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Enhancement of Squeezing in a Coherently Driven Degenerate Three-Level Laser with a Closed Cavity

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Keywords: *operator dynamics; quadrature squeezing; power spectrum.*

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Samuel Mosisa [α], Tamirat Abebe [σ], Milkessa Gebeyehu [ρ] & Gelana Chibsa [ω]

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I. Introducion

Squeezed states of light has played a crucial role in the development of quantum physics. Squeezing is one of the nonclassical features of light that have been extensively studied by several authors [1-8]. In a squeezed state the quantum noise in one quadrature is below the vacuum-state level or the coherent-state level at the expense of enhanced fluctuations in the conjugate quadrature, with the product of the uncertainties in the two quadratures satisfying the uncertainty relation [1, 2, 4, 9]. Because of the quantum noise reduction achievable below the vacuum level, squeezed light has potential applications in the detection of week signals and in low-noise communications [1, 2]. Squeezed light can be generated by various quantum optical processes such as subharmonic generations [1-5, 10-12], four-wave mixing [13, 14], resonance fluorescence [6, 7], second harmonic generation [8, 15], and three-level laser under certain conditions [1, 3, 4,9, 16-27]. Hence it proves useful to find some convenient means of generating a bright squeezed light.

A three-level laser is a quantum optical device in which light is generated by three-level atoms in a cavity usually coupled to a vacuum reservoir via a single-port mirror. In one model of a three-level laser, three-level atoms initially prepared in a coherent superposition of the top and bottom levels are injected into a cavity and then removed from the cavity after they have decayed due to spontaneous emission [9, 16-21]. In another model of a three-level laser, the top and bottom levels of the three-level atoms injected into a cavity are coupled by coherent light [22-27]. It is found that a three-level laser in either model generates squeezed light under certain conditions [28-34]. The superposition or the coupling of the top and bottom levels is responsible for the squeezed of the generated light [35-38]. It appears to be quite difficult to prepare the atoms in a coherent superposition of the top and bottom levels before they are injected into the cavity. In addition, it should certainly be hard to find out that the atoms have decayed spontaneously before they are removed from the cavity.

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In order to avoid the aforementioned problems, Fesseha [28] have considered that N two-level atoms available in a closed cavity are pumped to the top level by means of electron bombardment. He has shown that the light generated by this laser operating well above threshold is coherent and the light generated by the same laser operating below threshold is chaotic light. In addition, Fesseha [28] has studied the squeezing and statistical properties of the light produced by a degenerate three-level laser with the atoms in a closed cavity and pumped by electron bombardment. He has shown that the maximum quadrature squeezing of the light generated by the laser, operating far below threshold, is 50% below the vacuum-state level.

In this paper, we investigate the steady-state analysis of the squeezing and statistical properties of the light generated by a coherently driven degenerate three-level laser with a closed cavity which is coupled to a single-mode vacuum reservoir via a single-port mirror. We carry out our calculation by putting the noise operators associated with the vacuum reservoir in normal order and by taking into consideration the interaction of the three-level atoms with the vacuum reservoir.

II. The Master Equation

Let us consider a system of N degenerate three-level atoms in cascade configuration are available in a closed cavity and interacting with the two (degenerate) cavity modes. The top and bottom levels of the three-level atoms are coupled by coherent light. When a degenerate three-level atom in cascade configuration decays from the top level to the bottom levels via the middle level, two photons of the same frequency are emitted. For the sake of convenient, we denote the top, middle, and bottom levels of these atoms by $|a\rangle_k$, $|b\rangle_k$, and $|c\rangle_k$, respectively. We wish to represent the light emitted from the top level by $\hat{a}_1$ and the light emitted from the middle by $\hat{a}_2$. In addition, we assume that the two cavity modes a_1 and a_2 to be at resonance with the two transitions $|a\rangle_k \rightarrow |b\rangle_k$ and $|b\rangle_k \rightarrow |c\rangle_k$, with direct transitions between levels $|a\rangle_k$ and $|c\rangle_k$to be dipole forbidden.

The interaction of one of the three-level atoms with light modes a_1 and a_2 can be described at resonance by the Hamiltonian

$$\hat{H} = ig[\hat{\sigma}_a^{\dagger k}\hat{a}_1 - \hat{a}_1^{\dagger}\hat{\sigma}_a^k + \hat{\sigma}_b^{\dagger k}\hat{a}_2 - \hat{a}_2^{\dagger}\hat{\sigma}_b^k], \tag{1}$$

where

$$\hat{\sigma}_a^k = |b\rangle_{k\ k}\langle a|, \tag{2}$$

$$\hat{\sigma}_b^k = |c\rangle_{k\ k}\langle b|, \tag{3}$$

are the lowering atomic operators, g is the coupling constant between the atom and the light mode a_1 or light mode a_2, and $\hat{a}_1$ and $\hat{a}_2$ are the annihilation operators for light modes a_1 and a_2. And the interaction of the three-level atom with the driving coherent light can be described at resonance by the Hamiltonian

$$\hat{H} = \frac{i\Omega}{2}[\hat{\sigma}_c^{\dagger k} - \hat{\sigma}_c^k], \tag{4}$$

where $\hat{\sigma}_c^k = |c\rangle_k\, {}_k\langle a|$, and $\Omega = 2\varepsilon\xi$, in which ε considered to be real and constant, is the amplitude of the driving coherent light, and ξ is the coupling constant between the driving coherent light and the three-level atom.

Thus upon combining Eqs. (1) and (4), the interaction of a degenerate three-level atom with the coherent light and with the light modes a_1 and a_2 can be described by the Hamiltonian

$$\hat{H} = ig[\hat{\sigma}_a^{\dagger k}\hat{a}_1 - \hat{a}_1^\dagger\hat{\sigma}_a^k + \hat{\sigma}_b^{\dagger k}\hat{a}_2 - \hat{a}_2^\dagger\hat{\sigma}_b^k] + \frac{i\Omega}{2}[\hat{\sigma}_c^{\dagger k} - \hat{\sigma}_c^k]. \tag{5}$$

We assume that the laser cavity is coupled to a vacuum reservoir via a single-port mirror. In addition, we carry out our calculation by putting the noise operators associated with the vacuum reservoir in normal order. Thus, the noise operators will not have any effect on the dynamics of the cavity mode operators [1, 28, 29]. Therefore, with the help of the expression (1), one can drop the noise operators and write the quantum Langevin equations for the operators $\hat{a}_1$ and $\hat{a}_2$ as

$$\frac{d\hat{a}_1}{dt} = -\frac{\kappa}{2}\hat{a}_1 - i[\hat{a}_1, \hat{H}], \tag{6}$$

$$\frac{d\hat{a}_2}{dt} = -\frac{\kappa}{2}\hat{a}_2 - i[\hat{a}_2, \hat{H}], \tag{7}$$

where κ is the cavity damping constant. With the aid of Eq. (1), one can easily obtain

$$\frac{d\hat{a}_1}{dt} = -\frac{\kappa}{2}\hat{a}_1 - g\hat{\sigma}_a^k, \tag{8}$$

$$\frac{d\hat{a}_2}{dt} = -\frac{\kappa}{2}\hat{a}_2 - g\hat{\sigma}_b^k. \tag{9}$$

III. Equations of Evolution of Atomic Oprators

The procedure of normal ordering the noise operators renders the vacuum reservoir to be a noiseless physical entity. We uphold the view point that the notion of a noiseless vacuum reservoir would turn out to be compatible with observation [29]. Furthermore, employing the relation

$$\frac{d}{dt}\langle\hat{A}\rangle = -i\langle[\hat{A}, \hat{H}]\rangle \tag{10}$$

along with Eq. (1), one can readily establish that

$$\frac{d}{dt}\langle\hat{\sigma}_a^k\rangle = g[\langle\hat{\eta}_b^k\hat{a}_1\rangle - \langle\hat{\eta}_a^k\hat{a}_1\rangle + \langle\hat{a}_2^\dagger\hat{\sigma}_c^k\rangle] + \frac{\Omega}{2}\langle\hat{\sigma}_b^{\dagger k}\rangle, \tag{11}$$

$$\frac{d}{dt}\langle\hat{\sigma}_b^k\rangle = g[\langle\hat{\eta}_c^k\hat{a}_2\rangle - \langle\hat{\eta}_b^k\hat{a}_2\rangle - \langle\hat{a}_1^\dagger\hat{\sigma}_c^k\rangle] - \frac{\Omega}{2}\langle\hat{\sigma}_a^{\dagger k}\rangle, \tag{12}$$

$$\frac{d}{dt}\langle\hat{\sigma}_c^k\rangle = g[\langle\hat{\sigma}_b^k\hat{a}_1\rangle - \langle\hat{\sigma}_a^k\hat{a}_2\rangle] + \frac{\Omega}{2}[\langle\hat{\eta}_c^k\rangle - \langle\hat{\eta}_a^k\rangle], \tag{13}$$

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$$\frac{d}{dt}\langle\hat{\eta}_a^k\rangle = g[\langle\hat{\sigma}_a^{\dagger k}\hat{a}_1\rangle + \langle\hat{a}_1^\dagger\hat{\sigma}_a^k\rangle] + \frac{\Omega}{2}[\langle\hat{\sigma}_c^k\rangle + \langle\hat{\sigma}_c^{\dagger k}\rangle], \tag{14}$$

$$\frac{d}{dt}\langle\hat{\eta}_b^k\rangle = g[\langle\hat{\sigma}_b^\dagger\hat{a}_2\rangle + \langle\hat{a}_2^\dagger\hat{\sigma}_b^k\rangle - \langle\hat{\sigma}_a^{\dagger k}\hat{a}_1\rangle - \langle\hat{a}_1^\dagger\hat{\sigma}_a^k\rangle], \tag{15}$$

$$\frac{d}{dt}\langle\hat{\eta}_c^k\rangle = -g[\langle\hat{\sigma}_b^\dagger\hat{a}_1\rangle + \langle\hat{a}_2^\dagger\hat{\sigma}_b^k\rangle] - \frac{\Omega}{2}[\langle\hat{\sigma}_c^k\rangle + \langle\hat{\sigma}_c^{\dagger k}\rangle], \tag{16}$$

where $\hat{\eta}_a^k = |a\rangle_{k\,k}\langle a|$, $\hat{\eta}_b^k = |b\rangle_{k\,k}\langle b|$, $\hat{\eta}_c^k = |c\rangle_{k\,k}\langle c|$.

