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**GJSFR-F Classification:** *MSC 2010: 11D09*



ANALYSIS AND APPLICATION OF QUADRATIC B-SPLINE INTERPOLATION FOR BOUNDARY VALUE PROBLEMS

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R<sub>ref</sub>

2. Birkhoff, G. and Boor, C. de. (1964). Error bounds for spline interpolation, *Journal of Mathematics and Mechanics*, Vol. 13, pp. 827–835.

# Analysis and Application of Quadratic B-Spline Interpolation for Boundary Value Problems

Md. Asaduzzaman <sup>a</sup>, Liton Chandra Roy <sup>a</sup> & Md. Musa Miah <sup>a</sup>

**Abstract-** B-splines interpolations are very popular tools for interpolating the differential equations under boundary conditions which were pioneered by Maria et.al.[16] allowing us to approximate the ordinary differential equations (ODE). The purpose of this manuscript is to analyze and test the applicability of quadratic B-spline in ODE with data interpolation, and the solving of boundary value problems. A numerical example has been given and the error in comparison with the exact value has been shown in tabulated form, and also graphical representations are shown. Maple soft and MATLAB 7.0 are used here to calculate the numerical results and to represent the comparative graphs.

**Keywords:** *B-splines interpolation, polynomial, approximate solution, curve approximation.*

## I. INTRODUCTION

B-splines play an important role in many areas such as mathematics, engineering and computer science in recent years. Specially B-splines were used for approximation purposes. I. J. Schoenberg [1] initiate the idea of a spline in 1946 and letter on, in the early 1970s, de Boor [2] defined the splines. At present, these are popular in computer graphing due to their velvetiness, tractability, and precision.

It is well-known that polynomial B-splines, particularly the quadratic and cubic B-splines, have gained widespread application and approximate solutions are obtain using different types of quadratic B-spline methods. S. Kutuay, et.al.[5] demonstrate the numerical solutions of the Burgers' equation by the least-squares quadratic B-spline finite element method. B. Saka et.al.[6] obtained a numerical solution of the Regularised Long Wave (RLW) equation using the quadratic B-spline Galerkin finite element method. A.A. Soliman & K.R. Raslan[7] presented Regularised Long Wave (RLW) equation by Collocation method using quadratic B-splines at mid points as element shape functions. Curve approximation with quadratic B-splines can broadly divided in two categories. First, curve approximation with data point interpolation. The use of arc length and curvature characteristics of the given curve to extract the interpolation points was presented in a method for knot placement of the piecewise polynomial approximation of curves was given in [8] and global reparametrization for curve approximation was also been published in [9]. Article on rational parametric curve approximation was published in [10] where the data points, in this case, may not be located on the curve and not much of the work on curve approximation is available using this technique.

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Farago and Horvath [12] (1999) obtained numerical solutions of the heat equation using the finite difference method. Bhatti and Bracken [13] (2006) presented approximate solutions to linear and nonlinear ordinary differential equations using Bernstein polynomials. Bhatta and Bhatti [14] (2006) obtained a numerical solution of the KdV equation using modified Bernstein polynomials via Galerkin method. Munguia *M. and Bhatta. D.* [15] al. (2014) discussed the usage of cubic B-spline functions in interpolation.

The rest of the manuscript is designed as: in section 2, materials and methods are furnished. An illustrative example is shown in section 3 conclusions are prescribed in section 4.

## II. MATERIALS AND METHODS

**Theorem-1:** If  $f(x)$  is defined at  $a = x_0 < x_1 < x_2 < \dots < x_N = b$  then  $f$  has a unique natural interpolation on the nodes  $x_0, x_1, x_2, \dots, x_N$  that is a spline interpolation that satisfies the boundary conditions  $s''(0) = 0$  and  $s''(b) = 0$

**Proof:** The boundary conditions  $c_0 = 0$  and  $c_N = 0$  together with the following equations

$$\frac{c}{h_j}(a_{j+1} - a_j) - \frac{3}{h_{j-1}}(a_j - a_{j-1}) = h_{j-1}c_{j-1} + 2(h_{j-1} + h_j)c_j + h_jc_{j+1} \quad (2.1)$$

are produced by a linear system described by the vector equation  $\bar{A}\bar{X} = \bar{B}$  where  $\bar{A}$  is the  $(n+1)$  by  $(N+1)$  matrix.

