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Economic Determinants Affecting Military Expenditures: Panel Data Analysis

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Economic Determinants Affecting Military Expenditures: Panel Data Analysis

Mahmoud Mourad [α] & Bilal Nehme [σ]

Abstract- Our research revolves around the topic of considering the military expenditures per capita as a dependent variable and the GDP per capita and CO2 Emissions per capita as two explanatory variables. The study is made up of ten sections addressing several points, each of which clarifies the research method in order to reach a conclusion revealing the importance of the findings. Beginning with the basic statistical characteristics, such as averages, standard deviations, minimums, maximums and the Compound Annual Growth Rate (CAGR), a benefit use of the graph of each variable for each country has been highlighted for a better understanding of the rising and falling during its temporal evolution. The various aspects of the panel analysis have been completed as the questions of individual specific heterogeneity in panel data, the panel unit root tests using the most famous from the first and second generations, and the co-integration analysis according to the Pedroni's approach, which has led to the rejection of the null hypothesis of no co-integration for each country and for the group as a whole. The long-run equilibrium relationships are carried out using both the FMOLS and the DOLS estimators. The performance of these relationships has been measured over the period 2014−2017 by considering a linear adjustment that links the forecast and the observed values associated with the ten countries of linear correlation coefficients as positive and near to one. This research attempts to deal with a point considered to be innovative that consists of using Principal Component Analysis (PCA) applied to the residuals of the ADF equations used in the panel unit root tests. In this respect, an algorithm is being proposed showing the importance of a certain ordering of countries which could be informative on the degree of heterogeneity of the panel vis-à-vis the masked factors of military expenditures.

This link between PCA is considered as econometric without model and the panel's analysis with adequate model. This link can be better exploited by considering a panel with a large number of individuals.

Keywords: *principal component analysis, panel co-integration, long-run equilibrium, forecasts.*

I. Introduction

After World War II (1939-1945) and the eruption of the Cold War (1947-1991), the strategies of the political leaders were devoted to defend their own territories and those of their allies. This opened the door to ensure that the military expenditure to be used for national self-defence to face any eventual danger from outside the country. However, where did the arms race amongst all developed and developing countries come from ? Unfortunately, instead of spending the necessities for the welfare of the whole planet, the states regrouped

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themselves and built alliances (ANZUS [1], NATO [2], EDC [3], WEU [4],...). Moreover, the conflicts on the planet did not come to an end, but they are clearly seen in the Middle East conflict between the Arab states and the Israel, in southern Asia between Pakistan and India, and in eastern Asia between South-Korea and Vietnam. All these conflicts have created an impulse that boosted the military expenditure in all countries. In this regard, several factors mask the military expenditure among different states, for instance, the factors that masked the US military expenditure were not the same in Singapore, because for the former it is the whole planet that was being targeted, while the latter wants to protect the society's welfare. For this reason, we cannot generalize or suggest the same hidden factors for different countries, but certainly there will be some common hidden factors such as the self-defence against a possible external danger. Thus studying a panel of countries should take into account a certain degree of heterogeneity in their individual behaviors with respect to any economic variable, military expenditure or other factor. For example, the economic growth of a country depends on many factors such as the power of the industry sector, the degree of corruption, and others. A panel is an importer of heterogeneity and the experts must reduce this heterogeneity by a suitable choice of this panel.

The purpose of this article is to carry out an in-depth analysis of the panel of three macroeconomic variables, where Military Expenditure per Capita (MEXPC) is considered as a dependent variable, and both the Gross Domestic Product per Capita (GDPPC) and the CO2 Emissions per Capita (EMCO2PC) are independent variables. The research aims at performing a unit root panel analysis using first generation tests, without taking into account any dependence between the panel's sections (Harris and Tzavalis (1999)-HT, Breitung (2000)-λ, Im et al. (2003)-IPS) and the second generation, while considering the dependence using the Cross-sectionally Augmented IPS Panel Unit Root Test (CIPS). It is true that in any study of the panel, researchers start by testing the homogeneity, i.e. can we stay in the context of Pooled Regression Model, fixed model or random effect model. However econometric methodologies have progressed especially with Pesaran (2004, 2007); Im et al. (2003) and Pedroni (1995, 1996, 1999, 2001, 2004, 2007) in the analysis of co-integration at the level of a country taken alone and with the whole group. That is there may be two types of co-integration relationship : one relationship associated with each country in the panel and another for the entire panel. This leads us to be careful in making a quick decision about the heterogeneity of the panel.

The use of Principal Component Analysis (PCA) is now becoming essential to clarify the aspect of heterogeneity. So we will take the residues associated with the panel - Unit Root test (see Table (3)) to perform two types of analysis : The first is to use the cross section dependence proposed by (brush and Pagan 1980) CD_{BP} and the famous test CD_P proposed by Pesaran (2004). The second exploits the importance of masked factors that are fixed at two only, due to the two explanatory variables GDPPC and EMCO2PC that we have imposed. With this use of the PCA, we will propose an algorithm that could enrich

1. The Australia, New Zealand, United States Security Treaty (ANZUS or ANZUS Treaty) is the 1951 collective security non-binding agreement between Australia and New Zealand and, separately, Australia and the United States, to co-operate on military matters in the Pacific Ocean region, although today the treaty is taken to relate to conflicts worldwide.
2. The North Atlantic Treaty Organization also called the North Atlantic Alliance, is an intergovernmental military alliance between 29 North American and European countries. The organization implements the North Atlantic Treaty that was signed on April 4, 1949.
3. The Treaty establishing the European defence Community, also known as the Treaty of Paris, is an unratified treaty signed on 27 May 1952 by the six inner countries of European integration : the Benelux countries, France, Italy and West Germany.
4. The Western European Union was the international organization and military alliance that succeeded the Western Union (WU) after the 1954 amendment of the 1948 Treaty of Brussels.

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the panel econometric methods with model by better exposing the aspect of heterogeneity with a suitable choice of the panel ordering, see Appendix B. The seven co-integration tests proposed by Pedroni will be used in this paper and the long-term equilibrium relationships will be estimated by FMOLS and DOLS methods taking into account this heterogeneity.

This paper is organized as follows : Section 1 comprises the introduction and Section 2 presents a descriptive statistical study of the three random variables to better appreciate their temporal evolution and to highlight the interesting events that have affected the evolution. In Section 3, we test the homogeneity of the panel according to the Fisher test by taking the raw data, the log-transformed data and the first difference data. Section 4 is dedicated for the panel unit root tests. In Section 5, the Panel co-integration tests of Pedroni will be performed. In Section 6, we address the estimation of the long-run equilibrium relationship according to the FMOLS and DOLS estimators. While the section 7, the predictive model performance is performed using the findings of the FMOLS Estimator. The conclusion and discussions are presented in 8.

Notes

II. Statistical Description of Variables

For the purpose of the analysis, it is very useful to perform a basic statistical description when we have a panel of several countries in order to see the temporal evolution of each variable of the panel on one hand, and to check whether an individual effect is found among the countries on another hand. These are the basic statistical characteristics such as averages, standard deviations, minimums, maximums and the Compound Annual Growth Rate (CAGR) which is a specific term for the geometric progression ratio that provides a constant rate of evolution over the time period $(1968, 2014)$. For a time series X_t, the CAGR between 1968 and 2014 is calculated by :

$$\text{CAGR (1968,2014)} = \left(\frac{X_{2014}}{X_{1968}}\right)^{\frac{1}{46}} - 1.$$

It is clear that this description would have a meaning especially if the individual time series are realized with the stationary Gaussian random processes. Any way, we hope that this section will better explain the temporal evolution of each of the variables, especially if we can repair rupture of time due to a political or economic intervention that had an impact on the variables in question. As we stated in the introduction, we have a panel of (N=10) countries and three variables of which two are explanatory that are studied over the period $1968 - 2014$ (47 years).These countries are :

Arab world, Israel, USA, Canada, Japan, South Korea (Korea), France, United Kingdom, India and Pakistan. Figures of the individual in primary time series and in log-transformed data are presented in Appendix (A).

Let us have a close look at the elementary statistics in Appendix (A), Tables (8, 9, 10, 11, 12, 13). The lowest values are observed in India ($14), Pakistan ($25) and Arab world ($168). Israel and USA showed the highest military expenditure per capita of averages $1361 and $1117 respectively. The important difference between the Min and Max supports the idea that the two countries follow a very similar defence policy because each of them has enormous concerns about national security and domination by force. Both countries apply a policy based on the importance of military power to impose control over other states or enemies, ensuring the superiority of their military strength. The high standard deviations reflect a large variation over time. The annual growth over the period $1968 - 2014$ measured by CAGR is of the order of 4.58% in Israel and 3.45% in the USA which is the lowest rate.

These growth rates are relatively low when compared to rates in other countries. The highest CAGR are found in South Korea (10.05%), Japan (7.82%) and Arab world (7.57%). These results tell us about the situation of the conflict in the Korean Peninsula where South Korea, Japan and their allies line up on one side, and North Korea and its allies line up on the other side, not to mention the reality of the Arab-Israeli conflict, which has increased military expenditure in both directions.

Regarding GDP Per Capita, the three countries having the highest averages are USA ($26285), Japan ($24122) and Canada ($21945) while the lowest are located in India ($486), Pakistan ($501) and the Arab world ($2525). Considering the CAGR values, the highest value is recorded in South Korea (11.34%) followed by Arab world (7.93%). Indeed, South Korea has experienced an exceptional growth and integration in the world economy over the past fifty years Carroué (1997). For the Arab world, it is an informal way of admitting that the oil boom of the 1970s is responsible for this high value of CAGR. The lowest CAGR values are recorded in Pakistan (4.88%) and USA (5.48%).

For the third variable EMIPCA, since high values of this variable will have negative impacts on human life on the terrestrial globe, let us try to read carefully the CAGR values. Negative values indicate a decrease in the period 1968 – 2014. They were observed in USA (0.32%), France (1.07%) and United Kingdom (1.14%). This is a positive sign for these three countries, but also insufficient given the high averages measured by metric tons per capita of the variable EMIPCA, which are (19.51), (6.92) and (9.61) respectively. The three lowest positive values of CAGR are (0.07%) in Canada, (1.02%) in Israel, and (1.18%) in Japan. We choose this variable in the belief that it has an effect (positive or negative) on military expenditures per capita in a given country.

For raw data and logarithm transformed data, the elementary statistics such as Average, Std.Dev, Min and Max provide almost similar information for all variables. There will be only a difference in the CAGR because the trend evolution is generally weak in log-transformed data. For the MEXPCA variable, in raw data, the Max and Min were in South Korea and Pakistan, while the log-transformed data are observed in India and the USA. For GDPPCA, the highest CAGR values remain in South Korea and Arab world for raw data, while in log-transformed data, the lowest CAGR values appear in USA and Canada. For EMIPCA variable, in both raw data and log-transformed data, the negative CAGR are observed in the same countries : USA, France and United Kingdom, while the two highest positive values of CAGR are (5.73%) in South Korea and (2.52%) in Arab world.

Let us review the graphs of the time series associated with each country and start with the Military expenditures per capita (MEXPCA). The first graph shows the gap between the Arab world and Israel. A year after the Arab-Israeli War that took place between 5 and 10 of June 1967 between the Israel and the neighbouring states (Egypt, Jordan and Syria), the Israeli military expenditure per capita rose from ($287.19) in 1968 to ($823.62) in 1973, when the war known as the 1973 Arab-Israeli war erupted in October (6^{th} - 25^{th}), then it reached ($1145.35) in 1974. This graph shows two interesting peaks, the first ($1709.33) took place in 1988 marking the ending of the Iran-Iraq War (20 August 1988) and the second ($2114.21) in 1991 marking the Gulf War (17 January-28 February 1991). The MEXPCA of Arab world grew very slowly from ($16.44) in 1968 to ($224.92) in 1982 then he reached ($472.04) in 2014 while for Israel, the graph shows the value ($2249.5) at the end of this year.

