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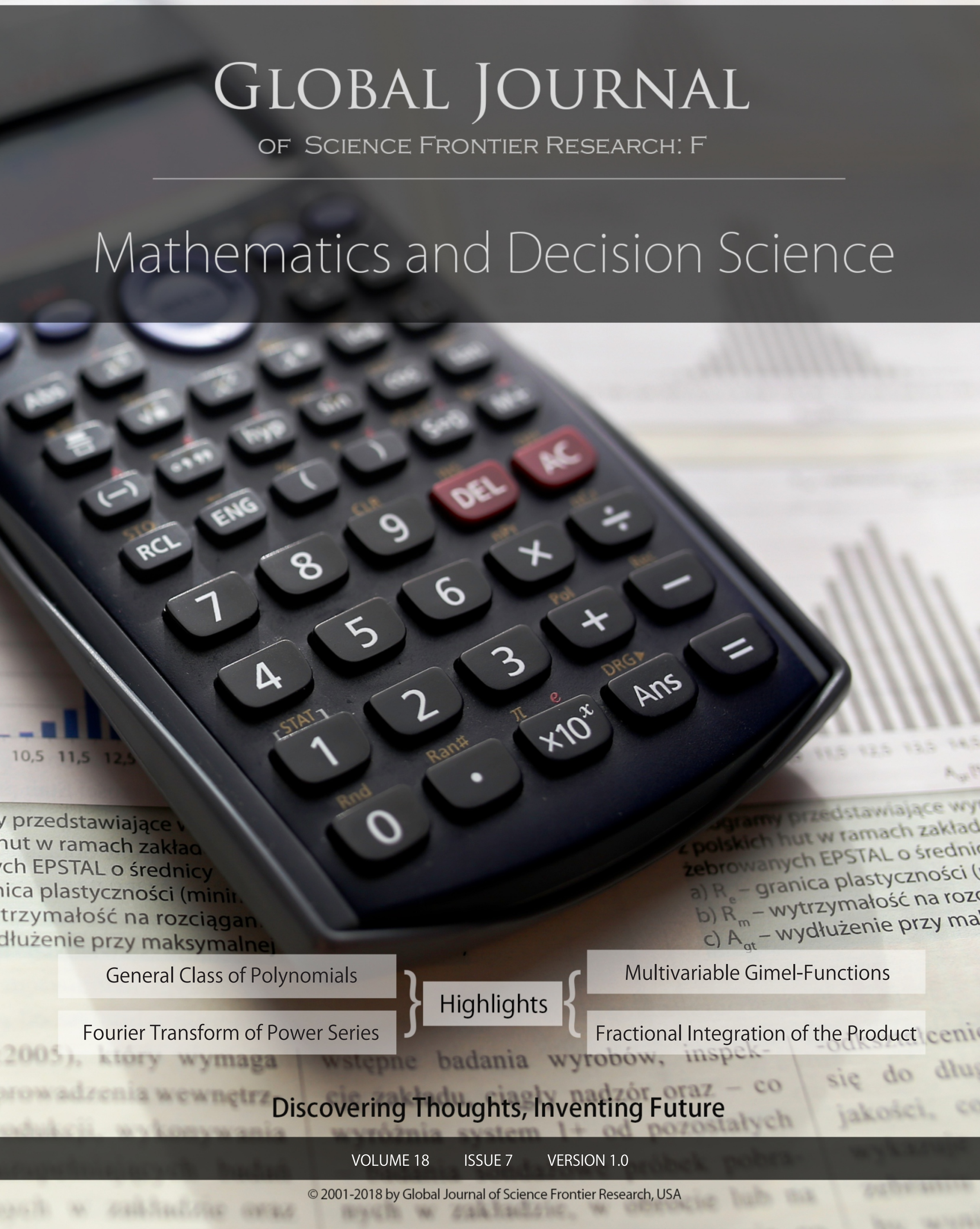

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Highlights

- General Class of Polynomials
- Fourier Transform of Power Series
- Multivariable Gimel-Functions
- Fractional Integration of the Product

Discovering Thoughts, Inventing Future

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On a General Class of Multiple Eulerian Integrals with Multivariable Gimel-Function

By Frederic Ayant

Abstract- Recently, Raina and Srivastava [11] and Srivastava and Hussain [16] have provided closed-form expressions for a number of a Eulerian integral about the multivariable H-functions. Motivated by these recent works, we aim at evaluating a general multiple Eulerian integrals involving the product of multivariable I-function defined by Prathima et al. [10], a class of multivariable polynomials, a generalization of the Mittag-Leffler functions and multivariable Gimel-function.

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Abstract- Recently, Raina and Srivastava [11] and Srivastava and Hussain [16] have provided closed-form expressions for a number of a Eulerian integral about the multivariable H-functions. Motivated by these recent works, we aim at evaluating a general multiple Eulerian integrals involving the product of multivariable I-function defined by Prathima et al. [10], a class of multivariable polynomials, a generalization of the Mittag-Leffler functions and multivariable Gimel-function.

Keywords: multivariable gimel-function, multiple eulerian integral, multivariable polynomials, generalization of the mittag-leffler function, multivariable i-function.

I. Introduction and Prerequisites

The well-known Eulerian Beta integral

$$\int_a^b (z-a)^{\alpha-1}(b-t)^{\beta-1}\mathrm{d}t = (b-a)^{\alpha+\beta-1}B(\alpha,\beta)(Re(\alpha)>0, Re(\beta)>0, b>a) \tag{1.1}$$

Is a basic result for evaluation of numerous other potentially useful integrals involving various special functions and polynomials. The authors Raina and Srivastava [11], Saigo and Saxena [12], Srivastava and Hussain [16], Srivastava and Garg [15] etc. have established a number of Eulerian integrals involving a various general class of polynomials, Meijer's G-function and Fox's H-function of one and more variables with general arguments. Recently, several authors study some multiple Eulerian integrals, see Bhargava et al. [4], Goyal and Mathur [6] , Ayant [1] and others. In this paper we obtain general multiple Eulerian integrals of the product of multivariable I-function defined by Prathima et al. [10], a class of multivariable polynomials, a generalization of the Mittag-Leffler functions and multivariable Gimel function.

For this study, we need the following function appointed Generalized multiple-index Mittag-Leffler function

A further generalization of the Mittag-Leffler functions is proposed recently in Paneva-Konovska [7]. These are 3mparametric Mittag-Leffler type functions generalizing the Prabhakar [8] 3-parametric function, defined as:

$$E_{(\alpha_i),(\beta_i)}^{(\gamma_i),m}(z) = \sum_{k=0}^{\infty}\frac{(\gamma_1)_k\cdots(\gamma_m)_k}{\Gamma(\alpha_1 k+\beta_1)\cdots\Gamma(\alpha_m k+\beta_m)}\frac{z^k}{k!} \tag{1.2}$$

where $\alpha_i,\beta_i,\gamma_i \in \mathbb{C}, i=1,\cdots,m, Re(\alpha_i)>0$

We shall note

$$E_k = \frac{(\gamma_1)_k\cdots(\gamma_m)_k}{\Gamma(\alpha_1 k+\beta_1)\cdots\Gamma(\alpha_m k+\beta_m)} \tag{1.3}$$

The generalized polynomials defined by Srivastava [14], is given in the following manner:

$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}[y_1,\cdots,y_u] \;=\; \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_u)_{M_uK_u}}{K_u!}$$

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Ref

8. T. R. Prabhakar, A singular integral equation with a generalizedMittag-Leffler function in the kernel.Yokohama Math. J.19(1971), 7–15.

$$A[N_1,K_1;\cdots;N_u,K_u]y_1^{K_1}\cdots y_u^{K_u} \tag{1.4}$$

Where $M_1,\cdots,M_u$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_u,K_u]$ are arbitrary constants, real or complex.

We shall note

$$A_u = \frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_u)_{M_uK_u}}{K_u!}A[N_1,K_1;\cdots;N_u,K_u] \tag{1.5}$$

The multivariable I-function defined by Prathima et al. [10] have expressed in term of multiple Mellin-Barnes types integrals :

$$\bar{I}(z_1,\cdots,z_s) = I^{0,N:M_1,N_1;\cdots;M_r,N_r}_{P,Q:P_1,Q_1;\cdots;P_r,Q_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(s)};A_j)_{1,P}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_s & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(s)};B_j)_{1,Q}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,N_1},(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{N_1+1,P_1};\cdots;(c_j^{(s)},\gamma_j^{(s)};C_j^{(r)})_{1,N_r},(c_j^{(s)},\gamma_j^{(s)};C_j^{(r)})_{N_s+1,P_s} \\ \\ (\mathrm{d}_j^{(1)},\delta_j^{(1)};1)_{1,M_1},(d_j^{(1)},\delta_j^{(1)};D_1)_{M_1+1,Q_1};\cdots;(d_j^{(s)},\delta_j^{(s)};1)_{1,M_s},(d_j^{(s)},\delta_j^{(s)};D_s)_{M_s+1,Q_s} \end{array}\right) \tag{1.6}$$

$$= \frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\phi(t_1,\cdots,t_s)\prod_{i=1}^{s}\theta_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.7}$$

where $\phi(t_1,\cdots,t_s),\theta_i(s_i), i=1,\cdots,r$ are given by :

$$\phi(t_1,\cdots,t_s) = \frac{\prod_{j=1}^{N}\Gamma^{A_j}\left(1-aj+\sum_{i=1}^{s}\alpha_j^{(i)}t_j\right)}{\prod_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{s}\alpha_j^{(i)}t_j\right)\prod_{j=1}^{Q}\Gamma^{B_j}\left(1-bj+\sum_{i=1}^{s}\beta_j^{(i)}t_j\right)} \tag{1.8}$$

$$\phi_i(t_i) = \frac{\prod_{j=1}^{N_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}t_i\right)\prod_{j=1}^{M_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}t_i\right)}{\prod_{j=N_i+1}^{P_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}t_i\right)\prod_{j=M_i+1}^{Q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}t_i\right)} \tag{1.9}$$

For more details, see Prathima et al. [10].

We can obtain the series representation and behavior for small values for the function $\bar{I}(z_1,\cdots,z_s)$ defined and represented by (1.16). The series representation may be given as follows :

which is valid under the following conditions:

$$\delta_i^{(h)}[d_i^{(j)}+r] \neq \delta_i^{(j)}[d_i^{(h)}+\mu] \text{ for } j\neq h, j,h=1,\cdots,M_i\,,s,\mu=0,1,2,\cdots$$

$$U_i = \sum_{j=1}^{P}A_j\alpha_j^{(i)} - \sum_{j=1}^{Q}B_j\beta_j^{(i)} + \sum_{j=1}^{P_i}C_j^{(i)}\gamma_j^{(i)} - \sum_{j=M_i+1}^{Q_i}D_j^{(i)}\delta_j^{(i)} \leqslant 0, i=1,\cdots,s \quad \text{and } z_i\neq 0$$

and if all the poles of (1.7) are simple. Then the integral (1.7) can be evaluated with the help of the Residue theorem to give

$$\bar{I}(z_1,\cdots,z_s) = \sum_{h_i=1}^{M_i}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^{s}\phi_i z_i^{\eta_{h_i,g_i}}(-)^{\sum_{i=1}^{s}g_i}}{\prod_{i=1}^{s}\delta_{h^{(i)}}^{(i)}\prod_{i=1}^{s}g_i!} \tag{1.10}$$

10. J. Prathima, V. Nambisan and S.K. Kurumujji, A Study of I-function of Several Complex Variables, International Journal of Engineering Mathematics Vol (2014), 1-12.

where ϕ_1 and ϕ_i are defined by

$$\phi_1=\frac{\prod_{j=1}^{N}\Gamma^{A_j}\left(1-aj+\sum_{i=1}^{s}\alpha_j^{(i)}\eta_{h_i,g_i}\right)}{\prod_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{s}\alpha_j^{(i)}\eta_{h_i,g_i}\right)\prod_{j=1}^{Q}\Gamma^{B_j}\left(1-bj+\sum_{i=1}^{s}\beta_j^{(i)}\eta_{h_i,g_i}\right)} \tag{1.11}$$

and

$$\phi_i=\frac{\prod_{j=1}^{N_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}\eta_{h_i,g_i}\right)\prod_{j=1}^{M_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}\eta_{h_i,g_i}\right)}{\prod_{j=N_i+1}^{P_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}\eta_{h_i,g_i}\right)\prod_{j=M_i+1}^{Q_i}\Gamma^{(D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}\eta_{h_i,g_i}\right)},i=1,\cdots,s \tag{1.12}$$

where $\eta_{h_i,g_i}=\dfrac{d_{h^{(i)}}^{(i)}+g_i}{\delta_{h^{(i)}}^{(i)}},i=1,\cdots,s.$

Throughout this paper, let $\mathbb{C},\mathbb{R}$ and $\mathbb{N}$ be set of complex numbers, real numbers and positive integers respectively.

Also, $\mathbb{N}_0=\mathbb{N}\cup\{0\}$. We define a generalized transcendental function of several complex variables, see Ayant [2] for more details,

$$\gimel(z_1,\cdots,z_r)=\gimel^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)}}_{p_{i_2},q_{i_2},\tau_{i_2};R_2;p_{i_3},q_{i_3},\tau_{i_3};R_3;\cdots;p_{i_r},q_{i_r},\tau_{i_r}:R_r:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|}z_1\\ \cdot\\ \cdot\\ \cdot\\ z_r\end{array}\right.$$

$$\begin{array}{c}[(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}};[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},\\ [\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}};\end{array}$$

$$\begin{array}{l}[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)};A_{rj})]_{1,n_r},\\ [\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;\qquad\cdots\end{array}$$

$$\begin{array}{l}[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)};A_{rji_r})]_{n_r+1,p_r}:[(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})]_{1,n^{(1)}},[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})]_{n^{(1)}+1,p_i^{(1)}}\\ [\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)};B_{rji_r})]_{1,q_r}:[(\mathrm{d}_j^{(1)}),\delta_j^{(1)};D_j^{(1)})]_{1,m^{(1)}},[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})]_{m^{(1)}+1,q_i^{(1)}}\end{array}$$

$$\left.\begin{array}{l};\cdots;[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})]_{1,m^{(r)}},[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})]_{m^{(r)}+1,p_i^{(r)}}\\ ;\cdots;[(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})]_{1,n^{(r)}},[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_{ji^{(r)}}^{(r)}]_{n^{(r)}+1,q_i^{(r)}}\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.13}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{n_2}\Gamma^{A_{2j}}(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k)}{\sum_{i_2=1}^{R_2}[\tau_{i_2}\prod_{j=n_2+1}^{p_{i_2}}\Gamma^{A_{2ji_2}}(a_{2ji_2}-\sum_{k=1}^{2}\alpha_{2ji_2}^{(k)}s_k)\prod_{j=1}^{q_{i_2}}\Gamma^{B_{2ji_2}}(1-b_{2ji2}+\sum_{k=1}^{2}\beta_{2ji2}^{(k)}s_k)]}$$

$$\frac{\prod_{j=1}^{n_3}\Gamma^{A_{3j}}(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k)}{\sum_{i_3=1}^{R_3}[\tau_{i_3}\prod_{j=n_3+1}^{p_{i_3}}\Gamma^{A_{3ji_3}}(a_{3ji_3}-\sum_{k=1}^{3}\alpha_{3ji_3}^{(k)}s_k)\prod_{j=1}^{q_{i_3}}\Gamma^{B_{3ji_3}}(1-b_{3ji3}+\sum_{k=1}^{3}\beta_{3ji3}^{(k)}s_k)]}$$

$$\cdot\qquad\cdot\qquad\cdot\qquad\cdot$$

Ref

2. F. Ayant, An expansion formula for multivariable Gimel-function involving generalized Legendre Associated function, International Journal of Mathematics Trends and Technology (IJMTT), 56(4) (2018), 223-228.

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$$\frac{\prod_{j=1}^{n_r}\Gamma^{A_{rj}}(1-a_{rj}+\sum_{k=1}^{r}\alpha_{rj}^{(k)}s_k)}{\sum_{i_r=1}^{R_r}[\tau_{i_r}\prod_{j=n_r+1}^{p_{i_r}}\Gamma^{A_{rji_r}}(a_{rji_r}-\sum_{k=1}^{r}\alpha_{rji_r}^{(k)}s_k)\prod_{j=1}^{q_{i_r}}\Gamma^{B_{rji_r}}(1-b_{rji_r}+\sum_{k=1}^{r}\beta_{rji_r}^{(k)}s_k)]} \tag{1.14}$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m^{(k)}}\Gamma^{D_j^{(k)}}(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n^{(k)}}\Gamma^{C_j^{(k)}}(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m^{(k)}+1}^{q_{i^{(k)}}}\Gamma^{D_{ji^{(k)}}^{(k)}}(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n^{(k)}+1}^{p_{i^{(k)}}}\Gamma^{C_{ji^{(k)}}^{(k)}}(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.15}$$

1) $[(c_j^{(1)};\gamma_j^{(1)}]_{1,n_1}$ stands for $(c_1^{(1)};\gamma_1^{(1)}),\cdots,(c_{n_1}^{(1)};\gamma_{n_1}^{(1)})$.

2) $n_2,\cdots,n_r,m^{(1)},n^{(1)},\cdots,m^{(r)},n^{(r)},p_{i_2},q_{i_2},R_2,\tau_{i_2},\cdots,p_{i_r},q_{i_r},R_r,\tau_{i_r},p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}},R^{(r)}\in\mathbb{N}$ and verify :

$0\leqslant m_2,\cdots,0\leqslant m_r,0\leqslant n_2\leqslant p_{i_2},\cdots,0\leqslant n_r\leqslant p_{i_r},0\leqslant m^{(1)}\leqslant q_{i^{(1)}},\cdots,0\leqslant m^{(r)}\leqslant q_{i^{(r)}}$

$0\leqslant n^{(1)}\leqslant p_{i^{(1)}},\cdots,0\leqslant n^{(r)}\leqslant p_{i^{(r)}}$.

3) $\tau_{i_2}(i_2=1,\cdots,R_2)\in\mathbb{R}^+;\tau_{i_r}\in\mathbb{R}^+(i_r=1,\cdots,R_r);\tau_{i^{(k)}}\in\mathbb{R}^+(i=1,\cdots,R^{(k)}),(k=1,\cdots,r)$.

4) $\gamma_j^{(k)},C_j^{(k)}\in\mathbb{R}^+;(j=1,\cdots,n^{(k)});(k=1,\cdots,r);\delta_j^{(k)},D_j^{(k)}\in\mathbb{R}^+;(j=1,\cdots,m^{(k)});(k=1,\cdots,r)$.

$\mathrm{C}_{ji^{(k)}}^{(k)}\in\mathbb{R}^+,(j=m^{(k)}+1,\cdots,p^{(k)});(k=1,\cdots,r);$

$\mathrm{D}_{ji^{(k)}}^{(k)}\in\mathbb{R}^+,(j=n^{(k)}+1,\cdots,q^{(k)});(k=1,\cdots,r)$.

$\alpha_{kj}^{(l)},A_{kj}\in\mathbb{R}^+;(j=1,\cdots,n_k);(k=2,\cdots,r);(l=1,\cdots,k)$.

$\alpha_{kji_k}^{(l)},A_{kji_k}\in\mathbb{R}^+;(j=n_k+1,\cdots,p_{i_k});(k=2,\cdots,r);(l=1,\cdots,k)$.

$\beta_{kji_k}^{(l)},B_{kji_k}\in\mathbb{R}^+;(j=m_k+1,\cdots,q_{i_k});(k=2,\cdots,r);(l=1,\cdots,k)$.

$\delta_{ji^{(k)}}^{(k)}\in\mathbb{R}^+;(i=1,\cdots,R^{(k)});(j=m^{(k)}+1,\cdots,q_{i^{(k)}});(k=1,\cdots,r)$.

$\gamma_{ji^{(k)}}^{(k)}\in\mathbb{R}^+;(i=1,\cdots,R^{(k)});(j=n^{(k)}+1,\cdots,p_{i^{(k)}});(k=1,\cdots,r)$.

5) $c_j^{(k)}\in\mathbb{C};(j=1,\cdots,n^{(k)});(k=1,\cdots,r);d_j^{(k)}\in\mathbb{C};(j=1,\cdots,m^{(k)});(k=1,\cdots,r)$.

$a_{kji_k}\in\mathbb{C};(j=n_k+1,\cdots,p_{i_k});(k=2,\cdots,r)$.

$b_{kji_k}\in\mathbb{C};(j=1,\cdots,q_{i_k});(k=2,\cdots,r)$.

$d_{ji^{(k)}}^{(k)}\in\mathbb{C};(i=1,\cdots,R^{(k)});(j=m^{(k)}+1,\cdots,q_{i^{(k)}});(k=1,\cdots,r)$.

$\gamma_{ji^{(k)}}^{(k)}\in\mathbb{C};(i=1,\cdots,R^{(k)});(j=n^{(k)}+1,\cdots,p_{i^{(k)}});(k=1,\cdots,r)$.

The contour L_k is in the $s_k(k=1,\cdots,r)$- plane and runs from $\sigma-i\infty$ to $\sigma+i\infty$ where σ if is a real number with loop, if necessary to ensure that the poles of $\Gamma^{A_{2j}}\left(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k\right)(j=1,\cdots,n_2),\ \Gamma^{A_{3j}}\left(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k\right)$

Notes

$(j=1,\cdots,n_3),\cdots,\Gamma^{A_{rj}}\left(1-a_{rj}+\sum_{i=1}^{r}\alpha_{rj}^{(i)}\right)(j=1,\cdots,n_r),\Gamma^{C_j^{(k)}}\left(1-c_j^{(k)}+\gamma_j^{(k)}s_k\right)(j=1,\cdots,n^{(k)})(k=1,\cdots,r)$ to the right of the contour L_k and the poles of $\Gamma^{D_j^{(k)}}\left(d_j^{(k)}-\delta_j^{(k)}s_k\right)(j=1,\cdots,m^{(k)})(k=1,\cdots,r)$ lie to the left of the contour L_k. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained of the corresponding conditions for multivariable H-function given by: as

$|arg(z_k)|<\frac{1}{2}A_i^{(k)}\pi$ where

$$A_i^{(k)}=\sum_{j=1}^{m^{(k)}}D_j^{(k)}\delta_j^{(k)}+\sum_{j=1}^{n^{(k)}}C_j^{(k)}\gamma_j^{(k)}-\tau_{i^{(k)}}\left(\sum_{j=m^{(k)}+1}^{q_i^{(k)}}D_{ji^{(k)}}^{(k)}\delta_{ji^{(k)}}^{(k)}+\sum_{j=n^{(k)}+1}^{p_i^{(k)}}C_{ji^{(k)}}^{(k)}\gamma_{ji^{(k)}}^{(k)}\right)+$$

$$-\tau_{i_2}\left(\sum_{j=n_2+1}^{p_{i_2}}A_{2ji_2}\alpha_{2ji_2}^{(k)}+\sum_{j=1}^{q_{i_2}}B_{2ji_2}\beta_{2ji_2}^{(k)}\right)-\cdots-\tau_{i_r}\left(\sum_{j=n_r+1}^{p_{i_r}}A_{rji_r}\alpha_{rji_r}^{(k)}+\sum_{j=1}^{q_{i_r}}B_{rji_r}\beta_{rji_r}^{(k)}\right)\tag{1.16}$$

Following the lines of Braaksma ([5] p. 278), we may establish the asymptotic expansion in the following convenient form:

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,),max(\,|z_1|,\cdots,|z_r|\,)\rightarrow 0$$

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,),min(\,|z_1|,\cdots,|z_r|\,)\rightarrow\infty \text{ where } i=1,\cdots,r:$$

$$\alpha_i=\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] \text{ and } \beta_i=\max_{1\leqslant j\leqslant n^{(i)}}Re\left[C_j^{(i)}\left(\frac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right)\right]$$

Remark 1.
If $n_2=\cdots=n_{r-1}=p_{i_2}=q_{i_2}=\cdots=p_{i_{r-1}}=q_{i_{r-1}}=0$ and $A_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=A_{rji_r}=B_{rji_r}=1$ $A_{rj}=A_{rji_r}=B_{rji_r}=1$, then the multivariable Gimel-function reduces in the multivariable Aleph- function defined by Ayant [3].

