Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 20 Issue 1 Version 1.0 Year 2020
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

On the Standard Cliffordian Hamiltonian Formulation of Symplectic Mechanics using Frame and Co-Frame Fields

By Gebreel Mohammed Khur Baba Gebreel & Mohammed Ali Bashir
West Kordufan University

Abstract- In the standard Cliffordian Hamiltonian mechanics we use canonical local basis $\{J_1^*, J_2^*, J_3^*\}$. In this paper using the frame fields $i_X = X^{an+i} \frac{\partial}{\partial x_{an+i}}, a = 0,1,2,\dots,7$ instead of the Hamiltonian vector field in the Cliffordian Hamiltonian formulation we verified the generalized form of Hamiltonian equation which is in conformity with the results that have been obtained previously.

GJSFR-F Classification: *MSC 2010: 37K05*

ONTHESTANDARDCLIFFORDIANHAMILTONIANFORMULATIONOFSYMPLECTICMECHANICSUSINGFRAMEANDCOFRAMEFIELDS

Strictly as per the compliance and regulations of:

On the Standard Cliffordian Hamiltonian Formulation of Symplectic Mechanics using Frame and Co-Frame Fields

Gebreel Mohammed Khur Baba Gebreel [α] & Mohammed Ali Bashir [σ]

Abstract- In the standard Cliffordian Hamiltonian mechanics we use canonical local basis $\{J_1^*, J_2^*, J_3^*\}$. In this paper using the frame fields $i_X = X^{an+i}\frac{\partial}{\partial x_{an+i}}$, $a = 0, 1, 2, \ldots, 7$ instead of the Hamiltonian vector field in the Cliffordian Hamiltonian formulation we verified the generalized form of Hamiltonian equation which is in conformity with the results that have been obtained previously.

I. Introduction

It is well-known that Modern differential geometry explains explicitly the dynamics of Hamiltonian's. Therefore, if Q is an $m-dimensional$ configuration manifold and $H: T^*Q \to R$ is a regular Hamiltonian function, then there is a unique vector field $X\ on\ T^*Q$ such that dynamic equations are given by

$$i_X\Phi = \mathrm{dH} \quad \rightarrow \quad (1)$$

Where Φ indicates the symplectic form. The triple (T^*Q, Φ, X) is called Hamiltonian system on the cotangent bundle T^*Q.

Nowadays, there are a lot of studies about Hamiltonian mechanics, formalisms, systems and equations [1,2] and there in. There are real, complex, paracomplex and other analogues. We say that in order to obtain different analogous in different spaces is possible.

Quaternions were invented by Sir William Rowan Hamiltonian as an extension to the complex numbers. Hamiltonian's defining relation is most succinctly written as:

$$i^2 = j^2 = k^2 = ijk = -1 \quad \rightarrow \quad (2)$$

If it is compared to the calculus of vectors, quaternions have slipped into the realm of obscurity. They do however still find use in the computation of rotations. A lot of physical laws in classical, relativistic, and quantum mechanics can be written pleasantly by means of quaternions. Some physicists hope they will find deeper understanding of the universe by restating basic principles in terms of quaternion algebra. It is well-known that quaternions are useful for representing rotations in both

Ref

1. M. De Leon, P.R. Rodrigues, Methods of Differential Geometry in Analytical Mechanics, North-Holland Mathematics Studies, vol.152, Elsevier, Amsterdam, 1989.

Author α: Department of Mathematics, Faculty of Education, West Kordufan University, Khartoum-Sudan.
e-mail: gebreel6600@gmail.com
Author σ: Academy of Engineering Sciences Khartoum-Sudan. e-mail: info@aes.edu.sd

quantum and classical mechanics [3]. We say that Cliffordian manifold is a quaternion manifold. Therefore, all properties defined on quaternion manifold of *dimension* $8n$ also is valid for Cliffordian manifold. Thus, it is possible to construct mechanical equations on Cliffordian *Kähler* manifold.

II. Preliminaries

Hereafter, all mappings and manifolds are assumed to be smooth, i.e. infinitely differentiable and the sum is taken over repeated indices. By $\mathcal{F}(M), \mathcal{X}(M)$ and $\Lambda^1(M)$ we denote the set of functions on M, the set of vector fields on M and the set of $1-forms\ on\ M$, respectively.

Theorem

Let f be differentiable $\phi\,,\psi$ are 1-form,then [4]:

- $d(f\phi) = df\Lambda\phi + fd\phi$
- $d(\phi\Lambda\psi) = d\phi\Lambda\psi - \phi\Lambda d\psi$

III. Frame Fields [5]

If U, x is a chart on a smooth $n-manifold$ then written $\mathrm{x} = (x^1, \dots, x^n)$ we have vector fields defined on U by

$$\frac{\partial}{\partial x^i} : p \to \left.\frac{\partial}{\partial x^i}\right|_p$$

Such that the together the $\frac{\partial}{\partial x^i}$ form a basis at each tangent space at point in U. We call the set of fields $\frac{\partial}{\partial x^1}, \dots, \frac{\partial}{\partial x^n}$ a holonomic frame field over U. If X is a vector field defined on some set including this local chart domain U then for some smooth functions X^i defined on U we have

$$X(p) = \sum X^i(p) \left.\frac{\partial}{\partial x^i}\right|_p$$

Or in other words

$$X|_U = \sum X^i \frac{\partial}{\partial x^i}$$

Notice also that $dx^i\!:p \to dx^i\big|_p$ defines a field of co-vectors such that $dx^1|_p, \dots dx^n|_p$ forms a basis of $T_p^*M\ for\ each\ p \in U$. The fields form what is called a holonomic co-frame over U. In fact, the functions X^i are given by $dx^i(X)\!:p \to dx^i\big|_p(X_p)$.Type equation here.

IV. Cliffordian *Kähler* Manifolds

Here, we recalled the main concepts and structures given in [6,7]. Let Let M be a real smooth manifold of dimension m. Suppose that there is a $6-dimensional$ vector bundle $V\ consisting\ of\ F_i(i = 1,2, \dots ,6)$ tensors of $type(1,1)\ over\ M$. Such a local basis $\{F_1, F_2, \dots, F_6\}$ is called a canonical local basis of the bundle V in a neighborhood $U\ of\ M$.

