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Temperatures and Altitudes of Isothermal Layers of Earth's, Titan's and Jupiter's Atmospheres in a Simultaneous Proportion of the Golden Ratio Constant, 1.6180339

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Temperatures and Altitudes of Isothermal Layers of Earth's, Titan's and Jupiter's Atmospheres in a Simultaneous Proportion of the Golden Ratio Constant, 1.6180339

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I. Introduction

The golden ratio constant φ=1.6180339…is a constant of the ancient times. The only constant older than φ is π (Rhind's mathematical papyrus, XIX century BC). First concrete application of this constant is noted in the proportions of the famous Parthenon (constructed in 447 BC and completed in 432 BC). Known algebra form of the constant φ was obtained on the basis of the Euclidean definition and it sums to: $\frac{1+\sqrt{5}}{2}=1.618...$

Constant φ has a unique unit fraction: $1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+...}}}=1.61803398...$

Value of the constant is also found in the convergence condition for consecutive order members $F_n=F_{n-1}+F_{n-2}$ with arbitrary initial numbers, of which certainly the best known one is the Fibonacci sequence with initial numbers 0 and 1:

$$0,\ 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ 55,\ 89,\ 134,\ ... \qquad \lim_{n\to\infty}\frac{F_n}{F_{n-1}}=1.61803398...$$

Although most of the current literature that deals with the topic of golden ratio constant is still of a predominantly sensationalistic character, last 20 years were marked

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by the production of a significant number of scientific papers in which the golden ratio constant had a central role. Systematization of these papers is not yet possible to perform, because the golden ratio constant is becoming a phenomenon found simultaneously in numerous areas. From quantum mechanics (El Nashie, 1994), chemical reactions (Heyrovska, 2006; Heyrovska, 2007; Yablonsky et al., 2010) in DNA structures (Yamagishi et al., 2008; Perez, 2010), the human brain functions (Conte et al., 2009; Pletzer et al., 2010), the structure of the human heart (Henein et al., 2011), the human facial features and facial attractiveness ratings (Schmid et al., 2008; Mizumoto et al., 2009) and structure of plants (Mathai and Davis, 1974; Ridley, 1982; Lanling and Wang, 2009). From micro and meta structures, the special mathematical phenomenon has multiple application in mathematics, science and engineering (Stakhov, 1989, Stakhov 2005), realistic networks (Estrada, 2007), the production of textiles (Gao et al., 2007), Special relativity theory (Sigalotti and Mejias, 2006) and cosmology (Stakhov and Rozin, 2007).

Golden ratio constant still has no significance in applied mathematics in a level of the constants e and π. Therefore, the established relation of the hyperbolic functions and formations of Fibonacci and Lucas sequences is of particular importance (Stakhov and Rozin, 2005) as well as relations of constants e and π with φ (Tanackov et al., 2011). In addition to the above listed and many not mentioned scientific papers related to the golden ratio constant, it should be noted that the existence of golden ratio constantis established in studies of the Earth's atmosphere (Willoughby, 2011).

II. Golden Ratio in the Earth's Atmosphere

99% of the Earth's atmosphe reconsists of two chemical elements, 78% nitrogen and 21% oxygen. The troposphere extends upwards from right above the boundary layer, and ranges in height from an average of 9 km (5.6 mi; 30000 ft) at the poles, to 17 km (11 mi; 56,000 ft) at the Equator (Gettelman et al., 2002).

In the absence of inversions and not considering moisture, the temperature lapse rate for this layer is 6.5° C/km, on average, according to the U.S. Standard Atmosphere. Boundary layer of troposphere and stratosphere is the tropopause. Isothermal phenomenon of the tropopause is, on average, around –60°C.

With increasing height, temperature rises through the stratosphere at a negative lapse rate until the next inversion, which is achieved through stratopause. The stratopause is the level of the atmosphere which is the boundary between stratosphere and mesosphere.

