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Parametrically Excited Anharmonic Oscillator

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Abstract- Earlier [0] analyzed the behavior of the “dynamic point” - the harmonic oscillator. But there are phenomena for which even the damped harmonic oscillator is not elementary, but primitive. ELEMENTARY is an oscillator, which was previously called simply parametric, but, as shown in this work, strictly speaking, should be called parametrically excited anharmonic oscillator. As the analysis showed, this oscillator has stationary solutions for a harmonic oscillator at a doubled resonant frequency and for only one strictly defined level of attenuation, the deviation from which leads to a catastrophic increase, or to full cancellation of the oscillations. As shown in the elementary model, the doubled resonant frequency of the excitation occurs with orthogonal (transverse) oscillation at the frequency of the longitudinal resonance. This analysis was done to describe the anomalous non-transmission band in boron nitride.

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Parametrically Excited Anharmonic Oscillator

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Abstract- Earlier [0] analyzed the behavior of the "dynamic point" - the harmonic oscillator. But there are phenomena for which even the damped harmonic oscillator is not elementary, but primitive. ELEMENTARY is an oscillator, which was previously called simply parametric, but, as shown in this work, strictly speaking, should be called parametrically excited anharmonic oscillator. As the analysis showed, this oscillator has stationary solutions for a harmonic oscillator at a doubled resonant frequency and for only one strictly defined level of attenuation, the deviation from which leads to a catastrophic increase, or to full cancellation of the oscillations. As shown in the elementary model, the doubled resonant frequency of the excitation occurs with orthogonal (transverse) oscillation at the frequency of the longitudinal resonance. This analysis was done to describe the anomalous non-transmission band in boron nitride.

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I. Mate-Solutions for the Natural Oscillations of a Parametric Oscillator

The general dependence of the resonant frequency of a harmonic oscillator ω_0 on changes in its parameters: mass or stiffness, is described by the Hill equation:

$$\frac{d^2y}{dt^2} + p(t)\,y = 0 \qquad (1)$$

The Hill equation does not have a common analytical solution and even its approximate solutions are practically not used. In practice, its particular case is used with the harmonic dependence of the oscillator resonant frequency deviation, which is expressed by the Mathieu formula

$$\frac{d^2y}{dt^2} + \left(1+\gamma^2/\omega_0\cos(\omega\cdot t)\right)\cdot\omega_0^2\cdot y = 0\,, \qquad (2)$$

where γ^2/ω_0 - the relative modulation depth of the resonant frequency.

The Mathieu formula has a common analytical solution, expressed through the above and named Mathieu functions, but because of its cumbersomeness, and it is not strictly rigorously analyzed purely mathematically, which resulted in the lack of rigor of the physical models built on it.

Both equations cited correspond to an ideal, non-damping harmonic oscillator. Although some kind of confusion brought the index used in the Mathieu formula to denote the depth of modulation and, thereby, unconsciously, tied it to the attenuation. Therefore, for the "harmonic" parametric oscillations, the base models, which are beautiful but inadequate to the actual physical processes (Figure 1 from [1, 2]), are rarely used in practice and in quantitative calculations. And during the initial analysis of the stopband in boron nitride, they gave us nothing but the obvious in Figure 1 — orthogonal, transverse oscillations parametrically "feel" the longitudinal resonance.

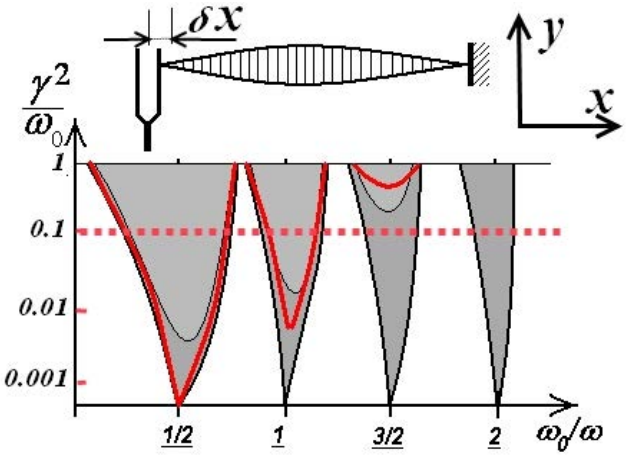

Fig. 1: The basic "picture of excitation" of parametric oscillations — the gray areas and its "refinement" —the red lines (above the 0.1 level, the Mathieu model does not work)

In depicted in Figure 1.the model as the statement of the analysis problem is precisely the practical need to detect "parasitic" vibrations. But the analysis of the oscillations of the actually modified, anharmonic oscillator (which actually gives Mathieu functions) was not carried out, but the set of modulating frequencies shown in Fig. 1 was taken for the OWN frequencies of this oscillator.

So the model shown in Figure 1 and we needed only as a seed in the direction of the search - what is the result we are looking for? And the analysis of the oscillator frequencies themselves had to start from "zero" - with Mathieu, with its functions, namely with OWN transverse (in the figure) modified vibrations, and not longitudinal (in the figure), excited by a tuning fork in Fig.1.

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To bring it into conformity with reality, we immediately introduce the damping into the Mathieu equation, which we will need later on and without which in describing the harmonic oscillator we cannot do without invoking an abstract singularity in the form of the Dirac delta function. And in order to avoid confusion, let us return the attenuation designation γ, and denote the modulation coefficient k. We also assume, without loss of generality, as in the analysis of a simple harmonic oscillator, its own resonant frequency $\omega_0 = 1$. Then equation (2) takes the following form:

$$\frac{d^2y}{dt^2} + \gamma \cdot \frac{dy}{dt} + \left(1 + k\cos\left(\omega \cdot t\right)\right) \cdot y = 0 \,, \quad (3)$$

Both in form and in fact, equation (3) corresponds to a harmonic oscillator with a single Eigenfrequency and with a driving force of an arbitrary frequency:

$$\frac{d^2y}{dt^2} + \gamma \cdot \frac{dy}{dt} + y = -k\cos\left(\omega \cdot t\right) \cdot y \,, \quad (4)$$

This elementary rewriting (4) indicates a fundamental point, without regard to which the analysis is not connected with reality and is meaningless: the Mathieu equation describes such a parameter change that is not just a parameter change for OWN (transverse in Figure 1) vibrations, but let but a real driving bias / force, parallel to the amplitude of the bias of OWN oscillations. But this compelling force is specific - it is itself proportional to the magnitude of the resulting displacement it initiates. This leads to the nonlinearity of its solutions, which gives grounds to call expression (4) the Mathieu equation of the anharmonic oscillator. We will analyze its decisions: $Y(\omega,\ t) =$ (5)

$$= e^{-\frac{t\gamma}{2}} C[1]\, MathieuC\left[-\frac{-4+\gamma^2}{\omega^2}, -\frac{2k}{\omega^2}, \frac{t\omega}{2}\right] + e^{-\frac{t\gamma}{2}} C[2]\, MathieuS\left[-\frac{-4+\gamma^2}{\omega^2}, -\frac{2k}{\omega^2}, \frac{t\omega}{2}\right]$$

where the constants C [1], C [2] are given by two initial conditions precisely for Y. It is they that give the initial impulse for Y, which determines the starting amplitude and phase, but the OWN fluctuation in Y, subject to, according to formula (3) amplitude and phase deviations.