Ref

3. D. Stahlke, Quaternions in Classical Mechanics, Phys 621. http:// www.stahlke.org/dan/phys-papers/quaternion-paper.pdf

Notes

Then V is called an almost Cliffordian structure in M. The pair (M, V) is named an almost Cliffordian manifold with V. Hence, an almost Cliffordian manifold M *is of dimension* $m = 8n$. If there exists on (M, V) a global basis $\{F_1, F_2, \dots, F_6\}$, then (M, V) is said to be an almost Cliffordian manifold; the basis $\{F_1, F_2, \dots, F_6\}$ is called a global basis for V.

An almost Cliffordian connection on the almost Cliffordian manifold (M, V) is a linear connection ∇ *on* M which preserves by parallel transport the vector bundle V. This means that if Φ is a cross-section (local-global) of the bundle V. Then $\nabla_x \Phi$ is also a cross-section (local-global, respectively) of V, X being an arbitrary vector field of M.

If for any canonical basis $\{J_i\}, i = \overline{1,6}$ *of* V in a coordinate neighborhood U, the identities

$$g(J_i X, J_i Y) = g(X, Y), \ \forall X, Y \in \chi(M), i = 1,2, \dots, 6 \quad \rightarrow \tag{3}$$

Hold, the triple$(M, g, V)$is named an almost Cliffordian Hermitian manifold or metric Cliffordian manifold denoting by V an almost Cliffordian structure V and by g a Riemannian metric and by (g, V) an almost Cliffordian metric structure.
Since each $J_i (i = 1,2, \dots, 6)$ is almost Hermitian structure with respect to g, setting

$$\Phi_i(X, Y) = g(J_i X, Y), \quad i = 1,2, \dots, 6 \qquad \rightarrow \tag{4}$$

For any vector fields X *and* Y, we see that Φ_i are 6-local 2-forms.

If the Levi-Civita connection $\nabla = \nabla^g$ *on* (M, g, V) preserves the vector bundle V by parallel transport, then (M, g, V) is called a Cliffordian *Kähler* manifold, and an almost Cliffordian structure Φ_i of M is called a Cliffordian *Kähler* structure. A Clifford *Kähler* manifold is Riemannian manifold (M^{8n}, g). For example, we say that R^{8n} is the simplest example of Clifford *Kähler* manifold. Suppose that let

$$\{x_i, x_{i+n}, x_{i+2n}, x_{i+3n}, x_{i+4n}, x_{i+5n}, x_{i+6n}, x_{i+7n}\}, i = \overline{1, n}$$

be a real coordinate system on R^{8n}.

The frame field represents the natural bases over R of the tangent space $T(R^{8n})$*of* R^{8n} and can be written:

$$\left\{\frac{\partial}{\partial x_{an+i}}\right\}, \quad a = 0,1,2,3, \dots 7 \quad \rightarrow \tag{5}$$

The co-frame field represents the natural bases over R of the cotangent space $T^*(R^{8n})$ *of* R^{8n} and can be written:

$$\{dx_{an+i}\}, \quad a = 0,1,2,3, \dots, 7 \quad \rightarrow \tag{6}$$

By structures $\{J_1, J_2, J_3\}$ the following expressions are obtained

$$J_1\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial x_{n+i}} \qquad J_2\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial x_{2n+i}} \qquad J_3\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial x_{3n+i}}$$

$$J_1\left(\frac{\partial}{\partial x_{n+i}}\right) = -\frac{\partial}{\partial x_i} \quad J_2\left(\frac{\partial}{\partial x_{n+i}}\right) = -\frac{\partial}{\partial x_{4n+i}} \quad J_3\left(\frac{\partial}{\partial x_{n+i}}\right) = -\frac{\partial}{\partial x_{5n+i}}$$

Global Journal of Science Frontier Research (F) Volume XX Issue I Version I Year 2020 3

$$J_1\left(\frac{\partial}{\partial x_{2n+i}}\right)=\frac{\partial}{\partial x_{4n+i}}\quad J_2\left(\frac{\partial}{\partial x_{2n+i}}\right)=-\frac{\partial}{\partial x_{i}}\quad J_3\left(\frac{\partial}{\partial x_{2n+i}}\right)=-\frac{\partial}{\partial x_{6n+i}}$$
$$J_1\left(\frac{\partial}{\partial x_{3n+i}}\right)=\frac{\partial}{\partial x_{5n+i}}\quad J_2\left(\frac{\partial}{\partial x_{3n+i}}\right)=\frac{\partial}{\partial x_{6n+i}}\quad J_3\left(\frac{\partial}{\partial x_{3n+i}}\right)=-\frac{\partial}{\partial x_{i}}$$
$$J_1\left(\frac{\partial}{\partial x_{4n+i}}\right)=-\frac{\partial}{\partial x_{2n+i}}\quad J_2\left(\frac{\partial}{\partial x_{4n+i}}\right)=\frac{\partial}{\partial x_{n+i}}\quad J_3\left(\frac{\partial}{\partial x_{4n+i}}\right)=\frac{\partial}{\partial x_{7n+i}}\quad\rightarrow\quad(7)$$
$$J_1\left(\frac{\partial}{\partial x_{5n+i}}\right)=-\frac{\partial}{\partial x_{3n+i}}\quad J_2\left(\frac{\partial}{\partial x_{5n+i}}\right)=-\frac{\partial}{\partial x_{7n+i}}\quad J_3\left(\frac{\partial}{\partial x_{5n+i}}\right)=\frac{\partial}{\partial x_{n+i}}$$
$$J_1\left(\frac{\partial}{\partial x_{6n+i}}\right)=\frac{\partial}{\partial x_{7n+i}}\quad J_2\left(\frac{\partial}{\partial x_{6n+i}}\right)=-\frac{\partial}{\partial x_{3n+i}}\quad J_3\left(\frac{\partial}{\partial x_{6n+i}}\right)=\frac{\partial}{\partial x_{2n+i}}$$
$$J_1\left(\frac{\partial}{\partial x_{7n+i}}\right)=-\frac{\partial}{\partial x_{6n+i}}\quad J_2\left(\frac{\partial}{\partial x_{7n+i}}\right)=\frac{\partial}{\partial x_{5n+i}}\quad J_3\left(\frac{\partial}{\partial x_{7n+i}}\right)=-\frac{\partial}{\partial x_{4n+i}}$$