On Earth, the stratopause is 50 to 55 kilometers. Altitude of temperature inversions in the stratopause may vary, there may be significant fluctuations in the isothermal zone of the stratosphere with several peaks (Prakash Raju et al., 2011) of medium temperatures around ˜–8° C or ˜–5° C at altitudes of 46 km or 53.5 km respectively, in tropical regions, or significantly lower temperatures that in polar regions generate the stratopause temperature up to –20° C (France et al., 2012). Temperature variations of the stratopause during the year can be found in a wide range of –20°C to +2°C at altitudes of 55 km to 42 km respectively (Alexander et al., 2011).

Further altitude increase is characterized by positive lapse rate until the next inversion, which is realized through mesopause. Intervals of minimum temperature of about –90° C at altitudes of 85 km to100 km were measured at the latitude of 23°S (Clemesha et al., 2011). Detailed analysis of temperatures and altitudes of mesopause in the tropical zone (13.5°N79.2°E) next to the clear influence of the season, introduces

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also the function of latitude and emphasizes the mesopause altitude of 100 km (Ratnam et al., 2010). In the polar area (80flS and 223flE), established altitudes of mesopause were 92 km (Sheese et al., 2011).

The same mesopause altitudes of 92 km were established at lowerl atitudes (68flN and 21flE). Mesopause is the coldest part of the atmosphere, with fascinating temperatures that may be lower than100° K. However, such low temperatures are characteristic for the specific conditions in polar areas–polar mesospheric cloud season (Schmidlin, 1992). Thermal structure at the midlatitude of mesopause (41°N, 105°W) has the established minimum temperatures from–100° C (172° K at summer solstice) and a high of–61°C (212°K at nearly one month following winter solstice) (She et al., 1993), or from −100°C (165°K) to −59°C (214°K) in Wuppertal (51°N,7°E) (Höppner and Bittner, 2007). These temperatures are established at altitudes from 87 km to 99 km. Overall, altitudes and temperatures of the tropopause, stratopause and mesopause depend on the season, latitude, local weather (Clemesha et al., 2011), volcanic eruptions (Hampson et al., 2006), planetary and gravity waves (Limpasuvan et al., 2012), solar cycles (She et al., 2002), etc.

Conventional average representation of the relations between mean temperatures of mean altitudes of atmosphere layers is shown in Figure 1. Typical values of temperatures and altitudes have an assumed simultaneous proportion of the golden ratio.

