $L_{kd}$, and capacity is equal to the static dielectric constant of dielectric minus the capacity of the equal dielectric constant of vacuum. In this case outline itself proves to be that shunted by the capacity, equal to the specific dielectric constant of vacuum. For the proof of this let us examine the sequential oscillatory circuit, when the inductance $L$ and the capacity $C$ are connected in series.

The connection between the current $I_C$, which flows through the capacity $C$, and the voltage of $U_C$, applied to it, is determined by the relationships:

$$U_C=\frac{1}{C}\int I_C dt,$$

$$I_C=C\frac{dU_C}{dt}. \qquad (5.10)$$

This connection will be written down for the inductance:

$$I_L=\frac{1}{L}\int U_L dt,$$

$$U_L=L\frac{dI_L}{dt}.$$

If the current, which flows through the series circuit, changes according to the law $I=I_0\sin\omega t$ then a voltage drop across inductance and capacity they are determined by the relationships

Notes

$$U_L = \omega L I_0 \cos\omega t,$$

$$U_C = -\frac{1}{\omega C} I_0 \cos\omega t,$$

Notes

and total stress applied to the outline is equal

$$U_\Sigma = \left(\omega L - \frac{1}{\omega C}\right) I_0 \cos\omega t.$$

In this relationship the value, which stands in the brackets, presents the reactance of sequential resonant circuit, which depends on frequency. The stresses, generated on the capacity and the inductance, are located in the reversed phase, and, depending on frequency, outline can have the inductive, the whether capacitive reactance. At the point of resonance the summary reactance of outline is equal to zero.

It is obvious that the connection between the total voltage applied to the outline and the current, which flows through the outline, will be determined by the relationship

$$I = -\frac{1}{\omega\left(\omega L - \frac{1}{\omega C}\right)} \frac{\partial U_\Sigma}{\partial t}. \quad (5.11)$$

The resonance frequency of outline is determined by the relationship

$$\omega_0 = \frac{1}{\sqrt{LC}},$$

therefore let us write down

$$I = \frac{C}{\left(1 - \frac{\omega^2}{\omega_0{}^2}\right)} \frac{\partial U_\Sigma}{\partial t}. \quad (5.12)$$

Comparing this expression with relationship (5.12) and (5.10) it is not difficult to see that the sequential resonant circuit, which consists of the inductance $L$ and capacity $C$, it is possible to present to the capacity of in the form dependent on the frequency

$$C(\omega) = \frac{C}{1 - \frac{\omega^2}{\omega_0{}^2}} \quad (5.13)$$

The inductance is not lost with this idea, since it enters into the resonance frequency of the outline $\omega_0$. Relationships (5.12) and (5.11) are equivalent. Consequently, value $C(\omega)$ is not the physical capacitance value of outline, being the certain composite mathematical parameter.

Relationship (5.11) can be rewritten and differently:

$$I = -\frac{1}{L\left(\omega^2 - \omega_0{}^2\right)} \frac{\partial U_\Sigma}{\partial t}$$

and to consider that

$$C(\omega) = -\frac{1}{L\left(\omega^2 - \omega_0{}^2\right)}. \tag{5.14}$$

Notes

Is certain, the parameter $C(\omega)$, introduced in accordance with relationships (5.13) and (5.14) no to capacity refers.

Let us examine relationship (9.12) for two limiting cases:

*c) When $\omega << \omega_0$,, we have*

$$I = C\frac{\partial U_\Sigma}{\partial t}.$$

This result is intelligible, since. at the low frequencies the reactance of the inductance, connected in series with the capacity, is considerably lower than the capacitive and it is possible not to consider it.

The equivalent the schematic of the dielectric, located between the planes of long line is shown in Fig. 3

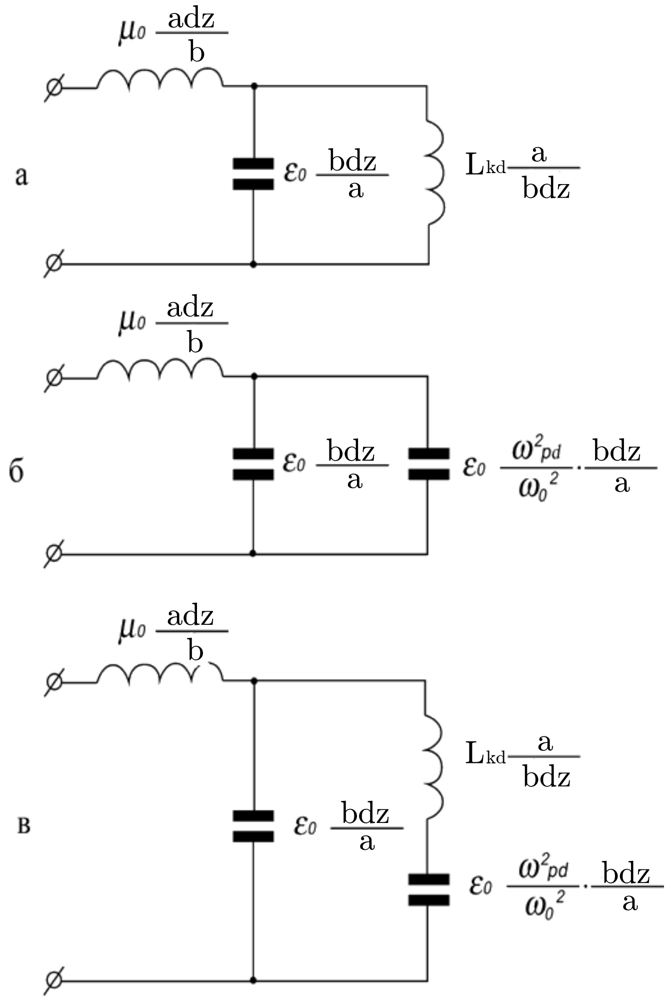


*Fig. 3:* a - equivalent the schematic of the section of the line, filled with dielectric, for the case $\omega ¿¿ \omega_0$;

б - equivalent the schematic of the section of line for the case $\omega ¡¡ \omega_0$;

в - the equivalent the schematic of the section of line for entire frequency band.

*d) When $\omega >> \omega_0$, we have*

$$I = -\frac{1}{\omega^2 L}\frac{\partial U_\Sigma}{\partial t} \tag{5.15}$$

Taking into account that for the harmonic signal

$$\frac{\partial U_\Sigma}{\partial t} = -\omega^2 \int U_\Sigma dt ,$$

we obtain from (5.2):

$$I_L = \frac{1}{L}\int U_\Sigma dt .$$

In this case the reactance of capacity is considerably less than in inductance and chain has inductive reactance.

The carried out analysis speaks, that is in practice very difficult to distinguish the behavior of resonant circuits of the inductance or of the capacity. For understanding of true design of circuits it is necessary to remove its amplitude and phase response in the range of frequencies. In the case of resonant circuit this dependence will have the typical resonance nature, when on both sides resonance the nature of reactance is different. However, this does not mean that real circuit elements: capacity or inductance depend on frequency.

Notes

In Fig. 3 a and 5 б are shown two limiting cases. $\omega$¿¿$\omega_0$, when the properties of dielectric correspond to conductor; $\omega$¡¡$\omega_0$ , when - to dielectric with the static dielectric constant

$$\varepsilon = \varepsilon_0 \left( 1 + \frac{{\omega_{pd}}^2}{{\omega_0}^2} \right).$$

Thus, the use of a term "dielectric constant of dielectrics" in the context of its dependence on the frequency is not completely correct. If the discussion deals with the dielectric constant of dielectrics, with which the accumulation of potential energy is connected, then correctly examine only static permeability, which is been the constant, which does not depend on the frequency. Specifically, it enters into all relationships, which characterize the electrodynamic characteristics of dielectrics.

Application of such new approaches most interestingly precisely for the dielectrics. Then each connected pair of charges is a separate unitary unit with its individual characteristics, and its interaction with the electromagnetic field (without taking into account the connections between the pairs) is strictly individual. Certainly, in the dielectrics not all dipoles have different characteristics, but there are different groups with similar characteristics, and each group of bound charges with the identical characteristics will resound at its frequency. Moreover the intensity of absorption, and in the excited state and emission, at this frequency will depend on a relative quantity of pairs of this type. Therefore it is possible to introduce the appropriate partial coefficients. Furthermore, these processes will influence the anisotropy of the dielectric properties of molecules themselves, which have the specific electrical orientation in crystal lattice. By these circumstances is determined the variety of resonances and their intensities, which is observed in the dielectric media. With the electric coupling between the separate groups of emitters the lines of absorption or emission can be converted into the strips. Such individual approach to the types of the connected pairs of charges is absent from the available theories.

Let us emphasize the important circumstance, which did not receive thus far proper estimation. In all relationships for any material media (conductors and dielectrics) together with the dielectric and magnetic constant figures the kinetic inductance of the charges, which indicates not less important role of this parameter.

In the works [3-6] the role of the kinetic inductance of charges in the electrodynamic processes, which occur in the conductors and the plasmo-like media is in sufficient detail opened, but the role of this parameter in the electrodynamics of dielectrics is not opened. This parameter in the electrodynamics of dielectrics plays not less important role, than in the electrodynamics of conductors. In this division the electrodynamics of dielectrics taking into account the kinetic inductance of the charges,

Notes

which form part of their atoms or molecules is examined. This most important question fell out from the field of the sight of scientists, and this article completes this deficiency. Let us emphasize this important circumstance, which did not receive thus far proper estimation. In all relationships for any material media (conductors and dielectrics) together with the dielectric and magnetic constant figures the kinetic inductance of the charges, which indicates not less important role of this parameter.

## VI. Conclusion

In the article are examinednew approaches to the solution of the problem of the propagation of electrical energy fluxes in the material media and the long lines it is examined the electrodynamics of plasma and shown that the absolute value of the vector Of poytinga can be obtained with the examination of the motion of specific electric field energy and kinetic kinetic energy of the charges, concentrated in the single volumes of plasma. Is obtained wave of equation for the plasma. The electrodynamics of dielectrics is examined and is obtained wave equation for them. Are examined processes occurring in the long lines, filled with plasma or dielectrics and predicted new phenomenon transverse plasma resonance in the limited nonmagnetized plasma. The use of transverse plasma resonance opens the possibility of designing of the lasers of large power.

## References Références Referencias


1. Арцимович Л А Что каждый физик должен знать о плазме (М.: Атомиздат, 1976)
2. Менде Ф Ф Спицын А И Поверхностный импеданс сверхпроводников(Киев, Наукова думка, 1985)
3. Менде Ф Ф Существуют ли ошибки в современной физике (Харьков, Константа, 2003)
4. Менде Ф Ф Непротиворечивая электродинамика (Харьков, НТМТ, 2008)
5. Mende F FarXiv: physics/0402084
6. Менде Ф. Ф. Роль и место кинетической индуктивности зарядов в классической электродинамике,Инженерная физика, №11, 2012. с. 10-19.
7. Александров А Ф Богданкевич Л С Рухадзе А АКолебания и волны в плазменных средах (Изд. Московского университета, 1990)
8. Ландау Л Д Лифшиц Е М Электродинамика сплошных сред (М: Наука, 1982)
9. Гинзбург В Л Распространение электромагнитных волн в плазме (М.: Наука. 1967)
10. Ахиезер А И Физика плазмы (М: Наука, 1974)
11. Тамм И Е Основы теории электричества (М.: Наука, 1989)
12. London F Superfluids. Vol.1. Microscopic theory of superconductivity (Nev York: Dower publ., 1950)
13. Mende F. F. Transversal plasma resonance in a nonmagnetized plasma and possibilities of practical employment of it. arXiv.org/abs/physics//0506081.
14. Никольский В. В., Никольская Т. И. Электродинамика и распространение радиоволн М: Наука, 1989.- 543 с.

