



GLOBAL JOURNAL OF SCIENCE FRONTIER RESEARCH: A
PHYSICS AND SPACE SCIENCE

Volume 20 Issue 13 Version 1.0 Year 2020

Type : Double Blind Peer Reviewed International Research Journal

Publisher: Global Journals

Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Ether Dynamics and Unification of Gravitational and Electromagnetic Forces

By Ling Jun Wang

University of Tennessee at Chattanooga

Abstract- Recently we have presented a theory of unification of gravitational and electromagnetic fields based on the generalization of Newton's law to include a dynamic term similar to the Lorentz force of electrodynamics[1]. The unification is convincing. The generalization based on similarity of Newton's law and Coulomb's law, however, is speculative although reasonable and compelling. In this article, we have presented a derivation of the dynamic term of gravitation based on our newly proposed ether dynamics, which removes the speculative nature of dynamic term and perfects the unification theory. It turns out that the gravitational interaction is transmitted through the space medium ether. An object in ether is in direct contact with the ether, causing it to move like a highly viscous and incompressible fluid. The movement of ether propagates thorough space like a continuous medium, exerting a force on any object in ether.

Keywords: *unification of gravitational and electromagnetic forces, ether dynamics, gravitation, biot-savart law, lorentz force.*

GJSFR-A Classification: FOR Code: 260203



Strictly as per the compliance and regulations of:



Ether Dynamics and Unification of Gravitational and Electromagnetic Forces

Ling Jun Wang

Abstract- Recently we have presented a theory of unification of gravitational and electromagnetic fields based on the generalization of Newton's law to include a dynamic term similar to the Lorentz force of electrodynamics[1]. The unification is convincing. The generalization based on similarity of Newton's law and Coulomb's law, however, is speculative although reasonable and compelling. In this article, we have presented a derivation of the dynamic term of gravitation based on our newly proposed ether dynamics, which removes the speculative nature of dynamic term and perfects the unification theory. It turns out that the gravitational interaction is transmitted through the space medium ether. An object in ether is in direct contact with the ether, causing it to move like a highly viscous and incompressible fluid. The movement of ether propagates through space like a continuous medium, exerting a force on any object in ether.

Not only neutral objects can disturb the fluid ether, the charged objects can also disturb the ether as well. Applying the fluid dynamics of ether on charged particles, we have derived the empirical Biot-Savart law and Lorentz force of electrodynamics. The significance of theoretic derivation of these empirical and fundamental laws is similar to the derivation of the empirical Kepler's laws by Newton's theory of gravitation. It turns out that a moving charged particle would disturb the fluid ether. The disturbance spreads into the space as a continuous medium, causing local vorticity. The magnetic field is linearly proportional to the local vorticity of ether, exerting a force on another moving particle in space. The vorticity of ether is responsible for the dynamic gravitation and Lorentz force of electrodynamics.

The Ether Dynamics consummates our theory of unification of gravitational and electromagnetic forces.

Keywords: unification of gravitational and electromagnetic forces, ether dynamics, gravitation, biot-savart law, lorentz force.

1. INTRODUCTION

Recently, we have developed a unification theory of gravitational and electromagnetic fields based on generalization of Newton's Law of gravitation to include a dynamic term similar to the Lorentz force of electromagnetism:[1,2]

$$\mathbf{F}_2 = -G \frac{mm'}{r^2} \frac{1}{c^2} \mathbf{v}' \times (\mathbf{v} \times \hat{\mathbf{r}}) \quad (1)$$

where \mathbf{v} and \mathbf{v}' are the velocities of the masses m and m' , r the distance between the two masses, $\hat{\mathbf{r}}$ the unit vector from mass m to mass m' , G the gravitational

constant and c the speed of light. Without a dynamic term, there is no way one can explain the propagation of gravitational interaction, and the spooky action-at-distance is inevitable. The fact that Eq(1) alone is sufficient to yield a complete theory of field equations and the gravitational wave equation lends us confidence in such generalization. However, a generalization based on mathematical similarity between Newton's law of gravitation and Coulomb's law of electrostatics is speculative without theoretical foundation of physics. In this article, we will provide a theoretical derivation of the dynamic term, Eq(1), based on the fluid dynamics of ether. It turns out that the space medium ether is a highly viscous incompressible fluid. An object, be it neutral or charged, will disturb the fluid ether in contact with it, causing fluid-dynamic movement. The dynamic movement of ether then propagates into the space, exerting a force on objects in space near and far. We will produce the mathematic details of such fluid-dynamic movements, and derive the empirical Biot-Savart law and Lorentz force that govern both gravitational and electromagnetic interactions.

Before starting the mathematics, we must first justify the concept of ether. Physicists believed that the electromagnetic waves were propagating through a medium ether before Einstein proposed his theory of relativity in 1905. It was unimaginable that any interaction could propagate without a medium. Einstein believed that the propagation of a electromagnetic wave was realized through field instead of ether. Physics community was then lead to believed that the field is matter. However, Einstein never disproved, scientifically or philosophically, the existence of a universal medium ether. His believing of field as matter was merely a subjective opinion. It is now a common knowledge that the space is filled with interstellar and intergalactic materials. The cosmic microwave background is the experimental evidence of the existence of the interstellar and intergalactic materials. The author would like to distant himself from the concept of omnipresent dark matter or dark energy. Our point is simply that space is filled with an interstellar and intergalactic material that is historically called ether.

We do challenge the concept that the field is matter. What is field? The gravitational field is the force felt by unit mass, which is acceleration. To say that field is matter is to say that acceleration is matter, which is absurd. There are many other fields, such as velocity

Author: University of Tennessee at Chattanooga, 615 McCallie Av., Chattanooga, TN 37403. e-mail: Lingjun-wang@utc.edu

field or temperature field. If field is matter, then the velocity and the temperature would also be matter. Moreover, if field is matter, are the gravitational field and the electromagnetic field the same matter or different matters?

A mistake the early physicists made is to assume that ether was an absolutely static medium at rest, without regarding the possibility of ether being a fluid capable of local movement. The concept of an absolutely static ether run into a conflict with the null result of the famous Michelson-Morley's ether drift experiment. Wang [3] has pointed out that if the local ether is rotating with the solar system, the null result of both the first order ether drift experiment of Wang and the second order ether drift experiment of Michelson-Morley are naturally explained.

In this article, we will assume ether to be a highly viscous incompressible fluid, and derive the dynamic gravitation, the Biot-Savart Law and the Lorentz force based on fluid dynamics of ether.

II. VISCOUS INCOMPRESSIBLE FLUID MOVING AROUND A SOLID SPHERE

The equation of motion of a fluid is described by the Navier-Stokes equation [4]. For an incompressible fluid, the continuity equation is:

$$\nabla \cdot \mathbf{v} = 0 \tag{2}$$

where \mathbf{v} is the velocity. The Navier-Stokes equation for incompressible fluid reduces to [4]

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{X} - \frac{\nabla p}{\rho} + \frac{\mu}{\rho} \nabla^2 \mathbf{v} \tag{3}$$

where ρ is the mass density, μ the viscosity coefficient, p the pressure, and \mathbf{X} the external body force per unit mass. For fluids of high viscosity or flowing at very slow speeds (Reynolds number $\ll 1$) the inertia-force terms on the left-hand side of Eq(3) can be neglected in comparison with the friction-force terms. The body force \mathbf{X} external to the ether is non-existent. Eq(3) then reduces to [4]

$$\nabla p = \mu \nabla^2 \mathbf{v} \tag{4}$$

Taking divergence of both sides of Eq(4), we have

$$\nabla^2 p = \mu \nabla \cdot (\nabla^2 \mathbf{v}) = \mu \nabla^2 (\nabla \cdot \mathbf{v}) = 0 \tag{5}$$

Eq(2) is applied in the last step in yielding Eq(5). Eq(5) indicates that the pressure p satisfies the Laplace equation, hence, for a very slow motion the pressure is a harmonic function.

Equations (2), (4) and (5) can be applied to the problem of a steady uniform flow around a sphere at rest. This problem was first solved by Stokes and is often referred to as Stokes' Law [4,5]. Referring to Fig.1,

the origin is chosen at the center of the sphere, the z axis in the direction opposite to a uniform flow velocity U far away from the sphere. At the spherical surface $r=R$, the velocity of the fluid must be zero due to high viscosity:

$$v_x = v_y = v_z = 0 \quad (r = R) \tag{6}$$

$$v_z = -V, \quad v_x = v_y = 0, \quad p = p_0 \quad (r \rightarrow \infty) \tag{7}$$

where R is the radius of the sphere. V and p_0 are the velocity and the pressure far away from the sphere.

