Bi-Directional Infinity Box

By Jayant Hooda

Abstract- The following article introduces a new concept of Bi-directional Box. The concept is explained by simultaneously calculating a new result: sum of all the natural multiples of each natural number upto infinity. The concept of Bi-directional Box helps to organise multiple infinite series and study different patterns in multiple infinite series. This Bi-directional box can be converted into a triangle by rearranging the already organised terms of the initial box. Similar to Pascal's triangle, this box has many patterns and properties instilled in it too. Along with the initial standard box, infinite such boxes can be made depending upon the sequence/series in which the pattern is to be observed: an example with different sequences is provided at the end of the article.

Keywords: *bi-directional infinite box, infinite series, sum of all natural multiples, pascal's triangle.*

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BIDIRECTIONALINFINITYBOX

Strictly as per the compliance and regulations of:

Bi-Directional Infinity Box

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Abstract- The following article introduces a new concept of Bi-directional Box. The concept is explained by simultaneously calculating a new result: sum of all the natural multiples of each natural number upto infinity. The concept of Bi-directional Box helps to organise multiple infinite series and study different patterns in multiple infinite series. This Bi-directional box can be converted into a triangle by rearranging the already organised terms of the initial box. Similar to Pascal's triangle, this box has many patterns and properties instilled in it too. Along with the initial standard box, infinite such boxes can be made depending upon the sequence/series in which the pattern is to be observed: an example with different sequences is provided at the end of the article.

Keywords: bi-directional infinite box, infinite series, sum of all natural multiples, pascal's triangle.

I. Result

The Sum of all multiples combined of every natural number upto infinity is 1/120. The series looks this:

$$(1+2+3+4\ldots) + (2+4+6+8\ldots) + (3+6+9+12\ldots) + (4+8+12+16\ldots) \ldots$$

In other words, the sum of infinite terms of a series, each of whose terms is further an infinite sum - the first term being the sum of all natural multiples of 1, second term being the sum of all natural multiple of 2, the third term being the sum of all natural multiple of 3, and so on - is 1/120.

To solve such kind of series and observe patterns in them *bi-directional infinity boxes* can be used.

To understand this particular series, first, we need to understand the concept of *bi-directional infinity box.*

(Col 1)

↓

Row 1: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ..., infinity
Row 2: 2, 4, 6, 8, 10, 12, ..., infinity
3, 6, ,9, 12, 15, 18, ..., infinity
4, 8, 12, 16, 20, 24, 28, ..., infinity
5, 10, 15, 20, 25, 30, 35, ..., infinity
6, 12, 18, 24, 30, 36, ..., infinity
.
.
.
Infinity

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Row 1 contains multiples of 1 upto infinity
Row 2 contains multiples of 2 upto infinity

And so on, neither the number of rows ends because we have to take multiple of each natural number upto infinity into consideration, nor the terms in a row ends because we have to take every multiple that exists(infinite) of each natural number into consideration. Thus, if considered as a box, and row 1 and column 1 as its sides, then we can call it a *bi-directional infinity box*(both sides extending to infinity).

In this way, we have organised all the multiples upto infinity of each natural number upto infinity.

This *bi-directional infinity box* can be rearranged into a triangle in 2 ways, one of which will help us to get our result.

(Col 1)

↓

Row 1:   1, 2, 3, 4, 5, 6, 7, 8, 9, 10, …, infinity
Row 2:   2, 4, 6, 8, 10, 12, …, infinity
         3, 6, ,9, 12, 15, 18, …, infinity
         4, 8, 12, 16, 20, 24, 28, …, infinity
         5, 10, 15, 20, 25, 30, 35, …, infinity
         6, 12, 18, 24, 30, 36, …, infinity
         .
         .
         .
Infinity

↓

```
              1                                              1

          2    2                                         2  4  2

        3   4   3                                      3  6  9  6  3

      4   6   6   4                                  4  8 12 16 12  8  4

    5   8   9   8   5                              5 10 15 20 25 20 15 10  5

  6  10  12  12  10  6                           6 12 18 24 30 36 30 24 18 12  6
```

             Form 1.   Form 2.