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# On A Subclass of Certain Convex Harmonic Univalent Functions Related to Q-Derivative

By Hamid Shamsan & S. Latha
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GJSFR-F Classification: *FOR Code: MSC 2010: 35E10*

ONASUBCLASSOFCERTAINCONVEXHARMONICUNIVALENTFUNCTIONSRELATEDTOQDERIVATIVE

*Strictly as per the compliance and regulations of:*

# On A Subclass of Certain Convex Harmonic Univalent Functions Related to Q-Derivative

Hamid Shamsan [α] & S. Latha [σ]

*Abstract-* We define and investigate a new class of harmonic functions defined by q -derivative. We give univalence criteria and sufficient coefficient conditions for normalized q -harmonic functions that are convex of order $\beta, 0 \leq \beta < 1$. We obtain coefficient inequalities, extreme points distortion bounds, convolution and convex combination condition, and covering theorems for these functions. Further, we obtain the closure property of this class under integral operator.

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## I. Introduction

Harmonic functions are famous for their use in the study of minimal surfaces and also play important roles in a variety of problems in applied mathematics (e.g. see Choquet [2], Dorff [4] , Duren [5]). A continuous complex-valued function $f = u + iv$ defined in a domain $\mathscr{D} \subseteq \mathbb{C}$ is a harmonic in $\mathscr{D}$ if $u$ and $v$ are real harmonic in $\mathscr{D}$. We call $h$ the analytic part and $g$ the co-analytic part of $f$. In any simply connected domain we can write $f = h + \bar{g}$, where $g$ and $h$ are analytic and $\bar{g}$ denotes the function $z \to \overline{g(z)}$. Clunie and Sheil-Small [3] pointed out that a necessary and sufficient condition for $f$ to be locally univalent and sense preserving in $\mathscr{D}$ is that $|h'(z)| > |g'(z)|$ in $\mathscr{D}$. Denote by $H$ the class of functions $f$ of the form

$$h(z) = z + \sum_{k=2}^{\infty} a_k z^k \,, \qquad g(z) = \sum_{k=1}^{\infty} b_k z^k, \qquad |b_1| < 1, \tag{1}$$

that are harmonic univalent and sense-preserving in the unit disk $\mathscr{U} = \{z : z \in \mathbb{C} \,,\ |z| < 1\}$ for which $f(0) = f_z(0) - 1 = 0$.

We note that the family $H$ of orientation preserving, normalized harmonic univalent functions reduces to the well known class $S$ of normalized univalent functions in $\mathscr{U}$ , if the co-analytic part of $f$ is identically zero, that is $g \equiv 0$. For $0 \leq \beta < 1$, Let $K_H(\beta)$ be the subclass of $H$ consisting of harmonic convex functions of order $\beta$. We further denote by $K_{\overline{H}}(\beta)$ the subclass of $K_H(\beta)$ such that the functions $h$ and $g$ in $f = h + \overline{g}$ are of the form

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2. G. Choquet, Sur un type de transformation analytique gnralisant la reprsentation conforme et dfinie au moyen de fonctions harmoniques, Bull. Sci. Math., 89(1945), 156-165.

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$$h(z) = z - \sum_{k=2}^{\infty} |a_k| z^k, \qquad g(z) = -\sum_{k=1}^{\infty} |b_k| z^k, \quad |b_1| < 1. \tag{2}$$

Jackson[7] initiated $q$-calculus and developed the concept of the $q$-integral and $q$-derivative. For a function $f \in S$ given by (1) and $0 < q < 1$, the $q$-derivative of $f$ is defined by

**Definition 1.1.**

$$\partial_q f(z) = \begin{cases} \frac{f(z)-f(qz)}{z(1-q)}, & z \neq 0, \\ f'(0), & z = 0. \end{cases} \quad , \quad where\ (0 < q < 1) \tag{3}$$

Equivalently (3), may be written as

$$\partial_q f(z) = 1 + \sum_{k=2}^{\infty} [k]_q a_k z^{k-1}, \quad z \neq 0$$

where

$$[k]_q = \begin{cases} \frac{1-q^k}{1-q}, & q \neq 1 \\ k, & q = 1 \end{cases}$$

Note that as $q \to 1$, $[k]_q \to k$.

As a right inverse, Jackson[6] presented the $q$-integral of a function $f$ as

$$\int_0^z f(t) d_q t = z(1-q) \sum_{k=0}^{\infty} q^k f(zq^k),$$

provided that the series converges. For a function $f(z) = z^k$, we note that

$$\int_0^z f(t) d_q t = \int_0^z t^k d_q t = \frac{z^{k+1}}{[k+1]_q} \qquad (k \neq -1)$$

and

$$\lim_{q \to 1^-} \int_0^z f(t) d_q t = \lim_{q \to 1^-} \frac{z^{k+1}}{[k+1]_q} = \frac{z^{k+1}}{k+1} = \int_0^z f(t) dt,$$

Ref

7. F. H. Jackson, On q-functions and a certain difference operator, Trans. Royal Soc. Edinburgh., 46(1909), 253-281.

where $\int_0^z f(t)dt$ is the ordinary integral.

The Jacobian of $f$ by $q$-derivative is given by

$$J_f(z) = |\partial_q h(z)|^2 - |\partial_q g(z)|^2$$

Notes

The mapping $z \to f(z)$ is locally one-to-one if $J_f(z) \neq 0$ in $\mathscr{U}$. Also the converse is true for harmonic mappings, and therefore $z \to f(z)$ is locally one-to-one and sense preserving if, and only if, $|\partial_q h(z)| > |\partial_q g(z)|$.

## II. Main Results

**Definition 2.1.** *Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a continuous function of two variables and $0 < q < 1$, the partial q-derivatives at $(x,y) \in \mathbb{R}^2$ can be defined as follows*

$$\frac{\partial_q f(x,y)}{\partial_q x} = \frac{f(qx,y)-f(x,y)}{x(q-1)},$$

$$\frac{\partial_q f(x,y)}{\partial_q y} = \frac{f(x,qy)-f(x,y)}{y(q-1)},$$

*and*

$$\frac{\partial_q f(x,y)}{\partial_q x \partial_q y} = \frac{f(qx,qy)-f(qx,y)-f(x,qy)+f(x,y)}{xy(q-1)^2}.$$

*The function $f : \mathbb{R}^2 \to \mathbb{R}$ is said to be partially q-differentiable on $\mathbb{R}^2$ if $\frac{\partial_q f(x,y)}{\partial_q x}$, $\frac{\partial_q f(x,y)}{\partial_q y}$ and $\frac{\partial_q^f(x,y)}{\partial_q x \partial_q y}$ exist for all $(x,y) \in \mathbb{R}^2$. We can similarly define higher order partial q-derivatives.*

For $0 \leq \beta < 1$, let $K_H(\beta,q)$ denote the subclass of $H$ consisting of $q$-harmonic convex functions of order $\beta$.

**Definition 2.2.** *A function $f$ of the form (1) is said to be in the class $K_H(\beta,q)$ $\beta$, $0 \leq \beta < 1$, for $|z| = r < 1$ if*

$$\frac{\partial_q}{\partial_q \theta}\left\{\arg\left(\partial_q f(re^{i\theta})\right)\right\} = Im\left\{\frac{\partial_q}{\partial_q \theta}\log\left(f(re^{i\theta})\right)\right\}$$

$$= Re\left(\frac{\lambda_q|\ln q|z\partial_q h(z) + \lambda_q|\ln q|z^2\partial_q^2 h(z) + \overline{\lambda_q|\ln q|z\partial_q g(z) + \lambda_q|\ln q|z^2\partial_q^2 g(z)}}{(1-q)z\partial_q h(z) - \overline{(1-q)z\partial_q g(z)}}\right) \geq \beta, \tag{4}$$

*where $\lambda_q = \sum_{k=0}^{\infty}\frac{(i\theta(1-q))^n}{(n+1)!}$, $\lambda_q \to 1$ as $q \to 1^-$, $z = re^{i\theta}$, $0 \leq \theta < 2\pi$, and $\partial_q f = \partial_q h + \overline{\partial_q g}$.*

**Theorem 2.3.** *Let $f = h + \overline{g}$ be given by (1). If*

$$\sum_{k=1}^{\infty}\left(\frac{[k]_q\left[\lambda_q|\ln q|[k]_q - \beta(1-q)\right]}{\lambda_q|\ln q| - \beta(1-q)}|a_k| + \frac{[k]_q\left[\lambda_q|\ln q|[k]_q + \beta(1-q)\right]}{\lambda_q|\ln q| - \beta(1-q)}|b_k|\right) \leq 2, \tag{5}$$

*where* $\lambda_q = \sum_{k=0}^{\infty} \frac{(i\theta(1-q))^n}{(n+1)!}$, $\lambda_q \to 1$ *as* $q \to 1^-$, $a_1 = 1$, $0 \le \beta < 1$ *and* $0 < q < 1$. *Then* $f$ *is* $q$*-harmonic univalent in* $\mathscr{U}$, *and* $f \in K_H(\beta, q)$.

*Proof.* Note first that

$$|\partial_q h(z)| \ge 1 - \sum_{k=1}^{\infty} [k]_q |a_k| r^{k-1} > 1 - \sum_{k=1}^{\infty} [k]_q |a_k| \ge 1 - \sum_{k=2}^{\infty} \frac{[k]_q \left[\lambda_q |\ln q| [k]_q - \beta(1-q)\right]}{\lambda_q |\ln q| - \beta(1-q)} |a_k|$$

$$\ge \sum_{k=1}^{\infty} \frac{[k]_q \left[\lambda_q |\ln q| [k]_q + \beta(1-q)\right]}{\lambda_q |\ln q| - \beta(1-q)} |b_k| \ge \sum_{k=1}^{\infty} [k]_q |b_k| > \sum_{k=1}^{\infty} [k]_q |a_k| r^{k-1} \ge |\partial_q g(z)|.$$

So that $f$ is locally univalent and sense-preserving in $\mathscr{U}$. If $g \equiv 0$, the univalence of $f$ will follow from its starlikeness. Otherwise if $z_1, z_2 \in \mathscr{U}$, $z_1 \ne z_2$. Since $\mathscr{U}$ is simply connected and convex, we have $z(t) = (1-t)z_1 + tz_2 \in \mathscr{U}$, where $0 \le t \le 1$. So we can write

$$f(z_2) - f(z_1) = \int_0^1 \left[(z_2 - z_1)\partial_q h(z(t)) + \overline{(z_2 - z_1)\partial_q g(z(t))}\right] d_q t.$$

Therefore

$$Re \frac{f(z_2) - f(z_1)}{z_2 - z_1} = \int_0^1 Re \left[\partial_q h(z(t)) + \frac{\overline{z_2 - z_1}}{z_2 - z_1} \overline{\partial_q g(z(t))}\right] d_q t$$

$$> \int_0^1 \left[Re \partial_q h(z(t)) - \left|\partial_q g(z(t))\right|\right] d_q t. \tag{6}$$

Moreover

$$Re \partial_q h(z(t)) - \left|\partial_q g(z(t))\right| \ge Re \partial_q h(z(t)) - \sum_{k=1}^{\infty} [k]_q |b_k|$$

$$\ge 1 - \sum_{k=2}^{\infty} [k]_q |a_k| - \sum_{k=1}^{\infty} [k]_q |b_k|$$

$$\ge 1 - \sum_{k=2}^{\infty} \frac{[k]_q \left[\lambda_q |\ln q| [k]_q - \beta(1-q)\right]}{\lambda_q |\ln q| - \beta(1-q)} |a_k| - \sum_{k=1}^{\infty} \frac{[k]_q \left[\lambda_q |\ln q| [k]_q + \beta(1-q)\right]}{\lambda_q |\ln q| - \beta(1-q)} |b_k|$$

$$\ge 0, \qquad by (5). \tag{7}$$

Hence (6) and (7) leads to the univalence of $f$.

