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Performance Assessment of Mean Methods in Estimating Process Capability for Non-Normal Process for Weibull Family Life Distribution

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Keywords: *process control, capability indices, performance index, standard error, skewed.*

GJSFR-F Classification: *MSC 2010: 11H60*

Ref

14. Montgomery, D. C and Runger, G. C. (2009). Applied Statistics and Probability for Engineers.3Ed. John Wiley & Sons, New York.

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Keywords: process control, capability indices, performance index, standard error, skewed.

I. Introduction

Statistical Process Control is the application of statistical tools and techniques in monitoring variation in a continuous process in order to detect variations that are of assignable causes, and therefore make recommendations for corrective check on the process. Control charts are used to monitor processes in order to detect assignable cause(s) that change the process parameters. [6, 7] emphasized the importance of identification of assignable cause. When the distribution of the output quality of the process variable is continuous, the combination of two control charts such as an X -chart and an R-chart are usually required to monitor both the process mean and the process variance [14]. However, recently [17] have shown that the two combined charts are not always reliable in identifying the nature of the change.

Measuring a process performance and acting upon the assessments based on the measurements are critical elements of any continuous quality improvement efforts [15], however, companies make assessments of process performance based on different indicators. Most common of these indicators can be described in terms of process yield, process expected loss and capability indices of a particular process characteristic [4]. Among these indicators, Process Capability Indices (PCIs) have gained substantial attention both in academic community and several types of manufacturing industries

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since 1980s [13]. The first process capability index proposed in the literature more than three decades ago is the C_p index, which is defined as:

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

where USL and LSL denote the upper and lower specification limits respectively and σ is the standard deviation of the process characteristic of interest [2]. In order to overcome this problem, a second generation PCI, the Cpk index, is introduced. The Cpk is defined as:

$$C_{pk} = \min\left[\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right] \tag{2}$$

where μ and σ are the mean and the standard deviation of the quality characteristic studied, respectively. The mean of the process characteristic has an influence on the C_{pk} index and therefore it is more sensitive to departures from centrality than the Cp index [1, 11].

Pp and Ppk are measures of process performance from a customer perspective [4, 12].

Non-normally distributed processes are not uncommon in practice. Combining this fact with the misleading results of applying basic PCIs to non-normal processes while treating them as normal distributions forced academicians and practitioners to investigate the characteristics of process capability indices with non-normal data [10, 16, 20].

There two approaches adopted in estimating PCI for non-normal process situation include:

(1) Data Transformation Approach: Data transformation approach is aimed at transforming the non-normal process data into normal process data [3, 5, 10].

(2) Distribution Fitting Method for Empirical Data: Distribution fitting methods use the empirical process data, of which the distribution is unknown [10]. These methods later fit the empirical data set with a non-normal distribution based on the parameters of the empirical distribution. Clements' Method is one of the most popular distribution approaches. Therefore, the percentile-based Cp is obtained by:

$$C_p = \frac{USL - LSL}{\xi_{0.99865} - \xi_{0.00135}} \tag{3}$$

where $\xi_{0.99865}$ and $\xi_{0.00135}$ denote the upper and lower 0.135th percentiles of the process distribution, respectively.

Following the same logic, the Cpkindex can be obtained using a percentile approach:

$$C_{pk} = \min\left[\frac{USL - \xi_{0.5}}{\xi_{0.99865} - \xi_{0.5}}, \frac{\xi_{0.5} - LSL}{\xi_{0.5} - \xi_{0.00135}}\right] \tag{4}$$

where $\xi_{0.5}$ is the median of the process distribution, which is used instead of the process mean, because the process mean is not indicative of the centrality of a non-normal distribution specially when skewness of the distribution is taken into account [1].

The mean difference is independent of any central measure of localization, which can be seen from its definition.

$$\Delta_1 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |x - y|\, dF(x)\, dF(y) \tag{5}$$

When the random variable X is discrete (a case more often considered) the formula has the form

Ref

10. Kotz, S and Johnson, N.L (1993). *Process Capability Indices.* 2nd Ed. Chapman & Hall, Suffolk.

$$\Delta_1 = \sum_{i=-\infty}^{i=+\infty} \sum_{j=-\infty}^{j=+\infty} |x-y| p_i p_j \quad (6)$$

The analytic investigation of the discussed characteristic is made difficult because of the absolute value occurring in the formula. However, it facilitates the computations on numerical data, which also concerns, as is well known, the mean deviation.

This paper therefore compares the performances of Gini Mean, Clements and Box- Cox transformation methods for estimating process capability Indices for a non-normal case.