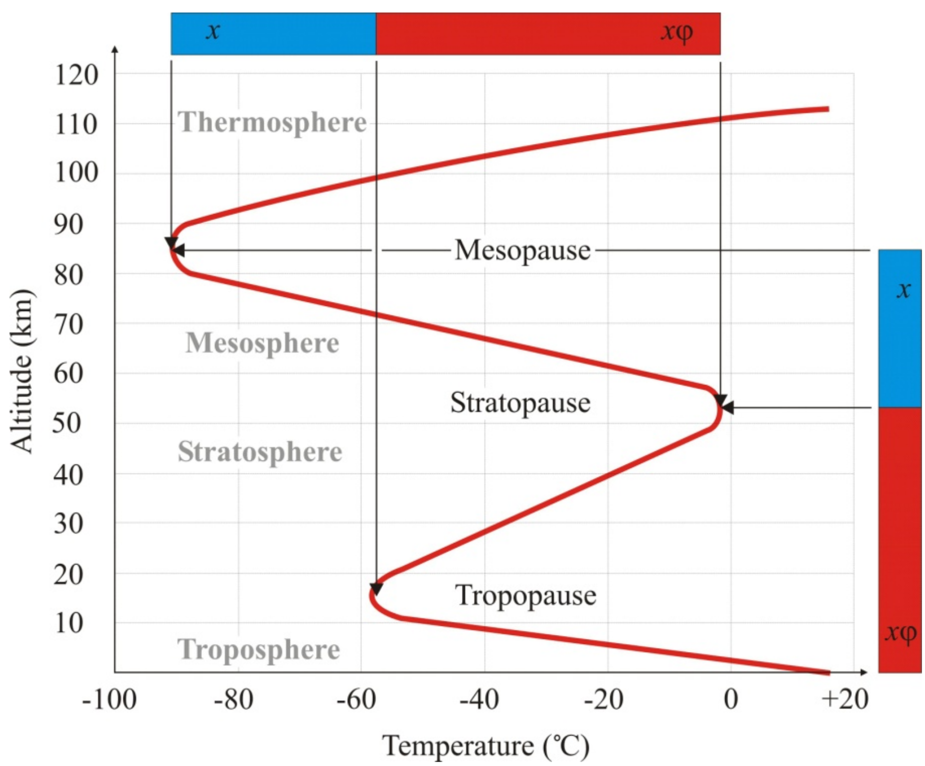

Fig. 1: Mean temperatures and mean altitudes of Earth's atmosphere layers

III. Golden Ratio in the Atmosphere of Titan

After Jupiter's satellite Ganymede, Saturn's satellite Titan is the second largest satellite in the solar system and the only known moon with a dense atmosphere, with an average surface pressure of 146.7 kPa, with nitrogen (98.4%) and methane (1.6%) structure.

According to the data for 79.8°N, temperature on the surface of Titan, at 0.0 km altitude is 90.15°K. A drop in temperature to 48 km altitude achieves the isothermal minimum of 66.45°K. Growth of temperature to 80km reaches isothermal maximum of 119.39°C, and at 100 km the isothermal minimum of 109.23°K (Schinder et al., 2012).

Relations in altitudes of 0 km, 48 km, 80 km and 100 km where are the established isothermal minima and maxima of temperatures of the Titan's atmosphere, have departures from the golden ratio constant of 3.00% (1) and 0.43% (2), respectively:

$$\frac{80-0}{48-0}=1.666 \Leftrightarrow \frac{1{,}666666-\varphi}{\varphi}=0.03005 \quad (1)$$

$$\frac{100-80}{100-48}=1.625 \Leftrightarrow \frac{1{,}625-\varphi}{\varphi}=0.00432 \quad (2)$$

The temperature values of characteristic altitudes T(0 km)=90.15°K, T(48 km)=66.45°K, T(80 km)=119.3°K and T(100 km)=109.23°K have numerical coincidences with values of linear combinations of the golden ratio constant. All these numerical coincidences result in deviations of less than 0.5% (3), (4), (5)!

$$\frac{109.23-66.45}{90.15-66.45}=1.805063 \Leftrightarrow \frac{1.805063-\left(\frac{\varphi}{2}+1\right)}{\left(\frac{\varphi}{2}+1\right)}=-0.002184 \quad (3)$$

$$\frac{119.39-66.45}{109.23-66.45}=1.237495 \Leftrightarrow \frac{1.237495-\frac{2}{\varphi}}{\frac{2}{\varphi}}=0.001153 \quad (4)$$

$$\frac{119.39-66.45}{90.15-66.45}=2{,}233755 \Leftrightarrow \frac{2{,}233755-(2\varphi-1)}{(2\varphi-1)}=-0.001034 \quad (5)$$

Relations of altitudes and temperatures of isothermal layers of the Titan's atmosphere, have a larger number of numerical coincidences with the golden ratio constant or its linear combinations (Figure 2.).