Now we show that $f \in K_H(\beta, q)$. It suffices to show that $\frac{\partial_q}{\partial_q \theta}\left\{\arg\left(\frac{\partial_q}{\partial_q \theta} f(re^{i\theta})\right)\right\} \ge \beta$, $0 \le \beta < 1$, $0 \le \theta < 2\pi$ and $0 < r < 1$.

Using the fact that $Re\omega \ge \beta$ if and only if $|1 - \beta + \omega| \ge |1 + \beta - \omega|$, it suffices to show that

$$|A(z) + (1-\beta)B(z)| - |A(z) - (1+\beta)B(z)| \ge 0, \tag{8}$$

where $A(z)=\lambda_q|\ln q|z\partial_q h(z)+\lambda_q|\ln q|z^2\partial_q^2 h(z)+\overline{\lambda_q|\ln q|z\partial_q g(z)+\lambda_q|\ln q|z^2\partial_q^2 g(z)}$

and $B(z)=(1-q)z\partial_q h(z)-\overline{(1-q)z\partial_q g(z)}$. Substituting for $A(z)$ and$B(z)$ in (8), we get

$|A(z)+(1-\beta)B(z)|-|A(z)-(1+\beta)B(z)|$

Notes

$$=\left|(\lambda_q|\ln q|+(1-\beta)(1-q))\,z\partial_q h(z)+\lambda_q|\ln q|z^2\partial_q^2 h(z)+\overline{(\lambda_q|\ln q|+(1-\beta)(1-q))\,z\partial_q g(z)+\lambda_q|\ln q|z^2\partial_q^2 g(z)}\right|$$

$$-\left|(\lambda_q|\ln q|-(1+\beta)(1-q))\,z\partial_q h(z)+\lambda_q|\ln q|z^2\partial_q^2 h(z)+\overline{(\lambda_q|\ln q|+(1+\beta)(1-q))\,z\partial_q g(z)+\lambda_q|\ln q|z^2\partial_q^2 g(z)}\right|$$

$$=\left|(-q+\beta q-\beta+\lambda_q|\ln q|+1)z+\sum_{k=2}^{\infty}[k]_q\left(\lambda_q|\ln q|[k]_q-q+\beta q-\beta+1\right)a_k z^k\right.$$
$$\left.+\overline{\sum_{k=1}^{\infty}[k]_q\left(\lambda_q|\ln q|[k]_q+q-\beta q+\beta-1\right)b_k z^k}\right|$$

$$-\left|(q+\beta q-\beta+\lambda_q|\ln q|+1)z+\sum_{k=2}^{\infty}[k]_q\left(\lambda_q|\ln q|[k]_q+q+\beta q-\beta-1\right)a_k z^k\right.$$
$$\left.+\overline{\sum_{k=1}^{\infty}[k]_q\left(\lambda_q|\ln q|[k]_q-q-\beta q+\beta+1\right)b_k z^k}\right|$$

$$\geq(-q+\beta q-\beta+\lambda_q|\ln q|+1)|z|-\sum_{k=2}^{\infty}[k]_q\left(\lambda_q|\ln q|[k]_q-q+\beta q-\beta+1\right)|a_k||z^k|$$

$$-\overline{\sum_{k=1}^{\infty}[k]_q\left(\lambda_q|\ln q|[k]_q+q-\beta q+\beta-1\right)|b_k||z^k|}+(q+\beta q-\beta+\lambda_q|\ln q|+1)|z|$$

$$-\sum_{k=2}^{\infty}[k]_q\left(\lambda_q|\ln q|[k]_q+q+\beta q-\beta-1\right)|a_k||z^k|-\overline{\sum_{k=1}^{\infty}[k]_q\left(\lambda_q|\ln q|[k]_q-q-\beta q+\beta+1\right)b_k z^k}$$

$$=2\left[\lambda_q|\ln q|-\beta(1-q)\right]|z|\left\{1-\sum_{k=2}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q-\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}|a_k||z|^{k-1}\right.$$
$$\left.-\sum_{k=1}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q+\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}|b_k||z|^{k-1}\right\}$$

$$\geq 2\left[\lambda_q|\ln q|-\beta(1-q)\right]|z|\left\{1-\left(\sum_{k=2}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q-\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}|a_k|\right.\right.$$
$$\left.\left.+\sum_{k=1}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q+\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}|b_k|\right)\right\}\geq 0,\ by(5).$$

The starlike harmonic mappings

$$f(z)=z+\sum_{k=2}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q-\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}x_k z^k+\sum_{k=1}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q+\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}\bar{y}_k\bar{z}^k, \tag{9}$$

where $\sum_{k=2}^{\infty}|x_k|+\sum_{k=2}^{\infty}|y_k|=1$, show that the coefficient bound given by (5) is sharp. Therefore

$$\sum_{k=1}^{\infty}\left(\frac{[k]_q\,[\lambda_q|\ln q|[k]_q-\beta(1-q)]}{\lambda_q|\ln q|-\beta(1-q)}|a_k|+\frac{[k]_q\,[\lambda_q|\ln q|[k]_q+\beta(1-q)]}{\lambda_q|\ln q|-\beta(1-q)}|b_k|\right)=1+\sum_{k=2}^{\infty}|x_k|+\sum_{k=2}^{\infty}|y_k|=2.$$

Hence $f\in K_H(\beta,q)$.

Note that when $\beta=0$ in the above theorem, we get the following corollary

**Corollary 2.4.** *Let $f=h+\overline{g}$ be given by* (1). *If*

$$\sum_{k=1}^{\infty}[k]_q^2\left(|a_k|+|b_k|\right)\leq 2, \tag{10}$$

*where $a_1=1$ and $0<q<1$. Then $f$ is q-harmonic univalent in $\mathscr{U}$, and $f\in K_H(q)$.*

Special choices, $\beta=b_1=0$, and as $q\to 1^-$ yield the following result, proved by Avci and Zlotkiewicz [1]

**Corollary 2.5.** *Let $f=h+\overline{g}$ be given by* (1). *If*

$$\sum_{k=1}^{\infty}k^2\left(|a_k|+|b_k|\right)\leq 2, \tag{11}$$

*where $a_1=1$ and $b_1=0$, Then $f$ is harmonic univalent in $\mathscr{U}$, and $f\in K_H^0$.*

As $q\to 1^-$ and for we get the following result, proved by J. M. Jahangiri [8]

**Corollary 2.6.** *Let $f=h+\overline{g}$ be given by* (1). *If*

$$\sum_{k=1}^{\infty}\left(\frac{k(k-\beta)}{1-\beta}|a_k|+\frac{k(k+\beta)}{1-\beta}|b_k|\right)\leq 2, \tag{12}$$

*where $a_1=1$, $0\leq\beta<1$ Then $f$ is q-harmonic univalent in $\mathscr{U}$, and $f\in K_H(\beta)$..*

**Theorem 2.7.** *Let $f=h+\overline{g}$ be given by* (2). *Then $f\in K_{\overline{H}}(\beta,q)$ if and only if*

$$\sum_{k=1}^{\infty}\left(\frac{[k]_q\,[\lambda_q|\ln q|[k]_q-\beta(1-q)]}{\lambda_q|\ln q|-\beta(1-q)}|a_k|+\frac{[k]_q\,[\lambda_q|\ln q|[k]_q+\beta(1-q)]}{\lambda_q|\ln q|-\beta(1-q)}|b_k|\right)\leq 2, \tag{13}$$

*where $\lambda_q=\sum_{k=0}^{\infty}\frac{(i\theta(1-q))^n}{(n+1)!}$, $\lambda_q\to 1$ as $q\to 1^-$, $a_1=1$, $0\leq\beta<1$ and $0<q<1$.*

*Proof.* Since $f\in K_{\overline{H}}(\beta,q)\subset K_H(\beta,q)$, we only need to prove the necessary part of the theorem. Assume that $f\in K_{\overline{H}}(\beta,q)$, then by (4) we have

$$Re\left(\frac{\lambda_q|\ln q|z\partial_q h(z)+\lambda_q|\ln q|z^2\partial_q^2 h(z)+\overline{\lambda_q|\ln q|z\partial_q g(z)+\lambda_q|\ln q|z^2\partial_q^2 g(z)}}{(1-q)z\partial_q h(z)-\overline{(1-q)z\partial_q g(z)}}\right)-\beta$$

Ref

1. Y. Avci and E. Zlotkiewicz, On harmonic univalent mappings, Ann. Univ. Mariae CurieSklodowska Sect. 44(1990), 1-7.

$$= Re\frac{[\lambda_q|\ln q|-\beta(1-q)]z-\sum_{k=2}^{\infty}[k]_q[\lambda_q|\ln q|[k]_q-\beta(1-q)]|a_k|z^k-\sum_{k=1}^{\infty}[k]_q[\lambda_q|\ln q|[k]_q+\beta(1-q)]|b_k|\bar{z}^k}{(1-q)z-(1-q)\sum_{k=2}^{\infty}[k]_q|a_k|z^k+(1-q)\sum_{k=1}^{\infty}[k]_q|b_k|\bar{z}^k}$$

$$\geq 0.$$

The above condition must hold for all values of $z \in \mathscr{U}$. Upon choosing the values of $z$ on the positive real axis where $0 \leq z = r < 1$ we must have

$$\frac{[\lambda_q|\ln q|-\beta(1-q)]-\sum_{k=2}^{\infty}[k]_q[\lambda_q|\ln q|[k]_q-\beta(1-q)]|a_k|r^{k-1}-\sum_{k=1}^{\infty}[k]_q[\lambda_q|\ln q|[k]_q+\beta(1-q)]|b_k|r^{k-1}}{(1-q)(1-\sum_{k=2}^{\infty}[k]_q|a_k|r^{k-1}+\sum_{k=1}^{\infty}[k]_q|b_k|r^{k-1})} \geq 0. \quad (14)$$

If (13) does not hold, then the numerator in (14) is negative for $r$ sufficiently close to 1. Therefore, there exists a point $z_0 = r_0$ in $(0,1)$ for which the quotient in (14) is negative. This contradicts our assumption that $f \in K_{\overline{H}}(\beta,q)$. This completes the proof of Theorem.

As $q \to 1^-$ and for we get the following result, proved by J. M. Jahangiri [8]

**Corollary 2.8.** *Let $f = h + \overline{g}$ be given by* (2). *Then $f \in K_{\overline{H}}(\beta)$ if and only if*

$$\sum_{k=1}^{\infty}\left(\frac{k(k-\beta)}{1-\beta}|a_k| + \frac{k(1+\beta)}{1-\beta}|b_k|\right) \leq 2, \quad (15)$$

*where $a_1 = 1$ and $0 \leq \beta < 1$.*

Now we determine the extreme points of the closed convex hulls of $K_{\overline{H}}(\beta,q)$, denoted by $clcoK_{\overline{H}}(\beta,q)$.

**Theorem 2.9.** *Let $f$ given by* (2). *Then $f \in clcoK_H(\beta,q)$ if and only if*

$$f(z) = \sum_{k=1}^{\infty}(X_k h_k + Y_k g_k) \quad (16)$$

*where* $h_1(z) = z$, $h_K(z) = z - \frac{[\lambda_q|\ln q| - \beta(1-q)]}{[k]_q[\lambda_q|\ln q|[k]_q - \beta(1-q)]}z^k$ $(k = 2,3,...)$

$$\text{and } g_K(z) = z - \frac{[\lambda_q|\ln q| - \beta(1-q)]}{[k]_q[\lambda_q|\ln q|[k]_q + \beta(1-q)]}\bar{z}^k$$

$(k = 1,2,3,...)$, $\sum_{k=1}^{\infty}(X_k + Y_k) = 1$, $X_k \geq 0$ *and* $Y_k \geq 0$. *In particular, the extreme points of* $K_{\overline{H}}(\beta,q)$ *are* $\{h_k\}$ *and* $\{g_k\}$.