Notes

II. Methodology

a) *Process Capability Indices*

The process capability index, the Cp index, which is defined as:

$$\text{Cp} = \frac{USL-LSL}{6\sigma} \quad (7)$$

The Cpk can be defined as:

$$min\left[\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right] \quad (8)$$

where USL and LSL denote the upper and lower specification limits, respectively, and σ is the standard deviation of the process characteristic of interest.
Process Capability relative to one sided specification limit

$C_{pu}= \frac{\text{USL-}\mu}{3\sigma}$ Process Capability relative to Upper specification limit

$C_{pl}= \frac{\mu\text{-LSL}}{3\sigma}$ Process Capability relative to lower specification limit

$P_{pu}= \frac{\text{USL-}\mu}{3\sigma}$ Process performance relative to Upper specification limit

$P_{pl}= \frac{\mu\text{-LSL}}{3\sigma}$ Process performance relative to lower specification limit

b) *Clements Method (CM)*

For non-normal Pearsonian distribution (which includes a wide class of "populations" with non-normal characteristics), [3,18] proposed a method of non-normal percentiles to calculate process capability Cp and process capability for off center process Cpk indices based on the mean, standard deviation, skewness and kurtosis. Clements utilized the table of the family of Pearson curves as a function of skewness and kurtosis [8, 9].

Clements replaced 6σ by (U_P - L_P) in the below equation,

$$\text{Cp} = \frac{USL-LSL}{U_P-L_P} \quad (9)$$

where, U_P is the 99.865 percentile and L_P isthe 0.135 percentile, For Cpk, the process mean u is estimated by median M, and thetwo 305 are estimated by (U_P — M) and (M—L_P) respectively,

$$\text{Cpk} = \min\left[\frac{USL-M}{U_P-M}, \frac{M-LSL}{M-L_P}\right] \quad (10)$$

i. *Algorithm for calculating PCIs using Clements method*

(1) Obtain the specification limits USL and LSL for a given quality characteristic

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(2) Estimate sample statistics for the given sample data: sample size, mean, standard deviation, skewness and kurtosis Calculate estimated 0.135percentile L_P
(3) Calculate estimated 99.865 percentile U_P
(4) Calculate estimated median M
(5) Calculate non-normal process capability indices using equations.

$$\mathrm{Cp} = \frac{USL-LSL}{U_P-L_P} \tag{11}$$

$$\frac{\text{USL- M}}{U_P - M}, \frac{\text{M-LSL}}{M - L_P}$$

$$\mathrm{Cpu} = \frac{\text{USL- M}}{U_P-M} \tag{12}$$

$$\mathrm{Cpl} = \frac{\text{M-LSL}}{M-L_P} \tag{13}$$

c) *Box-Cox power Transformation (BCT)*

The Box-Cox transformation was proposed by Box and Cox in 1964 and used for transforming non-normal data [9]. The Box-Cox transformation uses the parameter λ. In order to transform the data as closely as possible to normality, the best possible transformation should be performed by selecting the most appropriate value of λ. In order to obtain the optimal λ value, Box-Cox transformation method requires maximization of a log-likelihood function. After the transformation, process capability can be evaluated. They proposed a useful family of power transformations on the necessarily positive response variable *X*.

$$X^{(\lambda)} = \begin{cases} \frac{X^{\lambda}-1}{\lambda}, for\ \lambda \neq 0 \\ \ln X, for\ \lambda = 0 \end{cases} \tag{14}$$

where the variable X takes positive values. If the variable X takes negative values, then a constant value will be added in order to make the values positive. This continuous family depends on a single parameter λ that can be estimated by using maximum likelihood estimation.

Firstly, a value of λ from a pre-assigned range is collected. Then L_{max} is computed as in

$$L_{max} = -\frac{1}{2}\ln\hat{\sigma}^2 + \ln J(\lambda, X) = -\frac{1}{2}\ln\hat{\sigma}^2 + (\lambda-1)\sum_{i=1}^{n}\ln X_i \tag{15}$$

For all $\lambda, J(\lambda, X)$is evaluated as in Equation

$$J(\lambda, X) = \prod_{i-}^{n}\frac{\partial W_i}{\partial X_i} = \prod_{i=1}^{n} X_i^{\lambda-1} \tag{16}$$

$$\ln J(\lambda, X) = (\lambda-1)\sum_{i=1}^{n}\ln X_i \tag{17}$$

For fixed λ, σ^2 is estimated by using S(λ), which is the residual sum of squares of $X^{(\lambda)}$. σ^2 is estimated by the formula in the equation below [15].

$$\hat{\sigma}^2 = \frac{S(\lambda)}{n} \tag{18}$$

Ref

9. Hoerl, R.W and Snee, R. D. (2010). Flexible Process Capability Indices *The American Statistician*, 64(1), pp.10-14.

d) Gini's Mean Difference (GM)

