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Solitary Wave Solutions of Chafee-Infante Equation and (2+1)-Dimensional Breaking Soliton Equation by the Improved Kudryashov Method

By Umme Habiba, Md. Abdus Salam, Md. Babul Hossain & Mousumi Datta
Mawlana Bhashani Science and Technology University

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Keywords: improved kudryashov method; chafee-infante equation; (2+1) dimensional breaking soliton equation; solitary wave solutions.

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Ref

4. Chun C.Soliton and periodic solutions for the fifth-order KDV equation with the Exp - function method. Physics Letters A 2008; 372: 2760 -2766.

Solitary Wave Solutions of Chafee-Infante Equation and (2+1)-Dimensional Breaking Soliton Equation by the Improved Kudryashov Method

Umme Habiba α, Md. Abdus Salam σ, Md. Babul Hossain ρ & Mousumi Datta ω

Abstract- In this paper, we apply the improved Kudryashov method for finding exact solution and then solitary wave solutions of the Chafee-Infante equation and (2+1)-dimensional breaking soliton equation, where mathematical software Maple-13 is used as an important mathematical tool for removing calculation complexity, justification of the solutions and its graphical representations.

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I. Introduction

Nonlinear partial differential equations (NPDEs) describe many complex physical phenomena in different fields of science and engineering especially in fluid mechanics, plasma physics, chemical kinematics, chemical physics and geochemistry. It is important to note that many equations contain empirical parameters or empirical functions. Exact solutions allow us to determine these parameters or functions by using various techniques. So many techniques of obtaining exact and then solitary wave solutions have been explored and developed, such as $\exp(\Phi(\xi))$-expansion[1], Exp-function method[2]-[4], F-expansion method[5], modified Kudryashov method[6], modified Simple equation method[7]-[9], the extended tan-method[10], simplest equation method[11] and so on. The objective of this paper is to apply improved Kudryashov method [12] and to explore new exact solutions of nonlinear partial differential equations. This paper is organized as follows: in section 2, we give the description of the improved Kudryashov method. In section 3, we use this method to find the solitary wave solutions of nonlinear partial differential equations pointed out above. In section 4, we try to write the results and future directions. Last of all in section 5 conclusion is given.

II. Description of the Improved Kudryashov Method

The algorithm of the improved Kudryashov method for finding exact solutions of nonlinear partial differential equations is given below

Author α σ ρ ω: Department of Mathematics, Mawlana Bhashani Science and Technology University, Santosh, Tangail-1902, Bangladesh.
e-mail: mahinspr@gmail.com

Step-1: Suppose the nonlinear PDE in the following form:

$$p(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{xx}, u_{xxy} \dots\dots\dots\dots) = 0 \quad (2.1)$$

Now we use the traveling wave variable

$$u(x,t) = u(\xi),\ \xi = kx - ct \text{ [for (1+1)-dimensional equations]} \quad (2.2)$$

$u(x,y,t) = u(\xi),\ \xi = kx + wy - ct$ [for (2+1)-dimensional equations]

Then eq. (2.1) can be converted to nonlinear ordinary differential equation (ODE) by using eq.(2.2)

$$p(u, -cu', u', u', c^2u'', -cu'', u''', \dots\dots\dots\dots) = 0 \quad (2.3)$$

Step-2: We seek for the exact solution of eq. (2.3) in the following form:

$$u(\xi) = \frac{\sum_{i=0}^{M} a_i Q^i}{\sum_{j=0}^{N} b_j Q^j},\ Q = Q(\xi) \quad (2.4)$$

where $a_i, b_j, i = 1,2,3, \dots\dots M$ and $j = 1,2,3, \dots\dots N$ are unknown constants and $Q(\xi)$ are the following functions: $Q(\xi) = 1/\sqrt{\lambda + c_1 e^{2\xi}}$ or, $Q(\xi) = -1/\sqrt{\lambda + c_1 e^{2\xi}}$ (2.5)

Above functions satisfy to the first order differential equation

$$\frac{dQ}{d\xi} = \lambda Q^3 - Q \quad (2.6)$$

To calculate the necessary number of derivatives of function $u(\xi)$, equation (2.6) is necessary. We can obtain the positive integers M and N by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in eq. (2.3).

