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A Comparative Study between Finite Difference Method and Finite Volume Method for Shallow Water-Dam Break Flow Problem

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Abstract- In this paper we perform numerical simulation of shallow water equations for dam break flow problem. Finite difference method and finite volume method are applied for the numerical solution of the shallow water equations. We estimate water height and water velocity for the test case of shallow water-dam break flow problem at different height ratio. The comparisons between the finite difference method and finite volume method for 1-D dam break problem have been shown. The agreement of numerical solution with analytic solution by finite volume method is better than finite difference method. We compare the computational efficiency for both schemes in subcritical flow.

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Abstract- In this paper we perform numerical simulation of shallow water equations for dam break flow problem. Finite difference method and finite volume method are applied for the numerical solution of the shallow water equations. We estimate water height and water velocity for the test case of shallow water-dam break flow problem at different height ratio. The comparisons between the finite difference method and finite volume method for 1-D dam break problem have been shown. The agreement of numerical solution with analytic solution by finite volume method is better than finite difference method. We compare the computational efficiency for both schemes in subcritical flow.

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I. Introduction

Many problems of river management and civil protection consist of the evaluation of the maximum water levels and discharges that may be attained at particular locations during the development of an exceptional meteorological event. There is also prevision of the scenario subsequent to the almost instantaneous release of a great volume of liquid. The situation is that of breaking of a man made of dam. Free surface flows common in hydraulics are usually described by means of the shallow water equations, provided that the representative vertical dimensions are small with respect to horizontal dimensions. Despite their simplicity, this description is valid in many practical applications, rendering worthwhile the efforts in developing good numerical methods to solve the corresponding system of differential equations. Numerical methods are nowadays a common tool to predict flow properties both for steady and unsteady situations of practical interest in hydraulics. The applications of finite differences and finite volumes have been widely reported in particular. The shallow water equations (SWEs) describe the evolution of hydrostatic homogeneous (constant density), incompressible fluid in response to gravitational and rotational accelerations and they are derived from the principles of conservation of mass and conservation of momentum. The SWEs (also called Saint- Venant equations) are one of the simplest form of the equations of motion that can be used to describe the horizontal structure of an atmosphere and ocean that model the propagation of disturbances in fluid. They are

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widely used to model the free surface water flows such as periodic (tidal) flows, transient wave phenomena [10] (tsunamis, flood waves and dam break waves) etc. In fluid dynamics the flow of the fluid is known as the Navier- Stocks equation. The shallow water equations are good approximation to the fluid motion equation when fluid density is homogeneous and depth is small in comparison to characteristic horizontal distance.

Most flows on the surface of the Earths, for examples in rivers, seas and the atmosphere, are Shallow water flows in which the horizontal length and velocity scales of interest are much larger than the vertical one's. The Mathematical formulation of this flows, the so called Shallow water equations, are already known for over a century. At presents, these models are used in all kinds of applications such as flood warning system, impact of changes of water system, climate predictions and reducing water pollution. The analysis of dam break flow is important to capture spatial and temporal evolution of flood event and safety analysis. The dam break basically is catastrophic failure of dam, leading to uncontrolled release of water causing flood in the downstream region. Many Numerical methods are available in the literature to solve the shallow water equations. Bellos et al. [1] reported a two dimensional numerical methods for dam break problem by using the combination of finite element and finite difference method. Fayssal and Mohammed [2] proposed a new simple finite volume method for the numerical solution of shallow water equations. Rahaman et al. [4] have studied the finite difference method to simulate the water height and velocity through shallow water equations. Bagheri and Das [5] developed an implicit high order compact scheme for shallow water equations with dam break problem. Saiduzzaman & Ray [7] examined some numerical methods for shallow water equations. Ahmed et al. [8] developed Godunov type finite volume method for dam break problem.

In section 2, provides shallow water dam break flow problem. We investigate the finite difference Lax Friedrichs method and finite volume method for the numerical solution of shallow water equations in section 3. Numerical results are presented in section 4. Finally the conclusions of the work are given in the last section.