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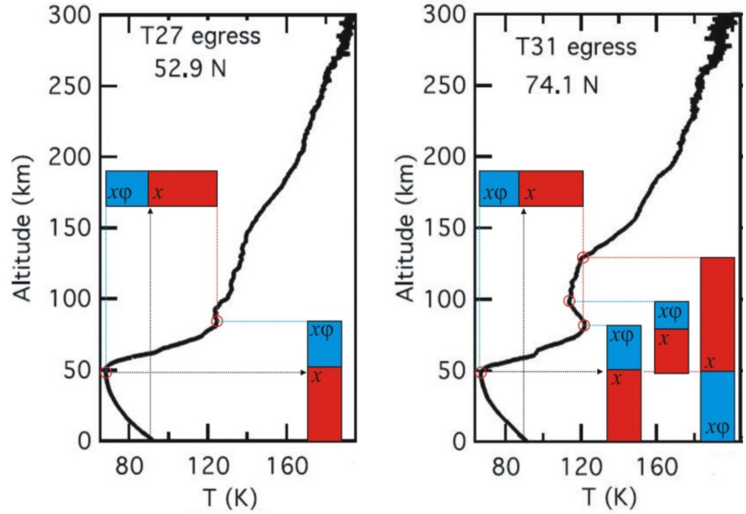

Fig. 2: Numerical coincidences with the golden ratio in Titan's atmosphere

IV. Golden Ratio in the Atmosphere of Jupiter

Jupiter's atmosphere has a dominant composition of hydrogen (86%) and helium (14%). Atmospheric pressure on Jupiteris is 70 kPa. Analogue to terrestrial terminology, Jupiter′s stratosphere extends from the tropopause at 28km (atmospheric pressure of 280 mbar) to the mesopause at ~350 km (atmospheric pressure of ~0001 mbar). Temperatures of Jupiter's upper stratosphere have been established with a Galileo Atmosphere Structure Instrument (ASI) (Young et al., 2005).

In regard to the overall temperature range of Jupiter's atmosphere, in the region between 90 km and 290 km, the mean temperature is essentially isothermal. At three different altitudes ~95 km, ~117 km and ~264 km, three extreme temperature values of the stratosphere were established, two temperature maxima and one temperature minimum. Data were obtained from two sensors (z_1 and z_2) at altitude of 6.5° North in December 1995. A detailed description of the resultsis available in the literature (Young et al., 2005).

Temperature of ~161°K at ~95 km altitude is the first extreme value of the isothermal zone of Jupiter's stratosphere. Average temperature value of Jupiter's stratosphere is 158.1°K and is close to the first extreme value.

The other extreme temperature value of Jupiter's stratosphere is ~148°K at ~117 km altitude which is also the established minimum temperature of Jupiter's stratosphere. Close value, but not the lowest, of 149.8°K at 262.736 km altitude (measured with sensor z_1) and 149.9°K at 260.785 km altitude (measured with sensor z_2) is still about 2°K higher. Altitude of ~117 km is reported to have a lower value of temperature fluctuations of Jupiter's stratosphere from ~–0.07, while the temperature fluctuation at altitude of ~261 km is lower and is around ~ –0.06.

Third extreme temperature value of Jupiter's stratosphere is ~170°K at altitude of ~164 km. This extreme value of temperature is the very maximum of the entire stratosphere. The following table distinguishes characteristic accelerometer data for sensors $z_1$and $z_{2.}$ (Table 1.).

Table 1: *Characteristic accelerometer data for sensors z_1 and z_2*

Sensors	Measured temperatures (°K)		Measured Altitudes (km)	
$P_1(z_1)$	169,6	Average P_{1a} 169.9	165,147	Average P_{1a} 164,261
$P_1(z_2)$	170,2		163,376	
$P_2(z_1)$	148,1	Average P_{2a} 148.0	116,513	Average P_{2a} 117,21
$P_2(z_2)$	147,9		117,907	
$P_3(z_1)$	160,6	Average P_{3a} 161.0	95,594	Average P_{3a} 95,130
$P_3(z_2)$	161,4		94,665	

Hypothesis on the temperature and altitude proportion of the golden ratio in the upper stratosphere of Jupiter can be confirmed by the ANOVA test. Hypotheses on the proportion in characteristic values of temperatures and latitudes of Jupiter's stratosphere, is proposed with the calculation of the third value based on choosing two values and function context of the constant φ=1.6180339.