And so, as a consequence of what has been said, the real OWN, described by the Mathieu functions, the oscillations of the anharmonic oscillator, in contrast to the purely sinusoidal oscillations of the harmonic oscillator, also have a similar sinusoidal mode of oscillations at OWN frequency $\omega_0 = 1$, BUT! their amplitude and phase are not constant (as with harmonic oscillations), but change in time, which is described by Mathieu functions. By setting the maximum parameters: an unrealistically large modulation factor of 10% and an unrealistically small zero attenuation and using the boundary conditions: y (0) = 0, y '(0) = 1, for a parametric modulation frequency equal to twice ω_0 we get its function of time and its sinusoidal approximation in the initial section (Fig.2).

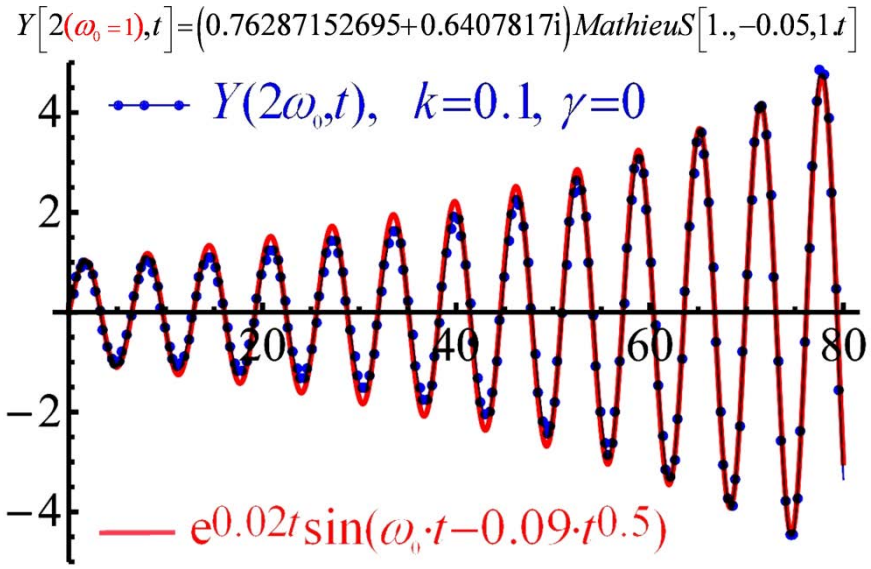


Fig. 2: Solution of the Mathieu equation for doubled modulation frequency and its approximation in the initial section by sine at single frequency

The time dependence of the amplitude and phase of OWN oscillations at a frequency ω_0 shown in Fig. 2 makes the frequency pattern shown in Fig. 1 ambiguous and incomplete. First, in Fig. 1, the modulation frequencies are indicated, and not the frequencies of OWN SPECIES. Secondly, on the asymptotics, with time tending to infinity, there is no exit to saturation and a third, time coordinate is required, the change along which is specific for each frequency of parameter change. At the same time, the picture shown is conditional, since it is not determined from which area of the three-dimensional space, at what point in time, each projection onto the drawing plane shown in the figure. So conditional that from the "received" in the classical works [1, 2] and the series of "own" parametric oscillations used in Fig. 1 ω^*

$$\omega^* = \frac{\omega}{\omega_0} = \frac{2}{n}, \quad n = 1, 2, 3..., \Rightarrow 2,\ 1,\ \frac{2}{3},\ \frac{1}{2},\ \frac{2}{5},\ \frac{1}{3},\ \frac{2}{7},\ \frac{1}{4},. \tag{6}$$

even with zero attenuation conditionally threshold, in the sense that capable of leading to an increase in oscillations over time, only two modulation frequencies can be considered:

$$\omega^* = 2,\ 1.$$

Those. the doubled modulation frequency shown in Fig. 2, and a single modulation frequency with a smaller amplitude rise rate of about 300 times (Fig. 3).

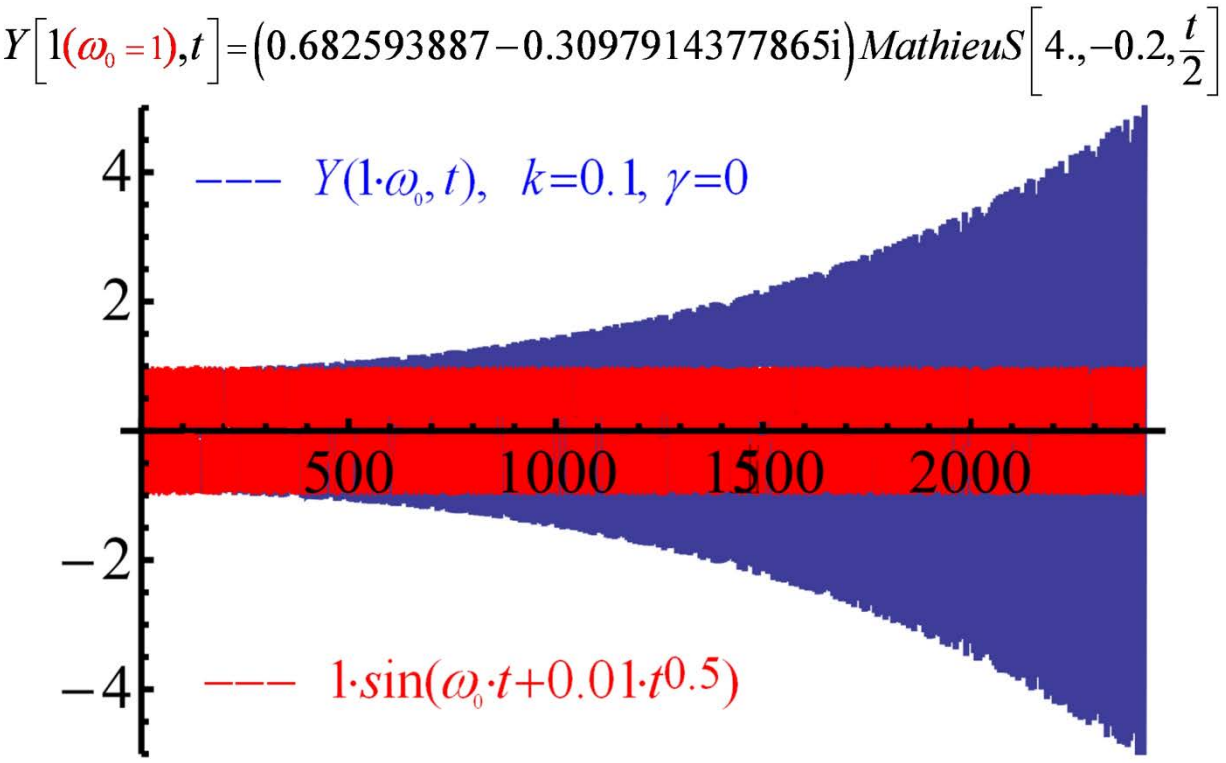


Fig. 3: Solving the Mathieu equation for a single modulation frequency and the original sine

At the next "threshold" frequency $2/3\omega_0$, even with zero attenuation, we have only weak pre-excitation (Fig. 4).