Here, the diagonal of the bi - directional box is transitioned into perpendicular height form: where the terms at the diagonal of initial box are kept adjacent while conversion, and the terms to the finite ends from a particular diagonal term are symmetrically arranged on both sides parallel to the base of triangle in straight line. In the other form of conversion, the diagonal terms are not adjacent, and they are

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Notes

arranged in a way similar to that of in pascal's triangle. Each of these forms contain several unique patterns instilled in them, but for now, we will use the 1st form.
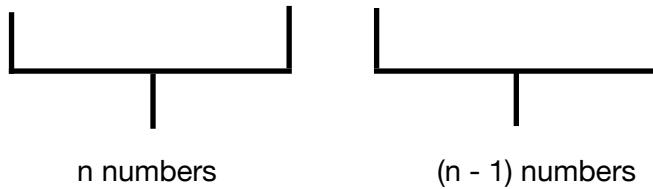
The sum of each row of this triangle is n ˆ3.

	Sum of Numbers In a row	
1	= 1	= 1^3
2 4 2	= 8	= 2^3
3 6 9 6 3	= 27	= 3^3
4 8 12 16 12 8 4	= 64	= 4^3
5 10 15 20 25 20 15 10 5	= 125	= 5^3
6 12 18 24 30 36 30 24 18 12 6	= 216	= 6^3

Proof:

S_n = Sum of numbers in nth row

$$= n + 2n + 3n + \ldots + n^2 \quad + \quad (n-1)n + (n-2)n + \ldots + n$$

n numbers   (n - 1) numbers

$$= n(1 + 2 + 3 + \ldots + n) \quad + \quad n[1 + 2 + 3 + \ldots + (n-1)]$$

$$= n \cdot n(n + 1)/2 \quad + \quad n(n-1)(n)/2 \quad \text{(sum of first n natural numbers = n(n+1)/2)}$$

$$= (n^2)[n + 1 + n - 1]/2$$

$$= n^3$$

To get the sum of all the terms of this triangle, we will now add the sum of all rows. According to the proof above:

Sum of terms in nth row = nˆ3

Sum of row 1 = 1ˆ3

Sum of row 2 = 2ˆ3

And so on.

On adding all the rows, we get:

1ˆ3 + 2ˆ3 + 3ˆ3 + …. = 1/120

We have now reduced the initial Bi-directional box into this$(1^3 + 2^3 + 3^3 + \ldots)$

We know that the result of this series is 1/120(initially stated by Ramanujan, and later it was proved in many ways). Some of the proofs can be found here:

https://math.stackexchange.com/questions/2052233/how-does-one-get-that-13233343-cdots frac1120/2052282

From one of the given proofs, we end up with the following form for the Zeta function whenever $s \in N$, $s > 0$:

$$\zeta(-s) = \frac{1}{1 - 2^{1+s}} \lim_{r \to 1^-} \left(\underbrace{r \cdot \frac{d}{dr} r \cdot \frac{d}{dr} \ldots r \cdot \frac{d}{dr} \frac{-1}{1+r}}_{s} \right)$$

Below are some worked out values; however, for our purpose, we will go with s = 3

$$\zeta(-1) = \frac{1}{1 - 2^{1+1}} \lim_{r \to 1^-} \left(r \cdot \frac{d}{dr} \frac{-1}{1+r} \right) = -\frac{1}{3} \lim_{r \to 1^-} \left(\frac{r}{(1+r)^2} \right) = -\frac{1}{12}$$

$$\zeta(-2) = \frac{1}{1 - 2^{1+2}} \lim_{r \to 1^-} \left(r \cdot \frac{d}{dr} r \cdot \frac{d}{dr} \frac{-1}{1+r} \right) = -\frac{1}{7} \lim_{r \to 1^-} \left(r \cdot \frac{d}{dr} \frac{r}{(1+r)^2} \right)$$