Remark 2.
If $n_2=\cdots=n_r=p_{i_2}=q_{i_2}=\cdots=p_{i_r}=q_{i_r}=0$ and $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2=\cdots=R_r=R^{(1)}=\cdots=R^{(r)}=1$, then the multivariable Gimel-function reduces in a multivariable I-function defined by Prathima et al. [10].

Remark 3.
If $A_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=A_{rji_r}=B_{rji_r}=1$ and $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2=\cdots=R_r=R^{(1)}=\cdots=R^{(r)}=1$, then the generalized multivariable Gimel-function reduces in multivariable I-function defined by Prasad [9].

Remark 4.
If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the multivariable H-function defined by Srivastava and Panda [18,19].

II. Integral Representation of Generalized Hypergeometric Function

The following generalized hypergeometric function regarding multiple integrals contour is also [12,p. 39, Eq. (30)]

$$\frac{\prod_{j=1}^{P}\Gamma(A_j)}{\prod_{j=1}^{Q}\Gamma(B_j)}\,{}_PF_Q\left[(A_P);(B_Q);-(x_1+\cdots+x_r)\right]$$

Ref

3. F. Ayant, An integral associated with the Aleph-functions of several variables. International Journal of Mathematics Trends and Technology (IJMTT), 31(3) (2016), 142-154.

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \frac{\prod_{j=1}^{P} \Gamma\left(A_j + \sum_{i=1}^{r} s_i\right)}{\prod_{j=1}^{Q} \Gamma\left(B_j + \sum_{i=1}^{r} s_i\right)} \Gamma(-s_1)\cdots\Gamma(-s_r) x_1^{s_1} \cdots x_r^{s_r} \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{2.1}$$

where the contours are of Barnes type with indentations, if necessary , to ensure that the poles $\Gamma\left(A_j + \sum_{i=1}^{r} s_i\right)$ are separated from those of $\Gamma(-s_j), j = 1, \cdots, r$. The above result (2.1) is easily established by an appeal to the calculus of residues by calculating the residues at the poles of $\Gamma(-s_j), j = 1, \cdots, r$.

III. Main Integral

We shall use the following notations :

$$\mathbb{A} = [(\mathrm{a}_{2j}; \alpha_{2j}^{(1)}, \alpha_{2j}^{(2)}; A_{2j})]_{1,n_2}, [\tau_{i_2}(a_{2ji_2}; \alpha_{2ji_2}^{(1)}, \alpha_{2ji_2}^{(2)}; A_{2ji_2})]_{n_2+1,p_{i_2}}, [(a_{3j}; \alpha_{3j}^{(1)}, \alpha_{3j}^{(2)}, \alpha_{3j}^{(3)}; A_{3j})]_{1,n_3},$$

$$[\tau_{i_3}(a_{3ji_3}; \alpha_{3ji_3}^{(1)}, \alpha_{3ji_3}^{(2)}, \alpha_{3ji_3}^{(3)}; A_{3ji_3})]_{n_3+1,p_{i_3}}; \cdots ; [(\mathrm{a}_{(r-1)j}; \alpha_{(r-1)j}^{(1)}, \cdots, \alpha_{(r-1)j}^{(r-1)}; A_{(r-1)j})]_{1,n_{r-1}},$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}}; \alpha_{(r-1)ji_{r-1}}^{(1)}, \cdots, \alpha_{(r-1)ji_{r-1}}^{(r-1)}; A_{(r-1)ji_{r-1}})]_{n_{r-1}+1,p_{i_{r-1}}} \tag{3.1}$$

$$\mathbf{A} = [(\mathrm{a}_{rj}; \alpha_{rj}^{(1)}, \cdots, \alpha_{rj}^{(r)}, \underbrace{0, \cdots, 0}_{l+T}; A_{rj})]_{1,n_r}, [\tau_{i_r}(a_{rji_r}; \alpha_{rji_r}^{(1)}, \cdots, \alpha_{rji_r}^{(r)}, \underbrace{0, \cdots, 0}_{l+T}; A_{rji_r})]_{\mathfrak{n}+1,p_{i_r}} \tag{3.2}$$

$$A_1 = \left[1 + \sigma_j^{(1)} - \sum_{l=1}^{u} K_l \rho_j^{\prime\prime(1,l)} - \sum_{i=1}^{s} \eta_{G_i,g_i} \rho_j^{(1,i)} - \theta_j^{(1)} k; \rho_j^{\prime(1,1)}, \cdots, \rho_j^{\prime(1,r)} \tau_j^{(1,1)}, \cdots, \tau_j^{(1,l)}, 1, \underbrace{0, \cdots, 0}_{T-1}; 1\right]_{1,s}, \cdots,$$

$$\left[1 + \sigma_j^{(T)} - \sum_{l=1}^{u} K_l \rho_j^{\prime\prime(T,l)} - \sum_{i=1}^{s} \eta_{G_i,g_i} \rho_j^{(T,i)} - \theta_j^{(T)} k; \rho_j^{\prime(T,1)}, \cdots, \rho_j^{\prime(T,r)} \tau_j^{(T,1)}, \cdots, \tau_j^{(T,l)}, 1, \underbrace{0, \cdots, 0}_{T-1}; 1\right]_{s1,},$$

$$[1 - A_j; \underbrace{0, \cdots, 0}_{r}, \underbrace{1, \cdots, 1}_{l}, \underbrace{0, \cdots, 0}_{T}; 1]_{1,P},$$

$$\left[1 - \alpha_j - \sum_{l=1}^{u} K_l \delta_j^{\prime\prime(l)} - \sum_{i=1}^{s} \eta_{G_i,g_i} \delta_j^{(i)} - \zeta_j^{(1)} k; \delta_j^{\prime(1)}, \cdots, \delta_j^{\prime(r)} \mu_i^{(1)}, \cdots, \mu_j^{(l)}, \underbrace{1, \cdots, 1}_{W}, \underbrace{0, \cdots, 0}_{W+1,T}; 1\right]_{1,s},$$

$$\left[1 - \beta_j - \sum_{l=1}^{u} K_l \eta_j^{\prime\prime(l)} - \sum_{i=1}^{s} \eta_{G_i,g_i} \eta_j^{(i)} - \lambda_j^{(1)} k; \eta_j^{\prime(1)}, \cdots, \eta_j^{\prime(r)} \theta_i^{(1)}, \cdots, \theta_j^{(l)}, \underbrace{1, \cdots, 1}_{W}, \underbrace{0, \cdots, 0}_{W+1,T}; 1\right]_{1,s} \tag{3.3}$$

$$A = [(\mathrm{c}_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)}]_{1,n^{(1)}}, [\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}, \gamma_{ji^{(1)}}^{(1)}; C_{ji^{(1)}}^{(1)})]_{n^{(1)}+1,p_i^{(1)}}; \cdots ;$$

$$[(c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})]_{1,m^{(r)}}, [\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}, \gamma_{ji^{(r)}}^{(r)}; C_{ji^{(r)}}^{(r)})]_{m^{(r)}+1,p_i^{(r)}} \underbrace{(1,0;1), \cdots, (1,0;1)}_{l}, \underbrace{(1,0;1), \cdots, (1,0;1)}_{T} \tag{3.4}$$

$$\mathbb{B} = [\tau_{i_2}(b_{2ji_2}; \beta_{2ji_2}^{(1)}, \beta_{2ji_2}^{(2)}; B_{2ji_2})]_{1,q_{i_2}}, [\tau_{i_3}(b_{3ji_3}; \beta_{3ji_3}^{(1)}, \beta_{3ji_3}^{(2)}, \beta_{3ji_3}^{(3)}; B_{3ji_3})]_{1,q_{i_3}}; \cdots ;$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}}; \beta_{(r-1)ji_{r-1}}^{(1)}, \cdots, \beta_{(r-1)ji_{r-1}}^{(r-1)}; B_{(r-1)ji_{r-1}})]_{1,q_{i_{r-1}}} \tag{3.5}$$

Notes

Notes

$$B_1=\left[1+\sigma_j^{(1)}-\sum_{l=1}^{u}K_l\rho_j^{\prime\prime(1,l)}-\sum_{i=1}^{s}\eta_{G_i,g_i}\rho_j^{(1,i)}-\theta_j^{(1)}k;\rho_j^{\prime(1,1)},\cdots,\rho_j^{\prime(1,r)}\tau_j^{(1,1)},\cdots,\tau_j^{(1,l)},\underbrace{0,\cdots,0}_{T};1\right]_{1,s},\cdots,$$

$$\left[1+\sigma_j^{(T)}-\sum_{l=1}^{u}K_l\rho_j^{\prime\prime(T,l)}-\sum_{i=1}^{s}\eta_{G_i,g_i}\rho_j^{(T,i)}-\theta_j^{(T)}k;\rho_j^{\prime(T,1)},\cdots,\rho_j^{\prime(T,r)}\tau_j^{(T,1)},\cdots,\tau_j^{(T,l)},\underbrace{0,\cdots,0}_{T};1\right]_{1,s},\cdots,$$

$$[1-B_j;\underbrace{0,\cdots,0}_{r},\underbrace{1,\cdots,1}_{l},\underbrace{0,\cdots,0}_{T};1]_{1,Q},$$

$$\left[1-\alpha_j-\sum_{l=1}^{u}K_l\delta_j^{\prime\prime(l)}-\sum_{i=1}^{s}\eta_{G_i,g_i}\delta_j^{(i)}-\zeta_j^{(1)}k;\delta_j^{\prime(1)},\cdots,\delta_j^{\prime(r)}\mu_i^{(1)},\cdots,\mu_j^{(l)},\underbrace{1,\cdots,1}_{W},\underbrace{0,\cdots,0}_{W+1,T};1\right]_{1,s},$$

$$\left[1-\alpha_j-\beta_j-\sum_{l=1}^{u}K_l(\delta_j^{\prime\prime(l)}+\eta_j^{\prime\prime(l)})-\sum_{i=1}^{s}\eta_{G_j,g_j}(\delta_j^{(k)}+\eta_j^{(k)})-k(\zeta_j+\lambda_j),\right.$$

$$\left.(\delta_j^{\prime(1)}+\eta_j^{\prime(1)}),\cdots,(\delta_j^{\prime(r)}+\eta_j^{\prime(r)}),(\mu_j^{(1)}+\theta_j^{(1)}),\cdots,(\mu_j^{(l)}+\theta_j^{(l)}),\underbrace{1,\cdots,1}_{T};1\right] \tag{3.6}$$

$$\mathbf{B}=\ [\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{l+T};B_{rji_r})]_{1,q_{i_r}} \tag{3.7}$$

$$\mathrm{B}=[(\mathrm{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})]_{1,m^{(1)}},[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})]_{m^{(1)}+1,q_i^{(1)}};\cdots;$$

$$[(\mathrm{d}_j^{(r)},\delta_j^{(r)};D_j^{(r)})]_{1,m^{(r)}},[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_{ji^{(r)}}^{(r)})]_{m^{(r)}+1,q_i^{(r)}};\underbrace{(0,1;1),\cdots,(0,1;1)}_{l};\underbrace{(0,1;1),\cdots,(0,1;1)}_{T} \tag{3.8}$$

$$U=0,n_2;0,n_3;\cdots;0,n_r;V=m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)};\underbrace{(1,0),\cdots,(1,0)}_{l};\underbrace{(1,0),\cdots,(1,0)}_{T}; \tag{3.9}$$

$$X=p_{i_2},q_{i_2},\tau_{i_2};R_2;\cdots;p_{i_{r-1}},q_{i_{r-1}},\tau_{i_{r-1}}:R_{r-1};Y=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}$$

$$\underbrace{(0,1),\cdots,(0,1)}_{l};\underbrace{(0,1),\cdots,(0,1)}_{T} \tag{3.10}$$

Theorem

$$\int_{u_1}^{v_1}\cdots\int_{u_t}^{v_t}\prod_{i=1}^{t}\left[(x_i-u_i)^{\alpha_i-1}(v_i-x_i)^{\beta_i-1}\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\sigma_i^{(j)}}\right]$$

$$\bar{I}\begin{pmatrix}\mathrm{z}_1\prod_{i=1}^{t}\frac{(x_i-u_i)^{\delta_i^{(1)}}(v_i-x_i)^{\eta_i^{(1)}}}{\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\rho_i^{(j,1)}}}\\ \cdot\\ \cdot\ \cdot\\ \mathrm{z}_s\prod_{i=1}^{t}\frac{(x_i-u_i)^{\delta_i^{(s)}}(v_i-x_i)^{\eta_i^{(s)}}}{\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\rho_i^{(j,s)}}}\end{pmatrix}\gimel\begin{pmatrix}\mathrm{z}_1'\prod_{i=1}^{t}\frac{(x_i-u_i)^{\delta_i^{\prime(1)}}(v_i-x_i)^{\eta_i^{\prime(1)}}}{\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\rho_i^{\prime(j,1)}}}\\ \cdot\\ \cdot\ \cdot\\ \mathrm{z}_r'\prod_{i=1}^{t}\frac{(x_i-u_i)^{\delta_i^{\prime(r)}}(v_i-x_i)^{\eta_r^{\prime(r)}}}{\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\rho_i^{\prime(j,r)}}}\end{pmatrix}$$

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$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}\begin{pmatrix} \mathrm{z}_1'' \prod_{i=1}^t \frac{(x_i-u_i)^{\delta_i''^{(1)}}(v_i-x_i)^{\eta_i''^{(1)}}}{\prod_{j=1}^T (U_i^{(j)}x_i+V_i^{(j)})^{\rho_i''^{(j,1)}}} \\ \cdot \\ \cdot\ \cdot \\ \mathrm{z}_u'' \prod_{i=1}^t \frac{(x_i-u_i)^{\delta_i''^{(u)}}(v_i-x_i)^{\eta_r''^{(u)}}}{\prod_{j=1}^T (U_i^{(j)}x_i+V_i^{(j)})^{\rho_i''^{(j,u)}}} \end{pmatrix} E_{(\alpha_i),(\beta_i)}^{(\gamma_i),m}\left[z\prod_{j=1}^{j}\left[\frac{(x_i-u_i)^{\zeta_i}(v_i-x_i)^{\lambda_i}}{\prod_{j=1}^T (U_i^{(j)}x+V_i^{(j)})^{\theta_i^{(j)}}}\right]\right]$$

$${}_PF_Q\left[(A_P);(B_Q);-\sum_{k=1}^{l} g_k\left[\prod_{i=1}^{t}\left[(x_i-u_i)^{u_i^{(k)}}(v_i-x_i)^{\theta_i^{(k)}}\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\theta_i^{(j)}}\right]\right]\right]$$

$$=\frac{\prod_{j=1}^Q\Gamma(B_j)}{\prod_{j=1}^P\Gamma(A_j)}\prod_{j=1}^{t}\left[(v_i-u_i)^{\alpha_i+\beta_i+1}\prod_{j=1}^{W}(u_iU_i^{(j)}+v_i^{(j)})^{\sigma_i^{(j)}}\prod_{j=W+1}^{T}(u_iU_i^{(j)}+v_i^{(j)})^{\sigma_i^{(j)}}\right]$$

$$\sum_{k=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{h_i=1}^{M_i}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^s\phi_i z_i^{\eta_{h_i,g_i}}(-)^{\sum_{i=1}^s g_i}}{\prod_{i=1}^s\delta_{h^{(i)}}^{(i)}\prod_{i=1}^s g_i!}\frac{z^k}{k!}E_kA_u z_1''^{K_1}\cdots z_u''^{K_u}A_{ij}$$

$$\gimel_{X;p_{i_r}+sT+2s,q_{i_r}+sT+s,\tau_{i_r}:R_r:Y}^{U;0,n_r+sT+P+2s:V}\left(\begin{array}{c|c} \mathrm{z}_1'w_1 & \\ \cdot & \mathbb{A};\mathrm{A}_1,\mathbf{A}:A \\ \cdot & \cdot \\ \mathrm{z}_r'w_r & \cdot \\ \mathrm{g}_1W_1 & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{g}_lW_l & \cdot \\ \mathrm{G}_1 & \cdot \\ \cdot & \\ \cdot & \mathbb{B};\mathbf{B},\mathrm{B}_1:B \\ \mathrm{G}_T & \end{array}\right) \tag{3.11}$$

where

$$A_{ij}=\frac{1}{\prod_{i=1}^t\prod_{j=1}^W(u_iU_i^{j)}+V_i^{(j)})^{\sum_{k'=1}^u\rho_i''^{(j,k')}K_{k'}+\sum_{k''=1}^s\rho_i^{(j,k'')}+\theta_i^{(j)}k}}$$

$$\frac{(v_i-u_i)^{\sum_{k'=1}^u(\delta_i''^{(k')}+\eta_i''^{(k')})K_{k'}+\sum_{k''=1}^s(\delta_i^{(k'')}+\eta_i^{(k'')})\eta_{G_{k''},g_{k''}}+(\zeta_i+\lambda_i)k}}{\prod_{i=1}^t\prod_{j=W+1}^T(u_iU_i^{j)}+V_i^{(j)})^{\sum_{k'=1}^u\rho_i''^{(j,k')}K_{k'}+\sum_{k''=1}^s\rho_i^{(j,k'')}+\theta_i^{(j)}k}} \tag{3.12}$$

$$w_K=\prod_{i=1}^{t}\left[(v_i-u_i)^{\delta_i'^{(K)}+\eta_i'^{(K)}}\prod_{j=1}^{W}(u_iU_i^{(j)}+v_i^{(j)})^{-\rho_i'^{(j,K)}}\prod_{j=W+1}^{T}(u_iU_i^{(j)}+v_i^{(j)})^{-\rho_i'^{(j,K)}}\right]:K=1,\cdots,s \tag{3.13}$$

$$W_L=\prod_{i=1}^{t}\left[(v_i-u_i)^{\mu_i'^{(L)}+\theta_i'^{(L)}}\prod_{j=1}^{W}(u_iU_i^{(j)}+v_i^{(j)})^{-\tau_i^{(j,L)}}\prod_{j=W+1}^{T}(u_iU_i^{(j)}+v_i^{(j)})^{-\tau_i^{(j,L)}}\right]:L=1,\cdots,l \tag{3.14}$$

$$G_j=\prod_{i=1}^{t}\left[\frac{(v_j-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right];j=1,\cdots,W \tag{3.15}$$

Notes

$$G_j = -\prod_{i=1}^{t}\left[\frac{(v_j-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right]; j=W+1,\cdots,T \tag{3.16}$$

provided

$W\in[0,T]; u_i,v_i\in\mathbb{R}; i=1,\cdots,t$

$min\{\delta_i^{(g)},\eta_i^{(g)},\delta_i'^{(h)},\eta_i'^{(h)},\delta_i''^{(k)},\eta_i''^{(k)},\zeta_i,\eta_i\}\geqslant 0; g=1,\cdots,s; i=1,\cdots,t; h=1,\cdots,r; k=1,\cdots,u$

$min\{\rho_i^{(j,g)},\rho_i'^{(j,h)},\rho_i'^{(j,k')},\theta_i^{j)},\tau_i^{(j,k)}\}\geqslant 0; j=1,\cdots,T; i=1,\cdots,t; g=1,\cdots,s; h=1,\cdots,r; k'=,\cdots,v; k=1,\cdots,l.$

$\sigma_i^{(j)}\in\mathbb{R},U_i^{(j)},V_i^{(j)}\in\mathbb{C},z_{i'},z_{j'}z_k'',G_j\in\mathbb{C},i=1,\cdots,t;j=1,\cdots,T;i'=1,\cdots,s,j'=1,\cdots,r;k'=1,\cdots,v;k=1,\cdots,l.$

$\alpha_i,\beta_i,\gamma_i\in\mathbb{C},i=1,\cdots,m,Re(\alpha_i)>0$

$$max\left[\left|\frac{(v_j-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right|\right]<1; i=1,\cdots,s; j=1,\cdots,W \quad \text{and}$$

$$max\left[\left|\frac{(v_j-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right|\right]<1; i=1,\cdots,s; j=W+1,\cdots,T$$

$$\left|arg\left(z_i'\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\rho_i'^{(j,k)}}\right)\right|<\frac{1}{2}\left(A_i'^{(k)}-\delta_i'^{(k)}-\eta_i'^{(k)}-\sum_{j=1}^{T}\rho_i'^{(j,k)}\right)\pi>0$$

where $A_i^{(k)}$ is defined by (1.16).

$$Re\left(\alpha_i+\zeta_ik+\sum_{j=1}^{r}\delta_i^{(j)}\eta_{G_j,g_j}\right)+\sum_{K=1}^{r}\delta_i'^{(K)}\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0 \text{ and}$$

$$Re\left(\beta_i+\lambda_ik+\sum_{j=1}^{s}\eta_i^{(j)}\eta_{Gj,g_j}\right)+\sum_{K=1}^{r}\eta_i'^{(K)}\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0 \text{ for } i=1,\cdots,t.$$

$P\leqslant Q+1$. The equality holds, also,

$$\text{either } P>P \text{ and } \sum_{k=0}^{l}\left|g_k\left(\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\tau_i^{(i,k)}}\right)\right|^{\frac{1}{Q-P}}<1 \quad (u_i\leqslant x_i\leqslant v_i; i=1,\cdots,t)$$

$$P\leqslant Q \text{ and } \max_{1\leqslant k\leqslant l}\left|g_k\left(\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{\tau_i^{(i,k)}}\right)\right|<1 \quad (u_i\leqslant x_i\leqslant v_i; i=1,\cdots,t)$$

Proof

To establish the main theorem, we first express the class of multivariable polynomials $S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}[.]$, the multivariable Ifunction defined by Prathima et al. [10], the 3m-parametric Mittag-Leffler type functions in series with the help of (1.4), (1.10) and (1.2) respectively, Further, using the Melin-Barnes multiple integrals contour representation for the multivariable Gimel-function and use the multiple integrals contour representation with the help of (2.1) for the generalized hypergeometric function ${}_PF_Q$. Interchanging the order of integrations and summations suitably, which ispermissible under the conditions stated above. Now we write

Ref

10. J. Prathima, V. Nambisan and S.K. Kurumujji, A Study of I-function of Several Complex Variables, International Journal of Engineering Mathematics Vol (2014), 1-12.

$$\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}=\prod_{j=1}^{W}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}\prod_{j=W+1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}} \tag{3.17}$$

Where

$$K_i^{(j)}=v_i^{(j)}-\theta_i^{(j)}k-\sum_{l=1}^{s}\rho_i^{(j,l)}\eta_{G_l,g_l}-\sum_{l=1}^{r}\rho_i'^{(j,l)}\psi_l-\sum_{l=1}^{u}\rho_i''^{(j,l)}K_l \text{ where } i=1,\cdots t; j=1,\cdots,T$$

and express the factor occurring in right hand side of (3.11), regarding the following Mellin-Barnes integrals, we obtain,

$$\prod_{j=1}^{W}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}=\prod_{j=1}^{W}\left[\frac{(U_i^{(j)}u_i+V_i^{(j)})^{K_i^{(j)}}}{\Gamma(-K_i^{(j)})}\right]\frac{1}{(2\pi\omega)^W}\int_{L_1'}\cdots\int_{L_W'}\prod_{j=1}^{W}\left[\Gamma(-\xi_j')\Gamma(-K_i^{(j)}+\xi_j')\right.$$

$$\prod_{j=1}^{W}\left[\frac{U_i^{(j)}(x_i-u_i)}{u_iU_i^{(j)}+V_i^{(j)}}\right]^{\xi_i'}\mathrm{d}\xi_1'\cdots\mathrm{d}\xi_W' \tag{3.18}$$

and

$$\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}=\prod_{j=W+1}^{T}\left[\frac{(U_i^{(j)}u_i+V_i^{(j)})^{K_i^{(j)}}}{\Gamma(-K_i^{(j)})}\right]\frac{1}{(2\pi\omega)^{T-W}}\int_{L_{W+1}'}\cdots\int_{L_{L_T'}}\prod_{j=W+1}^{T}\left[\Gamma(-\xi_j')\Gamma(-K_i^{(j)}+\xi_j')\right.$$

$$\prod_{j=1}^{W}\left[\frac{U_i^{(j)}(x_i-u_i)}{u_iU_i^{(j)}+V_i^{(j)}}\right]^{\xi_i'}\mathrm{d}\xi_1'\cdots\mathrm{d}\xi_W' \tag{3.19}$$

We apply the Fubini theorem for multiple integrals. Evaluating the innermost x-integral with the help of (1.1) and finally, reinterpreting the multiple Mellin-Barnes integrals contour regarding of multivariable Gimel-function of $(r+l+T)$, we obtain the desired result.