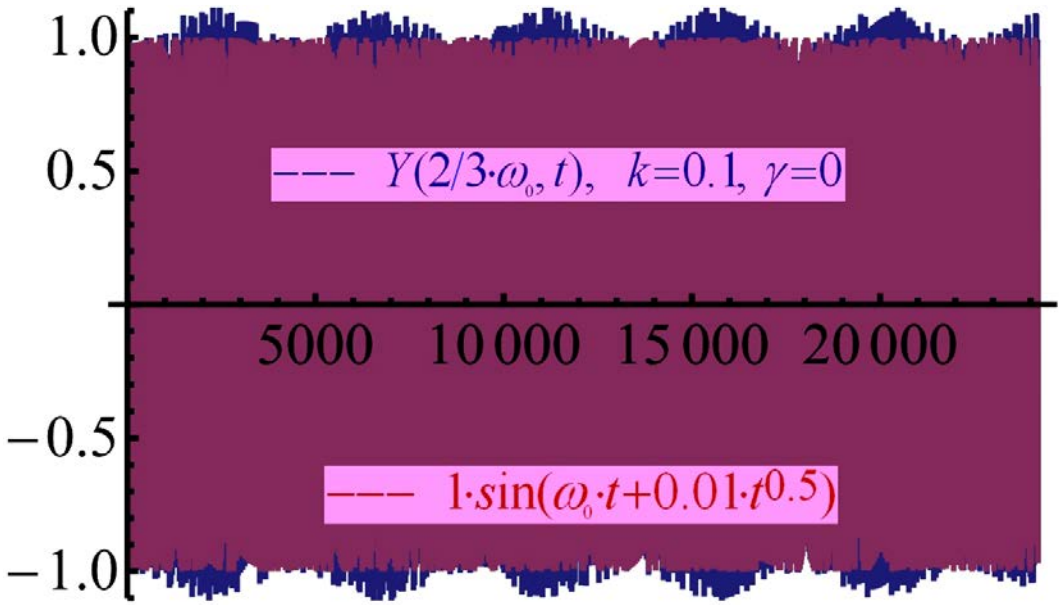


Fig. 4: Solution of the Mathieu equation for the modulation frequency of 2/3 and the original sine

And at lower modulation frequencies (at higher, shown in Fig. 1, inverse modulation frequencies), the excitation of OWNEED FUNCTIONS can only be achieved formally by increasing the modulation factor beyond the applicability limit of the Mathieu model.

And so, at any modulation frequencies, the OWN oscillator oscillations occur at the same OWN frequency (with some phase deviation), but either with a sharp increase and with almost no threshold (at zero attenuation), with a double modulation frequency, or with a weak increment at a single modulation frequency, or do not occur at all. T. h. To guess that when building a picture in Fig. 1, the author meant and I see no point in correcting it strictly, but qualitatively, conditionally, I showed her corrections with red lines. But the main feature is the unlimited increase of OWN on the marked two frequencies of parametric excitation with zero attenuation the same as that of the elementary harmonic oscillator BUT! without attenuation strictly at the resonant frequency.

A more attentive analysis of the dependence on the attenuation of OWN oscillations of a parametrically excited oscillator at the most sensitive, twice the modulation frequency will be carried out with a reasonable 1% modulation:

$$y''[t]+\gamma y'[t]+\left(1+k\cos[2t]\right)y[t]=0,\quad y'[0]=1,\quad y[0]=0\rightarrow\quad y_{k,\gamma}[t]=$$

$$/k=0.01,\qquad \gamma=0,\ \ 0.0001,\ \ 0.0005,\ \ 0.001,\ \ 0.002,\ \ 0.004,\ \ 0.008,\ \ 0.016$$

$$\begin{aligned}
&(0.7129898053724146+0.7007201267120846\mathrm{i})\ \ e^{-0.00000t}\,MathieuS[1.0000000000,-0.005,1.t]\\
&(0.7129898090019725+0.7007204806821420\mathrm{i})\ \ e^{-0.00005t}\,MathieuS[0.99999999750,-0.005,1.t]\\
&(0.7129898961113674+0.7007289760173282\mathrm{i})\ \ e^{-0.00025t}\,MathieuS[0.99999993750,-0.005,1.t]\\
&(0.7129901683283144+0.7007555246028571\mathrm{i})\ \ e^{-0.0005t}\,MathieuS[0.999999750000,-0.005,1.t]\\
&(0.7129912571974346+0.7008617289935156\mathrm{i})\ \ e^{-0.0010t}\,MathieuS[0.999999000000,-0.005,1.t]\\
&(0.7129956126952366+0.7012867074583439\mathrm{i})\ \ e^{-0.0020t}\,MathieuS[0.999996000000,-0.005,1.t]\\
&(0.7130130350275985+0.7029892037981437\mathrm{i})\ \ e^{-0.0040t}\,MathieuS[0.9999839999999999,-0.005,1.t]\\
&(0.7130827298158775+0.7098410287143199\mathrm{i})\ \ e^{-0.0080t}\,MathieuS[0.9999360000000,-0.005,1.t]
\end{aligned}\tag{7}$$

Under the same conditions, it is possible to calculate the dependences for attenuation close to the threshold

$$/\ k=0.01, \qquad \gamma \quad = \quad 0.0047, \quad 0.005, \quad 0.0055$$

$$y_{k,\gamma}[t]=$$

$$\left(0.71299782312342+0.70150248261\mathrm{i}\right)\mathrm{e}^{-0.00235t}\,MathieuS\left[0.9999944775,-0.005,1.t\right] \qquad (8)$$

$$\left(0.712998879340976+0.70160561045512\mathrm{i}\right)\mathrm{e}^{-0.0025t}\,MathieuS\left[0.99999375,-0.005,1.t\right]$$

$$\left(0.7130007848932+0.7017917042448\mathrm{i}\right)\mathrm{e}^{-0.00275t}\,MathieuS\left[0.9999924375,-0.005,1.t\right]$$

The dependence of the amplitude of these OWN oscillations on time for different attenuations corresponds to the upper limit of the corresponding (superimposed in color) absolute values (Fig. 5, 6)