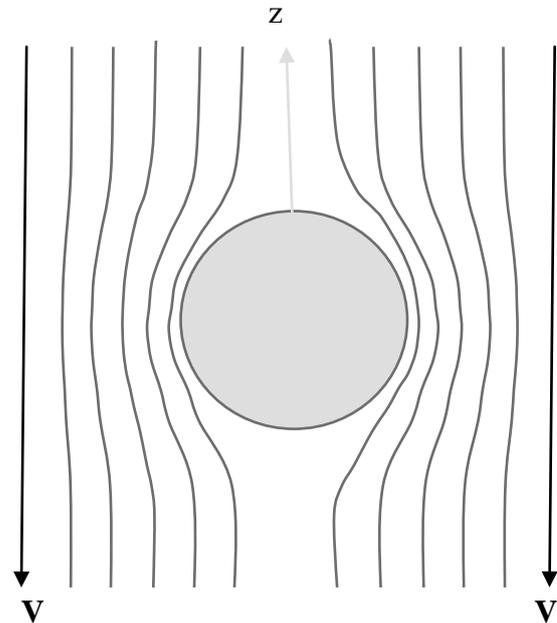


Figure 1: A fluid moving with a uniform velocity V opposite the z -axis against a solid sphere

The solution to Eq(5) is the first order spherical harmonics (Legendre polynomial):

$$p = p_0 - \frac{A \cos \theta}{r^2} = p_0 - \frac{Az}{r^3} \tag{8}$$

Substituting Eq (8) into Eq (4), we obtain

$$\left\{ \begin{aligned} \nabla^2 v_x &= \frac{A}{\mu} \frac{3zx}{r^5} \\ \nabla^2 v_y &= \frac{A}{\mu} \frac{3zy}{r^5} \\ \nabla^2 v_z &= \frac{A}{\mu} \left(\frac{3z^2}{r^5} - \frac{1}{r^3} \right) \end{aligned} \right. \tag{9}$$

The solution to Eq (9) is [4-6]:

$$\left\{ \begin{array}{l} v_x = -V \left[\frac{3 R z x}{4 r^3} \left(\frac{R^2}{r^2} - 1 \right) \right] \\ v_y = -V \left[\frac{3 R z y}{4 r^3} \left(\frac{R^2}{r^2} - 1 \right) \right] \\ v_z = -V \left[\frac{3 R z^2}{4 r^3} \left(\frac{R^2}{r^2} - 1 \right) + 1 - \frac{3 R}{4 r} - \frac{1 R^3}{4 r^3} \right] \end{array} \right. \quad (10)$$

The pressure is

$$p = p_o + \frac{3}{2} \mu V R \frac{z}{r^3} \quad (11)$$

It can be readily verified that the velocity components in Eq(10) satisfy Eq(9) and the boundary condition Eqs(6) and (7). Translating Eq(10) into the spherical coordinate system, the velocity components are:

$$\left\{ \begin{array}{l} v_r = -V \cos \theta \left(1 - \frac{3 R}{2 r} + \frac{1 R^3}{2 r^3} \right) \\ v_\theta = V \sin \theta \left(1 - \frac{3 R}{4 r} - \frac{1 R^3}{4 r^3} \right) \\ v_\phi = 0 \end{array} \right. \quad (12)$$

III. A SOLID SPHERE MOVING IN VISCOUS UNCOMPRESSIBLE FLUID

Eq(12) gives the velocity of a fluid moving against a solid sphere at rest with the velocity V in the direction of negative z -axis. If a velocity of $\mathbf{V}_0 = \mathbf{0}\hat{\mathbf{k}} = V \cos \theta \hat{\mathbf{r}} - V \sin \theta \hat{\boldsymbol{\theta}}$ is added to Eq(12), as shown in Fig.(2), it gives the velocities of a fluid ether at rest disturbed by a solid sphere moving with of a velocity of $\mathbf{v} = V\hat{\mathbf{k}}$ in the positive z -direction:

$$\left\{ \begin{array}{l} v_r = V \cos \theta \left(\frac{3 R}{2 r} - \frac{1 R^3}{2 r^3} \right) \\ v_\theta = -V \sin \theta \left(\frac{3 R}{4 r} + \frac{1 R^3}{4 r^3} \right) \\ v_\phi = 0 \end{array} \right. \quad (13)$$

The radius R in Eq (14) should be understood as the *effective radius* to be determined later. It may or may not be equal to the actual radius of the sphere. We do not know if a "solid" would remain "solid" with

respect to the fluid ether. A solid to the air might well be "porous" with respect to ether. As a matter of fact, the solutions Eq(10) and (13) are obtained for the field satisfying condition Eq(6). The radius R is the radius of a sphere on the surface of which the velocity is zero. This sphere may or may not have the same radius of the particle.

The stream function ψ can be calculated according to reference[4]:

$$\left\{ \begin{array}{l} v_r = \frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta} \\ v_\theta = -\frac{1}{r \sin \theta} \frac{\partial \psi}{\partial r} \end{array} \right. \quad (14)$$

Integrating the second equation of Eq(14) yields

$$\psi = -\int v_\theta r \sin \theta dr = \frac{V \sin^2 \theta}{4} \left(3Rr - \frac{R^3}{r} \right) + f(\theta) \quad (15)$$

Taking partial differentiation of Eq(15) with respect to θ , we obtain

$$\frac{\partial \psi}{\partial \theta} = \frac{V \sin \theta \cos \theta}{2} \left(3Rr - \frac{R^3}{r} \right) + \frac{df}{d\theta}$$

$$v_r = \frac{1}{r^2 \sin \theta} \left[\frac{V \sin \theta \cos \theta}{2} \left(3Rr - \frac{R^3}{r} \right) + \frac{df}{d\theta} \right]$$

$$v_r = V \cos \theta \left(\frac{3R}{2r} - \frac{R^3}{2r^3} \right) + \frac{1}{r^2 \sin \theta} \left[\frac{df}{d\theta} \right] \quad (16)$$

Comparing Eq(16) to the first equation of Eq(13), we have $\frac{df}{d\theta} = 0$, f is a constant. We conveniently choose $f = 0$, and obtain:

$$v_r = V \cos \theta \left(\frac{3R}{2r} - \frac{R^3}{2r^3} \right) \quad (17)$$

Figure 2 shows a plot of the stream lines of the ether disturbed by a sphere moving with velocity v .

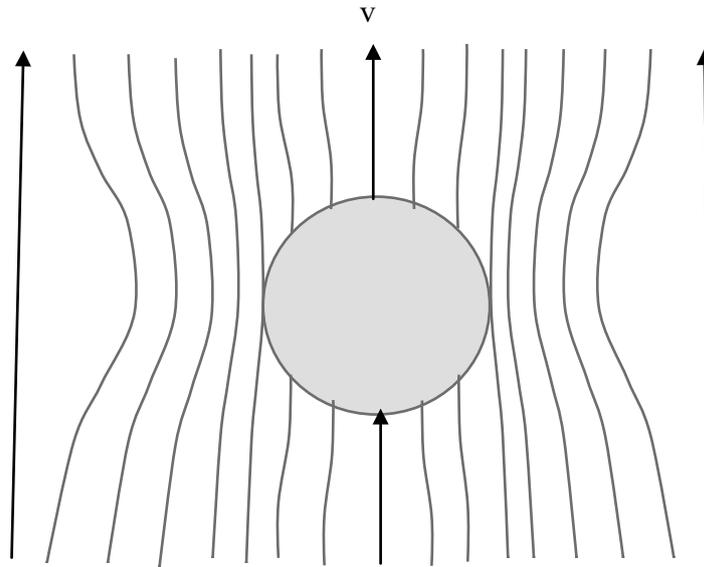


Figure 2: The stream lines of ether disturbed by a solid sphere moving with velocity v

Now let us consider the situations when $R \ll r$. Such would be the case either the radius R is very small for the electrical charges, or the distance r is very great for the astronomical movements of heavenly bodies. Under such condition, the last term in Eq(13) is negligibly small and can be dropped. We then have

$$\begin{cases} v_r = \frac{3R}{2r}V \cos \theta \\ v_\theta = -\frac{3R}{4r}V \sin \theta \\ v_\phi = 0 \end{cases} \quad (18)$$

IV. VORTICITY AND DYNAMIC FIELD

The vorticity Ω can be easily calculated [4]:

$$\begin{aligned} \Omega &= \nabla \times \mathbf{v} \\ &= \frac{1}{r \sin \theta} \left(\frac{\partial(v_\phi \sin \theta)}{\partial \theta} - \frac{\partial v_\theta}{\partial \phi} \right) \hat{r} + \frac{1}{r} \left(\frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial(rv_\phi)}{\partial r} \right) \hat{\theta} + \frac{1}{r} \left(\frac{\partial(rv_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta} \right) \hat{\phi} \end{aligned} \quad (19)$$

$$\Omega = \frac{3RV}{2r^2} \sin \theta \hat{\phi} = \frac{3R}{2r^2} \mathbf{v} \times \hat{r} \quad (20)$$

The vorticity Ω causes the local fluid to rotate. The angular velocity is related to the vorticity. Referring to Fig. 3, suppose a fluid is rotating with angular velocity ω , the linear velocity v at the radius r is

$$v = r\omega \quad (21)$$

The vorticity Ω is

$$\Omega = |\nabla \times \mathbf{v}| = \lim_{r \rightarrow 0} \frac{\left| \oint \mathbf{v} \cdot d\mathbf{l} \right|}{\pi r^2} = \lim_{r \rightarrow 0} \frac{v 2\pi r}{\pi r^2} = 2\omega \quad (22)$$

Using Eq(20),

$$\omega = \frac{\Omega}{2} = \frac{3RV}{4r^2} \sin \theta \hat{\phi} = \frac{3R}{4r^2} \mathbf{v} \times \hat{r} \quad (23)$$