*Proof.* Let f be written as (16). Then we have

$$f(z) = \sum_{k=1}^{\infty}(X_k h_k + Y_k g_k) = \sum_{k=1}^{\infty}(X_k + Y_k)z - \sum_{k=2}^{\infty}\frac{\lambda_q|\ln q| - \beta(1-q)}{[k]_q\,[\lambda_q|\ln q|[k]_q - \beta(1-q)]}x_k z^k$$

Ref

8. J. M. Jahangiri, Coefficient bounds and univalence criteria for harmonic functions with negative coefficients. Ann. Univ. Mariae Curie-Skodowska Sect. A, 52(2) (1998), 57-66.

$$-\sum_{k=1}^{\infty}\frac{\lambda_q|\ln q|-\beta(1-q)}{[k]_q\left[\lambda_q|\ln q|[k]_q+\beta(1-q)\right]}\bar{y}_k\bar{z}^k.$$

$$=z-\sum_{k=2}^{\infty}a_nz^n-\sum_{k=1}^{\infty}b_n\bar{z}^n.$$

Then

$$\sum_{k=2}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q-\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}\left(\frac{\lambda_q|\ln q|-\beta(1-q)}{[k]_q\left[\lambda_q|\ln q|[k]_q-\beta(1-q)\right]}x_k\right)$$

$$+\sum_{k=2}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q+\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}\left(\frac{\lambda_q|\ln q|-\beta(1-q)}{[k]_q\left[\lambda_q|\ln q|[k]_q+\beta(1-q)\right]}y_k\right).$$

$$=\sum_{k=2}^{\infty}X_k+\sum_{k=1}^{\infty}Y_k=1-X_1\leq 1$$

and so $f\in clcoK_{\overline{H}}(\beta,q)$. Conversely, assume that $f\in clcoK_{\overline{H}}(\beta,q)$. Putting

$$X_k=\frac{[k]_q\left[\lambda_q|\ln q|[k]_q-\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}|a_k|,\qquad k=2,3,\ldots$$

$$Y_k=\frac{[k]_q\left[\lambda_q|\ln q|[k]_q+\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}|b_k|,\qquad k=1,2,3,\ldots$$

and

$$X_1=1-\sum_{k=2}^{\infty}X_k-\sum_{k=1}^{\infty}Y_k,$$

then $\sum_{k=1}^{\infty}(X_k+Y_k)=1,\ 0\leq X_k\leq 1(k=2,3,\ldots),0\leq Y_k\leq 1(k=1,2,3,\ldots)$. Consequently, we obtain $f(z)=\sum_{k=1}^{\infty}(X_kh_k+Y_kg_k)$ as required.

Finally we give the distortion bounds for functions in $K_{\overline{H}}(\beta,q)$ which yield a covering result for $K_{\overline{H}}(\beta,q)$.

**Theorem 2.10.** *If $f\in K_{\overline{H}}(\beta,q)$ then*

$$|f(z)|\leq(1+|b_1|)r+\frac{1}{[2]_q}\left(\frac{\lambda_q|\ln q|-\beta(1-q)}{[2]_q\lambda_q|\ln q|-\beta(1-q)}-\frac{\lambda_q|\ln q|+\beta(1-q)}{[2]_q\lambda_q|\ln q|-\beta(1-q)}|b_1|\right)r^2,\quad |z|=r<1,$$

*and*

$$|f(z)|\geq(1-|b_1|)r-\frac{1}{[2]_q}\left(\frac{\lambda_q|\ln q|-\beta(1-q)}{[2]_q\lambda_q|\ln q|-\beta(1-q)}-\frac{\lambda_q|\ln q|+\beta(1-q)}{[2]_q\lambda_q|\ln q|-\beta(1-q)}|b_1|\right)r^2,\quad |z|=r<1.$$

*Proof.* Assume that $f\in K_{\overline{H}}(\beta,q)$. Then we have

Notes

$$|f(z)| \le (1+|b_1|)r + \sum_{k=2}^{\infty}(|a_k|+|b_k|)r^k$$

$$\le (1+|b_1|)r + \sum_{k=2}^{\infty}(|a_k|+|b_k|)r^2$$

$$= (1+|b_1|)r + \frac{\lambda_q|\ln q| - \beta(1-q)}{[2]_q([2]_q\lambda_q|\ln q| - \beta(1-q))}\sum_{k=2}^{\infty}\left(\frac{[2]_q([2]_q\lambda_q|\ln q| - \beta(1-q))}{\lambda_q|\ln q| - \beta(1-q)}|a_k| + \frac{[2]_q([2]_q\lambda_q|\ln q| - \beta(1-q))}{\lambda_q|\ln q| - \beta(1-q)}|b_k|\right) r^2$$

$$\le (1+|b_1|)r + \frac{\lambda_q|\ln q| - \beta(1-q)}{[2]_q([2]_q\lambda_q|\ln q| - \beta(1-q))}\sum_{k=2}^{\infty}\left(\frac{[k]_q(\lambda_q|\ln q|[k]_q - \beta(1-q))}{\lambda_q|\ln q| - \beta(1-q)}|a_k| + \frac{[k]_q(\lambda_q|\ln q|[k]_q + \beta(1-q))}{\lambda_q|\ln q| - \beta(1-q)}|b_k|\right) r^2$$

$$\le (1+|b_1|)r + \frac{\lambda_q|\ln q| - \beta(1-q)}{[2]_q([2]_q\lambda_q|\ln q| - \beta(1-q))}\left(1 - \frac{\lambda_q|\ln q| + \beta(1-q)}{\lambda_q|\ln q| - \beta(1-q)}|b_1|\right) r^2, \quad by (5),$$

$$= (1+|b_1|)r + \frac{1}{[2]_q}\left(\frac{\lambda_q|\ln q| - \beta(1-q)}{[2]_q\lambda_q|\ln q| - \beta(1-q)} - \frac{\lambda_q|\ln q| + \beta(1-q)}{[2]_q\lambda_q|\ln q| - \beta(1-q)}|b_1|\right) r^2.$$

and

$$|f(z)| \ge (1-|b_1|)r - \sum_{k=2}^{\infty}(|a_k|+|b_k|)r^k$$

$$\ge (1-|b_1|)r - \sum_{k=2}^{\infty}(|a_k|+|b_k|)r^2$$

$$= (1-|b_1|)r - \frac{\lambda_q|\ln q| - \beta(1-q)}{[2]_q([2]_q\lambda_q|\ln q| - \beta(1-q))}\sum_{k=2}^{\infty}\left(\frac{[2]_q([2]_q\lambda_q|\ln q| - \beta(1-q))}{\lambda_q|\ln q| - \beta(1-q)}|a_k| + \frac{[2]_q([2]_q\lambda_q|\ln q| - \beta(1-q))}{\lambda_q|\ln q| - \beta(1-q)}|b_k|\right) r^2$$

$$\ge (1-|b_1|)r - \frac{\lambda_q|\ln q| - \beta(1-q)}{[2]_q([2]_q\lambda_q|\ln q| - \beta(1-q))}\sum_{k=2}^{\infty}\left(\frac{[k]_q(\lambda_q|\ln q|[k]_q - \beta(1-q))}{\lambda_q|\ln q| - \beta(1-q)}|a_k| + \frac{[k]_q(\lambda_q|\ln q|[k]_q + \beta(1-q))}{\lambda_q|\ln q| - \beta(1-q)}|b_k|\right) r^2$$

$$\ge (1-|b_1|)r - \frac{\lambda_q|\ln q| - \beta(1-q)}{[2]_q([2]_q\lambda_q|\ln q| - \beta(1-q))}\left(1 - \frac{\lambda_q|\ln q| + \beta(1-q)}{\lambda_q|\ln q| - \beta(1-q)}|b_1|\right) r^2, \quad by (5),$$

$$= (1-|b_1|)r - \frac{1}{[2]_q}\left(\frac{\lambda_q|\ln q| - \beta(1-q)}{[2]_q\lambda_q|\ln q| - \beta(1-q)} - \frac{\lambda_q|\ln q| + \beta(1-q)}{[2]_q\lambda_q|\ln q| - \beta(1-q)}|b_1|\right) r^2.$$

As $q \to 1^-$ and for we get the following result, proved by J. M. Jahangiri [8]

**Corollary 2.11.** *If $f \in K_{\overline{H}}(\beta)$ then*

$$|f(z)| \le (1+|b_1|)r + \frac{1}{2}\left(\frac{1-\beta}{2-\beta} - \frac{1+\beta}{2-\beta}|b_1|\right) r^2, \qquad |z| = r < 1,$$

*and*

$$|f(z)| \ge (1-|b_1|)r - \frac{1}{2}\left(\frac{1-\beta}{2-\beta} - \frac{1+\beta}{2-\beta}|b_1|\right) r^2, \qquad |z| = r < 1.$$

Ref

8. J. M. Jahangiri, Coefficient bounds and univalence criteria for harmonic functions with negative coefficients. Ann. Univ. Mariae Curie-Skodowska Sect. A, 52(2) (1998), 57-66.

The bounds given in Theorem 2.10 for the function $f = h + \bar{g}$ of the form (2) also hold for functions of the form (1) if the coefficient condition (5) is satisfied. The functions

$$f(z) = (1+|b_1|)\bar{z} + \frac{1}{[2]_q}\left(\frac{\lambda_q|\ln q| - \beta(1-q)}{[2]_q\lambda_q|\ln q| - \beta(1-q)} - \frac{\lambda_q|\ln q| + \beta(1-q)}{[2]_q\lambda_q|\ln q| - \beta(1-q)}|b_1|\right)\bar{z}^2$$

and

$$f(z) = (1-|b_1|)z - \frac{1}{[2]_q}\left(\frac{\lambda_q|\ln q| - \beta(1-q)}{[2]_q\lambda_q|\ln q| - \beta(1-q)} - \frac{\lambda_q|\ln q| + \beta(1-q)}{[2]_q\lambda_q|\ln q| - \beta(1-q)}|b_1|\right)z^2$$

for $|b_1| \leq \frac{1}{[2]_q}\left(\frac{\lambda_q|\ln q| - \beta(1-q)}{[2]_q\lambda_q|\ln q| - \beta(1-q)} - \frac{\lambda_q|\ln q| + \beta(1-q)}{[2]_q\lambda_q|\ln q| - \beta(1-q)}|b_1|\right)$ show that the bounds given in Theorem 2.6 are sharp.

**Theorem 2.12.** *If $f \in K_{\overline{H}}(\beta, q)$ then*

$$\left\{\omega : |\omega| < \frac{1}{[2]_q}\frac{\lambda_q|\ln q|([2]_q-1)[([2]_q+1)-\beta(1-q)]}{[2]_q\lambda_q|\ln q| - \beta(1-q)} - \frac{1}{[2]_q}\frac{\lambda_q|\ln q|([2]_q+1)[([2]_q-1)-\beta(1-q)]}{[2]_q\lambda_q|\ln q| - \beta(1-q)}|b_1|\right\} \subset f(\mathscr{U}). \tag{17}$$

*Proof.* Letting $r \to 1^-$ in the left hand inequality in Theorem 2.10 and collecting the like terms we obtain (17).

