Hydrodynamics of Underwater «Physical Explosion»

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Hydrodynamics of Underwater «Physical Explosion»

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Abstract- The modern scientific and technological revolution has led to profound changes in the technique and technology of marine seismic research, ensuring an increase in labor productivity, a significant increase in the volume of work, as well as an increase in their efficiency. At the same time, along with the use of floating piezoelectric seismograph, digital recording equipment, modern electronic computing equipment and more accurate satellite navigation systems, one of the important factors that contributed to improving the efficiency of seismic exploration was the introduction of a new generation of seismic signal excitation devices – non-explosive sources – into the practice of marine seismic research. Of these, the most widely used in seismic exploration in water areas around the world are pneumatic sources, in which elastic waves are excited by underwater exhaust of compressed air.

The purpose of this work is to select and justify a theoretical model that adequately describes the process of pulsation in the water of the air cavity, as well as its acoustic radiation. The model chosen as a result can be used for mathematical modeling of new designs of pneumatic sources that are being developed.

Keywords: pneumatic source, physical explosion, hydrodynamics of anoscillating bubble, pressure field of anoscillating bubble, rayleigh equation, herring equation, keller-kolodner equation, kirkwood-be the approximation.

I. Equations of Motion in a Liquid of Spherical Gaseous Cavity. Pressure Field of The Oscillating bubble. Basic Quantitative Ratios

From the point of view of hydrodynamics, a pneumatic source belongs to the class of sources of the «oscillating bubble» type. In the theory of sound radiation of a oscillating bubble in an unlimited extent of water, the main problem is the description of its motion. A significant number of works related to the study of acoustic cavitation [1, 2, 14], the calculation of the pressure field from the explosion of condensed explosives [8, 9, 15], from electric discharges in water [10, 11], from the exhaust of compressed air into water by a pneumatic source are devoted to this issue[4, 12, 13, 16, 20].

In the study of the problem of describing the motion of a cavity, the classical solution of the problem of collapse of a spherical cavity in an unlimited volume of a non-viscous incondensable and devoid of surface tension liquid under the influence of constant pressure, given by Rayleigh [9], became fundamental. All further decisions are essentially reduced to analyzing the influence of Rayleigh's assumptions and taking into account the actual properties of the liquid.

Several types of approximations can be used to describe the motion of the cavity. All of them take into account the compressibility of the liquid in different ways, and each of them leads to a certain nonlinear differential equation of motion of the cavity interface.

a) Zero-order approximation (Rayleigh`s equation)

Assuming the fluid is incompressible $(\rho_0 = const)$, it is easy to show that the motion of the interface is described by a second-order nonlinear differential equation [8]:

$$R'' + \frac{3}{2} \cdot \frac{R'^2}{R} = \frac{1}{\rho_0 R}(P - P_0), \qquad (1)$$

where $R(t)$– the radius of the cavity; ρ_0– the density of the liquid; $P(t)$ – the pressure in the liquid at the interface; P_0– the hydrostatic pressure.

If it is necessary to study the sound radiation of the cavity, the incompressibility condition must be modified so that the speed of sound in the liquid is finite. In cases where the speed of expansion of the sphere is small compared to the speed of sound in a divergent medium, the density perturbations caused by the expansion of the sphere will also be small. Therefore, for such processes, the propagation of a divergent wave can be described by a solution of linear acoustics that satisfies the boundary condition of velocity continuity on the surface of a sphere. Then the pressure distribution in the liquid is given by the equation [9, 11]:

$$p(r,t) - P_0 = \rho_0 \frac{R^2 R'' + 2R'^2 R}{r} - \rho_0 \frac{R'^2}{2} \frac{R^4}{r^4}, \quad (2)$$

Or excluding R'' using the equation(2.1),

$$p(r,t) - P_0 = \rho_0 \frac{R}{r}\left(\frac{P - P_0}{\rho_0} + \frac{1}{2}R'^2\right) - \rho_0 \frac{R'^2}{2} \frac{R^4}{r^4}, \quad (3)$$

where r – radial coordinate.