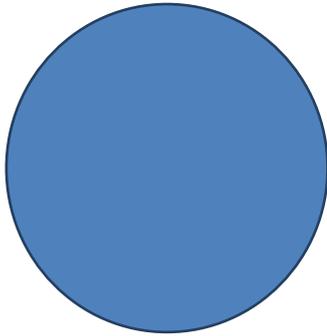


Figure 3: Relationship between vorticity and angular velocity

V. THE FORCE OF ETHER ON A MOVING BODY

Referring to Fig.4, if a mass m' is moving with velocity v' in a rotating fluid with angular velocity ω , its velocity would change due to rotation. The component of v' parallel to ω will not change. The component of v' perpendicular to ω will change. Suppose the angle between ω and v' is θ , the change in velocity (the acceleration a) caused by the rotation is

$$a = \frac{d}{dt}(\alpha\theta v' \sin \theta) = \alpha\omega v' \sin \theta \quad (24)$$

where α is the *dragging coefficient* dependent on the friction between the ether and the moving object with which the ether is dragging the object:

$$0 < \alpha \leq 1 \quad (25)$$

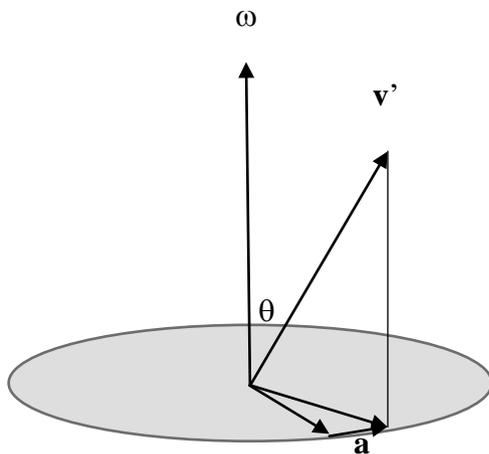


Figure 4: Acceleration of a moving object due to local rotation of fluid

In vector form

$$\mathbf{a} = \frac{d\mathbf{v}'}{dt} = \alpha\boldsymbol{\omega} \times \mathbf{v}' \quad (26)$$

The force exerted on the mass m' is

$$\mathbf{F} = m'\mathbf{a} = m'\alpha\boldsymbol{\omega} \times \mathbf{v}' = -\frac{3\alpha Rm'}{4r^2} \mathbf{v}' \times (\mathbf{v}' \times \hat{\mathbf{r}}) \quad (27)$$

This is exactly the dynamic gravitational force given by Wang [1,2]:

$$\mathbf{F} = -\frac{Gmm'}{c^2 r^2} \mathbf{v}' \times (\mathbf{v}' \times \hat{\mathbf{r}}) \quad (28)$$

Comparing Eqs(27) and (28) gives

$$\alpha \cdot R = \frac{4Gm}{3c^2} \quad (29)$$

The product of the two constants α and R are determined by G and m . However, we could not naively assume that the effective radius R is radius of mass m only. Qualitatively, we can see that both α and R could be dependent on the mass as well as the nature of the interaction.

The dynamic field (or, rotational field) h given by Wang [2] is:

$$\mathbf{h} = -\frac{Gm}{cr^2} \mathbf{v}' \times \hat{\mathbf{r}} \quad (30)$$

Comparing to Eq(23) and (29), we have

$$\mathbf{h} = -c\alpha\boldsymbol{\omega} = -\frac{c\alpha}{2} \boldsymbol{\Omega} \quad (31)$$

Eq(31) says that the dynamic field h is essentially the measure of the local vorticity of the ether.

VI. THE ORIGIN OF LORENTZ FORCE AND BIOT-SAVART LAW

If a charged particle is moving in ether, it will disturb the ether in the same way that a mass does, because any charged particle has a mass and volume. The only difference is that the effective radius is going to be different, and the local angular velocity $\boldsymbol{\Omega}_{em}$ is given by:

$$\boldsymbol{\Omega}_{em} = \frac{3 R_{em} V}{2 r^2} \sin \theta \hat{\boldsymbol{\phi}} = \frac{3 R_{em}}{2 r^2} \mathbf{v}' \times \hat{\mathbf{r}} = 2\boldsymbol{\omega}_{em} \quad (32)$$

where R_{em} is the *electromagnetic effective radius* of the charged particle. The change in velocity (the acceleration a_{em}) of a moving charge q' caused by the local rotation of ether is

$$a_{em} = \frac{d}{dt}(\alpha_{em} \theta v' \sin \theta) = \alpha_{em} \omega_{em} v' \sin \theta \quad (33)$$

The force exerted on the charge q' is

$$\mathbf{F}_{em} = m' \mathbf{a}_{em} = m' \alpha_{em} \boldsymbol{\omega}_{em} \times \mathbf{v}' = -\frac{3\alpha_{em} R_{em} m'}{4r^2} \mathbf{v}' \times (\mathbf{v} \times \hat{r}) \quad (34)$$

Comparing Eq(34) to the Lorentz force of electrodynamics:

$$\mathbf{F}_{em} = q' \mathbf{v} \times \mathbf{B} = \frac{k_1 q' q}{c^2 r^2} \mathbf{v}' \times (\mathbf{v} \times \hat{r}) \quad (35)$$

We have

$$\alpha_{em} \cdot R_{em} = -\frac{4k_1 q' q}{3c^2 m'} \quad (36)$$

Comparing Eqs (29) and (36), we notice two differences: 1) There is a negative sign in Eq(36), which reflects the fact that the same mass attract while the same charge repel. 2) In Eq(36) there is a factor of specific charge (q'/m'), simply because Newton's second law describes the linear relation between the force and the mass, not the force and the charge. But the essence of Eq(36) basically says that the product of the *dragging coefficient* and the *effective radius* ($\alpha_{em} \cdot R_{em}$) is determined by the source q and the nature of electromagnetic interaction represented by the Coulomb constant k_1 and the specific charge (q'/m'). The Biot-Savart law of electrodynamics is

$$\mathbf{B} = \frac{k_1 q}{c^2 r^2} \mathbf{v} \times \hat{r} \quad (37)$$

Comparing Eq(37) with Eq(32), using Eq(36), we have

$$\mathbf{F}_2 = -G \frac{mm'}{r^2} \frac{1}{c^2} \mathbf{v}' \times (\mathbf{v} \times \hat{r}) = -G \frac{mm'}{r^2} [(\boldsymbol{\beta}' \cdot \hat{r}) \boldsymbol{\beta} - (\boldsymbol{\beta}' \cdot \boldsymbol{\beta}) \hat{r}] = \mathbf{M} \cdot m' \boldsymbol{\beta}' \quad (42)$$

where $\boldsymbol{\beta}$ and $\boldsymbol{\beta}'$ are the ratios of velocities over the speed c of gravitational wave in vacuum, The tensor \mathbf{M} is a second rank anti symmetric tensor constructed by the usual rule of dyadic of two vectors:

$$\mathbf{M} = \frac{Gm}{cr^3} (\vec{r}\mathbf{v} - \mathbf{v}\vec{r}) = \frac{Gm}{cr^3} \begin{pmatrix} 0 & v_y x - v_x y & v_z x - v_x z \\ v_x y - v_y x & 0 & v_z y - v_y z \\ v_x z - v_z x & v_y z - v_z y & 0 \end{pmatrix} \quad (43)$$

Since the angular momentum

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = m\mathbf{r} \times \mathbf{v} \quad (44)$$

We have

$$\mathbf{B} = \frac{\alpha_{em} m'}{2 q'} \boldsymbol{\Omega}_{em} \quad (38)$$

Eq(38) states that the magnetic field is a measure of the local ether vorticity, just like the dynamic gravitational field is as expressed in Eq(31).

VII. DYNAMIC GRAVITATION

Eqs.(1) and (28) obtained above is the dynamic gravitational force that should be added to the static gravitational force in Newton's law, giving a complete description of gravitation:

$$\mathbf{F} = -G \frac{mm'}{r^2} \left[\hat{r} + \frac{1}{c^2} \mathbf{v}' \times (\mathbf{v} \times \hat{r}) \right] \quad (39)$$

where \mathbf{v} and \mathbf{v}' are the velocities of the masses m and m' , respectively. The constant c is the speed of gravitational wave. It must be noted that the theoretical development does not depend on the particular value of the speed of gravitational wave. Later on we will show that Eq(39) alone is sufficient to yield a complete theory of gravitational wave propagation, which further justifies and strengthens our confidence in the dynamic theory of gravitation.

The first term of the gravitational force is static:

$$\mathbf{F}_1 = -G \frac{mm'}{r^2} \hat{r} \quad (40)$$

We can define a static field:

$$\mathbf{g} = \frac{\mathbf{F}_1}{m'} = -G \frac{m}{r^2} \hat{r} \quad (41)$$

The second term can be written as

Define a vector \mathbf{h} :

$$\mathbf{M} = \frac{G}{cr^3} \begin{pmatrix} 0 & L_z & -L_y \\ -L_z & 0 & L_x \\ L_y & -L_x & 0 \end{pmatrix} \quad (45)$$

$$\mathbf{h} = \frac{G}{cr^3} \mathbf{L} = \frac{Gm}{cr^2} \hat{r} \times \mathbf{v} = -\mathbf{g} \times \boldsymbol{\beta} \quad (46)$$

We have

$$\mathbf{M} = \begin{pmatrix} 0 & h_z & -h_y \\ -h_z & 0 & h_x \\ h_y & -h_x & 0 \end{pmatrix} \quad (47)$$

Since L is proportional to r, Eqs (46) and (47) manifest inverse square law of h and M. We will call h the *rotational field*, and M the *dynamic field tensor*.

The physical meaning of the vector h can be appreciated if we recall the magnetic field B in *Biot-Savart* law:

$$\mathbf{B} = \frac{\mu_0 q}{4\pi r^2} \mathbf{v} \times \hat{r} = \frac{k_1 q}{c^2 r^2} \mathbf{v} \times \hat{r} \quad (48)$$

There are two differences between the magnetic field B and the gravitational rotational field h: 1) There is a difference of a factor of c due to the definition of M in Eq (43); 2) There is a difference of a negative sign due to the fact that the gravitational force between two masses is attractive while the electric force between two charges of the same sign is repulsive. The advantage of our definition of h is that M and h have the same dimension as that of the static field g.