The condition (17) for $\beta = b_1 = 0$ yields the following

**Corollary 2.13.** *if $f \in K^0_{\overline{H}}(0, q)$ then*

$$\{\omega : \omega < \frac{[2]_q^2 - 1}{[2]_q^2}\} \subset f(\mathscr{U}).$$

As $q \to 1^-$ and $\beta = b_1 = 0$, we get the following result, proved by Jay M. Jahangiri [8]



**Corollary 2.14.** *if $f \in K^0_{\overline{H}}(0)$ then*

$$\{\omega : \omega < \frac{3}{4}\} \subset f(\mathscr{U}).$$

For harmonic functions $f(z) = z + \sum_{k=2}^{\infty} a_k z^k + \sum_{k=1}^{\infty} b_k \bar{z}^k$ and $g(z) = z + \sum_{n=2}^{\infty} A_k z^k + \sum_{k=1}^{\infty} B_n \bar{z}^k$ we define the Hadamard product of $f$ and $g$ as

$$(f * g)(z) = z + \sum_{k=2}^{\infty} a_k A_k z^k + \sum_{k=1}^{\infty} b_k B_k \bar{z}^k.$$

**Theorem 2.15.** *For $0 \leq \alpha \leq \beta < 1$, let $f \in K_{\overline{H}}(\beta, q)$ and $g \in K_{\overline{H}}(\alpha, q)$. Then $f * g \in K_H(\beta, q) \subset K_H(\alpha, q)$.*

Ref

8. J. M. Jahangiri, Coefficient bounds and univalence criteria for harmonic functions with negative coefficients. Ann. Univ. Mariae Curie-Skodowska Sect. A, 52(2) (1998), 57-66.

*Proof.* Putting $f(z)=z-\sum_{k=2}^{\infty}|a_k|z^k-\sum_{k=1}^{\infty}|b_k|\overline{z}^k$ and $g(z)=z-\sum_{k=2}^{\infty}|A_k|z^k+\sum_{k=1}^{\infty}|B_k|\overline{z}^k$. Then the Hadamard product of $f$ and $g$ is

$$(f*g)(z)=z+\sum_{k=2}^{\infty}|a_k||A_k|z^k+\sum_{k=1}^{\infty}|b_k||B_k|\overline{z}^k.$$

Notes

Since $|A_k|\leq 1$ and $|B_k|\leq 1$, we can write

$$\sum_{k=2}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q-\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}|a_k||A_k|+\sum_{k=1}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q+\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}|b_k||B_k|$$

$$\leq\sum_{k=2}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q-\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}|a_k|+\sum_{k=1}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q+\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}|b_k|.$$

The right hand side of the above inequality is bounded by 1 because $f\in K_{\overline{H}}(\beta,q)$ .Therefore $f*g\in K_H(\beta,q)\subset K_H(\alpha,q)$.

**Theorem 2.16.** *The class $K_{\overline{H}}(\beta,q)$ is closed under convex combination.*

*Proof.* For $i=1,2,3,\ldots$ assume that $f_i\in K_{\overline{H}}(\beta,q)$ where $f_i$ is given by

$$f_i(z)=z-\sum_{k=2}^{\infty}|a_{i_k}|z^k-\sum_{k=1}^{\infty}|b_{i_k}|\overline{z}^k.$$

Then by (13)

$$\sum_{k=2}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q-\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}|a_{i_k}|+\sum_{k=1}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q+\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}|b_{i_k}|\leq 1. \tag{18}$$

For $\sum_{i=1}^{\infty}t_i=1$, $0\leq t_i\leq 1$, the convex combination of $f_i$ may be written as

$$\sum_{i=1}^{\infty}t_if_i(z)=z-\sum_{k=2}^{\infty}\left(\sum_{i=1}^{\infty}t_i|a_{i_k}|\right)z^n-\sum_{k=1}^{\infty}\left(\sum_{i=1}^{\infty}t_i|b_{i_k}|\right)\overline{z}^n.$$

Then by (18),

$$\sum_{k=2}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q-\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}\left|\sum_{i=1}^{\infty}t_i|a_{i_k}|\right|+\sum_{k=1}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q+\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}\left|\sum_{i=1}^{\infty}t_i|b_{i_k}|\right|$$

$$\sum_{i=1}^{\infty}t_i\left\{\sum_{k=2}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q-\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}|a_{i_k}|+\sum_{k=1}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q+\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}|b_{i_k}|\right\}\leq\sum_{i=1}^{\infty}t_i=1,$$

and so $\sum_{i=1}^{\infty}t_if_i(z)\in K_{\overline{H}}(\beta,q)$.

Now, we consider the closer property of the class $K_{\overline{H}}(\beta,q)$ under the Bernardi integral operator $F_a(z)$, which is defined by

$$F_a(z) = \frac{[a+1]_q}{z^a}\int_0^z t^{a-1}f(t)d_qt + \overline{\frac{[a+1]_q}{z^a}\int_0^z t^{a-1}f(t)d_qt}\ (a>-1). \tag{19}$$

**Theorem 2.17.** *Let $f \in K_{\overline{H}}(\beta,q)$. Then $F_a(z) \in K_{\overline{H}}(\beta,q)$.*

*Proof.* From the representation of $F_a(z)$, we have

$$F_a(z) = \sum_{k=1}^{\infty}\left(\frac{[a+1]_q}{[a+n]_q}\right)a_kz^k + \sum_{k=1}^{\infty}\left(\frac{[a+1]_q}{[a+k]_q}\right)\overline{b_k}\overline{z}^k \tag{20}$$

Now

$$\sum_{k=2}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q-\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}\left(\frac{[a+1]_q}{[a+k]_q}|a_k|\right) + \sum_{k=1}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q+\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}\left(\frac{[a+1]_q}{[a+k]_q}|b_k|\right) \tag{21}$$

$$\leq \sum_{k=2}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q-\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}|a_k| + \sum_{k=1}^{\infty}\frac{[k]_q\left[\lambda_q|\ln q|[k]_q+\beta(1-q)\right]}{\lambda_q|\ln q|-\beta(1-q)}|b_k| \leq 1.$$

Therefore $F_a(z) \in K_{\overline{H}}(\beta,q)$.

## References Références Referencias


1. Y. Avci and E. Zlotkiewicz, On harmonic univalent mappings, Ann. Univ. Mariae CurieSklodowska Sect. 44(1990), 1-7.
2. G. Choquet, Sur un type de transformation analytique gnralisant la reprsentation conforme et dfinie au moyen de fonctions harmoniques, Bull. Sci. Math., 89(1945), 156-165.
3. J. Clunie and T. Sheil-Small, Harmonic univalent functions, Ann. Acad. Aci. Fenn. Ser. AI Math., 9(1984), 3-25.
4. M. Dorff, Minimal graphs in R3 over convex domains, Proc. Am. Math. Soc., 132(2)(2003), 491-498.
5. P. Duren, Harmonic Mappings in the Plane, Cambridge Tracts in Math. Cambridge University Press, Cambridge., 156(2004).
6. F. H. Jackson, On q-definite integrals, Quarterly J. Pure Appl. Math., 41 (1910), 193-203. ON A SUBCLASS OF CERTAIN CONVEX HARMONIC UNIVALENT FUNCTIONS RELATED TO q-DERIVATIVE13
7. F. H. Jackson, On q-functions and a certain difference operator, Trans. Royal Soc. Edinburgh., 46(1909), 253-281.
8. J. M. Jahangiri, Coefficient bounds and univalence criteria for harmonic functions with negative coefficients. Ann. Univ. Mariae Curie-Skodowska Sect. A, 52(2) (1998), 57-66.


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# Induction and Parametric Properties of Radio-Technical Elements and Chains and Property of Charges and their Flows

By F. F. Mende

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GJSFR-F Classification: *FOR Code: MSC 2010: 35L05*

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*Strictly as per the compliance and regulations of:*

# Induction and Parametric Properties of Radio-Technical Elements and Chains and Property of Charges and their Flows

F. F. Mende

*Abstract-* In the article the electrical and current self-induction of radio-technical elements and chains is examined and it is shown that such elements can present the effective resistance, which depends on the time. Is introduced the concept of parametricself-induction. On the basis of the concepts indicated is obtained the wave equation for the long lines, which gives the possibility to establish the velocity of propagation of the front of stress with the connection to the line of dc power supply. The concept of the potential and kinetic flows of charges is introduced.

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## I. Electrical And Current Self-Induction

To the laws of self-induction should be carried those laws, which describe the reaction of such elements of radio-technical chains as capacity, inductance and resistance with the galvanic connection to them of the sources of current or voltage. To such elements let us carry capacities, inductances, effective resistance and long lines.

By self-induction of reactive elements we will understand the reaction of such elements as capacity and inductance with the constant or changing parameters to the connection to them of the sources of voltage or current. Subsequently we will use these concepts: as current generator and the voltage generator. By ideal voltage generator we will understand such source, which ensures on any load the lumped voltage, internal resistance in this generator equal to zero. By ideal current generator we will understand such source, which ensures in any load the assigned current, internal resistance in this generator equally to infinity. The ideal current generators and voltage in nature there does not exist, since both the current generators and the voltage generators have their internal resistance, which limits their possibilities.

If the capacity $C$ is charged to a potential difference $U$, then the charge o$Q$, accumulated in it, is determined by the relationship

$$Q_{C,U} = CU \ .$$

When the discussion deals with a change in the charge, determined by relationship, then this value can change with the method of changing the potential difference with a constant capacity, either with a change in capacity itself with a constant potential difference, or and that and other parameter simultaneously.

If the value of a voltage drop across capacity or capacity itself depends on time, then the strength of current, which flows in the chain, which includes the voltage source and capacity, is determined by the relationship:

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$$I(t)=\frac{dQ_{C,U}}{dt}=C\frac{\partial U}{\partial t}+U\frac{\partial C}{\partial t}.$$

This expression determines the law of electrical self-induction. Thus, current in the circuit, which contains capacitor, can be obtained by two methods, changing voltage across capacitor with its constant capacity either changing capacity itself with constant voltage across capacitor, or to produce change in both parameters simultaneously.

When the capacity $C_0$ is constant, we obtain expression for the current, which flows in the chain:

$$I(U)=C_0\frac{\partial U}{\partial t} \tag{1.1}$$

when changes capacity, and at it is supported the constant stress $U_0$, we have:

$$I(C)=U_0\frac{\partial C}{\partial t}. \tag{1.2}$$

This case to relate to the parametric capacitive self-induction, since the current strength it is connected with a change in the capacitance value.

Let us examine the consequences, which escape from relationship (1.1).

If we to the capacity connect the direct-current generator $I_0$, then stress on it will change according to the law:

$$U(t)=\frac{I_0 t}{C_0}. \tag{1.3}$$

Using to this relationship Ohm's law

$$U=IR,$$

we obtain the value of the effective resistance of the chain in question

$$R(t)=\frac{t}{C_0}.$$

Thus the capacity, connected to the current source, plays the role of the effective resistance, which linearly depends on the time. Thes it should be noted that obtained result is completely obvious; however, such properties of capacity, which customary to assume by reactive element they were for the first time noted in the work [1].

From a physical point of view this property of capacity is connected with the fact that, charging capacity, current source to expend energy. Capacity itself in this case performs the role of storage battery.

Charging capacity, current source expends the power

$$P(t)=\frac{I_0^2 t}{C_0} \tag{1.4}$$

the energy, accumulated by capacity in the time $t$, we will obtain, after integrating relationship (1.4) with respect to the time:

$$W_C(t)=\frac{I_0^2 t^2}{2C_0}.$$

Ref

1. Ф. Ф. Менде. Непротиворечивая электродинамика. Харьков, НТМТ, 2008, – 153 с.

