Motion of Elastic Particles and Spectrum of Hydrogen Atoms

By Zhong-Cheng Liang

Nanjing University of Posts and Telecommunications

Abstract- This article analyzes the spectral structure of hydrogen atoms according to the motion theory of elastic particles. The results demonstrate that optical radiation originates from the elastic vibration of atoms or molecules. The quantum state is the equilibrium feature of the classically statistical system, and the quantum transition is the process of conversion between different motion modes.

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I. Introduction

Atomic spectra are discrete line spectra, which are information for understanding the composition of matters. In 1913, Niels Bohr proposed the hypothesis of the quantum state (stationary state) to explain the atomic stability and the hydrogen spectrum [1]. The success of Bohr's atom theory promoted the establishment of quantum mechanics. The hypothesis of the quantum state is the foundation of quantum mechanics. Although quantum mechanics has made significant achievements, the interpretation of the quantum state has always been controversial [1-3]. The essence of quantum has become the ultimate mystery of nature. Four years ago, the author put forward a physical theory based on the elastic particle model [4], made a unique interpretation to the nature of quantum by classical mechanics [4-6], and developed the theories of particle fields [4,7,8], motion states [4,5], and statistical thermodynamics [4,9]. This article analyzes the spectral structure of hydrogen atoms based on the motion state theory, and further proves the classical nature of the quantum state.

II. Theory of Elastic Particles

a) Elastic particle

Objects are particle systems composed of elastic particles [4-9]. Elastic particles are three-dimensional objects that have mass and volume, that can spin and deform. Electrons, protons, neutrons, atoms, and molecules are all elastic particles. Protons and electrons are indecomposable primary particles, while neutrons are composite particles containing one proton and one electron. The composition of any object is protons and electrons.

The spatial object structure is the nesting of particles at different levels [5,6]. Nucleus and electrons constitute atoms; atoms constitute molecules; molecules constitute supra molecule; and so on. The general model describing the nested structure is

Author: College of Electronic and Optical Engineering, Nanjing University of Posts and Telecommunications Nanjing 210046, China.
e-mail: zcliang@njupt.edu.cn

1. G. Ludwig, *An Axiomatic Basis for Quantum Mechanics (Vol.2) Quantum Mechanics and Macrosystems* (Springer, Berlin, 1987). ISBN 978-3-642-71899-1.

$$\text{Top-particle} \supseteq \text{meso-particles} \supseteq \text{base-particles} \supseteq \text{sub-particles} \qquad (1)$$

A top-particle is the object to be studied, while base-particles are the statistical units of number conservation. An upper-level particle comprises all particles in lower-levels. The more levels of particles an object contains, the more complex the structure of the object is. The interaction of the elastic particles originates from two macroscopic constraints: the conservation of particle number, and the repulsion of particle volume. The elasticity of upper-level particles comes from the motion of lower-level particles. For example, the elasticity of atoms mainly comes from the movement of electrons outside the nucleus.

b) Spatial state

The spatial state of an elastic object includes position, posture, and profile. The position is represented by the position vector of the center-of-mass, the posture is represented by three principal inertia axes, and the profile is represented by three principal rotary inertias [5,6].

Let an object contain N particles, in which the particle P_i has mass M_i, and its position in the laboratory coordinate system $(O - XYZ)$ is

$$\boldsymbol{r}_i = r_s \cdot \tilde{\boldsymbol{r}}_i = \overrightarrow{OP_i} = (x_i, y_i, z_i). \qquad (2)$$

Where r_s is the scale of space length, also known as space quantum. The position vector $(\boldsymbol{r}_c)$ of the center-of-mass (P_c) of the object is

$$\boldsymbol{r}_c = \overrightarrow{OP_c} = (x_c, y_c, z_c) = \sum_{i=1}^{N} \left(\frac{M_i}{M}\right) \boldsymbol{r}_i \,, M = \sum_{i=1}^{N} M_i \,. \qquad (3)$$

