



Rectifications of Einstein's Errors Such as $E = Mc^2$ and Necessity of Extending General Relativity

By C. Y. Lo

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Keywords: anti-gravity coupling; principle of causality, repulsive gravitation.

A foolish faith in authority is the worst enemy of truth, --
A. Einstein.

I. INTRODUCTION

In the world, first we are aware of the distances between things and the time needed to travel the distance from one object to another. To measure these, we invent the rule and the clock. We are also aware that objects can be very light that we can easily carry or heavy that is difficult to move. Then we invent the scale to measure their different weights. The first natural force is gravity that we are against when we lift up things; otherwise an object will fall to earth with acceleration. At first we believed that a heavier object will drop with faster acceleration because a feather

would fall very slowly. However, Galileo taught us that most objects drop with the same acceleration. A theory must be tested by observation. Thus, a science called physics was born.

It was based on the three basic laws on planets of Kepler from observing the stars [1], Newton discovered that the gravitational force F_g between two particles with masses m_1 and m_2 and a distance r as

$$F_g = G \frac{m_1 m_2}{r^2}, \quad (1)$$

where G is the Newtonian gravitational coupling constant. Moreover, Newton defined the mass of a object with

$$F = ma \quad (2)$$

where F is the force acting on an object, a is the acceleration of the object and m is the mass of the object. His theory was successful to explain mechanics and the orbit of the planets. The only exception was the perihelion of Mercury.

The deficiency of Newtonian mechanics started to show with electromagnetism when the velocity can be getting close to light speed. Then, the theory of special relativity was proposed. A problem is that according special relativity, influence of physics must be propagated with finite speed, but the Newtonian gravitational force is instantaneous. For solving this issue, Einstein proposed the general theory of relativity, in which gravity would be propagated with a finite speed. Einstein's theory was verified by his prediction on the bending of light and red shifts of light [2].

In physics, there are repulsive forces related to attractive forces. In electromagnetism there is a repulsive force between two particles of the same charges, and there is an attractive force between different charges. In nuclear physics, there are strong nuclear attractive force and also the weak force that lead to the nuclear decay. However, since there was no repulsive gravitational force, Einstein claimed that the inertial mass and the gravitational mass were equivalent. Moreover, there is no bounded dynamic solution for the Einstein equation [3, 4]. Thus, his theory is actually incomplete.

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In fact, general relativity has never reached a verifiable prediction on dynamic problems with the non-linear Einstein equation [4]. Hawking's theory was in the central development of general relativity, but his theories have never been verified [5]. There were questions on Einstein's theory, but none had a clear experimental support [4]. Now, the formula $E = mc^2$ has been proven invalid theoretically as well as experimentally [6], it is the time to review the whole theory of general relativity. In particular, the necessity of extending general relativity must be addressed [7].

Because of mathematical inadequacy, the deficiency of the Einstein equation was not aware that [4] Einstein incorrectly assumed that the bounded solution from the linearized equation assures the existence of a bounded dynamic solution for the non-linear equation [4]. Moreover, based on the defective Einstein equation, the singularity theorems of Penrose and Hawking were derived and accepted although it is well-known in physics, a singularity would mean a deficiency of the assumptions. Because gravity seems to be always attractive and the deficiency of Einstein equation was not aware, based on simulation, the notion of black holes was proposed [8].

However, Einstein's claim of $E = mc^2$ actually has never been proven [9] and now it is proven wrong in theory and experiments [6]. A problem is that Einstein relies too much on his physical intuition, and thought experiments. He did not check his claims with explicit examples [4]. He also failed to see the need to do the real experiments to support $E = mc^2$ [10]. As we shall show that $E = mc^2$ is a major source of errors in Einstein's theory. It is also responsible for Einstein's failure in showing that the unification of gravitation and electromagnetism is valid since he did not accept the charge-mass interaction [7]. We shall show the existence of the repulsive gravitational force, and explain related issues.

Einstein also did not check the consistency of his theory [4]. In physics, even a proposal is confirmed by an experiment, there could still be imperfections that can be discovered only from later developments. For example, although his notion of photons was verified and Einstein won a Nobel Prize, no experiments had shown that the photons consist of only electromagnetic energy. These deficiencies are responsible for the failures of Einstein's unification.

In summary, there are six kinds of errors in Einstein's general relativity: 1) the implicit invalid physical assumptions such as the photons include only electromagnetic energy and the gravitation is independent of temperature; 2) the mathematical errors such as the linearization being as always valid and the existence of bounded dynamic solutions; 3) Acceptance of singularities as valid physical solutions; 4) the invalid unverified beliefs such as the covariance principle and the embedded Riemannian geometry; 5) the

inappropriate applications of theories such as the special relativity and Riemannian geometry; 6) the erroneous choice of $E = mc^2$ instead of the existence of repulsive gravitation.¹⁾ Moreover, Einstein has the habit of using thought experiments. This would lead to implicit usage of invalid assumptions. Nevertheless, Einstein is still an outstanding physicist since he started the difficult but successful pioneer works.

Currently, many theories are based on speculations. However, we base on facts and logic in this paper.

II. THE INCONSISTENCY OF THE EINSTEIN EQUATION AND THE FORMULA $E = mc^2$

To see the inconsistency of Einstein's theory, we need to do only some simple algebra. From the Einstein equation,

$$G_{\mu\nu} = R_{\mu\nu} - (1/2)g_{\mu\nu}R = -KT_{\mu\nu} \quad (3)$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, $g_{\mu\nu}$ is the space time metric, and $R = g^{\mu\nu}R_{\mu\nu}$, we have

$$R = Kg^{\mu\nu}T_{\mu\nu}. \text{ because } g^{\mu\nu}g_{\mu\nu} = 4. \quad (4)$$

Since the electromagnetic energy-momentum tensor $T(E)_{\mu\nu}$ is traceless ($g^{\mu\nu}T(E)_{\mu\nu} = 0$), it cannot affect the Ricci curvature R . Thus the electromagnetic energy cannot be equivalent to mass since the mass can affect R . Note that the validity of eq. (4) depends only on the static Einstein equation, which has produced many impressive and accurate predictions. Naturally, the problem would only be the inadequately verified formula [9].

Moreover, eq. (4) was first derived by Einstein himself [11]. Thus the failure of seeing this inconsistency is clearly his oversight. This also illustrates that a major problem of Einstein is his inadequate checking for the internal consistency of his theory. This is why he did not discover that there is no bounded dynamic solution for the Einstein equation. Nevertheless, theorists such as Will [12] incorrectly claim that $E = mc^2$ had been proven.

III. THE PROBLEM OF THE PHOTONS AND THE ANTI-GRAVITY COUPLING

The fact is that the equivalence of energy and mass has never been generally verified [9]. However, the equivalence between the photonic energy and mass has been verified theoretically by Einstein [13] and this is supported by the experimental fact that a π_0 meson can decay into two photons.²⁾

This paradox can be solved only if the photons contain energy other than the electromagnetic energy. This would mean that Einstein's proposal on the notion of photon [14] is only partially correct.³⁾ Experimentally, the photoelectric effect has verified that the photons contain mainly electromagnetic energy. However, it has

not been verified that the photons contain only electromagnetic energy.

According to Einstein, the gravity generated by an electromagnetic wave can be derived from the Einstein equation with the electromagnetic wave energy-stress tensor as the source. In the calculation of the bending of light, it is implicitly required that such gravity generated by an electromagnetic wave is negligible.

However, for a plane electromagnetic wave, the metric obtained by Penrose [15] is the following:

$$ds^2 = du dv + H du^2 - dx_i dx_i, \text{ where } u = ct - z, v = ct + z. \quad (5)$$

$$H = h_{ij}(u) x_i x_j,$$

where $h_{ij}(u)$ is the energy-momentum tensor.

Since the solution (5) is not bounded and has unphysical parameters, it violates the principle of causality (see Appendix A). This also exposed a deficiency because this violates the requirement for the calculation of the bending of light [2].

However, according to the old standard of the Physical Review [16] the solution of Penrose was perfectly valid. This shows that the editors of APS had a deficiency in physics. This deficiency was exposed because they are facing a choice of accepting the solution of Penrose or rejecting Einstein's theory. Moreover since such gravity is very weak in physics, journals such as the Chinese Physics B, in agreement with Einstein, believed that such gravity can be calculated with the perturbation approach, but they did not do it.⁽⁴⁾

Note, however, that mathematically for a perturbation approach to be valid, a necessary condition is that this problem has a bounded solution. This compatibility between mathematics and physics is crucial for the validity of a theory in physics.⁽⁵⁾ However, explicit calculation shows that it is impossible to have a bounded solution for the gravity of an electromagnetic wave [17]. This shows that Einstein does not understand general relativity adequately.

Fortunately, general relativity shows that the photon contains also gravitational energy. This is actually very natural because any charged particle is always massive and the electromagnetic wave is generated from the motion of charged particles. A gravitational wave component was not included in because general relativity had not been invented.

In order for Einstein's theory to make sense, the related Einstein equation with an electromagnetic wave as the source, must include the photonic energy-stress tensor with the anti-gravity coupling [17, 18].

For this case, the related modified Einstein equation is the following:⁽⁶⁾

$$G_{ab} \equiv R_{ab} - \frac{1}{2} g_{ab} R = -K[T(E)_{ab} - T(p)_{ab}], \quad (6)$$

and

$$T_{ab} = -T(g)_{ab} = T(E)_{ab} - T(P)_{ab},$$

where $T(E)_{ab}$ and $T(P)_{ab}$ are respectively the energy-stress tensors for the electromagnetic wave and the related photons. *Thus the photonic energy must include also the energy of its gravitational wave component as previous analysis shows.* Note that it is natural to include the gravitational wave energy in a photon since a charged particle is always massive.

There is a conflict if the photons consist of only electromagnetic energy. *Thus, the need and existence of a gravitational wave was discovered as early as 2006 [17, 18].⁽⁷⁾* Now, this conflict is resolved since the photonic energy is the sum of electromagnetic energy and gravitational energy, and thus it is established that $E = mc^2$ can be invalid.

In Einstein's 1905 proof [13] for the equivalence of mass and photonic energy, he implicitly assumed that the photonic energy is the energy of the massless particles. *Apparently he was not aware that the energy of massless particles is inconsistent with the energy-stress tensor of an electromagnetic wave. Moreover, to establish this, a modified Einstein equation must be used [6].* Ohanian [19] incorrectly credited von Laue for a complete proof of the equivalence of mass and photonic energy. However, the fact is that both von Laue and Einstein failed [6].

IV. EXPERIMENTAL VERIFICATION ON THE INVALIDITY OF $E = mc^2$

Theoretically it has been shown that the energy may not always be equivalent to mass. However, a theoretical conclusion may not be valid unless it is supported by experiments because in physics implicit assumptions could be used without knowing it. For examples, an implicit assumption in the space-time singularity theorems of Penrose and Hawking is that all the coupling constants have the same sign [20] and Einstein's claim of $E = mc^2$.

Einstein [10] explained that $E = mc^2$ means that a piece of heated-up metal would have an increment of weight since if an increment of energy for matter implies and increment of mass, this would result in the increment of weight. Therefore, if a piece of heated-up metal has a reduced weight, it has proven that the formula $E = mc^2$ is not valid. Note that it has been shown from eq. (6) that Einstein's understanding of general relativity is inadequate.

Moreover, Dmitriev, Nikushchenko, & Snegov [21] showed in 2003 that a piece of heated-up brass has reduced weight. This experiment is done inside a Dewar vessel that separates the influence of outside heat (see Appendix B). Fan Liangzao, Feng Jinsong, & Liu Wu Qing [22] also show in 2010 that six kinds of metal, after heating-up, have reduced weight. Thus, as

shown by the Einstein equation, energy may not be equivalent to mass (see Section 2).

However, they have mistaken instead that these experiments [21, 22] show a reduction of mass. Note that, it has been firmly established that mass is equivalent to energy from the atomic bomb although energy may not be equivalent to mass. Therefore, their claim is inconsistent with established experiments. Moreover, they must explain where the lost mass has become. Understandably their results were incorrectly rejected by many as due to errors.

As Einstein pointed out, the acceleration mass is related to the resistance to acceleration and gravitational mass is related to the attraction to a mass. Thus, acceleration mass and gravitational mass should be distinguishable. Einstein was surprised since he can define the acceleration mass as equivalent to the gravitational mass. *This is due to that the existence of repulsive gravity has not been recognized.* However, it is possible that the mass and gravity can be distinguished with the first approximation of a formula for the period T of a pendulum as follows [23]:

$$T \approx 2\pi \sqrt{\frac{l}{g}}, \quad (7)$$

where l is the length of the pendulum and g is the gravity. Thus, the change of mass of the pendulum would not change the period of the pendulum, but the change of g , the period of the pendulum will be changed.

Since a piece metal is a solid, a reduction of its mass or gravity can be distinguished by using it as a pendulum. In fact, it has been verified by Liu [24] that the mass is essentially the same as Einstein [10] and Lo [25] predicted, but the period is extended after heating-up. Moreover, recently it has been verified by Lo [26] with a torsion balance scale that the lead balls have reduced gravitation after heated-up. *Thus, measuring mass through gravity is no longer valid.*

Also, it has been verified by Tsipenyuk & Andreev [27] that a charged metal ball has reduced weight,⁸⁾ and a charged capacitor also reduces weight [28].⁹⁾ These experiments support the existence of a charge-mass repulsive force [29] that has been derived from the Reissner-Nordstrom metric of a particle with charge q and mass M [30],

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right) dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2, \quad (8)$$

(with $c = 1$) where r is the radial distance (in terms of the Euclidean-like structure [31]) from the particle center.¹⁰⁾ In this metric (8), the gravitational components generated by electricity have not only a very different radial coordinate dependence but also a different sign that makes it a new repulsive gravity [7]. This repulsion

implies that the basic assumption for black holes of gravity being always attractive [8] is invalid, and general relativity must be extended.¹¹⁾

For an elementary charged particle, the repulsive gravitational force would be very small. However, a similar metric can be derived for a charged ball. The only change is that r becomes R , the distance from the center of the ball and q becomes Q the total charge of the ball. Thus, for a charged ball with sufficient charges Q , the repulsive gravitational force can be macroscopically observed [32]¹²⁾.

V. A CHARGED CAPACITOR AND THE CURRENT-MASS INTERACTION

The study of charging capacitor was initiated by Thomas T. Brown and later was joined by Paul A. Biefeld [33, 34].

They are known as the B-B effects. A problem is that they cannot be explained with current theories. Thus, many regarded the B-B effects as just experimental errors. Note that physical laws are obtained from observations.

For instance, it is known that a charged capacitor has reduced weight. Moreover, after being charged with a high voltage (about 40 kilovolts), but without continuous supply of electric energy, the lifter (a light capacitor) is able to lift its own weight plus a payload hovering on earth. Also a lifter could get to work by charging the wire to either a positive or a negative potential. It has been determined that the lift is not due to ion wind effects [33]. Thus, the lift is generated by changing something inside the lifter with one high voltage charge.

In a charged capacitor, the only change is the state of motion of some electrons that have become statically concentrated instead of moving in orbits. Then, a repulsive force appears. Since such a force did not appear before, it is clear that such a force was cancelled out by the force created by the motion of the electrons. In other words, the repulsive force generated by the charges of protons and the electrons was cancelled by the force generated by the motion of the initially moving charges of the electrons.

However, this repulsive force cannot be proportional to the charge density. We have equal numbers of negatively charged electrons and positively charged protons with equal charge. This would lead to the cancellation of the forces generated by particles charges. However, if such a force is proportional to the charge density square, then these two kinds of forces would be added up, instead of cancelled out. Moreover, since the lifter has a limited height, one should expect that this repulsive gravitational force would diminish faster than the attractive gravitational force. Thus, if we assume that the force is proportional to mass as usual, the static charge-mass interaction

would be a repulsive force between particles with charge density D_q and another particle of mass m would have the following form,

$$F_r \approx K m D_q^2 / r^n \quad \text{where } n > 2, \quad (9)$$

r is the distance between the two particles, and K is the coupling constant. In formula (9), the coupling constant K and n the power of r can be determined by experiments. The simplest case would be $n = 3$.

Formula (9) is derived from the observations with common physical sense. The experimental results are that the charged capacitors have reduced weight. If the lift force is large enough, it will hover over the earth [33, 34].

According to general relativity, if the electric energy leads to a repulsive force toward a mass, the magnetic energy would lead to an attractive force from a current toward a mass [8, 35]. Due to that a charged capacitor has reduced weight, it is necessary to have the current-mass interaction to be cancelled out by the effect of the charge-mass interaction. Thus, the existence of the current-mass attractive force would solve a puzzle, i.e., why a charged capacitor exhibits the charge-mass repulsive force since a charged capacitor has no additional electric charges. In fact, the charge-mass repulsive force would be cancelled by the current-mass force as Galileo, Newton and Einstein implicitly assumed.