For USA, we observe a graph with a growing cycle of length (23 or 24 years) : from 1968 to 1990 where the variable MEXPCA increased from ($402.24) in 1968 to ($1226.53) in 1990, then from 1991 to 2014 where it arose from ($1107.96) to ($1914.22) with a very large

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growth peak between 2001 (September 11 attacks) and 2011 from (\$1097.46) to (\$2282.53). For Canada, there is a similarity with the USA but a much lower variability in the time series going from (\$86.64) in 1968 to (\$502.42) in 2014. The military expenditures per capita, in Japan and Korea, were very close to each other until the year 2004, and a net difference in growth was observed between the two countries to reach (\$740) and (\$368.52) in 2014 respectively. This growth is due to the buildup of Chinese military expenditure, which, in turn, encourages defence expenditure of some possible adversaries (e.g., Japan and Republic of Korea Todd and Justin (2016). France and United Kingdom followed a very similar policy of military expenditure per capita over the period 1968 − 2014, going from (\$119.55) to (\$959.25) in France and from (\$100.76) to (\$915.31) in the United Kingdom, with (4.63%) and (4.93%) CAGR values respectively. The interstate war that took place between India and Pakistan (2001−2003) has encouraged defence expenditure for each of them. The graphs show that in Pakistan, the variable MEXPCA was stronger than in India and the two graphs intersect in 2011 reaching the value around (\$40).

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For Israel, an investigation of the GDP per capita graph shows that important growth took place after the 2006 Lebanon-Israel war, also called the 2006 Israel-Hezbollah war, a 34-day military conflict. The GDPPCA moved from (\$21827.82) in 2006 to (\$37539.95) in 2014 while, in the Arab world, the overview shows a CAGR (7.93%) going from (\$222.62) in 1968 to (\$7452.81) in 2014. For the USA, an almost linear trend is noticed indicating an increase from (\$4695.92) in 1968 to (54696.73) in 2014. However, in Canada, we witness a convex evolution over 1991−2008 from (\$21664.6) to (\$46596.34) with a very increasing trend between 2002 and 2008 due to the largest increases for agricultural products such as wheat, corn and canola. With an overview of the Japanese economy, we can distinguish between three phases : 1968−1985 (\$1450.62−\$11584.65), 1986−1995 (\$17111.85−\$43440.37), 1996−2014 (\$38436.93−\$38109.41) with a maximum of (\$48603.48) in 2012. While in Korea, there were small fluctuations over the entire period from (\$198.37) in 1968 to (\$27811.37) in 2014. In fact, the GDPPCA in Japan began a period of expansion in 1986 that continued until 1995, marking the start of the end of the Cold War with the arrival of Mikhail Gorbachev as leader of the Soviet Union (1985 − 1991). For France and United Kingdom, there were two small troughs in 1984 and 2001 (September 11 attacks) where GDPPCA went from (\$9397.495 to \$22433.56) and (\$8179.194 to \$27427.59) respectively. In France, economic instability marked the Giscard d'Estaing government and the early years of the presidency of François Mitterrand. Moreover, in United Kingdom, a substantial increase in unemployment from 5.3% in 1979 to a peak at nearly 11.9% in 1984 Bell and Blanchflower (2010). Finally, for India and Pakistan, the GDPPCA are very comparable in their trends from 1968 to 2014, going from (\$98.83) to (\$1576.00) and from (\$146.98) to (\$1316.98) respectively.

We still need to quickly describe the third variable CO2 emissions measured by metric tons per capita. The fluctuations are small in all countries, with a net decrease in both France and United Kingdom passing between 1968 and 2014 from 7.50 to 4.57 and from 10.99 to 6.5 respectively. The highest values are observed in both USA and Canada with a slight decline in the period 1968 − 2014, for the USA (from 19.09 to 16.49) and for Canada (from 14.63 to 15.12). A simple comparison between the Israel and the Arab world shows a wide gap between the two parties : at the beginning of the period, we observe the values 4.93, 1.54, and at the end of the period we read the values 7.86 and 4.86. Among the ten countries, Korea and India recorded a net increase over the period from 1.21 to 11.57 in Korea, and from 0.35 to 1.73 in India. Japan reveals a growth of 5.57 to 9.54 with CAGR of 1.18%. Finally, in Pakistan, the variable EMIPCA had the lowest values compared with the other countries.

The evolution moved from 0.45 in 1968 to 0.9 in 2014. For the logarithm data graphs, the readers are left to appreciate the temporal evolution of each variable in each country.

III. Homogeneous or Heterogeneous Panel ?

Considering the following model :

$$Y_{it} = \beta_{0,i} + \sum_{j=1}^{k} \beta_{1,i} X_{j(it)} + \epsilon_{it}, \quad k = 2, i \in [1, N], \ t \in [1968, 2014] = [1, T]. \tag{1}$$

Notes

In matrix form $Y_{it} = \beta_{0,i} + \mathcal{X}_{it}' \mathcal{B}^i + \epsilon_{it}$, such that $\mathcal{X}_{it} = \left(X_{1i,t}, X_{2i,t}\right)', \mathcal{B}_i = \left(\beta_{0,1}, \mathcal{B}^i\right)'$ and $\mathcal{B}^i = \left(\beta_{1,i}, \beta_{2,i}\right)$. In the literature Hsiao (1986); Hurlin (2010) and (Mourad (2019), p. 150-154), we have three tests which represent the first steps in a panel data study. Indeed, the researcher in this field is invited to ask questions about the nature of the panel : Is it a homogeneous or heterogeneous panel ? Indeed, the three tests will lead together to a decision around the existence of a panel structure or take each country separately without taking care of the panel itself.

First test :

$$\begin{cases} H_0^1 : \beta_{0,i}, \mathcal{B}^i = \mathcal{B}, \ \forall i = 1, N \\ H_a^1 : \exists (i,j) \in [1, N]; \ \beta_{0,i} \neq \beta_{0,j} \text{ or } \mathcal{B}^i \neq \mathcal{B}^j \end{cases}$$

Under the null hypothesis H_0^1, we consider a pooled Regression Model (PRM) with respect to the number of the imposed restrictions $[\nu_1 = (k+1)(N-1)]$. Using the OLS method, we estimate the PRM and we save the residual sum of squares $\text{RSS}_{(\text{pooled},r_1)}$, where r_1 designates the restrictions under H_0^1. Then we consider N models of multiple linear regressions, with a model for each country, and we keep the residual sum of squares associated with each country $\text{RSS}_i^1, \ i = 1, \ldots, N$ and finally we calculate $\text{RSS}_{1,\nu_2} = \sum_{i=1}^{N} \text{RSS}_1^i$, where $[\nu_2 = N(T-K-1)]$ degrees of freedom. If the null hypothesis is true, then we calculate the F-statistic given by :

$$F_1 = \frac{\frac{\left[\text{RSS}_{\text{pooled},r_1} - \text{RSS}_{1,\nu_2}\right]}{\nu_1}}{\frac{\text{RSS}_{1,\nu_2}}{\nu_2}}$$

And we compare it to the tabulated value $F_{0.05;\nu_1;\nu_2}$. If $F_1 < F_{0.05;\nu_1;\nu_2} \approx 1.51$, then we accept H_0^1 and by consequence we obtain a homogeneous panel data model. If we reject H_0^1, we move on to the second step which consists of determining whether the heterogeneity comes from the coefficients $\mathcal{B}^i$ or not.

Second test :

$$\begin{cases} H_0^2 : \mathcal{B}^i = \mathcal{B}, \ \forall i = 1, N \\ H_a^2 : \exists (i,j) \in [1, N]; \ \mathcal{B}^i \neq \mathcal{B}^j \end{cases}$$

In this test, no restriction is imposed on the parameters $(\beta_{0,i}, \ i = 1, N)$. Using the so-called method (Within estimation) we obtain the residual sum of squares $\text{RSS}_{(\text{pooled},r_2)}$, where r_2 designates the restrictions under H_0^2 with restrictions $[\nu_1 = K(N-1)]$. Like our path in the first test, by the same method we estimate a model for each country and we retain the $(\text{RSS}_2 = \sum_{i=1}^{N} \text{RSS}^i_{\text{within}})$, with $[\nu_2 = (NT - (K+1)N)]$ as degrees of freedom. Under the null hypothesis H_0^2, we calculate the following F-statistic :

$$F_2 = \frac{\frac{\left[\text{RSS}_{\text{within},r_2} - \text{RSS}_{2,\nu_2}\right]}{\nu_1}}{\frac{\text{RSS}_{2,\nu_2}}{\nu_2}}$$

Notes

If $F_2 > F_{0.05;\nu_1;\nu_2} \approx 1.63$, then we reject the panel structure and, by consequence, the estimated vector $\mathcal{B}^i$ will be made for the countries one by one. If we accept H_0^2, then we retain the panel structure and we then seek to determine in a third step if the coefficients $(\beta_{0,i})$ have an individual dimension.

Third test :

$$\begin{cases} H_0^3 : \beta_{0,i} = \beta_0, \ \forall i = 1, N \\ H_a^3 : \exists (i,j) \in [1,N]; \ H_a^3 : \beta_{0,i} \neq \beta_{0,j} \end{cases}$$

Under H_0^3, we impose $(\mathcal{B}^i = \mathcal{B}, \ \forall i = 1, N)$. There will be available $(\nu_1 = (N-1))$ of restrictions. Under H_a^3, the $(\mathcal{B}^i, i = 1, N)$ are the same, but the $\beta_{0,i}$ differs according to the countries. Using the Pooled estimation method, we guarantee $\text{RSS}_{\text{pooled},r_3}$ and using Within Estimation Method, we retain $\text{RSS}_3 = \text{RSS}_{2,r_2}$, with $(\nu_2 = NT - N - K = N(T-1) - K)$ as degrees of freedom. Under the null hypothesis H_0^3, we calculate the following F-statistic :

$$F_2 = \frac{\frac{\left[\text{RSS}_{3,r_3} - \text{RSS}_{2,r_2}\right]}{\nu_1}}{\frac{\text{RSS}_{2,r_2}}{\nu_2}}$$

If $F_3 > F_{0.05;\nu_1;\nu_2} \approx 1.90$, then we reject H_0^3 and we get a panel model with individual effects. Contrariwise, if we accept H_0^3, we retrieve an homogeneous panel data model. The findings of the three tests above are given in the following Table (1) :

Table 1: Homogeneous or Heterogeneous panel : Fisher's test

Null hypothesis	Primary data$^{(1)}$	Data in logarithm$^{(2)}$	Data in first difference$^{(1)}$	Data in first difference$^{(2)}$
H_0^1	253.76^r	346.00^r	2.209^r	0.705^a
H_0^2	45.25^r	23.14^r	2.532^r	0.249^a
H_0^3	45.25^r	23.14^r	2.532^a	0.249^a

^{r} rejection the null hypothesis at a 5% significant level.
^{a} acceptance the null hypothesis at a 5% significant level.

This change in the responses of the H_0^i, $i = 1, 2, 3$ tests led us directly to examine the Panel Unit Root Tests.