The Gini's mean difference for a set of n ordered observations, $\{x_1, x_{2,\dots,}x_n\}$, of a random variable X is defined as:

$$\text{Gn} = \frac{2}{n(n-1)}\sum_{j=1}^{n}\sum_{i=1}^{n}\left|x_i - x_j\right| \quad (18)$$

$$\text{Gn} = \frac{2}{n(n-1)}\sum_{i=1}^{n}[(x_i - x_1) + (x_i - x_2) + \cdots + (x_i - x_{n-1})] \quad (19)$$

$$\text{Gn} = \frac{2}{n(n-1)}\sum_{i=1}^{n}(2i_i - n - 1)x_{(i)} \quad (20)$$

If the random variable X follows normal distribution with mean μ and variance σ^2, then [21] suggests a possible unbiased estimator of standard deviation (σ) as:

$$\sigma * = \text{c}\ \frac{\sum_{i=1}^{n}[(2i_i - n - 1)x_i]}{n(n-1)} \quad (21)$$

where $c = \sqrt{\pi} = 1.77245, \sigma^* = 0.8862$ G_n is an unbiased measure of variability. Gini's mean difference can be rewritten as:

$$\text{Gn} = \frac{2}{n(n-1)}\sum_{i=1}^{n}(2i - n - 1)x_{(i)} \quad (23)$$

If we write this as

$$\text{Gn} = \frac{2}{n(n-1)}\sum_{i=1}^{n}[(i-1) - (n-1)]x_{(i)} \quad (24)$$

$$\text{Gn} = \frac{2}{n(n-1)}\left[\sum_{i=1}^{n}(i-1)x_{(i)} - \sum_{i=1}^{n}(n-1)x_{(i)}\right] \quad (25)$$

$$\text{Gn} = \frac{2}{n(n-1)}[U - V] \quad (26)$$

where U= and V=

The unbiased estimator of Gini Mean difference for Weibull distribution is

$$E(\text{Gn}) = \left(2 - 2^{1-\frac{1}{\beta}}\right)\frac{\Gamma\left(1+\frac{1}{\beta}\right)}{\lambda} = \sigma_{gw} \quad (27)$$

The Weibull probability density function is given as:

$$f(x) = \lambda\beta(\lambda x)^{\beta 00000-1}e^{-(\lambda x)^{\beta}} \quad (28)$$

To compute Cp and Cpk using Gini's mean difference as a measure of variability when the data follow a Weibull distribution

$$C_{npg} = \frac{USL - LSL}{5.3172\sigma_{gw}} \quad (29)$$

$$C_{npkg} = \frac{\min(USL - m, m - LSL)}{2.6586\sigma_{gw}}$$

$$C_{npug} = \frac{USL - m}{2.6586\sigma_{gw}} \quad (30)$$

$$C_{nplg} = \frac{m - LSL}{2.6586\sigma_{gw}} \quad (31)$$

Ref

21. Yitzhaki, S (2010). Gini's mean difference: A superior measure of variability for non-normal distributions, *Metron*, 61(2), pp. 285-316.

e) *Performance Comparison of the Clements, Box-Cox transformation and the Gini Methods*

The performance comparison is carried out by generating Weibull data through simulation and for this reason, process performance indices (PPIs) are executed for computing process capability rather than process capability indices (PCIs).

Weibull distribution is used or modeling most industrial processes especially in reliability field which is concerned with the failure of a product or the time to failure of the product. Only one sided (USL) process performance index P_{P_u} is considered. The USL is computed from the equation below using a targeted *Ppu* of 1.0 and 1.5. The targeted Ppu of 1.0 is indicating the process is marginally capable of meeting the specifications and the Ppu of 1.5 is indicating the process is good and very capable of meeting the specification limits [14].

Box plots, descriptive statistics, the root-mean-square deviation (RMSD), which is used as a measure of error, are utilized for evaluating the performances of the methods. In addition, the bias of the estimated values is important as the efficiency measured by the mean square error.

f) *The Root-Mean-Square Deviation (RMSD)*

The root-mean-square deviation (RMSD) is used to measure the differences between the targeted Ppu values and the estimates obtained by BCT, Clements and Gini mean difference based methods.

$$\text{RMSD} = \sqrt{\frac{\sum_{i=1}^{r}(Estimated\ \ Ppu_i - Targeted\ \ Ppu_i)^2}{r}} \tag{32}$$

where *r* is the number of data sets generated randomly for each Weibull distribution with specified parameters. The RMSD serves to aggregate the magnitudes of the errors in the predictions for various times into a single measure of predictive power and a measure of accuracy [8].

III. RESULT AND DATA ANALYSIS

a) *The Descriptive Statistics*

The tables below show the corresponding quantiles, mean, median along with skewness and kurtosis based on the specified parameter values of Weibull distribution. Kurtosis gives information about the relative concentration of values in the center of the distribution as compared to the tails.