The posture and profile of an object are determined by the inertial matrix. The inertial matrix of the N-particle system in the laboratory coordinate system is [10]

$$\boldsymbol{I} = \begin{pmatrix} I_{11} & -I_{12} & -I_{13} \\ -I_{21} & I_{22} & -I_{23} \\ -I_{31} & -I_{32} & I_{33} \end{pmatrix}; \quad I_s = M_s r_s^2 \,. \qquad (4)$$

Where I_s is the scale of inertia. The elements of the inertial matrix are

$$I_{11} = \sum_{i=1}^{N} M_i(y_i^2 + z_i^2), \quad I_{22} = \sum_{i=1}^{N} M_i(z_i^2 + x_i^2), \quad I_{33} = \sum_{i=1}^{N} M_i(x_i^2 + y_i^2),$$

$$I_{12} = I_{21} = \sum_{i=1}^{N} M_i x_i y_i, \quad I_{31} = I_{13} = \sum_{i=1}^{N} M_i z_i x_i, \quad I_{23} = I_{32} = \sum_{i=1}^{N} M_i y_i z_i. \qquad (5)$$

Inertial matrix is a real symmetric matrix. It has three real eigenvalues I_1, I_2, I_3 and three eigenvectors $\hat{\boldsymbol{e}}_1, \hat{\boldsymbol{e}}_2, \hat{\boldsymbol{e}}_3$ according to the theory of linear algebra. We describe the profile by the eigenvalues of the inertial matrix, $\boldsymbol{I}_c = (I_1, I_2, I_3)$, and describe the posture by the angles between the eigenvectors and the laboratory coordinate axes, $\boldsymbol{\theta}_c = (\theta_1, \theta_2, \theta_3)$.

c) Motion energy

Motion is a process in which the spatial state of an object changes with time. Changes in position, posture, and profile are called translation, rotation, and vibration,

respectively. The translation is the movement of the center-of-mass in the laboratory reference system, the rotation is the spin around the center-of-mass, and the vibration is the extension and contraction relative to the center-of-mass. Translation, rotation, and vibration are three independent motion modes, and each mode has three degrees of freedom. An elastic particle has $3 \times 3 = 9$ degrees of freedom, and an object composed of N particles has $9N$ degrees of freedom.

If the three mode energies of a particle are $H_{i\alpha}$, $L_{i\alpha}$, $K_{i\alpha}$ ($\alpha = 1,2,3$), then the total energies of the three modes are [4-6]

$$H = \sum_{i=1}^{N} \sum_{\alpha=1}^{3} H_{i\alpha} \; ; \; H_{i\alpha} = \frac{V_i \chi_{i\alpha}^2}{2 Y_{i\alpha}},$$

$$L = \sum_{i=1}^{N} \sum_{\alpha=1}^{3} L_{i\alpha} \; ; \; L_{i\alpha} = \frac{s_{i\alpha}^2}{2 I_{i\alpha}},$$ (6)

$$K = \sum_{i=1}^{N} \sum_{\alpha=1}^{3} K_{i\alpha} \; ; \; K_{i\alpha} = \frac{p_{i\alpha}^2}{2 M_i}.$$

The total energy of each mode is the sum of $3N$ independent square terms and has positivity($H > 0, \; L > 0, \; K > 0$).

d) Energy space

The energy space is defined as the set of ordered array $\{\mathbb{E}^h, \mathbb{E}^l, \mathbb{E}^k\}$ [4-6]

$$\mathbb{E}^h = \langle H^h, L^h, K^h \rangle, \quad \mathbb{E}^l = \langle L^l, K^l, H^l \rangle, \quad \mathbb{E}^k = \langle K^k, H^k, L^k \rangle.$$ (7)

Where $x = h, l, k$ is the zone index. $\mathbb{E}^h, \mathbb{E}^l, \mathbb{E}^k$ is called gas zone, solid zone, and liquid zone, respectively.