II. Mathematical Model

The Mathematical model that describes the water flow in a river are defined as

$$\begin{aligned} &\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}(hv)=0 \\ &\frac{\partial v}{\partial t}+\frac{\partial}{\partial x}\left(\frac{1}{2}v^2+gh\right)=0 \end{aligned} \tag{1}$$

Where $h(x,t)$ is the water height at the time t and at the space x, g is the acceleration due to gravity, $v(x,t)$ is the flow velocity in the x-direction. With initial and Neumann boundary condition.

$$h(x,0)=\begin{cases} h_l & ,x<=L/2 \\ h_r & ,others \end{cases}$$

$$v(x,0)=v_0(x)\,;0\le x\le L$$

III. Numerical Method

We present here the discretization of the shallow water dam break flow model by finite difference formula, which leads to formulate explicit finite

Ref

1. Bellos V and Hrissanthou V, "Numerical simulation of Dam break flood wave". European water 33: Page: 45-53, 2011.

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difference methods for the numerical solution of the governing equation as a nonlinear partial differential equation. The finite volume method is a numerical technique that transforms the partial differential equations representing conservation laws over differential volumes into discrete algebraic equations over finite volumes or cells.

a) Finite difference method

The numerical discretization of (1) is as follows

$$h_i^{n+1} = \frac{1}{2}(h_{i+1}^n + h_{i-1}^n) - \frac{1}{2}\frac{\Delta t}{\Delta x}(h_{i+1}^n v_{i+1}^n - h_{i-1}^n v_{i-1}^n) \tag{3}$$

$$v_i^{n+1} = \frac{1}{2}(v_{i+1}^n + v_{i-1}^n) - \frac{1}{2}\frac{\Delta t}{\Delta x}[(\frac{1}{2}v_{i+1}^n v_{i+1}^n + gh_{i+1}^n) - (\frac{1}{2}v_{i-1}^n v_{i-1}^n + gh_{i-1}^n)] \tag{4}$$

b) Finite volume method

We consider conservation law to formulate finite volume method by the following PDE.

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = 0 \tag{5}$$

$$\frac{\partial U}{\partial t} + (\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y}) \bullet \vec{H} = 0$$

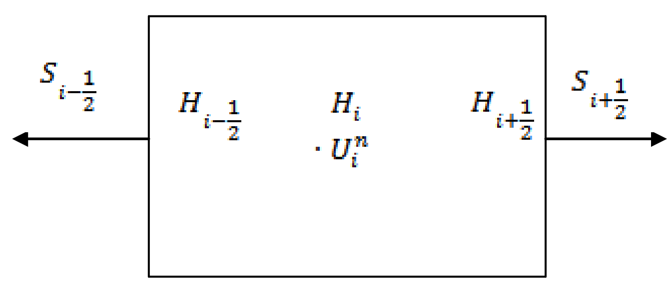
where $\vec{H} = F\vec{i} + 0\vec{j}$ is the flux density vector

$$\frac{\partial U}{\partial t} + \vec{\nabla} \bullet \vec{H} = 0$$

$$\iint_R \frac{\partial U}{\partial t} dR + \iint_R \nabla \bullet \vec{H} dR = 0$$

$$\iint_R \frac{\partial U}{\partial t} dR + \int_C \vec{H} \bullet \vec{n} ds = 0$$

$$\frac{\partial}{\partial t}\iint_R U dR + \int_C \vec{H} \bullet \vec{n} ds = 0$$



When $\vec{n}$ is outward pointing unit normal vector at each point on C. Δx is cell length in x direction and Δy cell length in y direction and area of the cell $A = \Delta x \Delta y$. For each cell the two side vectors in the x direction are $S_{i+\frac{1}{2}} = \Delta y\vec{i} + 0\vec{j}$ and $S_{i-\frac{1}{2}} = -\Delta y\vec{i} + 0\vec{j}$.

$$\frac{\partial}{\partial t} AU_i + \int_C \vec{H} \bullet \vec{n} ds = 0$$

$$\frac{\partial U_i}{\partial t} = -\frac{1}{A}\sum_{sides} \vec{H} \bullet \vec{s}$$

$$U_{next}(k,i) = 0.5(U(k,i-1) + U(k,i+1)) - \frac{\Delta t}{A}[H_{i+\frac{1}{2}} \bullet S_{i+\frac{1}{2}} + H_{i-\frac{1}{2}} \bullet S_{i-\frac{1}{2}}] \tag{6}$$

IV. Numerical Results and Discussion

We start by assuming that in both sides of the dam there are water with corresponding heights h_l and h_r, $h_l > h_r$. We use the three depth ratio (i) $h_r / h_l > 0.5$ (ii) $h_r / h_l < 0.5$ (iii) $h_r / h_l << 0.5$ for simulation. The dam is situated at 1000 m in channel and at time $t = 0$ the dam collapses. The flow consists of a shock wave travelling downstream and a rarefaction travelling upstream. Comparisons are carried out only for wet bed condition with respect to velocity and water height. The results are shown in the following figures.