The existence of such a current-mass attractive force has been discovered by Martin Tajmar and Clovis de Matos [36] from the European Space Agency. Martin et al found that a spinning ring of superconducting material increases its weight more than expected. Thus, they believed that general relativity was wrong. However, according to quantum theory, spinning superconductors should produce a weak magnetic field. Thus, they also measured the current-mass interaction to the earth! The current-mass interaction would generate a force which is perpendicular to the current.

Since the weight addition from a current-mass interaction is directional, the weight of a magnet is directional dependent as our experiment verified [36]. *This directional dependence of weight is a completely new phenomena that verify the existence of the current-mass interaction.*

One may ask what the formula for the current-mass force is. Unlike the charge-mass repulsive force, which can be derived from general relativity; this general force would be beyond general relativity since a current-mass interaction would involve the acceleration of a charge, this force would be time-dependent and generates electromagnetic radiation. Moreover, when the radiation is involved, the radiation reaction force and the variable of the fifth dimension must be considered [37]. Thus, we are not yet ready to derive the current-mass interaction. Nevertheless, we may assume that, for

a charged capacitor, the resulting force is the interaction of net macroscopic charges with the mass [28].

Experimentally, the repulsive force would be proportional to the potential square, V^2 where V is the electric potential difference of the capacitor ($Q = CV$, C is the capacitance and Q is the charge). This has been verified by the experiments of Musha [38, 39]. Thus, the factor of charge density square in heuristic Eq. (9) is correct. Moreover, the hovering of the lifter shows that the repulsive force would diminish faster than the gravitational force. However, even the $1/r^3$ factor in the repulsive force is verified, the calculation would still depend on the detailed modeling [40]. Although the initial thrust due to the electric field is directional, the weight reduction effect for charged capacitors is not directional and it stays if the potential does not change. This is verified by Liu [24] with the roll-up capacitors. Thus, the repulsive force on the charged capacitor is the same force that derived from general relativity [30].

One may ask, what would the weight of the capacitor be after it is discharged? It takes time for a capacitor to recover its weight after being discharged [41]. This was observed because the rolled-up capacitors keep heat better. A discharged capacitor needs time to dissipate the heat generated by discharging, and the motions of its charges would accordingly recover to the previous state. Thus, this connects the heat with a reduction of the weight, and this explains also the experiment on the weight reduction of a piece of heat-up metal. In other words, Einstein is clearly wrong.

VI. THE NON-LINEAR EINSTEIN EQUATION & ITS INCOMPATIBILITY WITH THE LINEARIZED EQUATION

The Einstein equation is the heart of general relativity. However, many results in general relativity, except the perihelion of Mercury, are actually derived from the linearized equation. A problem is that the Einstein equation and the linearized equation are consistent only for the static cases. For the dynamic case, they are actually independent equations. In particular, the Einstein equation has no dynamic solutions [42-44], but the linearized equation does.

Historically, it is Einstein's calculation of the perihelion in agreement with observation made him to have confidence on his theory [45]. However, A. Gullstrand [46], the chairman of the Nobel Committee (1920-1929) suspected that his calculation is invalid because Einstein failed to show that his calculation can be derived from the required perturbation approach. This is why Einstein obtained a Nobel Prize from his work in photoelectric effects, instead of general relativity as most theorists expected. It should be noted that D. Hilbert did not come to defend Einstein's calculation

[45] although he obtained the same result with the same method earlier than Einstein.

In 1993, D. Christodoulou and S. Klainerman [47] from Princeton University claimed that they have constructed dynamic solutions for the Einstein equation. The 1993 Nobel Committee was convinced that the Einstein equation had a dynamic solution, and accepted the invalid calculation of J. H. Taylor and T. Damour and awarded the 1993 Nobel Prize for physics to Hulse and Taylor [48].

However, in 1995 [42] it is found that the Einstein equation actually has no bounded dynamic solution. Moreover, Christodoulou and Klainerman actually have not completed a construction for any dynamics solution [49]. Nevertheless, the calculation of Taylor and Damour can be modified to justify their conclusion [43, 44]. It should be noted that the errors of Christodoulou and Klainerman was built on accumulated errors of Wheeler et al [30].

Earlier in 1973 Misner, Thorne, and Wheeler [30] erroneously claimed they have obtained a bounded wave solution because of their inadequacy in mathematics. They started from a metric of the form,

$$-ds^2 = c^2 dt^2 - dx^2 - L^2(e^{2\beta} dy^2 + e^{-2\beta} dz^2) \quad (10)$$

where $L = L(u)$, $\beta = \beta(u)$, $u = ct - x$, and c is the light speed. Then, the Einstein equation $G_{\mu\nu} = 0$ becomes

$$\frac{d^2 L}{du^2} + L \left(\frac{d\beta}{du} \right)^2 = 0 \quad (11)$$

Then, they [30] claimed that Eq. (11) has a bounded solution, compatible with a linearization of metric (10).

However, it has been shown with undergraduate mathematics [50] that Misner et al. are incorrect and Eq. (10) does not have a physical solution that satisfies Einstein's requirement on weak gravity. Moreover, Misner et al. [30] also make other serious errors in physics as shown in their eq. (40.14) for the proper time measured by an earth-based clock. It turns out that this is due to their mathematical errors at the undergraduate level [50].

On the other hand, from the Maxwell-Newton approximation, Einstein [51] obtained a solution as follows:

$$-ds^2 = c^2 dt^2 - dx^2 - (1 + 2\phi) dy^2 - (1 - 2\phi) dz^2 \quad (12)$$

where ϕ is a bounded function of $u (= ct - x)$. Note that metric (12) is the linearization of metric (10) if $\phi = \beta(u)$. Thus, the waves illustrate that the linearization is not valid for the dynamic case when gravitational waves are involved.

Also, in 1984 Wald [5] claimed that he can get a second order solution, but he did not provide one.

Many believed that the Einstein equation has bounded dynamic solutions because the linearized equation has. We shall give an example to show this is false. Consider the metric obtained by Bondi, Pirani, & Robinson [52] as follows:

$$ds^2 = e^{2\varphi} (d\tau^2 - d\xi^2) - u^2 \begin{bmatrix} \cosh 2\beta (d\eta^2 + d\zeta^2) \\ + \sinh 2\beta \cos 2\theta (d\eta^2 - d\zeta^2) \\ - 2 \sinh 2\beta \sin 2\theta d\eta d\zeta \end{bmatrix} \quad (13a)$$

where φ , β and θ are functions of $u (= \tau - \xi)$. It satisfies the differential equation (i.e., their Eq. [2.8]),

$$2\phi' = u(\beta'^2 + \theta'^2 \sinh^2 2\beta) \quad (13b)$$

which is a special cases of $G_{\mu\nu} = 0$. They claimed this is a wave from a distant source and weak gravity invalid. The metric is irreducibly unbounded due to the factor u^2 . Linearization does not make sense since u is not bounded.

Moreover, when gravity is absent, it is necessary to have $\phi = \sinh 2\beta = \sin 2\theta = 0$. These would reduce (13a) to

$$ds^2 = (d\tau^2 - d\xi^2) - u^2 (d\eta^2 + d\zeta^2) \quad (13c)$$

However, this metric is not equivalent to the flat metric. Thus, metric (13c) violates the principle of causality (see Appendix A). This challenges the view that both Einstein's notion of weak gravity and his covariance principle are valid because (13 c) cannot be transform to the flat metric and (13a) cannot be transformed into a metric of weak gravity.

These conflicting views are supported respectively by the editors of the "Royal Society Proceedings A" and the "Physical Review D"; thus there is no general consensus. Apparently Einstein did not know that, for a dynamic case, the Einstein equation and the linearized equation are independent equations [53]. Einstein was puzzled that he can have gravitational solution from the linearized equation, but he cannot obtain a bounded gravitational wave solution. So, when Einstein was asked about the existence of gravitational, his last words were that I do not know [54].

VII. THE NON-EXISTENCE OF A BOUNDED DYNAMIC SOLUTION FOR A TWO-BODY PROBLEM IN GENERAL RELATIVITY

According to the principle of causality, weak sources would produce a weak field, i.e.,

$$g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}, \text{ where } |\gamma_{\mu\nu}| \ll 1 \quad (14)$$

and $\eta_{\mu\nu}$ is the flat metric. However, eq. (14) is valid, only if the Einstein equation is valid. Since the strength of a source can be reduced, to show the non-existence of a

dynamic solution, it is sufficient to show the case of weak gravity.

Unfortunately, many believe that condition (14) for weak gravity is always valid for the Einstein equation. They believed that an approximate weak solution can be derived through the approach of the field equation being linearized. The linearized Einstein equation with the linearized harmonic gauge $\partial^\mu \bar{\gamma}_{\mu\nu} = 0$ is

$$\frac{1}{2} \partial^\alpha \partial_\alpha \bar{\gamma}_{\mu\nu} = \kappa T_{\mu\nu} \text{ where } \bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} (\eta^{\alpha\beta} \gamma_{\alpha\beta}), \quad (15)$$

Note that we have

$$G_{\mu\nu} = G_{\mu\nu}^{(1)} + G_{\mu\nu}^{(2)} \text{ and } G^{(1)}_{\mu\nu} = \frac{1}{2} \partial^\alpha \partial_\alpha \bar{\gamma}_{\mu\nu} + H^{(1)}_{\mu\nu}, \quad (16)$$

where

$$H^{(1)}_{\mu\nu} = -\frac{1}{2} \partial^\alpha [\partial_\mu \bar{\gamma}_{\nu\alpha} + \partial_\nu \bar{\gamma}_{\mu\alpha}] + \frac{1}{2} \eta_{\mu\nu} \partial^\alpha \partial^\beta \bar{\gamma}_{\alpha\beta}$$

The linearized vacuum Einstein equation means $G^{(1)}_{\mu\nu}[\gamma^{(1)}_{\alpha\beta}] = 0$. Thus, to have a solution of the second order we must correct $\gamma^{(1)}_{\mu\nu}$ by adding to it the term $\gamma^{(2)}_{\mu\nu}$ that satisfies

$$G^{(1)}_{\mu\nu}[\gamma^{(2)}_{\alpha\beta}] + G^{(2)}_{\mu\nu}[\gamma_{\alpha\beta}] = 0, \text{ where } \gamma_{\mu\nu} = \gamma^{(1)}_{\mu\nu} + \gamma^{(2)}_{\mu\nu} \quad (17)$$

which is the correct form of eq. (4.4.52) in Wald's book [5] (Wald did not distinguish $\gamma_{\mu\nu}$ from $\gamma^{(1)}_{\mu\nu}$).

However, detailed calculation shows that this equation (17) does not have a solution for the dynamic case [42-44]. In fact, as shown by the example in the last section, for a dynamic case, the linearized equation and the Einstein equation are independent equations [53].

It was believed that the linear Maxwell-Newton Approximation [42] (or the linearized Einstein equation),

$$\frac{1}{2} \partial^\alpha \partial_\alpha \bar{\gamma}_{\mu\nu} = \kappa T_{\mu\nu}, \text{ where } \bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} (\eta^{\alpha\beta} \gamma_{\alpha\beta}) \quad (18a)$$

and

$$\bar{\gamma}_{\mu\nu}(x^i, t) = \frac{K}{2\pi} \int \frac{1}{R} T_{\mu\nu}[y^i, (t-R)] d^3y,$$

$$\text{where } R^2 = \sum_{i=1}^3 (x^i - y^i)^2 \quad (18b)$$

provides the first-order approximation for the Einstein equation (3). However, this was verified for the static case only. The Cauchy data of eq. (3) must satisfy four constraint equations, $G_{\mu t} = -\kappa T_{\mu t}$ ($\mu = x, y, z, t$) since $G_{\mu t}$ contains only first-order time derivatives [55]. This

shows that (18a) would be dynamically incompatible with Einstein equation (3).

In 1957, Fock [56] pointed out that, in harmonic coordinates, there are divergent logarithmic deviations from expected linearized behavior of the radiation. This was misinterpreted to mean merely that the contribution of the complicated nonlinear terms in the Einstein equation cannot be dealt with satisfactorily following this method and that another approach is needed. Subsequently, vacuum solutions that do not involve logarithmic deviation were founded by Bondi, Pirani & Robinson [52] in 1959. Thus, the interpretation appears to be justified and the faith on the dynamic solutions maintained. In 1995 [42], it is recognized such a symptom shows the absence of bounded dynamic solutions.

Equation (18) shows that a gravitational wave is bounded and is related to the dynamic of the source. These are useful to prove that eq. (18), as the first-order approximation for a dynamic problem, is incompatible with the Einstein equation (3). According to the principle of causality, it is sufficient to consider the case of weak gravity. Consider, $G^{(2)}_{\mu\nu}$ ($G_{\mu\nu} \equiv G_{\mu\nu}^{(1)} + G_{\mu\nu}^{(2)}$) is at least of second order in terms of the metric elements. For an isolated system located near the origin of the space coordinate system, $G^{(2)}_{\mu t}$ at large r ($= [x^2 + y^2 + z^2]^{1/2}$) is of $O(K^2/r^2)$ [5, 55].

One may obtain some general characteristics of a dynamic solution for an isolated system as follows:

1) The characteristics of some physical quantities of an isolated system:

For an isolated system consisting of particles with typical mass $\bar{M}$, separation $\bar{r}$, and velocities $\bar{v}$, Weinberg [55] estimated, the power radiated at a frequency ω of order $\bar{v}/\bar{r}$ will be of order

$$P \approx \kappa (\bar{v}/\bar{r})^6 \bar{M}^2 \bar{r}^4 \text{ or } P \approx \bar{M} \bar{v}^8 / \bar{r}, \quad (19)$$

since $\kappa \bar{M}/\bar{r}$ is of order $\bar{v}^2$. The typical deceleration $\bar{a}_{\text{rad}}$ of particles in the system owing this energy loss is given by the power P divided by the momentum $\bar{M} \bar{v}$, or $\bar{a}_{\text{rad}} \approx \bar{v}^7/\bar{r}$. This may be compared with the accelerations computed in Newtonian mechanics, which are of order $\bar{v}^2/\bar{r}$, and with the post-Newtonian correction of $\bar{v}^4/\bar{r}$. Since radiation reaction is smaller than the post-Newtonian effects by a factor $\bar{v}^3$, if $\bar{v} \ll c$, the velocity of light, the neglect of radiation reaction is perfectly justified. This allows us to consider the motion of a particle in an isolated system as almost periodic.

Consider two particles of equal mass with an almost circular orbit in the x-y plane whose origin is the center of the circle (i.e., the orbits are a circle if radiation is neglected). Thus, the principle of causality implies that the metric $g_{\mu\nu}$ is weak and very close to the flat metric at

distance far from the source and that $g_{\mu\nu}(x, y, z, t')$ is an almost periodic function of $t' (= t - r/c)$.

2) The expansion of a bounded dynamic solution $g_{\mu\nu}$ for an isolated weak gravitational source:

According to eq. (18), a first-order approximation of metric $g_{\mu\nu}(x, y, z, t')$ is bounded and almost periodic since $T_{\mu\nu}$ is. Physically the principle of causality requires $g_{\mu\nu}$ to be almost periodic in time since the motion of a source particle is. Such a metric $g_{\mu\nu}$ is asymptotically flat for a large distance r , and the expansion of a bounded dynamic solution is:

$$g_{\mu\nu}(n^x, n^y, n^z, r, t') = \eta_{\mu\nu} + \sum_{k=1}^{\infty} f_{\mu\nu}^{(k)}(n^x, n^y, n^z, t')/r^k,$$

where $n^v = x^v/r$. (20a)

3) The non-existence of dynamic solutions:

It follows expansion (20a) that the non-zero time average of $G^{(1)}_{\mu t}$ would be of $O(1/r^3)$ due to

$$\partial_\mu n^v = (\delta^v_\mu + n^v n_\mu)/r, \quad (20b)$$

since the term of $O(1/r^2)$, being a sum of derivatives with respect to t' , can have a zero time-average. If $G^{(2)}_{\mu t}$ is of $O(K^2/r^2)$ and has a nonzero time-average, consistency can be achieved only if another term of time-average $O(K^2/r^2)$ at vacuum be added to the source of the Einstein equation (3). Note that there is no plane-wave solution for $G_{\mu\nu} = 0$.