A canonical local basis $\{J_1^*, J_2^*, J_3^*\}$ *of* V^* of the cotangent space $T^*(M)$ of manifold M satisfies the following condition as follows:

$$J_1^{*2}=J_2^{*2}=J_3^{*2}=J_1^*J_2^*J_3^{*2}J_2^*J_1^*=-I\,,\quad\rightarrow\quad(8)$$

Defining by

$$J_1^*(dx_i)=dx_{n+i}\quad J_2^*(dx_i)=dx_{2n+i}\quad J_3^*(dx_i)=dx_{3n+i}$$
$$J_1^*(dx_{n+i})=-dx_i\quad J_2^*(dx_{n+i})=-dx_{4n+i}\quad J_3^*(dx_{n+i})=-dx_{5n+i}$$
$$J_1^*(dx_{2n+i})=dx_{4n+i}\quad J_2^*(dx_{2n+i})=-dx_i\quad J_3^*(dx_{2n+i})=-dx_{6n+i}$$
$$J_1^*(dx_{3n+i})=dx_{5n+i}\quad J_2^*(dx_{3n+i})=dx_{6n+i}\quad J_3^*(dx_{3n+i})=-dx_i$$
$$J_1^*(dx_{4n+i})=-dx_{2n+i}\quad J_2^*(dx_{4n+i})=dx_{n+i}\quad J_3^*(dx_{4n+i})=dx_{7n+i}\quad\rightarrow\quad(9)$$
$$J_1^*(dx_{5n+i})=-dx_{3n+i}\quad J_2^*(dx_{5n+i})=-dx_{7n+i}\quad J_3^*(dx_{5n+i})=dx_{n+i}$$
$$J_1^*(dx_{6n+i})=dx_{7n+i}\quad J_2^*(dx_{6n+i})=-dx_{3n+i}\quad J_3^*(dx_{6n+i})=dx_{2n+i}$$
$$J_1^*(dx_{7n+i})=-dx_{6n+i}\quad J_2^*(dx_{7n+i})=dx_{5n+i}\quad J_3^*(dx_{7n+i})=-dx_{4n+i}$$

V. Hamiltonian Mechanics

Here, we obtain Hamiltonian equations and Hamiltonian mechanical system for quantum and classical mechanics structured on the standard Cliffordian *Kähler* manifold (R^{8n},V).

Firstly, let (R^{8n},V) be a standard Cliffordian *Kähler* manifold. Assume that a component of almost Cliffordian structure V^*, a Liouville form and a 1-form on the standard Cliffordian *Kähler* manifold (R^{8n},V) are shown by $J_1^*, \lambda_{J_1^*}$ *and* $\omega_{J_1^*}$, respectively.

Then

$$\omega_{J_1^*}=\frac{1}{2}(x_i dx_i+x_{n+i}dx_{n+i}+x_{2n+i}dx_{2n+i}+x_{3n+i}dx_{3n+i}+x_{4n+i}dx_{4n+i}+$$
$$x_{5n+i}dx_{5n+i}+x_{6n+i}dx_{6n+i}+x_{7n+i}dx_{7n+i})\quad\rightarrow\quad(10)$$

Notes

In this equation can be concise manner

$$\omega_{J_1^*} = \frac{1}{2}\sum_{a=0}^{7} x_{an+i}\, dx_{an+i} \quad \rightarrow \tag{11}$$

And

$$\lambda_{J_1^*} = J_1^*(\omega_{J_1^*}) = \frac{1}{2}(x_i dx_{n+i} - x_{n+i}dx_i + x_{2n+i}dx_{4n+i} + x_{3n+i}dx_{5n+i} - x_{4n+i}dx_{2n+i}$$
$$-x_{5n+i}dx_{3n+i} + x_{6n+i}dx_{7n+i} - x_{7n+i}dx_{6n+i})$$

Notes

It is well-known that if $\Phi_{J_1^*}$ is closed *Kähler* form on the standard Cliffordian *Kähler* manifold (R^{8n}, V), then $\Phi_{J_1^*}$ is also a symplectic structure on Cliffordian *Kähler* manifold (R^{8n}, V).

Can be written Hamilton vector field X associated with Hamilton energy H by using frame fields formula:

$$X = \sum_{a=0}^{7} X^{an+i} \frac{\partial}{\partial x_{an+i}} \quad \rightarrow \tag{12}$$

Then

$$\Phi_{J_1^*} = -d\lambda_{J_1^*} = dx_{n+i} \wedge dx_i + dx_{4n+i} \wedge dx_{2n+i} + dx_{5n+i} \wedge dx_{3n+i} + dx_{7n+i} \wedge dx_{6n+i} \rightarrow \tag{13}$$

Can be written i_X by using frame fields

$$i_X = X^{an+i} \frac{\partial}{\partial x_{an+i}} \quad a = 0,1,2,\ldots,7 \quad \rightarrow \tag{14}$$

If: $a = 0 \quad \Rightarrow \quad i_X = X^i \frac{\partial}{\partial x_i}$

$$i_X \Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^i \frac{\partial}{\partial x_i} dx_{n+i}.dx_i - X^i \frac{\partial}{\partial x_i} dx_i.dx_{n+i} + X^i \frac{\partial}{\partial x_i} dx_{4n+i}.dx_{2n+i}$$
$$-X^i \frac{\partial}{\partial x_i} dx_{2n+i}.dx_{4n+i} + X^i \frac{\partial}{\partial x_i} dx_{5n+i}.dx_{3n+i} - X^i \frac{\partial}{\partial x_i} dx_{3n+i}.dx_{5n+i} +$$
$$X^i \frac{\partial}{\partial x_i} dx_{7n+i}.dx_{6n+i} - X^i \frac{\partial}{\partial x_i} dx_{6n+i}.dx_{7n+i} \quad \Rightarrow$$
$$i_X \Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^{n+i}dx_i - X^i dx_{n+i} + X^{4n+i}dx_{2n+i} - X^{2n+i}dx_{4n+i}$$
$$+X^{5n+i}dx_{3n+i} - X^{3n+i}dx_{5n+i} + X^{7n+i}dx_{6n+i} - X^{6n+i}dx_{7n+i}$$