The energy space is confined to the first octant $(+,+,+)$ of the Cartesian space due to the positivity of motion energy. In the energy space, we defined the energy vector length (called entire energy) as

$$E = \sqrt{H^2 + L^2 + K^2}.$$ (8)

Three planes $\{H = K, K = L, L = H\}$ divide the energy space into six phases $\{\mathrm{B}[G_{\pm}^h], \mathrm{B}[G_{\pm}^l], \mathrm{B}[G_{\pm}^k]\}$. $\mathrm{B}[H] = \mathrm{B}[G_+^h] + \mathrm{B}[G_-^h]$ is the gas zone, $\mathrm{B}[L] = \mathrm{B}[G_+^l] + \mathrm{B}[G_-^l]$ is the solid zone, and $\mathrm{B}[K] = \mathrm{B}[G_+^k] + \mathrm{B}[G_-^k]$ is the liquid zone. There are six phase interfaces in the energy space, of which the J-type interfaces $\{\mathrm{S}[J_0^h], \mathrm{S}[J_0^l], \mathrm{S}[J_0^k]\}$ are the interfaces of zero potential energy, the G-type interfaces $\{\mathrm{S}[G_0^h], \mathrm{S}[G_0^l], \mathrm{S}[G_0^k]\}$ are the interfaces of zero chemical energy. The structure of the energy space is shown in Figure 1(a).
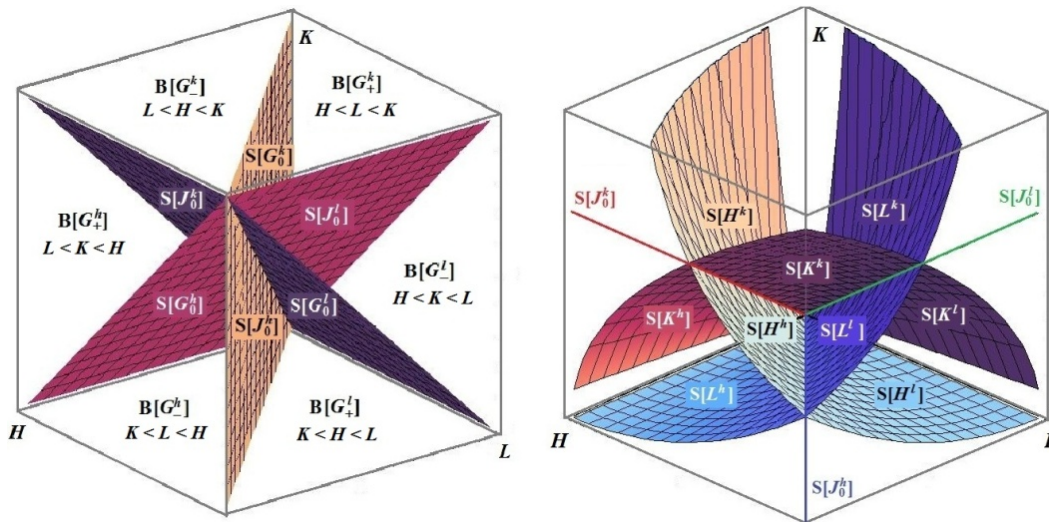
Figure 1: (a) Structure of energy space. (b) Equilibrium surfaces in energy space

e) Equilibrium surface

There are three equilibrium surfaces in the energy space: vibration surface $S[H]$, rotation surface $S[L]$, and translation surface $S[K]$. Their corresponding equilibrium equations are

$$H = \sqrt{2LK}, \quad L = \sqrt{2KH}, \quad K = \sqrt{2HL}. \tag{9}$$

$S[H]$, $S[L]$ and $S[K]$ represent vibration (radiative) equilibrium, rotation (magnetic) equilibrium, and translation (thermal) equilibrium, respectively. Each surface spans four phases and extends to three zones, as shown in Figure 1(b). A matrix describing the structure of the equilibrium surfaces is shown in Table 1. The diagonal elements of the matrix, $\{S[H^h], S[L^l], S[K^k]\}$, are stable areas and the rest are excited areas.