Remarks:
We obtain the same multiple Eulerian integrals about the functions cited in section I.
We obtain the same multiple eulerian integral about the general class of polynomials introduced, and studied by Srivastava [13].

$$S_V^U(x)=\sum_{\eta=0}^{[V/U]}\frac{(-V)_{U\eta}A_{V,\eta}}{\eta!}x^\eta \tag{3.20}$$

Where $V=0,1,\cdots$ and U is an arbitrary positive integer. The coefficients $A_{V,\eta}(V,\eta\geqslant 0)$ are arbitrary constants, real or complex. On suitably specializing the coefficients, $A_{V,\eta}$, $S_V^U(x)$, yields some of known polynomials, these include the Jacobi polynomials, Laguerre polynomials and others polynomials ([20], p. 158-161.)

IV. Conclusion

The importance of our all the results lies in their manifold generality. Firstly, in view of the multiple Eulerian integrals involving general class of multivariable polynomials, the multivariable I-function, the multivariable Gimel-function and the 3m-parametric Mittag-Leffler type functions with general arguments utilized in this study, we can obtain a large variety of single, double and several dimenionals Eulerian integrals. Secondly by specializing the various parameters as well as variables in the generalized multivariable Gimel-function, we get several formulae involving a remarkably wide variety of

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13. H.M. Srivastava, A contour integral involving Fox's H-function, Indian. J. Math. 14(1972), 1-6.

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useful functions (or product of such functions) which are expressible regarding the E, F, G, H, I, Alephfunction of one and several variables and simpler special functions of one and several variables. Hence the formulae derived in this paper are most general nature and may prove to be useful in several interesting cases appearing in the literature of Pure and Applied Mathematics and Mathematical Physics.

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Results and Conclusion of an Algorithm for Solving Indefinite QR-Programming Problems

By Awatif M. A. El Siddieg
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Abstract- In this paper we have two sections. In section (1), we write a Matlab program and apply it to solve chosen problems in general QP –problems, we use sub programs[11]. Section (2) conclude our work reported in this paper gave no account to the *special* structures that the matrix of constraints A might have. The *work* is ideal when A is dense, that is, full of non-zero elements. [19].

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I. Introduction

We solve a general quadratic programming problems[[15],[4],[17]] , obtaining a local minimum of a quadratic function subject to inequality constraints. The method terminates at a KKT-point in finite steps[8]. No effort is needed when the function is non- convex[[10],[8] ,[19]]. We give the general description of the matrices that uses in the program and tested the program by a number of problems.

Section(1)

II. Results

In this section, we write a Matlab program and apply it to solve the chosen problems. The program uses *sub programs*:-

1. htu(G,A): to evaluate the inverse of the active Lagrangian matrix, using the QR-factorization of the matrix of constraints when the tableau is complementary).[[13],[14]]. (We know that H,U and T define the inverse of the upper left partition of the basis matrix).This calls for making them available at every complementary tableau[[2],[19]].
2. init(A,G) to obtain an initial feasible point to the main algorithm.
3. solver (A,b), is used to solve a subsystem in the main algorithm.
4. lufactors (A), is used by solver(the above program). [17]

The Program

The program is designed to start with the Hessian matrix G, which is an $n \times n$ symmetric matrix, and A is an $n \times m$ matrix of the constraints, g the gradient of *f*, and b, the vector of right-hand coefficients b_i.[[6],[7],[9]].

Chosen Problems

The above program has been tested by some problems and proved to work adequately.

* Minimize : $-8x_1 - 16x_2 + x_1^2 + 4x_2^2$

Subject to: $-x_1 - x_2 \geq -5$

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Ref

19. http://www.iiste.org/journals/index.php/MTM.

$$-x_1 \geq -3$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

$$G=\begin{bmatrix}2 & 0\\ 0 & 8\end{bmatrix},\quad A=\begin{bmatrix}-1 & -1\\ -1 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix},\ b=\begin{bmatrix}-5\\ -3\\ 0\\ 0\end{bmatrix},\quad g=\begin{bmatrix}-8\\ -16\end{bmatrix}$$

x =

3

0

la =

2

-16

>> x =

x

3

2

>> la

la =

2

*. Minimize: $0.5x_1^2 + x_2^2 + 1.5x_3^2 + x_1x_2 + 25.5x_1 + 18x_2 + 29.875x_3$

Subject to : $-14x_1 - 2x_2 - 4x_3 \geq -12$

$$-x_1 - 2x_2 - x_3 \geq -4$$
$$-x_3 \geq -2$$
$$x_1 \geq 0$$
$$x_2 \geq 0,\ x_3 \geq 0$$

$$A=\begin{bmatrix}-14 & -2 & -4\\ -1 & -2 & -1\\ 0 & 0 & -1\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix},\qquad b=\begin{bmatrix}-12\\ -4\\ -2\\ 0\\ 0\\ 0\end{bmatrix}$$

Notes

$$G = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \qquad g = \begin{bmatrix} 25.5 \\ 18 \\ 29.875 \end{bmatrix}$$

```
x  =
   -0.0000
    1.5000
   -0.0000
la =
   -14.7500
     4.6563
     6.3333
x =
    -0.0000
    1.5000
    -0.0000
la =
    -14.7500
      4.6563
      6.3333
x =
     0.0000
          0
     1.0000
la =
     -1.2500
      5.5000
      4.7500
x =
     0.0000
          0
     1.0000
la =
      -1.2500
       5.5000
       4.7500
x =
     0
     5
     0
     2
```

Notes

la =

1.7500

-8.2500

16.0000

x =

0

0

2

la =

1.7500

-8.2500

16.0000

>> x

x =

0

1.5000

2.0000

>> la

la =

11.2188

6

20.5000

0.6563

Section(2)

III. Conclusion

The work reported in this paper gave no account to the special structures that the matrix of constraints A might have. The work is ideal when A is dense, that is, full of non-zero elements. In many problems the unknown variables) $x_i (i = 1,\ldots,n)$ are required to satisfy-bound restrictions, in which case we start the problem as follows:

$$\begin{aligned} &\min imize \;\; 0.5\, \underline{x}^T G \underline{x} + g^T \underline{x} \\ &subject\ to \\ &A^T \underline{x} \geq \underline{b} \\ &I_i \leq x_i \leq u_i \end{aligned} \quad (2.1)$$

Where I_i and u_i are respectively the lower and upper bounds for the variable x_i, A is $n \times m$ and assumed to be dense, $\underline{b}$ is an m vector, G is an $n \times n$ symmetric matrix and g is an n-vector. In (2.1) except in very *special* situations. A is dense since the bound constraints are separately considered. In this section we give our trial in treating, the case when $I_i = 0$ and u_i is infinite, that is when $x_i \geq 0 \;\; \forall\, i$, we do not give general proofs here, nor do we present a compact description of an algorithm. Instead we will show the steps to be followed in a similar way similar to those given in our work reported in this paper [19].

The problem to be treated is

Ref

19. http://www.iiste.org/journals/index.php/MTM.

$$\begin{aligned} &\min imize \;\; 0.5\,\underline{x}^T G \underline{x} + g^T \underline{x} \\ &subject\ to \quad A^T \underline{x} \geq \underline{b} \\ &\qquad\qquad \underline{x} \geq 0 \end{aligned} \tag{2.2}$$

let $\underline{\lambda}$ be the vector of multipliers corresponding to $A^T \underline{x} - \underline{b} \geq \underline{0}$ and $\underline{y}$ be the vector of multipliers[1], corresponding the bound constraints $\underline{x} \geq 0$.

The KKT – conditions to (2.2) we get

$$\begin{aligned} &G\underline{x} + \underline{g} - A\underline{\lambda} - \underline{y} = \underline{0} \\ &\underline{b} - A^T \underline{x} + \underline{v} = \underline{0} \\ &\underline{V}^T \underline{\lambda} = 0\,, \quad \underline{y}^T \underline{x} = 0 \\ &\underline{x}\,,\, \underline{y} \geq \underline{0},\, \underline{v},\, \lambda \geq \underline{0} \end{aligned} \tag{2.3}$$

where $\underline{v}$ is the vector of slack variables (2.3) could be put in the form: $M_1 \underline{W} + M_2 \underline{Z} = \underline{q}$

$$\underline{W} = \begin{bmatrix} \underline{Y} \\ \underline{V} \end{bmatrix} \;,\; \underline{q} = \begin{bmatrix} -\underline{g} \\ -\underline{b} \end{bmatrix} \;,\; M_1 = \begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix}$$

$$M_2 = \begin{bmatrix} G & -A \\ -A^T & 0 \end{bmatrix} \;,\; \underline{Z} = \begin{bmatrix} \underline{x} \\ \underline{\lambda} \end{bmatrix} \tag{2.4}$$

$$\begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \underline{Y} \\ \underline{V} \end{bmatrix} + \begin{bmatrix} G & -A \\ -A^T & 0 \end{bmatrix} \begin{bmatrix} \underline{x} \\ \underline{\lambda} \end{bmatrix} = \begin{bmatrix} -\underline{g} \\ -\underline{b} \end{bmatrix}$$

$$\underline{g} - G\underline{x} - A\underline{\lambda} - \underline{y} = \underline{0}$$

$$\underline{b} - A^T \underline{x} + \underline{v} = 0$$

$$\underline{v}^T \underline{\lambda} = 0$$

$$\underline{y}^T \underline{x} = 0$$

$$\underline{x}, \underline{y} \geq 0 \quad , \quad \underline{v}, \underline{\lambda} \geq 0$$

The general complementary tableau [2], will have the *form*:

$[M_B : M_N]$ with M_B having the *form*: $M_B = \begin{bmatrix} G_{12} & -A_{11} & -I & 0 \\ G_{22} & -A_{21} & 0 & 0 \\ -A_{21}^T & 0 & 0 & 0 \\ -A_{22}^T & 0 & 0 & I \end{bmatrix}$

and M_N having the *form*: $M_N = \begin{bmatrix} 0 & 0 & G_{11} & -A_{12} \\ -I & 0 & G_{22}^C & -A_{22} \\ 0 & I & -A_{11}^T & 0 \\ 0 & 0 & -A_{12}^T & 0 \end{bmatrix}$

Ref

1. AMO (2015). Advanced Modeling and Optimization, Volume 17, Number 2.

Here G_{11}, G_{12} and G_{22} define the following partition of G.

$$G = \begin{array}{c} n_1 \\ n_2 \end{array}\begin{bmatrix} G_{11} & G_{12} \\ G_{12}^T & G_{22} \end{bmatrix}, \quad n_1 + n_2 = n$$

and A_{11}, A_{12}, A_{21}, A_{22} define the following partition of A.

$$A = \begin{array}{c} m_1 \\ m_2 \end{array}\begin{bmatrix} A_{11} & A_{12} \\ A_{12} & A_{22} \end{bmatrix}, \quad m_1 + m_2 = m$$

corresponding : $\underline{x}, \underline{\lambda}, \underline{y}, \underline{v}, \underline{g}$ and $\underline{b}$ were partitioned to

$$\underline{x} = \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix}\begin{array}{c} n_1 \\ n_2 \end{array}, \; \underline{\lambda} = \begin{array}{c} n_1 \\ n_2 \end{array}\begin{bmatrix} \underline{\lambda}_1 \\ \underline{\lambda}_2 \end{bmatrix}, \quad \underline{v} = \begin{bmatrix} \underline{v}_1 \\ \underline{v}_2 \end{bmatrix}\begin{array}{c} m_1 \\ m_2 \end{array} \; \underline{g} = \begin{bmatrix} \underline{g}_1 \\ \underline{g}_2 \end{bmatrix}\begin{array}{c} n_1 \\ n_2 \end{array}$$

$$\underline{b} = \begin{bmatrix} \underline{b}_1 \\ \underline{b}_2 \end{bmatrix}\begin{array}{c} m_1 \\ m_2 \end{array}, \quad \underline{y} = \begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \end{bmatrix}\begin{array}{c} n_1 \\ n_2 \end{array}$$

Accordingly the *basic* variables are $\underline{x}_2$, $\underline{\lambda}_1, \underline{y}_1$ and $\underline{v}_2$. Their respective non-basic complements are $\underline{y}_2, \underline{v}_1, \underline{x}_1$ and $\underline{\lambda}_2$.

Omitting the superscripts, let q solve

$$\min \; \{y_{q1}, \lambda_{q2}\} \tag{2.5}$$

$$q \in \{q_1, q_2\}$$

Where q_1 and q_2 satisfy :

$$y_{q1} = \min_{1 \le i \le n_1} \; y_i \tag{2.6}$$

$$\lambda_{q2} = \min \; \lambda_i \tag{2.7}$$

$$1 \le i \le m_1$$

To carry on the description let $q = q_2$. If $\lambda_{q2} \ge 0$, then we are at a KKT- point. Otherwise the complement v_{q2} is chosen to be increased.

Accordingly the *basic* variables change by:

$$\underline{x}_2 = \underline{x}_2^{\backslash} - \underline{d}x\, v_{q2} \tag{2.8}$$

$$\underline{\lambda}_1 = \underline{\lambda}_1^{\backslash} - \underline{d}_x \lambda\, v_{q2} \tag{2.9}$$

$$y_1 = y_1^{\backslash} - \underline{d}_y v_{q2} \tag{2.10}$$

$$\underline{v}_1 = \underline{v}_1^{\backslash} - \underline{d}_x v_{q2} \tag{2.11}$$

Notes

where the dashes indicate the current values $\underline{d}_x, \underline{d}_\lambda, \underline{d}_y$ and $\underline{d}_v$ are the

$$\text{solution of:}\begin{bmatrix} G_{12} & -A_{11} & -I & 0 \\ G_{22} & -A_{11} & 0 & 0 \\ -A_{21} & -A_{21} & 0 & 0 \\ -A_{22} & 0 & 0 & I \end{bmatrix}\begin{bmatrix} \underline{d}_x \\ \underline{d}_\lambda \\ \underline{d}_y \\ d_v \end{bmatrix} = \begin{bmatrix} \underline{0} \\ \underline{0} \\ \underline{e}_{q2} \\ 0 \end{bmatrix}$$

$$\text{is solved in two steps:}\begin{bmatrix} G_{22} & -A_{12} \\ -A_{22}^T & 0 \end{bmatrix}\begin{bmatrix} \underline{dx} \\ \underline{d\lambda} \end{bmatrix} = \begin{bmatrix} \underline{0} \\ \underline{e}_{q2} \end{bmatrix}$$

$$\begin{aligned} & G_{22}\underline{dx} - A_{11}\underline{d}_\lambda = \underline{0} \\ & -A_{22}^T\underline{dx} = \underline{e}_{q2} \end{aligned} \tag{2.1 2}$$

$$\underline{dy} = G_{22}\underline{d}_x - A_{11}\underline{d}_\lambda \tag{2.1 3}$$

$$\underline{dv} = A_{22}^T\underline{d}_x \tag{2.15}$$

The increase of v_{q2} is continued until either λ_{q2} *increase* to zero or v_{q2} is blocked by either a basic x_{P1} decreasing to zero. The next step is to restart again if λ_{q2} decreases to zero first, in which case we are at another [19] complementary tableau[2]. Or one of the complements y_{P1} of x_{P1} or λ_{P2} of v_{P2} is to be changed in a similar way to that described in the main work of the paper. The process will keep on going until the solution is located. Also we point out another two incomplete features of our algorithm. They are:

1) It did not give any account to degeneracy.

2) Updating the factors of $G_A^{(K)}$ is not carried in all cases.

So, according to [[14],[15],[16]], is equivalent to active set methods in convex problems. When solving non-convex problems the method is more systematic than the variants of the active set methods[[8],[10]].

The latter methods need to change the strategy of choosing the direction of search from time to time, and some of them have no clue of what to do in the negative definite case[11]. In our work no change in the strategy is needed. In fact no check of indefiniteness of the reduced (generalized) Hessian is required.

Still we believe that our work should be tested in all aspects against the (modified) active set methods to reflect the major advantages and disadvantages of our work (i.e the active set methods) which dominated the scene for the last twenty years (of course to our knowledge). Also our work need to be compared with Beal's method [11] , [14]], since they are both constrained as simplex-like methods, although we feel that the general behavior of our work looks different. However, [14] referenced to the equivalence between the active set methods and Beale's method in convex problems. Orthoganalization methods are well known in the numerical analysis community for their numerical stability. Conversely, normal equation methods are known for their lack of numerical stability. QR- factorizations[[12], [17] ,[18]],can make very good use of sparsity of the problem.

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Modeling Heteroscedasticity of Discrete-Time Series in the Face of Excess Kurtosis

By Moffat, Imoh Udo & Akpan, Emmanuel Alphonsus
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Abstract- To tackle the influence of excess kurtosis (which is thought to be induced by outliers) on the distributions of the innovations, this study considered the presence of outliers in the data on daily closing prices of shares of Skye Bank, Sterling Bank, and Zenith Bank, starting from January 03, 2006 to November 24, 2016. The data consist of 2690 observations each obtained from the Nigerian Stock Exchange website. Our findings revealed that GARCH(1,1) model under normal distribution, EGARCH(1,1) model under normal distribution and EGARCH(1,1) model under student-t distribution fitted adequately to the returns of Skye Bank, Sterling Bank, and Zenith Bank, respectively. However, all the series possessed in their residuals excess kurtosis values of 132.8707, 80.3030, and 26.3794, respectively.

Keywords: *heteroscedasticity, outliers, volatility.*

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Modeling Heteroscedasticity of Discrete-Time Series in the Face of Excess Kurtosis

Moffat, Imoh Udo [α] & Akpan, Emmanuel Alphonsus [σ]

Abstract- To tackle the influence of excess kurtosis (which is thought to be induced by outliers) on the distributions of the innovations, this study considered the presence of outliers in the data on daily closing prices of shares of Skye Bank, Sterling Bank, and Zenith Bank, starting from January 03, 2006 to November 24, 2016. The data consist of 2690 observations each obtained from the Nigerian Stock Exchange website. Our findings revealed that GARCH(1,1) model under normal distribution, EGARCH(1,1) model under normal distribution and EGARCH(1,1) model under student-t distribution fitted adequately to the returns of Skye Bank, Sterling Bank, and Zenith Bank, respectively. However, all the series possessed in their residuals excess kurtosis values of 132.8707, 80.3030, and 26.3794, respectively. Conversely, when the returns for the three banks were adjusted for outliers, we discovered that GARCH(1,1) model under the normal distribution fitted well to the returns of Skye Bank, EGARCH(1,1) model under student-t fitted well to the returns of Sterling Bank and Zenith Bank with the respective kurtosis values of 2.9465, 3.6829, and 3.5746 in their residuals. Thus, with the outliers taken into consideration, the coefficients of kurtosis are, approximately, mesokurtic as required by the normal distribution. Hence, it could be deduced that the problem related to excess kurtosis and the choice of distribution of innovations in modeling heteroscedasticity of discrete-time series could be tackled by accounting for the presence of outliers.

Keyword: *heteroscedasticity, outliers, volatility.*

I. Background

Heteroscedasticity modeling of financial time series is based on the Generalized autoregressive conditional heteroscedastic (GARCH) model commonly specified under the assumption that the error follows a normal distribution. Actually, this assumption always appears to be insufficient in accommodating some characterizations of financial data especially fat-tailedness, which is due to excess kurtosis. Since kurtosis measures the degree of peakedness of distribution of real random variables, any distribution whose coefficient of kurtosis equals three is said to be mesokurtic as is the case with a normal distribution. Thus, distributions with heavy-tail probabilities compared to that of the normal are said to be heavy-tailed. If a distribution of returns has more returns clustered around the mean, it is referred to as leptokurtic or highly peaked, which leads to heteroscedasticity (changing variance). It is this stylized fact of stock returns that provides a more pragmatic reason for entertaining GARCH models (Franses and van Dijk, 2003). Again, the student-t distribution is traditionally specified to remedy the weakness of the normal distribution in accommodating the heavy-tailed property, yet it also failed in many applications to account for excess kurtosis and thus inadequate for capturing the fat-tailedness (Feng and Shi, 2017).

Notably, a heavy-tailed distribution is sensitive and allergic to outliers, which are observations that deviate from the overall pattern of the sample and are either the

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product of a data-reading or measurement error (Seefield and Lider, 2007). However, it could be argued that the presence of outliers is responsible for the existence of excess kurtosis in financial data. Hence, this study is aimed at curbing the effects of outliers to provide the needed stability in accommodating the excess kurtosis by the distribution of innovations in GARCH models.

The motivation for this study was drawn from the fact that previous studies in Nigeria such as Usman, Musa and Auwal , 2018; Diri *et al.*, 2018; Ibrahim, 2017; Moffat, Akpan and Abasiekwere, 2017; Akpan, Moffat and Ekpo, 2016; Onwukwe, Samson and Lipcsey, 2014 have not considered the effects of outliers on the distribution of errors while modeling heteroscedasticity in the face of excess kurtosis.

Notes

II. Materials And Methods

a) Return Series

The return series, R_t, can be obtained given that P_t is the price of a unit shares at the time, t and P_{t-1} is the price of shares at the time t−1. Thus, we have:

$$R_t = \nabla lnP_t = (1-B)lnP_t = ln\,P_t - ln\,P_{t-1} \tag{2.1}$$

In equation (2.1), R_t is regarded as a transformed series of the price (P_t) of shares meant to attain stationarity, such that, both the mean and the variance of the series are stable (Akpan and Moffat, 2017) while B is the backshift operator.

b) Autoregressive Integrated Moving Average (ARIMA) Model

Box, Jenkins and Reinsel (2008) considered the extension of the ARMA model to deal with homogenous non-stationary time series in which X_t, is non-stationary but its d^{th} difference is a stationary ARMA model. We denote the d^{th} difference of X_t by

$$\varphi(B) = \phi(B)\nabla^d X_t = \theta(B)\varepsilon_t\,, \tag{2.2}$$

where $\varphi(B)$ is the nonstationary autoregressive operator, such that, d of the roots of $\varphi(B) = 0$ is unity and the remainder lie outside the unit circle, while $\phi(B)$ is a stationary autoregressive operator.

c) Heteroscedastic Models

i. *Autoregressive Conditional Heteroscedastic (ARCH) Model*

The first model that provides a systematic framework for modeling heteroscedasticity is the ARCH model of Engle (1982). Specifically, an ARCH (q) model assumes that,

$$R_t = \mu_t + a_t, \quad a_t = \sigma_t e_t,$$

$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \cdots + \alpha_q a_{t-q}^2\,, \tag{2.3}$$

where $\{e_t\}$ is a sequence of independent and identically distributed (i.i.d.) random variables with mean zero and variance 1, $\omega > 0, and\ \alpha_1, .., \alpha_q \geq 0$ (Francq and Zakoian, 2010). The coefficients α_i, for $i > 0$, must satisfy some regularity conditions to ensure that the unconditional variance of a_t is finite. In practice, e_t is often assumed to follow the standard normal or a student-t distribution.

ii. *Generalized Autoregressive Conditional Heteroscedastic (GARCH) Model*

Although the ARCH model is simple, it often requires many parameters to adequately describe the volatility process of a share price return. An alternative model proposed by Bollerslev (1986) is a useful extension known as the generalized ARCH

(GARCH) model. For a return series, R_t, let $a_t = R_t - \mu_t$ be the innovation at time t. Then, a_t follows a GARCH (q, p) model if

$$a_t = \sigma_t e_t,$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 , \tag{2.4}$$

Notes

where again e_t is a sequence of i.i.d. random variance with mean, 0, and variance, 1, $\omega > 0,\ \alpha_i \geq 0,\ \beta_j \geq 0,\ and \sum_{i=1}^{max(p,q)} (\alpha_i + \beta_i) < 1$ (Tsay, 2010).