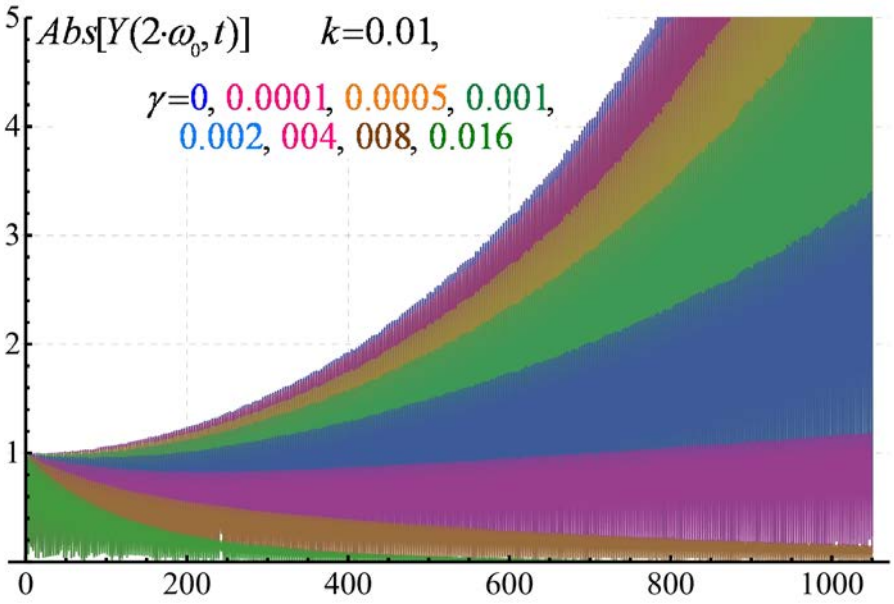
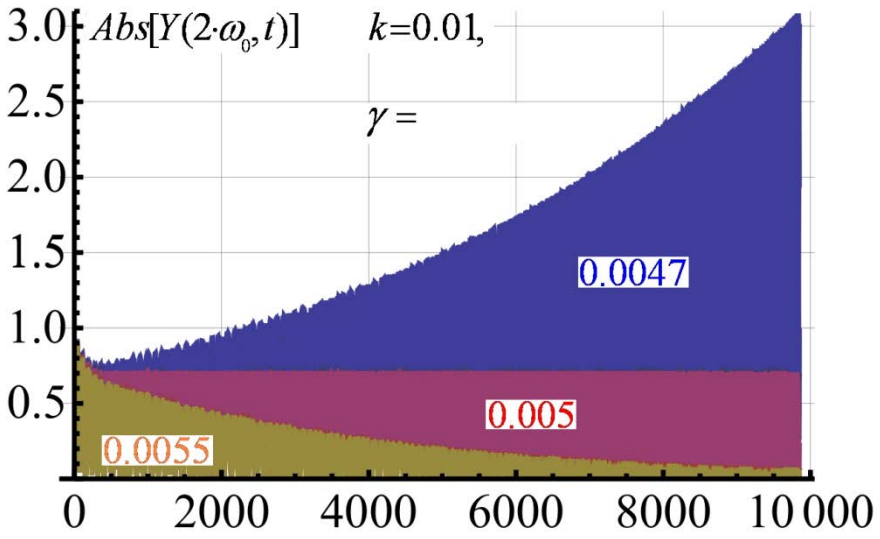


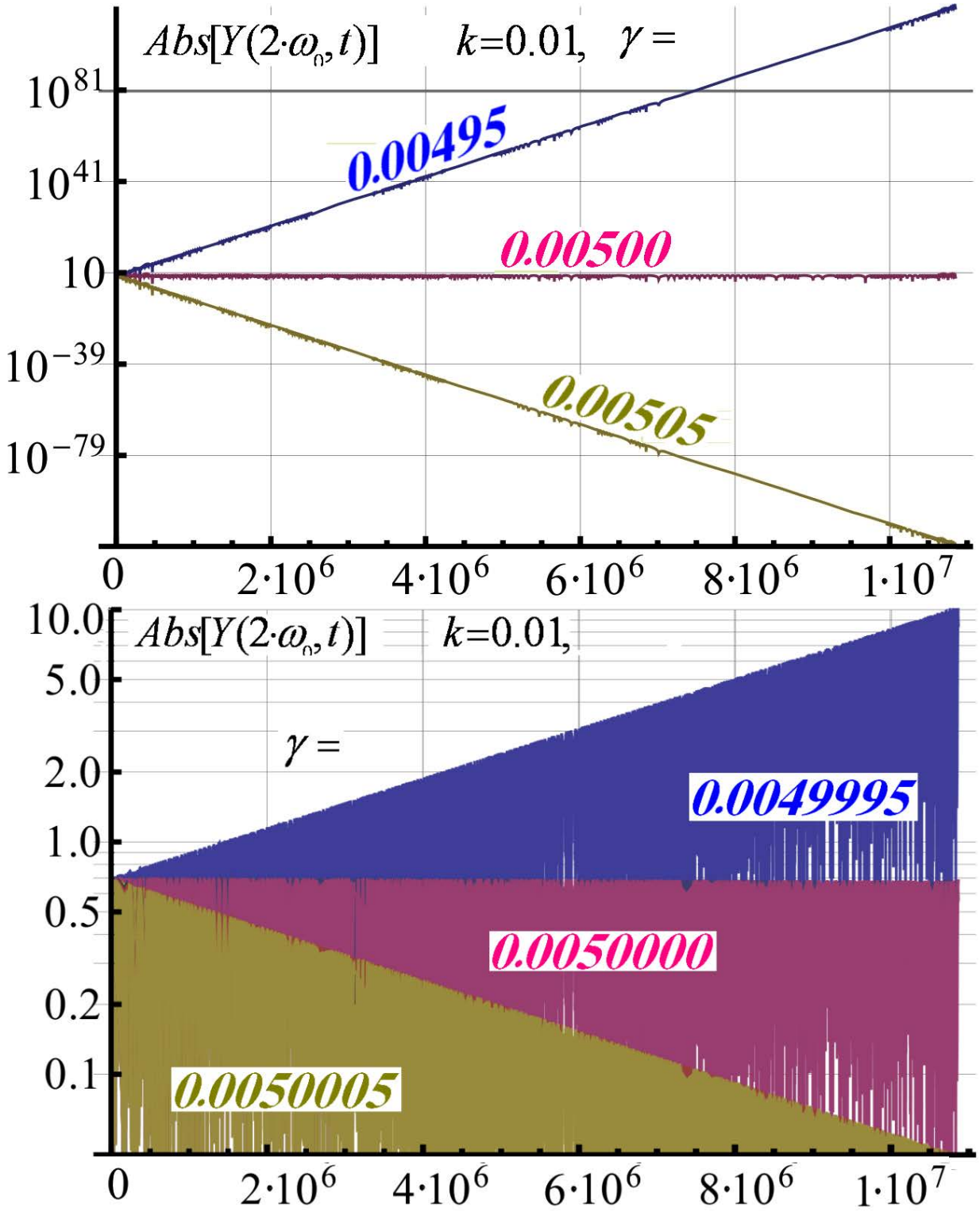
Fig. 5: The module of ω_0 oscillation is parametrically excited by a double frequency



Puc.6: The module of ω_0 oscillation is parametrically excited by a double frequency near the threshold of excitement

A characteristic feature of the obtained dependences of the Mathieu amplitude on time is a small decrease in the initial segment that is independent of attenuation due to de-phasing of the initial conditions followed by a catastrophic increase or decrease in amplitude over time dependent on damping.