It can be noted that expressions (11)-(16) are nonlinear and coupled differential equations. Therefore, it is not possible to obtain exact solutions. Then, employing the large-time approximation scheme on Eqs. (8) and (9), one obtains

$$\hat{a}_1 = -\frac{2g}{\kappa}\hat{\sigma}_a^k, \tag{17}$$

$$\hat{a}_2 = -\frac{2g}{\kappa}\hat{\sigma}_b^k. \tag{18}$$

Now introducing Eqs. (17) and (18) into (11)-(16) and sum over the N three-level atoms, it is possible to see that

$$\frac{d}{dt}\langle\hat{m}_a\rangle = -\gamma_c\langle\hat{m}_a\rangle + \frac{\Omega}{2}\langle\hat{m}_b^\dagger\rangle, \tag{19}$$

$$\frac{d}{dt}\langle\hat{m}_b\rangle = -\frac{\gamma_c}{2}\langle\hat{m}_b\rangle - \frac{\Omega}{2}\langle\hat{m}_a^\dagger\rangle, \tag{20}$$

$$\frac{d}{dt}\langle\hat{m}_c\rangle = -\frac{\gamma_c}{2}\langle\hat{m}_c\rangle + \frac{\Omega}{2}[\langle\hat{N}_c\rangle - \langle\hat{N}_a\rangle], \tag{21}$$

$$\frac{d}{dt}\langle\hat{N}_a\rangle = -\gamma_c\langle\hat{N}_a\rangle + \frac{\Omega}{2}[\langle\hat{m}_c\rangle + \langle\hat{m}_c^\dagger\rangle], \tag{22}$$

$$\frac{d}{dt}\langle\hat{N}_b\rangle = -\gamma_c\langle\hat{N}_b\rangle + \gamma_c\langle\hat{N}_a\rangle, \tag{23}$$

$$\frac{d}{dt}\langle\hat{N}_c\rangle = -\gamma_c\langle\hat{N}_b\rangle - \frac{\Omega}{2}[\langle\hat{m}_c\rangle + \langle\hat{m}_c^\dagger\rangle], \tag{24}$$

in which

$$\gamma_c = \frac{4g^2}{\kappa} \tag{25}$$

is the stimulated emission decay constant, $\hat{m}_a = \sum_{k=1}^N \hat{\sigma}_a^k$, $\hat{m}_b = \sum_{k=1}^N \hat{\sigma}_b^k$, $\hat{m}_c = \sum_{k=1}^N \hat{\sigma}_c^k$, $\hat{N}_a = \sum_{k=1}^N \hat{\eta}_a^k$, $\hat{N}_b = \sum_{k=1}^N \hat{\eta}_b^k$, $\hat{N}_c = \sum_{k=1}^N \hat{\eta}_c^k$, with the operators $\hat{N}_a$, $\hat{N}_b$, and $\hat{N}_c$ representing the number of atoms in the top, middle, and bottom levels, respectively.

Furthermore, employing the completeness relation

$$\hat{\eta}_a^k + \hat{\eta}_b^k + \hat{\eta}_c^k = \hat{I}, \tag{26}$$

one can easily arrive at

$$\langle \hat{N}_a \rangle + \langle \hat{N}_b \rangle + \langle \hat{N}_c \rangle = N. \tag{27}$$

Furthermore, applying the definition given by (2) and setting for any k

$$\hat{\sigma}_a^k = |b\rangle\langle a|, \tag{28}$$

we have

$$\hat{m}_a = N|b\rangle\langle a|. \tag{29}$$

Following the same procedure, one can easily find $\hat{m}_b = N|c\rangle\langle b|$, $\hat{m}_c = N|c\rangle\langle a|$, $\hat{N}_a = N|a\rangle\langle a|$, $\hat{N}_b = N|b\rangle\langle b|$, $\hat{N}_c = N|c\rangle\langle c|$.

Moreover, using the definition

$$\hat{m} = \hat{m}_a + \hat{m}_b \tag{30}$$

and taking into account the above relations, we observe that

$$\hat{m}^\dagger \hat{m} = N[\hat{N}_a + \hat{N}_b], \tag{31}$$

$$\hat{m}\hat{m}^\dagger = N[\hat{N}_b + \hat{N}_c], \tag{32}$$

$$\hat{m}^2 = N\hat{m}_c. \tag{33}$$

Now upon adding Eqs. (8) and (9), we have

$$\frac{d}{dt}\hat{a}(t) = -\frac{\kappa}{2}\hat{a}(t) - g[\hat{\sigma}_a^k(t) + \hat{\sigma}_b^k(t)], \tag{34}$$

where

$$\hat{a}(t) = \hat{a}_1(t) + \hat{a}_2(t). \tag{35}$$

In the presence of N three-level atoms, we can rewrite Eq. (34) as

$$\frac{d}{dt}\hat{a}(t) = -\frac{\kappa}{2}\hat{a}(t) + \lambda'\hat{m}(t), \tag{36}$$

in which λ' is a constant whose value remains to be determined. The steady-state solution of Eq (34) is

$$\hat{a}(t) = -\frac{2g}{\kappa}[\hat{\sigma}_a^k(t) + \hat{\sigma}_b^k(t)]. \tag{37}$$

Taking into account Eq. (37) and its adjoint, the commutation relation for the cavity mode operator is found to be

$$[\hat{a}, \hat{a}^\dagger] = \frac{\gamma_c}{\kappa}[\hat{\eta}_c - \hat{\eta}_a], \tag{38}$$

and on summing over all atoms, we have

$$[\hat{a}, \hat{a}^{\dagger}] = \frac{\gamma_c}{\kappa}[\hat{N}_c - \hat{N}_a], \tag{39}$$

where

$$[\hat{a}, \hat{a}^{\dagger}] = \sum_{k=1}^{N}[\hat{a}, \hat{a}^{\dagger}]_k \tag{40}$$

stands for the commutator $(\hat{a}, \hat{a}^{\dagger})$ when the superposed light mode a is interacting with all the N three-level atoms. On the other hand, using the steady-state solution of Eq. (36), one can verify that

$$[\hat{a}, \hat{a}^{\dagger}] = N\left[\frac{2\lambda'}{\kappa}\right]^2\left(\hat{N}_c - \hat{N}_a\right). \tag{41}$$

Thus inspection of Eqs. (39) and (41) show that

$$\lambda' = \pm\frac{g}{\sqrt{N}}. \tag{42}$$

Hence in view of this result, Eq. (36) can be rewritten as

$$\frac{d}{dt}\hat{a}(t) = -\frac{\kappa}{2}\hat{a}(t) + \frac{g}{\sqrt{N}}\hat{m}(t). \tag{43}$$

IV. Solutions of the Expectation Values of the Cavity and Atomic Mode Operators

In order to determine the mean photon number and the variance of the photon number, and the quadrature squeezing of a single-mode cavity light in any frequency interval at steady state, we first need to calculate the solution of the equations of evolution of the expectation values of the atomic operators and cavity mode operators. To this end, the expectation values of the solution of Eq. (43) is expressible as

$$\langle\hat{a}(t)\rangle = \langle\hat{a}(0)\rangle e^{-\kappa t/2} + \frac{g}{\sqrt{N}}e^{-\kappa t/2}\int_0^t dt' e^{-\kappa t'/2}\langle\hat{m}(t')\rangle. \tag{44}$$

We next wish to obtain the expectation value of the expression of $\hat{m}(t)$ that appear in Eq. (44). Thus applying the large-time approximation scheme to Eq. (20), we get

$$\langle\hat{m}_b\rangle = -\frac{\Omega}{\gamma_c}\langle\hat{m}_a^{\dagger}\rangle. \tag{45}$$

Upon substituting the adjoint of this into Eq. (19), we have

$$\frac{d}{dt}\langle\hat{m}_a(t)\rangle = -\mu\langle\hat{m}_a(t)\rangle, \tag{46}$$

where

$$\mu = \frac{2\gamma_c^2 + \Omega^2}{2\gamma_c}. \tag{47}$$

We notice that the solution of Eq. (46) for μ different from zero at steady state is

$$\langle\hat{m}_a(t)\rangle = 0. \tag{48}$$

In a similar manner, applying the large-time approximation scheme to Eq. (19), we obtain

$$\langle \hat{m}_a \rangle = -\frac{\Omega}{2\gamma_c} \langle \hat{m}_b^\dagger \rangle. \tag{49}$$

With the aid of the adjoint of Eq. (49), one can put Eq. (20) in the form

$$\frac{d}{dt}\langle \hat{m}_b(t) \rangle = -\frac{\mu}{2}\langle \hat{m}_b(t) \rangle. \tag{50}$$

We also note that for μ different from zero, the solution of Eq. (50) is found to be

$$\langle \hat{m}_b(t) \rangle = 0. \tag{51}$$

Upon adding Eqs. (46)and (50), we find

$$\frac{d}{dt}\langle \hat{m}(t) \rangle = -\frac{\mu}{2}\langle \hat{m}(t) \rangle - \frac{\mu}{2}\langle \hat{m}_a(t) \rangle. \tag{52}$$