Here, it is understood that $\alpha_i = 0$, for $i > p$, and $\beta_i = 0$, for $i > q$. The latter constraint on $\alpha_i + \beta_i$ implies that the unconditional variance of a_t is finite, whereas its conditional variance σ_t^2, evolves with time.

iii. *Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH) Model*

The EGARCH model represents a major shift from ARCH and GARCH models (Nelson, 1991). Rather than modeling the variance directly, EGARCH models the natural logarithm of the variance, and so no parameter restrictions are required to ensure that the conditional variance is positive. The EGARCH (q, p) is defined as,

$$R_t = \mu_t + a_t, \quad a_t = \sigma_t e_t,$$

$$ln\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \left| \frac{a_{t-i}}{\sqrt{\sigma_{t-i}^2}} \right| + \sum_{k=1}^{r} \gamma_k \left(\frac{a_{t-k}}{\sqrt{\sigma_{t-k}^2}} \right) + \sum_{j=1}^{p} \beta_j \, ln\sigma_{t-j}^2, \tag{2.5}$$

where e_t remains a sequence of i.i.d. That is, random variables with mean, 0, and variance, 1, while γ_k is the asymmetric coefficient.

d) Kurtosis

Kurtosis can be estimated by their sample counterparts. Let $-R_1, \ldots, R_T{''}$ be a random sample of returns, R, with T observations. The sample mean is:

$$\bar{R} = \frac{1}{T} \sum_{t=1}^{T} R_t, \tag{2.6}$$

The sample variance is

$$\sigma_R^2 = \frac{1}{T-1} \sum_{t=1}^{T} (R_t - \bar{R})^2. \tag{2.7}$$

And the sample kurtosis is:

$$K_R = \frac{1}{(T-1)\sigma_R^4} \sum_{t=1}^{T} (R_t - \bar{R})^4. \tag{2.8}$$

Under the normality assumptions, $K_R - 3$ is distributed asymptotically as normal with zero mean and variance $\frac{24}{T}$ (Tsay, 2010).

The excess kurtosis of GARCH(1, 1) model can be obtained as follows:

$$a_t = \sigma_t e_t, \tag{2.9}$$

$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{2.10}$$

Note that,

$\mathrm{E}(e_t) = 0$, $\mathrm{Var}(e_t) = 1$, and $\mathrm{E}(e_t^4) = K_e + 3$, where K_e is the excess kurtosis of the innovation, e_t.

Also,

$$\mathrm{Var}(a_t) = \mathrm{E}(\sigma_t^2) = \frac{\omega}{[1-(\alpha_1+\beta_1)]}. \tag{2.11}$$

$$\mathrm{E}(a_t^4) = (K_e + 3)\mathrm{E}(\sigma_t^4) \text{ provided that } \mathrm{E}(\sigma_t^4) \text{ exists.} \tag{2.12}$$

$$\text{But, } \mathrm{E}(\sigma_t^4) = \frac{\omega^2(1+\alpha_1+\beta_1)}{[1-(\alpha_1+\beta_1)][1-\alpha_1^2(K_e+2)-(\alpha_1+\beta_1)^2]}, \tag{2.13}$$

provided that $1>\alpha_1+\beta_1 \geq 0$ and $1 - \alpha_1^2(K_e + 2) - (\alpha_1+\beta_1)^2 > 0$. the excess kurtosis of a_t, if it exists, is then $K_a = \frac{\mathrm{E}(a_t^4)}{[\mathrm{E}(a_t^2)]^2} - 3$

$$= \frac{(K_e+3)[1-(\alpha_1+\beta_1)^2]}{1-2\alpha_1^2-(\alpha_1+\beta_1)^2-K_e\alpha_1^2} - 3. \tag{2.14}$$

This excess kurtosis can be written in an informative expression. Considering the case where e_t follows a normal distribution, $K_e = 0$,

$$K_a^{(g)} = \frac{6\alpha_1^2}{1-2\alpha_1^2-(\alpha_1+\beta_1)^2}, \tag{2.15}$$

where the superscript, *g*, is used to denote the Gaussian distribution. The same idea applies to other GARCH models (Tsay, 2010).

e) Outliers in Time Series

Generally, a time series might contain several outliers (say *k*) of different types, and we have the following general outlier model:

$$Y_t = \sum_{j=1}^{k} \tau_j V_j(\mathrm{B}) I_t^{(T)} + X_t, \tag{2.16}$$

where $X_t = (\theta(B)) / (\varphi(B))\, a_t$, $V_j(\mathrm{B}) = 1$ for an AO, and $V_j(\mathrm{B}) = \frac{\theta(B)}{\varphi(B)}$ for an IO at t $= T_j$, $V_j(\mathrm{B}) = (1-B)^{-1}$ for a LS, $V_j(\mathrm{B}) = (1-\delta B)^{-1}$ for an TC, and τ is the size of outlier. For more details on the types of outliers and estimation of the effects, see Moffat and Akpan, 2017; Sanchez and Pena, 2010; Box, Jenkins and Reinsel, 2008; Wei, 2006; Chen and Liu, 1993; Chang, Tiao and Chen, 1988.

Moreover, in financial time series, the residual series, a_t, is assumed to be uncorrelated with its past. In this case, the additive, innovative, temporary change and level shift outliers coincide, and where both the mean and the variance equations evolve together, we have, for example, GARCH(1, 1) model:

$$R_t - \mu_t = \tilde{a}_t + \tau I_t^{(T)}. \tag{2.17}$$

$$\tilde{a}_t = \sigma_t e_t. \tag{2.18}$$

$$\sigma_t^2 = \omega + \alpha_1\tilde{a}_{t-1}^2 + \beta_1\sigma_{t-1}^2, \tag{2.19}$$

where $\tilde{a}_t$ is the residual series contaminated by outliers.

Notes

III. Results and Discussion

Figures 1 - 3 represent the price series of shares for the three major banks in Nigeria. It is observed that their price series indicate the presence of a stochastic trend, which signifies non-stationarity of the series.

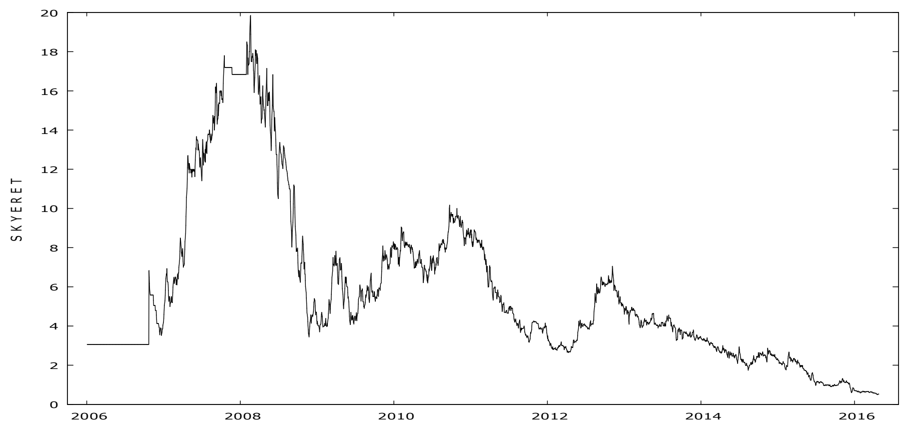

Figure 1: Price Series of Skye Bank shares

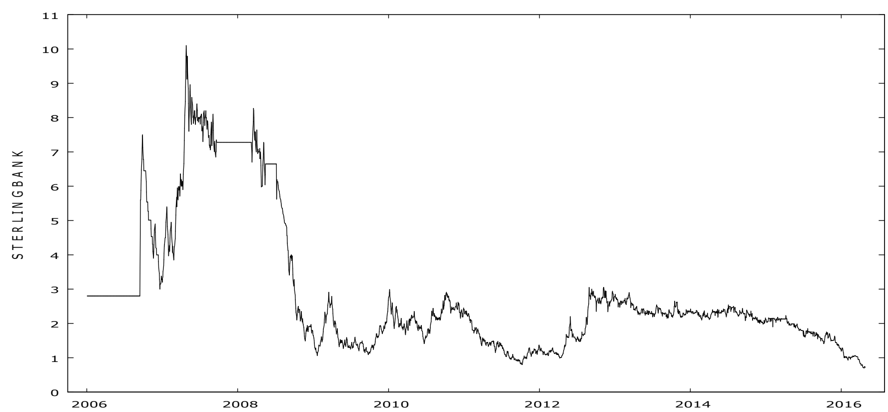

Figure 2: Price Series of Sterling Bank shares

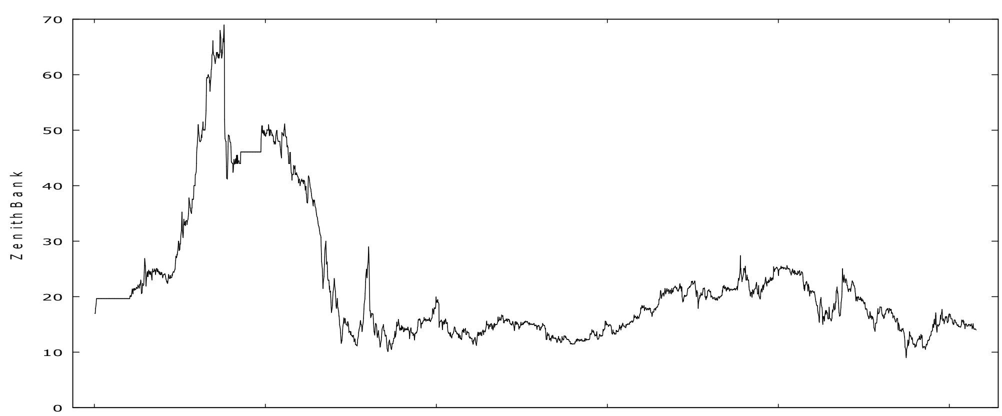

Figure 3: Price Series of Zenith Bank shares

However, the first difference of the logarithmic-transformed price series of the shares is carried out to ensure stationarity in both the mean and the variance [see Figures 4-6]. Meanwhile, the transformed series (which are the return series) appears to cluster around the common mean providing clear evidence of heteroscedasticity.

Notes

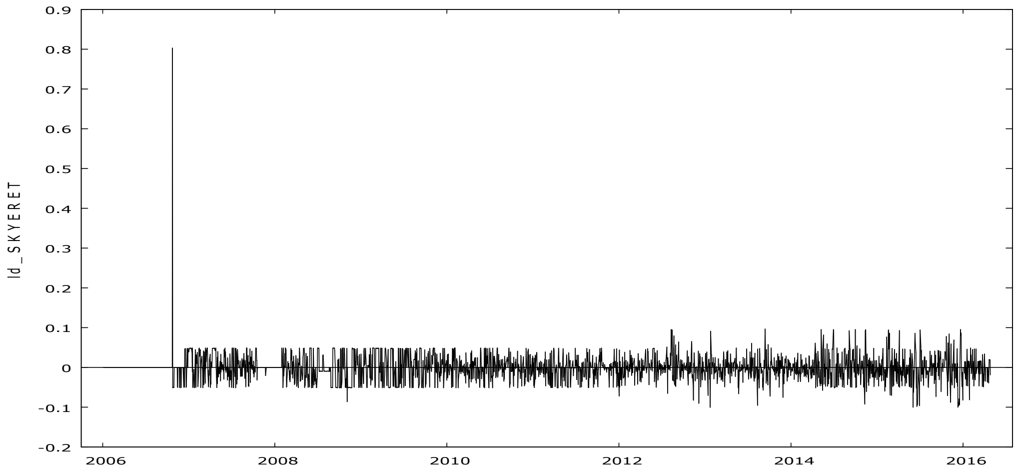

Figure 4: Return Series of Skye Bank

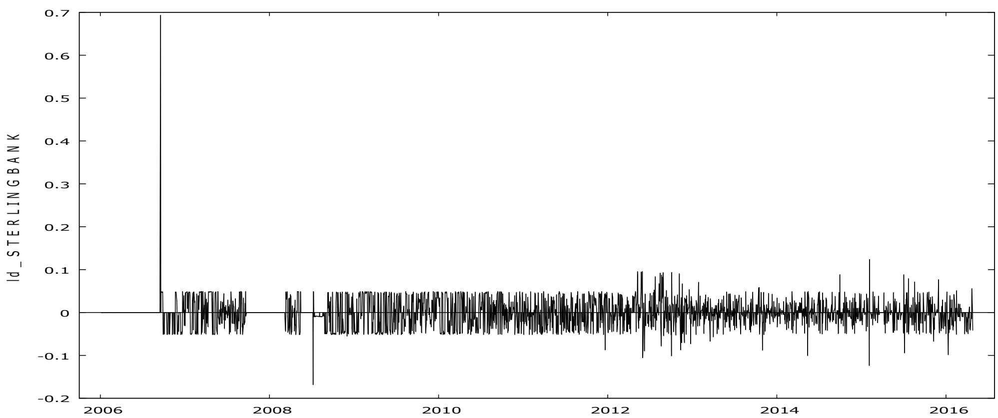

Figure 5: Return Series of Sterling Bank

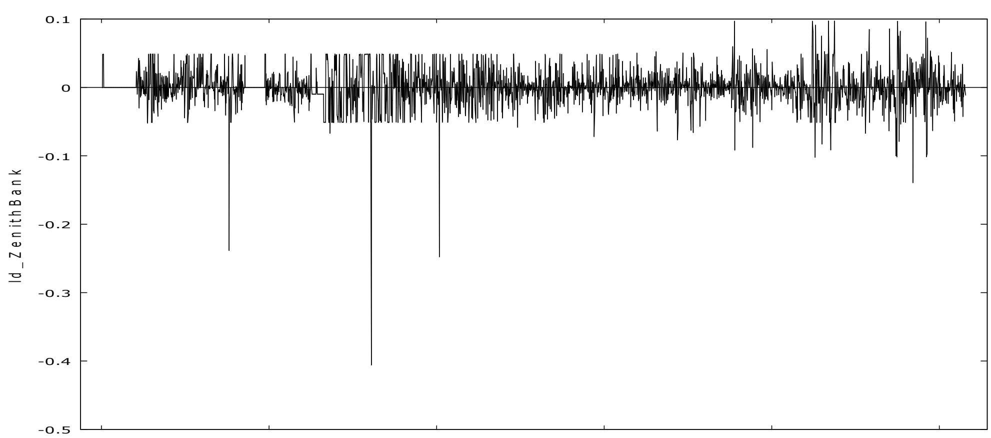

Figure 6: Return Series of Zenith Bank

The kurtoses of the returns series as indicated in Table 1 are in excess having exceeded the value accommodated by the normal distribution. Otherwise, it is evident that the distribution of the returns series is non-normal.

Notes

Table 1: Sample Kurtosis of Return Series

Bank	Kurtosis
Skye	123.5863
Sterling	73.6697
Zenith	27.7374

Notes

a) Modeling the Return Series of Skye Bank

From Table 2, ARIMA(1, 1, 0) model is selected based on the significance of the parameters and with minimum AIC.

Table 2: ARIMA Models for Return Series of Skye Bank

Model	Akaike Information Criteria (AIC)
*** ARIMA(1,1,0)	−10713.39
ARIMA(0,1,1)	−10711.03
ARIMA(1,1,1)	−10711.54

**** significance at 5% level*

Evidence from Ljung-Box Q-statistics shows that ARIMA(1, 1, 0) model is adequate at 5% level of significance given the Q-statistic at Lags 1, 4, 8 and 24, with Q(1) = 0.0050, Q(4) = 4.1838, Q(8) = 8.2689 and Q(24) = 22.469 with corresponding *p*-values given by $p = 0.9435$, $p = 0.3817$, $p = 0.4077$ and $p = 0.5513$.

On the other hand, evidence from ACF (Figure 7), PACF (Figure 8), Portmanteau-Q (PQ) statistics, and Lagrange-Multiplier (LM) test statistics in Table 3 shows that heteroscedasticity exists.

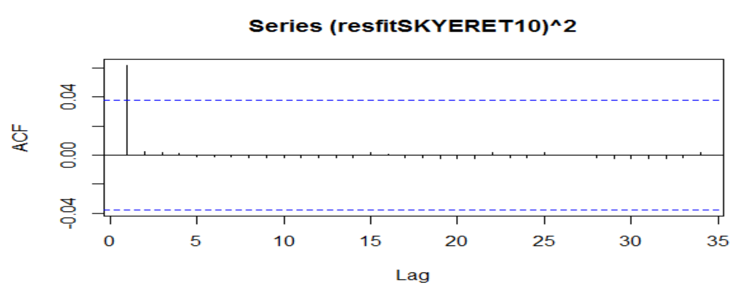

Figure 7: ACF of Squared Residuals of ARIMA (1, 1, 0) Model

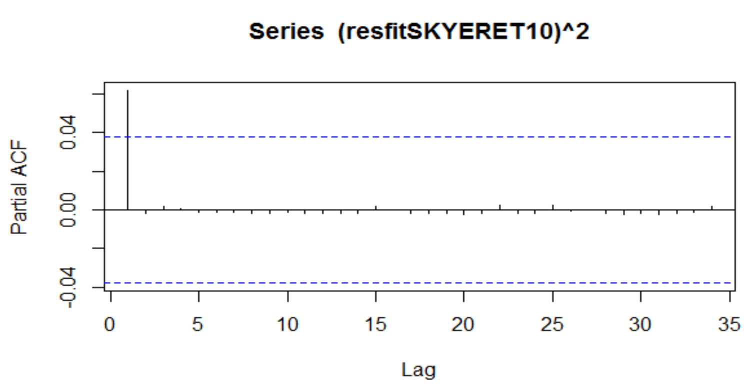

Figure 8: PACF of Squared Residuals of ARIMA (1, 1, 0) Model

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Table 3: ARCH Heteroscedasticity Test for Residuals of ARIMA(1,1,0) Model fitted to Return Series of Skye Bank

Lag(Order)	Portmanteau-Q Test		Lagrange -Multiplier Test	
	PQ Value	p-value	LM Value	p-value
4	10.3	0.036	57956	0.0000***
8	10.3	0.244***	28852	0.0000***
12	10.3	0.586***	19141	0.0000***
16	10.4	0.847***	14284	0.0000***
20	10.4	0.960***	11371	0.0000***
24	10.5	0.992***	9423	0.0000***

**** significance at 5% level*

Notes

Having detected the presence of heteroscedasticity in the residual series of ARIMA(1, 1, 0) model , the following models are considered to account for the heteroscedasticity with respect to Normal (norm) and Student-t (std) distributions: GARCH(1, 0), GARCH(2, 0), GARCH(1, 1), EGARCH(1, 1). Thus we have that: GARCH(1, 0)-norm, GARCH(1, 0)-std, GARCH(2, 0)-norm, GARCH(1, 1)-norm, GARCH(1, 1)-std, EGARCH(1, 1)-norm and EGARCH(1, 1)-std are successful except GARCH(2, 0)-std.

Comparing the values of the information criteria for the seven (7) models estimated as indicated in Table 4, GARCH(1, 1)-std has the smallest information criteria followed by EGARCH(1, 1)-std, though not adequate, and are constrained by several non-significant parameters. On the other hand, amongst the models that are adequate with only one non-significant parameter, GARCH (1, 1)-norm has the smallest information criteria and is selected as the appropriate heteroscedastic model for the return series of Skye bank with a kurtosis coefficient of 132.8707.

Table 4: Information Criteria for Heteroscedastic Models of the Return Series of Skye Bank

Model	Information Criteria		
	Akaike	Bayes	Hannan - Quinn
*GARCH(1,0)-norm	−4.2611	−4.2523	−4.2579
*GARCH(1,0)-std	−4.3304	−4.3195	−4.3265
*GARCH(2,0)-norm	37.512	37.523	37.516
*GARCH(1,1)-norm	−4.5293	−4.5183	−4.5253
GARCH(1,1)-std	−6.0009	−5.9878	−5.9962
EGARCH(1,1)-norm	−4.4125	−4.3993	−4.4077
EGARCH(1,1)-std	−4.6479	−4.6325	−4.6423

**Model that is adequate with one non-significant parameter.*

About twenty-six (26) different outliers are identified to have contaminated the residual series of ARIMA(1, 1, 0) model using the critical value, $C = 4$, namely: six (6) innovation outliers (IO), six (6) additive outliers (AO) and fourteen (14) temporary change (TC), as shown in Table 5.

Table 5: Types of Outliers Identified

	type	ind	coefhat	tstat
2	IO	211	-0.20150630	-8.487698
4	IO	1841	-0.10241849	-4.313995
5	IO	1843	0.09870872	4.157735
7	IO	2178	0.10295915	4.336768
8	IO	2263	0.09758512	4.110407
15	AO	210	0.81215294	34.804236
18	AO	1726	0.09492679	4.068020
20	AO	1984	-0.10371058	-4.444443

22	AO	2281	0.09978000	4.276001
23	AO	2414	0.10169110	4.357900
24	AO	2456	-0.09871728	-4.230459
27	TC	209	0.20948475	10.861708
30	TC	740	-0.09161349	-4.750126
31	TC	742	-0.07866550	-4.078778
32	TC	827	0.07862559	4.076708
33	TC	1723	0.08946532	4.638744
34	TC	2311	0.09068887	4.702185
35	TC	2381	0.09747559	5.054074
36	TC	2468	-0.10240036	-5.309421
37	TC	2590	-0.08395679	-4.353129
38	TC	2592	-0.12692535	-6.581033
41	TC	2599	0.10544854	5.467469
42	IO	2314	-0.10163346	-4.363007
10	TC	212	-0.10846069	-5.731470
9	TC	741	0.07648229	4.225631
101	TC	2589	0.07256176	4.009023

Having adjusted for outliers in the series, GARCH(1, 1)-norm is found to be adequate (with all the parameters significant) in capturing heteroscedasticity in the outlier-adjusted return series with kurtosis value of 2.9465, which is approximately the value accommodated by the normal distribution.

b) Modeling the Return Series of Sterling Bank

Using the same procedure as applied in modeling the return series of Skye Bank, ARIMA (2, 1, 0) is found to be adequate with significant parameters in modeling the linear dependence in the return series of Sterling bank. Also, heteroscedasticity is found to exist and is adequately captured by EGARCH(1, 1)-norm with a kurtosis coefficient of 80.3030.

However, about seven (7) different outliers are identified to have contaminated the series using the critical value, C = 5, namely: one (1) innovation outlier (IO), four (4) additive outliers (AO) and two (2) temporary change (TC) outliers, as shown in Table 6.

Table 6: Types of Outliers Identified

	type	ind	coefhat	tstat
4	AO	184	0.6913146	27.668748
5	AO	655	-0.1764822	-7.063413
6	AO	2371	-0.1398721	-5.598155
10	TC	183	0.2415834	11.950355
12	TC	1672	-0.1075351	-5.319415
3	IO	185	-0.1964258	-7.900298
8	AO	2372	0.1407112	5.678009

Cleaning the series of the detected outliers, ARIMA(2, 1, 2) model appeared to adequately fit outlier-adjusted series. Furthermore, the heteroscedasticity is captured by EGARCH(1, 1)-std with a kurtosis value of 3.6829.

c) Modeling the Return Series of Zenith Bank

Again, using the same procedure as applied in modeling the return series of Skye bank, ARIMA(2, 1, 1) model successfully captured the linear dependence in the return series while EGARCH(1, 1)-std adequately expressed the heteroscedasticity in the series with a kurtosis coefficient of 26.3794.

Meanwhile, about forty-two (42) different outliers are identified to have contaminated the series using the critical value of C = 5. They are thirteen (13) innovation outliers (IO), nine (9) additive outliers (AO) and twenty (20) temporary change (TC) outliers, as shown in Table 7.