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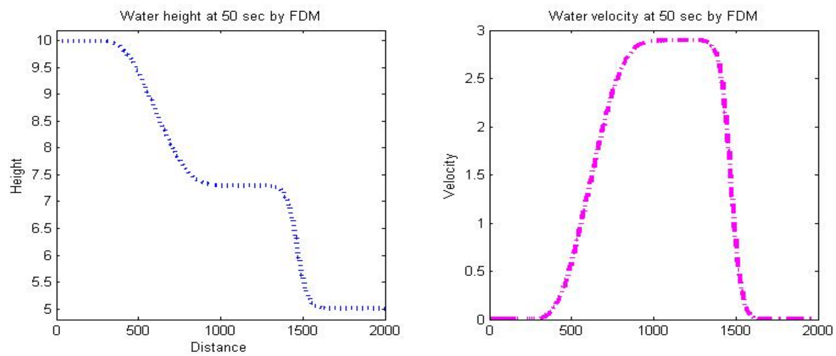

Fig. 1: Water height (left) and water velocity (right) for dam break on wet bed at 50 sec using height ratio 0.5

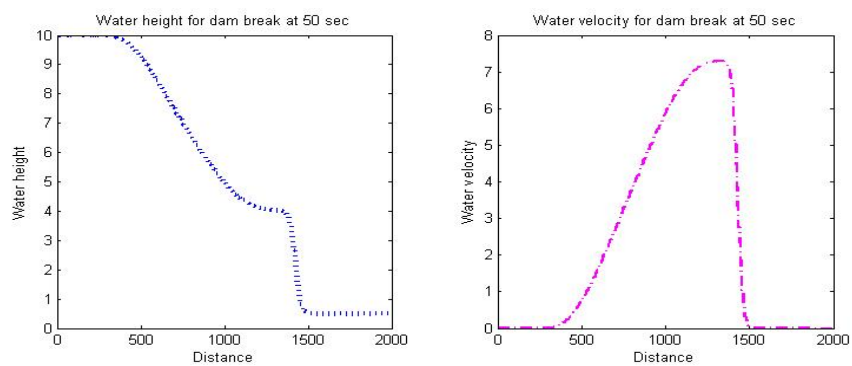

Fig. 2: Water height (left) and water velocity (right) in dam break at 50 sec using height ratio 0.05.

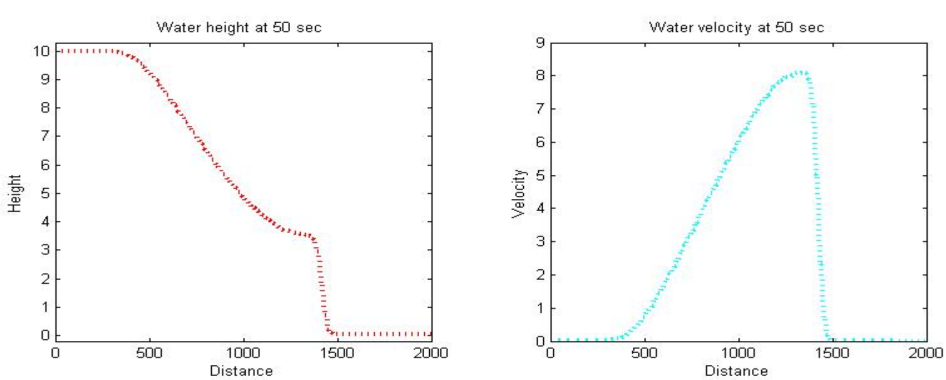

Fig. 3: Water height (left) and water velocity (right) in dam break at 50 sec using height ratio 0.005.

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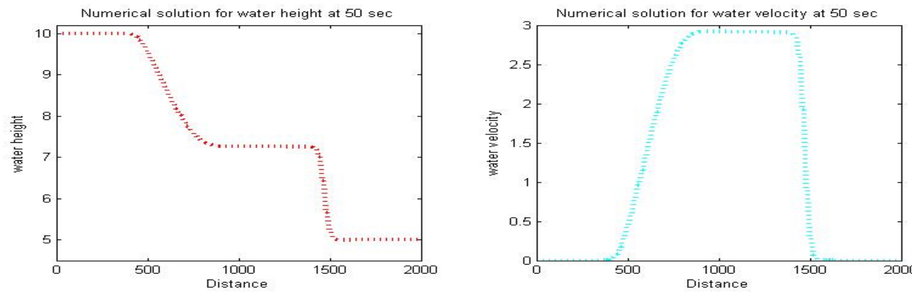

Fig. 4: Water height (left) and water velocity (right) in dam break at 50 sec using height ratio 0.5.