Table 1: Summary statistics of Weibull distribution at $\alpha = 1$ and $\beta = 1$ for different sample sizes

	Weibull(α,β)	$X_{0.99865}$	Median = $X_{0.50}$	Mean	Skewness	Kurtosis
n=25	Weibull(1,1)	3.8939	0.7469	1.0296	1.5138	2.7650
n=50	Weibull(1,1)	4.3501	0.6915	0.9989	1.6654	3.2510
n=75	Weibull(1,1)	4.8491	0.6803	0.9733	1.8448	4.7940
n=100	Weibull(1,1)	5.1722	0.7167	1.0130	1.8384	4.6222

Ref

14. Montgomery, D. C and Runger, G. C. (2009). Applied Statistics and Probability for Engineers.3Ed. John Wiley & Sons, New York.

Notes

Table 2: Summary statistics of Weibull distribution at $\alpha = 1$ and $\beta = 2$ for different sample sizes

	Weibull(α,β)	$X_{0.99865}$	Median = $X_{0.50}$	Mean	Skewness	Kurtosis
n=25	Weibull(1,2)	8.4542	1.4737	2.0537	1.7316	3.9214
n=50	Weibull(1,2)	8.9368	1.4228	2.0194	1.6610	3.7344
n=75	Weibull(1,2)	9.5834	1.3896	2.0026	1.7548	4.1068
n=100	Weibull(1,2)	10.2353	1.4067	2.0281	1.7980	4.2492

Table 3: Summary statistics of Weibull distribution at $\alpha = 2$ and $\beta = 1$ for different sample sizes

	Weibull(α,β)	$X_{0.99865}$	Median= $X_{0.50}$	Mean	Skewness	Kurtosis
n=25	Weibull(2,1)	1.9352	0.8284	0.8743	0.5722	0.3388
n=50	Weibull(2,1)	2.0577	0.8495	0.8916	0.5426	0.0124
n=75	Weibull(2,1)	2.0978	0.8295	0.8774	0.5650	0.0292
n=100	Weibull(2,1)	2.2503	0.8158	0.8719	0.6686	0.3788

Table 4: Summary statistics of Weibull distribution at $\alpha = 2$ and $\beta = 2$ for different sample sizes

	Weibull(α,β)	$X_{0.99865}$	Median = $X_{0.50}$	Mean	Skewness	Kurtosis
n=25	Weibull(2,2)	1.6916	1.7779	1.7770	0.5108	0.1892
n=50	Weibull(2,2)	1.7177	1.5458	1.7537	0.5820	0.1896
n=75	Weibull(2,2)	4.2828	1.6516	1.7703	0.5638	-0.0554
n=100	Weibull(2,2)	4.4739	1.6662	1.7733	0.5898	0.1090

The distribution plot of Weibull distribution for various shape and scale parameter is shown below.

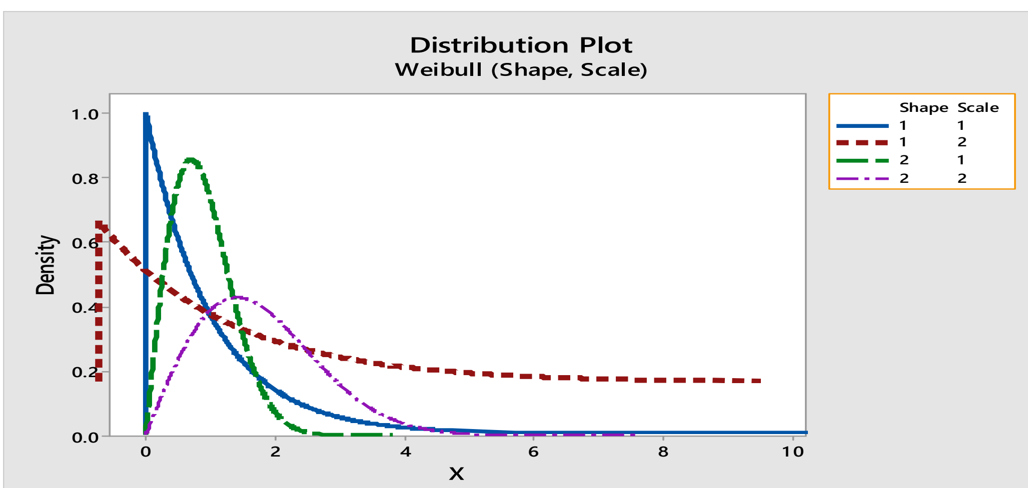

Figure 1: Distribution plot of Weibull distribution using different shape and scale parameters

From the distribution plot in Figure 1, the distribution plots are positively skewed (non-normal) for the combinations of the shape and scale parameters with Weibull (1, 1) the most peaked.

b) *Process Capability Analysis*

i. *Gini Mean Difference based Process Capability Analysis*

The table of parameter estimation is given below using the generated data from Weibull distribution with varying shape and scale parameters of (1,1), (1,2) (2,1) and (2,2) at different sample sizes of n = 25, 50,75 and 100.