If: $\quad a = 1 \quad \Rightarrow \quad i_X = X^{n+i} \frac{\partial}{\partial x_{n+i}}$

$$i_X \Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{n+i}.dx_i - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_i.dx_{n+i} + X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{4n+i}.dx_{2n+i} -$$
$$X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{2n+i}.dx_{4n+i} + X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{5n+i}.dx_{3n+i} - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{3n+i}.dx_{5n+i} +$$
$$X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{7n+i}.dx_{6n+i} - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{6n+i}.dx_{7n+i} \Rightarrow$$

$$i_X\Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^{n+i}dx_i - X^i dx_{n+i} + X^{4n+i}dx_{2n+i} - X^{2n+i}dx_{4n+i}$$
$$+X^{5n+i}dx_{3n+i} - X^{3n+i}dx_{5n+i} + X^{7n+i}dx_{6n+i} - X^{6n+i}dx_{7n+i}$$

If: $a = 2 \;\Rightarrow\; i_X = X^{2n+i}\frac{\partial}{\partial x_{2n+i}}$

$$i_X\Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{n+i}.dx_i - X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_i.dx_{n+i} +$$
$$X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{4n+i}.dx_{2n+i} - X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{2n+i}.dx_{4n+i} + X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{5n+i}.dx_{3n+i} -$$
$$X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{3n+i}.dx_{5n+i} + X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{7n+i}.dx_{6n+i} - X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{6n+i}.dx_{7n+i} \Rightarrow$$
$$i_X\Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^{n+i}dx_i - X^i dx_{n+i} + X^{4n+i}dx_{2n+i} - X^{2n+i}dx_{4n+i}$$
$$+X^{5n+i}dx_{3n+i} - X^{3n+i}dx_{5n+i} + X^{7n+i}dx_{6n+i} - X^{6n+i}dx_{7n+i}$$

⋮

If: $a = 7 \;\Rightarrow\; i_X = X^{7n+i}\frac{\partial}{\partial x_{7n+i}}$

$$i_X\Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{n+i}.dx_i - X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_i.dx_{n+i} +$$
$$X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{4n+i}.dx_{2n+i} - X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{2n+i}.dx_{4n+i} + X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{5n+i}.dx_{3n+i} -$$
$$X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{3n+i}.dx_{5n+i} + X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{7n+i}.dx_{6n+i} - X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{6n+i}.dx_{7n+i} \Rightarrow$$
$$i_X\Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^{n+i}dx_i - X^i dx_{n+i} + X^{4n+i}dx_{2n+i} - X^{2n+i}dx_{4n+i}$$
$$+X^{5n+i}dx_{3n+i} - X^{3n+i}dx_{5n+i} + X^{7n+i}dx_{6n+i} - X^{6n+i}dx_{7n+i} \quad \rightarrow \quad (15)$$

For all $a = 0,1,2,3,\ldots,7$ we obtain equation (15).

Furthermore, the differential of Hamilton energy is obtained as follows:

$$dH = \frac{\partial H}{\partial x_i}dx_i + \frac{\partial H}{\partial x_{n+i}}dx_{n+i} + \frac{\partial H}{\partial x_{2n+i}}dx_{2n+i} + \frac{\partial H}{\partial x_{3n+i}}dx_{3n+i} + \frac{\partial H}{\partial x_{4n+i}}dx_{4n+i}$$
$$+\frac{\partial H}{\partial x_{5n+i}}dx_{5n+i} + \frac{\partial H}{\partial x_{6n+i}}dx_{6n+i} + \frac{\partial H}{\partial x_{7n+i}}dx_{7n+i} \quad \rightarrow \quad (16)$$

According to Eq (1) if equaled Eq(15) and Eq(16), the Hamiltonian vector field is calculated as follows:

$$X = -\frac{\partial H}{\partial x_{n+i}}\frac{\partial}{\partial x_i} + \frac{\partial H}{\partial x_i}\frac{\partial}{\partial x_{n+i}} - \frac{\partial H}{\partial x_{4n+i}}\frac{\partial}{\partial x_{2n+i}} - \frac{\partial H}{\partial x_{5n+i}}\frac{\partial}{\partial x_{3n+i}} +$$
$$\frac{\partial H}{\partial x_{2n+i}}\frac{\partial}{\partial x_{4n+i}} + \frac{\partial H}{\partial x_{3n+i}}\frac{\partial}{\partial x_{5n+i}} - \frac{\partial H}{\partial x_{7n+i}}\frac{\partial}{\partial x_{6n+i}} + \frac{\partial H}{\partial x_{6n+i}}\frac{\partial}{\partial x_{7n+i}} \quad \rightarrow \quad (17)$$

Suppose that a curve

$$\alpha : R \rightarrow R^{8n} \quad \rightarrow \quad (18)$$

Be an integral curve of the Hamiltonian vector field, i.e. s

Notes

$$X\big(\alpha(t)\big) = \dot{\alpha}\ ,\ \ t \in R \qquad \rightarrow \qquad (19)$$

In the local coordinates , it is obtained that

$$\alpha(t) = (x_i, x_{n+i}, x_{2n+i}, x_{3n+i}, x_{4n+i}, x_{5n+i}, x_{6n+i}, x_{7n+i}) \qquad \rightarrow \qquad (20)$$

And

Notes

$$\dot{\alpha}(t) = \frac{dx_i}{dt}\frac{\partial}{\partial x_i} + \frac{dx_{n+i}}{dt}\frac{\partial}{\partial x_{n+i}} + \frac{dx_{2n+i}}{dt}\frac{\partial}{\partial x_{2n+i}} + \frac{dx_{3n+i}}{dt}\frac{\partial}{\partial x_{3n+i}} +$$

$$\frac{dx_{4n+i}}{dt}\frac{\partial}{\partial x_{4n+i}} + \frac{dx_{5n+i}}{dt}\frac{\partial}{\partial x_{5n+i}} + \frac{dx_{6n+i}}{dt}\frac{\partial}{\partial x_{6n+i}} + \frac{dx_{7n+i}}{dt}\frac{\partial}{\partial x_{7n+i}} \qquad \rightarrow \qquad (21)$$

Considering Eq (19) if equaled Eq (17) and Eq (21), it follows:

$$\frac{dx_i}{dt} = -\frac{\partial H}{\partial x_{n+i}},\quad \frac{dx_{n+i}}{dt} = \frac{\partial H}{\partial x_i},\quad \frac{dx_{2n+i}}{dt} = -\frac{\partial H}{\partial x_{4n+i}},\quad \frac{dx_{3n+i}}{dt} = -\frac{\partial H}{\partial x_{5n+i}},$$

$$\frac{dx_{4n+i}}{dt} = \frac{\partial H}{\partial x_{2n+i}},\quad \frac{dx_{5n+i}}{dt} = \frac{\partial H}{\partial x_{3n+i}},\quad \frac{dx_{6n+i}}{dt} = -\frac{\partial H}{\partial x_{7n+i}},\quad \frac{dx_{7n+i}}{dt} = \frac{\partial H}{\partial x_{6n+i}} \qquad \rightarrow \qquad (22)$$

Thus, the equations obtained in Eq. (22) are seen to be Hamiltonian equation with respect to component J_1^* of almost Cliffordian structure V^* on Cliffordian *Kähler* manifold (R^{8n}, V), and then the triple $(R^{8n}, \Phi_{J_1^*}, X)$ is seen to be a Hamiltonian mechanical system on Cliffordian *Kähler* manifold (R^{8n}, V).