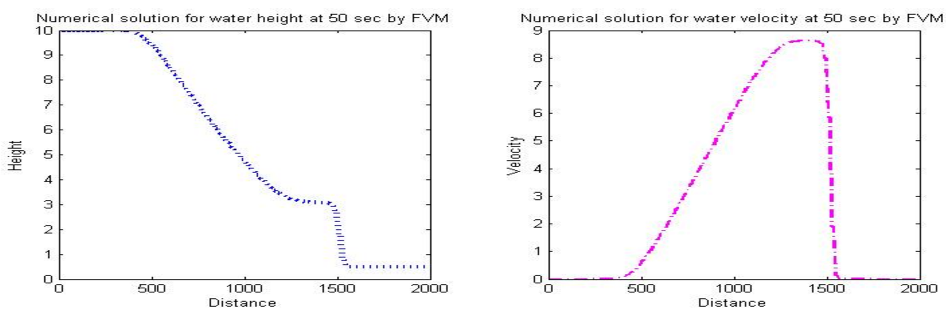

Fig. 5: Water height (left) and water velocity (right) in dam break at 50 sec using height ratio 0.05.

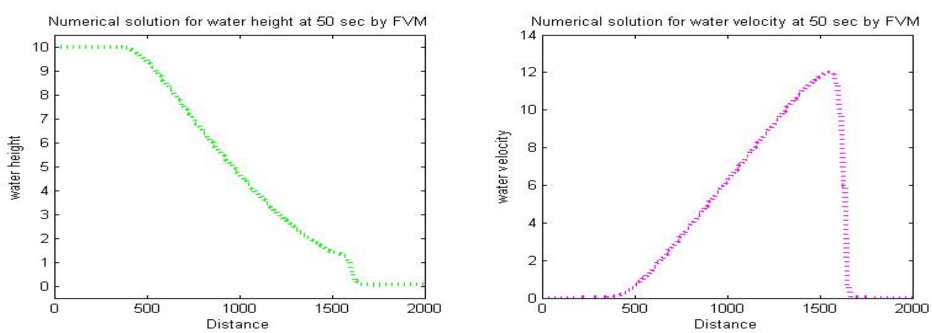

Fig. 6: Water height (left) and water velocity (right) in dam break at 50 sec for height ratio 0.005.

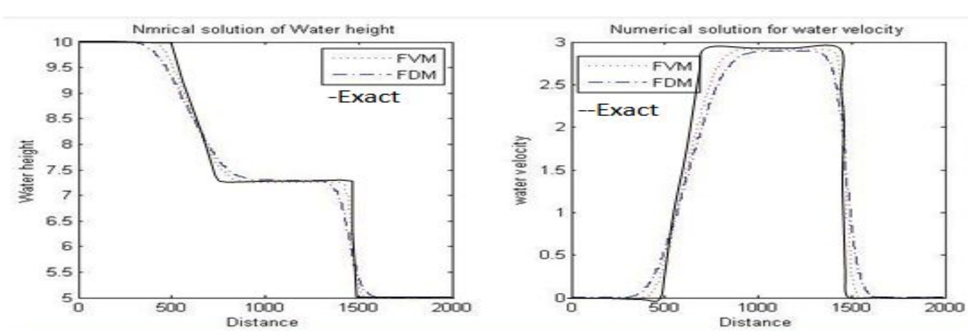

Fig. 7: Finite difference method, finite volume method and analytic solution ([2], [3]) of the dam break test problem for 50 sec using height ratio 0.5.

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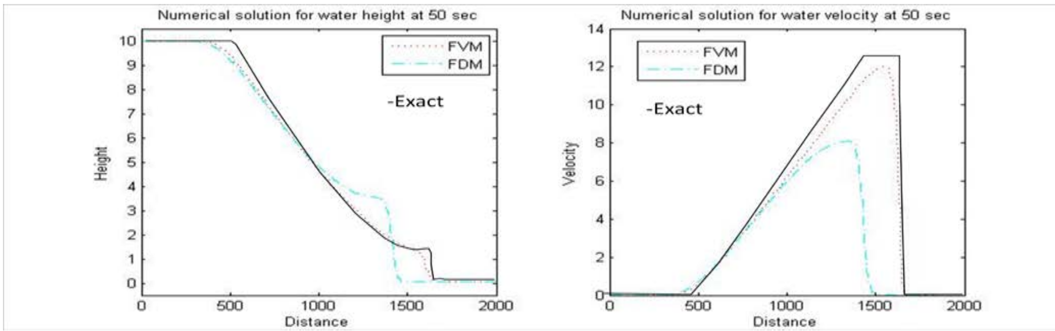

Fig. 8: Finite difference method, finite volume method and analytic solution ([2], [3]) of the dam break test problem for 50 sec using height ratio 0.005.