Table 5: Gini's Estimated USL obtained from the data

GMD	C_{npug}	USL FOR GINI			
		n=25	n=50	n=75	n=100
Weibull(1,1)	1.0	3.4055	3.3501	3.3389	3.3753
	1.5	4.7348	4.6794	4.6682	4.7046
Weibull(1,2)	1.0	3.8298	3.7789	3.7457	3.7629
	1.5	5.0079	4.9570	4.9238	4.9409
Weibull (2,1)	1.0	2.2086	2.2297	2.2096	2.1960
	1.5	2.8986	2.9198	2.8997	2.8861
Weibull (2,2)	1.0	3.1118	3.0668	3.0633	3.0778
	1.5	3.8176	3.7726	3.7691	3.7837

Table 6: Clements's Mean Difference based Process Capability Analysis

CA	C_{npug}	USL FOR CLEMENTS ANALYSIS			
		n=25	n=50	n=75	n=100
Weibull 1,1	1.0	3.8939	4.3501	4.8491	5.1722
	1.5	5.4674	6.1794	6.9335	7.3999
Weibull 1,2	1.0	8.4542	8.9368	9.5834	10.2353
	1.5	11.9445	12.6937	13.6802	14.6495
Weibull 2,1	1.0	1.9352	2.0577	2.0978	2.2503
	1.5	2.4886	2.6618	2.7319	2.9675
Weibull 2,2	1.0	1.6916	1.7177	4.2828	4.4739
	1.5	1.6484	1.8036	5.5983	5.8777

Table 7: Box - Cox's Mean Difference based Process Capability Analysis

BCT	C_{npug}	USL FOR BCT			
		n=25	n=50	n=75	n=100
Weibull 1,1	1	2.6189	2.3984	2.0027	1.8314
	1.5	3.7938	3.4562	2.7452	2.3774
Weibull 1,2	1	3.4239	3.0202	2.5266	2.1994
	1.5	4.7835	4.1252	3.3355	2.8110
Weibull 2,1	1	1.6913	1.6490	1.6384	1.6528
	1.5	2.2016	2.1041	2.0553	2.0790
Weibull 2,2	1	2.6310	2.3786	2.3879	2.3661
	1.5	3.3389	2.9853	2.9895	2.9084

Notes

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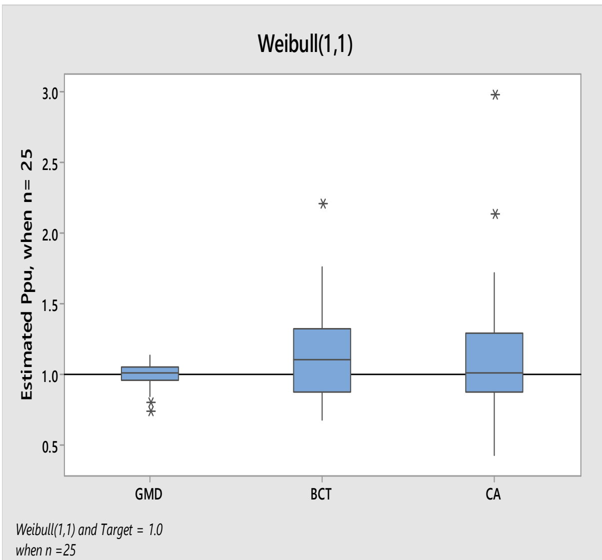
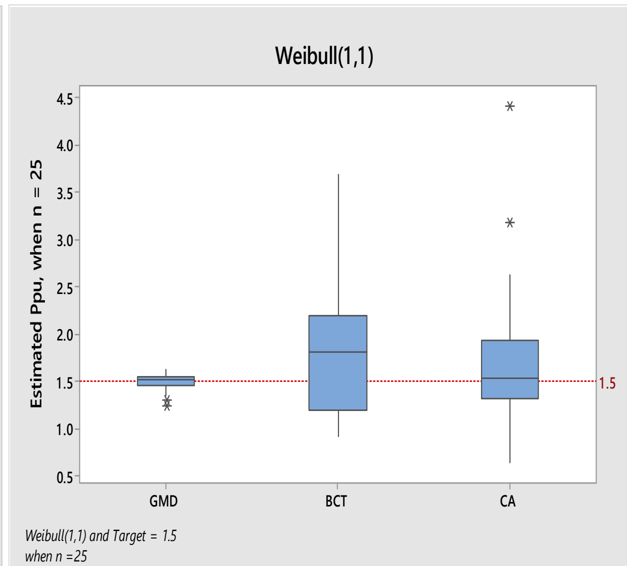
c) Graphical Comparison of the computed Process Capabilities