Denoting $V = 4/3\ \pi\ R^3$– the volume of the sphere, we get from (2)

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$$p(r,t) - P_0 = \rho_0 \frac{V''}{4\pi r} - \frac{1}{2} \frac{V'^2}{(4\pi)^2 r^4}.$$

In the wave zone, the second term on the right side of this formula is negligible, so we can write down

$$p(r,t) - P_0 = \rho_0 \frac{V''}{4\pi r}.$$

This shows that *the pressure in the liquid is proportional to the volume acceleration of the gas bubble*. The maximum acceleration will be at the minimum volume of the bubble. Thus, in the process of cavity pulsations, repeated pressure pulses are emitted at each minimum volume.

Differential equation (1) does not allow us to take into account the energy emitted by the compression wave, and is a zero-order approximation in which all terms of the order R'/c are ignored (where c is the speed of sound in a liquid). The elastic modulus is assumed to be infinitely large and, consequently, the speed of sound in water is infinitely large as well.

Solution of the Rayleigh equation, Rayleigh-Willis formula

Considering the collapse of a spherical cavity having at the initial moment the radius R_{max} and the pressure P inside, significantly less than the hydrostatic $P0$, and taking into account that this pressure ratio persists for most of the period of pulsations, Rayleigh obtained a simple solution to the equation (1) [14]:

$$R'^2 = \frac{2(P_0 - P)}{3\rho_0} \left(\frac{R_{max}^3}{R^3} - 1 \right) \approx \frac{2}{3} \frac{P_0}{\rho_0} \left(\frac{R_{max}^3}{R^3} - 1 \right). \quad (4)$$

This solution describes the collapse of the Rayleigh cavity. Based on it, Rayleigh determined the time t_{max}, required for complete collapse of the cavity under the condition $R_0 << R_{max}$:

$$t_{max} = \sqrt{\frac{3\rho_0}{2P_0}} \int_{R_0}^{R_{max}} \frac{dR}{\sqrt{\left(\frac{R_{max}}{R} \right)^3 - 1}}.$$

For $R_0 = 0$ this integral can be taken using Γ-function:

$$t_{max} = 0{,}915 R_{max} \left(\frac{\rho_0}{P_0} \right)^{\frac{1}{2}}.$$

This time is approximately equal to half the period of pulsation of the cavity. Hence

$$T \approx 2 t_{max} = 1{,}83 R_{max} \left(\frac{\rho_0}{P_0} \right)^{\frac{1}{2}}. \quad (5)$$

Expressing the maximum radius in terms of the total energy of the pulsating bubble, Willis obtained [9, 11, 18]:

$$T = 1{,}14 \rho_0^{\frac{1}{2}} \frac{E^{\frac{1}{3}}}{P_0^{\frac{5}{6}}}. \quad (6)$$

Formula (6) is often called the Rayleigh-Willis formula. It is very useful for estimating the relative energy and acoustic characteristics of various types of marine seismic sources. As can be seen from the Rayleigh-Willis formula, the pulsation period increases in proportion to the cube root of the total energy and decreases in proportion to the hydrostatic pressure in the power of 5/6. For processes accompanied by the formation of spherical gas bubbles in water (explosions of condensed explosives, underwater gas explosions, electric discharge, exhaust of compressed gas), this dependence is in better agreement with the experiment, the greater the depth of immersion of the source and the less heat exchange between the gas in the bubble and the surrounding liquid. Note, however, that formula (6) does not apply if the pulsating bubble is located near the liquid boundary due to a change in the nature of the spreading flows.

b) First-order approximation: Herring and Keller-Kolodner equations

The assumption that the speed of sound in water is permanent (and in this case the elastic modulus is considered as a constant and there is a linear dependence of pressure on density) leads to a first-order approximation, which contains terms of the order R'/c. Acoustic approximation allows us to take into account the energy loss due to radiation in this case.