Table 7: Types of Outliers Identified

	type	ind	coefhat	tstat
1	IO	396	-0.09816377	-13.221339
3	IO	840	0.04253167	5.728444
7	IO	2221	-0.03927918	-5.290377
9	IO	2263	0.04378397	5.897112
10	IO	2281	0.03787680	5.101495
12	IO	2473	-0.03936074	-5.301362
13	IO	2475	-0.04230834	-5.698364
14	IO	2525	-0.06416968	-8.642792
15	IO	2565	0.04357221	5.868590
16	IO	2568	-0.03892676	-5.242911
18	AO	839	-0.17023181	-23.610345
20	AO	1051	-0.10685231	-14.819909
22	AO	1971	-0.04930715	-6.838668
23	AO	2027	-0.04348949	-6.031786
24	AO	2223	0.04365213	6.054343
26	AO	2389	0.03931499	5.452803
27	AO	2453	0.04052320	5.620376
28	AO	2483	-0.03680867	-5.105188
31	TC	395	-0.04944665	-8.279793
33	TC	691	-0.03018987	-5.055264
34	TC	710	-0.03064893	-5.132133
35	TC	747	-0.03068035	-5.137395
37	TC	818	0.03346118	5.603041
44	TC	838	-0.05601975	-9.380451
49	TC	2477	0.03332783	5.580712
2	IO	1970	0.03696756	5.089711
32	IO	2269	-0.03667538	-5.049484
6	TC	698	0.03138467	5.372151
71	TC	754	0.03144461	5.382412
21	TC	802	0.02900192	5.029855
36	IO	2569	-0.04117313	-5.684147
72	TC	394	0.03079012	5.941387
101	TC	833	0.02933330	5.660273
141	TC	850	-0.02610763	-5.037836
151	TC	2212	0.02755604	5.317326
25	AO	2282	0.03601640	5.029504
311	TC	2484	0.02586343	5.048003
5	TC	824	0.02437288	5.000302
8	TC	890	-0.02533852	-5.198411
91	TC	2217	-0.02467081	-5.061424
102	TC	2450	-0.02471591	-5.070677
73	TC	919	0.02369936	5.067229

Adjusting the series for outliers, ARIMA(2, 1, 1) and EGARCH(1, 1) models are found to be adequate in capturing the linear dependence and heteroscedasticity, respectively, in the series with the kurtosis value of 3.5746, which is approximately the value occupied by the normal distribution.

So far, it is found that GARCH(1, 1) with respect to a normal distribution, could not capture the excess kurtosis in the return series of Skye bank. However, with outliers taken into consideration, the same GARCH(1, 1) is sufficient in capturing the excess kurtosis of the bank with respect a normal distribution. For Sterling bank, EGARCH (1, 1) model with respect to a normal distribution failed to contain the excess kurtosis

Notes

while EGARCH(1, 1) model with respect to a student-t distribution sufficiently captured the excess kurtosis when accounted for outliers. For Zenith bank, EGARCH (1, 1) model with respect to a student-t distribution could not capture the excess kurtosis but was successful when adjusted for outliers. The implication of our findings is that the existence of excess kurtosis is due to the presence of outliers and that the GARCH-type models, irrespective of the two distributions, are sufficient in capturing the excess kurtosis when outliers are taken into consideration.

Notes

IV. Conclusion

In summary, our study showed that the two types of distribution considered are not adequate in capturing the excess kurtosis while modeling heteroscedasticity. Also, given the fact that estimation of kurtosis in GARCH-type models is based on the fourth moment, the returns that are far from the mean would insert a huge impact on the kurtosis while the values that are close to the mean would have a less impact on the kurtosis. This very reason denies kurtosis coefficient the ability to describe the shape of different distributions and otherwise, provide a good measure of outliers. Hence, it is recommended that outliers be accounted for in order to overcome the conflict of the choice of distribution while applying GARCH-type models. It is also recommended that similar studies be conducted on other Nigerian stocks including those of other banks not considered in this study as further researches.

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Notes

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Fractional Integration of the Product of Two Multivariable Gimel-Functions and A General Class of Polynomials

By Frederic Ayant

Abstract- A significantly large number of earlier works on the subject of fractional calculus give the interesting account of the theory and applications of fractional calculus operators in many different areas of mathematical analysis (such as ordinary and partial differential equations, integral equations, special functions, the summation of series, etc.). The object of the present paper is to study and develop the Saigo-Maeda operators. First, we establish four results that give the images of the product of two multivariable Gimel-functions and a general class of multivariable polynomials in Saigo-Maeda operators. On account of the general nature of the Saigo-Maeda operators, multivariable Gimel-functions and a class multivariable polynomials a large number of new and known theorems involving Riemann-Liouville and Erdelyi-Kober fractional integral operators and several special functions.

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GJSFR-F Classification: FOR Code: MSC 2010: 30C10

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I. Introduction And Preliminaries

The fractional integral operator involving various special functions has found significant importance and applications in Various subfields of applicable mathematical analysis. Since the last four decades, some workers like Love [17], McBride [20], Kalla [8,9], Kalla and Saxena [10,11], Saxena et al. [29], Saigo [24,25], Kilbas [12], Kilbas and Sebastian [14] and Kiryakova [16,17] have studied in depth the properties, applications and different extensions of Various hypergeometric operators of fractional integration. A detailed account of such operators along with their properties and applications can be found in the research monographs by Samko, Kilbas, and Marichev [26], Miller and Ross [22], Kilbas, Srivastava, and Trujillo [15] and Debnath and Bhatta [6]. A useful generalization of the hypergeometric fractional integrals, including the Saigo operators [23,24], has been introduced by Marichev [18], see Samko et al. [28] and also see Kilbas and Saigo [13] for more details. The generalized fractional integral operator of arbitrary order, involving Appell function F_3 in the kernel defined and studied by Saigo and Maeda [27, p. 393, Eq (4.12)] and (4.13)] in the following manner:

Let $\alpha, \alpha', \beta, \beta', \eta$ be complex numbers and, $x, Re(\eta) > 0$, we have, see Saigo and Maeda [28, p. 393, Eq (4.12)]

Definition.1

$$I_{0,x}^{\alpha,\alpha',\beta,\beta',\eta} f(x) = \frac{z^{-\alpha}}{\Gamma(\eta)} \int_0^x (x-t)^{\eta-1} t^{-\alpha'} F_3\left[\alpha,\alpha',\beta,\beta';\eta;1-\frac{t}{x},1-\frac{x}{t}\right] f(t)\mathrm{d}t \tag{1.1}$$

and

Definition 2

$$I_{x,\infty}^{\alpha,\alpha',\beta,\beta',\eta} f(x) = \frac{x^{-\alpha}}{\Gamma(\eta)} \int_0^x (t-x)^{\eta-1} t^{-\alpha'} F_3\left[\alpha,\alpha',\beta,\beta';\eta;1-\frac{x}{t},1-\frac{t}{x}\right] f(t)\mathrm{d}t \tag{1.2}$$

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Ref

6. L. Debnath and D. Bhatta, Integral Transforms and Their Applications, Chapman and Hall/CRC Press, Boca Raton FL, 2006.

We have the following two results due to Saigo [25] where $Re(\eta) > 0$

Definition 3

$$I_{0^+}^{\alpha,\beta,\eta} f(z) = \frac{x^{-\alpha-\beta}}{\Gamma(\eta)} \int_0^x (x-t)^{\alpha-1} F\left[\alpha+\beta, -\eta; \alpha; 1-\frac{t}{x}\right] f(t)\mathrm{d}t \tag{1.3}$$

Definition 4

$$I_{0^-}^{\alpha,\beta,\eta} f(z) = \frac{1}{\Gamma(\eta)} \int_x^\infty t^{-\alpha-\beta}(x-t)^{\alpha-1} F\left[\alpha+\beta, -\eta; \alpha; 1-\frac{x}{t}\right] f(t)\mathrm{d}t \tag{1.4}$$

F is the Gaussian hypergeometric function. We obtain the following lemmas.

Lemma 1.

$$\left(I_{0,x}^{\alpha,\alpha',\beta,\beta',\eta} t^{\mu-1}\right) = \frac{\Gamma(u)\Gamma(\mu+\eta-\alpha-\alpha'-\beta)(\mu+\beta'-\alpha')}{\Gamma(\mu+\eta-\alpha-\alpha')\Gamma(\mu+\eta-\alpha'-\beta)\Gamma(\mu+\beta')} x^{\mu-\alpha-\alpha'+\eta-1} \tag{1.5}$$

where $\alpha, \alpha', \beta, \beta', \eta \in \mathbb{C}, Re(\mu) > max\{0, \Re(\alpha+\alpha'+\beta-\eta), Re(\alpha'-\beta')\}$

Lemma 2.

$$\left(I_{x;\infty}^{\alpha,\alpha',\beta,\beta',\eta} t^{\mu-1}\right) = \frac{\Gamma(1+\alpha+\alpha'-\eta-\mu)\Gamma(1+\alpha+\beta'-\eta-\mu)\Gamma(1-\beta-\mu)}{\Gamma(1-\mu)\Gamma(1-\mu-\eta+\alpha+\alpha'+\beta')\Gamma(1+\alpha-\beta-\mu)} x^{\mu-\alpha-\alpha'+\eta-1} \tag{1.6}$$

where $\alpha, \alpha', \beta, \beta', \eta \in \mathbb{C}, Re(\eta) > 0, Re(\mu) < min\{Re(-\beta), \Re(\alpha+\alpha'-\eta), Re(\alpha'+\beta'-\eta)\}$

Lemma 3.

$$\left(I_{0,x}^{\alpha,\beta,\eta} t^{\mu-1}\right) = \frac{\Gamma(u)\Gamma(\mu+\eta-\beta)}{\Gamma(\mu+\eta+\alpha+\eta)\Gamma(\mu-\beta)} x^{\mu-\beta-1} \tag{1.7}$$

$$\alpha, \beta, \eta \in \mathbb{C}, Re(> 0, Re(\mu) > max\{0, \Re(\beta-\eta), Re(\alpha'-\beta')\}$$

Lemma 4.

$$\left(I_{x;\infty}^{\alpha,\beta,\eta} t^{\mu-1}\right) = \frac{\Gamma(\beta-\mu+1)\Gamma(\eta-\mu+1)}{\Gamma(1-\mu)\Gamma(\alpha+\beta+\eta-\mu+1)} x^{\mu-\beta-1} \tag{1.8}$$

where $\alpha, \beta, \eta \in \mathbb{C}, Re(\alpha) > 0, Re(\mu) < 1 + min\{Re(\beta), \Re(\eta)\}$

Recently, Gupta et al. [7] have obtained the images of the product of two H-functions in Saigo operator given by (1.3) and (1.4) and thereby generalized several results obtained earlier by Kilbas, Kilbas and Sebastian [14] and Saxena et al. [29] as mentioned in this paper cited above. It has recently become a subject of interest for many researchers in the field of fractional calculus and its applications. Motivated by these avenues of applications, a number of workers have made use of the fractional calculus operators to obtain the image formulas. The aim of this paper is to obtain four results that give the theorems of the product of two multivariable Gimel functions and a general class of multivariable polynomials [30] in Saigo-Maeda operators and Saigo operators.

II. Multivariable Gimel-Function

We throughout this paper, let $\mathbb{C}, \mathbb{R}$, and $\mathbb{N}$ be set of complex numbers, real numbers and positive integers respectively. Also. $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. We define a generalized transcendental function of several complex variables.

Ref

7. K. C. Gupta, K. Gupta and A. Gupta, Generalized fractional integration of the product of two H-functions. J. Rajasthan Acad. Phy. Sci., 9(3), 203–212(2010).

Notes

$$\gimel(z_1,\cdots,z_r)=\gimel^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)}}_{p_{i_2},q_{i_2},\tau_{i_2};R_2;p_{i_3},q_{i_3},\tau_{i_3};R_3;\cdots;p_{i_r},q_{i_r},\tau_{i_r}:R_r:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|}\mathrm{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\end{array}\right.$$

$$[(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}};[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$
$$[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}};$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)};A_{rj})]_{1,n_r},$$
$$[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;\qquad\cdots$$

$$[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)};A_{rji_r})]_{n_r+1,p_r}:\ [(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})]_{n^{(1)}+1,p_i^{(1)}}$$
$$[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)};B_{rji_r})]_{1,q_r}:\ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)};D_j^{(1)})]_{1,m^{(1)}},[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})]_{m^{(1)}+1,q_i^{(1)}}$$

$$\left.\begin{array}{l};\cdots;[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})]_{1,m^{(r)}},[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})]_{m^{(r)}+1,p_i^{(r)}}\\ ;\cdots;[(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})]_{1,n^{(r)}},[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_{ji^{(r)}}^{(r)})]_{n^{(r)}+1,q_i^{(r)}}\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{2.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{n_2}\Gamma^{A_{2j}}(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k)}{\sum_{i_2=1}^{R_2}[\tau_{i_2}\prod_{j=n_2+1}^{p_{i_2}}\Gamma^{A_{2ji_2}}(a_{2ji_2}-\sum_{k=1}^{2}\alpha_{2ji_2}^{(k)}s_k)\prod_{j=1}^{q_{i_2}}\Gamma^{B_{2ji_2}}(1-b_{2ji2}+\sum_{k=1}^{2}\beta_{2ji2}^{(k)}s_k)]}$$

$$\frac{\prod_{j=1}^{n_3}\Gamma^{A_{3j}}(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k)}{\sum_{i_3=1}^{R_3}[\tau_{i_3}\prod_{j=n_3+1}^{p_{i_3}}\Gamma^{A_{3ji_3}}(a_{3ji_3}-\sum_{k=1}^{3}\alpha_{3ji_3}^{(k)}s_k)\prod_{j=1}^{q_{i_3}}\Gamma^{B_{3ji_3}}(1-b_{3ji3}+\sum_{k=1}^{3}\beta_{3ji3}^{(k)}s_k)]}$$

$$\cdot\qquad\cdot\qquad\cdot\qquad\cdot$$

$$\cdot\qquad\cdot\qquad\cdot\qquad\cdot$$

$$\cdot\qquad\cdot\qquad\cdot\qquad\cdot$$

$$\frac{\prod_{j=1}^{n_r}\Gamma^{A_{rj}}(1-a_{rj}+\sum_{k=1}^{r}\alpha_{rj}^{(k)}s_k)}{\sum_{i_r=1}^{R_r}[\tau_{i_r}\prod_{j=n_r+1}^{p_{i_r}}\Gamma^{A_{rji_r}}(a_{rji_r}-\sum_{k=1}^{r}\alpha_{rji_r}^{(k)}s_k)\prod_{j=1}^{q_{i_r}}\Gamma^{B_{rji_r}}(1-b_{rjir}+\sum_{k=1}^{r}\beta_{rjir}^{(k)}s_k)]} \tag{2.2}$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m^{(k)}}\Gamma^{D_j^{(k)}}(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n^{(k)}}\Gamma^{C_j^{(k)}}(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m^{(k)}+1}^{q_{i^{(k)}}}\Gamma^{D_{ji^{(k)}}^{(k)}}(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n^{(k)}+1}^{p_{i^{(k)}}}\Gamma^{C_{ji^{(k)}}^{(k)}}(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{2.3}$$

The contour L_k is in the $s_k(k=1,\cdots,r)$- plane and runs from $\sigma-i\infty$ to $\sigma+i\infty$ where σ if is a real number with loop, if necessary to ensure that the poles of $\Gamma^{A_{2j}}\left(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k\right)$ $(j=1,\cdots,n_2),\Gamma^{A_{3j}}\left(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k\right)$ $(j=1,\cdots,n_3),\cdots,\Gamma^{A_{rj}}\left(1-a_{rj}+\sum_{i=1}^{r}\alpha_{rj}^{(i)}\right)(j=1,\cdots,n_r)$, $\Gamma^{C_j^{(k)}}\left(1-c_j^{(k)}+\gamma_j^{(k)}s_k\right)(j=1,\cdots,n^{(k)})(k=1,\cdots,r)$ to the right of the contour L_k and the poles of $\Gamma^{D_j^{(k)}}\left(d_j^{(k)}-\delta_j^{(k)}s_k\right)(j=1,\cdots,m^{(k)})(k=1,\cdots,r)$ lie to the left of the contour L_k. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained of the corresponding conditions for multivariable H-function given by as

$|arg(z_k)|<\frac{1}{2}A_i^{(k)}\pi$ where

$$A_i^{(k)}=\sum_{j=1}^{m^{(k)}}D_j^{(k)}\delta_j^{(k)}+\sum_{j=1}^{n^{(k)}}C_j^{(k)}\gamma_j^{(k)}-\tau_{i^{(k)}}\left(\sum_{j=m^{(k)}+1}^{q_i^{(k)}}D_{ji^{(k)}}^{(k)}\delta_{ji^{(k)}}^{(k)}+\sum_{j=n^{(k)}+1}^{p_i^{(k)}}C_{ji^{(k)}}^{(k)}\gamma_{ji^{(k)}}^{(k)}\right) \tag{2.4}$$

$$-\tau_{i_2}\left(\sum_{j=n_2+1}^{p_{i_2}}A_{2ji_2}\alpha_{2ji_2}^{(k)}+\sum_{j=1}^{q_{i_2}}B_{2ji_2}\beta_{2ji_2}^{(k)}\right)-\cdots-\tau_{i_r}\left(\sum_{j=n_r+1}^{p_{i_r}}A_{rji_r}\alpha_{rji_r}^{(k)}+\sum_{j=1}^{q_{i_r}}B_{rji_r}\beta_{rji_r}^{(k)}\right)$$

Following the lines of Braaksma ([4] p. 278), we may establish the asymptotic expansion in the following convenient form

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,),max(\,|z_1|,\cdots,|z_r|\,)\to 0$$

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,),min(\,|z_1|,\cdots,|z_r|\,)\to\infty \text{ where } i=1,\cdots,r:$$

$$\alpha_i=\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] \text{ and } \beta_i=\max_{1\leqslant j\leqslant n^{(i)}}Re\left[C_j^{(i)}\left(\frac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right)\right]$$

Remark 1.

If $n_2=\cdots=n_{r-1}=p_{i_2}=q_{i_2}=\cdots=p_{i_{r-1}}=q_{i_{r-1}}=0$. $A_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=A_{rji_r}=B_{rji_r}=A_{rj}=A_{rji_r}=B_{rji_r}=1$, then the multivariable Gimel-function reduces in multivariable Aleph- function defined by Ayant [3].

Remark 2.

If $n_2=\cdots=n_r=p_{i_2}=q_{i_2}=\cdots=p_{i_r}=q_{i_r}=0$. $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2=\cdots=R_r=R^{(1)}=\cdots=R^{(r)}=1$, then the multivariable Gimel-function reduces in multivariable I-function defined by Prathima et al. [23].

Remark 3.

If $A_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=A_{rji_r}=B_{rji_r}=1$. $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2=\cdots=R_r=R^{(1)}=\cdots=R^{(r)}=1$, then the generalized multivariable Gimel-function reduces in multivariable I-function defined by Prasad [22].

Remark 4.

If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the H-function of several defined by Srivastava and Panda [32,33]. About the simplified notations, see Ayant ([4], page 248-255)

Now, we define the second Gimel function of s variables, the parameters are identical to the Gimel function of r variables with the prim sign and the validities conditions are equivalent.

The generalized polynomials of multivariable defined by Srivastava [30], is given in the following manner:

$$S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[y_1,\cdots,y_v]=\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_v)_{\mathfrak{M}_vK_v}}{K_v!}A[N_1,K_1;\cdots;N_v,K_v]y_1^{K_1}\cdots y_v^{K_v} \tag{2.5}$$

Ref

4. F.Y. Ayant, Some transformations and idendities form multivariable Gimel-function, International Journal of Matematics and Technology (IJMTT), 59(4) (2018), 248-255.

where $\mathfrak{M}_1, \cdots, \mathfrak{M}_\mathfrak{v}$ are arbitrary positive integers and the coefficients $A[N_1, K_1; \cdots; N_v, K_v]$ are arbitrary constants, real or complex.

We shall note $a_v = \dfrac{(-N_1)_{\mathfrak{M}_1 K_1}}{K_1!} \cdots \dfrac{(-N_v)_{\mathfrak{M}_\mathfrak{v} K_v}}{K_v!} A[N_1, K_1; \cdots; N_v, K_v]$

Notes

III. Main Results

We shall note

$$U = 0, n_2; 0, n_3; \cdots; 0, n_{r-1}; 0, n'_2; 0, n'_3; \cdots; 0, n'_{s-1} \tag{3.1}$$

$$V = m^{(1)}, n^{(1)}; m^{(2)}, n^{(2)}; \cdots; m^{(r)}, n^{(r)}; m'^{(1)}, n'^{(1)}; m'^{(2)}, n'^{(2)}; \cdots; m'^{(s)}, n'^{(s)} \tag{3.2}$$

$$X = p_{i_2}, q_{i_2}, \tau_{i_2}; R_2; \cdots; p_{i_{r-1}}, q_{i_{r-1}}, \tau_{i_{r-1}} : R_{r-1};\ p'_{i_2}, q'_{i'_2}, \tau'_{i_2}; R'_2; \cdots; p'_{i'_{s-1}}, q'_{i'_{s-1}}, \tau'_{i'_{s-1}} : R'_{s-1} \tag{3.3}$$

$$Y = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}; \cdots; p_{i^{(r)}}, q_{i^{(r)}}; \tau_{i^{(r)}}; R^{(r)}; p'_{i'^{(1)}}, q'_{i'^{(1)}}, \tau'_{i'^{(1)}}; R'^{(1)}; \cdots; p'_{i'^{(s)}}, q'_{i^{(s)}}; \tau'_{i'^{(s)}}; R'^{(s)} \tag{3.4}$$

Theorem 1.