At the same time, the attenuation threshold for double frequency with 1% modulation, as shown in Fig.6 and Fig.7, is 0.005 with great accuracy.



Puc.7: The module of ω_0 oscillation is parametrically excited by a double frequencynear the threshold of excitement

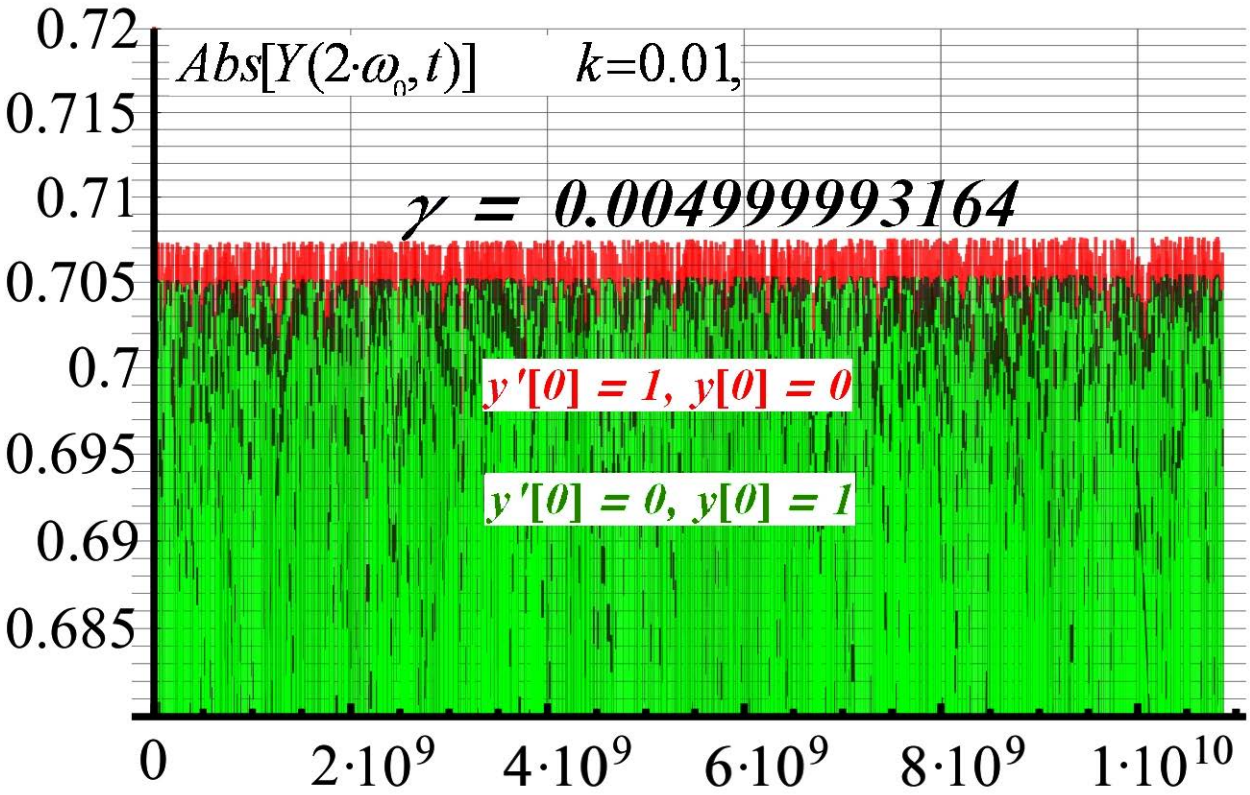


Fig. 8: A slight "instantaneous" drop in the amplitude of oscillations of the anharmonic oscillator to the saturation level with time at the threshold attenuation

The numerical value of the attenuation threshold is shown in the figure. In this damping, the oscillation amplitude weakly depends on the change in the initial conditions for the magnitude of the displacement and its derivative in a large time interval (Fig. 8).

However, the threshold amplitude increases by a factor of 200 if the unit acceleration is set to zero. But the characteristic asymptotic form of the time dependence of the amplitude is preserved. Thus, for the anharmonic oscillator, only for the threshold attenuation only, it is possible to construct, as for the harmonic oscillator, the frequency "resonance" characteristic. Whereas even an insignificant difference between attenuation and threshold leads to the time trend of the oscillation amplitude either to zero or to infinity.

And so, the numerical analysis of simple, but rigorous calculations of solutions to the Mathieu equation allows us to make qualitative conclusions and! allows them to be associated with an elementary physical model, which for a harmonic oscillator is in good agreement with many optical experiments and models.

II. Elementary Dynamicmodels

Elementary dynamic models of mechanics are used as basic in optics, in electricity, and in aero- and hydrodynamics, and static mechanical models are used as their asymptotics at zero frequency. But the development and refinement of dynamic models is often carried out at the expense of their complication and the introduction of additional, not rarely redundant dynamic parameters, which leads to their incorrectness - violation of the conditions of applicability of the original, basic static model. Therefore, we first consider the basic static model of a harmonic oscillator, the dynamic characteristics, which, describing well the normal lattice vibrations in Fedor's crystals, were previously presented in Chaos-Imagination-Orthogonality. In anisotropic crystals, it is necessary to consider not one oscillator, but at least two, corresponding to the orthogonal crystallographic directions of oscillators in a simple uniaxial crystal. Within the framework of this model, the frequencies of normal mechanical (excluding the Coulomb additive) lattice modes of an anisotropic crystal are associated (as shown in Fig. 9) only with the bond stiffness of ions in the lattice along the main crystallographic directions, for a uniaxial crystal:

$$\xi^{\perp C},\ \xi^{\parallel C}$$

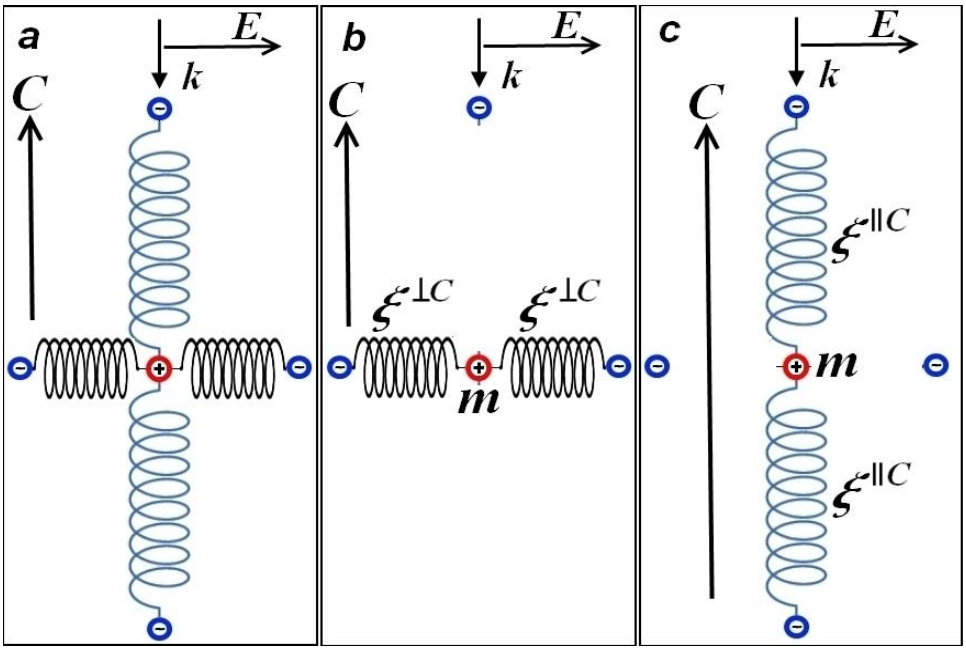


Fig. 9: Anelementary model of normal vibrations in an an isotropic medium, allowed and forbidden for light incidental on g the C axis

However, as shown for a separate chain of ions along the C axis (Fig. 9c and 10), the displacement of the central (positively conditional) along x increases the actual tension of the spring of the C axis along y, i.e. increases the resonant frequency of longitudinal oscillations propagating along C. So, the orthogonal spring stiffness $\xi^{\perp C}$, $\xi^{\| C}$ used in dynamics are some given values.

When the central ion is displaced along the C axis by an amount (Fig. 10a), an imbalance of forces F_1^y and F_2^y arises without an increase in the rigidity of the ion bonds:

$$F^y_{1/\Delta y=0} - F^y_{2/\Delta y=0} = 0 \rightarrow \Delta F^y = \left(l_0 + \Delta y\right)\cdot \xi^{\|} - \left(l_0 - \Delta y\right)\cdot \xi^{\|} = 2\cdot \Delta y \cdot \xi^{\|} \qquad (9)$$

where the effective length l_0 determines the equilibrium tension of the “springs” along y.

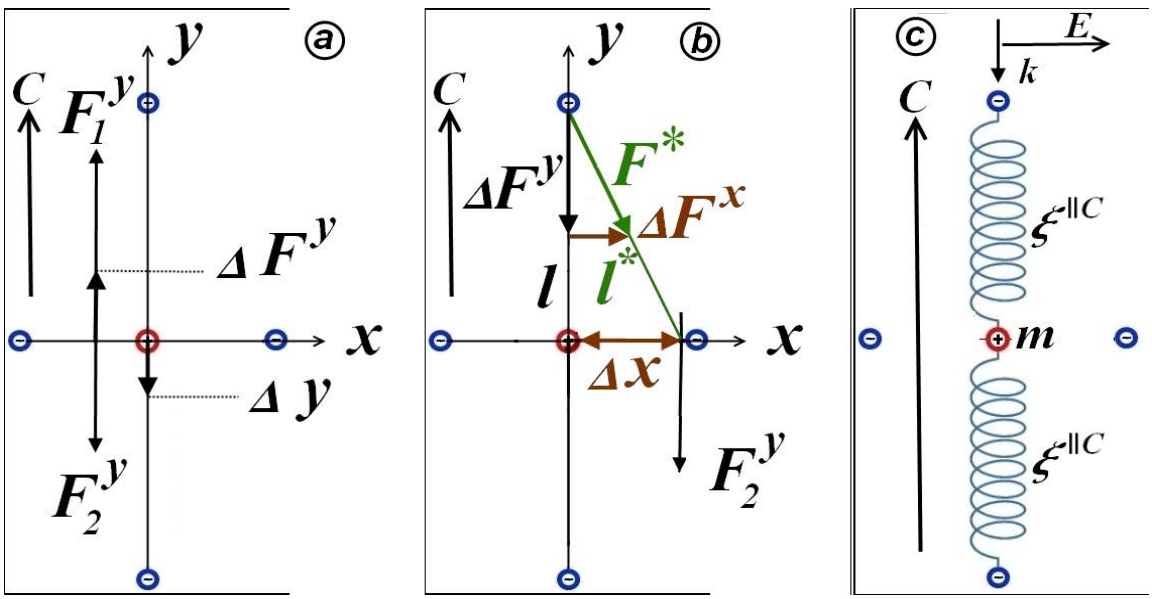


Fig. 10: Forces arising in a chain of atoms along the C axis with disregard of orthogonal C bonds of ions

And the total doubled stiffness $\xi^{||C}$ determines the resonant frequency (fig.10c):

$$\xi_{\Sigma}^{||c} = \Delta F^{y}/\Delta y = 2\cdot\xi^{||c} \rightarrow \omega_0^2 = 2\cdot\xi^{||c}/m \tag{10}$$

But in the highly ordered rhombohedral phase of boron nitride, with the detection of an optical anomaly in which the analysis presented itself began, the ions form pairs along the C axis, i.e. For the formation of the resonant frequency for oscillations along the C axis in boron nitride, the single stiffness is responsible:

$$\omega_0^{||c} = \sqrt{\xi^{||c}/m} = \omega_T^{||c} \tag{11}$$

When the central (positive, conditionally) ion perpendicular to the C axis is displaced by the amount (Fig.10b), the balance of the initially balanced forces F_1^y and F_2^y is maintained, and due to the additional lengthening of the "springs" they increase in magnitude by the amount F^* (Fig.10b). At the same time, given the elementary geometric relations

$$\begin{gathered}\Delta F^{y}/\Delta F^{x} = l/\Delta x \rightarrow \quad F^* = \sqrt{(\Delta F^{y})^2 + (\Delta F^{x})^2} = \Delta F^{y}\cdot\sqrt{1+(\Delta x/l)^2} \quad , \\ F^* = \xi^{||c}\xi\cdot\Delta l = \xi^{||c}\cdot(\sqrt{l^2+\Delta x^2} - l) = \xi^{||c}\cdot l\cdot(\sqrt{1+(\Delta x/l)^2} - 1)\end{gathered} \tag{12}$$

You can get the dependence on Δx - the magnitude of the displacement perpendicular to the C axis of the orthogonal forces - equivalent increments of the forces F_1^y and F_2^y

$$\Delta F^{y} = \xi^{||c}\cdot l\cdot\left(1-\frac{1}{\sqrt{1+(\Delta x/l)^2}}\right), \qquad \Delta F^{x} = \xi^{||c}\cdot\Delta x\cdot\left(1-\frac{1}{\sqrt{1+(\Delta x/l)^2}}\right) \tag{13}$$