$$\bar{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ h_0 & 2(h_0 + h_1) & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & h_{n-2} & 2(h_{n-2} + h_{n-1}) & h_{n-1} \\ 0 & \dots & \dots & 0 & 0 & 1 \end{bmatrix} \quad (2.2)$$

$$\bar{B} = \begin{bmatrix} 0 \\ \frac{3}{h_1}(a_2 - a_1) - \frac{3}{h_0}(a_1 - a_0) \\ \dots \\ \frac{3}{h_{n-1}}(a_n - a_{n-1}) - \frac{3}{h_{n-2}}(a_{n-1} - a_{n-2}) \\ 0 \end{bmatrix} \quad (2.3)$$

$$\text{And} \quad \bar{X} = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \dots \\ c_n \end{bmatrix} \quad (2.4)$$

The matrix is  $\bar{A}$  strictly diagonally dominant. So it satisfies the hypothesis. Therefore the linear system has a unique solution for  $c_0, c_1, \dots, c_n$ . The solution to the

Ref

12. Fargo, I. and Horvath, R. (1999). An optimal mesh choice in the numerical solutions of the heat equation, Computers and Mathematics with Applications, Vol. 38, pp. 79-85.

cubic spline problem with the boundary conditions  $s''(x_0) = s''(x_n) = 0$  can be obtained the theorem.

*Theorem-2:* If  $f(x)$  is defined at  $a = x_0 < x_1 < x_2 < \dots < x_n = b$  and differentiable at  $a$  and  $b$  then has a unique claimed interpolation on the nodes  $x_0, x_1, x_2, \dots, x_n$  that is a spline interpolation that satisfies the boundary conditions  $s'(a) = f'(a)$  and  $s'(b) = f'(b)$

*Proof:* It can be seen using the fact that  $s'(a) = s'(x_0) = b_0$

$$\text{Now we have } f'(a) = \frac{a_1 - a_0}{h_0} - \frac{h_0}{3}(2c_0 + c_1)$$

$$\text{Consequently, } 2h_0c_0 + h_0c_1 = \frac{h_0}{3}(2c_0 + c_1) - 3f'(a)$$

$$\text{Similarly } f'(b) = b_n = b_{n-1} + h_{n-1}(c_{n-1} + c_n)$$

$$\text{Now we have the equation } b_j = \frac{1}{h_j}(a_{j+1} - a_j) - \frac{h_j}{3}(2c_j + c_{j+1})$$

Putting  $j = n-1$  in the above equation, implies that

$$f'(b) = \frac{a_n - a_{n-1}}{h_{n-1}} - \frac{h_{n-1}}{3}(2c_{n-1} + c_n) + h_{n-1}(c_{n-1} + c_n) = \frac{a_n - a_{n-1}}{h_{n-1}} + \frac{h_{n-1}}{3}(c_{n-1} + 2c_n)$$

$$\text{And } h_{n-1}c_{n-1} + 2h_{n-1}c_n = 3f'(b) - \frac{3}{h_{n-1}}(a_n - a_{n-1})$$

$$\text{Also } 2h_0c_0 + h_0c_1 = \frac{3}{h_0}(a_1 - a_0) - 3f'(a)$$

determine the linear system  $\overline{AX} = \overline{B}$ , where

$$\overline{A} = \begin{bmatrix} 2h_0 & h_0 & 0 & 0 & \dots & 0 \\ h_0 & 2(h_0 + h_1) & h_1 & 0 & \dots & 0 \\ 0 & h_1 & 2(h_1 + h_2) & h_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & h_{n-2} & 2(h_{n-2} + h_{n-1}) & h_{n-1} \\ 0 & \dots & \dots & 0 & h_{n-1} & 2h_n \end{bmatrix} \quad (2.5)$$

$$\overline{B} = \begin{bmatrix} \frac{3}{h_0}(a_1 - a_0) - 3f'(a) \\ \frac{3}{h_1}(a_2 - a_1) - \frac{3}{h_0}(a_1 - a_0) \\ \dots \\ \frac{3}{h_{n-1}}(a_n - a_{n-1}) - \frac{3}{h_{n-2}}(a_{n-1} - a_{n-2}) \\ 3f'(b) - \frac{3}{h_{n-1}}(a_n - a_{n-1}) \end{bmatrix} \quad (2.6)$$

The matrix is  $A$  strictly diagonally dominant. So it satisfies the hypothesis. Therefore the linear system has a unique solution for  $c_0, c_1, \dots, c_n$ . The solution to the cubic spline problem with the boundary conditions  $s''(x_0) = s''(x_n) = 0$  can be obtained the theorem