We note that in view of Eq. (48) with the assumption the atoms initially in the bottom level, the solution of Eq. (52) turns out at steady state to be

$$\langle \hat{m}(t) \rangle = 0. \tag{53}$$

Now in view of Eq. (53) and with the assumption that the cavity light is initially in a vacuum state, Equation Eq. (44) goes over into

$$\langle \hat{a}(t) \rangle = 0. \tag{54}$$

Therefore, in view of the linear equations described by expressions (43) with (54), we claim that $\hat{a}(t)$ is a Gaussian variable with zero mean. We finally seek to determine the solution of the expectation values of the atomic operators at steady state. Moreover, the steady-state solution of Eqs. (21)-(24) yields

$$\langle \hat{N}_a \rangle_{ss} = \left[\frac{\Omega^2}{\gamma_c^2 + 3\Omega^2}\right] N, \tag{55}$$

$$\langle \hat{N}_b \rangle_{ss} = \left[\frac{\Omega^2}{\gamma_c^2 + 3\Omega^2}\right] N, \tag{56}$$

$$\langle \hat{N}_c \rangle_{ss} = \left[\frac{\gamma_c^2 + \Omega^2}{\gamma_c^2 + 3\Omega^2}\right] N, \tag{57}$$

$$\langle \hat{m}_c \rangle_{ss} = \left[\frac{\Omega\gamma_c}{\gamma_c^2 + 3\Omega^2}\right] N. \tag{58}$$

Up on setting $\eta = \frac{\Omega}{\gamma_c}$, we can rewrite Eqs. (55)-(58) as

$$\langle \hat{N}_a \rangle_{ss} = \left[\frac{\eta^2}{1 + 3\eta^2}\right] N, \tag{59}$$

$$\langle \hat{N}_b \rangle_{ss} = \left[\frac{\eta^2}{1 + 3\eta^2}\right] N, \tag{60}$$

$$\langle \hat{N}_c \rangle_{ss} = \left[\frac{1+\eta^2}{1+3\eta^2}\right] N, \tag{61}$$

$$\langle \hat{m}_c \rangle_{ss} = \left[\frac{\eta}{1+3\eta^2}\right] N. \tag{62}$$

Initially (when $\Omega = 0$), all the atoms are on the lower level ($\langle \hat{N}_c \rangle_{ss} = N$) while the number of atoms on the top and intermediate levels are zero.

V. Photon Statistics

Here we seek to obtain the global (local) mean photon number and the global (local) variance of the photon number for a single-mode cavity light beam at steady state.

a) *The Global Mean Photon Number*

To learn about the brightness of the generated light, it is necessary to study the mean number of photon pairs describing the two-mode cavity radiation that can be defined as

$$\bar{n} = \langle \hat{a}^\dagger \hat{a} \rangle. \tag{63}$$

On account of the steady state solution of (43) together with (31), the mean photon number of the two-mode cavity light is expressible as

$$\bar{n} = \frac{\gamma_c}{k}\left[\langle \hat{N}_a \rangle_{ss} + \langle \hat{N}_b \rangle_{ss}\right]. \tag{64}$$

With the aid of equations (59) and (60), one can readily show that

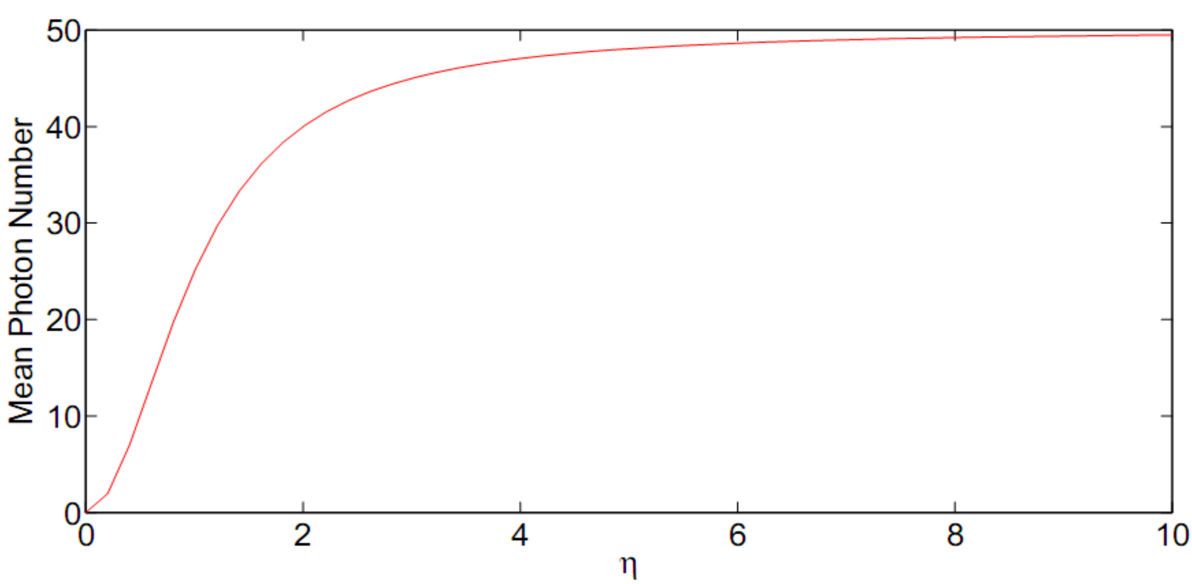

Figure 1: Plots of $\bar{n}$ vs. η for $\gamma_c = 0.4$, $\kappa = 0.8$, and $N = 50$

$$\bar{n} = \left(\frac{2\gamma_c}{k} N\right)\left[\frac{\eta^2}{1+3\eta^2}\right]. \tag{65}$$

It is not difficult to see, for $\Omega \gg \gamma_c$, that

$$\bar{n} = \frac{2\gamma_c}{3\kappa}N. \tag{66}$$

We see from Fig. (1) that the mean photon number of the two-mode light increases with η. In addition, as shown on Fig. (2) when Ω (the amplitude of coherent light) and γ_c (the stimulated emission decay constant) increase the global mean photon number also increases.

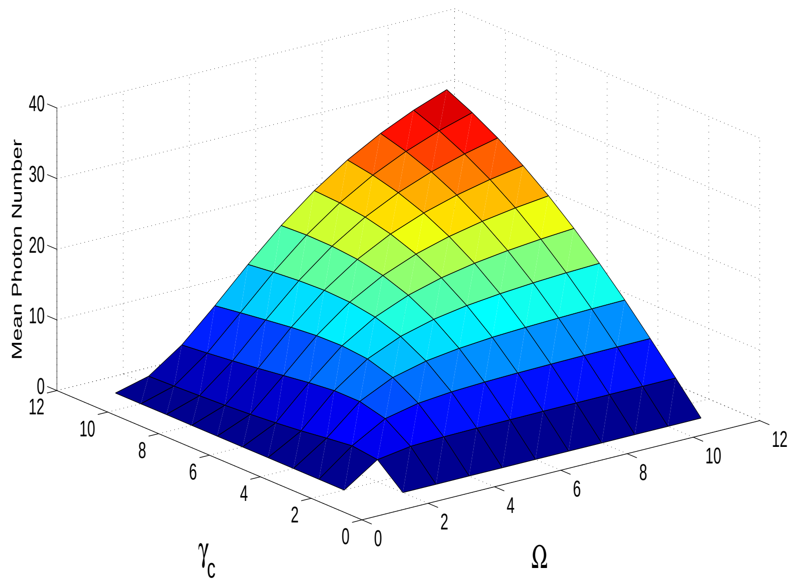

Figure 2: Plots of $\bar{n}$ vs. γ_c and Ω for $\kappa = 0.8$ and $N = 50$

b) The Local Mean Photon Number

We seek to determine the mean photon number in a given frequency interval, employing the power spectrum for the two-mode cavity light. The power spectrum of a two-mode cavity light with central common frequency ω_0 is defined as

$$\Gamma(\omega) = \frac{1}{\pi}Re\int_0^\infty d\tau e^{i(\omega-\omega_0)\tau}\langle\hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle_{ss}. \tag{67}$$

Next we seek to calculate the two-time correlation functions for the two-mode cavity light. To this end, we realize that the solution of Eq. (43) can write as

$$\hat{a}(t+\tau) = \hat{a}(t)e^{-\kappa\tau/2} + \frac{g}{\sqrt{N}}e^{-\kappa\tau/2}\int_0^\tau d\tau' e^{-\kappa\tau'/2}\hat{m}(t+\tau'). \tag{68}$$

On the other hand, one can put Eq. (52) in the form

$$\frac{d}{dt}\hat{m}(t) = -\frac{\mu}{2}\hat{m}(t) - \frac{\mu}{2}\hat{m}_a(t) + \hat{F}_m(t), \tag{69}$$

in which $\hat{F}_m(t)$ is a noise operator with zero mean. The solution of this equation is expressible as

$$\hat{m}(t+\tau) = \hat{m}(t)e^{-\mu\tau/2} + e^{-\mu\tau/2}\int_0^\tau d\tau' e^{-\mu\tau'/2}\Big[-\frac{\mu}{2}\hat{m}_a(t+\tau') + \hat{F}_m(t+\tau')\Big]. \tag{70}$$