Given the smallness of the displacement, it is possible to obtain simplified expressions for these forces.

$$\Delta F^{y} \cong \xi^{||c}\cdot\frac{Abs(\Delta x)}{2}, \qquad \Delta F^{x} \cong \xi^{||c}\cdot\frac{\Delta x}{l}\cdot\frac{Abs(\Delta x)}{2} \tag{14}$$

Thus, when the ion is displaced strictly perpendicular to the C axis, an additional, but balanced component of the force along the C axis arises, which leads to a change in stiffness (length of the initial tension l_0) of the spring along C, both at linear and harmonic displacements. For, again, boron nitride, where the ions form a pair, we have

$$\xi^{||*} \cong \xi^{||c}\cdot\left(1+\frac{Abs(\Delta x)}{2l_0}\right) \Rightarrow \xi^{||*} \cong \xi^{||c}\cdot\left(1+\frac{\Delta x_0}{2l_0}Abs(\cos(\omega t))\right) \tag{15}$$

If we set the frequency of harmonic displacements equal to the resonance ω_0, then decomposing the change of the parameter in a Fourier series, and, taking into account the first coefficients, we get

$$\xi^{||*} \cong \xi^{||c}\cdot\left(1+\frac{\Delta x_0}{l_0}\left[\frac{1}{\pi}+\frac{1}{3\pi}\cos(2\omega_0 t)-\frac{1}{15\pi}\cos(4\omega_0 t)\right]+\ldots\right) \tag{16}$$

Neglecting in the resulting expression (16) for the dependence of the longitudinal stiffness on the transverse oscillations of a small constant additive to the resonant frequency due to the zero term, we obtain the parametric Mathieu excitation at twice the most sensitive frequency

$$\xi^{||*} \cong \xi^{||c}\cdot\left(1+\frac{\Delta x_0}{l_0}\frac{1}{3\pi}\cos(2\omega_0 t)\right) \tag{17}$$

The excitations of the third and the following small Fourier coefficients in Φ.16 can be safely neglected, since it was shown above that the sensitivity of the excitation decreases sharply.

And so, if the ion displaced in x additionally begins to displace in y, then the returning force and the resonant frequency of oscillations along the C axis will be determined by the increased rigidity, in accordance with ϕ.16. But the main excitation at the doubled resonant frequency for longitudinal oscillations is a direct consequence of independence from the direction of displacement of the transverse oscillation strictly at the frequency of the longitudinal resonance.

For shear force of f. (14), since Since the transverse oscillations (at the longitudinal resonant frequency) are alternating, then there is, of course, no constant displacement. If we take into account the transverse stiffness $\xi^{\perp C}$ that was thrown out of this consideration, we obtain a small quadratic additive related to nonlinearity

$$F^x \cong \xi^{\perp C} \cdot \Delta x + \xi^{\|C} \cdot \frac{Abs(\Delta x)}{2l} \cdot \Delta x \tag{18}$$

and with harmonic excitation at the longitudinal resonant frequency we have a set of its odd harmonics falling down with the number

$$\begin{aligned} F^x &\cong \xi^{\perp C} \cdot \Delta x_0 \cdot \cos(\omega_0 t) + \xi^{\|C} \cdot \frac{\Delta x_0^2}{2l} \cdot Abs(\cos(\omega_0 t)) \cdot \cos(\omega_0 t) = \\ &\xi^{\perp C} \cdot \Delta x_0 \cdot \cos(\omega_0 t) + \xi^{\|C} \cdot \frac{\Delta x_0^2}{2l} \cdot \left(\frac{4}{3\pi} \cdot \cos(\omega_0 t) + \frac{4}{15\pi} \cdot \cos(3\omega_0 t) - \frac{4}{105\pi} \cdot \cos(5\omega_0 t) .... \right) \\ &\cong \left(\xi^{\perp C} + \xi^{\|C} \cdot \frac{2\Delta x_0}{3\pi l} \right) \cdot \Delta x_0 \cdot \cos(\omega_0 t) \end{aligned} \tag{19}$$

Those the oscillation stiffness perpendicular to the C axis has a small additive that increases linearly with the oscillation amplitude due to the stiffness along the C axis.

The above calculations can also be used to examine the ion chain in a plane perpendicular to the C axis. To do this, it is enough to swap formally $\xi^{\perp C}$, $\xi^{\| C}$, not formally, to take into account the features of each chain, specifically in boron nitride, it is necessary to take into account that in its hexagonal atomic layers each ion is surrounded symmetrically located three ions (Fig. 11)

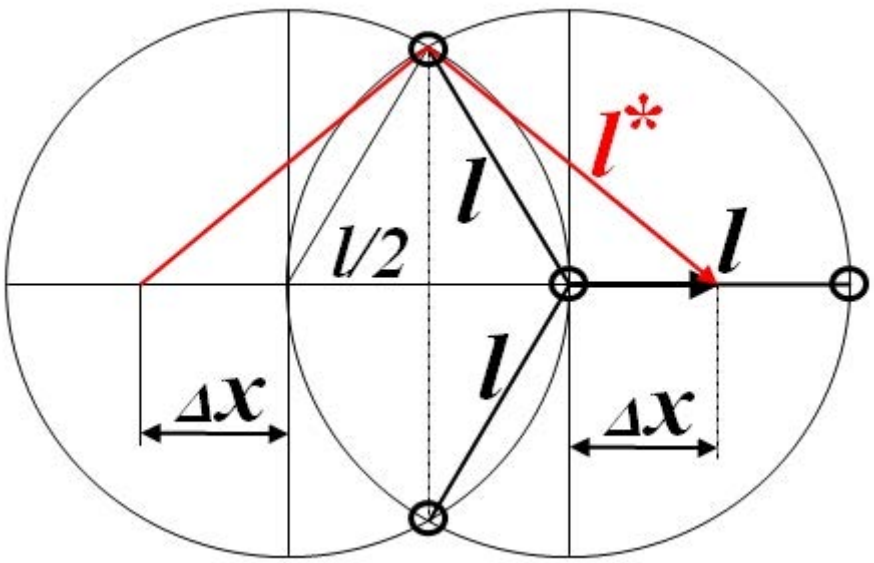


Fig. 11: Geometric construction demonstrating the ratio of displacements and forces

By displacing the central ion along x and reducing the parallel x bond, elementary geometric relations allow us to determine additionally the extension of two other bonds

$$(l^*)^2 = (l+\Delta l)^2 = (\Delta x)^2 + l^2 + 2\cdot\Delta x\cdot\frac{1}{2}l = l^2 + \Delta x\cdot l + (\Delta x)^2$$

$$\Delta l = \sqrt{l^2 + \Delta x\cdot l + (\Delta x)^2} - l = l\cdot\left(\sqrt{1+\frac{\Delta x}{l}+\left(\frac{\Delta x}{l}\right)^2} - 1\right) \approx \frac{\Delta x}{2} \quad (20)$$