IV. Panel Unit Root Tests (PURT)

Over the past two decades, research was carried out on the PURT. The first generation of testing was demonstrated by Levin and Lin (1992) as working papers at the University of

California, and then they were published by Proceeding with the application of LL (or LLC) technique. It is important to draw the reader's attention to the importance of individual and temporal dimensions in the unit root study of a panel data. The co-integration tests for short-time series are known to be inefficient in distinguishing between stationary and non-stationary time series. The issue of co-integration is complicated especially if the time series experienced a rupture in the trend. This is true of the time series associated with the exchange rates if we examine them over a period of time before and after the cancellation of the Bretton-Woods system. Therefore, the experts of econometrics propose to study a number of countries benefiting from the information related to each country, which contributes to the establishment of a broad analysis in the long and short run. Hence, the adoption of the panel data will provide a more objective analysis of the acceptance or rejection of the null hypothesis of co-integration, while we cannot do it at the level of each country separately. Another advantage of the use of the panel data, in both time and individual dimensions, is that the unit root test follows a normal distribution, while the latter is not available in the time dimension study alone. The researcher must move from the unit root test in a single time series to several multi-time series ; and therefore DF, ADF, PP, KPSS and the modified version of the DF test proposed by Elliott and Stock (1996) will need improvement to deal with the time and individual dimensions of the time series. Thus, even if the size of a time series is small for each country, the increase in the number of countries will increase the total number of observations and thus avoid falling into the rupture of trend. See a recent study Jaunky and Lundmark (2017).

Comparing the two approaches with the unit root, the traditional approach that takes only the time dimension and the approach that takes the time and individual dimensions Hurlin and Mignon (2006) reveals two fundamental differences :

- The first is related to the non-standard asymptotic distribution in the time dimension and how it varies with constant and/or trend in the deterministic component. In the case of panel data, the unit root tests will follow the normal distribution except for Fischer tests. This is a fundamental difference between the two approaches, and these normal distributions will remain conditionally related to the deterministic components of the model used.
- The second difference would be the possibility of heterogeneity among individuals in the case of panel data, whereas there is no such possibility in the case of time dimension.

Through this observation, we come to the following question : Can we use the same model in the case of time series for one individual, i.e. with only a time dimension, and in the case of a panel data ? If yes, this means that we have assumed a homogeneity among individuals in terms of dynamic characteristics and their consequences for the stationarity or non-stationarity of the time series.

The random use of the same model to test the unit root on all individuals will often lead to spurious results. Therefore, we must first resolve the issue of heterogeneity among individuals before embarking on testing the panel unit root. This is secured through what we have studied in the previous section. There are many tests in literature review that talk about testing the panel unit root, where the most famous of the first generation are Levin and Lin (1992); Harris and Tzavalis (1999); Maddala (1999); Maddala and Wu (1999), (λ test Breitung (2000)), (Lagrange multiplier (LM) test Hadri (2000)), Levin et al. (2002)

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and Im et al. (2003). The heterogeneity also affects the alternative hypothesis in terms of the panel unit tests. If we study the GDP per capita in several countries over a certain period of time, we can accept the existence of the unit root in a group and reject it in other countries. Given the importance of studying the heterogeneity among individuals, it was necessary to ask this question : Is it logical to consider the null hypothesis of cross-sectional independence among individuals ? This hypothesis is considered as a nuisance parameter for the researchers. Thus rejecting it suggests the use of appropriate new panel unit root test as a second generation. Many tests have been proposed, for example, Choi (2001); Breuer et al. (2002); Phillips and Hansen (1990); Chang (2002); Moon and Perron (2004); J. and S (2004); Breitung and Das (2005); Hurlin and Mignon (2006) and the most famous test is the Cross-Sectionally Augmented IPS (CIPS) Panel Unit Root proposed by Pesaran (2007). For more information about these tests see (Mourad (2019), p. 255-288). In our approach to the PURT, we will limit ourselves to the four tests : IPS, HT, λ and CIPS.

The critical values of the HT technique change with the change of the deterministic i.e. without intercept, with intercept only, and with intercept and trend. If we investigate Tables 1a, 1b and 1c of Harris and Tzavalis (1999) (p. 211, 212 and 213 respectively), we find by interpolation the critical values of the Z statistic as shown in the following table :

For λ and IPS, the output statistics are compared to the 1%, 5%, and 10% significance levels with the one-tailed (negative) of a standard Normal with the critical values of (-2.326), (-1.645), and (-1.282) correspondingly. For the CIPS test, the critical values of average of individual cross-sectionally Augmented Dickey-Fuller are around -2.55, -2.33 and -2.21 at 1%, 5% and 10% significant levels (constant only) respectively (Source : Table 3b-Pesaran (2007)).

	Significance levels $T = 47,\ N = 10$		
Critical values	1%	5%	10%
Without intercept	−3.96	−2.41	−1.79
Intercept only	−3.15	−2.09	−1.65
Intercept + trend	−2.82	−1.97	−1.54

Cross section dependence (CD) test :

From the findings in Table (1), for each variable, we have decided the measurement of the correlation coefficients among the **Cross-sectional analysis** using the ADF at order $(p = 3)$ in the level :

$$\Delta X_{it} = \alpha_i + \gamma_i X_{it-1} + \sum_{k=1}^{p-1} \varphi_{ik} \Delta X_{i,t-k} + \epsilon_{it} \tag{2}$$

Retaining the estimate residues $(e_{it},\ t \geq 4)$ for each section, for each country, we calculate the Pearson's correlation coefficient $\hat{\rho}_{ij}$ (correlations between panel units) :

Notes

$$corr(e_{it}, e_{jt}) = \hat{\rho}_{ij} = \frac{\sum_{t=1}^{T} e_{it}e_{jt}}{\sqrt{\left(\sum_{t=1}^{T} e_{it}^2\right)^2 \times \left(\sum_{t=1}^{T} e_{jt}^2\right)^2}}, \quad i, j = 1, \ldots, N \text{ and } i \neq j. \tag{3}$$

If $\hat{\rho}_{ij} = 0, \forall t, i \neq j$, then there is no correlation between e_i and e_j. For that, we test the null hypothesis H_0 versus the alternative H_a :

$$\begin{cases} H_0 : \text{there is no correlation between } e_i \text{ and } e_j \\ H_a : \text{there is correlation between } e_i \text{ and } e_j \end{cases}$$

By consulting the literature review Rafael E. De Hoyos and Sarafidis (2006) and (Mourad (2019), p. 355-357), if N is relatively small and T is large enough, it is possible to estimate the model above using the OLS method and saving the associated residues as proposed Breusch and Pagan (1980), and then we calculate the statistic :

$$\mathrm{CD}_{\mathrm{BP}} = T \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}_{ij}^2$$

$$\mathrm{CD}_{\mathrm{BP}} \xrightarrow[T \to \infty]{} \chi^2_{\frac{N(N-1)}{2};N=10}$$

df	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.01$
45	57.505	61.656	69.957

One disadvantage of this test is that it is inappropriate when N is large ($N \to \infty$). To treat better the cross-sectional analysis, Pesaran (2004) proposed the following statistic :

$$\mathrm{CD}_{\mathrm{P}} = \sqrt{\frac{2T}{N(N-1)}} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}_{ij} \Rightarrow \mathrm{CD}_{\mathrm{P}} \xrightarrow[T \to \infty, N \to \infty]{} N(0,1),$$

with the two-tailed of a standard normal, the critical values are (1.96), (2.58), and (3.29) for $10\%, 5\%$ and 1% respectively.

In Table (2), for each variable both $\mathrm{CD}_{\mathrm{BP}}$ and CD_{P} statistics are highly significant and suggest to take into account highly correlated countries.

Table 2: Testing for cross-section dependence in panel

	$\mathrm{CD}_{\mathrm{BP}}$		CD_{P}	
Variables	X	ΔX	X	ΔX
Y	109.19^a	115.38^a	6.12^a	07.05^a
X_1	170.93^a	187.23^a	10.17^a	10.26^a
X_2	122.60^a	144.43^a	04.53^a	05.21^a

^{a}, ^{b} and ^{c} indicate that the test is significant at $1\%, 5\%$ and 10% significant levels respectively.

Notes

CIPS test :

Pesaran suggests the following equation :

$$\text{CADF}_{p_i} : \Delta Y_{it} = a_i + b_i Y_{it-1} + c_i \bar{Y}_{t-1} + \sum_{j=0}^{p_i} d_{ij} \Delta \bar{Y}_{t-j} + \sum_{j=1}^{p_i} \delta_{ij} \Delta Y_{it-j} + e_{it}, \tag{4}$$

Notes

where $\bar{Y}_t = N^{-1} \sum_{j=1}^{N} Y_{jt}$; $\bar{Y}_{t-k} = N^{-1} \sum_{j=1}^{N} Y_{jt-k}$, $\bar{Y}_0 = N^{-1} \sum_{j=1}^{N} Y_{j0}$ and Y_{j0} is fixed or random, considering that the data generating process (DGP) is a simple dynamic linear heterogeneous panel data model. This test is entitled also *Cross-sectionally Augmented version of the IPS Panel Unit Root Test* entitled (CIPS), which is a simple average of the individual CADF-tests. In fact, practically, we maintain the t-statistics $\widehat{\text{CADF}}_{p_i}$ of the estimate parameters $(\hat{b}_i, i = 1, \ldots, N)$ then we calculate $\text{CIPS} = \overline{\text{CADF}} = \frac{1}{N} \sum_{i=1}^{N} \widehat{\text{CADF}}_{p_i}$.

The findings in Table (3) reveal that all variables are stationary in first difference.

V. Panel Co-Integration Tests of Pedroni

In this section, the methodology carried out by Pedroni (1995, 1996, 1999, 2001, 2004, 2007) will be used. The use of co-integration techniques to test the presence of long run

Table 3: Panel unit root test-Data in natural logarithm Individua l Specific Components : Constant Average p chosen from 3 by AIC

	HT		λ		IPS		CIPS	
Variables	X	ΔX	X	ΔX	X	ΔX	X	ΔX
Y	0.40	-42.3^a	2.48	-9.05^a	-1.87^b	-7.73^a	-2.45^b	-4.31^a
X_1	1.18	-43.46^a	4.66	-10.90^a	-1.45^c	-8.51^a	-2.01	-3.88^a
X_2	1.53	-45.95^a	5.71	-07.14^a	1.12	-11.0^a	-1.72	-4.29^a

^{a}, ^{b} and ^{c} indicate that the test is significant at 1%, 5% and 10% significant levels respectively.

relationships among integrated variables has enjoyed growing popularity in the empirical literature (see Mourad (2019) and Mourad (2018a,b)). In this section, we focus on the long-run relationship which could exist between the military expenditures per capita ($Y_{i,t}$) as a dependent variable in ten counties from 1968 − 2014 and two independent variables GDP per capita ($X_{1i,t}$) and CO2 emissions per capita ($X_{2i,t}$). The Pedroni procedure will be used respecting the following steps (see Mourad (2019), p. 296-301).

Briefly, we consider the hypothesized long-run regression between the dependent variable Y_{it} and two independent variables ($M = 2$) as the following :

$$Y_{it} = \alpha_i + \beta_{1i} X_{1i,t} + \beta_{2i} X_{2i,t} + e_{it}, \; i = 1, \ldots, 10, \; t = 1, \ldots, T. \tag{5}$$

$$Y_{it} = \alpha_i + \mathcal{X}'_{it} \mathcal{B} + e_{it}, \; \mathcal{X}_{it} = (X_{1i,t}, X_{2i,t}), \; \mathcal{B} = (\beta_{1i}, \beta_{2i})'$$

Assuming that there is a homogeneity of the parameters of the long run relationship, i.e. $\mathcal{B}_i = \mathcal{B}$, $\forall i = 1, \ldots, N$. In the equation (5), T is the number of observations over time and N denotes the number of individual members in the panel. It is quite clear to assume that the slope coefficients (β_{ij}, $j = 1, 2$) and the member specific intercept α_i can vary across each cross-section. In the equation (5), we have adopted a regression equation with a heterogeneous intercept. Note that it could also be estimated without a heterogeneous intercept, or with time trend and/or common time dummies. By OLS method, we estimate the model in the equation (5) and we save the residues $\hat{e}_{it}$. Using the estimate residuals $\hat{e}_{it}$ in (5) to estimate the model :

$$\hat{e}_{it} = \gamma_i \hat{e}_{it-1} + u_{it} \tag{6}$$

Pedroni suggests the nearest integer $k_i = 4\left(\frac{T}{100}\right)^{2/9}$ as truncation lag parameter for the Newey-West kernel estimator recommended in Newey and West (1994). His tests take into account the heterogeneity through the parameters that may differ between individuals. Such heterogeneity can be located at both the long-run regression i.e. the co-integration relations, and the short-run dynamics. Pedroni accepts the null hypothesis of no intra-individual co-integration for both homogeneous and heterogeneous panels. Thus, under the alternative hypothesis, exists a co-integration relation which is specific for each individual. He proposes seven statistics, four of which are based on the within-dimension and three on the between-dimension. Statistically speaking, for all tests, the null hypothesis of no co-integration is :

$$\mathrm{H}_0 : \gamma_i = 1,\ \forall i = 1, \dots, N,$$

where the parameter γ_i is estimated in (6). Whereas the alternative hypothesis changes according to the within (intra) or between (inter) dimension vision.