In addition, one can rewrite Equation (134) as

$$\frac{d}{dt}\hat{m}_a(t) = -\mu\hat{m}_a(t) + \hat{F}_a(t), \tag{71}$$

where $\hat{F}_a(t)$ is a noise operator with vanishing mean. Employing the large-time approximation scheme to Equation (71), we see that

$$\hat{m}_a(t+\tau) = \frac{1}{\mu}\hat{F}_a(t+\tau). \tag{72}$$

Furthermore, introducing this into Equation (70), we have

$$\hat{m}(t+\tau) = \hat{m}(t)e^{-\kappa\tau/2} + e^{-\kappa\tau/2}\int_0^\tau d\tau' e^{-\kappa\tau'/2}\Big[-\frac{1}{2}\hat{F}_a(t+\tau') + \hat{F}_m(t+\tau')\Big]. \tag{73}$$

Now combination of Eqs (68) and (73) yields

$$\hat{a}(t+\tau) = \hat{a}(t)e^{-\kappa\tau/2} + \frac{g}{\sqrt{N}}e^{-\kappa\tau/2}\Big[\hat{m}(t)\int_0^\tau d\tau' e^{-(\kappa-\mu)\tau'/2} + \int_0^\tau d\tau' e^{-(\kappa-\mu)\tau'/2}$$

$$\times\int_0^{\tau'} d\tau'' e^{-\mu\tau''/2}\Big(-\frac{1}{2}\hat{F}_a(t+\tau'') + \hat{F}_m(t+\tau'')\Big)\Big]. \tag{74}$$

On multiplying both sides on the left by $\hat{a}^\dagger(t)$ and taking the expectation value of the resulting equation, we get

$$\langle\hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle = \langle\hat{a}^\dagger(t)\hat{a}(t)\rangle e^{-\kappa\tau/2} + \frac{g}{\sqrt{N}}e^{-\kappa\tau/2}\Big[\langle\hat{a}^\dagger(t)\hat{m}(t)\rangle\int_0^\tau d\tau' e^{-(\kappa-\mu)\tau'/2}$$

$$+\int_0^\tau d\tau' e^{-(\kappa-\mu)\tau'/2}\int_0^{\tau'} d\tau'' e^{-\mu\tau''/2}\Big(-\frac{1}{2}\langle\hat{a}^\dagger(t)\hat{F}_a(t+\tau'')\rangle + \langle\hat{a}^\dagger(t)\hat{F}_m(t+\tau'')\rangle\Big)\Big]. \tag{75}$$

Moreover, applying the large-time approximation scheme to Eq. (43), we obtain

$$\hat{m}(t) = \frac{\kappa\sqrt{N}}{2g}\hat{a}(t). \tag{76}$$

With this substituting into Eq.(75), there follows

$$\langle\hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle = \langle\hat{a}^\dagger(t)\hat{a}(t)\rangle e^{-\kappa\tau/2} + \frac{g}{\sqrt{N}}e^{-\kappa\tau/2}\Big[\frac{\kappa}{2}\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle\int_0^\tau d\tau' e^{-(\kappa-\mu)\tau'/2}$$

$$+\int_0^\tau d\tau' e^{-(\kappa-\mu)\tau'/2}\int_0^{\tau'} d\tau'' e^{-\mu\tau''/2}\Big(-\frac{1}{2}\langle\hat{a}^\dagger(t)\hat{F}_a(t+\tau'')\rangle + \langle\hat{a}^\dagger(t)\hat{F}_m(t+\tau'')\rangle\Big)\Big]. \tag{77}$$

Since the cavity mode operator and the noise operator of the atomic modes are not correlated, we see that

$$\langle \hat{a}^\dagger(t)\hat{F}_a(t+\tau'')\rangle = \langle \hat{a}^\dagger(t)\rangle\langle \hat{F}_a(t+\tau'')\rangle = 0, \tag{78}$$

$$\langle \hat{a}^\dagger(t)\hat{F}_m(t+\tau'')\rangle = \langle \hat{a}^\dagger(t)\rangle\langle \hat{F}_m(t+\tau'')\rangle = 0. \tag{79}$$

On account of these results and on carrying out the integration of Eq. (77) over τ' , we readily ge

$$\langle \hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle = \langle \hat{a}^\dagger(t)\hat{a}(t)\rangle\left[\frac{\kappa}{\kappa-\mu}e^{-\mu\tau/2} - \frac{\mu}{\kappa-\mu}e^{-\kappa\tau/2}\right]. \tag{80}$$

On introducing (80) into Eq. (67) and carrying out the integration, we readily get

$$\Gamma(\omega) = \bar{n}\left\{\left[\frac{\kappa}{\kappa-\mu}\right]\left[\frac{\mu/2\pi}{(\omega-\omega_0)^2+(\mu/2)^2}\right] - \left[\frac{\mu}{\kappa-\mu}\right]\left[\frac{\kappa/2\pi}{(\omega-\omega_0)^2+(\kappa/2)^2}\right]\right\}. \tag{81}$$

The mean photon number in the frequency interval between $\omega' = -\lambda$ and $\omega' = +\lambda$ is expressible as

$$\bar{n}_{\pm\lambda} = \int_{-\lambda}^{+\lambda}\Gamma(\omega')d\omega', \tag{82}$$

in which $\omega' = \omega - \omega_0$. Thus upon substituting (81) into Equation (82), we find

$$\bar{n}_{\pm\lambda} = \left[\frac{\kappa\bar{n}}{\kappa-\mu}\right]\int_{-\lambda}^{+\lambda}\left[\frac{\mu/2\pi}{(\omega-\omega_0)^2+(\mu/2)^2}\right]d\omega' - \left[\frac{\mu\bar{n}}{\kappa-\mu}\right]\int_{-\lambda}^{+\lambda}\left[\frac{\kappa/2\pi}{(\omega-\omega_0)^2+(\kappa/2)^2}\right]d\omega' \tag{83}$$

and on carrying out the integration over ω', applying the relation

$$\int_{-\lambda}^{+\lambda}\frac{dx}{x^2+a^2} = \frac{2}{a}\tan^{-1}\left(\frac{\lambda}{a}\right), \tag{84}$$

we arrive at

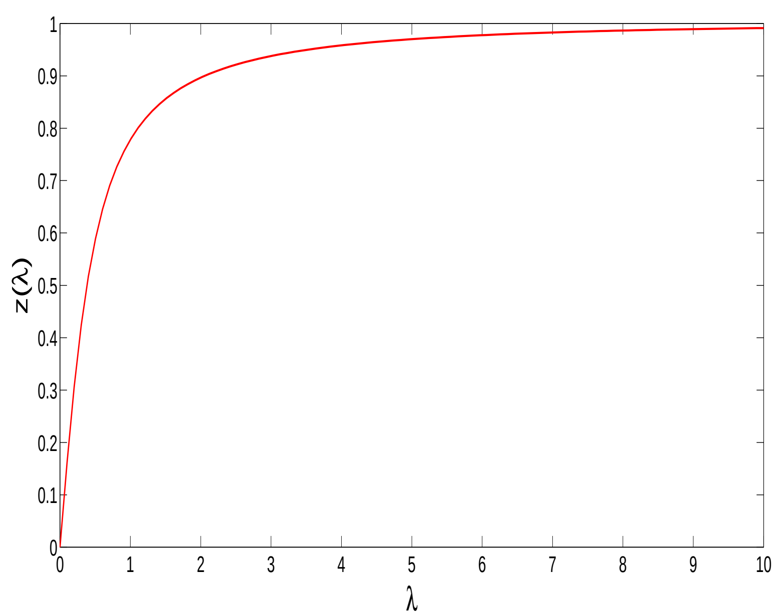

Figure 3: Plot of $z(\lambda)$ vs. λ for $\gamma_c = 0.4$, $\Omega = 3$, and $k = 0.8$

$$\bar{n}_{\pm\lambda} = \bar{n} z(\lambda), \tag{85}$$

where

$$z(\lambda) = \left[\frac{2\kappa/\pi}{\kappa-\mu}\right] \tan^{-1}\left(\frac{2\lambda}{\mu}\right) - \left[\frac{2\mu/\pi}{\kappa-\mu}\right] \tan^{-1}\left(\frac{2\lambda}{\kappa}\right). \tag{86}$$

One can readily get from Fig. (3) that $z(0.5) = 0.5891$, $z(1) = 0.7802$, and $z(2) = 0.8978$. Then combination of these results with Eq. (85) yields $\bar{n}_{\pm 0.5} = 0.5891\bar{n}$, $\bar{n}_{\pm 1} = 0.7802\bar{n}$, and $\bar{n}_{\pm 2} = 0.8978\bar{n}$. We therefore observe that a large part of the total mean photon number is confined in a relatively small frequency interval.

c) The Global Variance of the Photon Number

The variance of the photon number for the two-mode cavity light is expressible as

$$(\Delta n)^2 = \langle(\hat{a}^\dagger \hat{a})^2\rangle - \langle\hat{a}^\dagger \hat{a}\rangle^2. \tag{87}$$

Since $\hat{a}$ is Gaussian variable with zero mean, the variance of the photon number can be written as

$$(\Delta n)^2 = \langle\hat{a}^\dagger \hat{a}\rangle\langle\hat{a}\hat{a}^\dagger\rangle + \langle\hat{a}^{\dagger 2}\rangle\langle\hat{a}^2\rangle. \tag{88}$$

With the aid of the steady-stae solution of Eq. (43), one can easily establish that