Secondly, let (R^{8n}, V) be a cliffordian *Kähler* manifold.

Suppose that an element of almost cliffordian structure V^*, a Liouville form and a 1-form on cliffordian *Kähler* manifold (R^{8n}, V) are denoted by $J_2^*, \lambda_{J_2^*}$ *and* ωJ_2^*, respectively.

Putting:

$$\omega J_2^* = \frac{1}{2}(x_i dx_i + x_{n+i}dx_{n+i} + x_{2n+i}dx_{2n+i} + x_{3n+i}dx_{3n+i}$$

$$+x_{4n+i}dx_{4n+i} + x_{5n+i}dx_{5n+i} + x_{6n+i}dx_{6n+i} + x_{7n+i}dx_{7n+i})$$

In this equation can be concise manner

$$\omega_{J_2^*} = \frac{1}{2}\sum_{a=0}^{7} x_{an+i}\, dx_{an+i}$$

We have:

$$\lambda_{J_2^*} = J_2^*(\omega J_2^*) = \frac{1}{2}(x_i dx_{2n+i} - x_{n+i}dx_{4n+i} - x_{2n+i}dx_i + x_{3n+i}dx_{6n+i}$$

$$+x_{4n+i}dx_{n+i} - x_{5n+i}dx_{7n+i} - x_{6n+i}dx_{3n+i} + x_{7n+i}dx_{5n+i})$$

Assume that X is a Hamiltonian vector field related to Hamiltonian energy H and given by Eq(12). Take into consideration.

$$\Phi_{J_2^*} = -d\lambda_{J_2^*} = dx_{n+i}\wedge dx_{4n+i} + dx_{2n+i}\wedge dx_i + dx_{5n+i}\wedge dx_{7n+i} + dx_{6n+i}\wedge dx_{3n+i} \rightarrow \quad (23)$$

Then from Eq(14) we obtained

If: $a = 0 \quad \Rightarrow \quad i_X = X^i \frac{\partial}{\partial x_i}$

$$i_X \Phi_{J_2^*} = \Phi_{J_2^*}(X) = X^i \frac{\partial}{\partial x_i} dx_{n+i}.dx_{4n+i} - X^i \frac{\partial}{\partial x_i} dx_{4n+i}.dx_{n+i} + X^i \frac{\partial}{\partial x_i} dx_{2n+i}.dx_i - X^i \frac{\partial}{\partial x_i} dx_i.dx_{2n+i} + X^i \frac{\partial}{\partial x_i} dx_{5n+i}.dx_{7n+i} - X^i \frac{\partial}{\partial x_i} dx_{7n+i}.dx_{5n+i} + X^i \frac{\partial}{\partial x_i} dx_{6n+i}.dx_{3n+i} - X^i \frac{\partial}{\partial x_i} dx_{3n+i}.dx_{6n+i} \quad \Rightarrow$$

$$i_X \Phi_{J_2^*} = \Phi_{J_2^*}(X) = X^{n+i} dx_{4n+i} - X^{4n+i} dx_{n+i} + X^{2n+i} dx_i - X^i dx_{2n+i} + X^{5n+i} dx_{7n+i} - X^{7n+i} dx_{5n+i} + X^{6n+i} dx_{3n+i} - X^{3n+i} dx_{6n+i}$$

If: $a = 1 \quad \Rightarrow \quad i_X = X^{n+i} \frac{\partial}{\partial x_{n+i}}$

$$i_X \Phi_{J_2^*} = \Phi_{J_2^*}(X) = X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{n+i}.dx_{4n+i} - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{4n+i}.dx_{n+i} + X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{2n+i}.dx_i - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_i.dx_{2n+i} + X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{5n+i}.dx_{7n+i} - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{7n+i}.dx_{5n+i} + X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{6n+i}.dx_{3n+i} - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{3n+i}.dx_{6n+i} \Rightarrow$$

$$i_X \Phi_{J_2^*} = \Phi_{J_2^*}(X) = X^{n+i} dx_{4n+i} - X^{4n+i} dx_{n+i} + X^{2n+i} dx_i - X^i dx_{2n+i} + X^{5n+i} dx_{7n+i} - X^{7n+i} dx_{5n+i} + X^{6n+i} dx_{3n+i} - X^{3n+i} dx_{6n+i}$$

If: $a = 2 \quad \Rightarrow \quad i_X = X^{2n+i} \frac{\partial}{\partial x_{2n+i}}$

$$i_X \Phi_{J_2^*} = \Phi_{J_2^*}(X) = X^{2n+i} \frac{\partial}{\partial x_{2n+i}} dx_{n+i}.dx_{4n+i} - X^{2n+i} \frac{\partial}{\partial x_{2n+i}} dx_{4n+i}.dx_{n+i} + X^{2n+i} \frac{\partial}{\partial x_{2n+i}} dx_{2n+i}.dx_i - X^{2n+i} \frac{\partial}{\partial x_{2n+i}} dx_i.dx_{2n+i} + X^{2n+i} \frac{\partial}{\partial x_{2n+i}} dx_{5n+i}.dx_{7n+i} - X^{2n+i} \frac{\partial}{\partial x_{2n+i}} dx_{7n+i}.dx_{5n+i} + X^{2n+i} \frac{\partial}{\partial x_{2n+i}} dx_{6n+i}.dx_{3n+i} - X^{2n+i} \frac{\partial}{\partial x_{2n+i}} dx_{3n+i}.dx_{6n+i} \Rightarrow$$

$$i_X \Phi_{J_2^*} = \Phi_{J_2^*}(X) = X^{n+i} dx_{4n+i} - X^{4n+i} dx_{n+i} + X^{2n+i} dx_i - X^i dx_{2n+i} + X^{5n+i} dx_{7n+i} - X^{7n+i} dx_{5n+i} + X^{6n+i} dx_{3n+i} - X^{3n+i} dx_{6n+i}$$