The transformations performed by Herring [14] under the assumption $c = c_0 = = const$, give the equation:

$$\left(1 - \frac{2R'}{c_0} \right) R'' + \frac{3}{2} \left(1 - \frac{4}{3} \frac{R'}{c_0} \right) \frac{R'^2}{R} = \frac{1}{\rho_0 R} \left[P - P_0 + \frac{R}{c_0} \left(1 - \frac{R'}{c_0} \right) P' \right]. \quad (7)$$

Compatible equation was obtained by Keller and Kolodner [17]:

$$\left(R' - c_0 \right) \left(RR'' + \frac{3}{2} R'^2 - H_0 \right) - R'^3 + 2R'H_0 + RH_0' = 0, \quad (8)$$

where $H_0 = \dfrac{P - P_0}{\rho_0}$.

For the case of an incompressible fluid ($c_0 \to \infty$), equations (7) and (8) are transformed into the Rayleigh equation (1).

The expression for the pressure field around the oscillating bubbleis obtained in [17] (or equation (13.333) from [8]):

$$\frac{p(r,t) - P_0}{\rho_0} = -\frac{f'(t_1)}{r} - \frac{f^2(t_1)}{2r^4} - \frac{1}{2c_0}\left[\frac{f'^2(t_1)}{c_0 r^2} + \frac{2f(t_1)f'(t_1)}{r^3}\right], \qquad (9)$$

where$t_1 = t - r/c_0$; f and f' are given by the equations

$$f = -R^2 R' + \frac{R^2}{c_0}\left(\frac{P - P_0}{\rho_0} + \frac{R'^2}{2}\right), \qquad f' = -R\left(\frac{P - P_0}{\rho_0} + \frac{R'^2}{2}\right). \qquad (10)$$

In equation (9), Keller and Kolodner left the first two terms and omitted the second two, without examining their relative values [17]. In [21], an attempt was made to prove that the second term in equation (9) is not related to linear elastic theory and Newton's second law. Indeed, in deriving equation (9), Keller and Kolodner began their analysis with the wave equation, which applies only to a linearly compressible fluid with a constant speed of sound, when the particle velocity is sufficiently small. Then they linked the particle velocity to pressure via the Bernoulli equation, which has a much broader application than the wave equation. The authors of [21] correctly pointed out the incompatibility of the wave equation and the Bernoulli equation and the appearance of terms in equation (9) that are not related to linear elastic theory. However, in their analysis, they also omitted terms that are significant for linear theory when calculating the pressure field in the near zone.

Since the particle velocity in a linearly compressible fluid is quite small, we transform equation (9), by omitting the terms of the order$(R'/c)^2$. In the end we get:

$$p(r,t) - P_0 = \rho_0 \frac{R}{r}\left(\frac{P - P_0}{\rho_0} + \frac{R'^2}{2}\right)\left[1 - \frac{R'}{c_0}\frac{R^2}{r^2}\left(1 - \frac{R}{r}\right)\right] - \rho_0 \frac{R'^2}{2}\frac{R^4}{r^4}. \qquad (11)$$

If in (9) we neglect the term of the order R'/c_0, we get equation (3).

c) Second-order approximation (approximation of Kirkwood-Bethe)

For the case when the rate of expansion of the cavity in the liquid is high and the density perturbations caused by the expansion of the cavity are significant, the Kirkwood-Bethe approximation is applicable [8, 9]. This method, developed in the study of underwater explosions, consists in determining the invariant of motion, which is taken as the function

$$\Phi = r(h + u^2/2),$$

where h –specific enthalpy of the liquid; u –particle velocity.

The value of Φ propagates at a constant speed equal to $u + c$ (where c is the local speed of sound). Under this condition, the equation of motion of the interface takes the form:

$$\left(1 - \frac{R'}{c}\right)R'' + \frac{3}{2}\left(1 - \frac{R'}{3c}\right)\frac{R'^2}{R} = \left(1 + \frac{R'}{c}\right)\frac{H}{R} + \left(1 - \frac{R'}{c}\right)\frac{H'}{c}, \qquad (12)$$

where $H(t)$ and $c(t)$ are the specific enthalpy and speed of sound in the liquid at the interface, respectively. Both of these functions depend on the pressure $p(t)$. Equation (12) was first obtained by Gilmore [8, 9]. The relationship between pressure and enthalpy is found from the experimental dependence of pressure on density under isentropic compression, which is described by the formula [11]:

$$\frac{p + B}{P_0 + B} = \left(\frac{\rho}{\rho_0}\right)^n, \qquad (13)$$

where B and n –constants that depend on the type of liquid; for water $B = 2\,500$ atm, $n = 8$ [20], or $B = 3\,000$ atm, $n = 7$ [11]. Using the equation (13),

weget $c^2 = \dfrac{dp}{d\rho} = \dfrac{n(p + B)}{\rho} = \dfrac{n(p + B)}{\rho_0}\left(\dfrac{p + B}{P_0 + B}\right)^{-\frac{1}{n}}$,

hence

$$c = c_0\left(\frac{p + B}{P_0 + B}\right)^{\frac{n-1}{2n}}, \qquad (14)$$

where c_0–speed of sound propagation in an undisturbed liquid.

$$h(p) = \int_{P_0}^{P} \frac{dp}{d\rho} = \int_{P_0}^{P} \left(\frac{p+B}{P_0+B} \right)^{-\frac{1}{n}} \frac{dp}{\rho_0} = \frac{n(P_0+B)}{(n-1)\rho_0} \left[\left(\frac{p+B}{P_0+B} \right)^{\frac{n-1}{n}} - 1 \right].$$

(15)

On the surface of the sphere $h = H$, $c = C$, $p = P$, and from equations (14) and (15) we obtain:

$$C = c_0 \left(\frac{P+B}{P_0+B} \right)^{\frac{n-1}{2n}},$$

(16)

$$H = \frac{n(P_0+B)}{(n-1)\rho_0} \left[\left(\frac{P+B}{P_0+B} \right)^{\frac{n-1}{n}} - 1 \right].$$

(17)

To compare equation (12) with equations (7) and (8), transform equation (16)

$$c = c_0 \left(\frac{P+B}{P_0+B} \right)^{\frac{n-1}{2n}} = c_0 \left(1 + \frac{P-P_0}{P_0+B} \right)^{\frac{n-1}{2n}} = c_0 \left[1 + \frac{n-1}{2n} \frac{P-P_0}{P_0+B} - \frac{n^2-1}{(2n)^2 n!} \left(\frac{P-P_0}{P_0+B} \right)^2 + O\left(\left(\frac{P-P_0}{P_0+B} \right)^3 \right) \right]$$

and determine the order of value

$$\frac{P-P_0}{P_0+B} = \frac{\dfrac{P-P_0}{\rho_0}}{\dfrac{P_0+B}{\rho_0}} = \frac{\dfrac{P-P_0}{\rho_0}}{\dfrac{c_0^2}{n}}.$$

According to the approximate solution of equation (1) given by Rayleigh [9], the value $\dfrac{P-P_0}{\rho_0}$ is proportional to R'^2. Therefore, the expression $\dfrac{P-P_0}{P_0+B}$ has the order $(R'/c_0)^2$. Thus, up to terms of order $(R'/c_0)^2$

$$C = c_0.$$

(18)

Similarly, we can show that, neglecting the terms of order $(R'/c)^2$, for the enthalpy H we get

$$H = \frac{P-P_0}{\rho_0}.$$

(19)

Substituting the expressions (18) and (19) in equation (12) and neglecting the term $\dfrac{R'}{\rho c_0^2} P'$, having an order $(R'/c_0)^2$, we get the Keller and Kolodner equation (8).

We now transform equation (12) by multiplying each of its terms by c:

$$[(c+R') - 2R']R'' + \frac{3}{2} \left[(c+R') - \frac{4}{3} R' \right] \frac{R'^2}{R} = (c+R') \frac{H}{R} + [(c+R') - 2R'] \frac{H'}{c}.$$

Dividing each term of this equation by $c + R'$ and neglecting the terms of order $(R'/c)^2$, we arrive at the equation

$$\left(1 - 2 \frac{R'}{c_0} \right) R'' + \frac{3}{2} \left(1 - \frac{4}{3} \frac{R'}{c_0} \right) \frac{R'^2}{R} = \frac{1}{\rho_0 R} \left(P - P_0 + \frac{R}{c_0} P' \right),$$

(20)

which is accurate to the term $\dfrac{R'}{\rho c_0^2} P'$, having an order $(R'/c)^2$, coincides with the Herring equation (7).