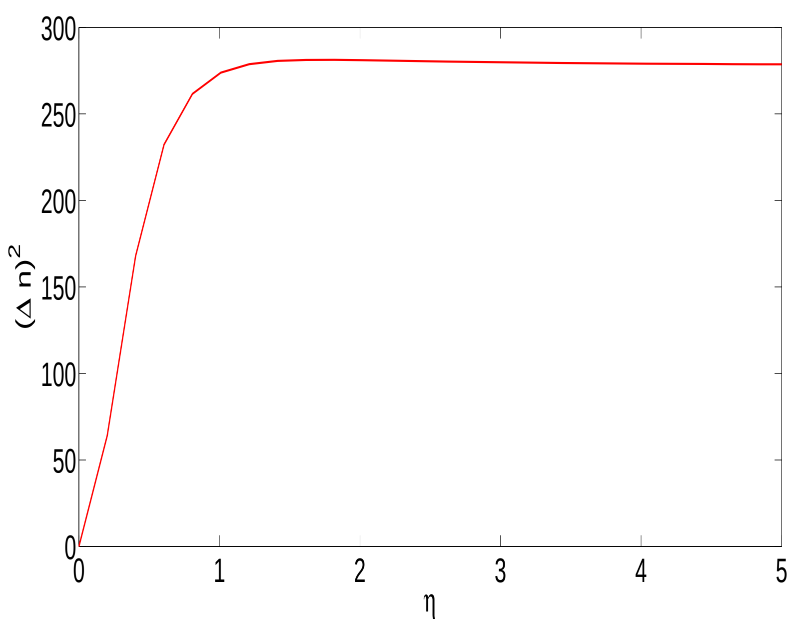

Figure 4: Plot of $(\Delta n)^2$ vs. η for $\gamma_c = 0.4$, $\kappa = 0.8$, and $N = 50$

$$\langle\hat{a}\hat{a}^\dagger\rangle = \frac{\gamma_c}{\kappa}[\langle\hat{N}_b\rangle + \langle\hat{N}_c\rangle] \tag{89}$$

and

$$\langle\hat{a}^2\rangle = \frac{\gamma_c}{\kappa}\langle\hat{m}_c\rangle. \tag{90}$$

Since $\langle \hat{m}_c \rangle$ is real, then $\langle \hat{a}^2 \rangle = \langle \hat{a}^{\dagger 2} \rangle$. Therefore, with the aid of Eqs. (64), (89) and (90), Eq. (88) turns out to be

$$(\Delta n)^2 = (\frac{\gamma_c}{\kappa})^2[(\langle \hat{N}_a \rangle + \langle \hat{N}_b \rangle)(\langle \hat{N}_b \rangle + \langle \hat{N}_c \rangle) + \langle \hat{m}_c \rangle^2]. \quad (91)$$

Furthermore, upon substituting of Eqs. (59)-(62) into Eq. (91), we see that

$$(\Delta n)^2 = \left(\frac{\gamma_c}{\kappa} N\right)^2 \left[\frac{3\eta^2 + 4\eta^4}{1 + 6\eta^2 + 9\eta^4}\right]. \quad (92)$$

This is the steady-state photon number variance of the two-mode light beam, produced by the coherently driven degenerate three-level laser with a closed cavity and coupled to a two-mode vacuum reservoir. Moreover, we note that for $\eta \gg 1$, Eq. (92) reduces to

$$(\Delta n)^2 = \left[\frac{2\gamma_c}{3\kappa} N\right]^2 \quad (93)$$

and in view of Eq. (66), we have

$$(\Delta n)^2 = \bar{n}^2, \quad (94)$$

which represents the normally-ordered variance of the photon number for chaotic light.

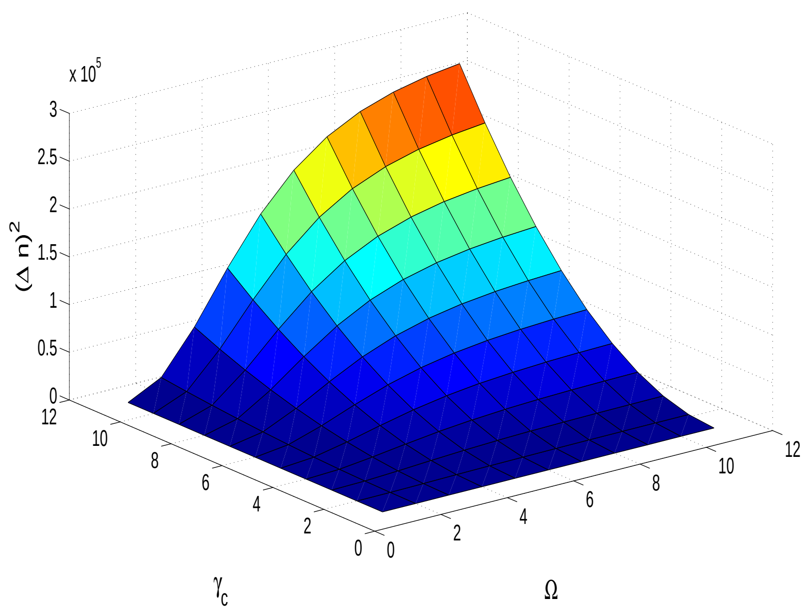

Figure 5: Plot of $(\Delta n)^2$ vs. γ_c and Ω for $\kappa = 0.8$ and $N = 50$

We see from Fig. (4) that the global photon number variance of the cavity light increases with η. In addition, as shown on Fig. (5) when Ω (the amplitude of coherent light) and γ_c (the stimulated emission decay constant) increase the global photon number variance also increases.

d) The Local Variance of the Photon Number

Here we wish to obtain the variance of the photon number in a given frequency interval, employing the spectrum of the photon number fluctuations for the superposition of light modes a_1 and

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a_2. We denote the central common frequency of these modes by ω_0. The spectrum of the photon number fluctuations for the superposed light modes can be expressed as

$$\Lambda(\omega) = \frac{1}{\pi} Re \int_0^\infty d\tau e^{i(\omega-\omega_0)\tau} \langle \hat{n}(t), \hat{n}(t+\tau)\rangle_{ss}, \tag{95}$$

where

$$\hat{n}(t) = \hat{a}^\dagger(t)\hat{a}(t), \tag{96}$$

$$\hat{n}(t+\tau) = \hat{a}^\dagger(t+\tau)\hat{a}(t+\tau). \tag{97}$$

Applying the realtion [39]

$$\langle \hat{n}(t), \hat{n}(t+\tau)\rangle = \langle \hat{n}(t)\hat{n}(t+\tau)\rangle - \langle \hat{n}(t)\rangle\langle \hat{n}(t+\tau)\rangle. \tag{98}$$

With the aid of Eqs. (96), (97) and (54), the photon number fluctuation can be expressed as

$$\Lambda(\omega) = \frac{1}{\pi} Re \int_0^\infty d\tau e^{i(\omega-\omega_0)\tau} [\langle \hat{a}^\dagger(t+\tau)\hat{a}(t+\tau)\rangle\langle \hat{a}(t+\tau)\hat{a}^\dagger(t+\tau)\rangle$$

$$+\langle \hat{a}^\dagger(t+\tau)\hat{a}^\dagger(t+\tau)\rangle\langle \hat{a}(t+\tau)\hat{a}(t+\tau)\rangle] \tag{99}$$

Following the same procedure to determine (80), one can readily get

$$\langle \hat{a}(t)\hat{a}^\dagger(t+\tau)\rangle = \langle \hat{a}(t)\hat{a}^\dagger(t)\rangle \left[\frac{\kappa}{\kappa-\mu} e^{-\mu\tau/2} - \frac{\mu}{\kappa-\mu} e^{-\kappa\tau/2}\right] \tag{100}$$

$$\langle \hat{a}(t)\hat{a}(t+\tau)\rangle = \langle \hat{a}^2(t)\rangle \left[\frac{\kappa}{\kappa-\mu} e^{-\mu\tau/2} - \frac{\mu}{\kappa-\mu} e^{-\kappa\tau/2}\right] \tag{101}$$

$$\langle \hat{a}^\dagger(t)\hat{a}^\dagger(t+\tau)\rangle = \langle \hat{a}^{\dagger 2}(t)\rangle \left[\frac{\kappa}{\kappa-\mu} e^{-\mu\tau/2} - \frac{\mu}{\kappa-\mu} e^{-\kappa\tau/2}\right] \tag{102}$$

Upon introducing (100)-(102) into Equation (99) and on carrying out the integration over τ, the spectrum of the photon number fluctuations for the two-mode cavity light is found to be

$$\Lambda(\omega) = (\Delta n)^2 \Bigg\{ \left[\frac{\kappa^2}{(\kappa-\mu}\right)^2\Bigg] \left[\frac{\mu/2\pi}{(\omega-\omega_0)^2 + (\mu/2)^2}\right] + \left[\frac{\mu^2}{(\kappa-\mu)^2}\right]\left[\frac{\kappa/\pi}{(\omega-\omega_0)^2 + (\kappa/2)^2}\right]$$

$$-\left[\frac{2\kappa\mu}{(\kappa-\mu)^2}\right]\left[\frac{(\kappa+\mu)/2\pi}{(\omega-\omega_0)^2 + (\kappa+\mu)^2/4}\right]\Bigg\}, \tag{103}$$

where $(\Delta n)^2$ is given by (92). Furthermore, upon integrating both sides of (103) over ω, we find

$$\int_{-\infty}^{\infty} \Lambda(\omega) d\omega = (\Delta n)^2_{ss}, \tag{104}$$

On the basis of Eq. (104), we observe that $\Lambda(\omega)d\omega$ represents the steady-state variance of the photon number for the two-mode cavity light in the interval between ω and $\omega + d\omega$. We thus realize that the photon- number variance in the interval between $\omega' = -\lambda$ and $\omega' = +\lambda$ can be written as

$$(\Delta n)^2_{\pm\lambda} = \int_{-\lambda}^{+\lambda} \Lambda(\omega)d\omega, \tag{105}$$