Step-3: Substitute $u(\xi)$ and its various derivatives in eq. (2.3) and then we collect all terms with the same powers of function $Q(\xi)$ and equate the resulting expression to zero. Then we obtain a system of algebraic equations. Solving this system, we get values for the unknown parameters.

Step-4: We put these values of unknown parameters and use the solutions of eq. (2.6) to construct the exact solutions of the eq. (2.1). And finally particular choices of arbitrary constants in exact solutions give many solitary wave solutions.

III. Applications

Now we will apply the improved Kudryashov method described in section 2 to find thesolitary wave solutions of nonlinear partial differential equations.

Example-1: Chafee-Infante equation

Here the improved Kudryashov method is used for finding the solitary wave solutions of the Chafee-Infante equation[13]

$$u_t - u_{xx} = \alpha u(1 - u^2) = 0 \quad (3.1)$$

Where α is an arbitrary constant. The parameter α adjust the relative balance of the diffusion term and the nonlinear term.

Applying the travelling wave variable $\xi = kx - ct$ we obtain the following ODE

$$-cu' - k^2u'' + \alpha(u^3 - u) = 0 \quad (3.2)$$

Ref

13. Sakthivel R and Chun C.New Soliton Solutions of Chaffee-Infante Equations Using the Exp-Function Method. Z. Naturforsch. 2010; 65a: 197–202.

Notes

where the prime denotes the differentiation with respect to ξ.
We suppose that eq. (3.2) has the travelling wave solution of the form

$$u(\xi)=\frac{\sum_{i=0}^{M} a_i Q^i}{\sum_{j=0}^{N} b_j Q^j}\,,\ Q=Q(\xi) \tag{3.3}$$

Considering the homogeneous balance between u'' and u^3 in eq. (3.2),we obtain $M=N+2$. Suppose $N=1$and then $M=3$.
Thus the exact travelling wave solution takes the following form:

$$u(\xi)=\frac{a_0+a_1Q+a_2Q^2+a_3Q^3}{b_0+b_1Q} \tag{3.4}$$

where $a_0,a_1,a_2,a_3$and b_0,b_1 are unknown constants. Substituting eq. (3.4) into eq. (3.2) and taking into account relations eq. (2.6), we get a polynomial of $Q(\xi)$. Collecting all the terms with the same power of $Q(\xi)$ together and equating each coefficient to zero, we can obtain a system of algebraic equations. Solving the resulting system by using Maple, we get the following sets of values of unknown constants.

Case-1: $c=\frac{3}{4}\alpha, k=\pm\frac{1}{2}\sqrt{\frac{\alpha}{2}}, a_0=0, a_1=0, a_2=a_2, a_3=\pm b_1\lambda, b_0=\pm\frac{a_2}{\lambda}, b_1=b_1$

The exact solution of eq. (3.1) is: $u(x,t)=\frac{\lambda}{\lambda+c_1e^{\sqrt{\frac{\alpha}{2}}x-\frac{3}{2}\alpha t}}$ or $\frac{-\lambda}{\lambda+c_1e^{\sqrt{\frac{\alpha}{2}}x-\frac{3}{2}\alpha t}}$ (3.5)

And for example, two of the solitary wave solutions and their corresponding graphs respectively are:

$u(x,t)=\frac{1}{1+e^{x-3t}}$ when $\lambda=1, c_1=1$and $\alpha=2$.

$u(x,t)=-\frac{5}{5+3e^{x-3t}}$ when $\lambda=-10, c_1=-6$ and $\alpha=2$.