Thus, the Kirkwood-Bethe approximation allows us to take into account terms of order $(R'/c)^2$ and is a second-order approximation. It can be considered proved that the Herring (7) and Keller-Kolodner (8) equations coincide with each other up to terms of order $(R'/c)^2$.

Taking into account the terms of the order $(R'/c)^2$ the pressure field of the oscillating bubble is calculated using the formulas [20]:

$$p(r,t) - P_0 = \rho_0 \left(\frac{y}{r} - \frac{u^2}{2} \right) + \frac{\rho_0}{2c_0^2} \left(\frac{y}{r} - \frac{u^2}{2} \right)^2 , \tag{21}$$

where $y = R\left(H + \dfrac{R'^2}{2} \right)$;

$$u = \frac{y}{c_0 r} + \frac{k_3 y^2}{c_0^3 r^2} \left(1 - \frac{y}{c_0^2 r} + \frac{k_3^2 y^4}{2c_0^8 r^4} \right); \tag{22}$$

$$k_3 = \frac{c_0^3 R^2 R'}{y^2} \left(1 - \frac{R'^2}{2c_0^2} \right) - \frac{c_0^2 R}{y} \left(1 - \frac{R'}{c_0} \right).$$

It is easy to show that, ignoring the terms of order $(R'/c)^2$, we find an expression for the pressure field from (21) that coincides with equation (11).

II. Results of Numerical Solution of Equations of the Motion of Spherical Cavity. Analysis of the Assumptions made in Relation to the Underwater Exhaust of Compressed Air

a) *Comparison of results of numerical solution of equations of the motion of a spherical gas cavity in a liquid*

So, to describe the motion of a pulsating spherical bubble in a liquid, we can apply three different differential equations, depending on the order to which the terms R'/c are taken into account. The assumption of incompressibility of the fluid leads to the differential equation (1), in which all terms of the order R'/c are ignored – this is an approximation of the zero order. The pressure field in a liquid is described by equation (3).

The assumption that the speed of sound in a liquid is constant leads to equation (7) or (8), which take into account the terms of the order R'/c and which coincide with each other up to the terms of the order $(R'/c)^2$. This is a first-order approximation. The pressure field is given by equation (11). Finally, the Kirkwood-Bethe approximation leads to differential equation (12), which takes into account second-order terms with respect to R'/c. This is a second-order approximation. The pressure field is given by equation (21).

Let's assume that a sphere with radius R_0 (initial volume of the sphere $V_{01} = (4/3)\pi R_0^3$) with gas pressure inside P_{01} exceeding the hydrostatic pressure is placed in an unlimited extent of liquid with density ρ_0 and hydrostatic pressure P_0. At the initial moment, the shell of the sphere is instantly removed and the bubble expands with acceleration under the influence of the pressure difference in the bubble and in the surrounding liquid. In this case, an acoustic signal is emitted, the maximum pressure of which is observed at $t = 0$. Assuming that the change in the state of the gas in the bubble occurs according to the adiabatic law, the pressure in the bubble as a function of its radius is described by the dependence

$$P = P_{01} \left(\frac{V_{01}}{V} \right)^\gamma = P_{01} \left(\frac{R_0}{R} \right)^{3\gamma} , \tag{23}$$

where γ–is the adiabatic exponent (for air $\gamma = 1,4$). Knowing the pressure in the bubble, we can solve differential equations (1), (7) (or (8)) and (12) by setting the following initial conditions:

$$R(0) = R_0, \quad R'(0) = 0, \text{при этом } P(0) = P_{01}. \tag{24}$$

Numerical solutions of these equations performed in MathCAD by the fourth-order Runge-Kutta-Gill method at $P_{01} = 12\ МПа$, $P_0 = 0,2\ МПа$, $R_0 = 0,089$ м ($V_{01} = 3\ дм^3$), are shown in Fig. 1. The solutions of equations (7), (8), representing the first-order approximation, and equation (12), describing the second-order approximation, as well as the pressure in the liquid, calculated respectively from equations (11) and (21), completely coincide and are represented by a