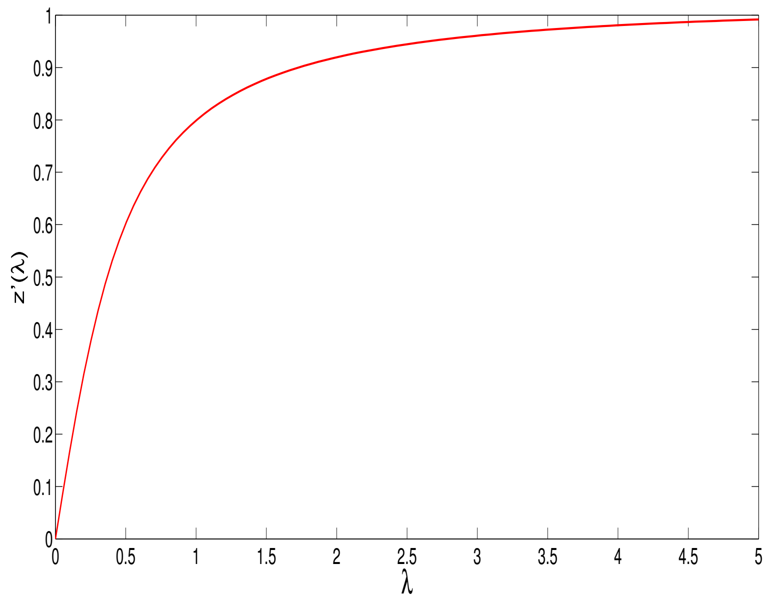

Figure 6: Plot of $z'(\lambda)$ vs. λ for $\gamma_c = 0.4$, $\Omega = 3$, and $k = 0.8$

in which $\omega' = \omega - \omega_0$. Thus upon substituting (103) into Eq. (105) and on carrying out the integration over ω', applying the relation described by Eq. (84), we readily get

$$(\Delta n)^2_{\pm\lambda} = (\Delta n)^2 z'(\lambda), \tag{106}$$

where

$$z'(\lambda) = \left[\frac{2\kappa^2/\pi}{(\kappa-\mu)^2}\right]\tan^{-1}\left(\frac{\lambda}{\mu}\right) + \left[\frac{2\mu^2/\pi}{(\kappa-\mu)^2}\right]\tan^{-1}\left(\frac{\lambda}{\kappa}\right) - \left[\frac{4\kappa\mu/\pi}{(\kappa-\mu)^2}\right]\tan^{-1}\left(\frac{2\lambda}{\kappa+\mu}\right), \tag{107}$$

One can readily get from Fig.(6) that $z'(0.5) = 0.6587$, $z'(1) = 0.8074$, and $z'(2) = 0.9254$. Then combination of these results with Eq. (106) yields $(\Delta n)^2_{\pm 0.5} = 0.6587(\Delta n)^2 z'(\lambda)$, $(\Delta n)^2_{\pm 1} = 0.8074(\Delta n$ and $(\Delta n)^2_{\pm 2} = 0.9254(\Delta n)^2$. We therefore observe that a large part of the total variance of the photon number is confined in a relatively small frequency interval.

VI. Quadrature Squeezing

In this section, we seek to obtain the quadrature variance and squeezing of the two-mode light in a closed cavity produced by a coherently driven nondegenerate three-level laser.

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a) Quadrature Variance

The squeezing properties of the two-mode cavity light are described by two quadrature operators

$$\hat{a}_{+} = \hat{a}^{\dagger} + \hat{a}, \tag{108}$$

$$\hat{a}_{-} = i(\hat{a}^{\dagger} - \hat{a}), \tag{109}$$

It can be readily established that

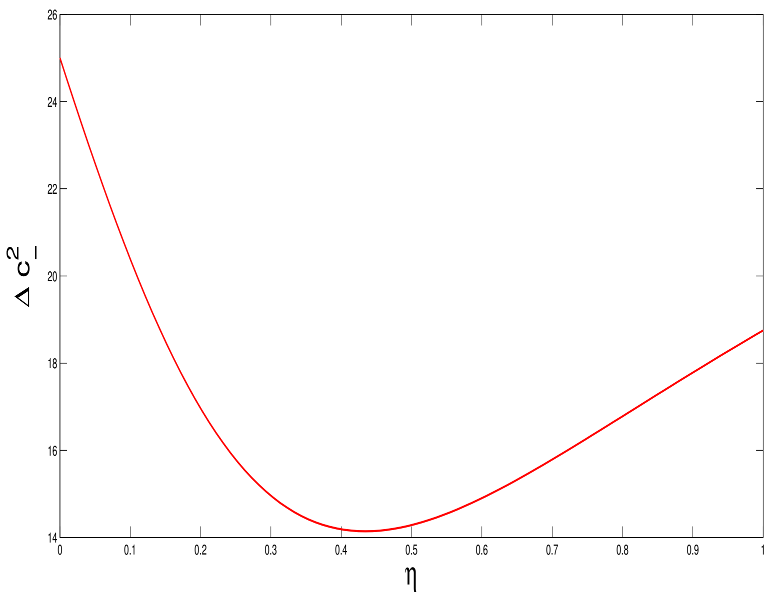

Figure 7: Plot of $(\Delta a_{-})^2$ vs. η for $\gamma_c = 0.4$, $k = 0.8$, and $N = 50$

$$[\hat{a}_{-}, \hat{a}_{+}] = 2i\frac{\gamma_c}{\kappa}[\hat{N}_a - \hat{N}_c], \tag{110}$$

It then follows that

$$\Delta a_{+}\Delta a_{-} \geq \frac{\gamma_c}{\kappa}\left|\langle\hat{N}_a\rangle - \langle\hat{N}_c\rangle\right|. \tag{111}$$

Now upon replacing the atomic operators that appear in Eq. (39) by their expectation values, the commutation relation for the two-mode light can be written as

$$[\hat{a}, \hat{a}] = \lambda, \tag{112}$$

in which

$$\lambda = \frac{\gamma_c}{\kappa}\left[\langle\hat{N}_c\rangle - \langle\hat{N}_a\rangle\right]. \tag{113}$$

Making use of the well-known definition of the variance of an operator, the variances of the quadrature operators (108) and (109) are found to have the form

$$(\Delta a_{\pm})^2 = \lambda + 2\langle \hat{a}^\dagger(t)\hat{a}(t)\rangle \pm \langle \hat{a}^2(t)\rangle \pm \langle \hat{a}^{\dagger 2}(t)\rangle \mp \langle \hat{a}(t)\rangle^2 \mp \langle \hat{a}^\dagger(t)\rangle^2 - 2\langle \hat{a}(t)\rangle\langle \hat{a}^\dagger(t)\rangle. \quad (114)$$

In view of Equation (54), one can put Equation (114) in the form

$$(\Delta a_{\pm})^2 = \lambda + 2\langle \hat{a}^\dagger(t)\hat{a}(t)\rangle \pm \langle \hat{a}^2(t)\rangle \pm \langle \hat{a}^{\dagger 2}(t)\rangle. \quad (115)$$

With the aid of Eqs. (64), (90), and (113) one can easily establish that

$$(\Delta a_{+})^2 = \frac{\gamma_c}{k}[N + \langle \hat{N}_b\rangle_{ss} + 2\langle \hat{m}_c\rangle_{ss}], \quad (116)$$

$$(\Delta a_{-})^2 = \frac{\gamma_c}{k}[N + \langle \hat{N}_b\rangle_{ss} - 2\langle \hat{m}_c\rangle_{ss}]. \quad (117)$$

Finally, on account of (60) and (62), the global quadrature variance of the two-mode cavity light

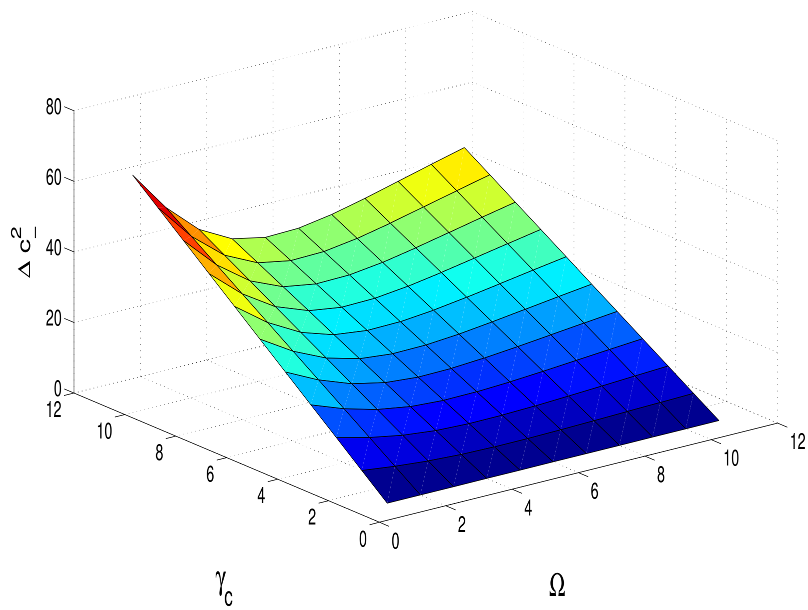

Figure 8: Plots of $(\Delta a_{-})^2$ vs Ω and γ_c for $k = 0.8$, $N = 50$.