Table 1: Structure of equilibrium surfaces

	B[H]	**B[L]**	**B[K]**
S[H]	$S[H^h]$	$S[H^l]$	$S[H^k]$
S[L]	$S[L^h]$	$S[L^l]$	$S[L^k]$
S[K]	$S[K^h]$	$S[K^l]$	$S[K^k]$

In Table 2, we list the energy names and parameter definitions on the three equilibrium surfaces. Where $\{X, Y, Z\}$ is motion energy. $\{E, Q, J, G\}$ is auxiliary energy. $\{a, b\}$ is order parameter, which has the relation $2ab = 1$. The relations $E = \sqrt{X^2 + Y^2 + Z^2} = Y + Z$ determine the equilibrium equation $X = \sqrt{2YZ}$.

Table 2: Energy names and parameter definition on equilibrium surfaces

Equilibrium surface	Definition	S[H]	S[L]	S[K]
Equilibrium equation	$X = \sqrt{2YZ}$	$H = \sqrt{2l}$	$L = \sqrt{2}$	$K = \sqrt{2HL}$
Major energy	X	H	L	K
Ahead energy	Y	H	L	K

Notes

9. Z. C. Liang, "Cluster ensemble statistics of body particle system", *New Horizons in Mathematical Physics* **3**, 53-73 (2019). DOI:10.22606/nhmp.2019.32002

Back energy	Z	K	H	L
Ahead parameter	$a = Y/X$	L/H	K/L	H/K
Back parameter	$b = Z/X$	K/H	H/L	L/K
Entire energy	$E = Y + Z$	$L + K$	$K + H$	$H + L$
Thermal energy	$Q = Z + X$	$K + H$	$H + L$	$L + K$
Potential energy	$J = Y - X$	$L - H$	$K - L$	$H - K$
Chemical energy	$G = Z - Y$	$K - L$	$H - K$	$L - H$
Energy quantum	X_s	$H_s = h\nu$	$L_s = lz$	$K_s = kT$

III. QUANTUM STATE AND TRANSITION

a) Energy quantum

According to the principle of objectivity [4-9], any physical quantity $\boldsymbol{x}$ can be expressed by the product of scale x_s and digit $\tilde{\boldsymbol{x}}$, namely $\boldsymbol{x} = x_s \cdot \tilde{\boldsymbol{x}}$. The scale is the identifier and metric of the physical quantity, and its essence is quantum. The energy quanta of the N-particle system are defined as the statistical average of three-mode energies as

$$H_s = H/N = h\nu, \quad L_s = L/N = lz, \quad K_s = K/N = kT. \tag{10}$$

In SI system, ν is the vibration intensity (frequency) with unit hertz (Hz), z is the rotation intensity (magnetic induction) with unit tesla (T), and T is the translation intensity (thermodynamic temperature) with unit kelvin (K). Taking the energy unit as joule (J), then, $h = 6.6260693 \times 10^{-34}$ J·Hz^{-1} is Planck constant, $l = 9.2740095 \times 10^{-24}$ J·T^{-1} is Bohr magneton constant, and $k = 1.3806506 \times 10^{-23}$ J·K^{-1} is Boltzmann constant.

b) Quantum state

On an equilibrium surface, the digits $\{\tilde{X}, \tilde{Y}, \tilde{Z}\}$ must be integers when using the scale of major energy $\{X_s\}$ [4-6]. The quantum state is defined as the state whose digits are integers on an equilibrium surface. According to this definition, we have the following theorem.

Quantum state theorem. Quantum state $\{\tilde{X}, \tilde{Y}, \tilde{Z}\}$ is the positive integer solution of the algebraic equations $\{X^2 = 2YZ, \ Y^2 = 2ZX, \ Z^2 = 2XY\}$.

The quantum state on equilibrium surface S[X] can be easily determined by algebraic equation $\tilde{X}^2 = 2\tilde{Y}\tilde{Z}$. For example, Table 3 lists the quantum states of $\tilde{Z} = 1$. In the table, the ahead parameter is $a = \tilde{Y}/\tilde{X}$.

Table 3: Quantum state of $\tilde{Z} = 1$ on equilibrium surface S[X]

$\tilde{X}$	2	4	6	8	10	12	14	16	18	20	...
$\tilde{Y}$	2	8	18	32	50	72	98	128	162	200	...
$\tilde{Z}$	1	1	1	1	1	1	1	1	1	1	...
a	1	2	3	4	5	6	7	8	9	10	...