Notes

*Quadratic B-spline:*

$$B_i^2(x) = \begin{cases} \frac{(x - x_i)^2}{(x_{i+2} - x_i)(x_{i+1} - x_i)} & \text{if } x_i \leq x < x_{i+1}, \\ \frac{(x - x_i)(x_{i+2} - x)}{(x_{i+2} - x_i)(x_{i+2} - x_{i+1})} + \frac{(x_{i+3} - x)(x - x_{i+1})}{(x_{i+3} - x_{i+1})(x_{i+2} - x_{i+1})} & \text{if } x_{i+1} \leq x < x_{i+2}, \\ \frac{(x_{i+3} - x)^2}{(x_{i+3} - x_{i+1})(x_{i+3} - x_{i+2})} & \text{if } x_{i+2} \leq x < x_{i+3}, \\ 0 & \text{otherwise} \end{cases} \quad (2.8)$$

The last equation is a quadratic spline with knots  $x_i, x_{i+1}, x_{i+2}, x_{i+3}$ . Note that the quadratic B-spline is zero except on the interval  $[x_i, x_{i+3}]$ . This is true for all B-splines. In fact,  $B_i^k(x) = 0$  if  $x \notin [x_i, x_{i+k+1}]$ , otherwise  $B_i^k(x) > 0$  if  $x \in (x_i, x_{i+k+1})$ .

Since we are only referring to B-splines of degree 2, we write  $B_i$  instead of  $B_i^2$ . Therefore, after including four additional knots, we assume that

$$\Delta: x_{-2} < x_{-1} < x_0 < x_1 < \dots < x_{N-1} < x_N < x_{N+1} < x_{N+2}$$

is a uniform grid partition.

Using (4) and letting  $h = x_{i+1} - x_i$  for any  $0 \leq i \leq N$ , we define the uniform quadratic B-spline  $B_i(x)$  as

$$B_i(x) = \frac{1}{2h^2} \begin{cases} (x - x_{i-2})^2 & \text{if } x_{i-2} \leq x < x_{i-1} \\ -2(x - x_{i-1})^2 + 2h(x - x_{i-1}) + h^2 & \text{if } x_{i-1} \leq x < x_i \\ (x_{i+1} - x)^2 & \text{if } x_i \leq x < x_{i+1} \\ 0 & \text{otherwise} \end{cases} \quad (2.9)$$

If we choose  $h=1$ , then in the interval  $[-2,1]$  we have the following

$$B_0(x) = \frac{1}{2} \begin{cases} (x+2)^2 & \text{if } -2 \leq x < -1 \\ -2x^2 - 2x + 1 & \text{if } -1 \leq x < 0 \\ (1-x)^2 & \text{if } 0 \leq x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (2.10)$$

and its graph is shown in Figure 1. We know that  $B_i$  lies in the interval  $[x_i, x_{i+1}]$  this interval has nonzero contributions from  $B_{i-1}, B_i, B_{i+1}$  and  $B_{i+2}$ . We have a better understanding of this from Figure 2. Next, we derive the quadratic B-spline method for approximating solutions to second-order linear equations.

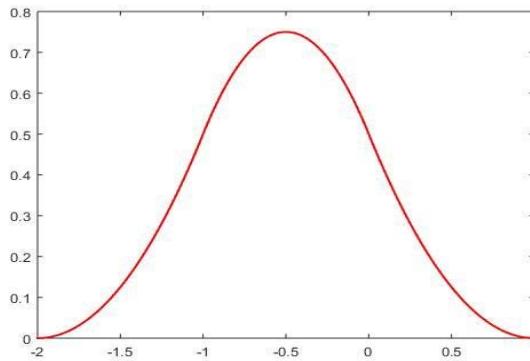


Fig. 1: Quadratic B-spline

#### Quadratic B-spline Solution Procedure:

To approximate the solution of this BVP using quadratic B-splines, we let  $Y(x)$  be a quadratic spline with knots  $\Delta$ . Then  $Y(X)$  can be written as linear combinations of  $B_i(x)$

$$Y(x) = \sum_{i=-1}^{N+1} c_i B_i(x) \quad (2.11)$$

Where the constants  $c_i$  are to be determined and the  $B_i(x)$  are defined in (2.9). It is required that (2.11) satisfies our BVP (3.1-3.2) at  $x = x_i$  where  $x_i$  is an interior point. That is