In order to compare the process capability methods graphically at each targeted Ppu (1.0 and 1.5), box plot or whisker plot is used to show the shape of the distribution, its central value (0.50), variability (0.75 – 0.25) and outliers by star symbol if it exists. The position of the median line in a box plot indicates the location of the values. The figures below shows the comparison



a.) Weibull(1,1), target Ppu 1.0 and n= 25



b.) Weibull(1,1), target Ppu 1.5 and n= 25

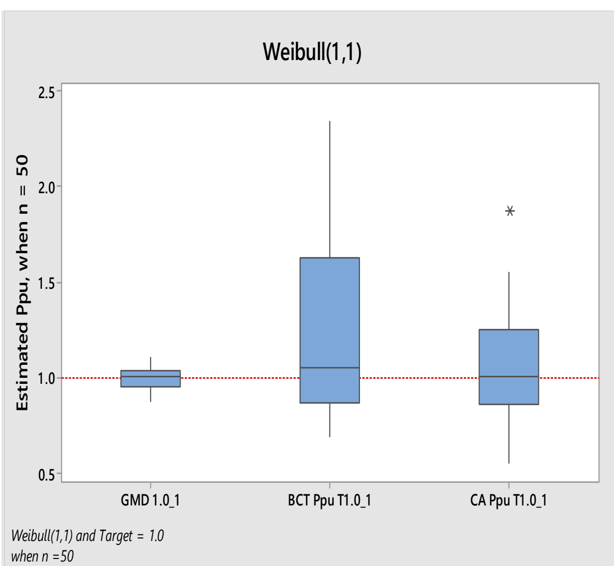


Figure 2: Weibull (1,1), target Ppu 1.0 and n= 50

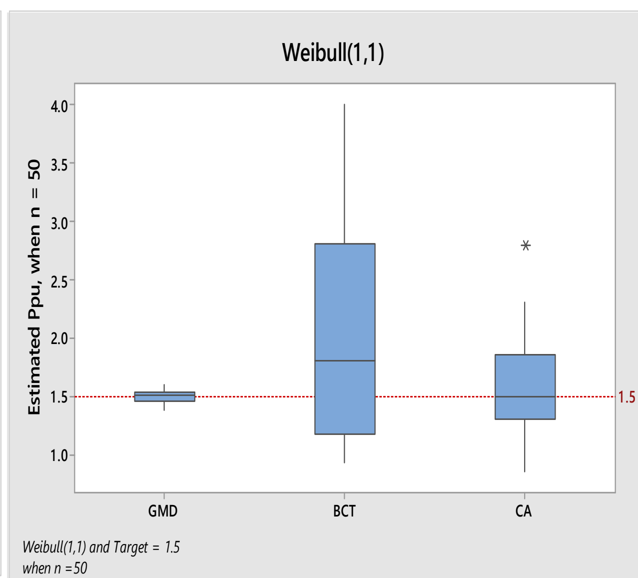


Figure 3: Weibull (1,1), target Ppu 1.5 and n= 50

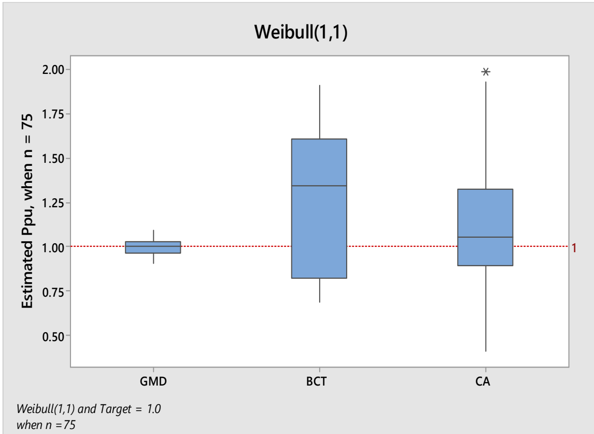


Figure 4: Weibull(1,1), target Ppu 1.0 and n= 75

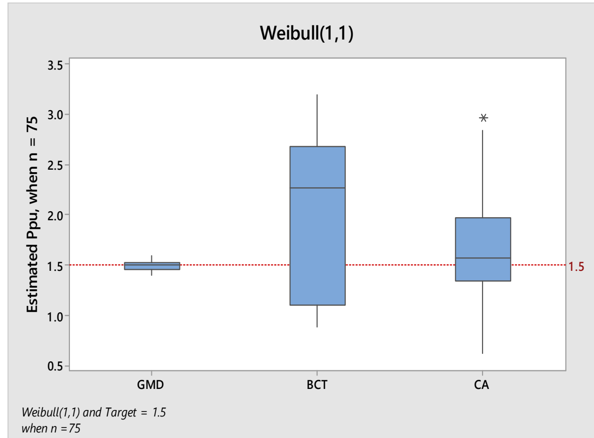


Figure 5: Weibull(1,1), target Ppu 1.5 and n= 75

From the boxplots, the results show that for different distribution parameters at different sample sizes, GMD methods is the best of the three methods for computing process capability for when the process is non-normal.