All quantum states on the surface S[X] can be plotted in the YZ plane with Z as the abscissa and Y as the ordinate, as shown in Figure 2. We marked the four straight

lines of $a = 1,2,3,4$ in the plot for easy identification. The order parameters outside the straight lines are rational numbers that are not integers.
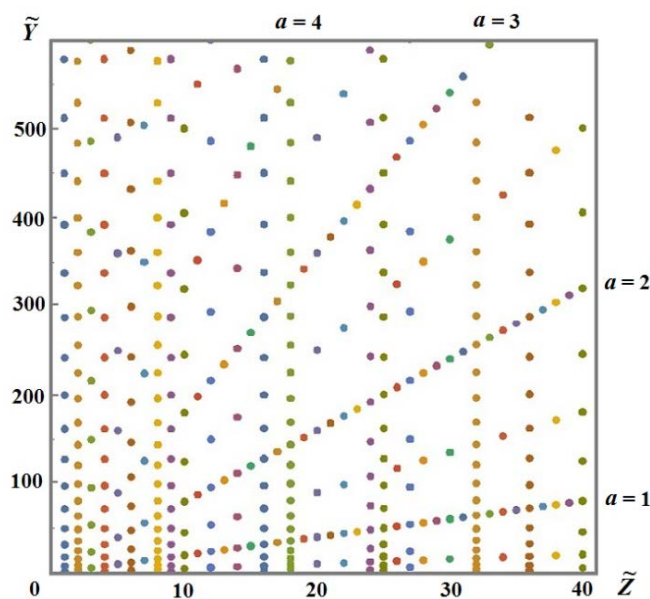

Figure 2: The plot of quantum state on equilibrium surface S[X]

c) Ground and excited state

Let's consider the quantum state in thermal equilibrium surface S[K]. In such case, it has $X_s = kT$; $\tilde{X} = \tilde{K}, \tilde{Y} = \tilde{H}, \tilde{Z} = \tilde{L}$. The quantum state equation is $\tilde{K}^2 = 2\tilde{H}\tilde{L}$, and the ahead parameter is $a = \tilde{H}/\tilde{K}$. Table 4 gives the quantum states of $\tilde{L} = 1\sim10$ on S[K] surface. In the table, S[K^h] represents the excited state of vibration. L[J_0^k] is the intersection of S[K] and S[H], representing a stable state. The first column with $\tilde{L} = 1$ is the ground state.

Table 4: Quantum states of $\tilde{L} = 1\sim10$ with ground state $\tilde{L} = 1$ on S[K] surface

State	L[J_0^k]	S[K^h]									
$\tilde{K}$	2	8	18	32	50	72	98	128	162	200	...
$\tilde{H}$	2	16	54	128	250	432	686	1024	1458	2000	...
$\tilde{L}$	1	2	3	4	5	6	7	8	9	10	...
a	1	2	3	4	5	6	7	8	9	10	...

d) Quantum transition

The quantum transition (or quantum jump) is a process of continuous conversion between quantum states. According to the continuous equilibrium equation $K^2 = 2HL$, the motion energy on S[K] can be expressed by the ahead parameter $a = H/K$ as

$$K(a) = 2aL(a), H(a) = 2a^2L(a), L = L(a). \qquad (11)$$

Therefore, we express the rotation energy as

$$L(a) = \frac{H(a)}{2a^2}. \qquad (12)$$

After the transition from a high-energy state (a') to a low-energy state (a), part of energy is converted to vibration radiation and lost. We express the lost rotation energy by the difference of the vibration frequency as

$$v_{a'a} = \frac{L(a) - L(a')}{h} = \frac{H(a)}{2ha^2} - \frac{H(a')}{2ha'^2}. \tag{13}$$

The frequency difference of lost energy is called emission spectrum, which can be decomposed into two terms

$$v_{a'a} = \bar{v}_{a'a} - \Delta v_{a'a}; \quad \bar{v}_{a'a} = \frac{H(a)}{2h}\left(\frac{1}{a^2} - \frac{1}{a'^2}\right), \quad \Delta v_{a'a} = \frac{\Delta H_{a'a}}{2ha'^2}. \tag{14}$$

$\bar{v}_{a'a} > 0$ is the spectral frequency caused by the change of ahead parameter. $\Delta v_{a'a} > 0$ is the spectral line-width caused by the loss of vibration energy, $\Delta H_{a'a} = H(a') - H(a)$. We can obtain the absorption spectrum $(v_{aa'})$ from the emission spectrum $(v_{a'a})$ by switching a and a', and changing the signs of frequency and line-width.