It will be shown that there is no dynamic solution for the Einstein equation with a massive source. Let us define

$$\gamma_{\mu\nu} = \gamma^{(1)}_{\mu\nu} + \gamma^{(2)}_{\mu\nu}; \quad \bar{\gamma}^{(i)}_{\mu\nu} = \gamma^{(i)}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} (\gamma^{(i)}_{cd} \eta^{cd}),$$

where $i = 1, 2$;

and

$$\frac{1}{2} \partial^\alpha \partial_\alpha \bar{\gamma}^{(1)}_{\mu\nu} = K T(m)_{\mu\nu}. \quad (21)$$

Then $\bar{\gamma}^{(1)}_{\mu\nu}$ is of a first-order; and $\gamma^{(2)}_{\mu\nu}$ is finite. On the other hand, from the Einstein equation (3), one has

$$\frac{1}{2} \partial^\alpha \partial_\alpha \bar{\gamma}^{(2)}_{\mu\nu} + H^{(1)}_{\mu\nu} + G^{(2)}_{\mu\nu} = 0. \quad (22)$$

For a dynamic case, equation (22) may not be satisfied. If (21) is a first-order approximation, $G^{(2)}_{\mu\nu}$ has a nonzero time-average of $O(K^2/r^2)$ [1] (but $[\partial^\alpha \partial_\alpha \bar{\gamma}^{(2)}_{\mu\nu}/2 + H^{(1)}_{\mu\nu}]$ would have); and thus $\bar{\gamma}^{(2)}_{\mu\nu}$ cannot have a solution.

However, if $\bar{\gamma}^{(2)}_{\mu\nu}$ is also of the first-order of K , one cannot estimate $G^{(2)}_{\mu\nu}$ by assuming that $\bar{\gamma}^{(1)}_{\mu\nu}$ provides a first-order approximation. For example, equation (18) does not provide the first approximation

for the static Schwarzschild solution, although it can be transformed to a form such that (18) provides a first-order approximation [22]. According to eq. (16), $\bar{\gamma}^{(2)}_{\mu\nu}$ will be a second order term if the sum $H^{(1)}_{\mu\nu}$ is of second order. From (16), this would require $\partial^\mu \bar{\gamma}_{\mu\nu}$ being of second order. For weak gravity, it is known that a coordinate transformation would turn $\partial^\mu \bar{\gamma}_{\mu\nu}$ to a second order term. (Eq. [22] implies that $\partial^c \partial_c \bar{\gamma}^{(2)}_{\mu\nu} - \partial^c [\partial_\nu \bar{\gamma}_{\mu c} + \partial_\mu \bar{\gamma}_{\nu c}] + \eta_{\mu\nu} \partial^\alpha \partial^\beta \bar{\gamma}_{\alpha\beta}$ would be of second order.) Thus, it is possible to turn (21) to become an equation for a first-order approximation for weak gravity.

Since it has been proven that (18) necessarily gives a first-order approximation [15], a failure of such a coordinate transformation means only that such a solution is not valid in physics. Moreover, for the dynamic of massive matter, experiment [22] supports the fact that Maxwell-Newton Approximation (11) is related to a dynamic solution of weak gravity [16]. Thus, theoretical considerations as well as experiments eliminate other unverified speculations thought to be possible since 1957.

As shown, the difficulty comes from the assumption of boundedness, which allows the existence of a bounded first-order approximation, which implies that a time-average of the radiative part of $G^{(2)}_{\mu\nu}$ is non-zero. The present method has an advantage over Fock's approach to obtaining logarithmic divergence [56] for being simple and clear.

In short, according to Einstein's radiation formula, a time average of $G^{(2)}_{\mu t}$ is non-zero and of $O(K^2/r^2)$ [5].

Although (18) implies $G^{(1)}_{\mu t}$ is of order K^2 , its terms of $O(1/r^2)$ can have a zero time average because $G^{(1)}_{\mu t}$ is linear on the metric elements. Thus, the Einstein equation (3) in vacuum cannot be satisfied. However, a static metric can satisfy (3), since both $G^{(1)}_{\mu\nu}$ and $G^{(2)}_{\mu\nu}$ are of $O(K^2/r^4)$ in vacuum. Note that $G_{\mu t} = K T(m)_{\mu t}$ are constraints on the initial data.

In conclusion, assuming the existence of dynamic solutions of weak gravity for Einstein equation (3) is invalid. This means that general relativity has not yet totally superseded Newtonian gravity, which has dynamic solutions [23]. Unfortunately, because of inadequacy in mathematics, many theorists carelessly follow the erroneous and groundless claims of Christodoulou and Klainerman [47]. However, nobody was able to produce a bounded dynamic solution. Furthermore, the positive mass theorem of Schoen & Yau, which eliminates all the dynamic solutions with a bounded assumption, misleads us that the Einstein equation had bounded dynamic solution.

VIII. A RECTIFIED EINSTEIN EQUATION AND THE ANTI-GRAVITY COUPLING

From the above analysis, there is a conflict between the Einstein equation, which has no dynamic solution and its linearized equation, which has a dynamic solution. The conflict is due to that the second order terms $G_{\mu\nu}^{(2)}$ cannot be eliminated in the Non-linear Einstein equation. Thus, a solution is the 1995 update of the equation [42] as follows:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = K [T(m)_{\mu\nu} - t(g)_{\mu\nu}], \quad (23)$$

where $t(g)_{\mu\nu}$ is the energy-stress tensors for gravity.¹⁴⁾ Then, from (23), the equation in vacuum is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -K t(g)_{\mu\nu} \quad (23')$$

Note that $t(g)_{\mu\nu}$ is equivalent to $G_{\mu\nu}^{(2)}$ (and Einstein's gravitational pseudotensor) in terms of his radiation formula.

When gravitational wave is present, the gravitational energy-stress tensor $t(g)_{\mu\nu}$ is non-zero. This explains Einstein's puzzle [54] why there is no bounded gravitational wave solution for $G_{\mu\nu} = 0$ as shown in solution (13).¹⁵⁾ For the dynamic case, the linear equation (18) is the linearization of (23) but not of the 1915 Einstein equation. This explains why the Einstein equation does not have a wave solution.

Note that the radiation of the binary pulsar can be calculated without detailed knowledge of $t(g)_{\mu\nu}$. From (23'), the approximate value of $t(g)_{\mu\nu}$ at vacuum can be calculated through $G_{\mu\nu}/K$ as before since the first-order approximation of $g_{\mu\nu}$ can be calculated through (18). In view of the facts that $Kt(g)_{\mu\nu}$ is of the fifth order in a post-Newtonian approximation, that the deceleration due to radiation is of the three and a half order in a post-Newtonian approximation [55] and that the perihelion of Mercury was successfully calculated with the second-order approximation from (3), the orbits of the binary pulsar can be calculated with the second-order post-Newtonian approximation of (23) by using (3). Thus, the approaches of Damour and Taylor [57, 58] would be essentially valid except that they did not realize the crucial fact that (18) is actually an approximation of the updated equation (23) [33].

In light of the above, the Hulse-Taylor experiments support the anti-gravity coupling being crucial to the existence of the gravitational wave [44], and (18) being an approximation of weak waves generated by massive matter. Thus, it has been verified that Einstein equation (3) is not compatible with radiation, but the updated Einstein equation is.

The 1995 updated Einstein equation actually was first proposed by Lorentz [59] and Levi-Civita [60] as follows;

$$\kappa t(g)_{ab} = G_{ab} + \kappa T_{ab} \quad (24)$$

where T_{ab} is the sum of other massive energy-stress tensors. Then, the gravitational energy-stress tensor $t(g)_{ab}$ takes a covariant form. However, Einstein [61] objected on the grounds that his field equation implies $t(g)_{ab} = 0$. Moreover, Einstein had treated the gravitational energy in the same coupling as other energy. Einstein is wrong since his equation is proven invalid for the dynamic case. Thus, eq. (23) should be called the Lorentz-Levi-Einstein equation. An independent evidence for unboundedness is that the calculated radiation depends on the approached chosen [62].

It should be noted that the anti-gravity coupling is a general feature that appears in where the gravitational wave is present. For instance, it has been shown that such coupling is necessary to appear in the Einstein equation for the gravitational waves generated by an electromagnetic wave [17, 18].

While eq. (23) is consistent with the linearized equation for the massive case and can do an approximate calculation for the gravitational radiation, it is still not yet clear that it is the exact equation. For this, our position is that this is the best we can get so far. Moreover, the non-unique signs of coupling is consistent with that $E = mc^2$ is conditionally valid.

IX. THE SPACE-TIME SINGULARITY THEOREMS AND THE ASSUMPTION OF UNIQUE SIGN OF COUPLINGS

In physics, the existence of singularities suggests problematic assumptions. Nevertheless, in current theory of general relativity, the existence of space-time singularities plays a central role on the notion of black holes and the expanding universe. However, these two speculations have not been firmly verified in spite of the efforts of generations of physicists. Thus, one may question the validity of general relativity in spite of earlier success.

The existence of space-time singularities is due to the spacetime singularity theorems of Hawking and Penrose [5]. The mathematical validity of these theorems is highly reliable because Penrose have won his arguments in mathematics against the theoretical physicist, E. M. Lifshitz [8] in a well-known long dispute. Accordingly, the problem should be in the common physical assumption of these theorems. Moreover, the static Einstein equation has passed various tests with surprises. Thus, the problem seems to be on the dynamic cases of the Einstein equation. This is related to the energy conditions of these theorems. These singularity theorems [5] are listed as the following:

Theorem 1. Let (M, g_{ab}) be a globally hyperbolic spacetime with $R_{ab}\xi^a\xi^b \geq 0$ for all timelike ξ^a , which will be the case if Einstein equation is satisfied with the

strong energy condition holding for matter. Suppose there exists a smooth (or at least C^2) spacelike Cauchy surface Σ for which the trace of the extrinsic curvature (for the past directed normal geodesic congruence) satisfies $0 > C \geq K$ everywhere C is a constant. Then no past directed timelike curve from Σ can have length greater than $3/|C|$. In particular, all past directed timelike geodesic are incomplete.

Theorem 2. Let (M, g_{ab}) be a strongly causal spacetime with $R_{ab}\xi^a\xi^b \geq 0$ for all timelike ξ^a , as will be the case if Einstein's equation is satisfied with the strong energy condition holding for matter. Suppose there exists a compact, edgeless, achronal smooth spacelike hypersurface S such that for the past directed normal geodesic congruence from S we have $0 > K$ everywhere on S . Let C denote the maximum value for K , so $0 > C \geq K$ everywhere on S . Then at least one inextendible past directed timelike geodesic from S has length no greater than $3/|C|$.

Theorem 3. Let (M, g_{ab}) be a connected, globally hyperbolic spacetime with a noncompact Cauchy surface Σ . Suppose $R_{ab}k^ak^b \geq 0$ for all null k^a , as will be the case if (M, g_{ab}) is a solution of Einstein's equation with matter satisfying the weak or strong energy condition. Suppose, further, that M contains a trapped surface T . Let $0 > \theta_0$ denote the maximum value of θ for both sets of orthogonal geodesic on T . Then at least one inextendible future directed orthogonal null geodesic from T has affine length no greater than $2/|\theta_0|$.

Theorem 4. Suppose a spacetime (M, g_{ab}) satisfies the following four conditions. (1) $R_{ab}v^av^b \geq 0$ for all timelike and null v^a , as will be the case if Einstein's equation is satisfied with the strong energy condition holding for matter. (2) The timelike and null generic conditions are satisfied. (3) No closed timelike curve exists. (4) At least one of the three properties holds: (a) (M, g_{ab}) possesses a compact achronal set without edge [i.e., (M, g_{ab}) is a closed universe], (b) (M, g_{ab}) possesses a trapped surface, or (c) there exists a point $p \in M$ such that the expansion of the future (or past) directed null geodesics emanating from p becomes negative along each geodesic in this congruence. Then (M, g_{ab}) must contain at least one incomplete timelike or null geodesic.

The energy condition is related to the energy-momentum tensor T_{ab} . According to the Einstein equation [2]

$$G_{ab} \equiv R_{ab} - (1/2)g_{ab}R = 4\pi T_{ab},$$

one would have

$$R_{ab} = 8\pi [T_{ab} - (1/2)g_{ab}T] \quad \text{where} \quad T = g^{ab}T_{ab} \quad (25)$$

Then,

$$R_{ab}\xi^a\xi^b = 8\pi [T_{ab} - (1/2)g_{ab}T]\xi^a\xi^b = 8\pi [T_{ab}\xi^a\xi^b + (1/2)T],$$

$$\text{for a unit timelike } \xi^a \quad (26)$$

It is believed that for all physically reasonable classical matter the energy condition is non-negative, i.e.,

$$T_{ab}\xi^a\xi^b \geq 0 \quad (27)$$

for all timelike ξ^a . This assumption is known as the weak energy condition. However, it also seems physically reasonable that the stress of matter will not become so large and negative as to make the right-hand side of eq. (3) negative. This assumption,

$$T_{ab}\xi^a\xi^b \geq -(1/2)T \quad (28)$$

for all unit timelike unit vector ξ^a , is known as the strong energy condition. *An implicit assumption of these energy-conditions (26)-(28) is that all the coupling constants have the same sign.*

However, it has been shown that this implicit assumption of unique sign for all the coupling constants is invalid in physics because it leads to the non-existence of dynamic solutions for the problem of photons and the dynamic case of massive sources. Although even Einstein admitted that Hawking's theory is based on general relativity, it is based on an oversight error of Einstein, $E = mc^2$. This explains why Hawking has no verified correct predictions.

Thus, an assumption that restricts to only bounded solutions would exclude all the dynamic solutions. This is why the positive energy theorem of Scheon and Yau [63] is invalid and misleading. However, the Fields Medal was awarded to Yau (1982) and Witten (1990) for their work including the misleading theorem. This is due to that mathematicians *such as Atiyah⁽⁷⁶⁾ and Faddeev⁽⁷⁷⁾, do not understand physics* [64]. Also, since the singularity theorems are irrelevant in physics, the claims of an expanding universe and the existence of black holes need new justifications.

It is interesting that Hawking is the only theorist who has no known verified correct prediction in his life. We are not sure that Hawking can be called a physicist because he failed to see that a physical solution must be bounded. In my opinion, he is essentially a mathematician, but does not understand physics.

X. PROBLEMS IN NEWTONIAN GRAVITY

Newton's inverse-square law of gravitation is the oldest standing mathematical description of a fundamental interaction. Experimental tests of gravity's distance-dependence define a frontier between our understanding of gravity and many proposed forms of new physics. As gravity is $\sim 10^{40}$ times weaker than electromagnetism, gravity remains hidden by experimental backgrounds at distances smaller than the diameter of a fine human hair. The recent talk of Charles Hagedorn [65] surveys the past, present, and near-future of the experimental field, with substantial

emphasis on precision sub-millimeter laboratory experiments. However, Hagedorn did not know as most of the APS members, the crucial fact that the measured weight of testing matter actually depends on its temperature [41].

Although Faller [66] is aware of that error budgets in the measurements of the Newtonian coupling constant are fundamentally flawed because they cannot make allowances for error sources that have not been thought of. However, he did not know that the measurements to obtain the Big G coupling constant could not be accurate due to ignorance on the influence of heat to weight [41]. Thus, the Newtonian coupling obtained by J. Luo (罗俊) is questionable [67].

Since the measured Newtonian gravity is actually temperature dependent [41], in principle, the temperature dependence must be understood before an accurate test of Newton's inverse square law. This would also mean that Newtonian gravity can be understood only if the repulsive gravity has been understood.

Einstein did not see that for the dynamic case, the Einstein equation does not have any bounded solution. Thus, the linearization actually cannot be executed, and the "linearized" equation is an unrelated equation. *Therefore, the Newtonian gravity has not been superseded yet.* Einstein was puzzled [53] why his equation does not produce the gravitational wave solution. This is due to that Einstein like most physicists incorrectly assumed that a physical solution always exists, once an accurate result has been obtained.

One might argue that the temperature dependence of gravity is expected since an increase of temperature means the increase of energy. The problem is, however, that an increase of temperature leads to a reduction of weight [41].¹⁸⁾

XI. THE REPULSIVE GRAVITATION AND THE NECESSITY OF EXTENDING GENERAL RELATIVITY

Newton and Einstein give an over-simplified picture for gravitation. In particular, the gravitational effects of the electromagnetic energy were neglected by Einstein. They overlooked the repulsive gravitation due to the mass-charge interaction and the attractive gravitation due to the current-mass interaction [7].

Due to theoretical errors, essentially nothing has been done on the energy of electromagnetism until 1997 [12]. Now, let us reexamine again the Reissner-Nordstrom metric [30] (with $c = 1$) for a particle P as follows:

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right) dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2, \quad (8)$$

where q and M are the charge and mass of a particle, and r is the radial distance from the particle center. In metric (8), the gravitational components generated by electricity have not only a very different radial coordinate dependence but also a different sign that makes it a new repulsive gravity in general relativity [7].