It must be noted that the sign in the definition of h and B is arbitrary. Either sign can be adopted in the definition without affecting the force that acts on the mass m' or the charge q'. The essential framework of electromagnetic and the gravitational theory would remain intact. The direction of the force is physical and uniquely determined no matter what sign is adopted in the definition of h and B. The arbitrariness simply manifests that h and B are merely intermediate quantities that provide convenience in mathematical presentation. Historically, the magnetic field B was defined according to the conventional right-hand rule. It was used by engineers and scientists for centuries. We will stick with this convention.

The gravitational force can be expressed as

$$\mathbf{F} = m' [\mathbf{g} + \mathbf{M} \cdot \boldsymbol{\beta}'] = m' [\mathbf{g} + \boldsymbol{\beta}' \times \mathbf{h}] \quad (49)$$

The total gravitational field f is defined as the total gravitational force per unit mass

$$\mathbf{f} = \mathbf{g} + \mathbf{M} \cdot \boldsymbol{\beta}' = \mathbf{g} + \boldsymbol{\beta}' \times \mathbf{h} \quad (50)$$

VIII. FIELD EQUATIONS AND WANG'S LAW

It is amazing that with the dynamic term included in the force law of gravitation we can develop a whole dynamic theory without any additional hypothesis.

Let us derive the field equations from the complete force law.

a) Gauss' Law and Wang's Law

Consider the closed surface integral of the static field g over a spherical surface s with radius r:

$$\oiint_s \mathbf{g} d\boldsymbol{\sigma} = -\oiint_s \frac{Gm}{r^2} \hat{r} d\boldsymbol{\sigma} = -\oiint_s Gm d\Omega = -4\pi Gm \quad (51)$$

This is the Gauss' law of the static field. If we allow the radius of the spherical surface approaching zero, we obtain the differential form of Gauss' Law:

$$\nabla \cdot \mathbf{g} = -4\pi G\rho \quad (52)$$

where
$$\rho = \frac{dm}{dV} \quad (53)$$

is the local mass density. V is the volume.

The similar closed surface integral of the dynamic field tensor M is

$$\oiint_s \mathbf{M} \cdot d\boldsymbol{\sigma} = \oiint_s \mathbf{M} \cdot \hat{r} d\boldsymbol{\sigma} \quad (54)$$

From Eq (43) we have

$$\oiint_s \mathbf{M} d\boldsymbol{\sigma} = \frac{Gm}{c} \oiint_s \frac{1}{r^2} \begin{bmatrix} v_y xy + v_z xz - v_x y^2 - v_x z^2 \\ v_x xy + v_z yz - v_y x^2 - v_y z^2 \\ v_x xz + v_y yz - v_z y^2 - v_z x^2 \end{bmatrix} d\Omega \quad (55)$$

where Ω is the solid angle. Note that the velocity of the mass is constant with respect to the integration. The integration (55) involves integrals like

$$\oiint \frac{x^2}{r^2} d\Omega = \oiint (\sin^3 \theta \cos^2 \varphi) d\theta d\varphi = \frac{4\pi}{3} \quad (56)$$

$$\oiint \frac{y^2}{r^2} d\Omega = \oiint (\sin^3 \theta \sin^2 \varphi) d\theta d\varphi = \frac{4\pi}{3}$$

$$\oiint \frac{z^2}{r^2} d\Omega = \oiint (\cos^2 \theta \sin \theta) d\theta d\varphi = \frac{4\pi}{3}$$

and

$$\oiint \frac{xy}{r^2} d\Omega = \oiint \frac{xz}{r^2} d\Omega = \oiint \frac{yz}{r^2} d\Omega = 0$$

Substituting these integrals into Eq(55) we have

$$\oiint_s \mathbf{M} \cdot d\boldsymbol{\sigma} = -\frac{8\pi Gm}{3c} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = -\frac{8\pi G}{3c} m\mathbf{v} = -\frac{8\pi G}{3c} \mathbf{p} \quad (60)$$

where $p = mv$ is the linear momentum of the mass m . Eq(60) is the dynamic counterpart of the static Gauss' Law of (51). The Gauss' Law is well known, but the dynamic counterpart was not known to the physics community before the publication of reference 1 in 2018. Eq (60) is called *Wang's Law*[1]. It says that the closed integration of the dynamic field tensor M is a constant proportional to the linear momentum of the moving mass. Wang's Law is therefore a statement that the total linear momentum transmitted through the gravitational field is conserved. The constant $-4\pi Gm$ in Gauss' Law will be called the *total statics flux*. The constant $-\frac{8\pi G}{3c}m\mathbf{v}$ will be called the *total dynamic flux*. We can

then speak of *conservation of the total static and dynamic fluxes* of the gravitational field.

Wang's Law reveals that the total linear momentum transmitted into the space through ether is conserved. It is a hint that the Gauss' Law must also be a manifestation of conservation of a physical quantity. As a matter of fact, the Gauss's law can be written, according to Eq(51), as

$$\oiint_s \mathbf{g} \cdot d\boldsymbol{\sigma} = -4\pi Gm \quad (61)$$

Eq(61) simply says that the total mass is conserved.

The conservation of the total static and dynamic fluxes reveals how the gravitational interaction is transmitted. Thus, a mass m at rest in space causes stress to the ether. The total stress flux is equal to $-4\pi Gm$. If the mass is moving with velocity v , it will cause additional dynamic stress to the ether. The total dynamic stress flux is $-\frac{8\pi}{3c}Gm\mathbf{v}$. The stress of the ether will then propagate into the space, with the total static and dynamic fluxes conserved and distributed over the whole solid angle. The static and the dynamic fields are simply the stress fluxes per unit area and therefore inversely proportional to the distance squared. It naturally explains the inverse square law of the electromagnetic and the gravitational forces because the total area of a spherical surface is inversely proportional to the radius (distance). Up to now, the inverse square law is an empirical law deduced from experimental observations. We know that the gravitational force gets weaker as the distance increases. We then assume the force to be proportional to r^s and determine the parameter s such that the

theory produces Kepler's third law. It turns out that s must be equal to 2 to do just right. Since Kepler's laws are all empirical laws, we then could not say that the inverse square law is absolutely accurate. The discovery of Wang's Law together with Gauss' law show that the inverse square law is as accurate as the surface area of a sphere is proportional to the square of its radius. The differential form of Wang's law, Eq(60), is

$$\nabla \cdot \mathbf{M} = -\frac{8\pi G}{3c} \mathbf{j} \quad (62)$$

where

$$\mathbf{j} = \rho \mathbf{v} = \frac{\mathbf{p}}{V} \quad (63)$$

is the *momentum density*, i.e., the momentum per unit volume. p is the total momentum of the mass contained in the volume V . j is also the current density (current per unit area).

Since $\nabla \cdot \mathbf{M} = -\nabla \times \mathbf{h}$, we have

$$\nabla \times \mathbf{h} = \frac{8\pi G}{3c} \mathbf{j} \quad (64)$$

It is straightforward to check that the divergence of h is zero:

$$\nabla \cdot \mathbf{h} = -\nabla \cdot \left(\frac{Gm}{cr^2} \mathbf{v} \times \hat{r} \right) = 0 \quad (65)$$

Substituting Eqs (52), (64) and (65) into Eq(50), we have

$$\nabla \cdot \mathbf{f} = \nabla \cdot \mathbf{g} + \nabla \cdot (\boldsymbol{\beta}' \times \mathbf{h}) = \nabla \cdot \mathbf{g} + \mathbf{h} \cdot (\nabla \times \boldsymbol{\beta}') - \boldsymbol{\beta}' \cdot (\nabla \times \mathbf{h}) \quad (66)$$

Since $\boldsymbol{\beta}'$ is a constant, $\nabla \times \boldsymbol{\beta}' = 0$. Using Eqs(52) and (64), we have

$$\nabla \cdot \mathbf{f} = \nabla \cdot \mathbf{g} - \boldsymbol{\beta}' \cdot (\nabla \times \mathbf{h}) = -4\pi G \left(\rho + \frac{2}{3c} \boldsymbol{\beta}' \cdot \mathbf{j} \right) \quad (67)$$

b) *Divergence of M and Curl of h in vacuum*
 From Eqs(43) and (47) we have

$$\nabla \cdot \mathbf{M} = -\nabla \times \mathbf{h} = \frac{Gm}{c} \nabla \times \left(\frac{\mathbf{v} \times \mathbf{r}}{r^3} \right) \quad (68)$$

But

$$\nabla \times \left(\frac{\mathbf{v} \times \mathbf{r}}{r^3} \right) = -\frac{\mathbf{r}}{r^3} (\nabla \cdot \mathbf{v}) + \mathbf{v} \left(\nabla \cdot \left(\frac{\mathbf{r}}{r^3} \right) \right) - (\mathbf{v} \cdot \nabla) \frac{\mathbf{r}}{r^3} + \left(\frac{\mathbf{r}}{r^3} \cdot \nabla \right) \mathbf{v}$$

The velocity v of the source is a constant with respect to the differential operation:

$$\nabla \cdot \mathbf{v} = 0 \text{ and } \left(\frac{\mathbf{r}}{r^3} \cdot \nabla \right) \mathbf{v} = 0 \quad (69)$$