And the same to define additional restoring forces due to these connections and their algebraic sum

$$\frac{F_\Sigma^x}{\xi^{\perp C}\cdot\Delta l} = \frac{l+2\Delta l}{l+\Delta l} \rightarrow F_\Sigma^x = \xi^{\perp C}\cdot\Delta l\cdot\left(1+\frac{1}{1+l/\Delta l}\right) \cong \xi^{\perp C}\cdot\Delta l\cdot\left(1+\frac{\Delta l}{l}\right) \quad (21)$$

$$F_\Sigma^x \cong \xi^{\perp C}\cdot\frac{\Delta x}{2}\cdot\left(1+\frac{\Delta x}{2l}\right) \approx \xi^{\perp C}\cdot\frac{\Delta x}{2}$$

So instead of the unbalance shown in f.9 we get

$$\Delta F^x \approx \xi^{\perp C}\cdot\frac{\Delta x}{2} + \xi^{\perp C}\cdot\Delta x = \frac{3}{2}\xi^{\perp C}\cdot\Delta x \quad (22)$$

And, thus, for the frequencies of orthogonal phonons in boron nitride (Fig. 12):

$$\omega_T^{\parallel C} = 757\ \text{cm}^{-1},\ \omega_L^{\parallel C} = 826.5\ \text{cm}^{-1},\ \omega_T^{\perp C} = 1372\ \text{cm}^{-1},\ \omega_T^{\perp C} = 1618\ \text{cm}^{-1}$$

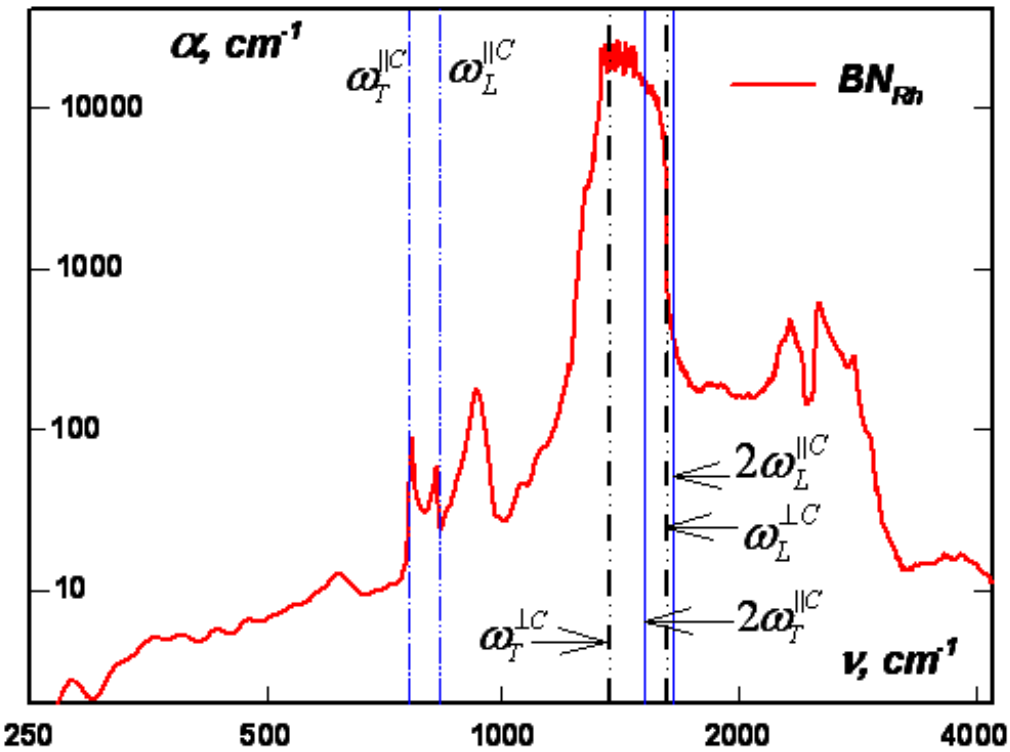


Fig. 12: The spectrum of "absorption" of radiation along the C axis in rhombohedral boron nitride

remembering f.11, we get the ratio

$$\frac{\omega_T^{\perp c}}{\omega_T^{\parallel c}} \simeq \sqrt{\frac{3\xi^{\perp c}}{2\xi^{\parallel c}}} \Rightarrow \frac{\xi^{\perp c}}{\xi^{\parallel c}} \simeq \frac{2}{3}\left(\frac{\omega_T^{\perp c}}{\omega_T^{\parallel c}}\right)^2 \simeq 2.2 \quad (23)$$

The spectrum shown in Figure 12 can be strictly (without any quotation marks) called the absorption spectrum in the entire range shown, except for the anomalous in shape and forbidden by symmetry band between the low-frequency phonons propagating along the C axis. Identified additionally in the experiments described below, its anomalous nature and the feat of strictly conducting the presented theoretical analysis. And along the way, as follows from f.21, it was shown that stiffnesses inside the layer and interlayer bonds differ only two times, which does not correspond to the standard concepts of the Van der Waals interaction between polyatomic layers.

III. Conclusion

Giant anisotropy of the electrical conductivity of graphite samples has pushed theorists to Van der Waals idealization of both graphite and boron nitride (C & BN). But, as was shown earlier, it was determined not by the properties of the graphite crystal itself, but by the texture of the samples. Both the erroneous discovery of "graphene" and the hype around it simply prompted the publication of experimental results and theoretical calculations of the real structure and real properties of C & BN. The real anisotropy of C & BN has nothing to do with theoretical "bad infinities", but it is large, moreover, it is extremely possible in crystals. And this makes it possible to use C & BN as a model material, in particular for the analysis of lattice vibrations.

On the one hand, the difference in the frequencies of their orthogonal normal lattice modes (parallel and perpendicular to the C axis) almost 2 times makes them weakly coupled in frequency. And this leads practically to the independence of the corresponding spectra of the lattice reflection and! to the rigorous description of each spectrum by its classical, with low attenuation, harmonic oscillator (which, in fact, was analyzed in previous work).

On the other hand, this large difference made it possible to experimentally reliably register at the longitudinal resonant frequency a "forbidden" spectral feature with anomalous properties of the stopband. The theory of parametric interaction developed earlier, although it was consistent with the observed effect, as such, but led to contradictions and questions, but which could not be answered. The analysis carried out in this work showed that there was simply no qualitative idea about the properties of parametric Mathieu solutions, and, therefore, there were not even correct quantitative estimates. And most importantly, this analysis showed that it is at the frequency of orthogonal oscillations equal to the frequency of the longitudinal resonance that this resonance leads to an unlimited (until we go beyond the scope of the model itself) increasing the amplitude of the associated longitudinal-transverse oscillations, which leads to skipping-scattering. A detailed experimental analysis of this "anomaly" will be presented in the next article "Parametric interaction of normal modes in C & BN.

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