Substituting here the value of current from relationship (1.4), we obtain the dependence of the value of the accumulated in the capacity energy from the instantaneous value of stress on it:

$$W_C(U)=\frac{1}{2}C_0U^2.$$

Notes

Now we will support at the capacity constant stress $U_0$, and change capacity itself, then

$$I(C)=U_0\frac{\partial C}{\partial t}.$$

Using to this relationship Ohm's law

$$R_C=\left(\frac{\partial C}{\partial t}\right)^{-1}$$

Value $R_C$ plays the role of the effective resistance. The derivative, entering this expression can have different signs. This result is intelligible. Since with a change in the capacity change the energy accumulated in it, capacity, it can extract energy in the current source, or return energy into the external circuit. The power, expended by current source, or output into the external circuit, is determined by the relationship:

$$P(C)=\frac{\partial C}{\partial t}U_0^{\,2}.$$

Let us examine one additional process, which earlier the laws of induction did not include, however, it falls under for our extended determination of this concept. If the charge $Q_0$, accumulated in the capacity, remains constant, then stress on it can be changed by changing the capacity. In this case the relationship will be carried out:

$$Q_0=C_0U_0=CU=const,$$

where $C$ and $U$ - instantaneous values, and $C_0$ and $U_0$ - initial values of these parameters. The stress on the capacity and the energy, accumulated in it, will be in this case determined by the relationships:

$$U=\frac{C_0U_0}{C},$$

$$W_c(C)=\frac{1}{2}\frac{(C_0U_0)^2}{C}. \qquad (1.5)$$

It is natural that this process of self-induction can be connected only with a change in capacity itself, and therefore it falls under for the determination of parametric self-induction.

Let us examine the processes, proceeding in the inductance. If the current strength through the inductance or inductance itself depend on time, then the value of stress on it is determined by the relationship:

$$U(t)=L\frac{\partial I}{\partial t}+I\frac{\partial L}{\partial t}.$$

Let us examine the case, when the inductance $L_0$ is constant then

$$U(I)=L_0\frac{\partial I}{\partial t}. \qquad (1.6)$$

After integrating expression (1.6) on the time, we will obtain:

$$I(t)=\frac{Ut}{L_0}. \qquad (1.7)$$

Using to this relationship Ohm's law, we obtain, that the inductance, connected to the dc power supply, presents for it the effective resistance

$$R(t)=\frac{L_0}{t}.$$

The power, expended in this case by source, is determined by the relationship:

$$P(t)=\frac{U^2t}{L_0}. \qquad (1.8)$$

After integrating relationship (1.8) on the time, we will obtain the energy, accumulated in the inductance

$$W_L(t)=\frac{1}{2}\frac{U^2t^2}{L_0}. \qquad (1.9)$$

After substituting into expression (1.9) the value of stress from relationship (1.7), we obtain the value of the energy, accumulated in the inductance:

$$W_L(I)=\frac{1}{2}L_0I^2.$$

Now let us examine the case, when the current $I_0$, which flows through the inductance, is constant, and inductance itself can change. In this case we obtain

$$U=I_0\frac{\partial L}{\partial t}. \qquad (1.10)$$

Consequently, the value

$$R(t)=\frac{dL}{dt}$$

as in the case the electric flux, effective resistance can be (depending on the sign of derivative) both positive and negative. This means that the inductance can how derive energy from without, so also return it into the external circuits.

If inductance is shortened outed, and made from the material, which does not have effective resistance, for example from the superconductor, then

$$L_0I_0=const\ ,$$

where $L_0$ and $I_0$- initial values of these parameters, which are located at the moment of the short circuit of inductance with the presence in it of current.

Notes

This regime we will call the regime of the frozen flow. In this case the relationship is fulfilled:

$$I_0 = \frac{I_1 L_1}{L_0},$$

where $I_1$ and $L_1$ - the instantaneous values of the corresponding parameters.

Notes

In flow regime examined of current induction remains constant, however, in connection with the fact that current in the inductance it can change with its change, this process falls under for the determination of parametric self-induction. The energy, accumulated in the inductance, in this case will be determined by the relationship

$$W_L(L) = \frac{1}{2}\frac{(L_0 I_0)^2}{L}.$$

where $L$ - the instantaneous value of inductance.

The capacity of the vacuum capacitor, which consists of the flat parallel plates, is determined by the relationship:

$$C = \frac{\varepsilon_0 S}{d},$$

where $\varepsilon_0, S$ and $d$ - dielectric constant of vacuum, the area of plates and the distance between them respectively. Substituting in this relationship equality (1.5), we obtain

$$W_c = \frac{1}{2}\frac{d\left(C_0 U_0\right)^2}{\varepsilon_0 S}. \tag{1.11}$$

Is evident that with the constant charge, stored up in the capacitor, an increase in the distance between the plates leads to an increase in its energy. This is connected with the fact that in order to increase the distance between the plates, it is necessary to spend the work, which will pass into the energy of its electrical pour on. As this occurs, evidently from Fig.1.

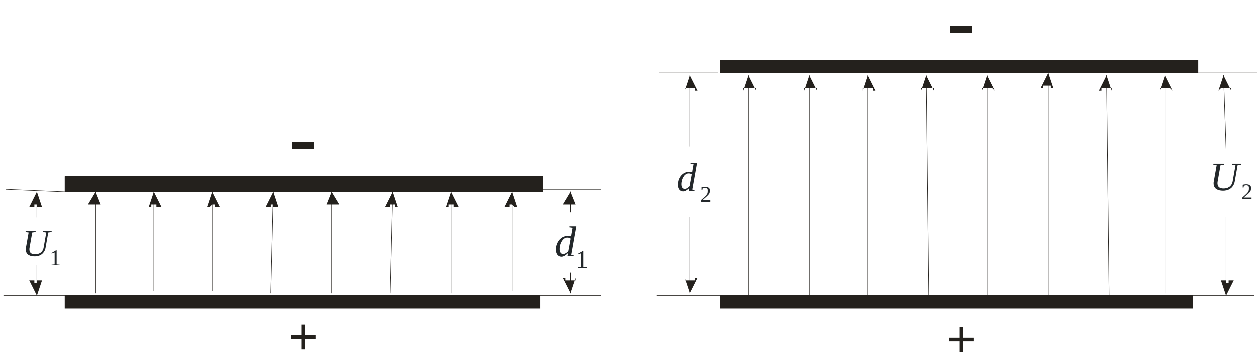


*Fig. 1:* The electric fields of parallel-plate capacitor with the different distance between its plates Taking into account that the work of capacity and stress is equal to charge, accumulated in the capacitor, relationship (1.9) can be rewritten

$$W_c = \frac{1}{2}\ \frac{d(Q_0)^2}{\varepsilon_0 S} = \frac{1}{2}\varepsilon_0 E^2 Sd \ , \qquad (1.12)$$

where $E$ - tension of electric field in the line.
From relationship (1.12) follows

$$E = \frac{Q_0}{\varepsilon_0 S}.$$

Thismeans that in the parallel-plate capacitor the field strength does not depend on the distance between the plates, but it is determined by the surface density of charge on them. Let us note that with this examination we do not consider edge effects that correctly when the distance between the plates much less than their length and width. Consequently, voltage across capacitor is determined by the distance between the plates

$$U_d = \frac{Q_0 d}{\varepsilon_0 S}.$$

From the carried out analysis escapes the interesting property of the electrons, which compose the charge $Q_0$. Their quantity is equal

$$N = \frac{Q_0}{e},$$

where $e$ is a charge of one electron. Thus, energy of one electron, which is located on the plate of capacitor, is equal

$$W_e = \frac{de}{\varepsilon_0 S} \ .$$

This energy depends on the distance between the plates, but since no limitations on $d$ they are superimposed, this energy can be as desired to large.

In the case examined the electric fields of each separate electron are located in the tube, located between the planes of capacitor. The cross-sectional area of this tube is equal and its height it is respectively equal: $\frac{S}{N}$ and $d$. When an increase in the size occurs $d$, volume of this tube increase, and, therefore, it grows and energy pour on. In this case the mechanical energy, spent on the displacement of the plate of capacitor, passes into the energy of electrical pour on electron. Analogous situation will be observed, also, in the coaxial capacitor. Difference will be only the fact that the fields of electron will occupy not tube with the constant section, but annular disk.

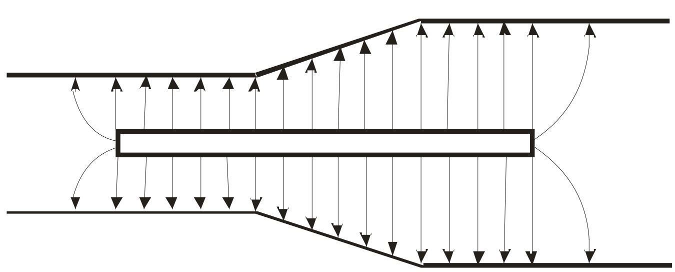

*Fig. 2:* Coaxial capacitor with the variable section

Notes

Notes

Let us load coaxial capacitor with the variable section, as shown in Fig. 2. If we move the charged rod from left to right, then the volume of electrical pour on it will be grow, and for this will have to expend energy. But if rod will be moved in the reverse direction, then volume pour on it will decrease, and rod will carry out external work. If we as the rod take the section of the moving electron beam, then picture not change. During the motion from left to right, kinetic energy of beam will pass into the energy of electrical pour on, and beam will slow down and vice versa. But when we deal concerning the moving electron beam, picture there will be somewhat different, since with the flow in the tubular part of the capacitor the return current will exist.

## II. Propagation Of Signals In The Long Lines

The processes of the propagation of voltages and currents in the long lines it is described with the aid of the wave equations

$$\frac{\partial^2 U}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 U}{\partial t^2}$$
$$\frac{\partial^2 I}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 I}{\partial t^2},$$

which are obtained from the telegraphic equations

$$\frac{\partial U}{\partial z} = -L \frac{\partial I}{\partial t}$$
$$\frac{\partial I}{\partial z} = -C \frac{\partial U}{\partial t}.$$

But as to enter, if to the line is connected dc power supply or source of voltage, which is changed according to the linear law, when the second derivatives of voltages and currents do be absent? In the existing literary sources the answer to this question is absent.

The processes, examined in two previous paragraphs, concern chains with the lumped parameters, when the distribution of potential differences and currents in the elements examined can be considered uniform.

We will use the results, obtained in the previous paragraph, for examining the processes, proceeding in the long lines, in which the capacity and inductance are the distributed parameters. Let us assume that the linear capacity and the inductance of line compose $C_0$ and $L_0$. If we to the line connect the dc power supply $U$ , thus will begin to charge the capacity of long line and the front of this stress will be extended along the line some by the speed $v$. The moving coordinate of this front will be determined by the relationship $z=vt$. In this case the total quantity of the charged capacity and the value of the summary inductance, along which it flows current, calculated from the beginning lines to the location of the front of stress, will change according to the law:

$$C(t) = zC_0 = vtC_0,$$
$$L(t) = zL_0 = vtL_0.$$

The source of voltage of will in this case charge the being increased capacity of line, for which from the source to the charged line in accordance with relationship (1.2) must leak the current:

$$I = U\frac{\partial C(t)}{\partial t} = UvC_0 \qquad (2.1)$$

This current there will be the leak through the conductors of line, that possess inductance. But, since the inductance of line in connection with the motion of the front of stress, also increases, in accordance with relationship (1.10), on it will be observed a voltage drop:

$$U_1 = I\frac{\partial L(t)}{\partial t} = IvL_0 = v^2UC_0L_0 .$$

But a voltage drop across the conductors of line in the absolute value is equal to the stress, applied to its entrance; therefore in the last expression should be placed $U = U_1$. We immediately find taking this into account that the rate of the motion of the front of stress with the assigned linear parameters and when, on, the incoming line of constant stress $U$ is present, must compose

$$v = \frac{1}{\sqrt{L_0C_0}} . \qquad (2.2)$$

This expression corresponds to the signal velocity in line itself. Consequently, if we to the infinitely long line connect the voltage source, then in it will occur the expansion of electrical pour on and the currents, which fill line with energy, and the speed of the front of constant stress and current will be equal to the velocity of propagation of electromagnetic vibrations in this line. This wave we will call electriccurent. It is interesting to note that the obtained result does not depend on the form of the function $U$, i.e., to the line can be connected both the dc power supply and the source, whose voltage changes according to any law. In all these cases the value of the local value of voltage on incoming line will be extended along it with the speed, which follows from relationship (2.2). This result could be, until now, obtained only by the method of solution of wave equations. This process occurs in such a way that the wave front, being extended with the speed of $v$, leaves after itself the line, charged to a potential difference $U_1$, which corresponds to the filling of line with electrostatic electric field energy. However, in the section of line from the voltage source also to the wave front flows the current $I_1$, which corresponds to the filling of line in this section with energy, which is connected with the motion of the charges along the conductors of line, which possess inductance.