Also, if $r \neq 0$, $\nabla \cdot \left(\frac{\mathbf{r}}{r^3} \right) = 0$,

Therefore,

$$\nabla \cdot \mathbf{M} = -\nabla \times \mathbf{h} = \frac{Gm}{c} \nabla \times \left(\frac{\mathbf{v} \times \mathbf{r}}{r^3} \right) = -\frac{Gm}{c} (\mathbf{v} \cdot \nabla) \frac{\mathbf{r}}{r^3} \quad (70)$$

Now let us examine the static field defined in Eq(41)

$$\mathbf{g} = \frac{\mathbf{F}_1}{m'} = -\frac{Gm}{r^2} \hat{r} = -Gm \frac{\mathbf{i}(x-x_1) + \mathbf{j}(y-y_1) + \mathbf{k}(z-z_1)}{\left((x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2 \right)^{3/2}} \quad (71)$$

$$\frac{\partial \mathbf{g}}{\partial t} = Gm \left[\frac{\partial}{\partial x} \left(\frac{\mathbf{r}}{r^3} \right) \frac{dx_1}{dt} + \frac{\partial}{\partial y} \left(\frac{\mathbf{r}}{r^3} \right) \frac{dy_1}{dt} + \frac{\partial}{\partial z} \left(\frac{\mathbf{r}}{r^3} \right) \frac{dz_1}{dt} \right] = Gm (\mathbf{v} \cdot \nabla) \left(\frac{\mathbf{r}}{r^3} \right) \quad (72)$$

Comparing Eq(70) and (72), we arrive at

$$\nabla \cdot \mathbf{M} = -\nabla \times \mathbf{h} = -\frac{1}{c} \frac{\partial \mathbf{g}}{\partial t} \quad (73)$$

Eq(73) gives the divergence of M when the point of interest is not at the origin, namely, $r \neq 0$.

At the origin where the source of gravitation m is located, $r = 0$, the divergence of M is given by Eqs(62) and (63). Combining Eq(62) and Eq(73), we have a general formula for the divergence of M (or curl of h):

$$-\nabla \cdot \mathbf{M} = \nabla \times \mathbf{h} = \frac{1}{c} \frac{\partial \mathbf{g}}{\partial t} + \frac{8\pi G}{3c} \mathbf{j} \quad (74)$$

where \mathbf{j} is the current density as defined in Eq(63).

The total gravitational field \mathbf{f} is related to the static field \mathbf{g} by Eqs(41), (46) and (50):

$$\mathbf{g} = \frac{\mathbf{F}_1}{m'} = -G \frac{m}{r^2} \hat{r} \quad (41)$$

$$\mathbf{h} = \frac{G}{cr^3} \mathbf{L} = \frac{Gm}{cr^2} \hat{r} \times \mathbf{v} = -\mathbf{g} \times \boldsymbol{\beta} \quad (46)$$

$$\mathbf{f} = \mathbf{g} + \mathbf{M} \cdot \boldsymbol{\beta}' = \mathbf{g} + \boldsymbol{\beta}' \times \mathbf{h} = \mathbf{g} + \boldsymbol{\beta}' \times (\boldsymbol{\beta} \times \mathbf{g}) \quad (50)$$

We have the relation of time derivatives

$$\frac{\partial \mathbf{f}}{\partial t} = \frac{\partial \mathbf{g}}{\partial t} + \boldsymbol{\beta}' \times \left(\boldsymbol{\beta} \times \frac{\partial \mathbf{g}}{\partial t} \right) \quad (75)$$

The difference between (71) and (41) is that we now place the mass m at a more general point (x_1, y_1, z_1) instead of the origin, and allow the mass m to move. Namely, its coordinates are functions of time t . To an observer at the point (x, y, z) , the field \mathbf{g} is a time-varying function $\mathbf{g}(x, y, z, t)$, where the time dependence is caused by the change in the coordinates (x_1, y_1, z_1) of the mass m . We can calculate this time derivative:

$$\frac{\partial \mathbf{g}}{\partial t} = -Gm \left[\frac{\partial}{\partial x_1} \left(\frac{\mathbf{r}}{r^3} \right) \frac{dx_1}{dt} + \frac{\partial}{\partial y_1} \left(\frac{\mathbf{r}}{r^3} \right) \frac{dy_1}{dt} + \frac{\partial}{\partial z_1} \left(\frac{\mathbf{r}}{r^3} \right) \frac{dz_1}{dt} \right]$$

But $\frac{\partial}{\partial x_1} \left(\frac{\mathbf{r}}{r^3} \right) = -\frac{\partial}{\partial x} \left(\frac{\mathbf{r}}{r^3} \right)$ and so on, we have

For most heavenly bodies and the objects on the earth, both $\boldsymbol{\beta}'$ and $\boldsymbol{\beta}$ are negligibly small, the second term of Eq(75) can be dropped. Namely,

$$\frac{\partial \mathbf{f}}{\partial t} = \frac{\partial \mathbf{g}}{\partial t} \quad (76)$$

We therefore have from Eq(76):

$$\nabla \times \mathbf{h} = \frac{1}{c} \frac{\partial \mathbf{f}}{\partial t} + \frac{8\pi G}{3c} \mathbf{j} \quad (77)$$

Eq(77) is to be compared to Ampere's law in electrodynamics:

$$\nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{j} \quad (78)$$

Since $\mu_0 \epsilon_0 = \frac{1}{c^2}$ and $\epsilon_0 = \frac{1}{4\pi k_1}$

$$\text{We have } \mu_0 = \frac{4\pi k_1}{c^2} \quad (79)$$

Substituting (79) into (78), we obtain

$$\nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi k_1 \mathbf{j}}{c^2} \quad (80)$$

Comparing (77) and (80), we see two differences: 1) The difference of a factor of c is due to our definition \mathbf{h} of Eq(46) so that \mathbf{h} , \mathbf{M} and \mathbf{g} all have the same dimension, while the magnetic field \mathbf{B} and the electric field \mathbf{E} differ in dimension by a speed. 2) The more significant difference between (77) and (80) is the

difference in the coefficient of the current density j in the second term of the right hand of the two equations. The difference can be traced to the derivation of Ampere's law. We know that the magnetic field of an infinitely long straight current is given by

$$\mathbf{B} = \frac{\mu_0 I}{2\pi r} \quad (81)$$

where I is the current and r is the distance from the point of interest to the straight current. Now consider the integration of \mathbf{B} along a circular contour on a plane perpendicular to and centered at the current, we have

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \quad (82)$$

Eq(82) is known as the Ampere's law. A little more math can prove that (82) is generally true even for non-circular contours as long as the current is infinitely long.

If the current is not infinitely thin but distributed over certain finite area, the right hand side of Eq(82) has to be replaced by an integral of the current density over the area:

$$\oint \mathbf{B} d\mathbf{l} = \mu_0 \iint \mathbf{j} \cdot d\mathbf{a} \quad (83)$$

Divide Eq(83) by the surface area enclosed by the contour, and let the size of the contour approach zero, we obtain the curl of \mathbf{B} :

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} = \frac{4\pi k_1 \mathbf{j}}{c^2} \quad (84)$$

which explains the coefficient of the second term on the right hand side of Eq(80). From our derivation above we see apparently that this coefficient of (4π) is the result of assuming the current to be infinitely long, while the result of Eq(77) is obtained without such assumption. In applications where the assumption of "infinitely-long current" does not apply, say, for a plasma or electromagnetic wave propagating in a dielectric medium, the correct coefficient of $(8\pi/3)$ should be used instead of (4π) . This difference is not noticed in many textbooks. In discussing the electromagnetic waves, the charge and current are usually assumed to be zero in free space, and the difference does not show. If in the

$$-Gm \oint \frac{1}{r^2 c^2} \mathbf{v}' \times (\mathbf{v} \times \hat{r}) \cdot d\mathbf{l} = \oint \frac{1}{c} (\mathbf{v}' \times \mathbf{h}) \cdot d\mathbf{l} = \frac{1}{c} \oint \mathbf{h} \cdot (d\mathbf{l} \times \mathbf{v}') = \frac{1}{c} \oint \mathbf{h} \cdot \left(d\mathbf{l} \times \frac{d\mathbf{r}}{dt} \right)$$

But $d\mathbf{r} \times d\mathbf{l} = d\mathbf{\tau}$, where $d\mathbf{\tau}$ is the area element according to the right-hand rule (Fig 5). We have

$$\oint \mathbf{f} \cdot d\mathbf{l} = \frac{1}{c} \oint \mathbf{h} \cdot \left(d\mathbf{l} \times \frac{d\mathbf{r}}{dt} \right) \mathbf{\tau} = -\frac{d}{dt} \left[\iint_s \frac{\mathbf{h} \cdot d\mathbf{\tau}}{c} \right] = -\frac{d\Phi}{dt} \quad (90)$$

boundary conditions where the infinite-current assumption does apply, say, the surface current on the inner surface of a wave guide, the coefficient of (4π) should be used.

c) *Induced motive potential*

The field \mathbf{f} is equal to the force per unit mass, i.e., $\mathbf{f} = \mathbf{F}/m'$. The testing mass cannot distinguish the dynamic force from the static force. If a mass m' is moved by the gravitational force \mathbf{F} from point a to point b , the work done by the force field is

$$W = \int_a^b \mathbf{F} \cdot d\mathbf{s} = -Gmm' \int_a^b \frac{1}{r^2} \left[\hat{r} + \frac{1}{c^2} \mathbf{v}' \times (\mathbf{v} \times \hat{r}) \right] \cdot d\mathbf{s} \quad (85)$$