$$a_1(x_i)Y''(x_i) + a_2(x_i)Y'(x_i) + a_3(x_i)Y(x_i) = f(x_i) \quad (2.12)$$

and the boundary conditions are

$$Y(x_0) = \alpha \text{ for } x_0 = a,$$

$$Y(x_N) = \beta \text{ for } x_N = b.$$

From (2.11), we have

$$Y(x_i) = c_{i-1}B_{i-1}(x_i) + c_iB_i(x_i) + c_{i+1}B_{i+1}(x_i) + c_{i+2}B_{i+2}(x_i),$$

$$Y'(x_i) = c_{i-1}B'_{i-1}(x_i) + c_iB'_i(x_i) + c_{i+1}B'_{i+1}(x_i) + c_{i+2}B'_{i+2}(x_i),$$

$$Y''(x_i) = c_{i-1}B''_{i-1}(x_i) + c_iB''_i(x_i) + c_{i+1}B''_{i+1}(x_i) + c_{i+2}B''_{i+2}(x_i),, \quad (2.13)$$

and these yield

$$c_{i-1}[a_1(x_i)B''_{i-1}(x_i) + a_2(x_i)B'_{i-1}(x_i) + a_3(x_i)B_{i-1}(x_i)] \\ + c_i[a_1(x_i)B''_i(x_i) + a_2(x_i)B'_i(x_i) + a_3(x_i)B_i(x_i)] \\ + c_{i+1}[a_1(x_i)B''_{i+1}(x_i) + a_2(x_i)B'_{i+1}(x_i) + a_3(x_i)B_{i+1}(x_i)] \\ + c_{i+2}[a_1(x_i)B''_{i+2}(x_i) + a_2(x_i)B'_{i+2}(x_i) + a_3(x_i)B_{i+2}(x_i)] = f(x_i), \quad (2.14)$$

also by the properties of quadratic B-spline functions, we obtain the following

$$B''_{i-1}(x_i) = 0, \quad B'_{i-1}(x_i) = 0, \quad B_{i-1}(x_i) = 0, \\ B''_i(x_i) = \frac{1}{h^2}, \quad B'_i(x_i) = -\frac{1}{h}, \quad B_i(x_i) = \frac{1}{2} \\ B''_{i+1}(x_i) = -\frac{2}{h^2}, \quad B'_{i+1}(x_i) = -\frac{1}{h}, \quad B_{i+1}(x_i) = \frac{1}{2}, \\ B''_{i+2}(x_i) = 0, \quad B'_{i+2}(x_i) = 0, \quad B_{i+2}(x_i) = 0. \quad (2.15)$$

If we combine (2.14) and (2.15), we obtain

$$c_i[2a_1(x_i) - 2ha_2(x_i) + a_3(x_i)h^2] \\ + c_{i+1}[-4a_1(x_i) - 2ha_2(x_i) + a_3(x_i)h^2] = 2h^2 f(x_i). \quad (2.16)$$

Now we apply the boundary conditions:

$$Y(x_0) = c_{-1}B_{-1}(x_0) + c_0B_0(x_0) + c_1B_1(x_0) + c_2B_2(x_0) = \alpha, \\ Y(x_N) = c_{N-1}B_{N-1}(x_N) + c_NB_N(x_N) + c_{N+1}B_{N+1}(x_N) + c_{N+2}B_{N+2} = \beta, \quad (2.17)$$

where the value of  $B_i(x)$  at  $x = x_0$  and  $x = x_N$  are given below

$$B_{-1}(x_0) = 0 = B_{N-1}(x_N), \\ B_0(x_0) = \frac{1}{2} = B_N(x_N), \\ B_1(x_0) = \frac{1}{2} = B_{N+1}(x_N), \\ B_2(x_0) = 0 = B_{N+2}(x_N). \quad (2.18)$$

Therefore,

$$c_0 + c_1 = 2\alpha \quad (2.19)$$

$$c_N + c_{N+1} = 2\beta \quad (2.20)$$

Notes

$$\begin{pmatrix} 0 & 0_1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & p_1 & q_1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & p_2 & q_2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & p_{N-2} & q_{N-2} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & p_{N-1} & q_{N-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0_2 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_{N-2} \\ c_{N-1} \\ c_N \end{pmatrix} = 2 \begin{pmatrix} z_0 \\ h^2 f(x_1) \\ h^2 f(x_2) \\ \vdots \\ h^2 f(x_{N-2}) \\ h^2 f(x_{N-1}) \\ z_N \end{pmatrix} \quad (2.21)$$

Where

$$p_i = 2a_1(x_i) - 2ha_2(x_i) + a_3(x_i)h^2,$$

$$q_i = -4a_1(x_i) - 2ha_2(x_i) + a_3(x_i)h^2,$$

$$0_1 = q_0 - p_0,$$

$$0_2 = p_N - q_N,$$

$$z_0 = h^2 f(x_0) - \alpha p_0,$$

$$z_N = h^2 f(x_N) - \beta q_N.$$

The quadratic B-spline approximation for the BVP (3.1-3.2) is obtained using (2.13), where the constant coefficients  $c_i$  satisfy the system defined in (2.21).