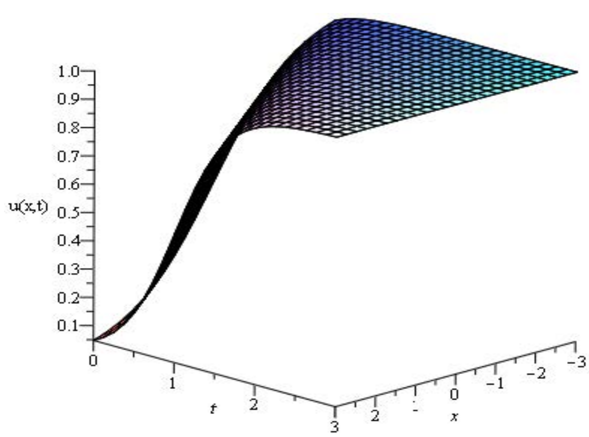

Fig. 1(3d plot): Kink type wave profile when $\lambda=1,\ c_1=1,\ \alpha=2$.

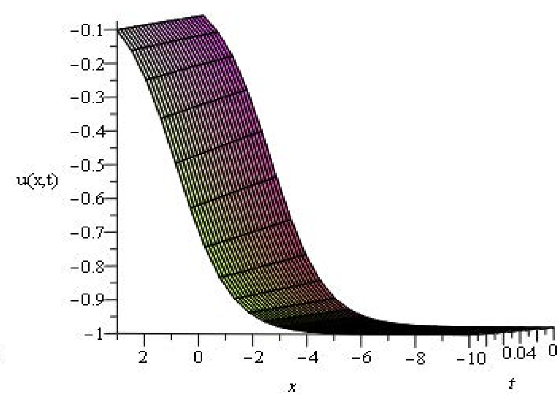

Fig. 2(3d plot): Kink type wave profile when $\lambda=-10,\ c_1=-6,\ \alpha=2$.

Case-2: $c=-\frac{3}{4}\alpha, k=\pm\frac{1}{2}\sqrt{\frac{\alpha}{2}}, a_0=-\frac{a_2}{\lambda}, a_1=\pm b_1, a_2=a_2, a_3=\pm b_1\lambda, b_0=\pm\frac{a_2}{\lambda}, b_1=b_1$

The exact solution of eq. (3.1) is: $u(x,t)=1-\frac{\lambda}{\lambda+c_1e^{\sqrt{\frac{\alpha}{2}}x+\frac{3}{2}\alpha t}}$ or $-1+\frac{\lambda}{\lambda+c_1e^{\sqrt{\frac{\alpha}{2}}x+\frac{3}{2}\alpha t}}$ (3.6)

And for example, two of the solitary wave solutions and their corresponding graphs respectively are:

$u(x,t) = 1 - \frac{1}{1+e^{x+3t}}$ when $\lambda = 1, c_1 = 1$ and $\alpha = 2$.

$u(x,t) = -1 + \frac{5}{5+3e^{x+3t}}$ when $\lambda = -10, c_1 = -6$ and $\alpha = 2$.

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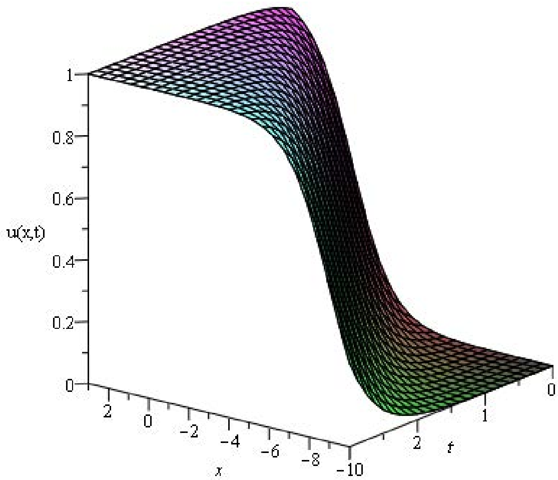