The field will move the particle from a point with higher potential energy to a point with lower potential energy. We have:

$$\mathbf{f} = \frac{\mathbf{F}}{m'} = -G \frac{m}{r^2} \left[\hat{r} + \frac{1}{c^2} \mathbf{v}' \times (\mathbf{v} \times \hat{r}) \right] \quad (86)$$

The potential difference between points a and b is the negative work done by the gravitational force per unit mass:

$$V_{ab} = -\frac{W}{m'} = -\int_a^b \mathbf{f} \cdot d\mathbf{l} = Gm \left[\int_a^b \frac{\hat{r}}{r^2} \cdot d\mathbf{l} + \int_a^b \frac{1}{r^2 c^2} \mathbf{v}' \times (\mathbf{v} \times \hat{r}) \cdot d\mathbf{l} \right] \quad (87)$$

The first term of integration is the electrostatic potential difference, which is conservative. The second term is the dynamic potential difference, which is non-conservative. Consider the contour integration

$$\oint \mathbf{f} \cdot d\mathbf{l} = -Gm \left[\oint \frac{\hat{r}}{r^2} \cdot d\mathbf{l} + \oint \frac{1}{r^2 c^2} \mathbf{v}' \times (\mathbf{v} \times \hat{r}) \cdot d\mathbf{l} \right] \quad (88)$$

The first term of the integration on the right side yields zero because

$$\oint \frac{\hat{r}}{r^2} \cdot d\mathbf{l} = \iint \nabla \times \left(\frac{\hat{r}}{r^2} \right) \cdot d\mathbf{\sigma} = 0 \quad (89)$$

The second term of the integration can be easily calculated for a dynamic field \mathbf{h} (Eq 46):

where s is the area enclosed by the contour, and

$$\Phi \equiv \iint_s \frac{\mathbf{h}}{c} \cdot d\mathbf{\tau} \quad (91)$$

Φ is called the *dynamic flux*.

Equation (90) is known as Faraday's law in electrodynamics, which says that the contour integration of the field is equal to the negative rate of change of the dynamic flux. We have presented a proof that Faraday's law can be derived from the Lorentz force. Faraday's

law, discovered by Faraday in 1880, is actually generally valid regardless if the change of the flux is caused by the change of contour, or the change of the dynamic field.

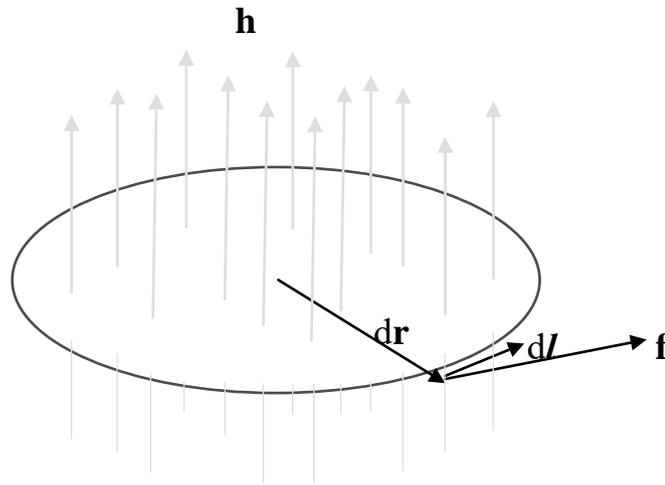


Figure 5: Faraday's law

According to the Stoker's theorem,

$$\oint \mathbf{f} \cdot d\vec{l} = \iint (\nabla \times \mathbf{f}) \cdot d\tau \quad (92)$$

Comparing Eq(90) and (92), we have

$$\nabla \times \mathbf{f} = -\frac{1}{c} \frac{\partial \mathbf{h}}{\partial t} \quad (93)$$

Eq (93) is the differential form of Faraday's law. The partial differentiation instead of total differentiation is used in Eq(93) because the value of h is fixed at the point of interest during the limiting process of letting the size of the contour approaching zero.

IX. WAVE EQUATION

We now have a set of equations that allows us to understand the propagation of gravitational wave. First, we have the divergence of h (Eq 65):

$$\nabla \cdot \mathbf{h} = 0 \quad (65)$$

The divergence of the field f is given by Eq(67):

$$\nabla \cdot \mathbf{f} = -4\pi G \left(\rho + \frac{2}{3c} \beta' \cdot \mathbf{j} \right) \quad (67)$$

The curl of h is given by Eq (77):

$$\nabla \times \mathbf{h} = \frac{1}{c} \frac{\partial \mathbf{f}}{\partial t} + \frac{8\pi G}{3c} \mathbf{j} \quad (77)$$

The curl of f is given by Eq (93):

$$\nabla \times \mathbf{f} = -\frac{1}{c} \frac{\partial \mathbf{h}}{\partial t} \quad (93)$$

The above equations constitute the fundamental equations of the gravitational field. In a free space where $\rho = 0$ and $\mathbf{j} = 0$, these equations take more simple and symmetric form:

$$\nabla \cdot \mathbf{f} = 0 \quad (94)$$

$$\nabla \cdot \mathbf{h} = 0 \quad (95)$$

$$\nabla \times \mathbf{h} = \frac{1}{c} \frac{\partial \mathbf{f}}{\partial t} \quad (96)$$

$$\nabla \times \mathbf{f} = -\frac{1}{c} \frac{\partial \mathbf{h}}{\partial t} \quad (97)$$

Eqs (94)-(97) form a complete set of equations that describes the propagation of the gravitational wave in vacuum. To obtain the wave equation, we take the curl of Eq (97):

$$\nabla \times (\nabla \times \mathbf{f}) = -\nabla \times \left(\frac{1}{c} \frac{\partial \mathbf{h}}{\partial t} \right) = -\frac{1}{c} \frac{\partial}{\partial t} (\nabla \times \mathbf{h}) \quad (98)$$

Using Eq(96), we have

$$\nabla \times (\nabla \times \mathbf{f}) = -\frac{1}{c} \frac{\partial}{\partial t} (\nabla \times \mathbf{h}) = -\frac{1}{c^2} \frac{\partial^2 \mathbf{f}}{\partial t^2} \quad (99)$$

But

$$\nabla \times (\nabla \times \mathbf{f}) = \nabla (\nabla \cdot \mathbf{f}) - \nabla^2 \mathbf{f} = -\nabla^2 \mathbf{f} \quad (100)$$

We have used Eq(94) in the last step. We have

$$\nabla^2 \mathbf{f} = \frac{1}{c^2} \frac{\partial^2 \mathbf{f}}{\partial t^2}$$

Or,

$$\nabla^2 \mathbf{f} - \frac{1}{c^2} \frac{\partial^2 \mathbf{f}}{\partial t^2} = 0 \quad (101)$$

This is the wave equation for the field \mathbf{f} . The solution is

$$\mathbf{f} = \mathbf{f}_0 \sin(\omega t - \mathbf{k} \cdot \mathbf{r}) \quad (102)$$

where

$$\omega = 2\pi f = \frac{2\pi}{T} \quad (103)$$

is the angular frequency. f and T are the frequency and the period. $\mathbf{k} = k\hat{\mathbf{k}}$ is the wave number:

$$k = \frac{2\pi}{\lambda} \quad (104)$$

And

$$\lambda = cT \quad (105)$$

is the wavelength. c is the speed of the gravitational wave.

X. UNIFICATION OF GRAVITATIONAL AND ELECTROMAGNETIC FORCES

The discussions above show that the gravitational force and the electromagnetic force can be described by exactly the same set of equations. The only difference here is that the mass m in the gravitational theory is replaced by the electric charge q in the electromagnetic theory. The different constants G and k are merely the indicators of strength of the interacting forces. As a matter of fact, we can combine the gravitational and electromagnetic forces into a unified equation:

$$\mathbf{F} = (k_1 qq' - Gmm') \frac{1}{r^2} \left[\hat{\mathbf{r}} + \frac{1}{c^2} \mathbf{v}' \times (\mathbf{v} \times \hat{\mathbf{r}}) \right] \quad (106)$$

Eq(106) states explicitly that the gravitational and electromagnetic waves travel at the same speed.

It is amazing that the force equation alone is sufficient to derive all the relevant laws governing the electromagnetic and gravitational interactions. It is a testimony of the consistency of our theory and justification of inclusion of a dynamic term in the law of gravitation.

XI. DISCUSSIONS

a) The speed of gravitational wave

The unification of gravitational and electromagnetic forces is so simple, and the identity of all the equations that govern the propagation of the interactions are so compelling that we believe the two interactions are propagating through the same universal medium ether, by causing static and dynamic stresses. We thus have good reason to predict that the speed of gravitational wave is the same as the speed of electromagnetic wave, or the speed of light. However, this is to be verified experimentally in the future. If the speed of gravitational wave turns out to be different from speed of light, the structure of our unification theory would remain intact and valid in the sense that both gravitational and electromagnetic interactions can be described by the same set of force law, field equations and wave equations, with different speeds of wave.

b) The theoretical derivation of gravitational wave equation

We have rigorously derived the gravitational wave equation first time ever from our unification theory. It is generally believed that Einstein's theory of general relativity predicted the gravitational wave. Such is not true. The gravitational wave equation is not derived from Einstein's general relativity but manufactured with linear approximation of Einstein's field equation and a few *ad hoc* hypotheses. It is proper and fitting here to give a brief account of how general relativity manufactured a gravitational wave equation. Einstein's field equation is

$$G_{\beta\delta} = \frac{8\pi G T_{\beta\delta}}{c^4} + \Lambda g_{\beta\delta} \quad (107)$$

Or, alternatively [7],

$$R_{\beta\delta} = \frac{8\pi G (T_{\beta\delta} - T_{\mu}^{\mu} g_{\beta\delta} / 2)}{c^4} - \Lambda g_{\beta\delta} \quad (108)$$

A number of *ad hoc* hypotheses are then inserted:

1. The first hypothesis: The space is supposed to be vacuum so the stress-energy tensor $T_{\beta\delta} = 0$.
2. The second hypothesis: The cosmological constant is zero: $\Lambda = 0$.