The current strength in the line can be obtained, after substituting the values of the velocity of propagation of the wave front, determined by relationship (2.2), into relationship (2.1). After making this substitution, we will obtain

$$I_1 = U_1\sqrt{\frac{C_0}{L_0}} ,$$

where $Z = \sqrt{\frac{L_0}{C_0}}$ - line characteristic.

The regularities indicated apply to all forms of transmission lines.

If we to the line with the length $z_0$ connect the effective resistance, equal to line characteristic, then the voltage of the power source will appear on it with the time delay

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$\Delta t = \frac{z_0}{v}$. This resistance will be coordinated with the line and entire energy, transferred by the line, will be in it absorbed. This connected with the fact that the current, which flows in the line is equal to the current, which flows through the resistance, when stress on it is equal to voltage on incoming line.

Thus, the processes of the propagation of a potential difference along the conductors of long line and current in it are connected and mutually supplementing each other, and to exist without each other they do not can. This process can be called elektriccurent spontaneous parametric self-induction. This name flow expansion they connected with the fact that occur spontaneously.

For different types of lines the linear parameters depend on their sizes. For an example let us examine the coaxial line, whose linear capacity and inductance are expressed by the relationships:

$$C_0 = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D}{d}\right)}, \quad L_0 = \frac{\mu_0}{2\pi}\ln\left(\frac{D}{d}\right);$$

where $D$ and $d$ - inside diameter of the cylindrical part of the coaxial and the outer diameter of central core, and $\varepsilon_0$ and $\mu_0$ - dielectric and magnetic constant of vacuum.

Exist coaxial lines with the variable section both the cylindrical part and the internal conductor. The sections of such coaxials are used as the matching devices devices between the coaxials with different diameters of cylindrical part and central core. Propagation of signals in such adapter has its specific character (Fig. 3).

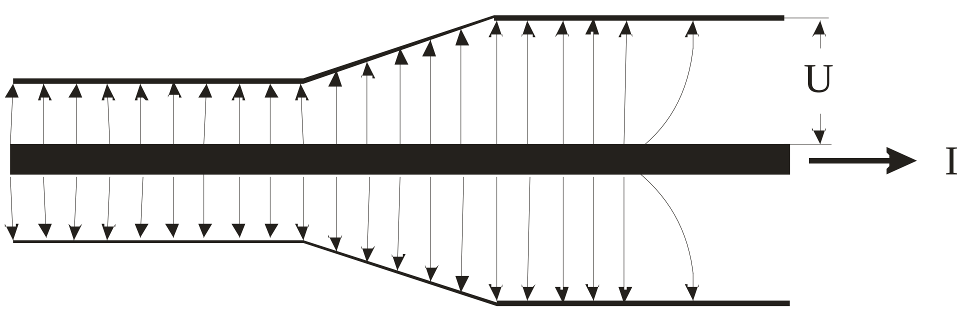


*Fig. 3:* Propagation of signal along the coaxial line with the variable section

A change in the dimensions of coaxial leads to the fact that the linear parameters begin to depend on coordinate. Begins to depend on coordinate and the wave drag

$$Z=\sqrt{\frac{L}{C}}=\ln\left(\frac{D}{d}\right)\sqrt{\frac{\mu_0}{\varepsilon_0}}.$$

At the same time velocity of propagation, both in the limits of the sections of coaxials and in the transition section it remains constant

$$v=\sqrt{\frac{1}{CL}}=\sqrt{\frac{1}{\varepsilon_0\mu_0}}.$$

Penetrating this adapter, signal changes its parameters.

Since wave drag gives the relation between the voltage and the current in the line

$$Z = \frac{U}{I}.$$

That changes the relationship between the voltage and the current in the initial and final section of coaxial. Consequently, such adapter is the current transformer and voltage. And this transformation occurs both with the propagation on the line of alternating voltage so and the constant. Thus this device is the configurative voltage transformer and currents. It is in the literature accepted to call such devices impedance transformers, but it is more correct them to call the voltage transformers and currents.

## III. Properties Of The Flows Of The Charges

If charges can move without the losses, then equation of motion takes the form:

$$m\frac{d\vec{v}}{dt} = e\vec{E},$$

where $m$ - mass electron, $e$ - electron charge, $\vec{E}$ - the tension of electric field, $\vec{v}$ - speed of the motion of charge.

Using an expression for the current density

$$\vec{j} = ne\vec{v},$$

we obtain the current density of the conductivity

$$\vec{j}_L = \frac{ne^2}{m}\int \vec{E}\ dt = \frac{1}{L_k}\int \vec{E}\ dt,$$

where

$$L_k = \frac{m}{ne^2},$$

where $L_k$ - kinetic inductance of charges [5,6].

In the real transmission lines kinetic inductance is not calculated on the basis of that reason, that their speed is small in view of the very high density of current carriers in the conductors and therefore field inductance always is considerably greater than kinetic. Let us show this based on simple example.

Let us examine processes in the line, which consists of two superconductive planes (Fig. 4).

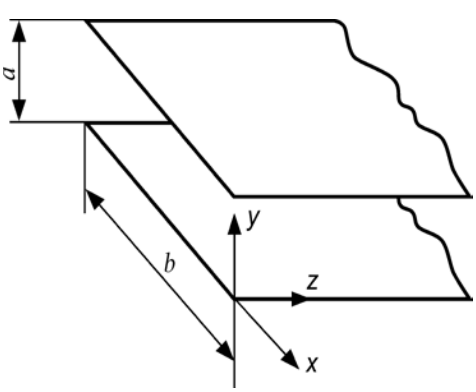


*Fig. 4:* The two-wire circuit, which consists of two ideally conducting planes

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5. Ф. Ф. Менде. Роль и место кинетической индуктивности зарядов в классической электродинамике, Инженерная физика, №11, 2012. с. 10-19.

The magnetic field on the internal surfaces of this line, equal to specific current, is determined from the relationship:

$$H = nev\lambda ,$$

where $n$, $e$, $v$ - density, charge and the velocity of the superconductive electrons, and $\lambda = \sqrt{\frac{m}{ne^2\mu}}$ - depth of penetration of magnetic field into the superconductor.

If we substitute the value of depth of penetration into the relationship for the magnetic field, then we will obtain:

$$H = v\sqrt{\frac{nm}{\mu}} .$$

Thus, specific kinetic the kinetic energy of charges in the skin-layer

$$W_H = \frac{1}{2}\mu H^2 = \frac{nmv^2}{2}$$

is equal to specific the energy of magnetic pour on. But the magnetic field, connected with the motion of current carriers in the skin-layer of superconductor, there is not only in it. If we designate the length of the line, depicted in Fig. 4 as $l$, then the volume of skin-layer in the superconductive planes of line will compose $2lb\lambda$. Energy of magnetic pour on in this volume we determine from the relationship:

$$W_{H,\lambda} = nmv^2 lb\lambda .$$

Energy of magnetic pour on, accumulated between the planes of line, it will comprise:

$$W_{H,a} = \frac{nmv^2 lba}{2} = \frac{1}{2} lba\mu_0 H .$$

If one considers that the depth of penetration of magnetic field in the superconductors composes several hundred angstroms, then with the macroscopic dimensions of line it is possible to consider that the total energy of magnetic pour on in it they determine by relationship.

Is obvious that the effective mass of electron in comparison with the mass of free electron grows in this case into $\frac{a}{2\lambda}$ of times. Thus, becomes clear nature of such parameters as inductance and the effective mass of electron, which in this case depend, in essence, not from the mass of free electrons, but from the configuration of conductors, on which the electrons move.

The kinetic flow of charges we will consider such flow, whose kinetic inductance is more than field. Let us examine this question in the concrete example [7,8].

For the evacuated coaxial line linear inductance is determined by the relationship

$$L_0 = \frac{\mu_0}{2\pi}\ln\left(\frac{D}{d}\right).$$

With the current $I$, which flows along the internal conductor, energy accumulated in the linear inductance will compose

$$W_L = \frac{1}{2}L_0 I^2 = \frac{\mu_0}{4\pi}\ln\left(\frac{D}{d}\right)I^2 .$$

Ref

7. Ф. Ф. Менде, А. С. Дубровин. Особые свойства реактивных элементов и потоков заряженных частиц. Инженерная физика, №11, 2016, с. 13-21.

The same relationship we can obtain by field method, using Maxwell's equations. since the magnetic field of straight wire, along which flows the current of , we determine by the relationship

$$\oint \vec{H} d\vec{l} = 2\pi r H = I$$

it is possible to find energy of magnetic field, concentrated between the central and external conductor of coaxial line. In this relationship $r$ there is a distance from the axis of center conductor to the observation point. Introducing cylindrical coordinate system and taking into account that the specific energy of magnetic field it is equal

$$W_0 = \frac{1}{2}\mu_0 H^2,$$

we find linear energy of the magnetic field

$$W_H = \frac{\mu_0 I^2}{8\pi^2}\int_d^D \int_0^{2\pi} \frac{r\ d\ rd\varphi}{r} = \frac{\mu_0}{4\pi}\ln\left(\frac{D}{d}\right) I^2.$$

As we see, the linear energy, calculated by field method, and with the aid of the linear inductance they coincide.

We will consider that the current is evenly distributed over the section of center conductor. Then kinetic energy of charges in the conductor of unit length composes

$$W_k = \frac{\pi d^2 n m v^2}{8},$$

where $n$, $m$, $v$ - electron density, their mass and speed respectively.

$$I = \frac{nev\pi d^2}{4}$$

it is possible to write down:

$$W_L = \frac{1}{2}L_0 I^2 = \frac{\mu_0}{4\pi}\ln\left(\frac{D}{d}\right)\frac{n^2 e^2 v^2 \pi^2 d^4}{16}.$$

From these relationships we obtain, that to the case, when

$$W_k \geq W_L$$

the condition corresponds

$$\frac{m}{ne^2} \geq \frac{\mu_0}{8}\ln\left(\frac{D}{d}\right) d^2.$$

From where we find for the charge density.

$$n \leq \frac{8m}{d^2 e^2 \mu_0}.$$

In such a way that the flow would be kinetic, is necessary that the specific kinetic inductance would exceed linear inductance, which is carried out with the observance of the given condition. From this relationship it is possible to estimate, what electron density in the flow corresponds to this of the case.

Notes

Let us examine the concrete example: $d = 1mm$ then we obtain

$$n \le \frac{8m}{e^2 \mu_0 \ln\left(\frac{D}{d}\right) d^2} \approx 10^{20} \frac{1}{m^3}.$$

Notes

With the observance of condition $W_k$ it is considerably more than $W_L$ field inductance it is possible not to consider. Specifically, this case is carried out in the case of using the electron beams for the electro-welding.

Such densities are characteristic to electron beams, and they are considerably lower than electron density in the conductors. Therefore electron beams should be carried to the kinetic flows, while electronic current in the conductors they relate to the potential flows.