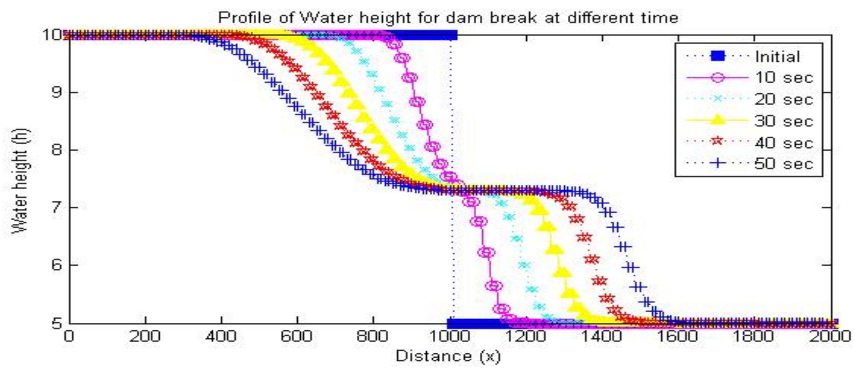

Fig. 9: Water heights in dam break at different time for subcritical flow by FDM.

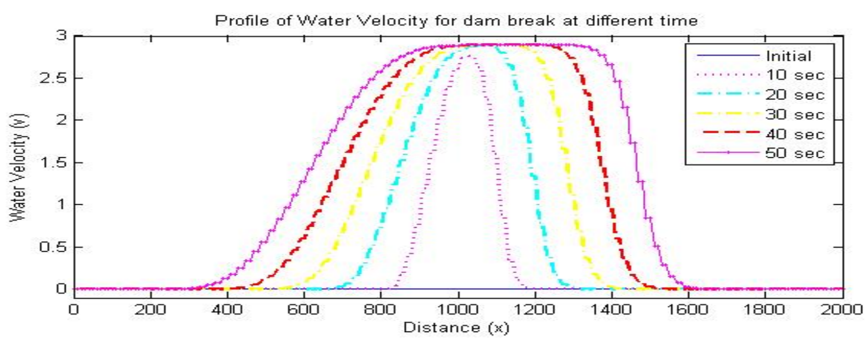

Fig. 10: Water velocity in dam break at different time for subcritical flow by FDM.

a) CPU times for FDM and FVM

We compare the computational efficiency for both numerical schemes.

Table 1: Efficiency test

	Time Step size	Stability condition	CPU time (sec)
FVM	0.05	0.0567	23.72
FDM	0.05	0.0567	2.75
FVM	0.5	0.567	2.586
FDM	0.5	0.567	0.43

Table 1 shows the CPU time of different method for different time step size. We observed that the CPU time of the problem is reduced very well. Table shows for fixed time step size the CPU time of FDM is less than the other method. FDM performs more efficiently than FVM.

Fig. 1-6 shows the water height and velocity profiles for different height ratio at time 50 sec after the dam-break for both schemes. Fig 7-8 shows comparison of height and velocity profiles for different height ratio of channel at time 50 sec after the dam-break. In fig.7-8, the comparison of numerical and analytical solution corresponding to water height as well as water velocity profile shows good agreement. Finite volume method provides more accurate results than Finite difference method. The analytical reference solutions for these test problems are due to [2], [3]. Therefore the above realistic phenomenons are well described by our implementation.

Ref

3. J. J. Stoker, Water Waves, Interscience Publishers, Inc., New York, 1986.

V. Conclusion

We have demonstrated numerical simulation using FDM and FVM for solving shallow water equations with initial boundary conditions. We have implemented the numerical scheme to simulate water height and water velocity at different condition through shallow water dam break flow after the dam break. We have shown that the numerical result based on the FDM and FVM agrees with some qualitative / realistic behavior of SWE. The solution of finite volume method shows good agreement with exact. Finite difference method is more efficient than finite volume method. The analysis of dam break flow is important to capture spatial and temporal evolution of flood event and safety analysis.

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