In accordance with the assumption on a "golden" ratio of altitude differences and the characteristic values of temperatures of isothermal layers of the Earth's atmosphere as well as numerical coincidences of altitude and temperature differences in isothermal layers in the atmosphere of Titan, a hypothesis is set on the simultaneous proportions of altitude and temperature differences of isothermal layers in the atmosphere of Jupiter.

The possibility of verifying the hypothesis by the ANOVA test was designed due to the existence of variance in the minimum statistical set of values of altitudes and temperatures. The minimum statistical set was formed with measurements from two sensors of the mission Galileo ASI (Young et al., 2005).

The basis of the hypothesis has a predictive character. It is estimated that based on the two selected values of temperatures or altitudes of isothermal layers of the Jupiter's atmosphere, it is possible to establish a functional context of the constant φ=1.6180339 and make a prediction of altitude and temperature of the following isothermal layer (Figure3).

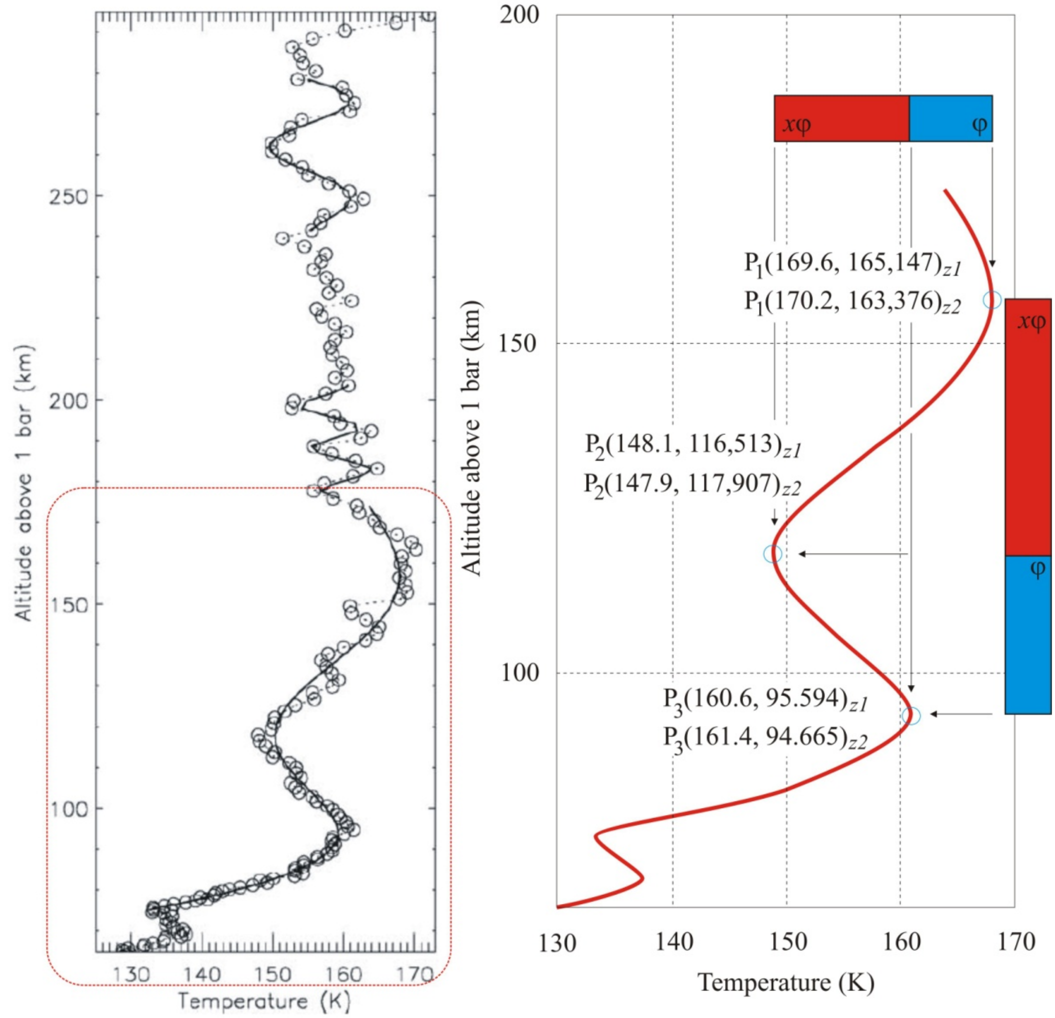

Fig. 3: Three wave trains in the Galileo ASI data, and numerical values of altitudes with extreme values of temperature in the stratosphere of Jupiter with a graphic proportion of golden mean