turns out at steady state to be

$$(\Delta a_{+})^2 = \frac{\gamma_c}{k}N\left[\frac{4\eta^2 + 2\eta + 1}{1 + 3\eta^2}\right], \quad (118)$$

$$(\Delta a_{-})^2 = \frac{\gamma_c}{k}N\left[\frac{4\eta^2 - 2\eta + 1}{1 + 3\eta^2}\right], \quad (119)$$

and for $\Omega \gg \gamma_c$

$$(\Delta a_{\pm})^2 = \frac{4\gamma_c}{3k}N = 2\bar{n}, \quad (120)$$

where $\bar{n}$ is given by equation (66). It can be seen that expression (120) represents the normally ordered quadrature variance for chaotic light. Moreover, for the case in which the deriving coherent light is absent, one can see that

$$(\Delta a_+)^2_v = (\Delta a_-)^2_v = \frac{\gamma_c}{k} N, \tag{121}$$

which is the normally ordered quadrature variance of the two-mode cavity light in vacuum state. It is also observed that, the uncertainty in the plus and minus quadratures are equal and satisfy the minimum uncertainty relation.

b) The quadrature squeezing

The quadrature squeezing of the two-mode cavity light relative to the quadrature variance of the two-mode vacuum light can be defined as

$$S = \frac{(\Delta a_\pm)^2_v - (\Delta a_-)^2}{(\Delta a_\pm)^2_v}, \tag{122}$$

where $(\Delta a_\pm)^2_v$ is the quadrature variance in vacuum state given by equation (121). Taking into

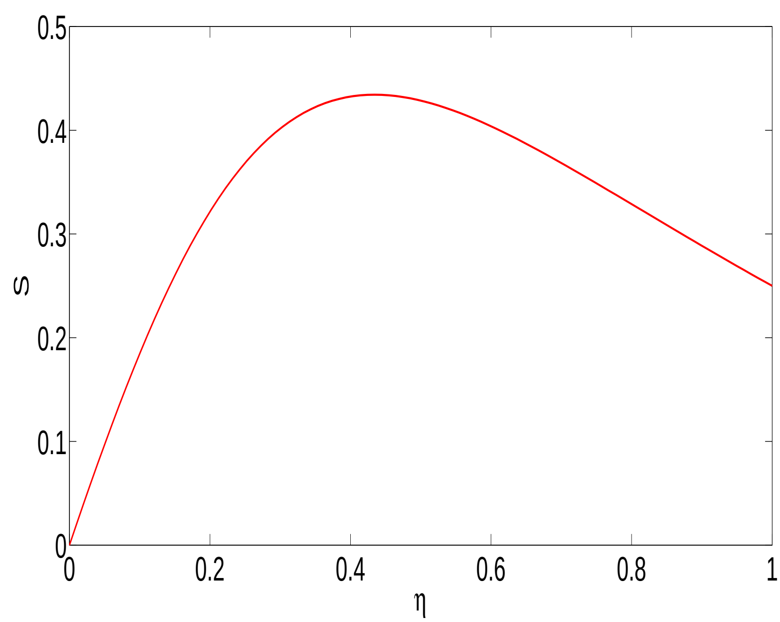

Figure 9: Plot of the quadrature squeezing vs. η for $\gamma_c = 0.4$.

account equations (118) and (121), (122) yields

$$S = \frac{2\eta - \eta^2}{1 + 3\eta^2}. \tag{123}$$

Equation (123) is indicates that the quadrature squeezing of the light produced by degenerate three-level laser with the N three-level atoms available inside a closed cavity pumped to the top level by electron bombardment which has been reported by Fesseha [1, 28].

We observe that in Eq. (123), unlike the mean photon number, the quadrature squeezing does not depend on the number of atoms. This implies that the quadrature squeezing of the cavity light is independent of the number of photons.

The plot in Figs. 9 shows that the maximum squeezing of the cavity light is 43% degree of squeezing and occurs when the three-level laser is operating at $\eta = 0.4$. Hence one can observe that a coherently driven light produced by a degenerate three-level laser can exhibit less than degree of squeezing when, for example, compared to the light generated by a three-level laser in which the three-level atoms available in a closed cavity are pumped to the top level by means of electron bombardment [1, 28, 29].

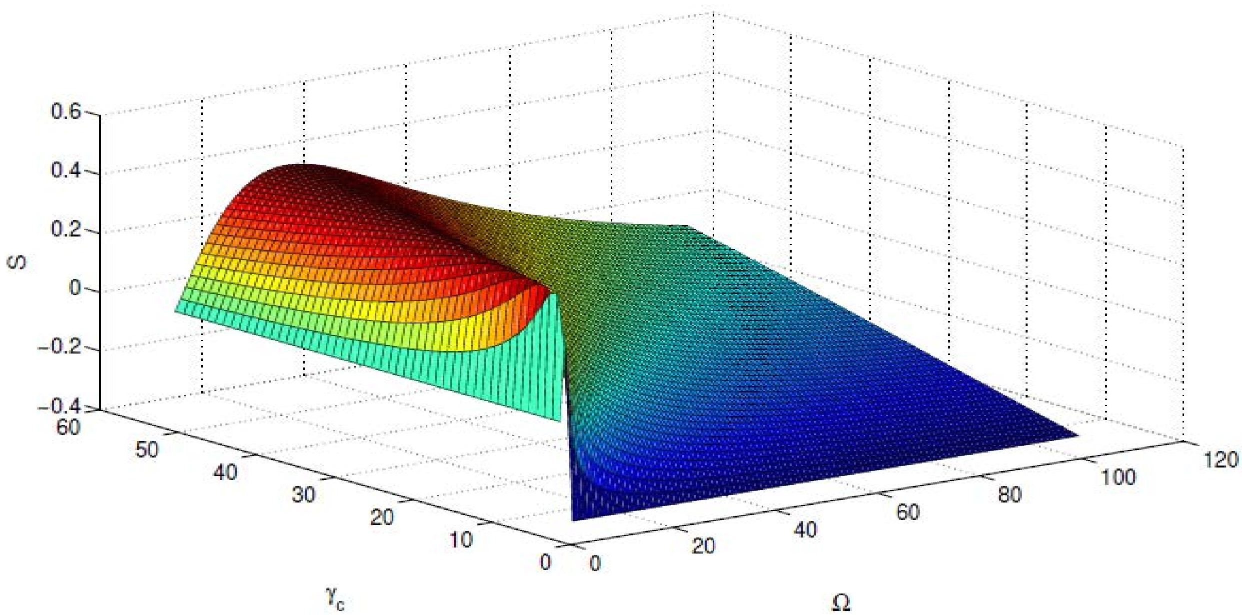

Figure 10: Plot of the quadrature squeezing vs. Ω and γ_c.

VII. Local Quadrature Squeezing

Here we wish to obtain the quadrature squeezing of a cavity light in a given frequency interval. To this end, we first obtain the spectrum of the quadrature fluctuations of the superposition of light modes a_1 and a_2. We define this spectrum for the two-mode cavity light by

$$S_{\pm}(\omega) = \frac{1}{\pi} Re \int_0^{\infty} d\tau e^{i(\omega-\omega_0)\tau} \langle \hat{a}_{\pm}(t), \hat{a}_{\pm}(t+\tau)\rangle_{ss}, \tag{124}$$

in which

$$\hat{a}_{+}(t+\tau) = \hat{a}^{\dagger}(t+\tau) + \hat{a}(t+\tau), \tag{125}$$

$$\hat{a}_{-}(t+\tau) = i(\hat{a}^{\dagger}(t+\tau) - \hat{a}(t+\tau)), \tag{126}$$

and ω_0 is the central frequency of the modes a_1 and a_2. In view of Eq. (54), we obtain

$$\langle \hat{a}_{\pm}(t), \hat{a}_{\pm}(t+\tau)\rangle = \langle \hat{a}_{\pm}(t)\hat{a}_{\pm}(t+\tau)\rangle. \tag{127}$$

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Then on account of Eqs. (108), (109), (125), and (126), one can write Equation (127) as

$$\langle \hat{a}_\pm(t), \hat{a}_\pm(t+\tau)\rangle = \langle \hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle + \langle \hat{a}(t)\hat{a}^\dagger(t+\tau)\rangle \pm \langle \hat{a}^\dagger(t)\hat{a}^\dagger(t+\tau)\rangle \pm \langle \hat{a}(t)\hat{a}(t+\tau)\rangle. \quad (128)$$

Upon substituting of Eqs. (80), (100)-(102) into Eq. (128), we arrive at

$$\langle \hat{a}_\pm(t), \hat{a}_\pm(t+\tau)\rangle = \left[\langle \hat{a}^\dagger(t)\hat{a}(t)\rangle + \langle \hat{a}(t)\hat{a}^\dagger(t)\rangle \pm \langle \hat{a}^\dagger(t)\hat{a}^\dagger(t)\rangle \pm \langle \hat{a}(t)\hat{a}(t)\rangle\right]$$

$$\times\left[\frac{\kappa}{\kappa-\mu}e^{-\mu\tau/2} - \frac{\mu}{\kappa-\mu}e^{-\kappa\tau/2}\right]. \quad (129)$$

This can be put in the form

$$\langle \hat{a}_+(t), \hat{a}_+(t+\tau)\rangle = (\Delta a_+)^2\left[\frac{\kappa}{\kappa-\mu}e^{-\mu\tau/2} - \frac{\mu}{\kappa-\mu}e^{-\kappa\tau/2}\right] \quad (130)$$

and

$$\langle \hat{a}_-(t), \hat{a}_-(t+\tau)\rangle = (\Delta a_-)^2\left[\frac{\kappa}{\kappa-\mu}e^{-\mu\tau/2} - \frac{\mu}{\kappa-\mu}e^{-\kappa\tau/2}\right]. \quad (131)$$