$$\left\{\left(I_{0,x}^{\alpha,\alpha',\beta,\beta',\eta} t^{\mu-1}(b-at)^{-v} S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_\mathfrak{v}} \begin{pmatrix} \mathrm{c}_1 t^{\lambda_1}(b-at)^{-\delta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{c}_v t^{\lambda_v}(b-at)^{-\delta_v} \end{pmatrix} \gimel \begin{pmatrix} \mathrm{z}_1 t^{\sigma_1}(b-at)^{-\omega_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r t^{\sigma_r}(b-at)^{-\omega_r} \end{pmatrix}\right.\right.$$

$$\left.\left.\gimel \begin{pmatrix} \mathrm{z}'_1 t^{\sigma'_1}(b-at)^{-\omega'_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}'_s t^{\sigma'_s}(b-at)^{-\omega'_s} \end{pmatrix}\right)\right\}(x) = b^{-v} x^{\mu-\alpha-\alpha'+\eta-1} \sum_{K_1=1}^{[N_1/M_1]} \cdots \sum_{K_v=1}^{[N_v/M_v]} a_v c_1^{K_1} \cdots c_v^{K_v}$$

$$v^{-\sum_{j=1}^{v} \delta_i K_j} x^{\sum_{j=1}^{v} \lambda_i K_j} \gimel^{U;0,n_r+n'_s+4:V,1,0}_{X;p_{i_r}+p'_{i'_s}+4,q_{i_r}+q'_{i'_s}+4,\tau_{i_r},\tau'_{i'_s}:R_r:R'_s:Y;0,1} \left(\begin{array}{c|c} \mathrm{z}_1 \frac{x^{\sigma_1}}{b^{\omega_1}} & \\ \cdot & \mathbb{A}, ,\mathrm{A}_1, \mathbf{A} : A \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r \frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot \\ \mathrm{z}_1 \frac{x^{\sigma'_1}}{b^{\omega'_1}} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}'_s \frac{x^{\sigma'_s}}{b^{\omega'_s}} & \mathbb{B},\mathbf{B},\ \mathrm{B}_1 : B, (0,1;1) \\ \frac{ax}{b} & \end{array} \right) \tag{3.5}$$

where

$$\mathbb{A} = [(\mathrm{a}_{2j}; \alpha_{2j}^{(1)}, \alpha_{2j}^{(2)}; A_{2j})]_{1,n_2}, [\tau_{i_2}(a_{2ji_2}; \alpha_{2ji_2}^{(1)}, \alpha_{2ji_2}^{(2)}; A_{2ji_2})]_{n_2+1,p_{i_2}}, [(a_{3j}; \alpha_{3j}^{(1)}, \alpha_{3j}^{(2)}, \alpha_{3j}^{(3)}; A_{3j})]_{1,n_3},$$

$$[\tau_{i_3}(a_{3ji_3}; \alpha_{3ji_3}^{(1)}, \alpha_{3ji_3}^{(2)}, \alpha_{3ji_3}^{(3)}; A_{3ji_3})]_{n_3+1,p_{i_3}}; \cdots; [(\mathrm{a}_{(r-1)j}; \alpha_{(r-1)j}^{(1)}, \cdots, \alpha_{(r-1)j}^{(r-1)}; A_{(r-1)j})]_{1,n_{r-1}},$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}}; \alpha_{(r-1)ji_{r-1}}^{(1)}, \cdots, \alpha_{(r-1)ji_{r-1}}^{(r-1)}; A_{(r-1)ji_{r-1}})]_{n_{r-1}+1,p_{i_{r-1}}},$$

$$[(\mathrm{a}'_{2j};\alpha'^{(1)}_{2j},\alpha'^{(2)}_{2j};A'_{2j})]_{1,n'_2},[\tau'_{i'_2}(a'_{2ji_2};\alpha'^{(1)}_{2ji'_2},\alpha'^{(2)}_{2ji'_2};A'_{2ji'_2})]_{n'_2+1,p'_{i'_2}},[(a'_{3j};\alpha'^{(1)}_{3j},\alpha'^{(2)}_{3j},\alpha'^{(3)}_{3j};A'_{3j})]_{1,n'_3},$$

$$[\tau'_{i'_3}(a'_{3ji'_3};\alpha'^{(1)}_{3ji'_3},\alpha'^{(2)}_{3ji'_3},\alpha'^{(3)}_{3ji'_3};A'_{3ji'_3})]_{n'_3+1,p'_{i'_3}};\cdots;[(\mathrm{a}'_{(s-1)j};\alpha'^{(1)}_{(s-1)j},\cdots,\alpha'^{(s-1)}_{(s-1)j};A'_{(s-1)j})]_{1,n'_{s-1}},$$

$$[\tau'_{i'_{s-1}}(a'_{(s-1)ji'_{s-1}};\alpha'^{(1)}_{(s-1)ji'_{s-1}},\cdots,\alpha'^{(s-1)}_{(s-1)ji'_{s-1}};A'_{(s-1)ji'_{s-1}})]_{n'_{s-1}+1,p'_{i'_{s-1}}} \tag{3.6}$$

$$A_1=(1-v+\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1),(1+\alpha+\alpha'+\beta-v-\eta-\sum_{j=1}^{K}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1;1)$$

$$(1-\mu+\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1),(1+\alpha'+\beta-\mu-\eta-\sum_{j=1}^{K}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1;1) \tag{3.7}$$

$$\mathbf{A}=[(\mathrm{a}_{rj};\alpha^{(1)}_{rj},\cdots,\alpha^{(r)}_{rj},\underbrace{0,\cdots,0}_{s+1},A_{rj})]_{1,n_r},[\tau_{i_r}(a_{rji_r};\alpha^{(1)}_{rji_r},\cdots,\alpha^{(r)}_{rji_r},\underbrace{0,\cdots,0}_{s+1};A_{rji_r})]_{\mathfrak{n}+1,p_{i_r}},$$

$$[(\mathrm{a}_{sj};\underbrace{0,\cdots,0}_{r}\,\alpha'^{(1)}_{sj},\cdots,\alpha'^{(r)}_{sj},0,A'_{sj})]_{1,n'_s},[\tau'_{i'_s}(a'_{sji'_s};\underbrace{0,\cdots,0}_{r}\,\alpha'^{(1)}_{sji'_s},\cdots,\alpha^{(s)}_{sji'_s},0;A'_{sji'_s})]_{n'_s+1,p'_{i'_s}} \tag{3.8}$$

$$A=[(\mathrm{c}^{(1)}_j,\gamma^{(1)}_j;C^{(1)}_j)_{1,n^{(1)}}],[\tau_{i^{(1)}}(c^{(1)}_{ji^{(1)}},\gamma^{(1)}_{ji^{(1)}};C^{(1)}_{ji^{(1)}})]_{n^{(1)}+1,p^{(1)}_i};\cdots;$$

$$[(c^{(r)}_j,\gamma^{(r)}_j;C^{(r)}_j)_{1,m^{(r)}}],[\tau_{i^{(r)}}(c^{(r)}_{ji^{(r)}},\gamma^{(r)}_{ji^{(r)}};C^{(r)}_{ji^{(r)}})]_{m^{(r)}+1,p^{(r)}_i},$$

$$[(\mathrm{c}'^{(1)}_j,\gamma'^{(1)}_j;C'^{(1)}_j)_{1,n'^{(1)}}],[\tau'_{i^{(1)}}(c'^{(1)}_{ji'^{(1)}},\gamma^{(1)}_{ji'^{(1)}};C'^{(1)}_{ji'^{(1)}})]_{n'^{(1)}+1,p'^{(1)}_i},$$

$$\cdots,[(c'^{(s)}_j,\gamma'^{(s)}_j;C'^{(s)}_j)]_{1,m'^{(s)}},[\tau'_{i'^{(s)}}(c'^{(s)}_{ji'^{(s)}},\gamma'^{(s)}_{ji'^{(s)}};C'^{(s)}_{ji'^{(s)}})]_{m'^{(s)}+1,p'^{(r)}_{i'}};-: \tag{3.9}$$

$$\mathbb{B}=[\tau_{i_2}(b_{2ji_2};\beta^{(1)}_{2ji_2},\beta^{(2)}_{2ji_2};B_{2ji_2})]_{1,q_{i_2}},[\tau_{i_3}(b_{3ji_3};\beta^{(1)}_{3ji_3},\beta^{(2)}_{3ji_3},\beta^{(3)}_{3ji_3};B_{3ji_3})]_{1,q_{i_3}};\cdots;$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}};\beta^{(1)}_{(r-1)ji_{r-1}},\cdots,\beta^{(r-1)}_{(r-1)ji_{r-1}};B_{(r-1)ji'_{r-1}})]_{1,q_{i_{r-1}}};$$

$$[\tau'_{i'_2}(b_{2ji'_2};\beta'^{(1)}_{2ji'_2},\beta'^{(2)}_{2ji'_2};B'_{2ji'_2})]_{1,q'_{i'_2}},[\tau'_{i'_3}(b'_{3ji'_3};\beta'^{(1)}_{3ji'_3},\beta'^{(2)}_{3ji'_3},\beta'^{(3)}_{3ji'_3};B'_{3ji'_3})]_{1,q'_{i'_3}};\cdots;$$

$$[\tau'_{i'_{s-1}}(b'_{(s-1)ji'_{s-1}};\beta'^{(1)}_{(s-1)ji'_{s-1}},\cdots,\beta'^{(s-1)}_{(s-1)ji'_{s-1}};B'_{(s-1)ji'_{s-1}})]_{1,q'_{i'_{s-1}}}$$

$$\mathbf{B}=[\tau_{i_r}(b_{rji_r};\beta^{(1)}_{rji_r},\cdots,\beta^{(r)}_{rji_r},\underbrace{0,\cdots,0}_{s+1};B_{rji_r})]_{1,q_{i_r}},\quad[\tau'_{i'_s}(b'_{sji'_s};\underbrace{0,\cdots,0}_{r};\beta'^{(1)}_{sji'_s},\cdots,\beta'^{(s)}_{sji'_s},0;B'_{sji'_s})]_{1,q'_{i'_s}} \tag{3.10}$$

$$B_1=(1-v-\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,0;1),(1+\alpha+\alpha'-v-\eta-\sum_{j=1}^{K}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'1,\cdots,\omega'_s,1;1)$$

$$(1-\mu-\beta'-\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1),(1+\alpha'+\beta-\mu-\eta-\sum_{j=1}^{K}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1;1) \tag{3.11}$$

Notes

$$\mathrm{B} = [(\mathrm{d}_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})]_{1,m^{(1)}}, [\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}, \delta_{ji^{(1)}}^{(1)}; D_{ji^{(1)}}^{(1)})]_{m^{(1)}+1,q_i^{(1)}}; \cdots ;$$

$$[(\mathrm{d}_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})]_{1,m^{(r)}}, [\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}, \delta_{ji^{(r)}}^{(r)}; D_{ji^{(r)}}^{(r)})]_{m^{(r)}+1,q_i^{(r)}},$$

$$[(\mathrm{d}'^{(1)}_j, \delta'^{(1)}_j; D'^{(1)}_j)]_{1,m'^{(1)}}, [\tau'_{i'^{(1)}}(d'^{(1)}_{ji'^{(1)}}, \delta'^{(1)}_{ji'^{(1)}}; D'^{(1)}_{ji'^{(1)}})]_{m'^{(1)}+1,q'^{(1)}_i}; \cdots ;$$

$$[(\mathrm{d}'^{(s)}_j, \delta'^{(s)}_j; D'^{(s)}_j)]_{1,m'^{(s)}}, [\tau'_{i'^{(s)}}(d'^{(s)}_{ji'^{(s)}}, \delta'^{(s)}_{ji'^{(s)}}; D'^{(s)}_{ji'(s)})]_{m'^{(s)}+1,q'^{(s)}_i}; . - . \qquad (3.12)$$

In our investigation, we will use these simplified notations cited above.

Provided

$$a, b, \alpha, \beta, \eta, \mu, \upsilon, \delta_k, \omega_i, \omega'_j \in \mathbb{C}, k = 1, \cdots, v; i = 1 \cdots, r; j = 1, \cdots, s$$

$$\lambda_k, \sigma_i, \sigma'_j > 0; \;\; k = 1, \cdots, v; i = 1 \cdots, r; j = 1, \cdots, s$$

$|arg\,(z_i)| < \frac{1}{2}\pi A_i^{(k)}$ and $A_i^{(k)}$ is defined by (2.4), $|arg\,(z'_i)| < \frac{1}{2}\pi A'^{(k)}_i; \left|\frac{a}{b}x\right| < 1$

$$Re(\mu) + \sum_{i=1}^{r} \sigma_i \min_{1 \leqslant j \leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] + \sum_{i=1}^{s} \sigma'_i \min_{1 \leqslant j \leqslant m'^{(i)}} Re\left[D'^{(i)}_j\left(\frac{d'^{(i)}_j}{\delta'^{(i)}_j}\right)\right] > max[0, Re(\alpha + \alpha' + \beta - \eta), Re(\alpha' - \beta')]$$

$$Re(\upsilon) + \sum_{i=1}^{r} \omega_i \min_{1 \leqslant j \leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] + \sum_{i=1}^{s} \omega'_i \min_{1 \leqslant j \leqslant m'^{(i)}} Re\left[D'^{(i)}_j\left(\frac{d'^{(i)}_j}{\delta'^{(i)}_j}\right)\right] > max[0, Re(\alpha + \alpha' + \beta - \eta), Re(\alpha' - \beta')]$$

Proof

To prove (3.1), we first express the class of multivariable polynomials $S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[y_1, \cdots, y_v]$ in series with the help of (2.13), the multivariable Gimel-functions regarding Mellin-Barnes type integrals contour with the help of (2.1). Now interchange the order of summations and two multiple Mellin-Barnes integrals contour, respectively and taking the fractional integral operator inside (which is permissible under the stated conditions) and make simplifications. Next, we express the terms $(b - ax)^{-\upsilon - \sum_{k=1}^{v} - \sum_{j=1}^{r} \omega s_i - \sum_{j=1}^{s} \omega' t_i}$ in terms of Mellin-Barnes integrals contour (Srivastava et al. [31], page 18, (2.6.3) and after algebraic manipulations, we obtain

$$\text{L.H.S} = b^{-\upsilon} \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u \leqslant L} y_1^{K_1} \cdots y_v^{K_v} c_1^{K_1} \cdots c_v^{K_v} b^{-\sum_{j=1}^{v} \delta_j K_j} \left(\frac{1}{(2\pi\omega}\right)^{r+s+1}$$

$$\int_{L_1} \cdots \int_{L_r} \int_{L'_1} \cdots \int_{L'_s} \psi(s_1, \cdots, s_r)\psi(t'_1, \cdots, t'_s) \prod_{k=1}^{r} \theta_k(s_k) z_k^{s_k} \prod_{j=1}^{s} \theta'_k(s_k) t_k^{t_k} \, b^{-\upsilon - \sum_{j=1}^{r} \omega s_i - \sum_{j=1}^{s} \omega' t_i}$$

$$\int_L \frac{\Gamma(\upsilon + \sum_{k=1}^{v} \delta_k K_k + \sum_{i=1}^{r} \omega_i s_i + \sum_{j=1}^{s} \omega'_i s t_i + u)}{\Gamma(\upsilon + \sum_{k=1}^{v} \delta_k K_k + \sum_{i=1}^{r} \omega_i s_i + \sum_{j=1}^{s} \omega'_i s t_i)\Gamma(1+u)} \left(-\frac{a}{b}\right)^u$$

$$\left(I_{0,x}^{\alpha,\alpha',\beta,\beta'\eta} t^{\mu + \sum_{k=1}^{v} \lambda_k K + \sum_{i=1}^{r} \sigma_i s_i + \sum_{i=1}^{r} \sigma'_i t_i}\right)(x) \mathrm{d}u \mathrm{d}s_1 \cdots \mathrm{d}s_r \mathrm{d}t_1 \cdots \mathrm{d}t_s$$

Now using the lemma 1. Finally interpreting the resulting Mellin-Barnes integrals contour as a multivariable Gimel-function of $(r + s + 1)$-variables, we obtain the desired result (3.1).

Let

Ref

31. H. M. Srivastava, K.C. Gupta and S. P. Goyal, The H-function of One and Two Variables with Applications, South Asian Publications, New Delhi, Madras, 1982. [32].

$$A_2 = (1 - \upsilon - \sum_{j=1}^{v} \lambda_j K_j; \sigma_1, \cdots, \sigma_r, \sigma'_1, \cdots, \sigma'_s, 1; 1), (\eta + \mu - \alpha - \alpha' + \sum_{j=1}^{K} \delta_j K_j; \omega_1, \cdots, \omega_r, \omega' 1, \cdots, \omega'_s, 1; 1)$$

$$(\beta + \mu + \sum_{j=1}^{v} \lambda_j K_j - \alpha - \alpha'; \omega_1, \cdots, \omega_r, \omega'_1, \cdots, \omega'_s, 1; 1), (\mu + \eta - \alpha - \beta' + \sum_{j=1}^{K} \delta_j K_j; \omega_1, \cdots, \omega_r, \omega'_1, \cdots, \omega'_s, 1; 1) \quad (3.13)$$

$$B_2 = (\mu + \sum_{j=1}^{v} \lambda_j K_j; \sigma_1, \cdots, \sigma_r, \sigma'_1, \cdots, \sigma'_s, 1; 1), (\beta + \mu - \alpha + \sum_{j=1}^{K} \delta_j K_j; \omega_1, \cdots, \omega_r, \omega' 1, \cdots, \omega'_s, 1; 1)$$

$$(1 - \mu - \beta' - \sum_{j=1}^{v} \lambda_j K_j; \sigma_1, \cdots, \sigma_r, \sigma'_1, \cdots, \sigma'_s, 1; 1), (1 + \alpha' + \beta - \mu - \eta - \sum_{j=1}^{K} \delta_j K_j; \omega_1, \cdots, \omega_r, \omega'_1, \cdots, \omega'_s, 1; 1) \quad (3.14)$$

We have the following resulting

Theorem 2.

$$\left\{ \left(I_{x,\infty}^{\alpha,\alpha',\beta,\beta',\eta} t^{\mu-1} (b-at)^{-\upsilon} S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v} \begin{pmatrix} \mathrm{c}_1 t^{\lambda_1} (b-at)^{-\delta_1} \\ . \\ . \\ . \\ \mathrm{c}_v t^{\lambda_v} (b-at)^{-\delta_v} \end{pmatrix} \gimel \begin{pmatrix} \mathrm{z}_1 t^{\sigma_1} (b-at)^{-\omega_1} \\ . \\ . \\ . \\ \mathrm{z}_r t^{\sigma_r} (b-at)^{-\omega_r} \end{pmatrix} \right.\right.$$

$$\left.\left. \gimel \begin{pmatrix} \mathrm{z}'_1 t^{\sigma'_1} (b-at)^{-\omega'_1} \\ . \\ . \\ . \\ \mathrm{z}'_s t^{\sigma'_s} (b-at)^{-\omega'_s} \end{pmatrix} \right)\right\} (x) = b^{-\upsilon} x^{\mu-\alpha-\alpha'+\eta-1} \sum_{K_1=1}^{[N_1/M_1]} \cdots \sum_{K_v=1}^{[N_v/M_v]} a_v c_1^{K_1} \cdots c_v^{K_v}$$

$$v^{-\sum_{j=1}^{v} \delta_i K_j} x^{\sum_{j=1}^{v} \lambda_i K_j} \gimel_{X; p_{i_r}+p'_{i'_s}+4, q_{i_r}+q'_{i'_s}+4, \tau_{i_r}, \tau'_{i'_s} : R_r : R'_s : Y; 0,1}^{U; 0, n_r+n'_s+4 : V, 1, 0} \left(\begin{array}{c|c} \mathrm{z}_1 \frac{x^{\sigma_1}}{b^{\omega_1}} & \\ . & \mathbb{A}, , \mathrm{A}_2, \mathbf{A} : A \\ . & . \\ . & . \\ \mathrm{z}_r \frac{x^{\sigma_r}}{b^{\omega_r}} & . \\ \mathrm{z}_1 \frac{x^{\sigma'_1}}{b^{\omega'_1}} & . \\ . & . \\ . & . \\ . & . \\ \mathrm{z}'_s \frac{x^{\sigma'_s}}{b^{\omega'_s}} & \mathbb{B}, \mathbf{B}, \mathrm{B}_2 : B, (0,1;1) \\ \frac{ax}{b} & \end{array} \right) \quad (3.14)$$

Provided

$a, b, \alpha, \beta, \eta, \mu, \upsilon, \delta_k, \omega_i, \omega'_j \in \mathbb{C}, k = 1, \cdots, v; i = 1 \cdots, r; j = 1, \cdots, s$

$\lambda_k, \sigma_i, \sigma'_j > 0; \quad k = 1, \cdots, v; i = 1 \cdots, r; j = 1, \cdots, s$

$|arg\,(z_i)| < \frac{1}{2}\pi A_i^{(k)}$ and $A_i^{(k)}$ is defined by (2.4), $|arg\,(z'_i)| < \frac{1}{2}\pi A_i'^{(k)}; \left|\frac{a}{b}x\right| < 1$

$$Re(\mu)-\sum_{i=1}^{r}\sigma_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]-\sum_{i=1}^{s}\sigma'_i \min_{1\leqslant j\leqslant m'^{(i)}} Re\left[D'^{(i)}_j\left(\frac{d'^{(i)}_j}{\delta'^{(i)}_j}\right)\right]<1+min[Re(-\beta),Re(\alpha+\alpha'-\eta),Re(\alpha+\beta'-\eta)]$$

$$Re(v)-\sum_{i=1}^{r}\omega_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]-\sum_{i=1}^{s}\omega'_i \min_{1\leqslant j\leqslant m'^{(i)}} Re\left[D'^{(i)}_j\left(\frac{d'^{(i)}_j}{\delta'^{(i)}_j}\right)\right]<1+max[Re(-\beta),Re(\alpha+\alpha'-\eta),Re(\alpha+\beta'-\eta)]$$

Notes

To prove the equation (3.14), we use the similar method that formula (3.5) by using the lemma 2.

Let

$$A_3=(1-\mu-\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1),=(1-v-\sum_{j=1}^{v}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1;1),$$

$$(1-\mu-\eta+\beta-\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1) \tag{3.15}$$

$$B_3=(1-v-\sum_{j=1}^{v}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,0;1),(1+\beta-\mu-\sum_{j=1}^{K}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1)$$

$$(1-\mu-\eta-\alpha-\sum_{j=1}^{K}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1) \tag{3.16}$$

Theorem 3.

$$\left\{\left(I_{0+}^{\alpha,\beta,\eta}t^{\mu-1}(b-at)^{-v}S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix}c_1t^{\lambda_1}(b-at)^{-\delta_1}\\ \cdot\\ \cdot\\ \cdot\\ c_vt^{\lambda_v}(b-at)^{-\delta_v}\end{pmatrix}\gimel\begin{pmatrix}z_1t^{\sigma_1}(b-at)^{-\omega_1}\\ \cdot\\ \cdot\\ \cdot\\ z_rt^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}\right.\right.$$

$$\left.\left.\gimel\begin{pmatrix}z'_1t^{\sigma'_1}(b-at)^{-\omega'_1}\\ \cdot\\ \cdot\\ \cdot\\ z'_st^{\sigma'_s}(b-at)^{-\omega'_s}\end{pmatrix}\right)\right\}(x)=b^{-v}x^{\mu-\alpha-\alpha'+\eta-1}\sum_{K_1=1}^{[N_1/M_1]}\cdots\sum_{K_v=1}^{[N_v/M_v]}a_vc_1^{K_1}\cdots c_v^{K_v}$$

$$v^{-\sum_{j=1}^{v}\delta_iK_j}x^{\sum_{j=1}^{v}\lambda_iK_j}\gimel_{X;p_{i_r}+p'_{i'_s}+3,q_{i_r}+q'_{i'_s}+3,\tau_{i_r},\tau'_{i'_s}:R_r:R'_s:Y;0,1}^{U;0,n_r+n'_s+3:V,1,0}\left(\begin{array}{c|c}z_1\frac{x^{\sigma_1}}{b^{\omega_1}} & \mathbb{A},\ ,\mathrm{A}_3,\mathbf{A}:A\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ z_r\frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot\\ z_1\frac{x^{\sigma'_1}}{b^{\omega'_1}} & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ z'_s\frac{x^{\sigma'_s}}{b^{\omega'_s}} & \\ \frac{ax}{b} & \mathbb{B},\mathbf{B},\ \mathrm{B}_3:B,(0,1;1)\end{array}\right) \tag{3.17}$$

Provided

$a,b,\alpha,\beta,\eta,\mu,v,\delta_k,\omega_i,\omega'_j\in\mathbb{C},k=1,\cdots,v;i=1\cdots,r;j=1,\cdots,s$

$\lambda_k,\sigma_i,\sigma'_j>0;\ \ k=1,\cdots,v;i=1\cdots,r;j=1,\cdots,s$

$|arg\,(z_i)| < \frac{1}{2}\pi A_i^{(k)}$ and $A_i^{(k)}$ is defined by (2.4), $|arg\,(z_i')| < \frac{1}{2}\pi A_i'^{(k)}; \left|\frac{a}{b}x\right| < 1$

$$Re(\mu) + \sum_{i=1}^{r} \sigma_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] + \sum_{i=1}^{s} \sigma_i' \min_{1\leqslant j\leqslant m'^{(i)}} Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right] > max[0, Re(\beta-\eta)]$$

$$Re(\upsilon) + \sum_{i=1}^{r} \omega_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] + \sum_{i=1}^{s} \omega_i' \min_{1\leqslant j\leqslant m'^{(i)}} Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right] > max[0, Re(\beta-\eta)]$$

To prove the formula (3.17), we use the similar method that the theorem 1 by using the lemma 3. Let

$$A_4 = (1-\mu-\sum_{j=1}^{v}\delta_j K_j;\omega_1,\cdots,\omega_r,\omega_1',\cdots,\omega_s',1;1),(-\eta+\mu+\sum_{j=1}^{v}\lambda_j K_j;\sigma_1,\cdots,\sigma_r,\sigma_1',\cdots,\sigma_s',1;1),$$

$$(1-\mu-\eta+\beta-\sum_{j=1}^{v}\lambda_j K_j;\sigma_1,\cdots,\sigma_r,\sigma_1',\cdots,\sigma_s',1;1) \tag{3.18}$$

$$B_4 = (-\upsilon-\sum_{j=1}^{v}\delta_j K_j;\eta_1,\cdots,\omega_r,\omega_1',\cdots,\omega_s',0;1),(\mu+\sum_{j=1}^{K}\lambda_j K_j;_1,\cdots,\sigma_r,\sigma_1',\cdots,\sigma_s',1;1)$$

$$(-\alpha-\beta-\eta+\mu+\sum_{j=1}^{K}\lambda_j K_j;\sigma_1,\cdots,\sigma_r,\sigma_1',\cdots,\sigma_s',1;1) \tag{3.19}$$

We have the formula.