Based on the maximum values of temperatures measured with sensor z_1 (169.6ﬂK and 160.6ﬂK), there is an estimation of the quotient of φ value of differences in maximum temperatures and minimum temperature of the isothermal layers of the Jupiter's atmosphere. From this assumption, replacing the value of the maximum, the predictive temperature of the minimum is calculated and it amounts to (6):

$$\frac{P_1(z_1)-P_2(z_1)}{P_3(z_1)-P_2(z_1)}=\varphi \Leftrightarrow P_1(z_1)-P_2(z_1)=\varphi P_3(z_1)-\varphi P_2(z_1)\Leftrightarrow$$

$$\Leftrightarrow \varphi^2 P_3(z_1)-\varphi P_1(z_1)=P_2(z_1)\Leftrightarrow 146{,}037=P_2(z_1)^{*} \qquad (6)$$

Analogously, for maximal values of temperature measured with z_2 sensor (170.2°K and 161.4°K), the second predictive value of the golden ratio proportion is obtained from (7):

$$\frac{P_1(z_2)-P_2(z_2)}{P_3(z_2)-P_2(z_2)}=\varphi \Leftrightarrow \varphi^2 P_3(z_2)-\varphi P_1(z_2)=P_2(z_2) \Leftrightarrow 147.161=P_2(z_2)^* \qquad (7)$$

Hypothesis on the equality of predictive and empirical values was verified by the Duncan test. Duncan test was chosen because of the liberality towards the error of the first kind and the introduction of the mean value factor. Data were analyzed with Microsoft Statistica 4.5.

Predictive value is consistent with the empirical measured value. With significance p=0.1337>0.05 it is accepted that mean values of predictive (146.59±0.7945°K) and empirical (148.00±0.1414°K) values of characteristic temperatures in isothermal layers of Jupiter's stratosphere are congruent. Proportion of the golden ratio constant in the relations of extreme temperature of Jupiter's stratosphere is significant (Table 2.).

Table 2: Predictive and empirical values of characteristic temperatures (°K) of Jupiter's stratosphere

Temeprature	Values	Means and standard deviations
$\varphi^2 P_3(z_1)-\varphi P_1(z_1)=P_2(z_1)^*$	146,037	146.59±0.7945
$\varphi^2 P_3(z_2)-\varphi P_1(z_2)=P_2(z_2)^*$	147,161	
$P_2(z_1)$	148,100	148.00±0,1414
$P_2(z_2)$	147,900	

For altitude, the following linear ratio was elected for values measured with z_1 sensor:

$$\frac{P_1(z_1)-P_3(z_1)}{P_2(z_1)-P_3(z_1)}=\varphi+\frac{3}{2} \Leftrightarrow \frac{P_1(z_1)-P_3(z_1)}{P_2(z_1)-P_3(z_1)}=\frac{2\varphi+3}{2} \Leftrightarrow$$

$$\Leftrightarrow P_1(z_1)-P_3(z_1)=\frac{2\varphi+3}{2}\left[P_2(z_1)-P_3(z_1)\right] \Leftrightarrow$$

$$\Leftrightarrow P_1(z_1)+\frac{2\varphi+3-2}{2}P_3(z_1)=\frac{2\varphi+3}{2}P_2(z_1) \Leftrightarrow$$

$$\Leftrightarrow 2P_1(z_1)+(2\varphi+1)P_3(z_1)=(2\varphi+3)P_2(z_1) \qquad (8)$$