Fig. 3(3d plot): Kink type wave profile when $\lambda = 1$, c_1=1,α=2

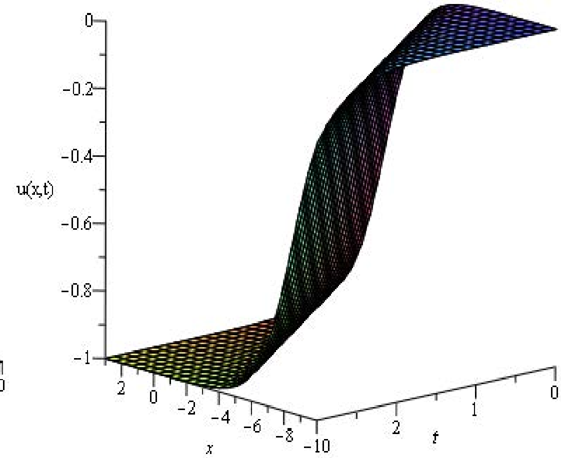
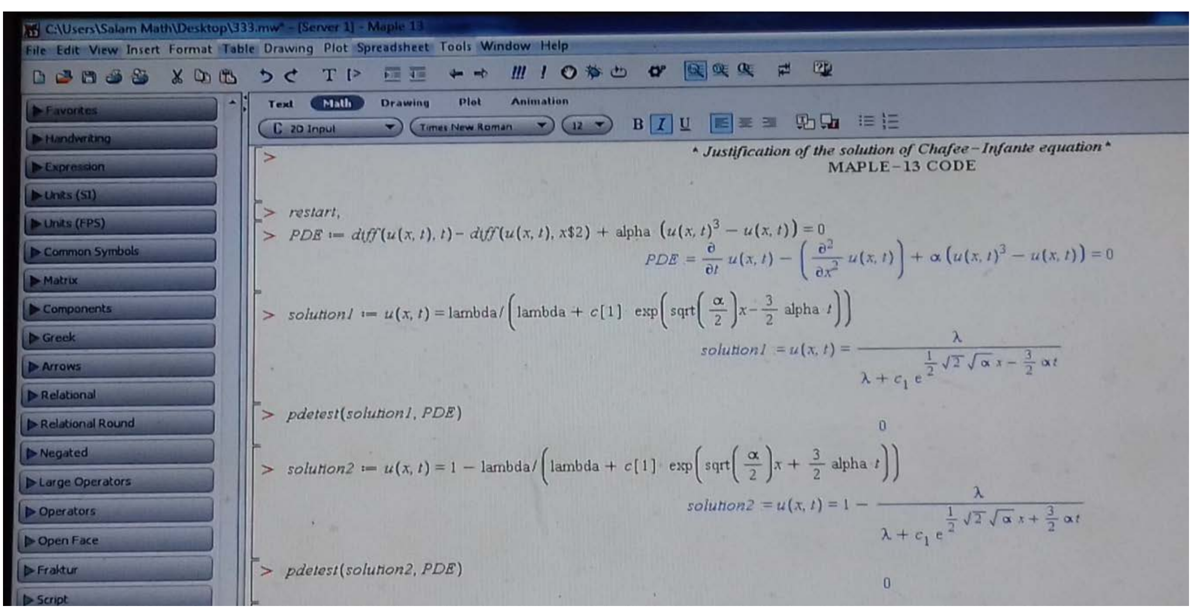


Fig. 4(3d plot): Kink type wave profile when λ =−10, c_1 =−6, α= 2.

<u>Justification of the solutions of Chafee-Infante equation by Maple-13</u>



Example 2: The (2+1)-dimensional Breaking Soliton (BS) equation

Now, we will investigate explicit solitary wave solutions of the following (2+1)-dimensional breaking soliton equations

$$u_t + \alpha u_{xxy} + 4\alpha(uv)_x = 0 \tag{3.7}$$

$$u_y = v_x \tag{3.8}$$

Notes

Where α is a nonzero constant. Equation (3.7) and eq. (3.8) describe the (2 + 1)-dimensional interaction of a Riemann wave propagation along the y-axis with a long wave propagated along the x-axis.

If we follow the similar solution procedure of example-1, we get the following sets of constants and corresponding exact solutions.