Theorem 4.

$$\left\{\left(I_{x,\infty}^{\alpha,\beta,\eta} t^{\mu-1}(b-at)^{-\upsilon} S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix} c_1 t^{\lambda_1}(b-at)^{-\delta_1} \\ \cdot \\ \cdot \\ \cdot \\ c_v t^{\lambda_v}(b-at)^{-\delta_v}\end{pmatrix} \gimel \begin{pmatrix} z_1 t^{\sigma_1}(b-at)^{-\omega_1} \\ \cdot \\ \cdot \\ \cdot \\ z_r t^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}\right.\right.$$

$$\left.\left.\gimel\begin{pmatrix} z_1' t^{\sigma_1'}(b-at)^{-\omega_1'} \\ \cdot \\ \cdot \\ \cdot \\ z_s' t^{\sigma_s'}(b-at)^{-\omega_s'}\end{pmatrix}\right)\right\}(x) = b^{-\upsilon} x^{\mu-\alpha-\alpha'+\eta-1} \sum_{K_1=1}^{[N_1/M_1]} \cdots \sum_{K_v=1}^{[N_v/M_v]} a_v c_1^{K_1} \cdots c_v^{K_v}$$

$$\upsilon^{-\sum_{j=1}^{v}\delta_i K_j} x^{\sum_{j=1}^{v}\lambda_i K_j} \gimel_{X;p_{i_r}+p'_{i_s'}+3,q_{i_r}+q'_{i_s'}+3,\tau_{i_r},\tau'_{i_s'}:R_r:R_s':Y;0,1}^{U;0,n_r+n_s'+3:V,1,0}\left(\begin{array}{c|c} z_1\frac{x^{\sigma_1}}{b^{\omega_1}} & \mathbb{A},\,,\mathrm{A}_4,\mathbf{A}:A \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r\frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot \\ z_1\frac{x^{\sigma_1'}}{b^{\omega_1'}} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ z_s'\frac{x^{\sigma_s'}}{b^{\omega_s'}} & \mathbb{B},\mathbf{B},\,\mathrm{B}_4:B,(0,1;1) \\ \frac{ax}{b} & \end{array}\right) \tag{3.20}$$

Notes

Provided

$$a,b,\alpha,\beta,\eta,\mu,\upsilon,\delta_k,\omega_i,\omega'_j\in\mathbb{C},k=1,\cdots,v;i=1\cdots,r;j=1,\cdots,s$$

$$\lambda_k,\sigma_i,\sigma'_j>0;\;\;k=1,\cdots,v;i=1\cdots,r;j=1,\cdots,s$$

$|arg\,(z_i)|<\frac{1}{2}\pi A_i^{(k)}$ and $A_i^{(k)}$ is defined by (2.4), $|arg\,(z'_i)|<\frac{1}{2}\pi A_i'^{(k)};\left|\frac{a}{b}x\right|<1$

$$Re(\mu)-\sum_{i=1}^{r}\sigma_i\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]-\sum_{i=1}^{s}\sigma'_i\min_{1\leqslant j\leqslant m'^{(i)}}Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right]<1+min[Re(-\beta),Re(\alpha+\alpha'-\eta),Re(\alpha+\beta'-\eta)]$$

$$Re(\upsilon)-\sum_{i=1}^{r}\omega_i\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]-\sum_{i=1}^{s}\omega'_i\min_{1\leqslant j\leqslant m'^{(i)}}Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right]<1+max[Re(-\beta),Re(\alpha+\alpha'-\eta),Re(\alpha+\beta'-\eta)]$$

Provided

$$a,b,\alpha,\beta,\eta,\mu,\upsilon,\delta_k,\omega_i,\omega'_j\in\mathbb{C},k=1,\cdots,v;i=1\cdots,r;j=1,\cdots,s$$

$$\lambda_k,\sigma_i,\sigma'_j>0;\;\;k=1,\cdots,v;i=1\cdots,r;j=1,\cdots,s$$

$|arg\,(z_i)|<\frac{1}{2}\pi A_i^{(k)}$ and $A_i^{(k)}$ is defined by (2.4), $|arg\,(z'_i)|<\frac{1}{2}\pi A_i'^{(k)};\left|\frac{a}{b}x\right|<1$

$$Re(\mu)-\sum_{i=1}^{r}\sigma_i\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]-\sum_{i=1}^{s}\sigma'_i\min_{1\leqslant j\leqslant m'^{(i)}}Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right]<1+min[Re(Re(\eta),Re(\beta)]$$

$$Re(\upsilon)-\sum_{i=1}^{r}\omega_i\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]-\sum_{i=1}^{s}\omega'_i\min_{1\leqslant j\leqslant m'^{(i)}}Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right]<1+max[Re(\beta),Re(\eta)]$$

To prove the theorem 4, we use the similar method that the equation (3.5) by using the lemma 4.

IV. Particular Cases

In this section, we shall see four particular cases.

If we put $\beta=-\alpha$ in the theorem three, we get

Corollary 1.

$$\{\big(I_-^{\alpha}t^{\mu-1}(b-at)^{-\upsilon}S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix}c_1t^{\lambda_1}(b-at)^{-\delta_1}\\ \cdot\\ \cdot\\ \cdot\\ c_vt^{\lambda_v}(b-at)^{-\delta_v}\end{pmatrix}\gimel\begin{pmatrix}z_1t^{\sigma_1}(b-at)^{-\omega_1}\\ \cdot\\ \cdot\\ \cdot\\ z_rt^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}$$

$$\gimel\begin{pmatrix}z'_1t^{\sigma'_1}(b-at)^{-\omega'_1}\\ \cdot\\ \cdot\\ \cdot\\ z'_st^{\sigma'_s}(b-at)^{-\omega'_s}\end{pmatrix}\Bigg)\Bigg\}(x)=b^{-\upsilon}x^{\mu-\beta-1}\sum_{K_1=1}^{[N_1/M_1]}\cdots\sum_{K_v=1}^{[N_v/M_v]}a_vc_1^{K_1}\cdots c_v^{K_v}$$

$$v^{-\sum_{j=1}^{v}\delta_i K_j} x^{\sum_{j=1}^{v}\lambda_i K_j} \gimel^{U;0,n_r+n'_s+2:V,1,0}_{X;p_{i_r}+p'_{i'_s}+2,q_{i_r}+q'_{i'_s}+2,\tau_{i_r},\tau'_{i'_s}:R_r:R'_s:Y;0,1}\left(\begin{array}{c|c} \mathrm{z}_1\frac{x^{\sigma_1}}{b^{\omega_1}} & \mathbb{A}, ,\mathrm{A}_5,\mathbf{A}:A \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r\frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot \\ \mathrm{z}_1\frac{x^{\sigma'_1}}{b^{\omega'_1}} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}'_s\frac{x^{\sigma'_s}}{b^{\omega'_s}} & \mathbb{B},\mathbf{B},\mathrm{B}_5:B,(0,1;1) \\ \frac{ax}{b} & \end{array}\right) \tag{4.1}$$

where

$$A_5 = (1-v-\sum_{j=0}^{v}\delta_j K_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1;1),(1-\mu-\sum_{j=1}^{v}\lambda_j K_j,\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1) \tag{4.2}$$

$$A_5 = (1-v-\sum_{j=0}^{v}\delta_j K_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,0;1), B_5 = (1-\mu-\alpha-\eta-\sum_{j=1}^{v}\lambda_j K_j,\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1;1) \tag{4.3}$$

under the same existence conditions that formula (3.17) with. $\beta=-\alpha$.

If $\beta = 0$ in theorem three, we have

Corollary 2.

$$\left(I^{+}_{\eta,\alpha}\, t^{\mu-1}(b-at)^{-v} S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}_{N_1,\cdots,N_v}\begin{pmatrix} \mathrm{c}_1 t^{\lambda_1}(b-at)^{-\delta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{c}_v t^{\lambda_v}(b-at)^{-\delta_v} \end{pmatrix} \gimel \begin{pmatrix} \mathrm{z}_1 t^{\sigma_1}(b-at)^{-\omega_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r t^{\sigma_r}(b-at)^{-\omega_r} \end{pmatrix}\right.$$

$$\left.\gimel \begin{pmatrix} \mathrm{z}'_1 t^{\sigma'_1}(b-at)^{-\omega'_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}'_s t^{\sigma'_s}(b-at)^{-\omega'_s} \end{pmatrix}\right)\Bigg\}(x) = b^{-v} x^{\mu-\beta-1} \sum_{K_1=1}^{[N_1/M_1]} \cdots \sum_{K_v=1}^{[N_v/M_v]} a_v c_1^{K_1}\cdots c_v^{K_v}$$

$$v^{-\sum_{j=1}^{v}\delta_i K_j} x^{\sum_{j=1}^{v}\lambda_i K_j} \gimel^{U;0,n_r+n'_s+2:V,1,0}_{X;p_{i_r}+p'_{i'_s}+2,q_{i_r}+q'_{i'_s}+2,\tau_{i_r},\tau'_{i'_s}:R_r:R'_s:Y;0,1}\left(\begin{array}{c|c} \mathrm{z}_1\frac{x^{\sigma_1}}{b^{\omega_1}} & \mathbb{A}, ,\mathrm{A}_6,\mathbf{A}:A \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r\frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot \\ \mathrm{z}_1\frac{x^{\sigma'_1}}{b^{\omega'_1}} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}'_s\frac{x^{\sigma'_s}}{b^{\omega'_s}} & \mathbb{B},\mathbf{B},\mathrm{B}_6:B,(0,1;1) \\ \frac{ax}{b} & \end{array}\right) \tag{4.4}$$

Notes

Notes

where

$$A_6 = (1 - v - \sum_{j=0}^{v} \delta_j K_j; \omega_1, \cdots, \omega_r, \omega_1', \cdots, \omega_s', 0; 1), (1 - \mu - \eta - \sum_{j=1}^{v} \lambda_j K_j, \sigma_1, \cdots, \sigma_r, \sigma_1', \cdots, \sigma_s', 1; 1) \tag{4.5}$$

$$B_6 = (1 - v - \sum_{j=0}^{v} \delta_j K_j; \omega_1, \cdots, \omega_r, \omega_1', \cdots, \omega_s', 0; 1), (1 - \mu - \alpha - \eta - \sum_{j=1}^{v} \lambda_j K_j, \sigma_1, \cdots, \sigma_r, \sigma_1', \cdots, \sigma_s', 1; 1) \tag{4.6}$$

provided that

$a, b, \alpha, \beta, \eta, \mu, v, \delta_k, \omega_i, \omega_j' \in \mathbb{C}, k = 1, \cdots, v; i = 1 \cdots, r; j = 1, \cdots, s$

$\lambda_k, \sigma_i, \sigma_j' > 0; \;\; k = 1, \cdots, v; i = 1 \cdots, r; j = 1, \cdots, s$

$|arg\,(z_i)| < \frac{1}{2}\pi A_i^{(k)}$ and $A_i^{(k)}$ is defined by (2.4), $|arg\,(z_i')| < \frac{1}{2}\pi A_i'^{(k)}; \left|\frac{a}{b}x\right| < 1$

$$Re(\mu) + \sum_{i=1}^{r} \sigma_i \min_{1 \leqslant j \leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] + \sum_{i=1}^{s} \sigma_i' \min_{1 \leqslant j \leqslant m'^{(i)}} Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right] > max[0, Re(-\eta)]$$

$$Re(v) + \sum_{i=1}^{r} \omega_i \min_{1 \leqslant j \leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] + \sum_{i=1}^{s} \omega_i' \min_{1 \leqslant j \leqslant m'^{(i)}} Re\left[D_j'^{(i)}\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right] > max[0, Re(-\eta)]$$

If we put $\beta = -\alpha$ in the equation (3.20), we get

Corollary 3.

$$\left\{\left(I_-^{\alpha} t^{\mu-1}(b-at)^{-v} S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix} c_1 t^{\lambda_1}(b-at)^{-\delta_1} \\ . \\ . \\ . \\ c_v t^{\lambda_v}(b-at)^{-\delta_v}\end{pmatrix} \gimel \begin{pmatrix} z_1 t^{\sigma_1}(b-at)^{-\omega_1} \\ . \\ . \\ . \\ z_r t^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}\right.\right.$$

$$\left.\left. \gimel \begin{pmatrix} z_1' t^{\sigma_1'}(b-at)^{-\omega_1'} \\ . \\ . \\ . \\ z_s' t^{\sigma_s'}(b-at)^{-\omega_s'}\end{pmatrix}\right)\right\}(x) = b^{-v} x^{\mu+\alpha-1} \sum_{K_1=1}^{[N_1/M_1]} \cdots \sum_{K_v=1}^{[N_v/M_v]} a_v c_1^{K_1} \cdots c_v^{K_v}$$

$$v^{-\sum_{j=1}^{v} \delta_i K_j} x^{\sum_{j=1}^{v} \lambda_i K_j} \gimel_{X; p_{i_r}+p'_{i'_s}+2, q_{i_r}+q'_{i'_s}+2, \tau_{i_r}, \tau'_{i'_s}: R_r: R'_s: Y; 0,1}^{U; 0, n_r+n'_s+2: V, 1, 0} \left(\begin{array}{c|c} z_1 \frac{x^{\sigma_1}}{b^{\omega_1}} & \mathbb{A}, \; , \mathrm{A}_7, \mathbf{A} : A \\ . & . \\ . & . \\ . & . \\ z_r \frac{x^{\sigma_r}}{b^{\omega_r}} & . \\ z_1 \frac{x^{\sigma_1'}}{b^{\omega_1'}} & . \\ . & . \\ . & . \\ . & . \\ z_s' \frac{x^{\sigma_s'}}{b^{\omega_s'}} & \mathbb{B}, \mathbf{B}, \; \mathrm{B}_7 : B, (0,1;1) \\ \frac{ax}{b} & \end{array}\right) \tag{4.7}$$

where

$$A_7 = (1 - v - \sum_{j=0}^{v} \delta_j K_j; \omega_1, \cdots, \omega_r, \omega'_1, \cdots, \omega'_s, 1; 1), (\alpha + \mu + \sum_{j=1}^{v} \lambda_j K_j, \sigma_1, \cdots, \sigma_r, \sigma'_1, \cdots, \sigma'_s, 1; 1) \tag{4.8}$$

$$A_7 = (1 - v - \sum_{j=0}^{v} \delta_j K_j; \omega_1, \cdots, \omega_r, \omega'_1, \cdots, \omega'_s, 0; 1), B_5 = (\mu + \sum_{j=1}^{v} \lambda_j K_j, \sigma_1, \cdots, \sigma_r, \sigma'_1, \cdots, \sigma'_s, 1; 1) \tag{4.9}$$

under the same existence conditions that formula (3.20) with. $\beta = -\alpha$

If $\beta = 0$ in theorem four, we have

Corollary 4.

Let

$$A_8 = (1 - v - \sum_{j=0}^{v} \delta_j K_j; \omega_1, \cdots, \omega_r, \omega'_1, \cdots, \omega'_s, 1; 1), (-\alpha - \eta + \mu + \sum_{j=1}^{v} \lambda_j K_j, \sigma_1, \cdots, \sigma_r, \sigma'_1, \cdots, \sigma'_s, 1; 1) \tag{4.10}$$

$$B_8 = (1 - v - \sum_{j=0}^{v} \delta_j K_j; \omega_1, \cdots, \omega_r, \omega'_1, \cdots, \omega'_s, 0; 1), (\mu - \alpha - \eta + \sum_{j=1}^{v} \lambda_j K_j, \sigma_1, \cdots, \sigma_r, \sigma'_1, \cdots, \sigma'_s, 1; 1) \tag{4.11}$$

$$\{K^{-}_{\eta,\alpha} t^{\mu-1}(b-at)^{-v} S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}_{N_1,\cdots,N_v}\begin{pmatrix} \mathrm{c}_1 t^{\lambda_1}(b-at)^{-\delta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{c}_v t^{\lambda_v}(b-at)^{-\delta_v} \end{pmatrix} \beth \begin{pmatrix} \mathrm{z}_1 t^{\sigma_1}(b-at)^{-\omega_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r t^{\sigma_r}(b-at)^{-\omega_r} \end{pmatrix}$$

$$\beth \begin{pmatrix} \mathrm{z}'_1 t^{\sigma'_1}(b-at)^{-\omega'_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}'_s t^{\sigma'_s}(b-at)^{-\omega'_s} \end{pmatrix} \Bigg) \Bigg\}(x) = b^{-v} x^{\mu-1} \sum_{K_1=1}^{[N_1/M_1]} \cdots \sum_{K_v=1}^{[N_v/M_v]} a_v c_1^{K_1} \cdots c_v^{K_v}$$

$$v^{-\sum_{j=1}^{v} \delta_i K_j} x^{\sum_{j=1}^{v} \lambda_i K_j} \beth^{U;0,n_r+n'_s+2:V,1,0}_{X;p_{i_r}+p'_{i'_s}+2,q_{i_r}+q'_{i'_s}+2,\tau_{i_r},\tau'_{i'_s}:R_r:R'_s:Y;0,1} \left(\begin{array}{c|c} \mathrm{z}_1 \frac{x^{\sigma_1}}{b^{\omega_1}} & \mathbb{A}, ,\mathrm{A}_8, \mathbf{A} : A \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r \frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot \\ \mathrm{z}_1 \frac{x^{\sigma'_1}}{b^{\omega'_1}} & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}'_s \frac{x^{\sigma'_s}}{b^{\omega'_s}} & \mathbb{B}, \mathbf{B}, \mathrm{B}_8 : B, (0,1;1) \\ \frac{ax}{b} & \end{array} \right) \tag{4.12}$$

Provided

$a, b, \alpha, \beta, \eta, \mu, v, \delta_k, \omega_i, \omega'_j \in \mathbb{C}, k = 1, \cdots, v; i = 1 \cdots, r; j = 1, \cdots, s$

$\lambda_k, \sigma_i, \sigma'_j > 0; \;\; k = 1, \cdots, v; i = 1 \cdots, r; j = 1, \cdots, s$

Notes

under the same existence conditions that equation (3.20) with. $\beta = 0$.

Remark: By the similar procedure, the results of this document can be extended to the product of any finite number of multivariable Gimel-functions and a class of multivariable polynomials defined by Srivastava [30].
Agarwal [1,2] has studied the fractional integration about the multivariable H-function.

V. Conclusion

In this paper, we have obtained several theorems of the generalized fractional integral operators given by Saigo-Maeda and Saigo. The images have been developed regarding the product of the two multivariable Gimel-functions and a general class of multivariable polynomials in a compact and elegant form with the help of Saigo-Maeda and Saigo operators. Most of the results obtained in this paper are useful in deriving the composition formulae involving Riemann–Liouville, Erdelyi–Kober fractional calculus operators and multivariable Gimel functions. The findings of this paper provide an extension of the results given earlier by Kilbas [12], Kilbas and Saigo [13], Kilbas and Sebastain [14], Saxena et al.[29] and Gupta et al.[7] as mentioned earlier.

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Fourier Transform of Power Series

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Abstract- The authors establish a set of presumably new results, which provide Fourier transform of power series. So in this paper the author try to evaluate Fourier transform of some challenging functions by expressing them as a sum of infinitely terms. Hence, the method is useful to find the Fourier transform of functions that difficult to obtain their Fourier transform by ordinary method or using definition of Fourier transformations.

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Fourier Transform of Power Series

Shiferaw Geremew Kebede [α], Awel Seid Gelete [σ], Dereje Legesse Abaire [ρ] & Mekonnen Gudeta Gizaw [ω]

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Keywords: fourier transforms, power series, taylor's and maclaurin series and gamma function.

I. Introduction

The Fourier transform is one of the most important integral transforms. Be-cause of a number of special properties, it is very useful in studying linear differential equations.

Fourier analysis has its most important applications in mathematical modeling, physical and engineering and solving partial differential equations (PDEs) related to boundary and initial value problems of Mechanics, heat flow, electro statistics and other fields. Daniel Bernoulli (1700-1782) and Leonhard Euler (1707-1783), Swiss mathematicians, and Jean-Baptiste D Alembert (1717-1783), a French mathematician, physicist, philosopher, and music theorist, were all prominent in the ensuing mathematical music debate. In 1751, Bernoullis memoir of 1741-1743 took Rameaus findings into account, and in 1752, DAlembert published Elements of theoretical and practical music according to the principals of Monsieur Rameau, clarified, developed, and simplified. DAlembert was also led to a differential equation from Taylors problem of the vibrating string,

$$\frac{\partial^2 y}{\partial x^2} = \alpha^2 \frac{\partial^2 y}{\partial^2 t^2}$$

The current widespread use of the transform (mainly in engineering) came about during and soon after World War II, although it had been used in the 19th century by Abel, Lerch, Heaviside, and Bromwich.

Joseph Fourier's method of Fourier series for solving the diffusion equation could only apply to a limited region of space because those solutions were periodic. In 1809, Laplace applied his transform to find solutions that diffused indefinitely in space.

a) Definition

The Fourier transform of the function $f(x)$ is given by:

$$F(f(x)) = \frac{1}{\sqrt{2\Pi}} \int_0^\infty f(x) e^{-ix\omega} dx$$

b) Definition

A Power series is a series defined of the form:

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$$f(x) = \Sigma_{n=0}^{\infty} a_n (x-c)^n$$

$$= a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + ... + a_n(x-c)^n$$

where c is any constant $c \in \Re$

c) Definition

If $f(x)$ has a power series expansion at c,where c is any constant $c \in \Re$. It's Taylor's series expansion is:

$$f(x) = \Sigma_{n=0}^{\infty} a_n f^{(n)}(c) \frac{(x-c)^n}{n!}$$

d) Definition

Maclaurin Series expansion of the function $f(x)$ is:

$$f(x) = \Sigma_{n=0}^{\infty} f^{(n)}(0) \frac{x^n}{n!}$$

$$= f(0) + f^{'}(0)x + \frac{f^{''}(0)}{2!}x^2 + \frac{f^{'''}(0)}{3!}x^3 + ...$$

e) Definition

The gamma function, whose symbol $\Gamma(s)$ is defined when $s > 0$ by the formula

$$\Gamma = \int_0^{\infty} e^{-x} x^{n-1} dx$$

II. Fourier Transform of Power Series

Theorem 1: (Fourier Transform of power series)

If$f(x)$ has a Power series expansion at c,where c is any constant $c \in \Re$.

$$f(x) = \Sigma_{n=0}^{\infty} a_n (x-c)^n$$

then the Fourier transform of $f(x)$ is given in the form of power series as:

$$F(f(x)) = F(\Sigma_{n=0}^{\infty} a_n (x-c)^n)$$

$$= \frac{1}{\sqrt{2\Pi} e^{ic\omega}} \Sigma_{n=0}^{\infty} a_n \frac{1}{(i\omega)^{n+1}} \frac{\Gamma(n+1)}{s^{n+1}}$$

Proof

Suppose $f(x)$ has a Power series expansion at c,where c is any constant $c \in \Re$.

i.e

$$f(x) = \Sigma_{n=0}^{\infty} a_n (x-c)^n$$

Then, By using the definition of Fourier transforms,

Notes

$$F(f(x)) = F(\Sigma_{n=0}^{\infty} a_n (x-c)^n)$$

$$= \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{\infty} [\Sigma_{n=0}^{\infty} a_n (x-c)^n] e^{-ix\omega} dx$$

$$= \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{\infty} \Sigma_{n=0}^{\infty} a_n (x-c)^n e^{-ix\omega} dx$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_{n=0}^{\infty} \int_{-\infty}^{\infty} a_n e^{-ix\omega} (x-c)^n dx$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_0^{\infty} a_n \int_{-\infty}^{\infty} e^{-ix\omega} (x-c)^n dx$$

Let,$x = t + c \Longleftrightarrow dx = dt$
So,

$$F(f(x)) = F(\Sigma_{n=0}^{\infty} a_n (x-c)^n)$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_0^{\infty} a_n \int_{-\infty}^{\infty} e^{-i(t+c)\omega} t^n dt$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_0^{\infty} a_n \int_{-\infty}^{\infty} e^{-it\omega} e^{-ic\omega} t^n dt$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_0^{\infty} a_n e^{-ic\omega} \int_{-\infty}^{\infty} e^{-it\omega} t^n dt$$