d) Mean and Standard deviation of Computed Capability Indices

To confirm the result shown from the boxplots above, the mean values and the standard deviation (which shows how concentrated the data are around the mean) of the computed process capabilities are computed in the tables below.

Table 8: Descriptive statistics for CA, BCT, and GMD methods when n = 25

n = 25						
Target Ppu	**Statistics**	**Method**	**Weibull(1,1)**	**Weibull(1,2)**	**Weibull(2,1)**	**Weibull(2,2)**
1	Mean	CA	1.1230	1.1930	1.0702	1.1163
		BCT	1.1314	1.0663	1.0125	1.1055
		GMD	1.0000	1.0000	0.9999	1.0000
	Standard Deviation	CA	0.4235	0.5112	0.3542	0.4386
		BCT	0.3097	0.2478	0.2126	0.3483
		GMD	0.0758	0.1706	0.0832	0.1696
1.5	Mean	CA	1.6849	1.7897	1.6191	1.6774
		BCT	1.7820	1.6351	1.5815	1.6583
		GMD	1.5000	1.5000	1.5000	1.5000
	Standard Deviation	CA	0.6275	0.7665	0.5182	0.6465
		BCT	0.6087	0.4404	0.3048	0.4828
		GMD	0.0758	0.1706	0.0832	0.1696

Notes

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Table 9: Descriptive statistics for CA, BCT, and GMD methods when n = 50

n = 50						
Target Ppu	**Statistics**	**Method**	**Weibull(1,1)**	**Weibull(1,2)**	**Weibull(2,1)**	**Weibull(2,2)**
1.0	Mean	CA	1.0577	1.1384	1.0262	1.0616
		BCT	1.2337	1.1833	0.9929	1.0356
		GMD	1.0000	1.0000	1.0000	1.0000
	Standard Deviation	CA	0.2644	0.3955	0.1658	0.2691
		BCT	0.4849	0.5377	0.1333	0.1778
		GMD	0.0525	0.1204	0.0603	0.1172
1.5	Mean	CA	1.5856	1.7068	1.5418	1.5930
		BCT	1.9822	1.8171	1.5461	1.5546
		GMD	1.5000	1.5000	1.5000	1.5000
	Standard Deviation	CA	0.3907	1.5000	0.2517	0.4002
		BCT	0.9365	0.8660	0.1953	0.2396
		GMD	0.0525	0.1204	0.0603	0.1172

Table 10: Descriptive statistics for CA, BCT, and GMD methods when n = 75

n = 75						
Target Ppu	**Statistics**	**Method**	**Weibull(1,1)**	**Weibull(1,2)**	**Weibull(2,1)**	**Weibull(2,2)**
1	Mean	CA	1.1044	1.0783	1.0396	1.0223
		BCT	1.2484	1.3056	0.9957	1.0203
		GMD	1.0000	1.0762	1.0000	1.0000
	Standard Deviation	CA	0.3418	0.3200	0.2112	0.1540
		BCT	0.3859	0.5790	0.0945	0.1513
		GMD	0.0459	0.3183	0.0523	0.1025
1.5	Mean	CA	1.6550	1.6168	1.5589	1.5345
		BCT	2.0129	2.0084	1.5295	1.5439
		GMD	1.5000	1.5000	1.5000	1.5055
	Standard Deviation	CA	0.5053	0.4764	0.3071	0.2295
		BCT	0.7514	0.9211	0.1434	0.2088
		GMD	0.0459	0.0965	0.0523	0.1025

Table 11: Descriptive statistics for CA, BCT, and GMD methods when n = 100

n = 100						
Target Ppu	**Statistics**	**Method**	**Weibull(1,1)**	**Weibull(1,2)**	**Weibull(2,1)**	**Weibull(2,2)**
1	Mean	CA	1.0627	1.0552	1.0311	1.0287
		BCT	1.1825	1.2099	0.9890	1.0056
		GMD	1.0000	1.0000	1.0000	1.0000
	Standard Deviation	CA	0.2580	0.2305	0.1834	0.1856
		BCT	0.3095	0.3763	0.0844	0.0865
		GMD	0.0394	0.0707	0.0436	0.0859
1.5	Mean	CA	1.5936	1.5829	1.5476	1.5428

		BCT	1.8554	1.8531	1.5156	1.5081
		GMD	1.5000	1.5000	1.5000	1.5000
	Standard Deviation	CA	0.3850	0.3449	0.2776	0.2700
		BCT	0.5458	0.5991	0.1000	0.1215
		GMD	0.0394	0.0707	0.0436	0.0859

At Weibull (1, 1) and Weibull (1, 2) at sample size of 25, 50, 75 and 100, the Gini Mean Difference based process capability estimates approximately the the target Ppu of 1.0 and 1.5, the Clements method estimates is also close to the target Ppu while the Box-Cox transformation method is at deviance from the target (overestimated) the Ppu of 1.0 and1.5 as the sample size increases.