$\vdots$

If: $a = 7 \quad \Rightarrow \quad i_X = X^{7n+i} \frac{\partial}{\partial x_{7n+i}}$

$$i_X \Phi_{J_2^*} = \Phi_{J_2^*}(X) = X^{7n+i} \frac{\partial}{\partial x_{7n+i}} dx_{n+i}.dx_{4n+i} - X^{7n+i} \frac{\partial}{\partial x_{7n+i}} dx_{4n+i}.dx_{n+i} + X^{7n+i} \frac{\partial}{\partial x_{7n+i}} dx_{2n+i}.dx_i - X^{7n+i} \frac{\partial}{\partial x_{7n+i}} dx_i.dx_{2n+i} + X^{7n+i} \frac{\partial}{\partial x_{7n+i}} dx_{5n+i}.dx_{7n+i} - X^{7n+i} \frac{\partial}{\partial x_{7n+i}} dx_{7n+i}.dx_{5n+i} + X^{7n+i} \frac{\partial}{\partial x_{7n+i}} dx_{6n+i}.dx_{3n+i} - X^{7n+i} \frac{\partial}{\partial x_{7n+i}} dx_{3n+i}.dx_{6n+i} \Rightarrow$$

Notes

$$i_X\Phi_{J_2^*} = \Phi_{J_2^*}(X) = X^{n+i}dx_{4n+i} - X^{4n+i}dx_{n+i} + X^{2n+i}dx_i - X^i dx_{2n+i}$$
$$+X^{5n+i}dx_{7n+i} - X^{7n+i}dx_{5n+i} + X^{6n+i}dx_{3n+i} - X^{3n+i}dx_{6n+i} \quad \rightarrow \quad (24)$$

For all $a = 0,1,2,3,\dots,7$ we obtain equation (24).
Furthermore, the differential of Hamiltonian energy is obtained as follows:

Notes

$$dH = \frac{\partial H}{\partial x_i}dx_i + \frac{\partial H}{\partial x_{n+i}}dx_{n+i} + \frac{\partial H}{\partial x_{2n+i}}dx_{2n+i} + \frac{\partial H}{\partial x_{3n+i}}dx_{3n+i} + \frac{\partial H}{\partial x_{4n+i}}dx_{4n+i}$$
$$+\frac{\partial H}{\partial x_{5n+i}}dx_{5n+i} + \frac{\partial H}{\partial x_{6n+i}}dx_{6n+i} + \frac{\partial H}{\partial x_{7n+i}}dx_{7n+i} \quad \rightarrow \quad (25)$$

According to Eq(1), if equaled Eq(24) and Eq(25), the Hamiltonian vector field is calculated as follows:

$$X = -\frac{\partial H}{\partial x_{2n+i}}\frac{\partial}{\partial x_i} + \frac{\partial H}{\partial x_{4n+i}}\frac{\partial}{\partial x_{n+i}} + \frac{\partial H}{\partial x_i}\frac{\partial}{\partial x_{2n+i}} - \frac{\partial H}{\partial x_{6n+i}}\frac{\partial}{\partial x_{3n+i}} -$$
$$\frac{\partial H}{\partial x_{n+i}}\frac{\partial}{\partial x_{4n+i}} + \frac{\partial H}{\partial x_{7n+i}}\frac{\partial}{\partial x_{5n+i}} + \frac{\partial H}{\partial x_{3n+i}}\frac{\partial}{\partial x_{6n+i}} - \frac{\partial H}{\partial x_{5n+i}}\frac{\partial}{\partial x_{7n+i}} \quad \rightarrow \quad (26)$$

Assume that a curve

$$\alpha : R \rightarrow R^{8n} \quad \rightarrow \quad (27)$$

Be an integral curve of the Hamiltonian vector field X, i.e.

$$X\big(\alpha(t)\big) = \dot{\alpha} \ , \ \ t \in R \quad \rightarrow \quad (28)$$

In the local coordinates, it is obtained that

$$\alpha(t) = (x_i, x_{n+i}, x_{2n+i}, x_{3n+i}, x_{4n+i}, x_{5n+i}, x_{6n+i}, x_{7n+i}) \quad \rightarrow \quad (29)$$

And

$$\dot{\alpha}(t) = \frac{dx_i}{dt}\frac{\partial}{\partial x_i} + \frac{dx_{n+i}}{dt}\frac{\partial}{\partial x_{n+i}} + \frac{dx_{2n+i}}{dt}\frac{\partial}{\partial x_{2n+i}} + \frac{dx_{3n+i}}{dt}\frac{\partial}{\partial x_{3n+i}} +$$
$$\frac{dx_{4n+i}}{dt}\frac{\partial}{\partial x_{4n+i}} + \frac{dx_{5n+i}}{dt}\frac{\partial}{\partial x_{5n+i}} + \frac{dx_{6n+i}}{dt}\frac{\partial}{\partial x_{6n+i}} + \frac{dx_{7n+i}}{dt}\frac{\partial}{\partial x_{7n+i}} \quad \rightarrow \quad (30)$$

Considering Eq(28) if equaled Eq(26) and Eq(30) it follows:

$$\frac{dx_i}{dt} = -\frac{\partial H}{\partial x_{2n+i}}, \frac{dx_{n+i}}{dt} = \frac{\partial H}{\partial x_{4n+i}}, \frac{dx_{2n+i}}{dt} = \frac{\partial H}{\partial x_i}, \frac{dx_{3n+i}}{dt} = -\frac{\partial H}{\partial x_{6n+i}},$$
$$\frac{dx_{4n+i}}{dt} = -\frac{\partial H}{\partial x_{n+i}}, \frac{dx_{5n+i}}{dt} = \frac{\partial H}{\partial x_{7n+i}}, \frac{dx_{6n+i}}{dt} = \frac{\partial H}{\partial x_{3n+i}}, \frac{dx_{7n+i}}{dt} = -\frac{\partial H}{\partial x_{5n+i}} \quad \rightarrow \quad (31)$$

The equations obtained in Eq(31) are known to be Hamiltonian equations with respect to component J_2^* of standard almost Cliffordian structure V^* on the standard Cliffordian *Kähler* manifold (R^{8n} , V) and then the triple (R^{8n}, $\Phi_{J_2^*}$, X) is a Hamiltonian mechanical system on the standard Cliffordian *Kähler* manifold (R^{8n} , V).