IV.   SPECTRUM OF HYDROGEN ATOMS

a) Spectral frequency

According to Table 4, we obtain the first three series of the emission spectrum as

$$\bar{v}_{a'1} = \frac{H(1)}{2h}\left(\frac{1}{1^2} - \frac{1}{a'^2}\right); \quad a' = 2,3,4,\cdots \tag{15a}$$

$$\bar{v}_{a'2} = \frac{H(2)}{2h}\left(\frac{1}{2^2} - \frac{1}{a'^2}\right); \quad a' = 3,4,5,\cdots \tag{15b}$$

$$\bar{v}_{a'3} = \frac{H(3)}{2h}\left(\frac{1}{3^2} - \frac{1}{a'^2}\right); \quad a' = 4,5,6,\cdots \tag{15c}$$

Compared with Rydberg formula of hydrogen atoms [1], spectral frequencies $\bar{v}_{a'1}, \bar{v}_{a'2}, \bar{v}_{a'3}$ are Lyman series, Balmer series, and Paschen series, respectively. Therefore, the emission spectrum with ground state $\tilde{L} = 1$ is the spectrum of hydrogen atoms.

The spectral frequency of hydrogen atoms can be generally expressed as

$$\bar{v}_{a'a} = \frac{H(a)}{2h}\left(\frac{1}{a^2} - \frac{1}{a'^2}\right), \quad (a = 1,2,3,\cdots; \quad a' = a+1, a+2, a+3, \cdots) \tag{16}$$

According to formula (16), we know that the ahead parameter is the principal quantum number in Bohr's atomic theory. From Table 4, we find that all three quantum numbers $\{\tilde{H}, \tilde{L}, \tilde{K}\}$ are changed after the transition. In other words, all three-mode energies $\{H, L, K\}$ are continuously changing during the transition.

b) Limit frequency

Substituting $a' \to \infty$ into formula (16) obtains the limit frequency of the a-serial spectrum.

$$\bar{v}_{\infty a} = \frac{H(a)}{2ha^2} = \frac{L(a)}{h} = \frac{\tilde{L}(a)kT}{h} \tag{17}$$

For examples, the limit frequencies of the Lyman series and the Balmer series are $\bar{v}_{\infty 1} = kT/h$ and $\bar{v}_{\infty 2} = 2kT/h$, respectively. According to formula (17), $\bar{v}_{\infty a}$ is a variable that depends on $H(a)$. The limit frequency given by the Rydberg formula of hydrogen is $\bar{v}_{\infty a} = cR_H/a^2$, where $R_H = 10967758 \text{ m}^{-1}$ is Rydberg constant [1].

Compared with (17), R_H should be $H(a)/(2hc)$. Therefore, regarding R_H as constant is an approximation.

We can express the spectral frequency by the limit frequency as

$$\bar{v}_{a'a} = \bar{v}_{\infty a}\left(1 - \frac{a^2}{a'^2}\right) \tag{18}$$

For example, the spectral frequency of the Lyman series is

$$\bar{v}_{a'1} = v_{\infty 1}\left(1 - \frac{1}{a'^2}\right), \quad a' = 2, 3, 4, \cdots \tag{19}$$

c) Spectral line-width

We have a differential relation in the continuous transition according to the expression of spectral line-width in formula (14)

$$\frac{d(v_{a'a})}{d(H_{a'a})} = \frac{1}{2ha'^2} \tag{20}$$

Roughly, the line-width is inversely proportional to the square of the ahead parameter.