Therefore for calculating the energy, transferred by electromagnetic fields they use Poynting's vector, and for calculating the energy, transferred by electron beams is used kinetic energy of separate charges, this all the more correctly, when the discussion deals with the calculation of the energy, transferred by ion beams, since. the mass of ions many times exceeds the mass of electrons.

Thus, the reckoning of the flows of charges to one or the other form depends not only on density and diameter of beam itself, but also on the diameter of that conducting tube, in which it is extended. It is obvious that in the case of potential beam, its front cannot be extended at a velocity, which exceeds the speed of light. It would seem that there are no such limitations for the purely kinetic beams. There is no clear answer to this question as yet. The mass of electron to usually connect with its electric fields and if we with the aid of the external conducting tube begin to limit these fields, then the mass of electron will begin to decrease, but the decrease of mass will lead to the decrease of kinetic inductance and beam will begin to lose its kinetic properties. And only when the part of the mass of electron does not have electrical origin, there is the hope to organize the purely kinetic electron beam, whose speed can exceed the speed of light. If we take the beam of protons, then picture will be the same. But here, if we take, for example, the nuclei of deuterium, which contain the neutron, whose mass is located, but electrical pour on no, then with the aid of such nuclei it is possible to organize purely kinetic beams, and it is possible to design for the fact that such beams can be driven away to the speeds of the large of the speed of light.

## IV. Conclusion

In the article the electrical and current self-induction of radio-technical elements and chains is examined and it is shown that such elements can present the effective resistance, which depends on the time. Is introduced the concept of parametric self-induction. On the basis of the concepts indicated is obtained the wave equation for the long lines, which gives the possibility to establish the velocity of propagation of the front of stress with the connection to the line of dc power supply. The concept of the potential and kinetic flows of charges is introduced.

## References Références Referencias

1. Ф. Ф. Менде. Непротиворечивая электродинамика. Харьков, НТМТ, 2008, – 153 с.
2. F. F. Mende. New Properties of Reactive Elements, Lines of Transmission of Energy and the Relaxation Properties of Electronic Fluxes and Conductors, AASCIT Journal of Physics, Vol.1, No. 3, Publication Date: June 12, 2015, Page: 190-200.

3. F. F. Mende, New Properties of Reactive Elements and the Problem of Propagation of Electrical Signals in Long Lines, American Journal of Electrical and Electronic Engineering, Vol. 2, No. 5, (2014), 141-145.
4. F. F. Mende. Nominal and Parametric Self-Induction of Reactive Elements and Long Lines, Engineering and Technology, Vol.2 , No. 2, Publication Date: April 3, 2015, Page: 69-73
5. Ф. Ф. Менде. Роль и место кинетической индуктивности зарядов в классической электродинамике, Инженерная физика, №11, 2012. с. 10-19.
6. F. F. Mende. Kinetic Induktance Charges and its Role in Classical Electrodynamics. Global Journal of Researches in Engineering (J), General Engineering, Volume 14, Issue 5, Version 1.0, Year 2014.
7. Ф. Ф. Менде, А. С. Дубровин. Особые свойства реактивных элементов и потоков заряженных частиц. Инжененрная физика, №11, 2016, с. 13-21.
8. F. F. Mende. Induction and Parametric Properties of Radio-Technical Elements and Lines and Property of Charges and Their Flows, AASCIT Journal of Physics Vol.1, No. 3, Publication Date: May 21, 2015, Page: 124-134.

Notes

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# Fellows

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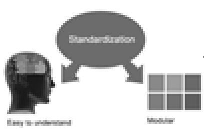


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*The following benefitscan be availed by you only for next three years from the date of certification.*

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We shall provide you intimation regarding launching of e-version of journal of your stream time to time.This may be utilized in your library for the enrichment of knowledge of your students as well as it can also be helpful for the concerned faculty members.

The MARSS member can apply for approval, grading and certification of standards of their educational and Institutional Degrees to Open Association of Research, Society U.S.A.


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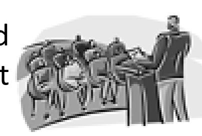

The Global Journals Incorporation (USA) at its discretion can also refer double blind peer reviewed paper at their end to the board for the verification and to get recommendation for final stage of acceptance of publication.

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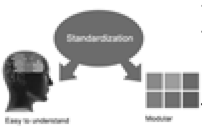


After nomination of your institution as "Institutional Fellow" and constantly functioning successfully for one year, we can consider giving recognition to your institute to function as Regional/Zonal office on our behalf.
The board can also take up the additional allied activities for betterment after our consultation.

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"

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**Acknowledgments**

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- Large images must be in one column.
- The names of first main headings (Heading 1) must be in Roman font, capital letters, and font size of 10.
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The recommended size of an original research paper is under 15,000 words and review papers under 7,000 words. Research articles should be less than 10,000 words. Research papers are usually longer than review papers. Review papers are reports of significant research (typically less than 7,000 words, including tables, figures, and references)

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f) Results which should be presented concisely by well-designed tables and figures.
g) Suitable statistical data should also be given.
h) All data must have been gathered with attention to numerical detail in the planning stage.

Design has been recognized to be essential to experiments for a considerable time, and the editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned unrefereed.

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j) There should be brief acknowledgments.
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Many researchers searching for information online will use search engines such as Google, Yahoo or others. By optimizing your paper for search engines, you will amplify the chance of someone finding it. In turn, this will make it more likely to be viewed and cited in further works. Global Journals has compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

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A major lynchpin of research work for the writing of research papers is the keyword search, which one will employ to find both library and internet resources. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining, and indexing.

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Although low-quality images are sufficient for review purposes, print publication requires high-quality images to prevent the final product being blurred or fuzzy. Submit (possibly by e-mail) EPS (line art) or TIFF (halftone/ photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Avoid using pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings). Please give the data for figures in black and white or submit a Color Work Agreement form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

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***2. Think like evaluators:*** If you are in confusion or getting demotivated because your paper may not be accepted by the evaluators, then think, and try to evaluate your paper like an evaluator. Try to understand what an evaluator wants in your research paper, and you will automatically have your answer. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

***3. Ask your guides:*** If you are having any difficulty with your research, then do not hesitate to share your difficulty with your guide (if you have one). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work, then ask your supervisor to help you with an alternative. He or she might also provide you with a list of essential readings.

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***6. Bookmarks are useful:*** When you read any book or magazine, you generally use bookmarks, right? It is a good habit which helps to not lose your continuity. You should always use bookmarks while searching on the internet also, which will make your search easier.

***7. Revise what you wrote:*** When you write anything, always read it, summarize it, and then finalize it.

***8. Make every effort:*** Make every effort to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in the introduction—what is the need for a particular research paper. Polish your work with good writing skills and always give an evaluator what he wants. Make backups: When you are going to do any important thing like making a research paper, you should always have backup copies of it either on your computer or on paper. This protects you from losing any portion of your important data.

***9. Produce good diagrams of your own:*** Always try to include good charts or diagrams in your paper to improve quality. Using several unnecessary diagrams will degrade the quality of your paper by creating a hodgepodge. So always try to include diagrams which were made by you to improve the readability of your paper. Use of direct quotes: When you do research relevant to literature, history, or current affairs, then use of quotes becomes essential, but if the study is relevant to science, use of quotes is not preferable.

***10. Use proper verb tense:*** Use proper verb tenses in your paper. Use past tense to present those events that have happened. Use present tense to indicate events that are going on. Use future tense to indicate events that will happen in the future. Use of wrong tenses will confuse the evaluator. Avoid sentences that are incomplete.

***11. Pick a good study spot:*** Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

***12. Know what you know:*** Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

***13. Use good grammar:*** Always use good grammar and words that will have a positive impact on the evaluator; use of good vocabulary does not mean using tough words which the evaluator has to find in a dictionary. Do not fragment sentences. Eliminate one-word sentences. Do not ever use a big word when a smaller one would suffice.

Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

***14. Arrangement of information:*** Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

***15. Never start at the last minute:*** Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

***16. Multitasking in research is not good:*** Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

***17. Never copy others' work:*** Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

***18. Go to seminars:*** Attend seminars if the topic is relevant to your research area. Utilize all your resources.

***19. Refresh your mind after intervals:*** Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.

***20. Think technically:*** Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

***21. Adding unnecessary information:*** Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

***22. Report concluded results:*** Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

***23. Upon conclusion:*** Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

## Informal Guidelines of Research Paper Writing

***Key points to remember:***

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

**Final points:**

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

*The introduction:* This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

**The discussion section:**

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

**General style:**

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

**To make a paper clear:** Adhere to recommended page limits.

*Mistakes to avoid:*

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

**Title page:**

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

**Abstract:** This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

*Reason for writing the article—theory, overall issue, purpose.*

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

**Approach:**

- o Single section and succinct.
- o An outline of the job done is always written in past tense.
- o Concentrate on shortening results—limit background information to a verdict or two.
- o Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

**Introduction:**

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.

*The following approach can create a valuable beginning:*

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

**Approach:**

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

**Procedures (methods and materials):**

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

**Materials:**

*Materials may be reported in part of a section or else they may be recognized along with your measures.*

**Methods:**

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

**Approach:**

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

**What to keep away from:**

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

**Results:**

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

**Content:**

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

**What to stay away from:**

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

**Approach:**

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

**Figures and tables:**

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

**Discussion:**

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- Give details of all of your remarks as much as possible, focusing on mechanisms.
- Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

**Approach:**

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

Describe generally acknowledged facts and main beliefs in present tense.

## The Administration Rules

Administration Rules to Be Strictly Followed before Submitting Your Research Paper to Global Journals Inc.

*Please read the following rules and regulations carefully before submitting your research paper to Global Journals Inc. to avoid rejection*.

*Segment draft and final research paper:* You have to strictly follow the template of a research paper, failing which your paper may get rejected. You are expected to write each part of the paper wholly on your own. The peer reviewers need to identify your own perspective of the concepts in your own terms. Please do not extract straight from any other source, and do not rephrase someone else's analysis. Do not allow anyone else to proofread your manuscript.

*Written material:* You may discuss this with your guides and key sources. Do not copy anyone else's paper, even if this is only imitation, otherwise it will be rejected on the grounds of plagiarism, which is illegal. Various methods to avoid plagiarism are strictly applied by us to every paper, and, if found guilty, you may be blacklisted, which could affect your career adversely. To guard yourself and others from possible illegal use, please do not permit anyone to use or even read your paper and file.

## CRITERION FOR GRADING A RESEARCH PAPER (COMPILATION) BY GLOBAL JOURNALS

**Please note that following table is only a Grading of "Paper Compilation" and not on "Performed/Stated Research" whose grading solely depends on Individual Assigned Peer Reviewer and Editorial Board Member. These can be available only on request and after decision of Paper. This report will be the property of Global Journals.**

| Topics | Grades | | |
|---|---|---|---|
| | **A-B** | **C-D** | **E-F** |
| ***Abstract*** | Clear and concise with appropriate content, Correct format. 200 words or below | Unclear summary and no specific data, Incorrect form<br><br>Above 200 words | No specific data with ambiguous information<br><br>Above 250 words |
| ***Introduction*** | Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited | Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter | Out of place depth and content, hazy format |
| ***Methods and Procedures*** | Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads | Difficult to comprehend with embarrassed text, too much explanation but completed | Incorrect and unorganized structure with hazy meaning |
| ***Result*** | Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake | Complete and embarrassed text, difficult to comprehend | Irregular format with wrong facts and figures |
| ***Discussion*** | Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited | Wordy, unclear conclusion, spurious | Conclusion is not cited, unorganized, difficult to comprehend |
| ***References*** | Complete and correct format, well organized | Beside the point, Incomplete | Wrong format and structuring |

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