Nevertheless, theorists such as Herrera, Santos, & Skea [68] argued that M in (8) involves the electric energy. Then they obtained a metric that would imply a charged ball would increase its weight as the charge q increased [7]. However, this is in disagreement with experiments [27]. Nevertheless, they are not alone. For instance, Nobel Laureates 't Hooft [69]¹⁹⁾ and Wilczek [70]²⁰⁾ also have mistaken that $m = E/c^2$ was universally true.

On the other hand, if the mass M is the inertial mass of the particle, the weight of a charged metal ball can be reduced [7]. Thus, experiments on two metal balls [27] supports that the mass does not include electric energy since a charged ball has a reduced weight. This is an experimental direct proof that the electric energy is not equivalent to mass. It will be shown that such a force leads to the necessity to extend the theoretical framework of general relativity.

To see the necessity of extending relativity, we consider the force on a test particle with mass m , and

$$\frac{d^2 x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0,$$

$$\text{where } \Gamma^\mu_{\alpha\beta} = (\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta}) g^{\mu\nu} / 2 \quad (29)$$

and $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$, according to Einstein. Note, the gauge affects only the second order approximation of g_{tt} [55].

Let us consider only the static case. For a test particle p with mass m at $\mathbf{r}$, the force on p is

$$-m \frac{M}{r^2} + m \frac{q^2}{r^3} \quad (30)$$

in the first order approximation because $g^{rr} \cong -1$. Thus, the second term is a repulsive force.

If the particles are at rest, then the force acting on the charged particle P has the same magnitude

$$\left(m \frac{M}{r^2} - m \frac{q^2}{r^3}\right) \hat{\mathbf{r}}, \text{ where } \hat{\mathbf{r}} \text{ is a unit vector} \quad (31)$$

because the action and reaction forces are equal and in the opposite directions. However, for the motion of the charged particle with mass M , if one calculates the metric according to the particle p of mass m , only the first term is obtained.

Then, it is necessary to have a repulsive force with the coupling q^2 to the charged particle P in a gravitational field generated by mass m . Thus, force (31) to particle P is beyond the framework of gravitation +

electromagnetism. As predicted by Lo, Goldstein, & Napier [37], general relativity would lead to the necessity of its extension.

The repulsive force in (8) comes from the electric energy [7]. An immediate question would be whether such a charge-mass repulsive force mq^2/r^3 is subjected to electromagnetic screening. Physically, this force, being independent of a charge sign, should not be subjected to such a screening. However, it could be according to Maxwell.

Note that this force can be considered as a result of q^2 interacting with a field created by the mass m . Thus such a field is independent of electromagnetism and is beyond general relativity, and the need of unification is established. To test such a possibility, one can measure whether there is such a repulsive force outside a charged capacitor.²¹⁾ When the charged particle P is moved, one must consider also the force due to attractive current-mass interaction.

The existence of such a current-mass attractive force has been verified by Martin Tajmar and Clovis de Matos [35].²²⁾ Moreover, such interaction could be identified as the cause for the anomaly of flybys.

$$\frac{d}{ds} \left(g_{ik} \frac{dx^k}{ds} \right) = \frac{1}{2} \frac{\partial g_{kl}}{\partial x^i} \frac{dx^k}{ds} \frac{dx^l}{ds} + \left(\frac{\partial g_{5k}}{\partial x^i} - \frac{\partial g_{5l}}{\partial x^k} \right) \frac{dx^5}{ds} \frac{dx^k}{ds} - \Gamma_{i,55} \frac{dx^5}{ds} \frac{dx^5}{ds} - g_{i5} \frac{d^2 x^5}{ds^2} \quad (32a),$$

$$\frac{d}{ds} \left(g_{5k} \frac{dx^k}{ds} + \frac{1}{2} g_{55} \frac{dx^5}{ds} \right) = \Gamma_{k,55} \frac{dx^5}{ds} \frac{dx^k}{ds} - \frac{1}{2} g_{55} \frac{d^2 x^5}{ds^2} + \frac{1}{2} \frac{\partial g_{kl}}{\partial x^5} \frac{dx^l}{ds} \frac{dx^k}{ds}, \quad (32b)$$

where $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$, $\mu, \nu = 0, 1, 2, 3, 5$ ($d\tau^2 = g_{kl} dx^k dx^l$; $k, l = 0, 1, 2, 3$).

If instead of ds , $d\tau$ is used in (32), for a test particle with charge q and mass M , the Lorentz force suggests

$$\frac{q}{Mc^2} \left(\frac{\partial A_i}{\partial x^k} - \frac{\partial A_k}{\partial x^i} \right) = \left(\frac{\partial g_{i5}}{\partial x^k} - \frac{\partial g_{k5}}{\partial x^i} \right) \frac{dx^5}{d\tau}. \quad (33a)$$

Thus,

$$\frac{dx^5}{d\tau} = \frac{q}{Mc^2} \frac{1}{K}, \quad K \left(\frac{\partial A_i}{\partial x^k} - \frac{\partial A_k}{\partial x^i} \right) = \left(\frac{\partial g_{i5}}{\partial x^k} - \frac{\partial g_{k5}}{\partial x^i} \right)$$

and

$$\frac{d^2 x^5}{d\tau^2} = 0 \quad (33b)$$

where K is a constant. It thus follows that (32) is reduced to

$$\frac{d}{d\tau} \left(g_{ik} \frac{dx^k}{d\tau} \right) = \frac{1}{2} \frac{\partial g_{kl}}{\partial x^i} \frac{dx^k}{d\tau} \frac{dx^l}{d\tau} + \left(\frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k} \right) \frac{q}{Mc^2} \frac{dx^k}{d\tau} - \Gamma_{i,55} \left(\frac{q}{Mc^2} \right)^2 \frac{1}{K^2}, \quad (34a)$$

$$\frac{d}{d\tau} \left(g_{5k} \frac{dx^k}{d\tau} + \frac{1}{2} g_{55} \frac{q}{KMc^2} \right) = \Gamma_{k,55} \frac{q}{KMc^2} \frac{dx^k}{d\tau} + \frac{1}{2} \frac{\partial g_{kl}}{\partial x^5} \frac{dx^l}{d\tau} \frac{dx^k}{d\tau}. \quad (34b)$$

One may ask what the physical meaning of the fifth dimension is. Our position is that the physical meaning of the fifth dimension is not yet very clear [37], except some physical meaning is given in the equation, $dx^5/d\tau = q/Mc^2 K$ where M and q are respectively the

In short, there are three factors that determine the weight of matter. They are; 1) the mass of the matter; 2) the charge-mass repulsive force; and 3) the attractive current-mass force. For a piece of a heated-up metal, the current-mass attractive force due to orbital electrons is reduced, but the charge-mass repulsive force would increase. Therefore, a net result is a reduction of weight [7] instead of increased weight as Einstein predicted. Thus, to test the inverse square law accurately, one must know exactly how temperature affects the weight.

XII. EINSTEIN'S CONJECTURE OF UNIFICATION AND THE FIVE-DIMENSIONAL RELATIVITY

The coupling with q^2 leads to a five-dimensional space of Lo et al. [37] since such a coupling does not exist in a four-dimensional theory, the five dimensional theories of Kaluza [71] or Einstein & Pauli [72].

Now let us give a brief introduction of the five-dimensional relativity. The five dimensional geodesic of a particle is

mass and charge of a test particle, and K is a constant. We shall denote the fifth axis as the w -axis. Our approach is to find out the full physical meaning of the w -axis as our understanding gets deeper.

For a static case, we have the forces on the charged particle P in the ρ -direction

$$-\frac{mM}{\rho^2} \approx \frac{Mc^2}{2} \frac{\partial g_{tt}}{\partial \rho} \frac{dct}{d\tau} \frac{d\tau}{d\tau} g^{\rho\rho}, \quad \text{and} \quad \frac{mq^2}{\rho^3} \approx -\Gamma_{\rho,55} \frac{1}{K^2} \frac{q^2}{Mc^2} g^{\rho\rho} \quad (35a)$$

and

$$\Gamma_{k,55} \frac{q}{KMc^2} \frac{dx^k}{d\tau} = 0, \quad \text{where} \quad \Gamma_{k,55} \equiv \frac{\partial g_{k5}}{\partial x^5} - \frac{1}{2} \frac{\partial g_{55}}{\partial x^k} = -\frac{1}{2} \frac{\partial g_{55}}{\partial x^k} \quad (35b)$$

in the $(-r)$ -direction. The meaning of (35b) is the energy momentum conservation. Thus,

$$g_{tt} = 1 - \frac{2m}{\rho c^2}, \quad \text{and} \quad g_{55} = \frac{mMc^2}{\rho^2} K^2 + \text{constant} \quad (\text{or} \quad \frac{1}{MK^2 c^2} g_{55} = \frac{m}{\rho^2} + \text{constant.}) \quad (36)$$

In other words, g_{55} is a repulsive potential, and g_{55}/M is also a function of a distance mass m .

On the other hand, because g_{55} is independent of q , this force would penetrate electromagnetic screening. From the above, it is also possible that a charge-mass repulsive potential would exist for a metric based on the mass M of the charged particle P . However, because p is neutral, there is no charge-mass repulsion force (from $\Gamma_{k,55}$) on p .

Thus, general relativity must be extended to accommodate the charge-mass interaction, as Einstein conjectured. For this, a five-dimensional relativity is a natural candidate. According to Lo et al. [37], the charge-mass interaction would penetrate a charged capacitor. To verify the five-dimensional theory, one can simply test the repulsive force on a charged capacitor. This has been experimentally confirmed [7]. On the other hand, from four-dimensional theory, we would not get a repulsive force acting on a test massive particle outside a capacitor since the electromagnetic field would be screened out. Thus, one may expect that there are surprises in five-dimensional theory.

However, journals such as the Physical Review and Proceedings of the Royal Society A, still have not recognized these important experiments due to inadequacy in nonlinear mathematics and blind faith toward Einstein. They all, like Hawking, believed in the invalid speculation $E = mc^2$.

XIII. APPLICATIONS OF THE CHARGE-MASS REPULSIVE FORCE AND ANOMALY OF THE SPACE PROBES

The Reissner-Nordstrom metric was first published in 1916, the same year that first paper on general relativity was published. Thus, the repulsive charge-mass interaction should have been discovered shortly afterward. However, this was not recognized until 1997 [36], because Einstein and his followers believed in his invalid speculation of $E = mc^2$.

However, the existence of repulsive gravitation was inadvertently verified by the charged metal ball

experiment [39] in 2005. One may ask whether the repulsive gravitation has some effects on astrophysics. In addition to the temperature and the composition of the test particle, the gravity also has some issues related to the sun.

Note that, the calculation of metric (8) is essentially based on general relativity. However, it is important to see this is crucial to establish a charge-mass repulsive force, which is independent of electromagnetism.

Then, the charge-mass repulsive force between a point charge q and a point mass m is

$$F = \frac{q^2 m}{r^3} \quad (37)$$

in the r -direction. The five-dimensional theory supports that it is not subjected to electromagnetic screening, and this is supported by the experiment of weighing charged capacitors [52] because a concentration of charges would provide such repulsion. This new force is different from Newtonian attractive force, which is inversely proportional to the square of the distance. Thus such a repulsive force would become weak faster than gravity at long distance.

Due to such a force, a capacitor lifter hovers on earth only in a limited height [50]. Note that the lifter does not need a continuation of power supply, and it is essentially due to the positive and negative static charges. This provides a theoretical basis for the reported phenomena that some monks can hover above the earth. Similarly, in the Chinese martial arts, there are speculations of the exceptional ability of high jump and walking on top of water and snow. Now, these are actually possible in terms of the law of physics although how these could be done is not yet clear. Previously, such exceptional abilities were simply disregarded as miracle since it would be against the law of physics.

The space probes also give a good opportunity to check the mass-charge interaction. If the repulsive force comes from the sun, then m in (37) would be m_p , the mass of the pioneer, and distance r would be R the distance between the sun and the space probe.

However, the charge term is not clear since for the sun we do not know what the non-linear term for the charge square should be.

Nevertheless, since such forces act essentially in the same direction, we could use a parameter P_s to represent the collective effect of the charges. Then, the effective repulsive force F_p would be ²³⁾

$$F_p = \frac{P_s m_p}{R^3}. \quad (38)$$

Since the neutral sun emits light and is in an excited state, the sun has many locally charged particles, whose effect exceed the attractive effects of motion of charge and thus P_s is not negligible. If the data fits well with a parameter P_s , then this would be a confirmation of the charge-mass interaction.

Since this force is much smaller than the gravitational force from the sun, in practice the existence of such a repulsive force would result in a very slightly smaller mass M_{ss} for the sun of mass M_s , i.e.

$$F = \frac{M_s m_p}{R^2} - \frac{P_s m_p}{R^3} \quad (39a)$$

and

$$\frac{M_s m_p}{R_0^2} - \frac{P_s m_p}{R_0^3} = \frac{M_{ss} m_p}{R_0^2} \quad (39b)$$

where R_0 is the distance from earth to the sun. Then, we have

$$F = \frac{M_{ss} m_p}{R^2} + \frac{P_s m_p}{R^2} \left(\frac{1}{R_0} - \frac{1}{R} \right) \quad (40)$$

Thus, it appears that there is an additional attractive force for $R \gg R_0$.

Moreover, such a force would not be noticeable from a closed orbit since the variation of the distance from the sun is small. However, for open orbits of the pioneers, there are great variations. When the distance is very large, the repulsive force becomes negligible, and thus an additional attractive force would appear as the anomaly. Such a force would appear as a constant over a not too long distance. Thus, the repulsive fifth force seems to be the only force that satisfies the overall requirements from the data [53]. However, this problem does not affect the gravity of the moon where there is no charged particles, thus the orbits around the moon are reliable.

Some claimed that the Pioneer Space-Probe Anomaly has been resolved by a heat-radiation model. However, a discoverer of the anomaly, Erik Anderson (April 1, 2011 at 12:57) commented, "...Science will have suffered the worst sort of dysfunction if the Pioneer Anomaly gets swept under the convenient rug of 'the plausible.' Even so, we will still have the Earth flyby anomalies and the so-called 'A.U.' anomaly left uncovered. All three anomalies seem to be

manifestations of a singular phenomenon — the latter two cannot be dismissed as heat radiation. Heat-radiation models, like string theory, can be customized to fit any set of observational parameters. There is no limit on sophistication. We should not be so easily impressed. Nothing has been resolved." I would like or add also that there is no evidence that can justify a heat-radiation model. It seems such modeling reflex only a blind faith on Einstein.

When the four planetary probes experienced unaccountable changes in velocity as they passed Earth, they experienced an additional repulsive force from the Earth because the core of the globe has charged currents. Moreover, depending on the way of approaching the globe, a planetary probe would also experience an additional attractive force due to current-mass interaction. Thus, a planetary probe would have an additional acceleration or de-acceleration. These cannot be modeled with a heat-radiation model.²⁴⁾

Therefore, there are two forces acting on a planet, one attractive and another repulsive with different strengths and distance dependencies. It is possible that these forces would have an effect on the spins of the planets. A speculation is that such a coupling would supply the energy that heats up planets internally. Current explanations for such heat as being due to radiation decay are not satisfactory since there has been no radioactive material discovered from volcanoes. Thus, a new area for experimental and theoretical development of the charge-mass interaction and higher dimensional unification are opened for physicists to explore. Now, fundamental physics will be more alive again.

XIV. INVALIDITY OF THE COVARIANCE PRINCIPLE

Because of the Bianchi Identity, the Einstein equation requires an additional condition to make the solution definite. Such a condition is called the gauge condition. The commonly used gauges are the harmonic gauge, the isotropic gauge, the Schwarzschild gauge, etc. The covariance principle states that all the gauges are invariance in physics. However, as Zhou Pei Yuan [73] pointed out, the covariance principle leads to "the concept that coordinates don't matter in the interpretation of Einstein's theory ... and necessarily leads to mathematical results which can hardly have a physical interpretation and are therefore a mystification of the theory." Thus, Zhou [73] proposed the harmonic gauge. It will be shown that this covariance principle is invalid in physics through explicit examples.

Einstein's principle of general relativity states "The law of physics must be of such a nature that they apply to systems of reference in any kind of motion. Einstein later extended this principle to unrestricted mathematical covariance and called it as the "principle

of covariance" [2]. He stated, "The general laws of nature are to be expressed by equations which hold good for all systems of co-ordinates, that is, are co-variant with respect to any substitutions whatever (generally co-variant)." However, the gauge conditions such as the harmonic gauge are known to be not a tensor condition. Thus, validity of this principle needs to be tested and verified.