Case-1: Values of constants

$$c = 4\alpha, a_0 = 0, a_1 = 0, a_2 = 6b_0, a_3 = 6b_1, a_4 = -6b_0, a_5 = -6b_1, b_0 = b_0, b_1 = b_1$$

The exact solution of eq. (3.7) and (3.8) are:

$$u(x, y, t) = v(x, y, t) = \frac{6}{\lambda + c_1 e^{2(x+y-4\alpha t)}} - \frac{6}{[\lambda + c_1 e^{2(x+y-4\alpha t)}]^2} \tag{3.9}$$

And for example, a solitary wave solution and its graphs is:

$$u = v = \frac{6}{1 + e^{2(x+4t)}} - \frac{6}{[1 + e^{2(x+4t)}]^2}$$

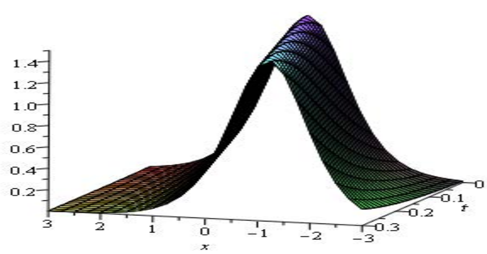


Fig. 5(3d plot): Kink Type wave profile at y=0 when $\lambda = 1, c_1 = 1, \alpha = -1$.

Case-2: Values of constants

$$c = -4\alpha, a_0 = -b_0, a_1 = -b_1, a_2 = 6b_0, a_3 = 6b_1, a_4 = -6b_0, a_5 = -6b_1, b_0 = b_0, b_1 = b_1$$

The exact solution of eq. (3.7) and (3.8) are:

$$u = v = -1 + \frac{6}{\lambda + c_1 e^{2(x+y+4\alpha t)}} - \frac{6}{[\lambda + c_1 e^{2(x+y+4\alpha t)}]^2} \tag{3.10}$$

And for example, a solitary wave solution and its graphs is:

$$u = v = -1 + \frac{6}{1 + 2e^{2(x-8t)}} - \frac{6}{[1 + 2e^{2(x-8t)}]^2}$$

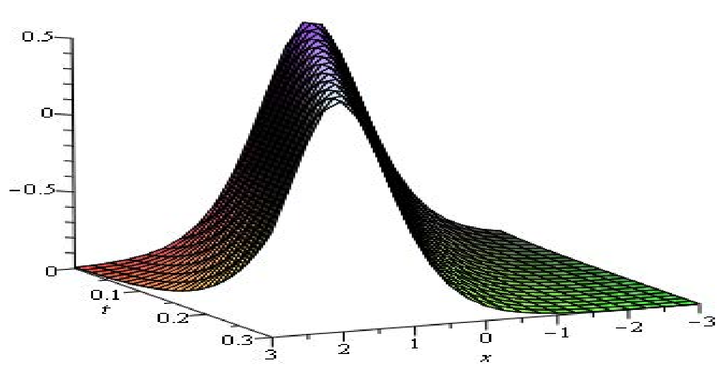


Fig.6(3d plot): Kink type wave profile at y=0 when $\lambda = 1, c_1 = 2, \alpha = -2$

IV. Results and Future Directions

In example-1 of section 3, we find the exact sloutions of Chafee-Infante equation by improved Kudryashov method. From fig. 1-fig.4 we get kink type wave profile for different particular values of parameters choosing in eq.(3.5) and eq.(3.6). These graphs increase or fall down from one asymptotic state to another. The kink solution approaches a constant at infinity. In example-2, using this method wesolve the (2+1)-dimensional breaking soliton equations and get also kink type wave profile. From fig. 5(3d plot) and fig.6(3d plot) give its graphical representations. In future, various partial differential equations of higher order can be solved by using the improved Kudryashov method. Besides, obtained results can be used for practical applications in later research.

V. Conclusion and Future Research

We have properly applied the improved Kudryashov method to establish exact solutions and then solitary wave solutions of the Chafee-Infante equation and the (2+1)-dimensional breaking soliton equation. The result discover that nonlinear partial differential equations can be easily handled by the improved Kudryashov method and that the performance of this method is authentic and efficient. The method is short and straightforward, and we can also apply this to other nonlinear problems. Also, thephysical interpretation of these solutions and actual applications in reality will be investigated in future papers.

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