single curve. Fig. 1 (a) shows the change in the bubble radius as a function of time. Fig. 1 (b) shows acoustic pressure signals in the far zone, reduced to a distance of 1 m from the emitter. As can be seen from figure 1, equation (1) gives a sustained solution. Equations (7), (8), and (12), which take into account the compressibility of the liquid, allow for energy losses due to radiation. In this case, along with the attenuation of the amplitude, the period of pulsations also decreases.
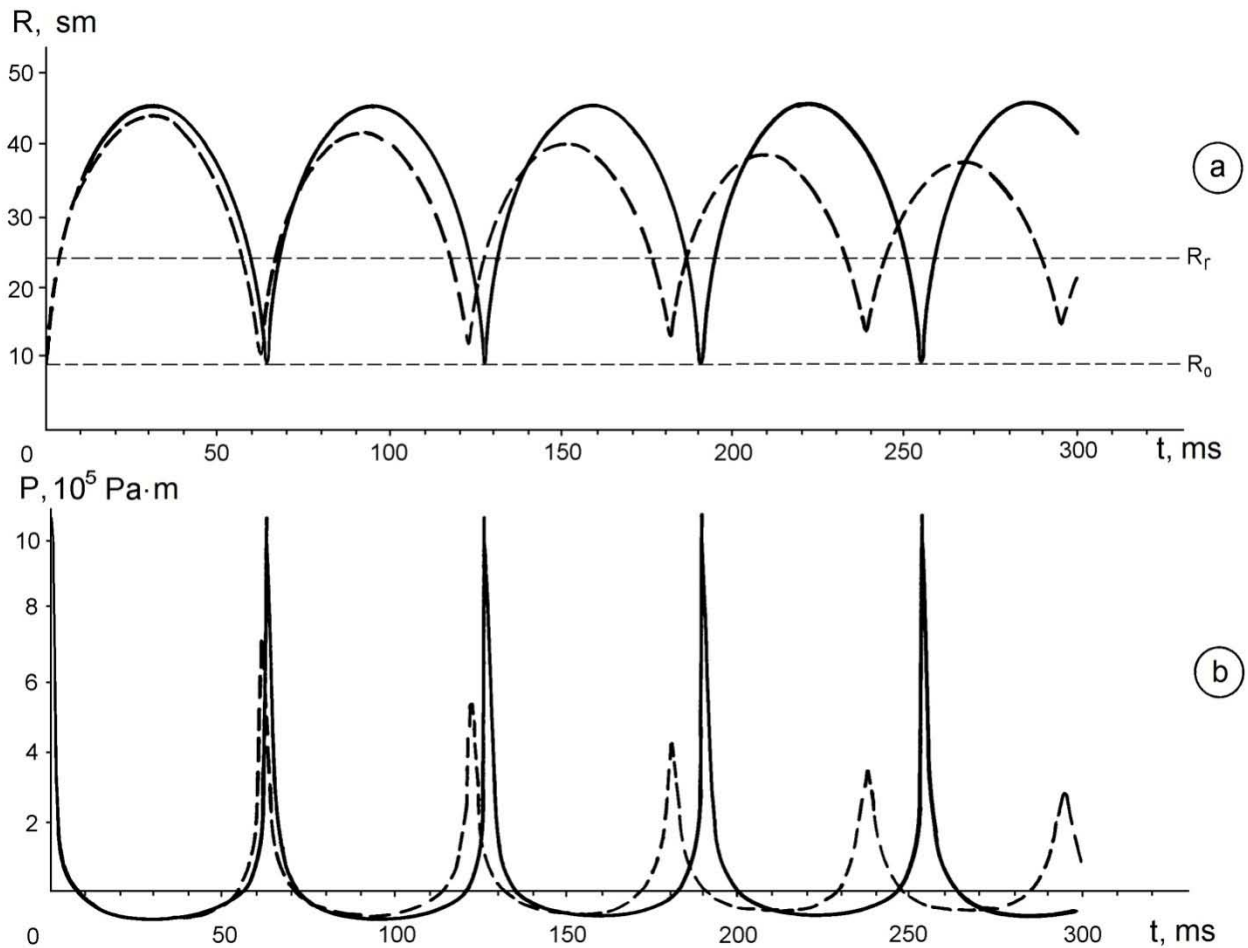
Fig. 1: Numerical solution of equations (1), (7), (8) and (12) at $P_{01} = 12$ *MPa*, $P_0 = 0,2$ *MPa*, $R_0 = 0,089$ *m*; (*a*) – dependence of the bubble radius on time; (*b*) – pressure in the compression wave; _____ equation (1); _ _ _ _ equations (7), (8) and (12)