A simple test of this principle is possible due to the fact that there are different gauges such as the harmonic, the isotropic and the Schwarzschild gauges for a given frame of reference. Since a physical quantity in a classical theory is uniquely determined by measurement, any physical quantity should be gauge invariant among those gauges of a given frame. Based on that both the Schwarzschild and the harmonic solution produced the same first order deflection of a light ray, Einstein [74] remarked, "It should be noted that this result, also, of the theory is not influenced by our arbitrary choice of a system of coordinates." However, such a claim is premature because this gauge invariance should have been supported by all physical quantities in all orders.

In Einstein's arguments for this principle, he emphasized that a physical theory is about the coincidences of the space-time points, but the meaning of measurements is crucially omitted [2]. However, to describe events, one must be able to relate events of different locations in a definite manner. Eddington [75] commented, "space is not a lot of points close together; it is a lot of distances interlocked." Moreover, as pointed out by Morrison, the "covariance principle" is invalid because it disrupts the necessary physical continuity from special relativity to general relativity [76, 77]. In fact, Einstein's "principle of covariance" has no theoretical basis in physics or observational support beyond allowed by the principle of general relativity [76].

Nevertheless, many still believe in this invalid "covariance principle", in part, because gauge invariance has a long history starting from classical electrodynamics. Later, the notion of gauge invariance

$$\alpha(b) = \frac{4GM}{c^2 b} \left[1 + \frac{15\pi}{16} \frac{GM}{c^2 b} \right] + O\left(\frac{m^3}{b^3}\right),$$

Here, $\kappa = G/c^2 = 7.425 \times 10^{-29} \text{cmg}^{-1}$, M is the total mass, " r_0 " is the closest (Euclidean-like) distance from the center of the sun to the light ray, and " b " is the impact parameter of the light ray. For the first order, as Einstein, we obtained,

$$\alpha(b) \sim 4m/b \sim 4m/r_0 \quad \text{since } A(r) \sim D(r) \sim 1. \quad (42b)$$

Thus, $\alpha(b)$ is gauge invariant. However, the impact parameter b can be measured only indirectly.

$$\xi = \rho \sin\theta \cos\phi, \quad \zeta = \rho \sin\theta \sin\phi, \quad \eta = \rho \cos\theta, \quad \text{and} \quad \rho > M\kappa. \quad (43b)$$

$$ds^2 = [(1 - M\kappa/2r)^2 / (1 + M\kappa/2r)^2] c^2 dt^2 - (1 + M\kappa/2r)^4 (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) \quad (44a)$$

has been developed to non-Abelian gauge theories such as the Yang-Mills theory [78].²⁵⁾ However, electromagnetisms are actually not gauge invariance since the electromagnetic potentials actually are physically effective, as Aharonov & Bohm [79] pointed out. In fact, all the physical Yang-Mills theories are not gauge invariant (Appendix C) since the invariance must be broken such that masses can be created for Fermions. These facts support the view that gauge invariance in a theory would be a manifestation that there are some deficiencies in the physical theory [80, 81].

Thus, it is clear that C. N. Yang does not understand gauge theory as Salam and Weinberg [82] pointed out. In fact his misunderstanding of gauge started from their paper on the gauge invariance in their paper on Yang-m-Mills theory. A consequence is that Yang believed in Einstein's covariance principle, and thus wrongly against Zou Pei-Yuan. This could be a main reason that Zhou's rectification of general relativity was not well accepted (see Appendix B).

a) Invalidity of covariance principle and the bending of light

It has been shown by Bodenner & Will [83] and Gérard & Piereaux [84] that the deflection angle is gauge invariant to the second order. However, the calculation of the deflection to the second order actually manifests intrinsic gauge non-invariance. They are based on the assumption that the metric for a spherical coordinate system takes the form,

$$ds^2 = A^2(r) c^2 dt^2 - B^2(r) dr^2 - D(r)^2 r^2 (d\theta^2 + \sin^2\theta d\phi^2), \quad (41)$$

This coordinate system is attached to a static Sun with spherical symmetry mass distribution. In fact, the spherical coordinate system (r, θ, ϕ) is necessarily defined on the Euclidean-like structure [85, 86] of the frame of reference.

From metric (41), for a weak field, the deflection angle is derived [83, 84] as

$$\text{where } b = \frac{D(r_0)}{A(r_0)} r_0 \quad \text{and} \quad m = \frac{GM}{c^2} \quad (42a)$$

To see the invariant b , let us consider the harmonic, the isotropic and the Schwarzschild metrics as follows:

$$ds^2 = \frac{\rho - M\kappa}{\rho + M\kappa} c^2 dt^2 - \frac{\rho + M\kappa}{\rho - M\kappa} d\rho^2 - (\rho + M\kappa)^2 (d\theta^2 + \sin^2\theta d\phi^2), \quad (43a)$$

where

$$\xi = \rho \sin\theta \cos\phi, \quad \zeta = \rho \sin\theta \sin\phi, \quad \eta = \rho \cos\theta, \quad \text{and} \quad \rho > M\kappa. \quad (43b)$$

$$ds^2 = [(1 - M\kappa/2r)^2 / (1 + M\kappa/2r)^2] c^2 dt^2 - (1 + M\kappa/2r)^4 (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) \quad (44a)$$

where

$$x = r \sin \theta \cos \phi, \quad y = r \sin \theta \sin \phi, \quad z = r \cos \theta, \quad \text{and} \quad r > M\kappa \quad (44b)$$

$$ds^2 = (1 - 2M\kappa/r')c^2 dt^2 - (1 - 2M\kappa/r')^{-1} dr'^2 - r'^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (45a)$$

where

$$x' = r' \sin \theta \cos \phi, \quad y' = r' \sin \theta \sin \phi, \quad z' = r' \cos \theta \quad \text{and} \quad r' > 2M\kappa. \quad (45b)$$

Since ds^2 is gauge invariant, from (43) and (45) one gets $[(\rho + M\kappa)/(\rho - M\kappa)]d\rho^2 = (1 - 2M\kappa/r')^{-1}dr'^2$. It follows that

$$\frac{\rho - M\kappa}{\rho + M\kappa} = (1 - 2M\kappa/r'), \quad (\rho + M\kappa)^2 = r'^2, \quad \text{and} \quad r' = \rho + M\kappa, \quad \text{for} \quad r' > 2M\kappa \quad (46a)$$

Similarly, from (44) and (45) one gets $(1 + M\kappa/2r)^4 dr^2 = (1 - 2M\kappa/r')^{-1}dr'^2$. It follows that

$$(1 - M\kappa/2r)^2/(1 + M\kappa/2r)^2 = (1 - 2M\kappa/r'), \quad (1 + M\kappa/2r)^4 r^2 = r'^2, \quad \text{and} \quad r' = r(1 + M\kappa/2r)^2 \text{ for } r' > 2M\kappa. \quad (46b)$$

Therefore, equations (46a) and (46b) show that the shortest distance from the sun center ρ_0 , r_0 , or r'_0 in terms of the Euclidean-like structure of the respective gauge are different. Thus, these structures are different for different gauges.

The first order of deflections are the same for the harmonic and the Schwarzschild gauge because $4m/\rho_0 \sim 4m/r'_0$ to the first order, according to eq. (46a). Accordingly, the shortest distance from the sun is a

$$b_s = \frac{D(d_s)}{A(d_s)} d_s = \frac{d_s}{(1 - 2m/d_s)^{1/2}} = d_h \frac{(1 + m/d_h)}{(1 - m/d_h)} = \frac{D(d_h)}{A(d_h)} d_h = b_h \quad (47a)$$

and

$$b_s = \frac{D(d_s)}{A(d_s)} d_s = \frac{d_s}{(1 - 2m/d_s)^{1/2}} = d_l \frac{(1 + m/2d_l)}{(1 - m/2d_l)} = \frac{D(d_l)}{A(d_l)} d_l = b_l \quad (47b)$$

Then, b is the same for all three gauges. In terms of the shortest distance, the deflection angle is respectively,

$$\alpha(b) = \frac{4m}{d_s} \left[1 + \left(\frac{15\pi}{16} - 1 \right) \frac{m}{d_s} \right] + O\left(\frac{m^3}{d_s^3} \right), \quad b = \frac{D(d_s)}{A(d_s)} d_s = \frac{d_s}{(1 - 2m/d_s)^{1/2}} \quad (48a)$$

$$\alpha(b) = \frac{4m}{d_h} \left[1 + \left(\frac{15\pi}{16} - 2 \right) \frac{m}{d_h} \right] + O\left(\frac{m^3}{d_h^3} \right), \quad b = \frac{D(d_h)}{A(d_h)} d_h = d_h \frac{(1 + m/d_h)}{(1 - m/d_h)} \quad (48b)$$

$$\alpha(b) = \frac{4m}{d_l} \left[1 + \left(\frac{15\pi}{16} - 2 \right) \frac{m}{d_l} \right] + O\left(\frac{m^3}{d_l^3} \right), \quad b = \frac{D(d_l)}{A(d_l)} d_l = d_l \frac{(1 + m/2d_l)}{(1 - m/2d_l)}. \quad (48c)$$

When the closest distance r_0 is obtained from a measurement, the relation between the distance r_0 and b would be

$$b \approx m + r_0, \quad (49a)$$

or

$$b \approx 2m + r_0 \quad (49b)$$

depending on what gauge (Schwarzschild, isotropic or harmonic) is valid in physics. Accordingly, one has to decide first what the actual gauge is in reality. Thus, this gauge invariance has no meaning in physics unless the first order of the metric is known that (49a) or (49b) can

physical quantity although it is not gauge invariant. Einstein's approach is necessary since the actual physical gauge in physics is not known.

If b_s , b_h and b_l respectively denote the impact parameters and if d_s , d_h , and d_l are respectively denote the closest distance (in their respective Euclidean-like structure [83, 84]) for different gauges, then according to eq. (42a), we have,

be chosen. This example shows that Zhou Pei-Yuan is right.

Some incorrectly claimed that " r_0 " is only an arbitrary label. However, according to Einstein [2, 74], r_0 is not arbitrary. Bodenner & Will [83] argued that b is the ratio of angular momentum J to energy E for the photon ($J = bE$). However, to measure J , similar problems would occur [87]. Thus, they do not have a valid argument.

Thus, general relativity is intrinsically not gauge invariant since the shortest distance from the sun center is gauge dependent although the impact parameter b is

the same. This is consistent with Einstein's equivalence principle, which requires the physical gauge being unique for a given frame [88] since measurements should be unique in a given frame.

Moreover, invalidity of the gauge invariance can be seen explicitly from the light speeds as follows:

$$\frac{dr'}{dt} = c \left(1 - \frac{2M\kappa}{r'} \right) = \frac{d\rho}{dt} = \frac{dr}{dt} \left(1 - \frac{M\kappa}{2r} \right) \left(1 + \frac{M\kappa}{2r} \right), \quad (50a)$$

$$\frac{r' d\theta}{dt} = c \left(1 - \frac{2M\kappa}{r'} \right)^{\frac{1}{2}} = \frac{\rho d\theta}{dt} \left(1 + \frac{M\kappa}{\rho} \right) = \frac{rd\theta}{dt} \left(1 + \frac{M\kappa}{2r} \right)^2, \quad (50b)$$

$$\sin \theta \frac{r' d\phi}{dt} = c \left(1 - \frac{2M\kappa}{r'} \right)^{\frac{1}{2}} = \sin \theta \frac{\rho d\phi}{dt} \left(1 + \frac{M\kappa}{\rho} \right) = \sin \theta \frac{rd\phi}{dt} \left(1 + \frac{M\kappa}{2r} \right)^2. \quad (50c)$$

Thus, the light speeds ($ds^2 = 0$), calculated from the Schwarzschild solution, the harmonic solution, and the isotropic solution, are explicitly not invariant although they are diffeomorphic solutions. Perhaps due to such a violation, some theorists [89] insist that the measured light speed in vacuum is always c although Einstein has remarked that the light speeds are no longer a constant since a light ray bends [2, 74]. This illustrates how the so-called experts change Einstein's

theory to cover up their errors. To determine the physical valid gauge, one may use Zhou's experiment [90].

b) Invalidity of Covariance Principle and the de Sitter Precession Formulas

Since r_0 is not gauge invariant, there should be other examples of such a violation. For instance, from the Maxwell-Newton Approximation [42, 44], one would obtain a formula for the *de Sitter* precession [89] as follows:

$$\frac{d\vec{S}}{d\tau} = -2(\vec{v} \cdot \vec{S})\vec{\nabla}\Phi + v(\vec{S} \cdot \vec{\nabla}\Phi) + \vec{S}(\vec{v} \cdot \vec{\nabla}\Phi) = \vec{v} \times (\vec{S} \times \vec{\nabla}\Phi) + \vec{S} \times (\vec{v} \times \vec{\nabla}\Phi), \text{ where } \phi = -\kappa M/r \quad (51a)$$

$\vec{v}$ is the velocity of the gyroscope, and $\vec{S}$ is the spin.

However, from the Kerr metric, one would have a formula [91] as follows:

$$\frac{d\vec{S}}{d\tau} = -3(\vec{v} \cdot \vec{S})\vec{\nabla}\Phi + 3\hat{r}(\vec{S} \cdot \hat{r})(\vec{v} \cdot \vec{\nabla}\Phi), \quad (51b)$$

where $\hat{r}$ is the unit vector in the r -direction.

For a circular orbit, the situation in Gravity Probe-B, since $(\vec{v} \cdot \vec{\nabla}\Phi) = 0$, from (51a) and (51b) we have

$$\frac{d\vec{S}}{d\tau} = -2(\vec{v} \cdot \vec{S})\vec{\nabla}\Phi + v(\vec{S} \cdot \vec{\nabla}\Phi) \quad (51a')$$

and

$$\frac{d\vec{S}}{d\tau} = -3(\vec{v} \cdot \vec{S})\vec{\nabla}\Phi, \quad (51b')$$

respectively. However, although they are different, it is difficult to distinguish them experimentally. Since the time average of $(\vec{S} \cdot \vec{v}) \nabla\Phi + v(\vec{S} \cdot \vec{\nabla}\Phi)$ is zero, there is no difference because the experiment measures only the time average.

Invalidity of the covariance principle implies that Einstein's theory is incomplete. Fortunately, it has been proven, *independent of the Einstein equation*, that the Maxwell-Newton Approximation, which uses the harmonic gauge, is valid for the first order approximation of a physical gauge for gravity induced by massive

sources [92] such that the binary pulsar experiment can be explained satisfactorily [42, 44]. This implies that the second order of the deflection angle can be obtained from measuring the shortest distance r_0 from the sun center since $b \approx 2m + r_0$ is valid. Clearly r_0 is not an arbitrary label although the Royal Society is correct in finding out inconsistency in Einstein's theory [52].

Looking back, one may wonder why many tried so hard to justify Einstein's covariance principle. Remember, Einstein justified his theory of measurement in his initial paper on general relativity [2] as if a natural consequence of extending special Relativity [2, 74]. Therefore, many believed such a theory could not be incorrect. Also, nobody asked why special relativity which has nothing to do with gravitation, leads to the need of a different measurement? However, Einstein also had to create the covariance principle to remedy the related problems. Thus, invalidity of this principle would imply also invalidity of his theory of measurement. *Recently, it has been found that Einstein's justification is actually based on invalid applications of special relativity [93]. Then, it would become clearer to many that his covariance principle can be incorrect. The popularity of this principle shows that many do not understand physics.*

A remaining problem is what the correct gauge is? For this we have no answer yet. However, Zhou has proposed the harmonic gauge, which is consistent with the Maxwell=Newton Approximation. Maybe Zhou is right on this too.

XV. THE EQUIVALENCE PRINCIPLE AND ITS MISINTERPRETATIONS

General relativity was started with the equivalence principle. Then, for massive sources, the Einstein equation was derived by extending the Newtonian equation for gravity [2]. However, it has been shown that Einstein missed the gravitational energy-stress tensor with an anti-gravitational coupling, and thus there is no dynamic solution. For the source of an electromagnetic wave, the Einstein equation is very different, and an anti-gravitational coupling for the photonic energy-stress tensor is necessary. Moreover, the covariance principle has been proven invalid by explicit examples. We shall show that Einstein's equivalence principle is supported by experiments, and thus valid.

Unfortunately, although many agree that Einstein's equivalence principle is the foundation of general relativity, there is no book or reference, other than Einstein's own work, that explains this principle correctly [88, 94]. Many failed to see the physical contents of Einstein's equivalence principle; and some even confused this principle with Einstein's invalid 1911 assumption of equivalence [95]. Here, we clarify first what his 1911 assumption is.