In the within-dimension :

$$\mathrm{H}_a : \gamma_i = \gamma < 1,\ \forall i = 1, \dots, N,$$

where γ is a common value. The alternate to no co-integration must be that if the individuals are co-integrated, then they will exhibit the same long run co-integrating relationships.

In the between-dimension :

$$\mathrm{H}_a : \gamma_i < 1,\ \forall i = 1, \dots, N,$$

where a common value γ is not required. Under this alternative hypothesis, the individual cross sections contain co-integrating relationships that are free to take on different values for different members of the panel. In other words, we allow the presence of heterogeneity between individuals. Since it is rare in practice to find an identical co-integration vectors for all individuals, because a considered heterogeneity through parameters may differ among individuals.

VI. FMOLS and DOLS Estimators of the Long-Run Equilibrium

When the residues of the co-integration relationship are correlated with the innovations of regressors, then the ordinary least squares estimators (OLS) of the co-integration vector parameters are biased. This bias entitled as long-term endogeneity or a bias of the second order implies non-standard distributions of the main usual tests statistics. Given the evidence of panel co-integration, the long-run relationships between the different variables can be further estimated by several methods proposed in the literature, e.g. the Fully-Modified Ordinary Least Squares (FMOLS) which is a semi-parametric procedure suggested by Phillips and Hansen (1990); Phillips (1995); Pedroni (1995) and the dynamic OLS (DOLS) estimator proposed by Stock and Watson (1993); Kao and Chiang (2000); Mark and Sul (2003). In both cases, the FMOLS and DOLS procedures estimate both individual-specific cointegrating vectors and aggregated estimator.

Notes

Notes

FMOLS procedure :

The Fully Modified Ordinary Least Square (FMOLS) method is one of the methods that permits a correction of the long term endogenous bias particularly for the finite sample size. The idea is to bring a new representation of the co-integration relationship in which the residues verify well the orthogonality properties. In other words, the FMOLS regression estimates a linear regression, then it adjusts the estimates and covariance matrix for endogeneity Mourad (2019). When the individual dimension is sufficiently large and even for the short time series, the FMOLS estimator is consistent and it has a relatively well performance controlling the likely endogeneity of the regressors and serial correlation.

Table 4: Pedroni Panel co-integration Tests Results
Natural logarithm of data in deviations from time period means

Alternative hypothesis : Common AR coefficients (within-dimension)		
	Statistics	
Tests	Det = constant $p = 6$	Det = trend $p = 4$
Panel-ν statistic (non-parametric)	0.31	0.85
Panel ρ-statistic (non-parametric)	-1.04	-0.73
Panel pp-statistic (non-parametric)	-1.58^c	-1.99^b
Panel ADF-statistic (parametric)	-1.58^c	-1.90^b
Alternative hypothesis : Common AR coefficients (within-dimension)		
Group ρ-statistic (non-parametric)	-0.14	0.15
Group pp-statistic (non-parametric)	-1.46^c	-1.65^b
Group ADF-statistic (parametric)	-2.10^b	-1.39^c

The variance ratio test is right-sided, while the other Pedroni tests are left-sided. All reported values are distributed $N(0,1)$ under the null unit root or no co-integration. For the left-sided tests, the rejection of the null will take place in the left tail. The critical values are -1.28, -1.64 and 2.33 at 10%, 5% and 1% significance levels respectively.
Conclusion : The estimation proceeds on the basis that the demeaned series are co-integrated.
^{b} and ^{c} indicate the rejection of the null hypothesis of no co-integration on the 5% and 10% significance levels respectively.
Note 1 : The data have been demeaned with respect to common time effects to accommodate some forms of cross-sectional dependency, so that in place of y_{it}, x_{1it} and x_{2it}, we use : $\tilde{y}_{it} = Y_{it} - \bar{Y}_t;\ \bar{Y}_t = \frac{1}{N}\sum_{i=1}^{N} y_{it}$ and $\tilde{x}_{jit} = x_{jit} - \bar{x}_{jt};\ \bar{x}_{jt} = \frac{1}{N}\sum_{i=1}^{N} x_{jit};\ j = 1, 2.$
Note 2 : A variable on the right hand side (RHS) of your model may be endogenous. This endogeneity means that the explanatory variable is correlated with the model's error term. The correlation of a RHS variable with the error term means that OLS is neither unbiased nor consistent.
Note 3 : Kernel width = 4.

DOLS procedure :

The DOLS procedure consists in including lags and leads of the regressors in the long-run equilibrium relationship to eliminate feedback effects and endogeneity. This has the consequence of eliminating the correlations between the explanatory variables and residues. Thus in our case, we obtain :

$$\tilde{Y}_{it} = \alpha_i + \beta_{1i}\tilde{X}_{1i,t} + \beta_{2i}\tilde{X}_{2i,t} + e_{it}.$$

Let's consider

$$\tilde{X}_{i,t} = \left(\tilde{X}_{1i,t}, \tilde{X}_{2i,t}\right)' \text{ and } \mathcal{B}_i = \left(\beta_{1i}, \beta_{2i}\right)'.$$

If we choose truncation at $lagp$, we obtain :

$$\tilde{Y}_{it} = \alpha_i + \tilde{X}_{it}'\mathcal{B}_i + \sum_{s=-p}^{p} c_{is}^{1}\Delta\tilde{X}_{1i,t+s} + \sum_{s=-p}^{p} c_{is}^{6}\Delta\tilde{X}_{2i,t+s} + e_{it},\ i \in [1, 10],\ t \in [1975, 2008].$$

The DOLS can very quickly exhaust the degrees of freedom in a data set. If we choose truncation at lag p, there are $2p+1$ added regressors in the differences for each right side endogenous variable, plus we lose $2p+1$ data points allowing for lags, leads and first differences. So with 47 observations per individual, the order $(p = 6)$ leaves us with 34 usable observations, and 29 regressors.

For all of the group mean FMOLS estimates and standard errors in Table (5), we have considered the case in which the data have been demeaned over the cross-sectional dimension in order to account for some of the likely cross-sectional dependence through common time effects. The FMOLS and DOLS group mean estimators for the panel as a whole provide credible estimates for the parameters using the RATS option (AVERAGE=sqrt)[5].

In Tables (5), we present the estimation results associated to the long-run equilibrium individually and aggregately according to the two methods FMOLS and DOLS.

VII. Evaluating Forecast Performance

The long-term relationships were estimated over the period $1968 - 2014$. We will make forecasts for the military expenditures per capita $(Y_{i,t})$ for the years $2015-2017$ and compare with the observed values that are available for these three years. To make these forecasts, we need the observed values for the variables $(X_{1i,t})$ and $(X_{2i,t})$. In fact the observations are available for $(X_{1i,t})$ and not available for $(X_{2i,t})$.

For this, we will predict the CO2 emissions per capita $(X_{2i,t})$ over the period $2015 - 2017$ and for each country taken separately using the ARIMA technique. The Augmented Dickey-Fuller test (ADF) will be used to test the null hypothesis which a unit root presents in a univariate time series in logarithm. The choice of the ADF order p was made by AIC ensuring that the residues behaved like a white noise for lags $6, 12$ and 18. In other words, the Ljung-Box statistic is significant for a level of 5% . The null hypothesis of unit root is tested against alternative of absence of unit root and the results are presented in Tables (6) and (7). The inspection of these tables shows the acceptance of the null hypothesis (there is a root unit in the level variables) but this hypothesis is rejected if we consider the data in first difference for both primary data and natural logarithm of data. For a sample size $(T = 50)$ and at 5% and 10% significance levels, the critical values for the ADF tests are respectively -2.93 and -2.60 for τ_μ test statistic (intercept only), -3.50 and -3.18 for τ_τ test statistic (intercept and trend). The findings of ADF tests support the idea of taking EMIPCA variables in first difference.

If we closely investigate the long-term equilibrium relationship for each country, we find different results for both FMOLS and DOLS techniques. For all 10 countries, Arab world, Israel, USA, Canada, Japan, South Korea, France, United Kingdom, India and Pakistan, we found when using FMOLS positive effect of GDPPCA on MEXPCA. Thus an increase of 1%

[5] Considering the Manual RATS, AVERAGE=SQRT weights each individual by the diagonal matrix formed by taking the square roots of the precision matrix (inverse covariance matrix) of the estimates for that individual. This matches up with the averaging done in computing the t-statistics, that is, the coefficients and covariance matrix from AVERAGE=SQRT will reproduce the average t-statistics.

Notes

in the variable GDPPCA leads to an increase in MEXPCA of $1.22\%, 2.77\%, 0.99\%, 1.14\%$ and $1.02\%, 0.27\%, 1.13\%, 0.65\%, 0.87\%$ and 0.66% respectively. However, using DOLS estimator, the effects will be positive and they are evaluated at $0.97\%, 4.7\%, 1.4\%, 1.06\%, 1.56\%$ and 1.27% for each of Arab World, Israel, Canada, Japan, France, and India successively. Moreover, we observe a negative impact (significant at level 10%) evaluated at 0.57% for USA and for the other countries, but we did not notice a significant effect of GDPPCA on MEXPCA for United Kingdom and Pakistan.

Table 5: Estimation of the long-run equilibrium

Country (i)	FMOLS			DOLS		
	Intercept	$\tilde{X}_{1it}$	$\tilde{X}_{2it}$	Intercept	$\tilde{X}_{1it}$	$\tilde{X}_{2it}$
Arab world	0.896	1.222	-0.192	0.172	0.967	-1.184
	(8.19)	(12.92)	(-0.97)	(3.55)	(29.80)	(-21.98)
Israel	0.803	2.768	-2.359	-0.400	4.697	-2.757
	(3.15)	(7.03)	(-4.59)	(-3.07)	(26.66)	(-31.35)
USA	0.536	0.991	-0.198	1.946	-0.574	0.291
	(1.59)	(2.46)	(-0.70)	(7.78)	(-1.80)	(2.07)
Canada	-0.849	1.139	-0.122	-1.111	1.397	-0.138
	(-6.32)	(5.79)	(-0.70)	(-18.66)	(12.72)	(-1.74)
Japan	-0.095	1.017	-2.367	-0.208	1.055	-2.293
	(-0.33)	(8.61)	(-6.13)	(-2.18)	(56.99)	(-12.58)
South Korea	-0.203	0.267	0.760	-0.950	-2.120	4.183
	(-2.41)	(1.02)	(2.29)	(-3.99)	(-3.02)	(3.58)
France	-0.263	1.129	-0.071	-0.703	1.558	-0.183
	(-3.23)	(14.56)	(-2.80)	(-16.87)	(39.40)	(-19.87)
United Kingdom	0.089	0.653	0.252	0.951	-0.037	-0.009
	(0.87)	(7.83)	(5.60)	(2.76)	(-0.15)	(-0.06)
India	0.254	0.867	0.406	1.494	1.268	0.435
	(0.90)	(10.91)	(6.07)	(12.94)	(55.90)	(12.37)
Pakistan	-0.925	0.660	-0.228	-2.723	-0.025	-0.301
	(-1.38)	(4.65)	(-1.22)	(-12.87)	(-0.72)	(-5.41)
Group	0.015	0.978	-0.030	-0.294	1.046	-0.345
	(0.32)	(23.96)	(-0.99)	(-9.68)	(68.23)	(-23.70)

Using both methods FMOLS and DOLS, the investigation of the impact of the EMIPCA variable on the MEXPCA variable revealed a negative effect for Arab World, Israel, Japan, France, and Pakistan. An increase of 1% in the variable EMIPCA leads to a decrease in MEXPCA of $0.19\%(1.18\%), 2.36\%(2.76\%)$, $2.37\%(2.29\%)0.07\%(0.18\%)$, and 0.3% (with DOLS only) respectively.