### III. NUMERICAL EXAMPLE

Let us consider a linear boundary value problem with constant coefficients

$$y'' + y' - 6y = x \quad \text{for } 0 < x < 1 \quad (3.1)$$

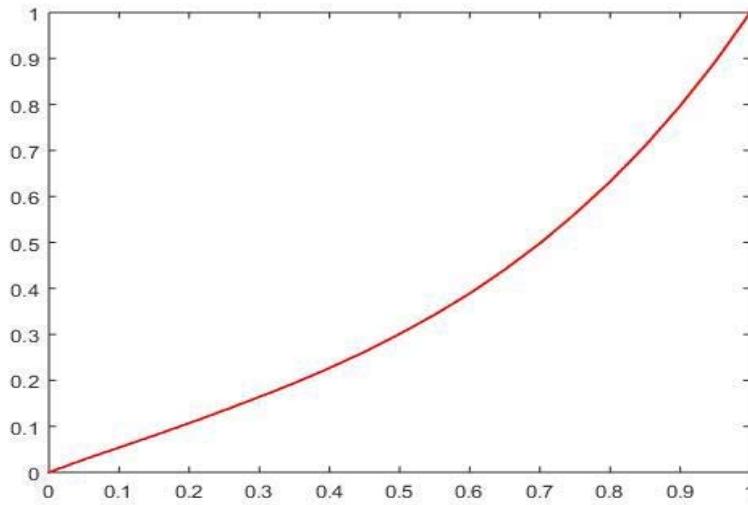
With boundary conditions

$$y(0) = 0, \quad y(1) = 1 \quad (3.2)$$

The exact solution to the boundary value problem is

$$y(x) = \frac{(43 - e^2)e^{-3x} - (43 - e^{-3})e^{2x}}{36(e^{-3} - e^2)} - \frac{x}{6} - \frac{1}{36} \quad (3.3)$$

The graph of the exact solution using MATLAB 7.0 is given below



*Fig. 2:* Exact solution curve

We approximate the solution of (3.1) with the boundary conditions (3.2) using the quadratic B-spline method with  $N = 20$ .

In order to use (2.13), we first need to find the constant coefficients  $c_i$  for  $i = -1, 0, 1, \dots, 21$  using the system of linear equations (2.21) where the coefficient matrix is an  $21 \times 21$  matrix and using (2.19) and (2.20) to find  $c_{-1}$  and  $c_{21}$  respectively. These coefficients are given below

$$\begin{array}{lllll}
 c_{-1} = 3.37050 & c_0 = -1.37050 & c_1 = -0.62963 & c_2 = -0.28932 & c_3 = -0.133041 \\
 c_4 = -0.61303 & c_5 = -0.28407 & c_6 = -0.133544 & c_7 = -0.64997 & c_8 = -0.34113 \\
 c_9 = -0.20532 & c_{10} = -0.00149 & c_{11} = -0.129210 & c_{12} = -0.12619 & c_{13} = -0.13087 \\
 c_{14} = -0.13910 & c_{15} = -0.14896 & c_{16} = -0.159565 & c_{17} = -0.17051 & c_{18} = -0.18161 \\
 c_{19} = -0.19279 & c_{20} = -0.20400 & c_{21} = 2.0020400 & & 
 \end{array}$$

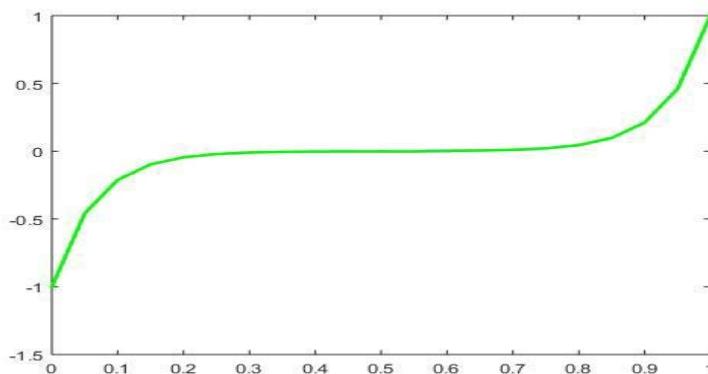
and therefore the quadratic polynomials are as follows