Let, $v = it\omega \Longleftrightarrow t = \frac{v}{i\omega} \Rightarrow dt = \frac{1}{i\omega} dv$
Hence,

$$F(f(x)) = \frac{1}{\sqrt{2\Pi}} \Sigma_0^{\infty} a_n e^{-ic\omega} \int_{-\infty}^{\infty} e^{-v} [\frac{v}{i\omega}]^n \frac{1}{i\omega} dv$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_0^{\infty} a_n e^{-ic\omega} \int_{-\infty}^{\infty} e^{-v} \frac{v^n}{(i\omega)^n} \frac{1}{i\omega} dv$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_0^{\infty} a_n \frac{1}{[i\omega]^{n+1}} e^{-ic\omega} \int_{-\infty}^{\infty} e^{-v} v^n dv$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_0^{\infty} a_n \frac{1}{[i\omega]^{n+1}} e^{-ic\omega} [2 \int_{0}^{\infty} e^{-v} v^n dv]$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_0^{\infty} a_n \frac{1}{[i\omega]^{n+1}} e^{-ic\omega} [2 \frac{\Gamma(n+1)}{s^{n+1}}]$$

$$= \frac{2}{\sqrt{2\Pi} e^{ic\omega}} \Sigma_0^{\infty} a_n \frac{1}{[i\omega]^{n+1}} \frac{\Gamma(n+1)}{s^{n+1}}$$

In particular, for $n = 1, 2, 3, ...$

$$\Gamma(n+1) = n!$$

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Such that,

$$F(\Sigma_{n=0}^{\infty}a_n(x-c)^n) = \frac{2}{\sqrt{2\Pi}e^{ic\omega}}\Sigma_0^{\infty}a_n\frac{1}{[i\omega]^{n+1}}\frac{n!}{s^{n+1}}$$

Theorem 2:) (Fourier Transform of Taylor's Series)
If $f(x)$ has a power series expansion at c,where c is any constant $c \in \Re$. It's Taylor's series expansion is:

$$f(x) = \Sigma_{n=0}^{\infty}a_n f^{(n)}(c)\frac{(x-c)^n}{n!}$$

then the Fourier transform of $f(x)$ is given in the form of power series as:

$$F(f(x)) = F[\Sigma_{n=0}^{\infty}a_n f^{(n)}(c)\frac{(x-c)^n}{n!}]$$

$$= \frac{2}{\sqrt{2\Pi}e^{ic\omega}}f^{(n)}(c)\Sigma_{n=0}^{\infty}a_n\frac{1}{n!(i\omega)^{n+1}}\frac{\Gamma(n+1)}{s^{n+1}}$$

Proof

Suppose $f(x)$ has a Power series expansion at c,where c is any constant $c \in \Re$.
Hence, the Taylor's series expansion of $f(x)$ is:

$$f(x) = \Sigma_{n=0}^{\infty}a_n f^{(n)}(c)\frac{(x-c)^n}{n!}$$

Then, By using the definition of Fourier transforms,

$$F(f(x)) = F(\Sigma_{n=0}^{\infty}a_n f^{(n)}(c)\frac{(x-c)^n}{n!})$$

$$= \frac{1}{\sqrt{2\Pi}}\int_{-\infty}^{\infty}[\Sigma_{n=0}^{\infty}a_n f^{(n)}(c)\frac{(x-c)^n}{n!}]e^{-ix\omega}dx$$

$$= \frac{1}{\sqrt{2\Pi}}\int_{-\infty}^{\infty}\Sigma_{n=0}^{\infty}a_n f^{(n)}(c)\frac{(x-c)^n}{n!}e^{-ix\omega}dx$$

$$= \frac{1}{\sqrt{2\Pi}}\Sigma_{n=0}^{\infty}\int_{-\infty}^{\infty}a_n f^{(n)}(c)\frac{1}{n!}e^{-ix\omega}(x-c)^n dx$$

$$= \frac{1}{\sqrt{2\Pi}}\Sigma_0^{\infty}a_n f^{(n)}(c)\frac{1}{n!}\int_{-\infty}^{\infty}e^{-ix\omega}(x-c)^n dx$$

Let,$x = t + c \Longleftrightarrow dx = dt$
So,

$$F(f(x)) = F(\Sigma_{n=0}^{\infty}a_n f^{(n)}(c)\frac{(x-c)^n}{n!}$$

Notes

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_0^\infty a_n f^{(n)}(c) \frac{1}{n!} \int_{-\infty}^{\infty} e^{-i(t+c)\omega} t^n dt$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_0^\infty a_n f^{(n)}(c) \frac{1}{n!} \int_{-\infty}^{\infty} e^{-it\omega} e^{-ic\omega} t^n dt$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_0^\infty a_n f^{(n)}(c) \frac{1}{n!} e^{-ic\omega} \int_{-\infty}^{\infty} e^{-it\omega} t^n dt$$

Notes

Let, $v = it\omega \iff t = \frac{v}{i\omega} \Rightarrow dt = \frac{1}{i\omega} dv$
Hence,

$$F(f(x)) = \frac{1}{\sqrt{2\Pi}} \Sigma_0^\infty a_n f^{(n)}(c) \frac{1}{n!} e^{-ic\omega} \int_{-\infty}^{\infty} e^{-v} [\frac{v}{i\omega}]^n \frac{1}{i\omega} dv$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_0^\infty a_n f^{(n)}(c) \frac{1}{n!} e^{-ic\omega} \int_{-\infty}^{\infty} e^{-v} \frac{v^n}{(i\omega)^n} \frac{1}{i\omega} dv$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_0^\infty a_n f^{(n)}(c) \frac{1}{n!} \frac{1}{[i\omega]^{n+1}} e^{-ic\omega} \int_{-\infty}^{\infty} e^{-v} v^n dv$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_0^\infty a_n f^{(n)}(c) \frac{1}{n!} \frac{1}{[i\omega]^{n+1}} e^{-ic\omega} [2 \int_0^{\infty} e^{-v} v^n dv]$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_0^\infty a_n f^{(n)}(c) \frac{1}{n!} \frac{1}{[i\omega]^{n+1}} e^{-ic\omega} [2 \frac{\Gamma(n+1)}{s^{n+1}}]$$

$$= \frac{2}{\sqrt{2\Pi} e^{ic\omega}} \Sigma_0^\infty a_n f^{(n)}(c) \frac{1}{n!} \frac{1}{[i\omega]^{n+1}} \frac{\Gamma(n+1)}{s^{n+1}}$$

In particular, for $n = 1, 2, 3, ...$

$$\Gamma(n+1) = n!$$

Such that,

$$F(\Sigma_{n=0}^\infty a_n f^{(n)}(c) \frac{(x-c)^n}{n!}) = \frac{2}{\sqrt{2\Pi} e^{ic\omega}} \Sigma_0^\infty a_n f^{(n)}(c) \frac{1}{n!} \frac{1}{[i\omega]^{n+1}} \frac{n!}{s^{n+1}}$$

Theorem 3:) (Fourier Transform of Maclaurin Series)
In particular if $f(x)$ has a power series expansion at 0, then, the power series expansion of f(x) is given by:

$$f(x) = \Sigma_{n=0}^\infty a_n x^n$$

which is known as Maclaurin series, then the Fourier transform of $f(x)$ is defined by:

$$F(f(x)) = F(\Sigma_{n=0}^\infty a_n x^n)$$

$$= \frac{2}{\sqrt{2\Pi}} f^{(n)}(c) \Sigma_{n=0}^\infty a_n \frac{1}{(i\omega)^{n+1}} \frac{\Gamma(n+1)}{s^{n+1}}$$

proof

suppose $f(x)$ has the power series expansion at 0
i.e

$$f(x) = \Sigma_{n=0}^{\infty} a_n x^n$$

By using the definition of Fourier transforms,

$$F(f(x)) = F(\Sigma_{n=0}^{\infty} a_n x^n)$$

$$= \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{\infty} [\Sigma_{n=0}^{\infty} a_n x^n] e^{-ix\omega} dx$$

$$= \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{\infty} \Sigma_{n=0}^{\infty} a_n x^n e^{-ix\omega} dx$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_{n=0}^{\infty} \int_{-\infty}^{\infty} a_n e^{-ix\omega} x^n dx$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_{0}^{\infty} a_n \int_{-\infty}^{\infty} e^{-ix\omega} x^n dx$$

Let, $t = ix\omega \Longleftrightarrow x = \frac{t}{i\omega} \Rightarrow dx = \frac{1}{i\omega} dt$ Hence,

$$F(f(x)) = \frac{1}{\sqrt{2\Pi}} \Sigma_{0}^{\infty} a_n \int_{-\infty}^{\infty} e^{-t} [\frac{t}{i\omega}]^n \frac{1}{i\omega} dt$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_{0}^{\infty} a_n \int_{-\infty}^{\infty} e^{-t} \frac{t^n}{(i\omega)^n} \frac{1}{i\omega} dt$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_{0}^{\infty} a_n \frac{1}{[i\omega]^{n+1}} \int_{-\infty}^{\infty} e^{-t} t^n dt$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_{0}^{\infty} a_n \frac{1}{[i\omega]^{n+1}} [2 \int_{0}^{\infty} e^{-t} t^n dt]$$

$$= \frac{1}{\sqrt{2\Pi}} \Sigma_{0}^{\infty} a_n \frac{1}{[i\omega]^{n+1}} [2 \frac{\Gamma(n+1)}{s^{n+1}}]$$

$$= \frac{2}{\sqrt{2\Pi}} \Sigma_{0}^{\infty} a_n \frac{1}{[i\omega]^{n+1}} \frac{\Gamma(n+1)}{s^{n+1}}$$

Note: In particular, for $n = 1, 2, 3,$

$$\Gamma(n+1) = n!$$

Hence,

$$F(\Sigma_{n=0}^{\infty} a_n x^n) = \frac{2}{\sqrt{2\Pi}} \Sigma_{0}^{\infty} a_n \frac{1}{[i\omega]^{n+1}} \frac{n!}{s^{n+1}}$$

Notes

III. Conclusion

The results on Fourier transform of power series are summarized as follows;

Some functions like e^{t^2}, $\frac{sint}{t}$ and son on are difficult to get their Fourier transform. Hence it is possible to find Fourier transform such functions by expanding them into power series, Taylor's series and Maclaurin series form as:

Notes

$$F(f(x)) = \frac{2}{\sqrt{2\Pi}e^{ic\omega}}\Sigma_0^\infty a_n \frac{1}{[i\omega]^{n+1}}\frac{\Gamma(n+1)}{s^{n+1}}$$

where

$$f(x) = \Sigma_{n=0}^\infty a_n(x-c)^n, c \in \Re$$

2.

$$F(f(x)) = \frac{2}{\sqrt{2\Pi}e^{ic\omega}}\Sigma_0^\infty a_n f^{(n)}(c)\frac{1}{n!}\frac{1}{[i\omega]^{n+1}}\frac{\Gamma(n+1)}{s^{n+1}}$$

where

$$f(x) = \Sigma_{n=0}^\infty a_n f^{(n)}(c)\frac{(x-c)^n}{n!}$$

3.

$$F(f(x)) = \frac{2}{\sqrt{2\Pi}}\Sigma_0^\infty a_n \frac{1}{[i\omega]^{n+1}}\frac{\Gamma(n+1)}{s^{n+1}}$$

where

$$f(x) = \Sigma_{n=0}^\infty a_n t^n$$

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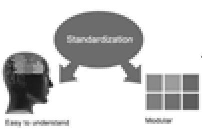

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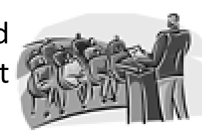

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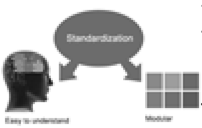

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- The Fellow can organize symposium/seminar/conference on behalf of Global Journals Incorporation (USA) and he/she can also attend the same organized by other institutes on behalf of Global Journals.
- The Fellow can become member of Editorial Board Member after completing 3yrs.
- The Fellow can earn 60% of sales proceeds from the sale of reference/review books/literature/publishing of research paper.
- Fellow can also join as paid peer reviewer and earn 15% remuneration of author charges and can also get an opportunity to join as member of the Editorial Board of Global Journals Incorporation (USA)
- • This individual has learned the basic methods of applying those concepts and techniques to common challenging situations. This individual has further demonstrated an in–depth understanding of the application of suitable techniques to a particular area of research practice.

Note :

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- In future, if the board feels the necessity to change any board member, the same can be done with the consent of the chairperson along with anyone board member without our approval.
- In case, the chairperson needs to be replaced then consent of 2/3rd board members are required and they are also required to jointly pass the resolution copy of which should be sent to us. In such case, it will be compulsory to obtain our approval before replacement.
- In case of "Difference of Opinion [if any]" among the Board members, our decision will be final and binding to everyone. "

Preferred Author Guidelines

We accept the manuscript submissions in any standard (generic) format.

We typeset manuscripts using advanced typesetting tools like Adobe In Design, CorelDraw, TeXnicCenter, and TeXStudio. We usually recommend authors submit their research using any standard format they are comfortable with, and let Global Journals do the rest.

Alternatively, you can download our basic template from https://globaljournals.org/Template.zip

Authors should submit their complete paper/article, including text illustrations, graphics, conclusions, artwork, and tables. Authors who are not able to submit manuscript using the form above can email the manuscript department at submit@globaljournals.org or get in touch with chiefeditor@globaljournals.org if they wish to send the abstract before submission.

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Authors must ensure the information provided during the submission of a paper is authentic. Please go through the following checklist before submitting:

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2. Authors must accept the privacy policy, terms, and conditions of Global Journals.
3. Ensure corresponding author's email address and postal address are accurate and reachable.
4. Manuscript to be submitted must include keywords, an abstract, a paper title, co-author(s') names and details (email address, name, phone number, and institution), figures and illustrations in vector format including appropriate captions, tables, including titles and footnotes, a conclusion, results, acknowledgments and references.
5. Authors should submit paper in a ZIP archive if any supplementary files are required along with the paper.
6. Proper permissions must be acquired for the use of any copyrighted material.
7. Manuscript submitted *must not have been submitted or published elsewhere* and all authors must be aware of the submission.

Declaration of Conflicts of Interest

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- Findings
- Writings
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- Printed material
- Graphic representations
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2. Drafting the paper and revising it critically regarding important academic content.
3. Final approval of the version of the paper to be published.

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The corresponding author should mention the name and complete details of all co-authors during submission and in manuscript. We support addition, rearrangement, manipulation, and deletions in authors list till the early view publication of the journal. We expect that corresponding author will notify all co-authors of submission. We follow COPE guidelines for changes in authorship.

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Unless specified in the notification, the Editorial Board's decision on publication of the paper is final and cannot be appealed before making the major change in the manuscript.

Acknowledgments

Contributors to the research other than authors credited should be mentioned in Acknowledgments. The source of funding for the research can be included. Suppliers of resources may be mentioned along with their addresses.

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Preparing your Manuscript

Authors can submit papers and articles in an acceptable file format: MS Word (doc, docx), LaTeX (.tex, .zip or .rar including all of your files), Adobe PDF (.pdf), rich text format (.rtf), simple text document (.txt), Open Document Text (.odt), and Apple Pages (.pages). Our professional layout editors will format the entire paper according to our official guidelines. This is one of the highlights of publishing with Global Journals—authors should not be concerned about the formatting of their paper. Global Journals accepts articles and manuscripts in every major language, be it Spanish, Chinese, Japanese, Portuguese, Russian, French, German, Dutch, Italian, Greek, or any other national language, but the title, subtitle, and abstract should be in English. This will facilitate indexing and the pre-peer review process.

The following is the official style and template developed for publication of a research paper. Authors are not required to follow this style during the submission of the paper. It is just for reference purposes.

Manuscript Style Instruction (Optional)

- Microsoft Word Document Setting Instructions.
- Font type of all text should be Swis721 Lt BT.
- Page size: 8.27" x 11'", left margin: 0.65, right margin: 0.65, bottom margin: 0.75.
- Paper title should be in one column of font size 24.
- Author name in font size of 11 in one column.
- Abstract: font size 9 with the word "Abstract" in bold italics.
- Main text: font size 10 with two justified columns.
- Two columns with equal column width of 3.38 and spacing of 0.2.
- First character must be three lines drop-capped.
- The paragraph before spacing of 1 pt and after of 0 pt.
- Line spacing of 1 pt.
- Large images must be in one column.
- The names of first main headings (Heading 1) must be in Roman font, capital letters, and font size of 10.
- The names of second main headings (Heading 2) must not include numbers and must be in italics with a font size of 10.

Structure and Format of Manuscript

The recommended size of an original research paper is under 15,000 words and review papers under 7,000 words. Research articles should be less than 10,000 words. Research papers are usually longer than review papers. Review papers are reports of significant research (typically less than 7,000 words, including tables, figures, and references)

A research paper must include:

a) A title which should be relevant to the theme of the paper.
b) A summary, known as an abstract (less than 150 words), containing the major results and conclusions.
c) Up to 10 keywords that precisely identify the paper's subject, purpose, and focus.
d) An introduction, giving fundamental background objectives.
e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition, sources of information must be given, and numerical methods must be specified by reference.
f) Results which should be presented concisely by well-designed tables and figures.
g) Suitable statistical data should also be given.
h) All data must have been gathered with attention to numerical detail in the planning stage.

Design has been recognized to be essential to experiments for a considerable time, and the editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned unrefereed.

i) Discussion should cover implications and consequences and not just recapitulate the results; conclusions should also be summarized.
j) There should be brief acknowledgments.
k) There ought to be references in the conventional format. Global Journals recommends APA format.

Authors should carefully consider the preparation of papers to ensure that they communicate effectively. Papers are much more likely to be accepted if they are carefully designed and laid out, contain few or no errors, are summarizing, and follow instructions. They will also be published with much fewer delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and suggestions to improve brevity.

Format Structure

It is necessary that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

All manuscripts submitted to Global Journals should include:

Title

The title page must carry an informative title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) where the work was carried out.

Author details

The full postal address of any related author(s) must be specified.

Abstract

The abstract is the foundation of the research paper. It should be clear and concise and must contain the objective of the paper and inferences drawn. It is advised to not include big mathematical equations or complicated jargon.

Many researchers searching for information online will use search engines such as Google, Yahoo or others. By optimizing your paper for search engines, you will amplify the chance of someone finding it. In turn, this will make it more likely to be viewed and cited in further works. Global Journals has compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Keywords

A major lynchpin of research work for the writing of research papers is the keyword search, which one will employ to find both library and internet resources. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining, and indexing.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy: planning of a list of possible keywords and phrases to try.

Choice of the main keywords is the first tool of writing a research paper. Research paper writing is an art. Keyword search should be as strategic as possible.

One should start brainstorming lists of potential keywords before even beginning searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in a research paper?" Then consider synonyms for the important words.

It may take the discovery of only one important paper to steer in the right keyword direction because, in most databases, the keywords under which a research paper is abstracted are listed with the paper.

Numerical Methods

Numerical methods used should be transparent and, where appropriate, supported by references.

Abbreviations

Authors must list all the abbreviations used in the paper at the end of the paper or in a separate table before using them.

Formulas and equations

Authors are advised to submit any mathematical equation using either MathJax, KaTeX, or LaTeX, or in a very high-quality image.

Tables, Figures, and Figure Legends

Tables: Tables should be cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g., Table 4, a self-explanatory caption, and be on a separate sheet. Authors must submit tables in an editable format and not as images. References to these tables (if any) must be mentioned accurately.

Figures

Figures are supposed to be submitted as separate files. Always include a citation in the text for each figure using Arabic numbers, e.g., Fig. 4. Artwork must be submitted online in vector electronic form or by emailing it.

Preparation of Eletronic Figures for Publication

Although low-quality images are sufficient for review purposes, print publication requires high-quality images to prevent the final product being blurred or fuzzy. Submit (possibly by e-mail) EPS (line art) or TIFF (halftone/ photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Avoid using pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings). Please give the data for figures in black and white or submit a Color Work Agreement form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution at final image size ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs): >350 dpi; figures containing both halftone and line images: >650 dpi.

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Tips for Writing a Good Quality Science Frontier Research Paper

Techniques for writing a good quality Science Frontier Research paper:

1. Choosing the topic: In most cases, the topic is selected by the interests of the author, but it can also be suggested by the guides. You can have several topics, and then judge which you are most comfortable with. This may be done by asking several questions of yourself, like "Will I be able to carry out a search in this area? Will I find all necessary resources to accomplish the search? Will I be able to find all information in this field area?" If the answer to this type of question is "yes," then you ought to choose that topic. In most cases, you may have to conduct surveys and visit several places. Also, you might have to do a lot of work to find all the rises and falls of the various data on that subject. Sometimes, detailed information plays a vital role, instead of short information. Evaluators are human: The first thing to remember is that evaluators are also human beings. They are not only meant for rejecting a paper. They are here to evaluate your paper. So present your best aspect.

2. Think like evaluators: If you are in confusion or getting demotivated because your paper may not be accepted by the evaluators, then think, and try to evaluate your paper like an evaluator. Try to understand what an evaluator wants in your research paper, and you will automatically have your answer. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

3. Ask your guides: If you are having any difficulty with your research, then do not hesitate to share your difficulty with your guide (if you have one). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work, then ask your supervisor to help you with an alternative. He or she might also provide you with a list of essential readings.

4. Use of computer is recommended: As you are doing research in the field of science frontier then this point is quite obvious. Use right software: Always use good quality software packages. If you are not capable of judging good software, then you can lose the quality of your paper unknowingly. There are various programs available to help you which you can get through the internet.

5. Use the internet for help: An excellent start for your paper is using Google. It is a wondrous search engine, where you can have your doubts resolved. You may also read some answers for the frequent question of how to write your research paper or find a model research paper. You can download books from the internet. If you have all the required books, place importance on reading, selecting, and analyzing the specified information. Then sketch out your research paper. Use big pictures: You may use encyclopedias like Wikipedia to get pictures with the best resolution. At Global Journals, you should strictly follow here.

6. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right? It is a good habit which helps to not lose your continuity. You should always use bookmarks while searching on the internet also, which will make your search easier.

7. Revise what you wrote: When you write anything, always read it, summarize it, and then finalize it.

8. Make every effort: Make every effort to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in the introduction—what is the need for a particular research paper. Polish your work with good writing skills and always give an evaluator what he wants. Make backups: When you are going to do any important thing like making a research paper, you should always have backup copies of it either on your computer or on paper. This protects you from losing any portion of your important data.

9. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several unnecessary diagrams will degrade the quality of your paper by creating a hodgepodge. So always try to include diagrams which were made by you to improve the readability of your paper. Use of direct quotes: When you do research relevant to literature, history, or current affairs, then use of quotes becomes essential, but if the study is relevant to science, use of quotes is not preferable.

10. Use proper verb tense: Use proper verb tenses in your paper. Use past tense to present those events that have happened. Use present tense to indicate events that are going on. Use future tense to indicate events that will happen in the future. Use of wrong tenses will confuse the evaluator. Avoid sentences that are incomplete.

11. Pick a good study spot: Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

12. Know what you know: Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

13. Use good grammar: Always use good grammar and words that will have a positive impact on the evaluator; use of good vocabulary does not mean using tough words which the evaluator has to find in a dictionary. Do not fragment sentences. Eliminate one-word sentences. Do not ever use a big word when a smaller one would suffice.

Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

14. Arrangement of information: Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

15. Never start at the last minute: Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

16. Multitasking in research is not good: Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

17. Never copy others' work: Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

18. Go to seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

19. Refresh your mind after intervals: Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.

20. Think technically: Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

21. Adding unnecessary information: Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

22. Report concluded results: Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

23. Upon conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

The introduction: This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear: Adhere to recommended page limits.

Mistakes to avoid:

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

Title page:

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

Abstract: This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

Approach:

- o Single section and succinct.
- o An outline of the job done is always written in past tense.
- o Concentrate on shortening results—limit background information to a verdict or two.
- o Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

Introduction:

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.

The following approach can create a valuable beginning:

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

Approach:

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

Procedures (methods and materials):

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

Materials may be reported in part of a section or else they may be recognized along with your measures.

Methods:

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

What to stay away from:

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

Figures and tables:

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

Discussion:

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- o You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- o Give details of all of your remarks as much as possible, focusing on mechanisms.
- o Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- o One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- o Recommendations for detailed papers will offer supplementary suggestions.

Approach:

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References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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