$$= -\frac{1}{7} \lim_{r \to 1^-} \left(\frac{r - r^2}{(1+r)^3} \right) = 0$$

$$\zeta(-3) = \frac{1}{1 - 2^{1+3}} \lim_{r \to 1^-} \left(r \cdot \frac{d}{dr} r \cdot \frac{d}{dr} r \cdot \frac{d}{dr} \frac{-1}{1+r} \right) = -\frac{1}{15} \lim_{r \to 1^-} \left(r \cdot \frac{d}{dr} \frac{r - r^2}{(1+r)^3} \right)$$

$$= -\frac{1}{15} \left(\frac{r - 4r^2 + r^3}{(1+r)^4} \right) = \frac{1}{120}$$
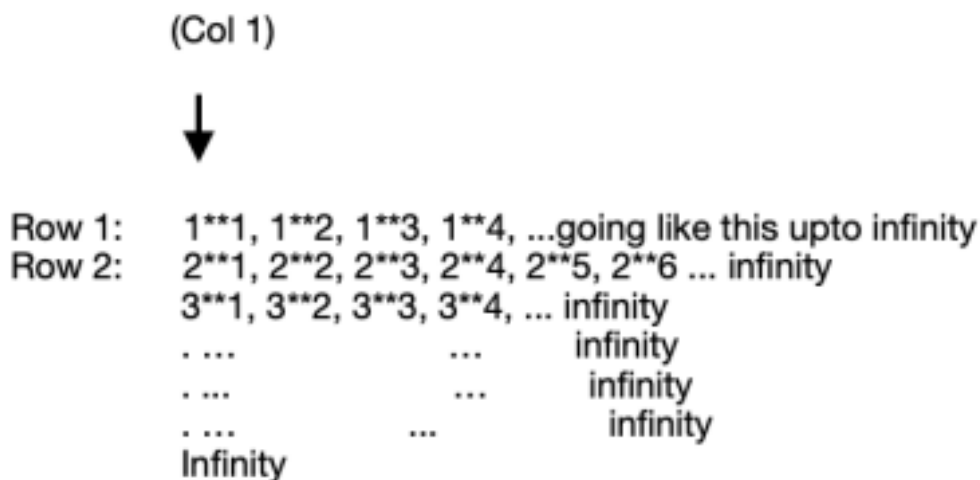
For s = 3, we get 1/120

Thus, we can say that:

The sum of all multiples upto infinity of each natural number upto infinity is 1/120

OR

Sum of all multiples combined of every natural number upto infinity is 1/120.

The terms in a bi-directional infinity box can be different according to the series which we are dealing with and the patterns we are trying to observe. The one we discussed above is the standard bi-directional infinity box. Another example with of series converted into bi-directional infinity box is

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Notes

(Col 1)

↓

Row 1:   1**1, 1**2, 1**3, 1**4, ...going like this upto infinity
Row 2:   2**1, 2**2, 2**3, 2**4, 2**5, 2**6 ... infinity
         3**1, 3**2, 3**3, 3**4, ... infinity
. ...   infinity
. ...   infinity
. ...   infinity
Infinity

Where Row 1 contains a sequence of $1\hat{\ }n$, Row 2 contains an individual sequence of $2\hat{\ }n$, Row 3 contains an individual sequence of $3\hat{\ }n$, and so on(n belongs to all the natural numbers).

II.   Conclusion

Such conversions can be helpful in organising multiple infinite series or for even doing an organised study of patterns and results when multiple series/sequences are combined. Along with it, the article helps us in understanding in a new form of numerical distribution(Bi-directional Infinite Boxes) and its two forms of triangle, on which further development can be done.

References Références Referencias

1. https://math.stackexchange.com/questions/2052233/how-does-one-get-that-13233343- cdots-frac1120/2052282
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