In 1911 Einstein assumed the equivalence of a uniformly accelerated system K' and a stationary system of coordinates K with a uniform Newtonian gravitational potential ϕ . Currently many assume the Newtonian metric form,

$$d\tau^2 = (1 + 2\phi) dt^2 - dx^2 - dy^2 - dz^2, \quad (52)$$

that later Fock [56] has proved to be impossible. From this metric (1), Einstein derived the gravitational redshifts, but an incorrect light velocity that leads to only one half of the observed light bending angle [96].

In 1916, however, Einstein assumed the equivalence of a uniformly accelerated system K' and a stationary system of coordinate K with an *unspecified* metric form that generates a uniform gravitation. In his book, Einstein [74] wrote:

"Let now K be an inertial system. Masses which are sufficiently far from each other and from other bodies are then, with respect to K , free from acceleration. We shall also refer these masses to a system of co-ordinates K' , uniformly accelerated with respect to K . Relatively to K' all the masses have equal and parallel accelerations; with respect to K' they behave just as if a gravitational field were present and K' were unaccelerated. Overlooking for the present the question as to the "cause" of such a gravitational field, which will occupy us later, there is nothing to prevent our conceiving this gravitational field as real, that is, the conception that K' is 'at rest' and a gravitational field is present we may

consider as equivalent to the conception that only K is an 'allowable' system of co-ordinates and no gravitational field is present. The assumption of the complete physical equivalence of the systems of coordinates, K and K' , we call the "principle of equivalence;" this principle is evidently intimately connected with the law of the equality between the inert and the gravitational mass, and signifies an extension of the principle of relativity to coordinate systems which are non-uniform motion relatively to each other."

In this statement, the gravitational mass is defined without considering the repulsive gravitation. However, the press release of the 1993 Nobel Committee [48], claimed the equivalence principle as the identity between gravitational and inertial mass (due to Newton and Galileo). This is incorrect because of the existence of repulsive gravity. Later, Einstein made clear that a gravitational field is generated from a space-time metric, but is not a Newtonian potential. (However, the latter was not explicitly stated.) Moreover, concurrent with Einstein's equivalence principle of 1916, Einstein makes the claim of the Einstein-Minkowski condition as a consequence [2].

A problem is that since Einstein did not provide an example to illustrate his equivalence principle, a careless reader could mistake the 1911 assumption of equivalence as the 1916 equivalence principle. It is not until 2007 that a metric for uniform gravity [88] was published as follows:

$$ds^2 = (c^2 - 2U) dt'^2 - (1 - 2U/c^2)^{-1} dx'^2 - (dy'^2 + dz'^2), \quad (53)$$

where $c^2/2 > U(x', t') = (at)^2/2$, " a " is the acceleration of system $K'(x' y' z')$ with respect to $K(x, y, z, t)$ in the x -direction. Metric (53) shows the time dilation and space contractions clearly. Here, dt' is defined locally by $cdt' = cdt - (at/c)dx'[1 - (at/c)^2]^{-1}$. Moreover, metric (2) is equivalent to the metric

$$ds^2 = (c^2 - a^2 t^2) dt^2 - 2at dt dx - dx^2 - (dy^2 + dz^2) \quad (53')$$

that was derived by Tolman [96]. It was a surprise that U is actually time dependent, and this explains the earlier failed derivation of such a metric [97]. Now, clearly the 1916 principle is different from the 1911 assumption.

To avoid the usual association of an elevator with the gravity of Earth, the equivalence of accelerated frame and uniform gravity is best described, as Einstein did, in terms of a uniformly accelerated chest [98]. Nevertheless, due to the popular "Einstein's elevator" of Bergmann [99], Einstein was often falsely accused of ignoring the tidal force [14].⁵⁾

To illustrate the equivalence principle further, consider a disk K' uniformly rotating w. r. t. an inertial system (x, y, z, t) , a metric for the disk of space $K'(x', y', z')$ is derived [100]. According to Landau & Lifshitz [101], the metric is

$$ds^2 = (c^2 - \Omega^2 r^2) dt^2 - 2\Omega r^2 d\phi dt - dr^2 - r^2 d\phi^2 - dz^2, \quad (54)$$

where Ω is an angular velocity relative to an inertial system $K(x, y, z, t)$, z and z' coincide with the rotating axis, and $r^2 = x^2 + y^2 = x'^2 + y'^2$. Metric (54) is equivalent to its canonical form,

$$ds^2 = (c^2 - \Omega^2 r'^2) dt'^2 - dr'^2 - (1 - \Omega^2 r'^2/c^2)^{-1} r'^2 d\phi'^2 - dz^2, \quad (54'a)$$

where

$$cdt' = cdt - (r\Omega/c)rd\phi[1 - (r\Omega/c)^2]^{-1}. \quad (54'b)$$

Then it is clear that the local light speed cannot be larger than c . However, (54'b) is not integrable [100] because local time dt' is related to different inertial systems at different r or time t . Thus, to obtain the correct space contractions, one must first transform the metric to a canonical form such that the space contractions are clear.

The fact that the local time t' is not a global time was a problem that leads to the rejection by the editorial of the Royal Society [100]. This rejection is incorrect since validity of metric (3') can be derived theoretically with special relativity. Experimentally, the time dilation from metric (54'a) for the local metric, $ds^2 = c^2 dT^2 - dX^2 - dY^2 - dz^2$, is

$$dT = [1 - (r\Omega/c)^2]^{1/2} dt'. \quad (54'c)$$

From (54'b) the local clock resting at K' , if observed from K , would have

$$dt' = dt. \text{ and } dT = [1 - (r\Omega/c)^2]^{1/2} dt. \quad (54'd)$$

Moreover, as Kundig [102] has shown, the time dilation (54'd) is valid for a local clock fixed at K' . Hence, Einstein's equivalence principle has experimental supports. Therefore, the 1993 Nobel Committee press release should not frivolously reject this principle; especially since it was done implicitly.

An earlier source of confusion is that Pauli's invalid version [103] has been mistaken as Einstein's equivalence principle although Einstein has made clear it is a misinterpretation [104]. Since Pauli was an outstanding physicist, and was often critical to theoretical errors, many still rely on his version, instead of the necessary supporting evidences.

For instance, in the book "Gravitation" [30] of Misner, Thorne and Wheeler, there is no reference to Einstein's equivalence principle (i. e. [2] and [74]). Instead, they misleadingly refer to Einstein's invalid 1911 assumption [95] and Pauli's invalid version [103]. Like Pauli, they also did not refer to the related mathematical theorems [105]. In addition, as shown in their Eq. (40.14), they even failed to understand the local time of a particle at free fall [30], a basic of general relativity. Due to their influence, Einstein's equivalence principle was often mistakenly regarded the same as the invalid 1911 assumption. The failure of understanding Einstein's equivalence principle is a source of current errors.²⁶⁾

Note that since the 1911 assumption has been proven invalid by observations in 1919, that Fock [56] misidentified it in 1955 as Einstein's equivalence principle of 1916, is *beyond just incompetence but a deliberate unethical distortion to discredit Einstein*. Unfortunately, many universities, research institutes, as well as the 1993 Nobel Committee are victims of such a distortion. This illustrates that a human folly can happen to Sciences, not just politics.

Moreover, many cannot tell the difference between the principle of 1916 and the assumption of 1911 [88, 106-108]. Einstein's equivalence principle is generally valid because a uniform gravity in the equivalence principle is generated by acceleration but not mass. However, experiments on the equivalence of inertial mass and gravitational mass have not been updated beyond the case when the mass-charge interaction is absent [109].

The mathematical theorems related to Einstein's equivalence principle are as follows:

Theorem 1. Given any point P in any Lorentz manifold (whose metric signature is the same as a Minkowski space) there always exist coordinate systems (x^μ) in which $\partial g_{\mu\nu}/\partial x^\lambda = 0$ at P .

Theorem 2. Given any time-like geodesic curve Γ there always exists a coordinate system (the so-called Fermi coordinates) (x^μ) in which $\partial g_{\mu\nu}/\partial x^\lambda = 0$ along Γ .

In these theorems, the local space of a particle is locally constant, but not necessarily Minkowski.

However, after some algebra, a local Minkowski metric exists at any given point and along any time-like geodesic curve Γ . In a uniformly accelerated frame, the local space in a free fall is a Minkowski space according to special relativity. What Einstein added to these theorems is that physically such a locally constant metric must be Minkowski. Such a condition is needed for the case of special relativity [88, 94]. This is also the theoretical basis of the Einstein-Minkowski condition that Einstein uses to derive the bending of light rays and the gravitational redshifts [2, 74].

Thus, Pauli's version [103] is a simplified but corrupted version of these theorems as follows:

"For every infinitely small world region (i.e. a world region which is so small that the space- and time-variation of gravity can be neglected in it) there always exists a coordinate system $K_0(X_1, X_2, X_3, X_4)$ in which gravitation has no influence either in the motion of particles or any physical process."

Pauli regards the equivalence principle as merely the existence of locally constant spaces. Then, Pauli's version is only a corrupted mathematical statement which may not be physically realizable because of the theorems.

A crucial error is that Pauli extended the removal of uniform gravity to the removal of gravity in a small region. This is simply incorrect in mathematics.

Because he does not understand mathematical analysis, he did not recognize that the removal of gravity in a small region, no matter how small, would be very different from a removal of gravity at one point. The correct statement should replace "no influence" with "approximately little influence". Then, the removal of gravity would be limited to essentially an isolated point as the mathematical theorems allow.

Moreover, Pauli [103], and Will [109, 110], overlooked Einstein's [2] remark, "For it is clear that, e.g., the gravitational field generated by a material point in its environment certainly cannot be 'transformed away' by any choice of the system of coordinates..." Apparently, neither Pauli [103] nor the Wheeler School [5, 30, 89, 110] understands the mathematics of the above theorems [105]. Misner et al. [5] claimed that Einstein's equivalence principle is as follows: -

"In any and every local Lorentz frame, anywhere and anytime in the universe, all the (non-gravitational) laws of physics must take on their familiar special-relativistic form. Equivalently, there is no way, by experiments confined to infinitesimally small regions of spacetime, to distinguish one local Lorentz frame in one region of spacetime frame from any other local Lorentz frame in the same or any other region."

They even claimed this as the Einstein's principle in its strongest form. The truth is, however, this version makes essentially another form of the misinterpretation of Pauli [103]. They do not understand the related mathematics [105], and their followers have similar problems. *This version of the Wheeler School combines errors of Pauli and the 1911 assumption, but ignores the Einstein-Minkowski condition that is the physical essence of Einstein's principle.*

In fact, their phrase, "must take on" should be changed to "must take on approximately". The phrase, "experiments confined to infinitesimally small regions of spacetime" does not make sense since experiments can be conducted only in a finite region. Moreover, in their eq. (40.14) they got an incorrect local time of the earth, in disagreement with Einstein. Thus, clearly these three theorists [30] failed to understand Einstein's equivalence principle [2, 74].

Furthermore, Thorne [8] criticized Einstein's principle with his own distortion²⁷⁾ as follows:

"In deducing his principle of equivalence, Einstein ignored tidal gravitational forces; he pretended they do not exist. Einstein justified ignoring tidal forces by imagining that you (and your reference frame) are very small."

However, Einstein has already explained these problems in his letter of 12 July 1953 to Rehtz [104] as follows:

"The equivalence principle does not assert that every gravitational field (e.g., the one associated with the Earth) can be produced by acceleration of the

coordinate system. It only asserts that the qualities of physical space, as they present themselves from an accelerated coordinate system, represent a special case of the gravitational field."

Perhaps, Thorne did not know that *the term "Einstein elevator" of Bergmann [99] is misleading.*

As Einstein [104] explained to Laue, "What characterizes the existence of a gravitational field, from the empirical standpoint, is the non-vanishing of the Γ^l_{jk} (field strength), not the non-vanishing of the R_{iklm} ," and no gravity is a special case of gravity. This allows Einstein to conclude that the geodesic equation is also the equation of motion of a massive particle under gravity, which made it possible to conceive a field equation for the metric.

Although Einstein's equivalence principle was clearly illustrated only recently [88, 94, 111], the Wheeler School should bear the responsibility of their misinformation on this principle [30] by ignoring both crucial work of Einstein, i.e., references [2] and [74], and related theorems [105], and giving an invalid version of such a principle. *A main problem is that the Einstein-Minkowski condition [2, 74], which plays a crucial role in measurement, is eliminated.* As shown by Zhou [73, 90], Einstein's equivalence principle is actually inconsistent with his covariance principle.

Einstein [2, 74] uses the satisfaction of his equivalence principle as an assumption to calculate the bending of light in the harmonic and the Schwarzschild gauges. Thus, the satisfactory of the equivalence principle is crucial that the time dilation and space contractions are unique. These errors are responsible for the mistakes in the press release of the 1993 Nobel Committee who was unaware of the non-existence of dynamic solutions and the experimental supports to Einstein's equivalence principle.

XVI. MEASUREMENTS AND EINSTEIN'S INVALID APPLICATIONS OF SPECIAL RELATIVITY

The exposition of general relativity started by considering a system of rotating coordinates [2]. However, few know that Einstein's errors started from there. Specifically, Einstein considered an inertial system of reference $K(x, y, z, ct)$ and a system $K'(x', y', z')$ in a uniform rotation Ω relatively to K . The origins of both systems and their z - and z' -axes (the axis of rotation) coincide. The flat metric of K is

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - dz^2$$

where $x = r \cos \phi$, and $y = r \sin \phi$, (55)

in the cylindrical coordinate system. The metric for $K'(x', y', z')$ has the following form:⁵⁴⁾

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \text{ where } dx^\mu = dx', dy', dz', cdt', \quad (56)$$

and dt' is the local time. For reasons of symmetry (i.e., based on the principle of causality¹⁵), a circle with center at the origin in the x - y plane of K may at the same time be regarded as a circle in the x' - y' plane of K' .

Einstein argued that if the circle is measured from K' because of Lorentz contraction, the circumference would be greater than $2\pi r'$ although the so measured radius $r' = r$. Moreover, based on special relativity, Einstein [2] claimed,

"An observer at the common origin of co-ordinates, capable of observing the clock at the circumference by means of light, would therefore see it lagging behind the clock beside him ... So, he will be obliged to define time in such a way that the rate of a clock depends upon where the clock may be."

Thus, Einstein defined a physical space-time coordinate system together with a metric that relates to local clock rates and local spatial measurements. Subsequently, Einstein concluded

"We therefore reach this result: In the general theory of relativity, space and time cannot be defined in such a way that differences of the spatial coordinates can be directly measured by the unit measuring-rod, or differences in the time co-ordinate by a standard clock."

However, Whitehead [112] considered this conclusion to be unacceptable. Moreover, Einstein continued, "The method hitherto employed for laying co-ordinate into the space and time continuum in a definite manner thus breaks down, and there seems to be no other way which would allow us to adapt systems of coordinates to the four-dimensional universe so that we might expect from their application a particularly simple formulation of the laws of nature. So there is nothing for it but to regard all imaginable systems of coordinates, on principle, as equally suitable for the description of nature. This comes to requiring that the general laws of nature are to be expressed by equations which hold good for all systems of coordinates, that is, are covariant with respect to any substitutions whatever (generally covariant)."

Nevertheless, many (including this author) had failed to see that his arguments are actually invalid.

To see his errors, consider a particle P resting at $K'(r', \phi', z', ct')$. The local space of P is $L^*(dr, dX, dz', cdT)$ with a Minkowski metric. In K , P has a position (r, ϕ, z) and its local space $(dr, rd\phi, dz, cdt)$ has the Minkowski metric. These two local spaces have a relative velocity $r\Omega$ in the ϕ -direction. Here X has the same direction as $rd\phi$.

From this example, we can show further that Einstein's justifications for his theory of measurement^{1,2} are invalid. According to special relativity, one has $dz = dz'$ and $dr = dR$, and the Lorentz transformations are as follows:

$$rd\phi = [1 - (r\Omega/c)^2]^{-1/2} [dX + r\Omega dT], \quad (57a)$$

and

$$cdt = [1 - (r\Omega/c)^2]^{-1/2} [cdT + (r\Omega/c)dX]; \quad (57b)$$

or

$$dX = [1 - (r\Omega/c)^2]^{-1/2} [rd\phi - r\Omega dt], \quad (58a)$$

and

$$dT = [1 - (r\Omega/c)^2]^{-1/2} [dt - (r\Omega/c^2) rd\phi]. \quad (58b)$$

It follows that if dX is measured simultaneously (i.e., $dt = 0$) from K , then from Eq. (58a) one has

$$dX = [1 - (r\Omega/c)^2]^{-1/2} [rd\phi]. \quad (59a)$$

This is a space contraction for L^* ($dX > r d\phi$). For a clock fixed at L^* (i.e., $dX = 0$), from Eq. (57b) we have

$$cdT = [1 - (r\Omega/c)^2]^{1/2} cdt \quad (59b)$$

if measured from K . This is a time dilation for L^* ($dt > dT$).