The quantum transition is a continuous conversion between different motion modes. Because energy conversion takes time, the quantum transition is not instantaneous. The transition time $(\Delta t_{a'a})$ is represented by the reciprocal of spectral line-width $(\Delta v_{a'a})$ as

$$\Delta t_{a'a} = \frac{1}{\Delta v_{a'a}} = \frac{\Delta \lambda_{a'a}}{c}. \tag{21}$$

Where $\Delta \lambda_{a'a}$ is the spectral line-width in terms of wavelength. For example, a spectral line-width $\Delta \lambda_{a'a} = 1\text{nm}$ corresponds to a transition time $\Delta t_{a'a} \approx 3 \times 10^{-18}\text{sec}$.

d) Hydrogen-like spectrum

Table 5 gives the quantum states of $\tilde{L} = 2 \sim 11$ on $S[K]$ surface. Where $L[J_0^l]$ is the intersection of $S[L]$ and $S[K]$. $L[J_0^l]$ and $L[J_0^k]$ are both stable states. In this case, $L[J_0^l]$ is ground state as $\tilde{L} = 2$ is the lowest rotation energy, and $L[J_0^k]$ is meta-stable state as it has higher rotation energy.

Table 5: Quantum states of $\tilde{L} = 2 \sim 11$ with ground state $\tilde{L} = 2$ on $S[K]$ surface

State	$L[J_0^l]$	$L[J_0^k]$	$S[K^h]$								
$\tilde{K}$	2	6	12	20	30	42	56	72	90	110	...
$\tilde{H}$	1	6	18	40	75	126	196	288	405	550	...
$\tilde{L}$	2	3	4	5	6	7	8	9	10	11	...
a	1/2	1	3/2	2	5/2	3	7/2	4	9/2	5	...

From Table 5, we obtain the first three series of emission spectrum

$$\bar{v}_{a'\frac{1}{2}} = \frac{H(1/2)}{2h}\left[\frac{1}{(1/2)^2} - \frac{1}{a'^2}\right]; \quad a' = \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \cdots \tag{22a}$$

$$\bar{v}_{a'\frac{2}{2}} = \frac{H(2/2)}{2h}\left[\frac{1}{(2/2)^2} - \frac{1}{a'^2}\right]; \quad a' = \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \cdots \tag{22a}$$

$$\bar{v}_{a'\frac{3}{2}} = \frac{H(3/2)}{2h}\left[\frac{1}{(3/2)^2} - \frac{1}{a'^2}\right]; \quad a' = \frac{4}{2}, \frac{5}{2}, \frac{6}{2}, \cdots \tag{22a}$$

Unlike the case of hydrogen atoms, the order parameter contains half integers, which is the spectral feature of hydrogen-like ions. Hydrogen-like spectral frequency can be uniformly expressed as

$$\bar{v}_{a'a} = \frac{H(a)}{2h}\left(\frac{1}{a^2} - \frac{1}{a'^2}\right), \left(a = \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \cdots; \ a' = a + \frac{1}{2}, a + \frac{2}{2}, a + \frac{3}{2}, \cdots\right) \qquad (23)$$

V. Conclusion

The elastic particle theory shows that particles have three independent motion modes of translation, rotation, and vibration. The three-mode energies $\{H, L, K\}$ form a Cartesian energy space. The energy quanta $\{K_s = kT, L_s = lz, H_s = hv\}$ are the statistical averages of $\{H, L, K\}$. There are three equilibrium surfaces in the energy space, respectively representing thermal equilibrium, magnetic equilibrium, and radiative equilibrium. Quantum states $\{\widetilde{H}, \widetilde{L}, \widetilde{K}\}$ are those on the equilibrium surface in which the energy takes integer.

The application of particle motion theory to atomic system shows that atoms are elastic particles and the optical radiation originates from the elastic vibration of atoms or molecules. The transition between quantum states produces a line spectrum. The main characteristics of the optical spectrum, including spectral frequency, limit frequency, spectral line-width, and transition time, are predicted theoretically, which are consistent with the existing observations. The results demonstrate that the quantum state is the equilibrium property of particle statistics, the quantum transition is the conversion process of particle motion modes, and the quantum randomness originates from the classical statistics of elastic particles.

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