For South Korea, United Kingdom, and India, we found a positive impact of EMIPCA on MEXPCA using FMOLS estimator, so an increase of 1% in the variable EMIPCA leads

Notes

Table 6: The Augmented Dickey-Fuller test (ADF)-Primary data

Country (i)		Intercept		Intercept & trend	
	p	$X_{2i,t}$	$\Delta X_{2i,t}$	$X_{2i,t}$	$\Delta X_{2i,t}$
Arab world	5	-0.28	-4.28^a	-1.86	-4.22^a
Israel	3	-1.22	-3.89^a	-0.42	-4.02^a
USA	5	-0.63	-4.41^a	-1.68	-4.30^a
Canada	4	-2.08	-4.16^a	-2.44	-4.25^a
Japan	5	-1.08	-3.71^a	-2.43	-3.55^a
South Korea	5	-0.42	-3.46^a	-1.96	$-3.35b$
France	5	-0.75	-3.71^a	-2.03	-3.62^a
United Kingdom	5	0.38	-3.72^a	-1.62	-3.78^a
India	3	3.53	-3.11^a	1.05	-4.79^a
Pakistan	3	-0.65	-3.20^a	-3.33	-3.08

^{a,b} : the null hypothesis of a unit root is rejected at 5% and 10% significance levels respectively.

Table 7: The Augmented Dickey-Fuller test (ADF)-log-transformed data

Country (i)		Intercept		Intercept & trend	
	p	$X_{2i,t}$	$\Delta X_{2i,t}$	$X_{2i,t}$	$\Delta X_{2i,t}$
Arab world	5	-1.52	-3.99^a	-3.00	-4.01^a
Israel	3	-1.21	-4.26^a	-0.30	-4.43^a
USA	5	-0.56	-4.14^a	-1.63	-4.07^a
Canada	4	-2.08	-4.13^a	-2.43	-4.22^a
Japan	5	-1.08	-3.89^a	-2.53	-3.70^a
South Korea	4	-2.80	-3.13^a	-0.76	-4.30^a
France	5	-0.33	-3.47^a	-2.09	-3.43^a
United Kingdom	5	1.08	-3.26^a	-0.57	-3.52^a
India	3	1.00	-4.38^a	-1.80	-4.56^a
Pakistan	3	-0.69	-3.45^a	-3.06	-3.37^b

^{a,b} : the null hypothesis of a unit root is rejected at 5% and 10% significance levels respectively.

to an increase in MEXPCA of 0.76%, 0.25% and 0.41% respectively. However, we found a positive impact of EMIPCA on MEXPCA while using DOLS estimator, so an increase of 1% in the variable EMIPCA leads to an increase in MEXPCA of 0.29%, 4.18% and 0.44% for USA, South Korea and India respectively. For the other countries not mentioned, no significant effect of EMIPCA on MEXPCA was found.

Finally, for the group of 10 countries, both FMOLS and DOLs methods revealed a positive impact of GDPPCA on MEXPCA. In this way, a growth of 1% in GDPPCA leads to a growth in MEXPCA of 0.99% and 1.05% respectively. The variable EMIPCA revealed a negative

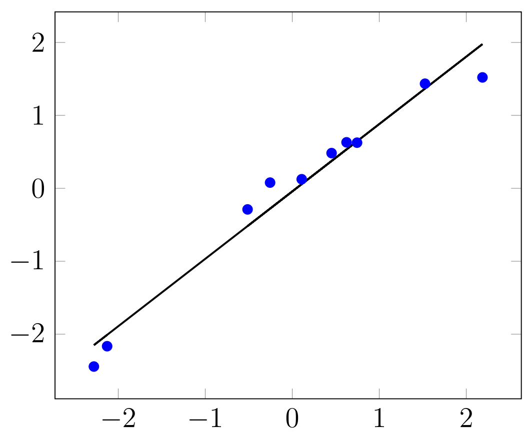
Notes

impact (0.35%) on MEXPCA if the DOLS estimator is used. Based on these results, it is necessary to choose between the two methods for each country calculating the forecasts for the period $2015-2017$, noting that the period data $1968-2014$ have been used to estimate the predictive models.

In the following, the forecasts will be calculated using the models estimated by FMOLS only. In fact with the DOLS method, we lose a lot of observations. Let's designate by P_t the forecasted value and A_t the real value of a time series. If $P_t = A_t$ then the forecasts are perfectly exact and the linear correlation coefficient between P_t and A_t is equal to 1. In the following, for each year on the period $2015-2017$, the forecasted and observed values associated with the ten countries are plotted and a simple regression model is estimated. If the coefficient of determination is near to 1, then the accuracy of the forecasts is considered as very good.

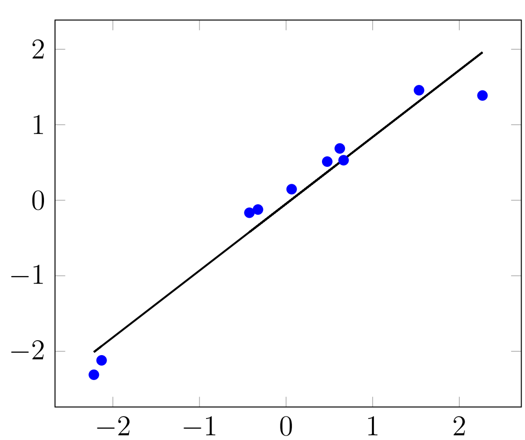
Notes

$$\left| \begin{array}{c} \hat{A}_t = 0.9242 P_t - 0.0431 \\ \text{t-stat} \quad (15.37) \quad (-0.53) \\ R^2 = 0.9673 \quad \hat{\rho} = 0.9835 \end{array} \right|$$



Observed (A_t) vs. predicted (P_t) values : 2015.

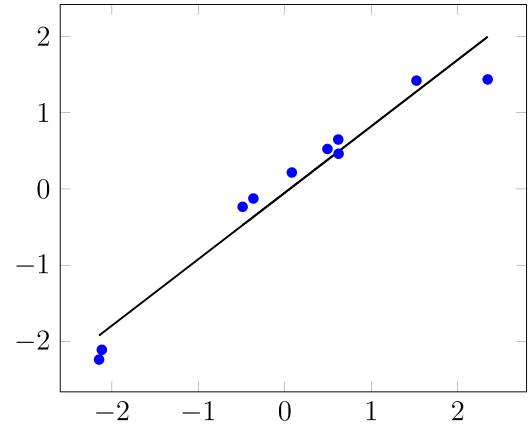
$$\left| \begin{array}{c} \hat{A}_t = 0.8847 P_t - 0.0469 \\ \text{t-stat} \quad (13.20) \quad (-0.52) \\ R^2 = 0.9561 \quad \hat{\rho} = 0.9778 \end{array} \right|$$



Observed (A_t) vs. predicted (P_t) values : 2016.

Figure 1: Forecast accuracy for Military Expenditure per Capita.

$$\left| \begin{array}{l} \hat{A}_t = 0.8716 P_t - 0.0498 \\ \text{t-stat} \quad (12.78) \quad (-0.54) \\ R^2 = 0.9533 \quad \hat{\rho} = 0.9764 \end{array} \right|$$



Observed (A_t) vs. predicted (P_t) values : 2017.

Notes

VIII. Conclusion

The model estimated by FMOLS estimator reveals such interesting results. In fact, the use of this nonparametric estimator is justified comparing with the DOLS estimator leading to a significant reduce in the degree of freedom in the size of panel data. A positive effect of GDP per capita appeared for all countries except Korea. Indeed, if GDP per capita increases 1%, then the military expenditure increases 1.22%, 2.77%, 0.99%, 1.14%, 1.02%, 1.13%, 0.65%, 0.87% and 0.66% respectively for the Arab word, Israel, USA, Canada, Japan, France, UK, India and Pakistan. It should be noted that the Israel comes in the first place among this group of 10 countries with regard to an increase military expenditure resulting in an increase in the GDP per capita. Furthermore, we have observed an almost similar behaviour regarding the impact of GDP per capita on the military expenditure, a different behaviour notices CO2 emissions per capita and its impact on military expenditure has been observed. Indeed, we found a positive effect for Korea, UK, India i.e. if the CO2 emissions per capita increases 1%, then the military expenditure increases 0.76%, 0.25% and 0.41% respectively. However, a negative effect is revealed for Israel, Japan, France only, an increase of 1% reduces the military expenditure per capita of 2.36%, 2.37% and 0.071% respectively, signalling the DOLS estimator yields the same findings for this countries. For other countries, a nonsignificant effect was observed. For the entire group, there is a very positive effect only for GDP per capita, and a 1% increase in GDP increases the military expenditure per capita of 0.98%. The importance of our proposal and especially the suggested *ordering algorithm* (see Appendix B) is the optimal choice of countries to have a high degree of homogeneity to be together in an econometric study of the panel. The problem is not limited to military expenditure, but for each economic variable we can use this algorithm leading to a better choice of such a tuple. Econometrics without model is seen for us as an approach of great interest to help the econometrics with model to better choose the panels by gaining homogeneity and consequently in the planning especially the predictions.