Now introducing (131) into Eq. (124) and on carrying out the integration over τ, we find the spectrum of the minus quadrature fluctuations for a two-mode cavity light to be

$$S_-(\omega) = (\Delta a_-)^2_{ss}\left\{\left[\frac{\kappa}{\kappa-\mu}\right]\left[\frac{\mu/2\pi}{(\omega-\omega_0)^2+(\mu/2)^2}\right] - \left[\frac{\mu}{\kappa-\mu}\right]\left[\frac{\kappa/2\pi}{(\omega-\omega_0)^2+(\kappa/2)^2}\right]\right\}. \quad (132)$$

Upon integrating both sides of (132) over ω, we get

$$\int_{-\infty}^{+\infty} S_-(\omega)d\omega = (\Delta a_-)^2. \quad (133)$$

On the basis of Equation (133), we observe that $S_-(\omega)d\omega$ is the steady-state variance of the minus quadrature in the interval between ω and $\omega + d\omega$. We thus realize that the variance of the minus quadrature in the interval between $\omega' = -\lambda$ and $\omega' = +\lambda$ is expressible as

$$(\Delta a_{\pm\lambda})^2 = \int_{-\lambda}^{+\lambda} S_-(\omega')d\omega', \quad (134)$$

in which $\omega - \omega_0 = \omega'$. On introducing (132) into Eq. (134) and on carrying out the integration over ω', employing the relation described by Eq. (84), we find

$$(\Delta a_-)^2_{\pm\lambda} = (\Delta a_-)^2 z(\lambda), \quad (135)$$

where $z(\lambda)$ is given by Eq. (86). We define the quadrature squeezing of the two-mode cavity light in the $\lambda_\pm$ frequency interval by

$$S_{\pm\lambda} = 1 - \frac{(\Delta a_-)^2_{\pm\lambda}}{(\Delta a_-)^2_{v\pm\lambda}}, \quad (136)$$

Furthermore, upon setting $\eta = 0$ in Eq. (135), we see that the local quadrature variance of a two-mode cavity vacuum state in the same frequency is found to be

$$(\Delta a_-)^2_{v\pm\lambda} = (\Delta a_-)^2_v z_v(\lambda), \tag{137}$$

in which

$$z_v(\lambda) = \left[\frac{2\kappa/\pi}{\kappa - \gamma_c}\right] \tan^{-1}\left(\frac{2\lambda}{\gamma_c}\right) - \left[\frac{2\gamma_c/\pi}{\kappa - \gamma_c}\right] \tan^{-1}\left(\frac{2\lambda}{\kappa}\right) \tag{138}$$

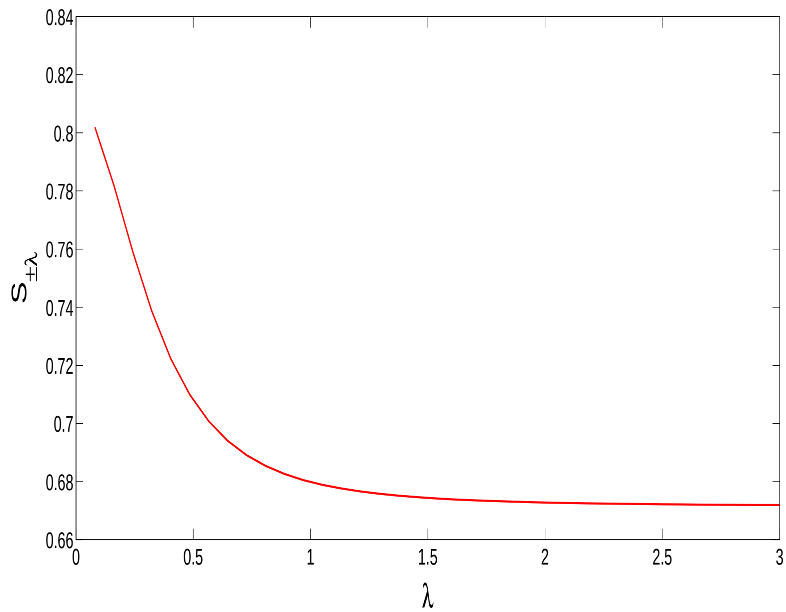

Figure 11: Plot of $S_{\pm\lambda}$ vs. λ for $\gamma_c = 0.4, \Omega = 0.1717$, and $k = 0.8$

and $(\Delta a_-)^2_v$ is given by (121). Finally, on account of Equations (119), (121), and (137) along with (136), we readily get

$$S_{\pm\lambda} = \frac{1}{z_v(\lambda)}\left\{z_v(\lambda) - z(\lambda) - \left[\frac{2\eta - \eta^2}{1 + 3\eta^2}\right] z(\lambda)\right\}. \tag{139}$$

This shows that the local quadrature squeezing of the two-mode cavity light beams is not equal to that of the global quadrature squeezing. Moreover, we found from the plots in Figure 6 that the maximum local quadrature squeezing is 80.2% (and occurs at $\lambda = 0.08$). Furthermore, we note that the local quadrature squeezing approaches the global quadrature squeezing as λ increases.

VIII. Conclusion

The steady-state analysis of the squeezing and statistical properties of the light produced by coherently pumped degenerate three-level laser with closed cavity and coupled to a single-mode vacuum reservoir is presented. We carry out our analysis by putting the noise operators associated with the vacuum reservoir in normal order and by taking into consideration the interaction of the three-level atoms with the vacuum reservoir inside the cavity. We observe that a large part of the total mean photon number (variance of the photon number) is confined in a relatively small frequency interval. In addition, we find that the maximum global quadrature squeezing of the light produced by the system under consideration operating at $\eta = 0.1717$ is 43.43%.

Moreover, we find that the maximum local quadrature squeezing is 80.2% (and occurs at $\lambda = 0.08$). Furthermore, our results have shown that unlike the local mean of the phonon number and photon number variance, the local quadrature squeezing does not increase as the value of λ increases. We observe that the light generated by this laser operating under the condition $\Omega \gg \gamma_c$ is in a chaotic light. And we have also established that the local quadrature squeezing is not equal to the global quadrature squeezing. Furthermore, we point out that unlike the mean photon number and the variance of the photon number, the quadrature squeezing does not depend on the number of atoms. This implies that the quadrature squeezing of a cavity light is independent of the number of photons.

References Références Referencias

1. Fesseha Kassahun (2014), *Refined Quantum Analysis of Light*, (Create Space Independent Publishing Platform).
2. D. F. Walls and J. G. Milburn (1994), *Quantum Optics*, (Springer-Verlag, Berlin).
3. M O Scully and M S Zubairy (1997), *Quantum Optics*, (Cambridge: Cambridge University Press).
4. Meystre, P. and Sargent III, M. (1997) *Elements of Quantum Optics*, (2nd Edition, Springer-Verlag, Berlin).
5. Barnett, S.M. and Radmore, P.M. (1997), *Methods in Theoretical Quantum Optics*, (Clarendon Press, Oxford).
6. Vogel, W. and Welsch, D.G. (2006) *Quantum Optics*, (Wiley-VCH, New York).
7. Collet, M.J. and Gardiner, C.W. (1984), *Phys. Rev. A*, **30**, 1386.
8. Leonhardt, U. (1997) *Measuring the Quantum Analysis of Light*, (Cambridge University Press, Cambridge).
9. Scully M.O., Wodkienicz, K., Zubairy, M.S., Bergou, J., Lu, N. and Meyer ter Vehn, J. (1988), *Phys. Rev. Lett.* **60** 1832.
10. Daniel, B. and Fesseha, K. (1998), *Opt. Commun.* **151** 384-394.
11. Teklu, B. (2006), *Opt. Commun.* **261**, 310-321.
12. Darge, T.Y. and Kassahun, F. (2010), *PMC Physics B*, **31**.
13. Anwar, J. and Zubairy, M.S. (1992), *Phys. Rev. A*, **45** 1804.
14. Plimark, L.I. and Walls, D.F. (1994), *Phys. Rev. A*, **50** 2627.
15. Drummond, P.D., McNeil, K.J. and Walls, D.F. (1980), *Opt. Acta*, **27** 321-335.
16. Scully, M.O. and Zubairy, M.S. (1988), *Opt. Commun.* **66** 303-306.23
17. Anwar, J. and Zubairy, M.S. (1994), *Phys. Rev. A*, **49** 481.
18. N Lu and S Y Zhu (1989), *Phys. Rev. A* **40** 5735.
19. N A Ansari (1993), *Phys. Rev. A* **48** 4686.
20. Fesseha, K. (2001), *Phys. Rev. A* **63** 033811.
27. S Tesfa (2006) *Phys. Rev. A* **74** 043816.
28. Fesseha Kassahun (2011) *Opt. Commun.* **284** 1357.
29. Tamirat Abebe and Tamiru Deressa (2018) *GJSFR: A, Physics and Space Science* **18**, 1, 19.
30. JMLiu, B S Shi, X F Fan, J Li and G C Guo. (2001) *J. Opt. B: Quant. Semiclass. Opt.* **3** 189.
31. S L Braunstein and H J Kimble (2000) *Phys. Rev. A* **61** 42302.
32. S Lloyd and S L Braunstein (1999) *Phys. Rev. Lett.* **82** 1784.
33. S L Braunstein (1998) *Nature* **394** 47.
34. T C Ralph (2000) *Phys.Rev. A* **61** 010302.
35. Eyob Alebachew (2007) *Opt. Commun.* **280** 133.
36. T. Abebe (2018) *Ukr. J. Phys.* **63** 733.
37. J Anwar and MS Zubairy (1994) *Phys. Rev. A* **49** 481.
38. H Xiong, MO Scully and MS Zubairy (2005) *Phys. Rev. Lett.* **94** 023601.
39. C. Gerry and P. L. Knight (2005) *Introductory Quantum Optics*, (Cambridge: Cambridge University Press).