The first predictive value of isothermal layer altitude of the Jupiter s atmosphere is (9):

$$\frac{2P_1(z_1)+(2\varphi+1)P_3(z_1)}{(2\varphi^2+3)}=P_2(z_1) \Leftrightarrow 117{,}900=P_2(z_1)^{**} \qquad (9)$$

Analogously, for the value of altitude measured withsensor z_2, the second predictive value of isothermal layer altitude of the Jupiter's atmosphere is (10):

$$\frac{2P_1(z_2)+(2\varphi+1)P_3(z_2)}{(2\varphi^2+3)}=P_2(z_2) \Leftrightarrow 116.702=P_2(z_2)^{**} \qquad (10)$$

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With extremely high significance p=0.9275>0.05, it is accepted that mean predictive (117.30±0.8478) and empirical (117.21±0.9857) values of characteristic altitudes of isothermal layers of Jupiter's stratosphere are congruent (Table 3.).

Table 3: Predictive and empirical values of characteristic altitudes (km) of Jupiter's stratosphere

Altitude	Values	Means and standard deviations
$2P_1(z_1)+(2\varphi+1)P_3(z_1)/(2\varphi^2+3)=P_2(z_1)^{**}$	117.900	117,30±0,8478
$2P_1(z_2)+(2\varphi+1)P_3(z_2)/(2\varphi^2+3)=P_2(z_2)^{**}$	116.702	
$P_2(z_1)$	116.513	117.21±0,9857
$P_2(z_2)$	117.907	

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V. Conclusion and Recommendations

The assumption about the part of the golden ratio constant φ=1.6180339... in the Earth's atmosphere, numerical coincidence is in the atmosphere of Titan and the prediction significance in the atmosphere of Jupiter, may represent an important incentive for further research in numerical models of the atmosphere. Though the part of golden ratio constants in natural phenomena has a primarily sensationalistic character, two basic mathematical fields should be emphasized, that have an important role in studies of the atmosphere, and where the golden ratio constant can have an exact mathematical importance.

The first mathematical field refers to the phenomena of fractals. Fractal phenomena have been multiply noted in the atmosphere of the Earth (Baryshnikova et al., 1989; Collins, and Rastogi, 1989) and the atmospheres of Titan and Jupiter (West and Smith, 1991; Cabane et al., 1993; Friedson et al., 2002; Rannou et al, 2003). Temperature amplitudes (Peusse et al., 2006), mean global potential energy distribution for vertical wavelengths (Fröhlich et al., 2007), air velocity in the atmospheric layers (Wrasse et al., 2006), etc. are largely self-similar and have properties of stochastic fractals. The final attractor does not exist because the system dynamics of the atmosphere is under the constant influence of cyclical phenomena (time of day, season, solar activity,etc.) or impulse phenomena (volcanic eruptions, etc.). The potential role of golden ratio constant is in the prognosis of equilibrium status of altitudes and temperatures of isothermal layers of the atmosphere. Based on the results presented in this paper, the final attractor can be determined in the proportion of the golden ratio constant, which is in the fractal concept simultaneously the initial fractal.

Another field of mathematics concerns the application of hyperbolic functions. General role of hyperbolic functions in studies of fluid dynamics is known, and specific application in atmospheric research (Kraginsky and Oparin, 2003), as well as the synthesis of hyperbolic and fractal mathematical concepts (Harlander and Maas, 2007), Binet's formulas for the calculation of Fibonacci and Lucas sequence members in a continuous domain have an exact mathematical relation with Riccati hyperbolic functions (Stakhov and Rozin, 2005). The spherical form of the atmosphere and the atmospheric layers is a space dominated by non-Euclidean geometry, i.e. hyperbolic geometry, which is based on the golden ratio constant (Stakhov, 2006). Substitution of classic analytic forms of hyperbolic function with classes of hyperbolic function based on the golden ratio constant, presents a possible way of introducing golden ratio constant in the numerical models of the atmosphere.

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