Appendix A

Table 8: Military Expenditure per Capita-Primary data

	Arab world	Israel	USA	Canada	Japan	Korea	France	UK	India	Pakistan
Average	168	1361	1117	307	225	256	580	559	14	25
Std.	100	494	578	152	148	205	283	288	10	10
Min	16	287	361	84	12	9	113	100	3	7
Max	472	2249	2283	623	475	740	1034	1076	39	46
CAGR	0.0757	0.0458	0.0345	0.0389	0.0782	0.1005	0.0463	0.0491	0.0576	0.0361

Notes

Table 9: Military Expenditure per Capita-Data in logarithm

	Arab world	Israel	USA	Canada	Japan	Korea	France	UK	India	Pakistan
Average	4.90	7.13	6.87	5.59	5.01	5.03	6.19	6.14	2.38	3.11
Std.	0.79	0.48	0.58	0.57	1.09	1.24	0.66	0.70	0.70	0.46
Min	2.80	5.66	5.89	4.43	2.45	2.20	4.73	4.61	1.10	1.97
Max	6.16	7.72	7.73	6.43	6.16	6.61	6.94	6.98	3.67	3.82
CAGR	0.0173	0.0068	0.0050	0.0072	0.0193	0.0242	0.0079	0.0085	0.0266	0.0122

Table 10: GDP per Capita-Primary Data

	Arab world	Israel	USA	Canada	Japan	Korea	France	UK	India	Pakistan
Average	2525	14556	26285	21945	24122	9064	20819	20769	486	501
Std.	2018	10126	15791	14704	15418	8467	13350	15178	410	336
Min	223	1648	4696	3411	1451	198	2532	1896	99	100
Max	7509	37540	54697	52497	48603	27811	45334	50134	1576	1317
CAGR	0.0793	0.0703	0.0548	0.0604	0.0736	0.1134	0.0635	0.0722	0.0620	0.0488

Table 11: GDP per Capita-Data in logarithm

	Arab world	Israel	USA	Canada	Japan	Korea	France	UK	India	Pakistan
Average	7.50	9.28	9.94	9.75	9.72	8.37	9.67	9.56	5.89	6.01
Std.	0.91	0.86	0.75	0.76	1.03	1.50	0.84	0.99	0.76	0.66
Min	5.41	7.41	8.45	8.13	7.28	5.29	7.84	7.55	4.59	4.61
Max	8.92	10.53	10.91	10.87	10.79	10.23	10.72	10.82	7.36	7.18
CAGR	0.0109	0.0077	0.0056	0.0062	0.0081	0.0144	0.0067	0.0077	0.0103	0.0079

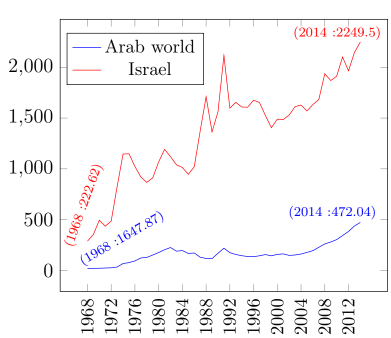
Table 12: Emission CO2 per Capita-Primary Data

	Arab world	Israel	USA	Canada	Japan	Korea	France	UK	India	Pakistan
Average	3.28	7.41	19.51	16.41	8.61	6.44	6.92	9.61	0.79	0.63
Std.	0.84	1.68	1.44	0.89	1.00	3.47	1.41	1.28	0.39	0.22
Min	1.54	4.93	16.30	14.62	5.57	1.21	4.57	6.50	0.35	0.31
Max	4.86	9.88	22.51	18.21	9.91	11.80	9.67	11.82	1.73	0.99
CAGR	0.0252	0.0102	-0.0032	0.0007	0.0118	0.0504	-0.0107	-0.0114	0.0351	0.0151

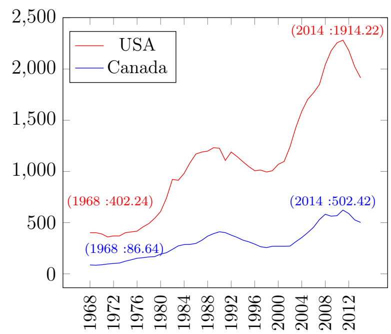
Table 13: Emission CO2 per Capita- Data in logarithm

	Arab world	Israel	USA	Canada	Japan	Korea	France	UK	India	Pakistan
Average	1.15	1.98	2.97	2.80	2.15	1.68	1.91	2.25	-0.35	-0.53
Std.	0.28	0.23	0.08	0.05	0.12	0.67	0.20	0.14	0.49	0.37
Min	0.43	1.59	2.79	2.68	1.72	0.19	1.52	1.87	-1.04	-1.18
Max	1.58	2.29	3.11	2.90	2.29	2.47	2.27	2.47	0.55	-0.01
CAGR	0.0285	0.0056	-0.0011	0.0003	0.0059	0.0573	-0.0061	-0.0054	(a)	(a)

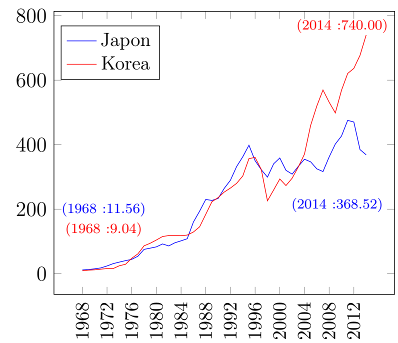
(a) : The data in natural logarithm contains negative values.



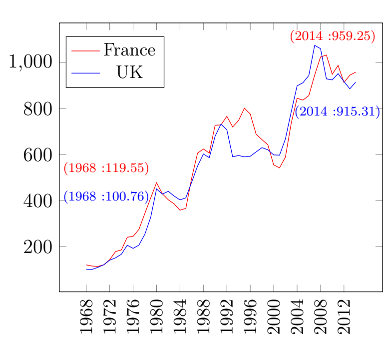
(a) MEXPC Ar vs. Is



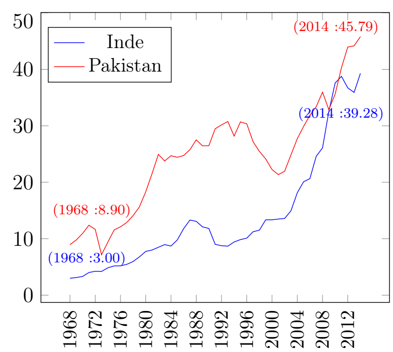
(b) MEXPC US vs. Ca



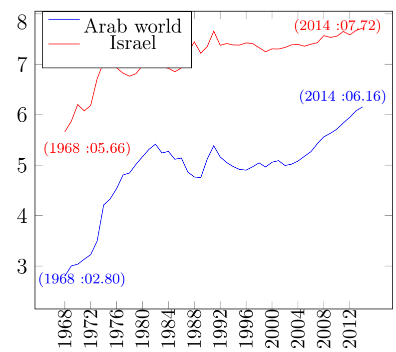
(c) MEXPC Ja vs. Ko



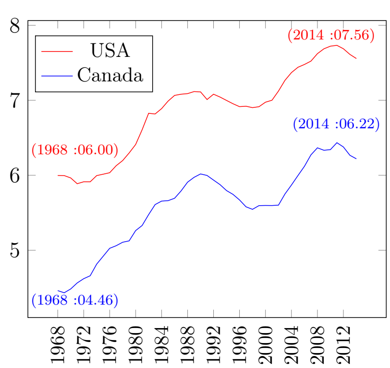
(d) MEXPC Fr vs. UK



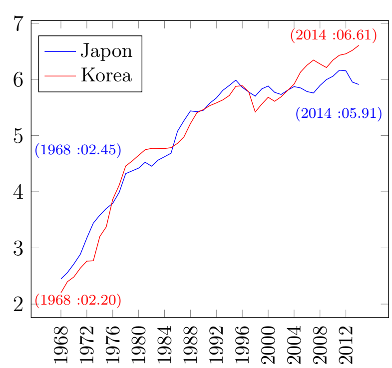
(e) MEXPC In vs. Pa



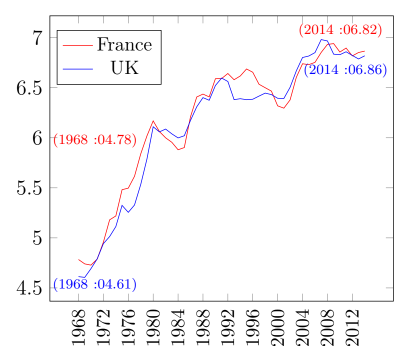
(f) LnMEXPC Ar vs. Is



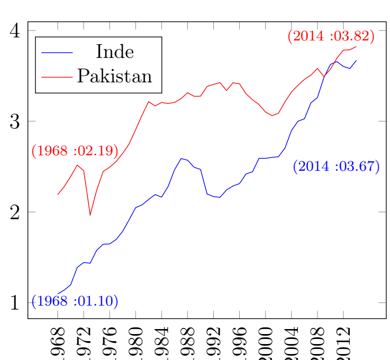
(g) LnMEXPC US vs. Ca



(h) LnMEXPC Ja vs. Ko



(i) LnMEXPC Fr vs. UK



(j) LnMEXPC In vs. Pa

Figure 2: Military Expenditures per Capita-Primary Data & Data in logarithm.

Notes

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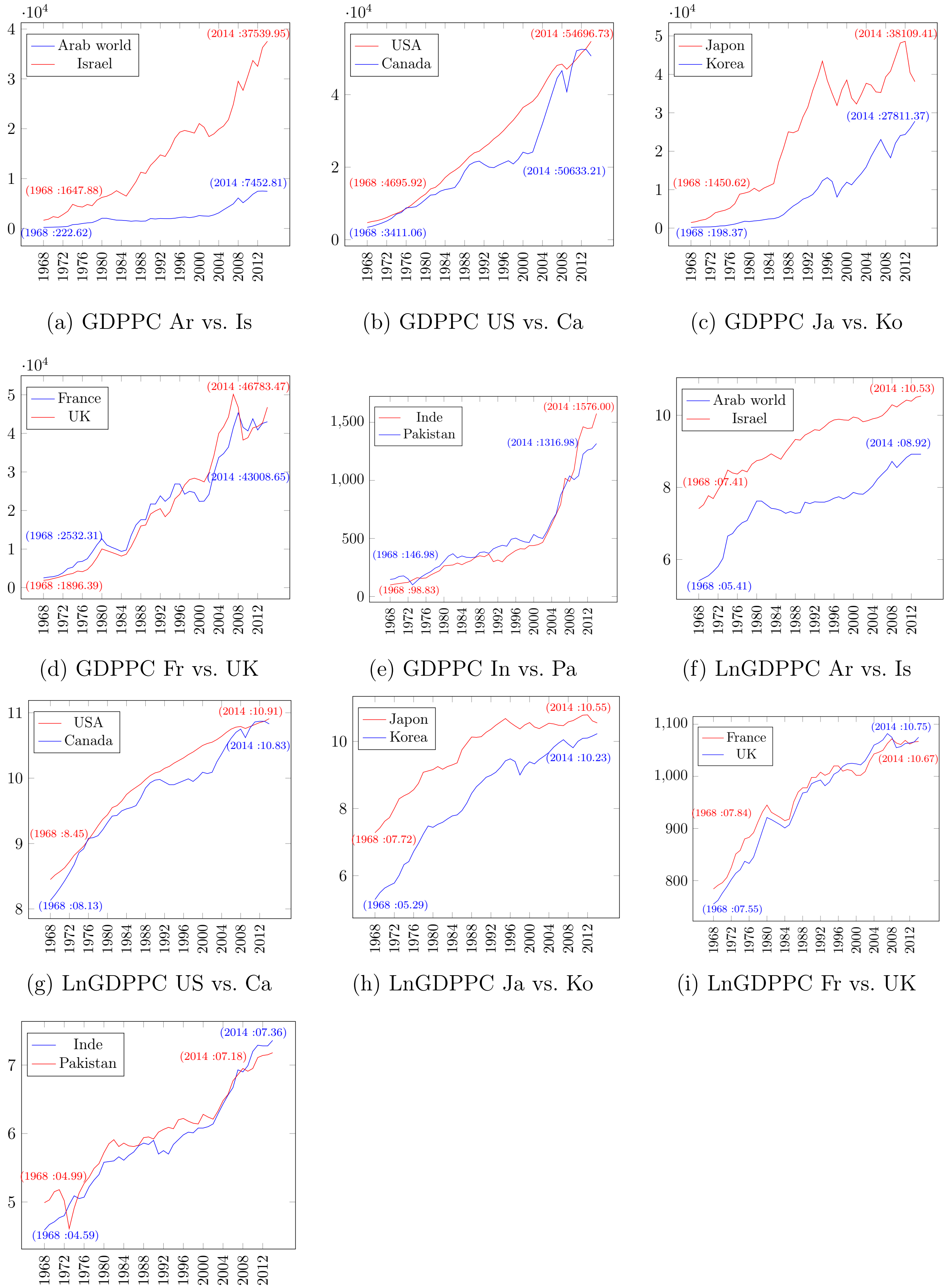