From Eq. (59a), Einstein concluded that $U/D > \pi$, where D is the diameter of a circle and U is its circumference. Since all the measurements in Eq. (59a) are done in K , Einstein has mistakenly considered that the integration

$$U = (1/2)[1 - (D\Omega/2c)^2]^{-1/2} \oint Dd\phi = \pi D[1 - (D\Omega/2c)^2]^{-1/2} \quad (60)$$

is valid. *The error is that the distance dX in Eq. (59a) is in a local space L^* , and all L^* s are under different accelerations.* Moreover, the space K is in a relative motion with respect to K_- . Space contractions and the time dilation are incompatible since the space S and such a local space L are at rest with each other. Thus, Eqs. (59a) and (59b) actually have nothing to do with Einstein's equivalence principle.

In other words, for this case, Einstein's claims for space contractions and the time dilation are supported with invalid arguments. Therefore, to clarify the issue of measurements, one should derive a space-time metric and show that such a metric satisfies Einstein's equivalence principle (see also Appendix D). For the case of a rotating disk, the transformation to a uniformly rotating reference frame $K'(x', y', z')$ with angular velocity Ω has the form

$$x = x' \cos \Omega t - y' \sin \Omega t, \quad y = x' \sin \Omega t + y' \cos \Omega t, \quad \text{and } z = z', \quad (61a)$$

or

$$r = r', \quad z = z', \quad \text{and } \phi = \phi' + \Omega t \quad (61b)$$

Then a metric in terms of the coordinates in $K'(x', y', z')$ can be obtained from (B7b); and

$$dr = dr', \quad dz = dz', \quad \text{and } d\phi = d\phi' + \Omega dt. \quad (61c)$$

The transformed metric in system $K'^*(x', y', z', ct)$ would then have the following form,

$$ds^2 = (c^2 - \Omega^2 r^2) dt^2 - 2\Omega r^2 d\phi' dt - dr^2 - r^2 d\phi'^2 - dz'^2, \quad (62)$$

$$g^{ct ct} = 1, \quad g^{r' r'} = g^{z' z'} = -1, \quad g^{\phi' \phi'} = -(1 - \Omega^2 r'^2/c^2)/r'^2,$$

and

$$g^{\phi' ct} = g^{ct \phi'} = -\Omega/c \quad (63)$$

are the non-zero elements of the inverse metric. The force acting on particle P at rest with mass m is then mv^2/r' . Moreover, Eq. (61a) implies that

$$r' = r, \quad x' = r \cos \phi', \quad \text{and} \quad y' = r \sin \phi', \quad (64)$$

Thus, (64) means that $K'(x', y' z')$ also has a Euclidean-like structure. Therefore, Einstein's claim is incorrect.

The metric (62) could have led to the "light speed" $rd\phi'/dt$ larger than c. To rectify this situation, one must have a metric with the local time t' of K' . Now, consider the local space L^* from Eqs. (58a), (58b), and (61c). We have

$$dX = [1 - (r\Omega/c)^2]^{-1/2} rd\phi', \quad (65a)$$

and

$$dT = [1 - (r\Omega/c)^2]^{1/2} \{dt - [1 - (r\Omega/c)^2]^{-1} (r\Omega/c^2) rd\phi'\} \quad (65b)$$

Then

$$ds^2 = (c^2 - \Omega^2 r^2) \{dt - [1 - (r\Omega/c)^2]^{-1} (r\Omega/c^2) rd\phi'\}^2 - dr^2 - [1 - (r\Omega/c)^2]^{-1} r^2 d\phi'^2 - dz'^2. \quad (65c)$$

Note that the space L^* is the local space of the Einstein-Minkowski condition. Consequently, we should have

$$ds^2 = g_{t't'} c^2 dt'^2 - dr'^2 - (1 - \Omega^2 r'^2/c^2)^{-1} r'^2 d\phi'^2 - dz'^2. \quad (66)$$

Now, (65a) implies that the metric has space contractions. According to Landau & Lifshitz⁶⁸, we should have

$$ds^2 = (c^2 - \Omega^2 r'^2) dt'^2 - dr'^2 - (1 - \Omega^2 r'^2/c^2)^{-1} r'^2 d\phi'^2 - dz'^2. \quad (67)$$

where

$$cdt' = cdt - (r\Omega/c)rd\phi'[1 - (r\Omega/c)^2]^{-1},$$

$$\text{and} \quad dT = [1 - (r\Omega/c)^2]^{1/2} dt' \quad (68)$$

Eq. (B1468), which is different from (59b), implies that for a local clock fixed at K' an observer at K would have

$$dt' = dt. \quad (69a)$$

Thus, Einstein's claim on this time dilation is clearly invalid. Moreover, as Kundig⁶⁹ has shown, the time dilation (68) is valid. For a local clock fixed at K , however, an observer at K' would have $dt' = [1 - (r\Omega/c)^2]^{-1} dt$.

Moreover, since $r = r'$, (65a) and (59a) imply

$$rd\phi[1 - (r\Omega/c)^2]^{-1/2} = dX = r'd\phi'[1 - (r\Omega/c)^2]^{-1/2},$$

$$\text{and} \quad rd\phi = r'd\phi' \quad (69b)$$

Thus, Einstein's claim of $U/D > \pi$ is also invalid. And $[1 - (r\Omega/c)^2]^{-1/2} rd\phi'$ is a distance measured in the system L^* .

Note that metric (62) and canonical metric (67) are related to each other by the relations (61c). However, in (68), dt' is related to dT of the local systems $L^*(dR, dX, dz', cdT)$ at different t, r , and ϕ' ; and from (68) we have

$$cdt' = cdt - (r\Omega/c)rd\phi'[1 - (r\Omega/c)^2]^{-1}. \quad (70)$$

Thus Eq. (70) is not integrable. Nevertheless, the Einstein-Minkowski condition is satisfied.

Therefore, the Euclidean-like structure (64) is physically realizable in terms of measurements.

Einstein stated that the light speed is measured "in the sense of Euclidean geometry,"² and all of Einstein's predictions are in terms of the Euclidean-like structure. For instance, a ray of light, traveling the shortest distance Δ from the center of sun with mass M will be deflected by an amount $M\kappa/2\pi\Delta$.^{1,2} The secular rotation of the elliptic orbit of the planet has the same sense as the revolution of the planet, amounting in radians per revolution of $24\pi^3 a^2/(1 - e^2)c^2 T^2$. In addition to Δ , e the numerical eccentricity and a the semimajor axis of the planetary orbit in centimeters are defined in terms of the Euclidean-like structure, and T the period of revolution in seconds is defined in terms of the time of a quasi-Minkowskian space. Thus, Einstein's theory of measurement has not been used in his calculations. Therefore, the criticisms of Whitehead [112] to Einstein's theory are actually supported by Einstein's own calculations.

In short, the Euclidean-like structure is used for his measurements. *In other words, the physical Riemannian space with an Euclidean-like structure is different from a mathematical Riemannian space embedded in a Euclidean space.* Thus, the Riemannian space in physics can be different from a Riemannian space in mathematics.

XVII. THE QUESTION OF EXPANDING UNIVERSE

The space-time singularity theorems were used to justify the existence of the black holes and the big bang theory. The speculation of back holes is no longer valid because of the existence of repulsive gravitation and the Einstein equation is not valid for the dynamic problems. However, the speculation of big bang and the expanding universe remains. It will be shown there is no evidence for the expanding universe. The speculation of the receding galaxies as evidence is actually due to a misinterpretation of the coordinates.

Hubble's law is often considered as the observational evidence for an expanding universe. It will be shown that Hubble's Law need not be related to the notion of Doppler redshifts of the light from receding Galaxies. Moreover, this notion of receding velocity is

incompatible with the local light speeds used in deriving the light bending.

The notion of an expanding universe is based on an unverified assumption that a local distance in a physical space is similar to that of a mathematical Riemannian space embedded in a higher dimensional flat space. However, this assumption has been proven as theoretically invalid. In fact, a physical Riemannian space necessarily has a frame of reference, which has a Euclidean-like structure that is independent of the yet to be determined physical metric [85].

Hubble's law is often considered as a manifestation of the Doppler red shift of the light from the receding Galaxies [5]. Thus, the further a galaxy is from the Milky Way, the faster it appears to be receding. However, Hubble himself rejected this interpretation and concluded in 1936 that the Galaxies are actually stationary [113].

It should be pointed out, in the derivation of the receding velocity, an implicit assumption, which implies no expansion for the frame of reference, must be used [5]. Moreover, the receding velocity is incompatible with the light speeds used in deriving the light bending. Thus, the notion of expanding universe is a production due to confusing notion of the coordinates and also inadequate understanding of a physical space.

a) Invalid Interpretations of Hubble's Law

Hubble discovered from light emitted by near by galaxies that the redshifts S are linearly proportion to the present distance L from the Milky Way as follows:

$$S = H L \quad (71)$$

where H is the Hubble constant although the redshifts of distant galaxies will deviate from this linear law with a slightly different constant. In general relativity, it is known that this law can be derived with the following metric [5],

$$ds^2 = -d\tau^2 + a^2(\tau)\{dx^2 + dy^2 + dz^2\}, \quad (72)$$

since

$$S = \frac{\lambda_2 - \lambda_1}{\lambda_1} = \frac{\omega_1}{\omega_2} - 1 = \frac{a(\tau_2)}{a(\tau_1)} - 1, \quad (73)$$

where ω_1 is the frequency of a photon emitted at event P_1 at time τ_1 , and ω_2 is the frequency of the photon observed at P_2 at time τ_2 . Furthermore, for nearby galaxies, one has

$$a(\tau_2) \approx a(\tau_1) + (\tau_2 - \tau_1)\dot{a}. \quad (74)$$

If

$$(\tau_2 - \tau_1) = L = \int_1^2 \sqrt{dx^2 + dy^2 + dz^2} \quad (75)$$

$$ds^2 = -[(1 - M\kappa/2r)^2/(1 + M\kappa/2r)^2] c^2 dt^2 + (1 + M\kappa/2r)^4(dx^2 + dy^2 + dz^2) \quad (80)$$

where $\kappa = G/c^2$ ($G = 6.67 \times 10^{-8}$ erg cm/gm²), M is the total mass, and $r = \sqrt{x^2 + y^2 + z^2}$. Then, if the

then

$$S = \frac{\dot{a}}{a} L = H L, \quad \text{and} \quad H = \frac{\dot{a}}{a}. \quad (76)$$

Formula (75) is compatible with the calculation in the bending of light. Please note that Hubble's Law need not be related to the Doppler redshifts. Understandably, Hubble rejected such an interpretation himself [113]. In fact, there is actually no receding velocity since L is fixed (i.e., $dL/d\tau = 0$).

On the other hand, if one chooses to define the distance between two points as Riemannian geometry

$$R = \int_1^2 a(\tau) \sqrt{dx^2 + dy^2 + dz^2} = a(\tau)L, \quad (77)$$

Then

$$v = \frac{dR}{d\tau} = \frac{da}{d\tau} L + \frac{dL}{d\tau} a = \frac{da}{d\tau} \frac{R}{a} = HR,$$

$$\text{if} \quad \frac{dL}{d\tau} = 0. \quad (78)$$

Note that according to (77), (75) would have to change into $(\tau_2 - \tau_1) = R$, and (71) into $S = H R$. Thus,

$$v = S. \quad (79)$$

This means that the redshifts could be superficially considered as a Doppler effect. Thus, whether Hubble's Law represents the effects of an expanding universe is a matter of the interpretation of the local distance. From the above analysis, the crucial point is what a valid physical velocity in a physical space is.

It should be noted that $dL/d\tau = 0$ means that the space coordinates are independent of the metric. In other words, the physical space has a Euclidean-like structure [85]. However, since L between any two space-points is fixed, the notion of an expanding universe, if it means anything, is just an illusion. Moreover, as pointed out by Whitehead [112], the validity of (77) as the physical distance has no known experimental supports. Another problem is that the velocity in (78) would be incompatible with the light speeds in the calculation of light bending experiment.

Consider a spherical mass distribution with the center at the origin, the metric with the isotropic gauge is,

$$\frac{\sqrt{dx^2 + dy^2 + dz^2}}{dt} = c \frac{1 - M\kappa/2r}{(1 + M\kappa/2r)^3} \quad (81)$$

However, such a definition of light speeds is incompatible with the definition of velocity (78) although compatible with (75). Since this light speed is supported by observations, (78) is invalid in physics.

Nevertheless, Liu [114] has defined light speeds, which is more compatible with (78), as

$$\frac{\sqrt{g_{ij}dx^i dx^j}}{dt} = c \frac{1 - M\kappa/2r}{1 + M\kappa/2r} \quad (82)$$

for metric (80). However, (82) implies only half of the deflection implied by (81) [2, 74].

The above analysis also explains why many current theorists insist that the light speeds are not defined even though Einstein defined them clearly in his 1916 paper as well as in his book, "The Meaning of Relativity". They might argued that the light speeds are not well defined since diffeomorphic metrics give different sets of light speeds for the same frame of reference. However, they should note that Einstein defines light speeds after the assumption that his equivalence principle is satisfied [2, 74]. Different metric for the same frame of reference means only that at most only one of such metrics is physically valid, and therefore the definition of light speeds are uniquely well-defined.

However, since the problem of a physical valid metric has not been solved, whether a light speed is valid remains a question. Nevertheless, it has been proven that the Maxwell-Newton Approximation gives the valid first order approximation of the physical metric, the first order of the physically valid light speeds are solved [42]. Since metric (80) is compatible with the Maxwell-Newton approximation, the first order of light speed (72) is valid in physics.

Thus, the groundless speculation that local light speeds are not well defined is proven incorrect. In essence, the velocity definition (78), which leads to the notion of the Doppler redshifts, has been rejected by experiments.

b) A Possible Explanation of Hubble's Law

A major problem in Einstein's theory, as pointed out by Whitehead [112] and Fock [56], is that the physical meaning of coordinates is ambiguous and confusing. In view of this, it is understandable that the notion in an embedded Riemannian space is used when the physical nature of the problem is not yet clear. A major difference between physics and mathematics is that the coordinates in physics must have physical meaning. Since Einstein is not a mathematician, his natural step would be to utilize the existing theory of Riemannian space. However, as Whitehead [112] saw,

this created a seemingly irreconcilable problem between coordinates of a curved space-time and physics.

Then, one may ask if the observed gravitational redshifts is not due to an expanding universe, what causes such redshifts that are roughly proportional to the distances from the observer. One possibility is that the scatterings of a light ray along its path to the observer. In physics, it is known that different scatterings are common causes for losing energy of a particle, and for the case of photons it means redshifts. However, to test such a conjecture is impossible since current theory is incapable of handling the inelastic scatterings of lights because Einstein's equation even does not have any dynamic solution [17, 43]. Thus, to solve this puzzle rigorously seems surely in the remote future.

Nevertheless, the assumption that observed redshifts could be due to inelastic scatterings may help to explain some puzzles of observed facts. For instance, it is known that younger objects such as star forming galaxies have higher intrinsic redshifts, and objects with the same path length to the observer have much different redshifts while all parts of the object have about the same amount of redshifts [115]. For alternative cosmology theories, there are others [116].

An advantage of the Euclidean-like structure [85] is that notions used in a Euclidean space could be adapted much easier. Many things would be calculated as if in a Euclidean space as Einstein did, and physics should return to normal.

XVIII. DISCUSSIONS AND CONCLUSIONS

In this paper, we solved at least two puzzles 1) Why Hawking has no verified predictions in spite of that he follows Einstein faithfully. 2) Would the lack of breakthrough in physics imply there is no longer any genius? For the first question, it is simply that Hawking follows Einstein's major error $E = mc^2$ that Einstein had mistaken as valid. But, now it is clear that is invalid in theory as well as incorrect that have been proven by three types of experiments. For the second question, currently we do not need a genius, but only careful derivations and examinations that will discover existing errors and eventually lead us to a break through. In fact, too many have attempted to be a genius already.

A problem in general relativity is that people are impressed by Einstein's achievements, but failed to see that even as a genius he too can also make mistakes because Einstein is also a human being. Because of the general inadequacy in mathematics among physicists, they failed to see Einstein's error in mathematics. Moreover, they regarded a Nobel Prize as a certificate for correctness, and thus failed to see that such a prize could mean only partially correct. Also, because most physicists do not understand general relativity, they incorrectly regarded errors in the theory such as the space-time singularity theorems as new achievements.

They even support such errors with dubious evidence based on their misinterpretations. They also did not see that not all the neutral objects fall with the same acceleration.