Thirdly, let (R^{8n} , V) be a standard Cliffordian *Kähler* manifold. By J_3^*, $\lambda_{J_3^*}$ *and* ωJ_3^* we denote a component of almost Cliffordian structure V^*, a Liouville form and a 1-form on Cliffordian *Kähler* manifold (R^{8n} , V) respectively.
Let ωJ_3^* be given by:

$$\omega J_3^* = \frac{1}{2}(x_i dx_i + x_{n+i} dx_{n+i} + x_{2n+i} dx_{2n+i} + x_{3n+i} dx_{3n+i}$$
$$+x_{4n+i} dx_{4n+i} + x_{5n+i} dx_{5n+i} + x_{6n+i} dx_{6n+i} + x_{7n+i} dx_{7n+i})$$

In this equation can be concise manner

$$\omega_{J_3^*} = \frac{1}{2}\sum_{a=0}^{7} x_{an+i}\, dx_{an+i}$$

Then it holds

$$\lambda_{J_3^*} = J_3^*(\omega J_3^*) = \frac{1}{2}(x_i dx_{3n+i} - x_{n+i} dx_{5n+i} - x_{2n+i} dx_{6n+i} - x_{3n+i} dx_i$$
$$+x_{4n+i} dx_{7n+i} + x_{5n+i} dx_{n+i} + x_{6n+i} dx_{2n+i} - x_{7n+i} dx_{4n+i})$$

It is well-known that if $\Phi_{J_3^*}$ is a closed *Kähler* form on the standard Cliffordian *Kähler* manifold (R^{8n}, V), then $\Phi_{J_3^*}$ is also a symplectic structure on Cliffordian *Kähler* manifold (R^{8n}, V).

Consider X. It is Hamiltonian vector field connected with Hamiltonian energy H and given by Eq(12) taking into:

$$\Phi_{J_3^*} = -d\lambda_{J_3^*} = dx_{3n+i} \wedge dx_i + dx_{n+i} \wedge dx_{5n+i} + dx_{2n+i} \wedge dx_{6n+i} + dx_{7n+i} \wedge dx_{4n+i} \quad \rightarrow \quad (32)$$

Then from Eq(14) we obtained

If: $a = 0 \quad \Rightarrow \quad i_X = X^i \frac{\partial}{\partial x_i}$

$$i_X \Phi_{J_3^*} = \Phi_{J_3^*}(X) = X^i \frac{\partial}{\partial x_i} dx_{3n+i}.dx_i - X^i \frac{\partial}{\partial x_i} dx_i.dx_{3n+i} + X^i \frac{\partial}{\partial x_i} dx_{n+i}.dx_{5n+i}$$
$$-X^i \frac{\partial}{\partial x_i} dx_{5n+i}.dx_{n+i} + X^i \frac{\partial}{\partial x_i} dx_{2n+i}.dx_{6n+i} - X^i \frac{\partial}{\partial x_i} dx_{6n+i}.dx_{2n+i}$$
$$+X^i \frac{\partial}{\partial x_i} dx_{7n+i}.dx_{4n+i} - X^i \frac{\partial}{\partial x_i} dx_{4n+i}.dx_{7n+i} \quad \Rightarrow$$

$$i_X \Phi_{J_3^*} = \Phi_{J_3^*}(X) = X^{3n+i} dx_i - X^i dx_{3n+i} + X^{n+i} dx_{5n+i} - X^{5n+i} dx_{n+i}$$
$$+X^{2n+i} dx_{6n+i} - X^{6n+i} dx_{2n+i} + X^{7n+i} dx_{4n+i} - X^{4n+i} dx_{7n+i}$$

If: $a = 1 \;\Rightarrow\; i_X = X^{n+i} \frac{\partial}{\partial x_{n+i}}$

$$i_X \Phi_{J_3^*} = \Phi_{J_3^*}(X) = X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{3n+i}.dx_i - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_i.dx_{3n+i} + X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{n+i}.dx_{5n+i}$$
$$-X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{5n+i}.dx_{n+i} + X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{2n+i}.dx_{6n+i} - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{6n+i}.dx_{2n+i} +$$
$$X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{7n+i}.dx_{4n+i} - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{4n+i}.dx_{7n+i} \quad \Rightarrow$$

$$i_X \Phi_{J_3^*} = \Phi_{J_3^*}(X) = X^{3n+i} dx_i - X^i dx_{3n+i} + X^{n+i} dx_{5n+i} - X^{5n+i} dx_{n+i}$$

Notes

$$+X^{2n+i}dx_{6n+i} - X^{6n+i}dx_{2n+i} + X^{7n+i}dx_{4n+i} - X^{4n+i}dx_{7n+i}$$

If: $a = 2 \quad \Rightarrow \quad i_X = X^{2n+i}\frac{\partial}{\partial x_{2n+i}}$

$$i_X\Phi_{J_3^*} = \Phi_{J_3^*}(X) = X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{3n+i}.dx_i - X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_i.dx_{3n+i} +$$

$$X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{n+i}.dx_{5n+i} - X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{5n+i}.dx_{n+i} + X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{2n+i}.dx_{6n+i}$$

$$-X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{6n+i}.dx_{2n+i} + X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{7n+i}.dx_{4n+i} - X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{4n+i}.dx_{6n+i} \Rightarrow$$

$$i_X\Phi_{J_3^*} = \Phi_{J_3^*}(X) = X^{3n+i}dx_i - X^i dx_{3n+i} + X^{n+i}dx_{5n+i} - X^{5n+i}dx_{n+i}$$

$$+X^{2n+i}dx_{6n+i} - X^{6n+i}dx_{2n+i} + X^{7n+i}dx_{4n+i} - X^{4n+i}dx_{7n+i}$$

⋮

If: $a = 7 \quad \Rightarrow \quad i_X = X^{7n+i}\frac{\partial}{\partial x_{7n+i}}$

$$i_X\Phi_{J_3^*} = \Phi_{J_3^*}(X) = X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{3n+i}.dx_i - X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_i.dx_{3n+i} +$$

$$X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{n+i}.dx_{5n+i} - X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{5n+i}.dx_{n+i} + X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{2n+i}.dx_{6n+i}$$

$$-X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{6n+i}.dx_{2n+i} + X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{7n+i}.dx_{4n+i} - X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{4n+i}.dx_{7n+i} \Rightarrow$$

$$i_X\Phi_{J_3^*} = \Phi_{J_3^*}(X) = X^{3n+i}dx_i - X^i dx_{3n+i} + X^{n+i}dx_{5n+i} - X^{5n+i}dx_{n+i}$$

$$+X^{2n+i}dx_{6n+i} - X^{6n+i}dx_{2n+i} + X^{7n+i}dx_{4n+i} - X^{4n+i}dx_{7n+i} \quad \rightarrow \quad (33)$$

For all $a = 0,1,2,3,\dots,7$ we obtain equation (33).