With these two hypotheses, the field equation reduces to

$$G_{\beta\delta} = R_{\beta\delta} = 0 \quad (109)$$

3. The third hypothesis: The field is weak and the metric tensor $g_{\alpha\beta}$ can be approximated as nearly Minkowski:

$$g_{\beta\delta} = \eta_{\beta\delta} + h_{\beta\delta} \quad (110)$$

Under linear approximation, Eq(109) reduces to

$$R_{\beta\delta} = \frac{1}{2} g^{\alpha\nu} (h_{\nu\delta,\beta\alpha} - h_{\delta\beta,\nu\alpha} + h_{\alpha\beta,\nu\delta} - h_{\alpha\nu,\beta\delta}) = 0 \quad (111)$$

Therefore,

$$h_{\nu\delta,\beta\alpha} - h_{\delta\beta,\nu\alpha} + h_{\alpha\beta,\nu\delta} - h_{\alpha\nu,\beta\delta} = 0 \quad (112)$$

4. The fourth hypothesis: It is assumed that each term in Eq(112) is separately zero:

$$h_{\beta\delta,\alpha}^\alpha = 0 \quad (113)$$

$$h_\alpha^\alpha = 0 \quad (114)$$

$$h_{\delta,\alpha}^\alpha = h_{\alpha\beta}^\alpha = 0 \quad (115)$$

$$h_{\alpha 0} = 0 \quad (116)$$

In general the symmetric tensor $h_{\alpha\beta}$ has 10 independent components. Only two of Eqs(113)-(116) are independent. These are interpreted as the two polarizations of the gravitational wave. Eq(113) is the familiar form of wave equation:

$$\frac{\partial^2 h_{\beta\delta}}{\partial x^2} + \frac{\partial^2 h_{\beta\delta}}{\partial y^2} + \frac{\partial^2 h_{\beta\delta}}{\partial z^2} - \frac{\partial^2 h_{\beta\delta}}{c^2 \partial t^2} = 0 \quad (117)$$

Now let us examine the hypotheses needed to yield the wave equation Eq(117). The first hypothesis (vacuum hypothesis) assumes that the universe is a vacuum with zero mass density. The second hypothesis assumes the cosmological term to be zero. These two hypotheses are in direct conflict with the current mainstream cosmology which claims that the cosmological constant is mainly responsible for the dark energy, and the universe is filled with dark energy and dark matter that count 97% of the total mass of the universe. How can one set 97% to zero? The third hypothesis is certainly invalid near the black holes because that is where the metric tensor diverges to infinity, not anything close to Minkowski metric. The fourth hypothesis cannot be justified in any way mathematically. There is no base for assuming each individual terms of an equation to be zero simply because the theory does not work otherwise.

Apparently, the gravitational wave equation is not “derived” from Einstein’s field equation. It is manufactured with a number of invalid hypotheses. The true derivation of gravitational wave equation is given first time ever by our unification theory published in 2018.

c) *The experimental verification*

The experimental detection of the dynamic term is extremely difficult at the current level of technology.

The relative strength of the dynamic term over the static term is at least a factor of (v/c^2) smaller at the most advantageous orientation of the velocities. The velocities must be measured with respect to the medium ether. We do not really know the velocities of the planets and the sun with respect to the ether. The orbital velocity of the earth is about 30 km/s. The velocity of the sun relative to the center of mass of the sun-earth system is about 10^{-4} km/s. If we assume that the center of mass of the solar system is at rest with ether, it would mean that the dynamic term of the gravitational force is about a factor of 3×10^{-14} smaller than the static term. The velocities of the objects on the earth are much smaller than the orbital velocity of the earth. The dynamic term of gravitation should be many orders of magnitude smaller than the above figure. Little wonder the dynamic term of gravitation escaped detection by astronomers, scientists and engineers. It is our hope that in the future the technology would be advanced to allow the detection of the dynamic term.

We do realize the importance of experimental support to the acceptance of a new theory. Historically, Urbain Le Verrier predicted in 1840 the position of the then-undiscovered planet Neptune and its position based on Newton's theory of gravitation after analyzing perturbations in the orbit of Uranus. Subsequent observations of Neptune in the late 19th century led astronomers to speculate that Uranus's orbit was being disturbed by another planet besides Neptune (Pluto). These predictions are celebrated as proof of Newton’s theory of gravitation. But the significance of such prediction is overemphasized. Newton’s law of gravitation was published in 1687 in his “Principia”. Urbain Le Verrier’s prediction came almost 200 years later. Newton’s theory of gravitation was accepted long before the predictions on Neptune and Pluto. The main reason for the acceptance was the success of Newton’s theory in explaining Kepler’s three empirical laws and in explaining the phenomena on Earth. It would be nice if some time down the road predictions based on our ether dynamics is supported by experiments. However, the value of our theory is evident even before such triumphant predictions are available. The credit of our theory of unification and ether dynamics rests on the detailed derivation of the dynamic term of gravitation, which is theoretically as significant as the discovery of Newton’s static term of gravitation, and completes the theory of gravitation with a rigorous wave equation and removes the spooky action-at-distance. The theoretical derivation of the empirical Biot-Savart law and Lorentz force based on ether dynamics is as significant as Newton’s derivation of Kepler’s empirical laws of planetary movements.

A significant achievement of our theory is the newly discovered Wang’s Law which says that the total momentum transmitted into space is conserved, a discovery not known before to physics community.

Another achievement of our theory is the revelation of the essence of the inverse-square law that governs both the electrodynamic and the gravitational interactions. We have demonstrated that the inverse square law is the result of the conservation of the total static and dynamic fluxes as expressed in Gauss' Law and Wang's Law. The inverse square law is as accurate as the surface area of a sphere is proportional to the square of the radius.

d) *The unification of gravitational and electromagnetic interactions*

The century long dream of the physics community to unify the gravitational and electromagnetic forces finally realized with our unification theory and ether dynamics in a rigorously classical way.

Ever since Maxwell unified the theory of electricity and magnetism, the unification of the gravitational and electromagnetic fields had become the dream of the physics community. Early attempts were made by Hermann Weyl, Arthur Eddington, Theodor Kaluza and Albert Einstein.

Hermann Weyl's theory of infinitesimal geometry [8] was based on general relativity. He believed that in addition to a metric field there could be additional degrees of freedom along a path between two points in a manifold. He introduced a gauge field as basic method for comparison of local size measures along such a path. It generalized Riemannian geometry in that there was a vector field Q in addition to the metric g . The vector field and the metric together generated both the electromagnetic and gravitational fields. His theory was mathematically complicated, resulting in high-order field equations. Weyl's theory was found physically unreasonable after extensive communication with Einstein and others.

Kaluza's approach of unification was to embed space-time into a five-dimensional cylindrical world, consisting of four space dimensions and one time dimension [9,10]. The extra dimension allowed the electromagnetic field vector to be incorporated into the geometry. After discussion with Einstein it was discovered that Kaluza's theory did not allow a non-singular, static, spherically symmetric solution, a critical test of the validity of the theory.

Being the most influential early promoter of Einstein's general theory of relativity, Sir Arthur Eddington [11] proposed an extension of the gravitational theory based on the affine connection as the fundamental structure of the gravitational field, instead of the metric tensor as the fundamental structure according to general relativity. Eddington believed that the stress-energy tensor in Einstein's field equations was provisional, and that in a unified theory the source term would automatically come up from the field equations. Eddington's theory were sketchy and difficult

to understand. Very few physicists followed up on his work.

In the spirit of his theory of relativity, Einstein considered the electromagnetic field energy being equivalent to mass according to his mass-energy relationship $E=mc^2$, and contributes to the stress tensor and to the curvature of space-time [12]. Namely, certain configurations of curved space-time should incorporate effects of an electromagnetic field. Einstein then treated both the metric tensor and the affine connection as fundamental fields. His unified-field equations were derived from a variational principle expressed in terms of the Riemann curvature tensor for the presumed space-time manifold. However, Riemannian geometry is unable to describe the properties of the electromagnetic field as a purely geometric phenomenon. The abstract nature and the lack of mathematical tools for analyzing nonlinear equations made it hard to connect such a theory with the physical reality. Einstein became isolated from physics community since then, and his attempts to unify gravity with electromagnetic field was not successful.

The unification theories of Einstein and his contemporaries are considered "classical unification theories". These theories were built around Einstein's general relativity with different ways of modification, all met with failure. After the 1930s, few scientists worked on classical unification, partially due to the failure of Einstein and others' theories, partially due to the emergence of quantum field theory. The unification of electromagnetic interaction with the weak nuclear interaction under the framework of the Standard Model [13-17] seems to be very encouraging, and many are hoping that the further development of quantum field theory might eventually lead to the unification to include the strong nuclear interaction and the gravitational interaction in a final Theory-of-Everything (TOE). The picture is not as rosy as the public is led to believe. It is fair to say that the chances for unifying these forces under the framework of Standard Model are extremely slim.