Numerical analysis of the equations of motion of the cavity shows that at an initial pressure $P_{01} = 10 \div 15 MPa$ for volumes $V_{01} = 0, 1 \div 10$ *dm³* (namely, at such pressures and volumes of operating chambers, pneumatic sources are used), approximations of the first and second orders give the same results. Since the vibrations of a free bubble represent an idealized case for a pneumatic source, when the energy losses associated with the flow of air through the exhaust windows of the source are ignored, the expansion rate of real bubbles will be at least no greater than the expansion rate calculated for free bubbles. Hence, we conclude that the rate of expansion of bubbles generated by pneumatic emitters is small and there is no need to use a second-order approximation.

The zero – order approximation – equation (1) – is the roughest approach to the problem of acoustic radiation and is mainly used to obtain semi-quantitative information about the oscillating bubble.

The first-order approximation remains, which leads to equations (7) and (8). In our opinion, the use of these equations is most advantageous, since with the same accuracy of the solution as equation (12), these equations are simpler. Since equations (7) and (8) coincide up to second-order terms with respect to R'/c, any of these equations can be used to describe the motion of a bubble formed by an underwater exhaust of compressed gas. The Keller-Kolodner equation (8) is most often used in the literature devoted to the theoretical study of underwater compressed air exhaust.

b) Analysis of the effect of viscosity and surface tension

When a fluid moves radially, viscosity and surface tension are not included in the equation of motion, but they appear under boundary conditions. If we assume that the radial stress in the liquid and the pressure in the cavity must be continuous at the interface, we get [2, 14, 20]:

$$P(t) = P_g + P_v - \frac{2\sigma}{R} - \frac{4\mu R'}{R}, \qquad (25)$$

where P_g и P_v–partial pressures of gas and water vapor, respectively; σ– surface tension; μ– shear viscosity. For water, the value is: $\sigma = 72,5 \cdot 10^{-3} N/m$, $\mu = 10^{-3} kg/m \cdot s$.

The effect of surface tension and viscosity can be significant only for a very small radius of the bubble and a high speed of its surface movement, which is observed, for example, in acoustic cavitation [2, 14, 19]. For bubbles generated by pneumatic sources with operating chamber volumes from 0.1 to 10.0 dm^3, according to calculations, we obtain the following order of values: $R \approx 10^{-1} m$, $R \approx 10$ m/s. Thus, the order of magnitude of the stress that occurs under the action of surface tension and viscosity is:

$$\frac{2\sigma}{R} + \frac{4\mu R'}{R} \approx 1,85 \, N/m^2.$$

Since the minimum value of the gas pressure in the bubble, reached at $R = R_{max}$, has an order $P_{g\,min} = 0, 2 \, atm = 2 \cdot 10^4 N/m^2$, which is 4 orders of magnitude greater ($\frac{2\sigma}{R} + \frac{4\mu R'}{R}$), the effect of surface tension and viscosity in this process can be ignored. Thus,

$$P = P_g + P_v.$$

The water vapor pressure in the bubble P_v will reach its maximum when the bubble expands to its maximum volume and the gas pressure of the P_g becomes minimal ($P_g = 0, 2 \, atm$). The partial pressure of water vapor never exceeds 10% of the P_g value [20], so $P_{v\,max} = 0, 02 \, atm$, i.e. this value can be ignored. As a result, we get

$$P = P_g, \qquad (26)$$

that is, the pressure inside the bubble is caused only by the air pressure.

c) Physical models of pneumatic sources

 i. *Model in the form of a spherical gas layer*

In some works (Schulze-Gattermann R. [22]; Gribanov A.M., Akentyev L. G.[13]), theoretical estimates of the parameters of the pressure signal emitted by a pneumatic source were achieved. Physical model of the source is a spherical layer with compressed gas containing an absolutely rigid sphere.