Furthermore, the differential of Hamiltonian energy is obtained as follows:

$$dH = \frac{\partial H}{\partial x_i}dx_i + \frac{\partial H}{\partial x_{n+i}}dx_{n+i} + \frac{\partial H}{\partial x_{2n+i}}dx_{2n+i} + \frac{\partial H}{\partial x_{3n+i}}dx_{3n+i} + \frac{\partial H}{\partial x_{4n+i}}dx_{4n+i}$$

$$+\frac{\partial H}{\partial x_{5n+i}}dx_{5n+i} + \frac{\partial H}{\partial x_{6n+i}}dx_{6n+i} + \frac{\partial H}{\partial x_{7n+i}}dx_{7n+i} \quad \rightarrow$$

According to Eq(1) if equaled Eq(33) and Eq(34), the Hamiltonian vector field given by

$$X = -\frac{\partial H}{\partial x_{3n+i}}\frac{\partial}{\partial x_i} + \frac{\partial H}{\partial x_{5n+i}}\frac{\partial}{\partial x_{n+i}} + \frac{\partial H}{\partial x_{6n+i}}\frac{\partial}{\partial x_{2n+i}} + \frac{\partial H}{\partial x_i}\frac{\partial}{\partial x_{3n+i}} -$$

$$\frac{\partial H}{\partial x_{7n+i}}\frac{\partial}{\partial x_{4n+i}} - \frac{\partial H}{\partial x_{n+i}}\frac{\partial}{\partial x_{5n+i}} - \frac{\partial H}{\partial x_{2n+i}}\frac{\partial}{\partial x_{6n+i}} + \frac{\partial H}{\partial x_{4n+i}}\frac{\partial}{\partial x_{7n+i}} \quad \rightarrow \quad (35)$$

Assume that a curve

$$\alpha : R \rightarrow R^{8n} \quad \rightarrow \quad (36)$$

Be an integral curve of the Hamiltonian vector field, i.e.

$$X(\alpha(t)) = \dot{\alpha}, \quad t \in R \quad \rightarrow \quad (37)$$

Notes

In the local coordinates, it is obtained that

$$\alpha(t) = (x_i, x_{n+i}, x_{2n+i}, x_{3n+i}, x_{4n+i}, x_{5n+i}, x_{6n+i}, x_{7n+i}) \qquad \rightarrow \qquad (38)$$

And

$$\dot{\alpha}(t) = \frac{dx_i}{dt}\frac{\partial}{\partial x_i} + \frac{dx_{n+i}}{dt}\frac{\partial}{\partial x_{n+i}} + \frac{dx_{2n+i}}{dt}\frac{\partial}{\partial x_{2n+i}} + \frac{dx_{3n+i}}{dt}\frac{\partial}{\partial x_{3n+i}} +$$

$$\frac{dx_{4n+i}}{dt}\frac{\partial}{\partial x_{4n+i}} + \frac{dx_{5n+i}}{dt}\frac{\partial}{\partial x_{5n+i}} + \frac{dx_{6n+i}}{dt}\frac{\partial}{\partial x_{6n+i}} + \frac{dx_{7n+i}}{dt}\frac{\partial}{\partial x_{7n+i}} \qquad \rightarrow \qquad (39)$$

Thinking out Eq(37) if equaled Eq(35) and Eq(39), it follows:

$$\frac{dx_i}{dt} = -\frac{\partial H}{\partial x_{3n+i}}, \frac{dx_{n+i}}{dt} = \frac{\partial H}{\partial x_{5n+i}}, \frac{dx_{2n+i}}{dt} = \frac{\partial H}{\partial x_{6n+i}}, \frac{dx_{3n+i}}{dt} = \frac{\partial H}{\partial x_i},$$

$$\frac{dx_{4n+i}}{dt} = -\frac{\partial H}{\partial x_{7n+i}}, \frac{dx_{5n+i}}{dt} = -\frac{\partial H}{\partial x_{n+i}}, \frac{dx_{6n+i}}{dt} = -\frac{\partial H}{\partial x_{2n+i}}, \frac{dx_{7n+i}}{dt} = \frac{\partial H}{\partial x_{4n+i}} \qquad \rightarrow \qquad (40)$$

Finally, the equations obtained in Eq(40) are obtained to be Hamiltonian equations with respect to component J_3^* of almost Cliffordian structure V^* on the standard Cliffordian *Kähler* manifold (R^{8n}, V), and then the triple $(R^{8n}, \Phi_{J_2^*}, X)$ is a Hamiltonian mechanical system on the standard Cliffordian *Kähler* manifold (R^{8n}, V).

References Références Referencias

1. M. De Leon, P.R Rodrigues, Methods of Differential Geometry in Analytical Mechanics, North-Holland Mathematics Studies, vol.152, Elsevier, Amsterdam, 1989.
2. M. Tekkoyun, On Para-Euler-Lagrange and Para-Hamiltonian Equations, Phys. Lett. A,vol.340, Issues 1-4, 2005,pp. 7-11.
3. D. Stahlke, Quaternions in Classical Mechanics, Phys 621. http://www.stahlke.org/dan/phys-papers/quaternion-paper.pdf
4. Gebreel Mohammed Khur Baba Gebreel, M.A. Bashir. On a new dynamics on *Kähler* manifold using local canonical basis. International Journal of information research and review. Vol.4. Issue.2.February 2017.
5. Jeffrey M.Lee, Differential Geometry, Analysis and physics, Φ 2000 Jeffrey Marc Lee.
6. K. Yano, M. Kon, Structures on Manifolds, Series in Pure Mathematics Volume 3, World Scientific Publishing co. Pte. Ltd., Singore, 1984.
7. I. Burdujan, Clifford *Kähler* Manifolds, Balkan Journal of Geometry and its Applications, Vol.13, No: 2, 2008, pp.12-23.

Notes