(a) GDPPC Ar vs. Is

(b) GDPPC US vs. Ca

(c) GDPPC Ja vs. Ko

(d) GDPPC Fr vs. UK

(e) GDPPC In vs. Pa

(f) LnGDPPC Ar vs. Is

(g) LnGDPPC US vs. Ca

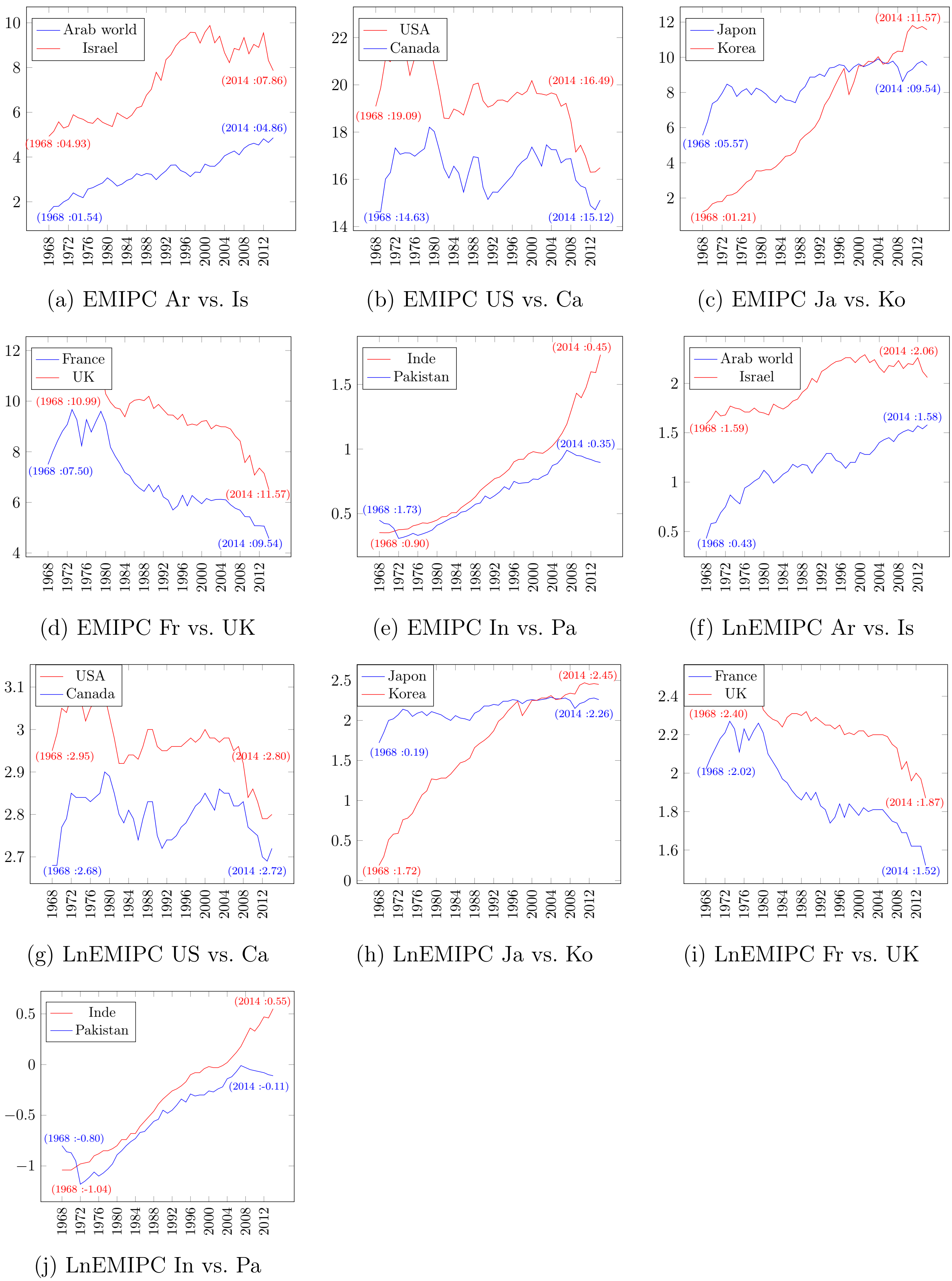
(h) LnGDPPC Ja vs. Ko

(i) LnGDPPC Fr vs. UK

(j) LnGDPPC In vs. Pa

Figure 3: GDP per Capita- Primary Data & Data in logarithm.

Notes



(a) EMIPC Ar vs. Is
(b) EMIPC US vs. Ca
(c) EMIPC Ja vs. Ko
(d) EMIPC Fr vs. UK
(e) EMIPC In vs. Pa
(f) LnEMIPC Ar vs. Is
(g) LnEMIPC US vs. Ca
(h) LnEMIPC Ja vs. Ko
(i) LnEMIPC Fr vs. UK
(j) LnEMIPC In vs. Pa

Figure 4: CO2 Emissions per Capita - Primary Data & Data in logarithm

Notes

Appendix B

Large data tables usually contain a large amount of information, which is partly hidden because the data are too complex to be easily interpreted. Principal Component Analysis (PCA) is a projection method that helps to extract the important information from the statistical data to represent it as a set of new orthogonal variables called *principal components*. So, in this way, the first principal component retains maximum variation that was present in the original components. The principal components are the *eigenvectors* of a covariance matrix, and hence they are orthogonal. The eigenvectors determine the directions of the new feature space, and the *eigenvalues* determine their magnitude. In other words, the eigenvalues explain the variance of the data along the new feature axes.

How do we choose the order of the countries using a PCA analysis? When we talk about a panel of several individuals, we do not give an order of individuals but we discuss the different topics of the analysis, such as the unit root tests, the individual and global cointegration, i.e. for the set of panel. After the estimation of residuals due to the Augmented Dickey-Fuller (ADF) equation and after using the Cross-sectional Dependence (CD) test, a strong dependence appeared between cross residuals and validated by CD_P and CD_{BP}. But this work did not take into account the order of the countries in the panel. Often, the PCA users look for hidden factors in a time series without respecting the order. In order to explain and understand the military expenditure per capita for a country, certainly we can consider many hidden factors to discriminate countries because the hidden factors are not the same. Cavatorta (2010) considers four factors that influence the military expenditure.

For example, in 2017, the United States, alone, has an annual military expenditure about 35.85% as a share of the total military expenditure of the world and its occupations go beyond its national borders to reach the whole planet. So there is a significant number of hidden factors, including GDP! It is not only the protection of its territory but the domination of the entire continent. A country like Singapore, for example, has military expenditure, but it is to protect its national achievements, especially economic development and the social welfare system. Briefly, the number of hidden factors is relative to each country. In other words, there is a *country effect* in the military expenditure. This effect would be fixed or not, and it needs a specific analysis considering the place of the country in the proposed ordering.

Since a factor reflects a criterion of homogeneity between individuals, and since we have fixed two explanatory variables (GDP per capita and CO2 emission per capita) to explain military expenditure, we dedicate this section to the PCA analysis applied on the cross residual data of the military expenditure, for 10 countries form 1968 to 2014, due to the ADF equation used in Section 5 by proposing an algorithm of the order for the panel components that we call *Ordering Algorithm*. In this section, the PCA method was used to extract a fixed number of components (two) and the computation of each ordering presented. The power to detect the heterogeneity i.e. how the two factors measure the two common factors among the ten countries. Without doubt, the two factors do not explain military expenditures in the same way, because it can have four factors for one country but only one factor for another.

The proposed Ordering Algorithm is a sequential procedure based on five steps. In this Algorithm, ${}^{i}_{j}\lambda_{max}$ (rsp. ${}^{i}_{j}\lambda_{min}$) represents the max (rsp. min) value of the Initial Eigenvalues-Cumulative Percentage Variance (CPV) for j-tuple of ordering i. Each eigenvalue represents the amount of variance in the original variables accounted for by each component. The Percentage of Variance is the ratio, expressed in percentage, of the variance accounted for by each component to the total variance in all of the variables. Finally, the Cumulative Percentage Variance (CPV) gives the percentage of variance represented by the first 2 components. The

Initial Eigenvalues-CPV presented in Tables 1 to 10 are computed by using the SPSS (Statistical Package for Social Sciences) program. The Ordering Algorithm involves the following five steps :

- **Step 1 :** We take the countries two by two with order, then we calculate ${}_2^1\lambda_{max}$ and ${}_2^1\lambda_{min}$ associated with all the 2-tuple; knowing that, in this step will be only one factor. Then we calculate ${}_2^1\lambda_{max}$ and ${}_2^1\lambda_{min}$ for the nine 2-tuple. We choose the 2-tuple corresponds to ${}_2^1\lambda_{max}$.

- **Step 2 :** Now, we set the number of factors to two. We take the couple chosen in step 1 then we introduce the remaining eight countries one by one. We calculate ${}_3^1\lambda_{max}$ and ${}_3^1\lambda_{min}$ for the eight 3-tuple. We choose the 3-tuple corresponds to ${}_3^1\lambda_{max}$.

- **Step 3 :** For each j-tuple, $j = 3, \ldots, 10$, we obtain the couples $({}_j^1\lambda_{min}, {}_j^1\lambda_{max})$.

- **Step 4 :** We repeat the same previous steps to choose the couple $({}_j^i\lambda_{min},\ {}_j^i\lambda_{max})$, for $i = 1, \ldots, 10$; $j = 3, \ldots, 10$, associated to the j-tuple from each i-ordering.

- **Step 5 :** We calculate the associated average of ${}_j\lambda_{min}$ and ${}_j\lambda_{max}$ by

$$
{}_j\bar{\lambda}_{min} = \frac{1}{10}\sum_{i=1}^{10} {}_j^i\lambda_{min} \text{ and } {}_j\bar{\lambda}_{max} = \frac{1}{10}\sum_{i=1}^{10} {}_j^i\lambda_{max}.
$$

Algorithm 1: Computation of $\left\{ {}_j\bar{\lambda}_{min},\ {}_j\bar{\lambda}_{max} \right\}_{j\in\{3,\ldots,n\}}$.

Data: The considered n-countries, the Initial Eigenvalues-CPV ${}_j^i\lambda$ and the couple $({}_j^i\lambda_{min},\ {}_j^i\lambda_{max})$, for $i \in \{1, \ldots, n\}$ and $j \in \{3, \ldots, n\}$.

Result: $\left\{ {}_j\bar{\lambda}_{min},\ {}_j\bar{\lambda}_{max} \right\}_{j\in\{3,\ldots,n\}}$.

```
begin
    for i = 1 to n do
        {}_j^i λ_min   0, {}_j^i λ_max   0, {}_j λ̄_min   0, {}_j λ̄_max   0
        for j = 3 to n do
            {}_j^i λ_min = min({}_j^i λ_1, ..., {}_j^i λ_{n-j+1})
            {}_j^i λ_max = max({}_j^i λ_1, ..., {}_j^i λ_{n-j+1})
            {}_j λ̄_min = {}_j λ̄_min + {}_j^i λ_min
            {}_j λ̄_max = {}_j λ̄_max + {}_j^i λ_max
        end
        {}_j λ̄_min = (1/n) {}_j^i λ_min
        {}_j λ̄_max = (1/n) {}_j^i λ_max
    end
end
```

Notes

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Notes

The Tables 1 to 10 represent, for each country, the Ordering graph and the computation of ${}^{i}_{j}\lambda_{max}$, ${}^{i}_{j}\lambda_{min}$ and the range for each j-tuple. We can based on the Tables 1 to 10 to compute, for example, the ${}_{3}\bar{\lambda}_{min}$. We have the following Table[6] :

3-tuple	${}^{i}_{3}\lambda_{min}$
(Ar, Is, Ja)	81.182
(Is, Ar, Ja)	81.182
(US, In, Fr)	74.639
(Ca, UK, Fr	80.392
(Ja, Fr, UK)	82.887
(Ko, Pa, Ja)	81.092
(Fr, UK, Ja)	89.213
(UK, Fr, Ja)	89.213
(In, Is, Ar)	76.959
(Pa, Ko, Ja)	81.092
	$\sum = 817.851$

Then ${}^{i}_{3}\bar{\lambda}_{min} = \dfrac{\sum_{i=1}^{10} {}^{i}_{3}\lambda_{min}}{10} = \dfrac{817.851}{10} \approx 81.785$. This means that, for a 3-tuple, for all applied ordering on the 10 countries, the power of discrimination of two imposed factors is of the order 81.78%.