Another problem is that many believed that general relativity can deal with only large scale problem. In particular, based on the erroneous space-time singularity theorems, Hawking and Penrose claimed that general relativity cannot deal with microscopic problems. This is completely nonsense since one must use general relativity to prove the existence of photons. Consequently, to verify general relativity, we look into the sky. This led to an unusual event that the verification of the gravitational waves without a valid Einstein equation that can generate them [54]. This not only makes such tests expensive but also ignores the possibility of experiments on earth.

For a long time nobody tests Einstein's claim that the weight of a piece of metal would increase after being heated-up. This led to the delay to verify and to confirm the repulsive gravitation, and thus the unification of gravitation and electromagnetism. In addition, editors such as Eric J. Weinberg of the Physical Review D, who does not understand general relativity, are against experiments to test Einstein's claims. The fact is, however, that gravity reduction not only depends on the temperature, but also on the metal used [21].

Currently, the most urgent task is to develop a field equation that can deal with the dynamic problems such as the two-body solution, the generation of gravitational waves, and the problem of repulsive gravitation. In particular, the repulsive gravitation is crucial for the unification of gravitation and electromagnetism. Moreover, since the charge-mass interaction is absent from quantum theory, Einstein is correct that it is not a final theory.

Now, we should have learned from Galileo, physics must be fully supported by real experiments. We must improve our mathematical skills. Above all, as philosopher Hu shih said, we must be careful in our proof even though we are allowed to have bold assumptions. It should be noted also that as experimental skills improved, we may find some of the old experimental tests such as those for Newtonian gravitation are actually inadequate. Since the repulsive gravity is discovered, the measurement of the mass based on gravitation is no longer valid.

However, many physicists just believed the "experts", who actually do not understand general relativity for various reasons. It is hoped that this paper would help our colleagues understanding of general relativity better.

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Appendix A: The Principle of Causality in Physics

There are two aspects in causality: its relevance and its time ordering. In time ordering, a cause event must happen before its effects. This is further restricted by relativistic causality that no cause event can propagate faster than the light speed in vacuum. The time-tested assumption that phenomena can be explained in terms of identifiable causes will be called the principle of causality. This is the basis of relevance for all scientific investigations. Thus, the principle of causality implies that any parameter in a solution for physics must be related to some physical causes. Moreover, the principle of causality implies a weak source would produce a weak gravity. Here this principle will be elucidated first in connection with symmetries of a field, the boundedness of a field solution, and the validity of a field equation.

In practice, when the considered field is absent, physical properties are ascribed to the space-time as in a "normal" state. For example, the electromagnetic field is zero in a normal state. Then, any deviation from the normal state must have physically identifiable causes. Thus, the principle of causality implies that the symmetry must be preserved if no cause breaks it. The implication of causality to symmetry has been used in deriving the inverse square law from Gauss's law. The normal state of a space-time metric is the flat metric in special relativity. Thus, if a metric does not possess a symmetry, then there must be physical cause(s) which has broken such a symmetry. For a spherically symmetric mass, causality requires that the metric is spherically symmetric and asymptotically flat. Also, a weak cause can lead to only weak gravity. Therefore, Einstein's notion of weak gravity is a consequence of the principle of causality.

However, the physical cause(s) should not be confused with the mathematical source term in the field equation. In general relativity, the cause of gravity is the physical matter itself, but not its energy tensors in the source term of Einstein's field equation. The energy-stress tensors (for example the perfect fluid model) may explicitly depend on the metric. Since nothing should be a cause of itself, such a source tensor does not represent the cause of a metric. For the accompanying gravitational wave of an electromagnetic wave, the physical cause is the electromagnetic wave. Thus, one should not infer the symmetries of the metric based on the source term instead of its causes.

Moreover, inferences based on the source term can be misleading because it may have higher symmetries than those of the cause and the metric. For instance, a transverse electromagnetic plane-wave is not rotationally invariant with respect to the z-direction of propagation. But the related electromagnetic energy-stress tensor component $T(E)_{tt}$ for a circularly polarized wave is. Such an assumption violates causality and results in theoretical difficulties.

Classical electrodynamics implies that the flat metric is an accurate approximation, caused by the presence of weak electromagnetic waves. This physical requirement is supported by the principle of causality which implies such a metric to be a bounded periodic function. However, this required boundedness is not satisfied by solutions in the literature [52, 117, 118]. These solutions also violate causality directly since they involve parameters without any physical cause [15].

They also do not satisfy the equivalence principle [119, 120] although they are Lorentz manifolds.

A necessary and sufficient condition for satisfying the equivalence principle is that a time-like geodesic represents a physical free falling; but this does

not mean the existence of Minkowski spaces in a neighborhood. Another problem in general relativity is that many theorists and journals do not understand physics, such as the principle of causality adequately. For instance, the Physical Review accepted unbounded solution as valid in physics. And the Royal Society (London) believed that Hawking is an outstanding physicist in the same status as Newton.

However, these journals are wrong, Hawking follows Einstein's error such as $E = mc^2$ and thus has no observationally confirmed prediction. Moreover, his space-time singularity theorems, which are based on implicit invalid assumptions in physics, are obviously against the principle of causality.

Appendix B: Influence of the Temperature of a Body on its Weight

Dmitriev, Nikushchenko, and Snegov [21] show that a brass metal rod heated by ultrasound confirms a dependence of the weight of the rods on their temperature. Their results can be shown by the following figures.

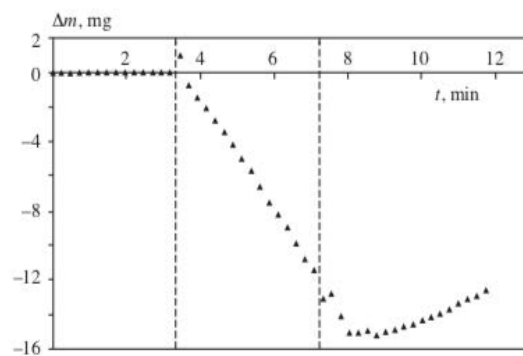


Figure 1: Change in mass of a brass rod mountend in an open holder. Ultrasound Frequency 131.25 kHz. The dashed lines indicate the moments when the ultrasound was switched on and off.

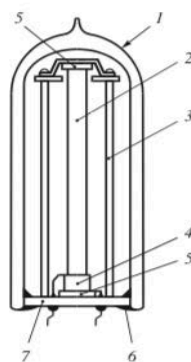


Figure 2: Arrangement of the air tight container: 1) Dewar vassel 2) metal rod 3) holder pillar(textolite cloth-based laminate) 4) piezoelectric transducer 5) foam plastic spacers 6) cold weld 7) holder base(ebonite)

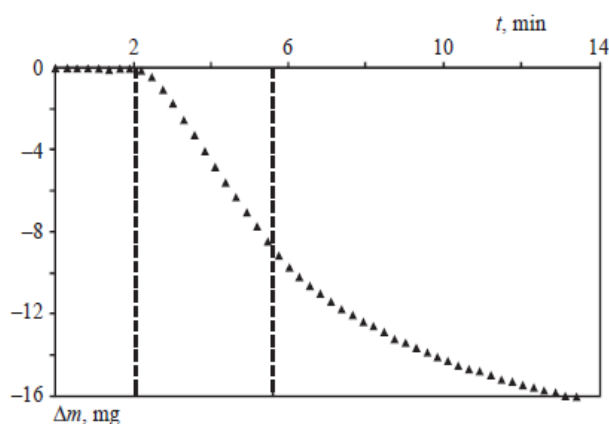


Figure 3: Arrangement of the air tight container: 1) Dewar vessel 2) metal rod 3) holder pillar(textolite cloth-based laminate) 4) piezoelectric transducer 5) foam plastic spacers 6) cold weld 7) holder base(ebonite)

Figure 1 is the change of weight for the brass rod mounted in an open holder. Figure 3 is for the arrangement in an air tight container. Figure 4 is the change of weight for the brass rod in a closed Dower vessel. The brass rod weighs 58.5 g before heated-up, with a length of 140.0 mm, and diameter of 8.0 mm. These figures show that the Dewar vessel is not essential for the weight reduction experiment.

It should be noted that the temperature dependence of gravity also depends on the metal involved. They have measured such dependence for the lead, Copper, Brass, and Duralumin, and find they are different. It would be interesting to find out the rule on such dependence.

A main problem is that they have not been able to correctly identify the cause of the weight reduction. They incorrectly regard the reduction of weight as a reduction of mass. Apparently, they do not understand that when the repulsive gravitation is present, to measure mass through gravitation is no longer valid.

Appendix C: Comments on C. N. Yang and Zhou Pei-Yuan

Nobel Prize has become a certificate for the truth because of the huge accompanying prestige. However, to determine whether a scientific work is actually related to the truth, is the supports of scientific evidence. For instance, Einstein obtained a Nobel Prize in Physics for his proposal of photons that are quantum of electromagnetic energy. However, later evidence shows that the photons include also gravitational energy in addition to electromagnetic energy because the electromagnetic energy is not equivalent to mass.

Thus, although a Nobel Prize in Physics is certainly an achievement, this could be only a partial truth. In other words, there could be errors even for a Nobel Prize. Therefore, a physicist should not be judged by the Prize alone. Besides, as a human institute, the Nobel Committee has also made mistakes. Therefore, if

a country cares about its scientific prestige, attention should be on the achievement rather than the Prize.

C. N. Yang is Nobel Laureate in physics, but in my opinion, his scientific achievement is inferior to another Chinese scientist Prof. Zhou Pei-Yuan of Peking University. Some people may object because Zhou has never gotten a Nobel Prize. However, this could be a problem of the Nobel Committee. In my opinion, whether one got a Prize are not the sole criteria for scientific excellence. It is more important whether a scientist can stand alone for the truth in front of an overwhelming objection, and/or his work would be delayed for a long time without him.

Einstein belongs to that category although he also made mistakes. It is very difficult to imagine general relativity without Einstein. Prof. Zhou has the same quality because in his 1983 paper he stood alone in front of an almost unified objection to point out that the Einstein's covariance principle is invalid. Although he did not provide a detailed proof, he is proven correct later in 2008 by concrete examples [87], 25 years later. In fact, Zhou studied general relativity for his Ph. D. in Caltech. In 1937, he had faced Einstein to discuss the problem of coordinates in general relativity. Einstein could not answer his questions on this issue. Besides, he is a well-known authority in fluid dynamics.

On the other hand, Yang is incorrectly convinced that Einstein's covariance principle was valid because of his misconception on gauge invariance. This is confirmed by Yang himself in our personal communications. Probably because of Yang's objection, Zhou's correct viewpoint was not well received in China.

However, since Yang is an author of the Yang-Mills theory, which starts the non-abelian gauge theory, many believed he should have no problem on the gauge invariance. Nevertheless, it should be noted that the notion of gauge invariance in their paper is invalid in physics. The gauge invariance simply does not make sense for particles with different masses. This is why

their theory had not produced anything meaningful until 1967 when the spontaneous broken symmetry is employed in the Weinberg's model [121].

It should be noted also almost exactly the same thesis of Yang-Mills was produced by Ron Shaw [122] in his Ph. D. thesis of 1955. Another major contribution of Yang is the Yang-Baxter equation, of which Yang published in 1967 and Baxter in 1972. In view of that the parity non-conservation was produced with T. D. Lee, it is safe to say Yang's work is good, but is not outstanding that cannot be produced for a long time without Yang.

It is on the above basis that I regard Zhou is a better scientist than Yang. Few nations have shown their anxiety for a genius more than the Chinese. Nevertheless, they failed to identify Zhou.

Endnotes

1. Einstein's errors were not generally accepted because the lack of clear evidence, in addition to the general inadequacy of mathematics among physicists, dominating misconceptions in physics and the mathematicians do not understand physics. However, there are three types of experiments that shows $E = mc^2$ is incorrect [29].
2. This π_0 meson experiment is also an evidence that a photon can be considered as a massless particle.
3. Since Einstein won a Nobel Prize for this work, many have incorrectly believed that the photons consist of only electromagnetic energy. Many even have incorrectly taken the Nobel Prize as the standard for truth. This also often leads to an over evaluation of the Prize and the opinions of Nobel Laureates in physics.
4. Some theorists often claimed a result without given one. For instance, Wald [5] claimed that he can obtain a bounded dynamic solution of second order approximation for the Einstein equation.
5. For a linear equation, this compatibility was not a problem, and thus some incorrectly believe this is automatic.
6. Equation (6) is a necessary extension of Einstein's general relativity that Einstein failed to see. A special feature of this equation is that an energy-stress tensor with an anti-gravity coupling must be included. It will be seen that such an anti-gravity coupling must be included for any dynamic case.
7. The purpose of general relativity is to establish that gravity must be propagated with a finite speed. Thus the existence of the gravitational wave is expected. However, Einstein failed to establish the need of a gravitational wave. Nevertheless, this was established in 2006 with a modified Einstein equation.
8. However, Tsipenyuk & Andreev was puzzled because they did not see the existence of a repulsive gravitation.
9. The weight reduction of a charged-capacity was considered as an experimental error, because it was believed that, according to Maxwell's theory, there is no electromagnetic force beyond a charged capacitor.
10. The discovery of the Euclidean-like structure in a physical space clarifies the difference between a physical Riemannian space and a mathematical Riemannian space embedded in a higher dimensional Euclidean space. This was the crucial point needed to settle the difference between Einstein and Whitehead [112].
11. Due to the existence of repulsive gravitation, Einstein no longer can claim that gravitational mass is equivalent to acceleration mass. This is why Einstein reject repulsive gravitation, in addition to his invalid belief on $E = mc^2$.
12. This leads to the settlement that the mass in this metric is just the acceleration mass without wrongly including the electromagnetic energy due to the charge. For this, even Nobel Laureate t' Hooft had mistaken.[69].
13. Note that Wheeler was the thesis advisor of D. Christodoulou. As we shall see that Wheeler's mathematics is erroneous even at the undergraduate level [50]
14. It has been derived from Einstein's equivalence principle that eq.(23) is valid [43].
15. Since the Einstein equation has no gravitational wave solutions, one may wonder how they do the calculation to compare with the data. It seems they simply use the linearized equation to do the calculation. Due to their inadequate mathematics, they are not even aware of this mistake. Thus, they "verified" the gravitational wave without a valid equation that generates such waves [54].
16. Michael Francis Atiyah has been leader of the Royal Society (1990-1995), master of Trinity College, Cambridge (1990-1997), chancellor of the University of Leicester (1995-2005), and President of the Royal Society of Edinburgh (2005-2008). Since 1997, he has been an honorary professor at the University of Edinburgh. However, Atiyah does not understand physics [64].
17. Ludwig D. Faddeev, the Chairman of the Fields Medal Committee, wrote ("On the work of Edward Witten"): "Now I turn to another beautiful result of Witten – proof of positivity of energy in Einstein's theory of gravitation. Hamiltonian approach to this theory proposed by Dirac in the beginning of the fifties and developed further by many people has led to the natural definition of energy. In this approach a metric γ and external curvature h on a space-like initial surface $S^{(3)}$ embedded in space-time $M^{(4)}$ are used as parameters in the corresponding phase space. These data are not independent. They satisfy Gauss-Codazzi

constraints – highly non-linear PDE, The energy H in the asymptotically flat case is given as an integral of indefinite quadratic form of $\nabla \gamma$ and h . Thus, it is not manifestly positive. The important statement that it is nevertheless positive may be proved only by taking into the account the constraints – a formidable problem solved by Yau and Schoen in the late seventy as Atiyah mentions, 'leading in part to Yau's Fields Medal at the Warsaw Congress'." Faddeev failed to see that the so-called 'natural definition of energy' actually excludes the dynamic cases by assuming all the dynamic solutions are bounded [64].

18. The reduction of weight while the temperature increase, is due to the increment of a repulsive gravitational force.
19. G. 't Hooft incorrectly believed that the mass of an electron includes its electric energy. This exposes that he does not understand Newtonian mechanics and also special relativity adequately.
20. Frank A. Wilczek incorrectly believed that $E = mc^2$ is unconditional [70]. Thus, their proof (Frank. A. Wilczek along with David Gross and H. David Politzer) for asymptotic freedom is actually incomplete.
21. Because this repulsive force is against Maxwell's theory, many disregard this repulsive force of gravity.
22. They failed to understand that the attractive gravitational force is due to current-mass interaction.
23. We assume that force from the current-mass interaction in the sun is comparatively very weak.
24. It is clear that there is much work on astrophysics to be done with a five-dimensional theory.
25. In the next year, the Ph. D. thesis of Ron Shaw [122] was produced with essentially the same content.
26. Pauli [103] and Misner et al. [30] also did not have adequate training in pure mathematics. The misinterpretation of Misner et al. [30] creates the so-called Lorentz invariance, being tested by Chu et al. [123].
27. This is due to Thorne's error. Since Thorne had won a Nobel Prize in 1975, his error could be mistaken as correct.
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