The unification of the strong interaction within the framework of the standard model seems to be the next logical step after the unification of electrodynamic and weak forces. The effort in this direction, however, has not been very successful. Sheldon Glashow and Howard George proposed in 1974 a model to include the strong interaction into the electro-weak theory, known as the Goerge-Glashow model [18]. It was the first Grand Unified Theory (GUT). The major problem with GUT is that the energy needed to check these theories is way beyond what the current technology could reach, in the order of 10^{16} GeV. It means that the accelerator needs to be bigger than the solar system. It is absolutely impossible. A theory that is not experimentally verifiable and falsifiable cannot be considered as a scientific theory.

Another problem with the GUT is that some of its predictions contradict the experimental findings. For instance, many GUT theories predict that the proton would decay, but the experiments show that the lifetime of the proton is at least 10^{35} years. This is 24 orders of magnitude longer than the lifetime of the universe predicted by the Big Bang cosmology. It is a heavy blow on the effort to unify the strong nuclear force.

The unification of gravitational force with other fundamental forces is even harder than the unification of the strong nuclear force. An unsurmountable obstacle is that the gravitational field is not renormalizable, which means that a unification theory including the gravitational force in the framework of the standard model is a divergent theory. Any divergent theory does not make any sense. It is now generally realized that general relativity is not compatible with quantum field theory.

From the point of view of energy scale, Theory of Everything requires an energy scale of the Plank energy of 10^{19} GeV. That is 1000 times higher than the energy scale of the Grand Unification Theory. It is far beyond the reach of modern accelerators.

The unification of gravitational force with other forces is a failure along the approaches of either general relativity or quantum field theory. However, for over a hundred years, the physics community has been educated to believe that any possible future theory of unifying gravity with other forces would have to be built upon general relativity or the standard model of quantum field theory. Whether or not confirming general relativity and quantum field theory has been used to judge and reject a manuscript for publication. Our unification theory shows that the unification of gravitational and electromagnetic forces could be done within classical framework without resorting to general relativity and quantum field theory. The simplicity, rigorousness and completeness of our unification theory are so compelling that it leaves no doubt on the correctness of the classical approach. It will certainly shake the confidence of physics community in the paradigm and doctrine of theoretical physics of the 20th century. The fundamental problems with General Relativity and the Standard Model of particle physics are analyzed by Wang in two review articles [21, 22].

e) *Existence of ether*

As we commented earlier on that Einstein never disproved the existence of ether. He just disliked it and believed that field is matter. We have shown that the concept of field as matter is illogical and invalid. The fact that our ether dynamics gives a theoretical derivation of the dynamic term of gravitation and the empirical Biot-Savart law and Lorentz force is a verdict that ether does exist, and it is a highly viscous incompressible fluid capable of local movements. Nowadays, the concept of a universal space medium is a common knowledge of

physics community, but the knowledge of such space medium as a highly viscous incompressible fluid, however, is a new discovery.

Our ether dynamics and unification of gravitational and electromagnetic forces would certainly open up a new landscape of physics research into many fundamental questions:

- 1) Are mass and charge internally related? Can the unification theory lead us to a deeper and more general understanding of the charge-mass relationship? If so, such relationship would reveal a whole new microscopic world. Mathematically, this question can be formulated as: Is it possible to reduce the two terms in the first bracket of Eq(106) into a single term only?
- 2) Why are the two interactions dramatically different in strength? The question can be asked differently: Are the gravitational constant G and the Coulomb constant k_1 internally related? We know that the Coulomb constant k_1 is related to another constant k_2 in the Biot-Savart law. It turns out that the k_1 is equal to $(c^2 k_2)$. This relationship manifests that the electrostatic field and the magnetic field are actually the two aspects of a single electromagnetic field, and these two constants determine the speed of electromagnetic wave. If we find the interrelationship between the gravitational constant G and the Coulomb constant k_1 , we could have much deeper understanding of the unification of the two well-known macroscopic forces.
- 3) Our theory predicts per Eq(106) that the speed of gravitational wave is the same as the speed of light. How to verify this experimentally?

Our theory of unification of gravitational and electromagnetic forces would have enormous impact on physical science in many ways. It will change our way of thinking, foster new assessment and evaluation of the approach that physics has been following in the last century, and raise questions on the entire edifice of the prevailing doctrine of theoretical physics.

XII. CONCLUSIONS

We have derived the dynamic term of the gravitational force based on fluid dynamics of ether. It provides a solid theoretical foundation for our unification theory of gravitational and electromagnetic forces based on generalization of Newton's law of gravitation to include a dynamic term similar in form to the Lorentz force of electromagnetic interaction. Such generalization is very logical and reasonable [1,2], but nonetheless speculative in nature. The ether dynamics presented in the present article has removed the speculative nature, completed the law of gravitation started by Newton and consummated our unification theory.

Our ether dynamics has proved the existence of ether by giving a rigorous derivation of Biot-Savart law

and Lorentz force and the dynamic term of gravitation. These laws are taken as empirical laws until this date. Many aspects of these laws, such as the inverse square dependence on the distance, the sinusoidal dependence on the angle between the velocity and the displacement vector, the direction of the magnetic field and the Lorentz force determined by the right hand rule, are none less mysterious than Kepler's three laws. Our rigorous derivation of Biot-Savart law and Lorentz force based on ether dynamics is a convincing proof of the existence of ether. There is simply no other way to explain Biot-Savart law and Lorentz force without accepting a highly viscous incompressible fluid ether serving as the medium to propagate these interactions. Our ether dynamics certainly deepens our understanding of the gravitational and electromagnetic interactions and justifies their unification.

REFERENCES RÉFÉRENCES REFERENCIAS

1. Wang, Ling Jun, "Unification of Gravitational and Electromagnetic Fields", *Physics Essays*, Vol. 31 No. 1, 2018.
2. Wang, Ling Jun, *Unification of Gravitational and Electromagnetic Forces*, Scholar's Press, ISBN 978-3-639-51331-8, 2019.
3. Wang, Ling Jun, "First-order ether drift experiment", *Physics Essays*, Vol. 23 No. 3(2010).
4. Yuan, S.W., *Foundations of Fluid Mechanics*, Civil Engineering and Engineering Mechanics Series, Prentice-Hall International, Inc., London, Sydney, Toronto, New Delhi, Tokyo; Lamb, Horace, *Hydrodynamics*, 6th ED., Dover Publications, New York 1945, ISBN0-486-60256-7.
5. Stokes, G. G., "On the Effects of the Internal Friction of Fluids on the Motion of Pendulums", *Cambridge Phil. Trans.* 9, 1851.
6. Lamb, Horace, *Hydrodynamics*, 6th ED., Dover Publications, New York 1945, ISBN0-486-60256-7.
7. I.R. Kenyon, *General Relativity*, Oxford University Press, Oxford, New York, Tokyo, 1990, ISBN 0 19 851996 6.
8. Weyl, H. (1918). "Gravitation und Elektrizität". *Sitz. Preuss. Akad. Wiss.:* 465.
9. Daniela Wuensch. *Kaluza—Klein Theory*. Compendium of Quantum Physics 2009, pp 328–331.
10. *Sitzungsberichte der Preussischen Akademie der Wissenschaften* (1918) (*Proceedings of the Prussian Academy of Sciences* (1918)). archive. org. pp. 966–974.
11. Eddington, A. S. (1924), *The Mathematical Theory of Relativity*, 2nd ed. Cambridge Univ. Press.
12. Einstein, A. (1956). *The Meaning of Relativity*. 5th ed. Princeton Univ. Press.
13. S.L. Glashow. "Partial-symmetries of weak interactions". *Nuclear Physics*. 22 (4): 579–588. Bibcode:1961NucPh.22.579G. doi:10.1016/0029-5582(61)90469-2.
14. A. Salam (1968). N. Svartholm, ed. *Elementary Particle Physics: Relativistic Groups and Analyticity*. Eighth Nobel Symposium. Stockholm: Almqvist and Wiksell. p. 367.
15. S. Weinberg (1967). "A Model of Leptons". *Physical Review Letters*. 19 (21): 1264–1266. Bibcode: 1967PhRvL.19.1264W. doi: 10.1103/PhysRevLett.19.1264.
16. P.W. Higgs (1964). "Broken Symmetries and the Masses of Gauge Bosons". *Physical Review Letters*. 13 (16): 508–509.
17. 't Hooft, G.; Veltman, M. (1972). "Regularization and renormalization of gauge fields". *Nuclear Physics B*. 44: 189. Bibcode: 1972NuPhB.44.189T. doi: 10.1016 /0550-3213(72)90279-9.
18. Georgi, Howard; Glashow, Sheldon (1974). "Unity of All Elementary-Particle Forces", *Physical Review Letters*. 32 (8): 438. Bibcode:1974PhRvL.32.438G. doi:10.1103/PhysRevLett.32.438.
19. Witten, Edward (1995). "String theory dynamics in various dimensions". *Nuclear Physics B*. 443 (1): 85–126.
20. M. Veltman, *Facts and Mysteries in Elementary Particle Physics*, World Scientific, New Jersey, London, Singapore, Hong Kong, (2003), ISBN 981-238-149-X.
21. Ling Jun Wang, "One Hundred Years of General Relativity – a Critical View", *Physics Essays*, Vol. 28 No. 4, 2015.
22. Ling Jun Wang, "Physics needs nothing less than a Renaissance – On the relation between physics and philosophy", 2020 *J. Phys.: Conf. Ser.* 1466 012002.