At Weibull (2,1) and Weibull (2,2) which indicate low symmetry and at sample size of 25, 50, 75 and 100, the three method estimates are all approximately target Ppu of 1.0 and 1.5 with the Gini Mean Difference based process capability estimates the best (closest).

e) The Root-Mean-Square Deviation (RMSD)

The root-mean-square deviation (RMSD) is used to measure the differences between the targeted Ppu values and the estimates obtained by Box-Cox Transformation, Clements and Gini mean difference based methods.

The tables below summaries the result obtained for each of the distribution parameter at different sample sizes

Table 12: The root-mean-square deviations for CA, BCT, and GMD methods when n = 25

n = 25					
Target Ppu	**Method**	**Weibull(1,1)**	**Weibull(1,2)**	**Weibull(2,1)**	**Weibull(2,2)**
1	CA	0.4369	0.5416	0.3576	0.4495
	BCT	0.4794	0.2541	0.2108	0.3606
	GMD	0.0750	0.1688	0.0824	0.1679
1.5	CA	0.6482	0.8122	0.5266	0.6641
	BCT	0.6653	0.4564	0.3125	0.5035
	GMD	0.0750	0.1688	0.0824	0.1679

Table 13: The root-mean-square deviations for CA, BCT, and GMD methods when n = 50

n = 50					
Target Ppu	**Method**	**Weibull(1,1)**	**Weibull(1,2)**	**Weibull(2,1)**	**Weibull(2,2)**
1	CA	0.3749	0.4153	0.1662	0.2734
	BCT	0.5339	0.5629	0.1321	0.1795
	GMD	0.0525	0.1192	0.0597	0.1160
1.5	CA	0.3961	0.6173	0.2526	0.4069
	BCT	1.0048	0.9141	0.1988	0.2434
	GMD	0.0525	0.1192	0.0597	0.1160

Notes

Table 14: The root-mean-square deviations for CA, BCT, and GMD methods when n = 75

n = 75						
Target Ppu	**Method**	**Weibull(1,1)**	**Weibull(1,2)**	**Weibull(2,1)**	**Weibull(2,2)**
1	CA	0.3541	0.3263	0.2128	0.1540
	BCT	0.4557	0.6496	0.0937	0.1512
	GMD	0.0455	0.0956	0.0518	0.1015
1.5	CA	0.5237	0.4859	0.3097	0.2298
	BCT	0.9036	1.0440	0.1450	0.2113
	GMD	0.0455	0.0956	0.0518	0.1016

Table15: The root-mean-square deviations for CA, BCT, and GMD methods when n = 100

n = 100					
Target Ppu	**Method**	**Weibull(1,1)**	**Weibull(1,2)**	**Weibull(2,1)**	**Weibull(2,2)**
1	CA	0.2630	0.2348	0.1841	0.1860
	BCT	0.3566	0.4276	0.0843	0.0858
	GMD	0.0390	0.0700	0.0432	0.0850
1.5	CA	0.3925	0.3514	0.2789	0.2707
	BCT	0.6467	0.6902	0.1002	0.1205
	GMD	0.0390	0.0700	0.0432	0.0850

Results from the root-mean-square deviation (RMSD) in Table 12 to Table 15 shows that the GMD methods have the lowest RMSDs across all the different distribution parameters and sample sizes

IV. Conclusion

In order to examine the impact of non-normal data, the parameter values of Weibull distribution were specified as (1, 1), (1, 2), (2, 1), and (2, 2) corresponding to (shape, scale) at different sample sizes of 25, 50, 75 and 100. These parameters of Weibull distributions are specified such that the effects of the tail behaviour on process capability could be examined. When the Weibull shape parameter is equal to 1, Weibull distribution reduces to Exponential distribution. Hence, this study covers all the Exponential family distributions as well.

Conclusively, from our results and findings, the Gini Mean difference based approach is the best among three methods in estimating process capability in skewed (non-normal) situations. In general, methods involving transformation seem more burdensome in terms of calculation, though it provide estimates of PCIs that truly reflect the capability of the process when there is low symmetry as in Weibull (2, 2).

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