Notes

## Notes

$$Y(x) = \begin{cases} -22.24775x^2 + 14.81739x - 1.00007 & \text{for } x \in [0.00, 0.05) \\ -10.19669x^2 + 7.82581x - 0.82528 & \text{for } x \in [0.05, 0.10) \\ -4.64808x^2 + 4.05526x - 0.57023 & \text{for } x \in [0.10, 0.15) \\ -2.08680x^2 + 2.06079x - 0.35934 & \text{for } x \in [0.15, 0.20) \\ -0.89796x^2 + 1.01712x - 0.21236 & \text{for } x \in [0.20, 0.25) \\ -0.33963x^2 + 0.47087x - 0.11737 & \text{for } x \in [0.25, 0.30) \\ -0.70978x^2 + 0.17968x - 0.57443 & \text{for } x \in [0.30, 0.35) \\ 0.64597x^2 + 0.16550x - 0.18661 & \text{for } x \in [0.35, 0.40) \\ 0.13903x^2 - 0.84064x + 0.86481 & \text{for } x \in [0.40, 0.45) \\ 0.18538x^2 - 0.15558x + 0.30699 & \text{for } x \in [0.45, 0.50) \\ 0.21882x^2 - 0.21486x + 0.51335 & \text{for } x \in [0.50, 0.55) \\ 0.24634x^2 - 0.27037x + 0.72908 & \text{for } x \in [0.55, 0.60) \\ 0.27113x^2 - 0.32629x + 0.90995 & \text{for } x \in [0.60, 0.65) \\ 0.29467x^2 - 0.38472x + 0.12422 & \text{for } x \in [0.65, 0.70) \\ 0.31764x^2 - 0.44666x + 0.15558 & \text{for } x \in [0.70, 0.75) \\ 0.34034x^2 - 0.51263x + 0.19149 & \text{for } x \in [0.75, 0.80) \\ 0.36292x^2 - 0.58286x + 0.23237 & \text{for } x \in [0.80, 0.85) \\ 0.38544x^2 - 0.65747x + 0.27861 & \text{for } x \in [0.85, 0.90) \\ 0.40794x^2 - 0.73653x + 0.33057 & \text{for } x \in [0.90, 0.95) \\ 0.43043x^2 - 0.82006x + 0.38861 & \text{for } x \in [0.95, 1.00) \end{cases}$$

The figure of quadratic B-spline by these polynomials using MATLAB 7.0 is given below



*Fig. 3:* Quadratic Polynomial

## Comparing Exact values with Quadratic B-spline for Example-1

$x_i$	Quadratic B - spline	Exact	Absolute Error
0.00	-1.00006612	0.0000000000	-1.00006612
0.05	-0.45947752	0.0275370031	0.487014523
0.10	-0.211182808	0.0542570003	0.265439808
0.15	-0.097172806	0.0807133503	0.177886156
0.20	-0.0448544	0.1074285617	0.152282961
0.25	-0.020879375	0.1349034523	0.155782827
0.30	-0.0099242	0.1636255435	0.173549743
0.35	-0.004955225	0.1940768511	0.199032076
0.40	-0.0027292	0.2267412146	0.229470414
0.45	-0.00178155	0.2621112965	0.263892846
0.50	-0.001385	0.3006953693	0.302080369
0.55	-0.00127565	0.3430239998	0.344299649
0.60	-0.0071772	0.3896567348	0.396833934
0.65	-0.001349925	0.4411888843	0.442538809
0.70	-0.0014384	0.4982584988	0.499696898
0.75	-0.001544125	0.5615536314	0.563097756
0.80	-0.0016492	0.6318199790	0.633469179
0.85	-0.0017591	0.7098689951	0.711628095
0.90	-0.0018756	0.7965865702	0.79846217
0.95	-0.001983925	0.8929423791	0.894926304
1.00	1.0000001	1.0000000000	-0.0000001

Notes

## IV. CONCLUSION

This paper presented the application of quadratic B-spline function to solve the boundary value problems. At first, we have derived the quadratic B-spline functions and hence derived the methods. Then we used these methods to solve second order linear boundary value problems. After solving these problems, we compare the numerical solution with the exact solution. The comparative graph have shown some error in quadratic B-spline in comparison to the exact solution of the same function.

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