Arab World Ordering	Initial Eigenvalues-CPV (%)		
Ar	${}^{1}_{j}\lambda_{max}$	${}^{1}_{j}\lambda_{min}$	Range
(Ar, Is)	71.773	50.698	21.075
(Ar, Is, Ja)	86.974	81.182	05.792
(Ar, Is, Ja, Fr)	73.607	67.124	06.483
(Ar, Is, Ja, Fr, UK)	66.781	59.071	07.710
(Ar, Is, Ja, Fr, UK, Ca)	60.541	55.841	04.700
(Ar, Is, Ja, Fr, UK, Ca, Pa)	54.595	52.174	02.421
(Ar, Is, Ja, Fr, UK, Ca, Pa, Ko)	50.431	48.370	02.061
(Ar, Is, Ja, Fr, UK, Ca, Pa, Ko, In)	46.435	45.287	01.148
(Ar, Is, Ja, Fr, UK, Ca, Pa, Ko, In, US)	42.391	42.391	00.000

Table 14: ${}^{1}_{j}\lambda_{max}$, ${}^{1}_{j}\lambda_{min}$ and the range for each j-tuple for Arab World Ordering.

6. In all Tables, we use the abbreviations : Ar for Arab Word ; Is for Israel ; US for United States ; Ca for Canada; Fr for France; UK for United Kingdom; Ja for Japan ; Ko for South-Korea ; In for India ; Pa for Pakistan.

Israel Ordering Is	Initial Eigenvalues-CPV (%) $^2_j\lambda_{max}$	 $^2_j\lambda_{min}$	 Range
(Is, Ar)	71.773	52.564	19.209
(Is, Ar, Ja)	86.974	81.182	05.792
(Is, Ar, Ja, Fr)	73.607	67.124	06.483
(Is, Ar, Ja, Fr, UK)	66.781	59.071	07.710
(Is, Ar, Ja, Fr, UK, Ca)	60.541	55.841	04.700
(Is, Ar, Ja, Fr, UK, Ca, Pa)	54.595	52.174	02.421
(Is, Ar, Ja, Fr, UK, Ca, Pa, Ko)	50.431	48.370	02.061
(Is, Ar, Ja, Fr, UK, Ca, Pa, Ko, In)	46.435	45.287	01.148
(Is, Ar, Ja, Fr, UK, Ca, Pa, Ko, In, US)	42.391	42.391	00.000

Table 15: $^2_j\lambda_{max}$, $^2_j\lambda_{min}$ and the range for each j-tuple for Israel Ordering.

United States Ordering US	Initial Eigenvalues-CPV (%) $^3_j\lambda_{max}$	 $^3_j\lambda_{min}$	 Range
(US, In)	61.839	52.910	08.929
(US, In, Fr)	77.672	74.639	03.033
(US, In, Fr, UK)	73.773	62.269	11.504
(US, In, Fr, UK, Is)	65.704	60.950	04.754
(US, In, Fr, UK, Is, Ar)	59.420	54.985	04.435
(US, In, Fr, UK, Is, Ar, Ca)	50.702	49.711	00.991
(US, In, Fr, UK, Is, Ar, Ca, Pa)	49.058	48.201	00.857
(US, In, Fr, UK, Is, Ar, Ca, Pa, Ko)	45.969	44.836	01.133
(US, In, Fr, UK, Is, Ar, Ca, Pa, Ko, Ja)	42.391	42.391	00.000

Table 16: $^3_j\lambda_{max}$, $^3_j\lambda_{min}$ and the range for each j-tuple for United States Ordering.

Canada Ordering Ca	Initial Eigenvalues-CPV (%) $^4_j\lambda_{max}$	 $^4_j\lambda_{min}$	 Range
(Ca, UK)	70.503	55.630	14.873
(Ca, UK, Fr)	90.193	80.392	09.801
(Ca, UK, Fr, Pa)	79.655	72.973	06.682
(Ca, UK, Fr, Pa, Ko)	72.638	65.081	07.557
(Ca, UK, Fr, Pa, Ko, In)	63.781	61.471	02.310
(Ca, UK, Fr, Pa, Ko, In, Ja)	55.915	54,843	01.072
(Ca, UK, Fr, Pa, Ko, In, Ja, Ar)	51.524	49.239	02.285
(Ca, UK, Fr, Pa, Ko, In, Ja, Ar, US)	46.695	46.435	00.260
(Ca, UK, Fr, Pa, Ko, In, Ja, Ar, US, Is)	42.391	42.391	00.000

Table 17: $^4_j\lambda_{max}$, $^4_j\lambda_{min}$ and the range for each j-tuple for Canada Ordering.

Notes

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Japan Ordering	Initial Eigenvalues-CPV (%)		
Ja	${}^{5}_{j}\lambda_{max}$	${}^{5}_{j}\lambda_{min}$	Range
(Ja, Fr)	68.470	52.510	15.960
(Ja, Fr, UK)	91.501	82.887	08.614
(Ja, Fr, UK, Ar)	76.876	72.241	04.635
(Ja, Fr, UK, Ar, Ca)	67.655	61.606	06.049
(Ja, Fr, UK, Ar, Ca, Pa)	61.668	57.900	03.768
(Ja, Fr, UK, Ar, Ca, Pa, Ko)	56.271	54.094	02.177
(Ja, Fr, UK, Ar, Ca, Pa, Ko, In)	51.524	50.019	01.505
(Ja, Fr, UK, Ar, Ca, Pa, Ko, In, US)	46.695	46.435	00.260
(Ja, Fr, UK, Ar, Ca, Pa, Ko, In, US, Is)	42.391	42.391	00.000

Table 18: ${}^{5}_{j}\lambda_{max}$, ${}^{5}_{j}\lambda_{min}$ and the range for each j-tuple for France Ordering.

Korea Ordering	Initial Eigenvalues-CPV (%)		
Ko	${}^{6}_{j}\lambda_{max}$	${}^{6}_{j}\lambda_{min}$	Range
(Ko, Pa)	71.628	52.093	19.535
(Ko, Pa, Ja)	83.760	81.092	02.668
(Ko, Pa, Ja, Ar)	72.324	64.052	08.272
(Ko, Pa, Ja, Ar, Ca)	64.995	58.763	06.232
(Ko, Pa, Ja, Ar, Ca, Fr)	58.271	55.278	02.993
(Ko, Pa, Ja, Ar, Ca, Fr, UK)	56.271	50.846	05.425
(Ko, Pa, Ja, Ar, Ca, Fr, UK, In)	51.524	50.019	01.505
(Ko, Pa, Ja, Ar, Ca, Fr, UK, In, US)	46.695	46.435	00.260
(Ko, Pa, Ja, Ar, Ca, Fr, UK, In, US, Is)	42.391	42.391	00.000

Table 19: ${}^{6}_{j}\lambda_{max}$, ${}^{6}_{j}\lambda_{min}$ and the range for each j-tuple for United Kingdom Ordering.

France Ordering	Initial Eigenvalues-CPV (%)		
Fr	${}^{7}_{j}\lambda_{max}$	${}^{7}_{j}\lambda_{min}$	Range
(Fr, UK)	83.814	52.834	30.980
(Fr, UK, Ja)	91.501	89.213	02.288
(Fr, UK, Ja, Ar)	76.876	72.241	04.635
(Fr, UK, Ja, Ar, Ca)	67.655	61.606	06.049
(Fr, UK, Ja, Ar, Ca, Is)	60.541	57.900	02.641
(Fr, UK, Ja, Ar, Ca, Is, Pa)	54.595	52.174	02.421
(Fr, UK, Ja, Ar, Ca, Is, Pa, Ko)	50.431	48.370	02.061
(Fr, UK, Ja, Ar, Ca, Is, Pa, Ko, In)	46.435	45.287	01.148
(Fr, UK, Ja, Ar, Ca, Is, Pa, Ko, In, US)	42.391	42.391	00.000

Table 20: ${}^{7}_{j}\lambda_{max}$, ${}^{7}_{j}\lambda_{min}$ and the range for each j-tuple for Japan Ordering.

United Kingdom Ordering	Initial Eigenvalues-CPV (%)		
UK	${}^{8}_{j}\lambda_{max}$	${}^{8}_{j}\lambda_{min}$	Range
(UK, Fr)	83.814	52.093	31.721
(UK, Fr, Ja)	91.501	89.213	02.288
(UK, Fr, Ja, Ar)	76.876	72.241	04.635
(UK, Fr, Ja, Ar, Ca)	67.655	61.606	06.049
(UK, Fr, Ja, Ar, Ca, Is)	60.541	57.900	02.641
(UK, Fr, Ja, Ar, Ca, Is, Pa)	54.595	52.174	02.421
(UK, Fr, Ja, Ar, Ca, Is, Pa, Ko)	50.431	48.370	02.061
(UK, Fr, Ja, Ar, Ca, Is, Pa, Ko, In)	46.435	45.287	01.148
(UK, Fr, Ja, Ar, Ca, Is, Pa, Ko, In, US)	42.391	42.391	00.000

Table 21: ${}^{8}_{j}\lambda_{max}$, ${}^{8}_{j}\lambda_{min}$ and the range for each j-tuple for Korea Ordering.

India Ordering	Initial Eigenvalues-CPV (%)		
In	${}^{9}_{j}\lambda_{max}$	${}^{9}_{j}\lambda_{min}$	Range
(In, Is)	65.286	50.698	14.588
(In, Is, Ar)	84.623	76.959	07.664
(In, Is, Ar, Ca)	72.771	68.012	04.759
(In, Is, Ar, Ca, UK)	65.135	58.758	06.377
(In, Is, Ar, Ca, UK, Fr)	61.616	54.775	06.841
(In, Is, Ar, Ca, UK, Fr, Pa)	54.731	53.974	00.757
(In, Is, Ar, Ca, UK, Fr, Pa, Ko)	50.745	49.058	01.687
(In, Is, Ar, Ca, UK, Fr, Pa, Ko, Ja)	46.435	45.969	00.466
(In, Is, Ar, Ca, UK, Fr, Pa, Ko, Ja, US)	42.391	42.391	00.000

Table 22: ${}^{9}_{j}\lambda_{max}$, ${}^{9}_{j}\lambda_{min}$ and the range for each j-tuple for India Ordering.

Pakistan Ordering	Initial Eigenvalues-CPV (%)		
Pa	${}^{10}_{j}\lambda_{max}$	${}^{10}_{j}\lambda_{min}$	Range
(Pa, Ko)	71.628	52.510	19.118
(Pa, Ko, Ja)	83.760	81.092	02.668
(Pa, Ko, Ja, Ar)	72.324	64.052	08.272
(Pa, Ko, Ja, Ar, Ca)	64.995	58.763	06.232
(Pa, Ko, Ja, Ar, Ca, Fr)	58.271	55.278	02.993
(Pa, Ko, Ja, Ar, Ca, Fr, Ca)	56.271	50.846	05.425
(Pa, Ko, Ja, Ar, Ca, Fr, Ca, In)	51.524	50.019	01.505
(Pa, Ko, Ja, Ar, Ca, Fr, Ca, In, US)	46.695	46.435	00.260
(Pa, Ko, Ja, Ar, Ca, Fr, Ca, In, US, Is)	42.391	42.391	00.000

Table 23: ${}^{10}_{j}\lambda_{max}$, ${}^{10}_{j}\lambda_{min}$ and the range for each j-tuple for Pakistan Ordering.

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