This model is somewhat closer to reality and takes into account the influence of the source body on the expansion of the cavity, but the description of the dynamics of the process, and especially the acoustic radiation at the initial stage of the exhaust, is also approximate and differs from the experimental data.

 ii. *The Schulze-Gattermann model*

In contrast to the spherical layer model considered by Gribanov A.M. and Akentiev L.G. based on the Rayleigh equation for adiabatic changes in the state of gas in a bubble, the Schulze-Gattermann spherical layer model is based on the Keller-Kolodner equation under the assumption of isothermality of the process of changing the state of air in a bubble during its pulsation in water.

Some people calculated characteristics obtained for the Schulze-Gattermann model are somewhat closer to the experimental ones (the period of pulsations), but other indicators (the amplitude and shape of the signal) differ from the characteristics of the real process.

 iii. *The Safar model*

In contrast to all other models in which the pneumatic source is modeled as an air sphere with an initial volume equal to the volume of the operating chamber, in the theoretical model of Safar (Safar M. H. [16]), it is proposed to approximate the shape of the bubble at the initial stage of the process with an equivalent sphere, surface area of which is equal to the total area of the exhaust windows of the source. In addition, it is assumed that the first pulse of the emitted acoustic signal reaches a peak value when the exhaust port is fully opened, and the movement of the movable piston occurs under constant pressure.

The calculated characteristics of the underwater exhaust process obtained using the Safar model allow us to more accurately estimate not only the amplitude, steepness, and shape of the first pressure peak of the emitted signal, but also the period of its pulsation. However, the main parameters that characterize the dynamics of the emitter itself are determined in the Safar model either too approximately or not at all.

 iv. *The Maksakov-Roy Model*

An important step in the study of the process of underwater exhaust of compressed air was the work (Maksakov A. A., Roy N. A. [3]), which considers a system of differential equations describing the impulse flow of gas into water through a hole, cross-sectional area of which changes in time according to any predetermined law. The air pressures in a constant volume chamber and in a bubble are expressed from the law of conservation of energy and from the equation of state and conservation of mass of gas as it flows from the chamber to the bubble. The flow rate of gas into the bubble during an isentropic flow is given by the Saint-Venant and Wantzel formula, in which the area of the

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hole was set to vary linearly over time, and the Rayleigh equation was used to describe the movement of the bubble walls.

This model already allows us to quantify the effect of the opening speed of the exhaust port, its maximum area and the initial gas pressure on the acoustic efficiency of the underwater exhaust process. However, in real designs of pneumatic sources, the exhaust area is a complex function of both the design parameters and the gas parameters in the operating and control chambers and in the cavity, and does not change in time according to a linear law.

v. *Model of sources«Signal»*

The most adequate description of the underwater exhaust process can be constructed only taking into account the dynamics of the moving element of the pneumatic source. This model was presented in the work (Gulenko V. I. [4], Gulenko V. I., Karpenko V. D. [5, etc.]).

This model is implemented as a system of ten differential equations that depend on very many parameters, and is a further development of the idea laid down in the Maksakov-Roy model. An experimental study of the dynamic and acoustic characteristics of pneumatic sources of the «Signal» series [6] has showed a very good agreement with the theory.

III.   Conclusion

1. The Rayleigh differential equation, which describes the simplest model of a pneumatic source in the form of a gas cavity, does not take into account the compressibility of water, the presence of a source, and the influence of processes occurring in it. Therefore, this model can be used only in the first approximation as a purely qualitative illustration of the process of pulsation in the liquid of an air bubble.
2. In the model described by the Keller-Kolodner equation, energy losses due to acoustic radiation are taken into account, so its solution is decaying and non-periodic. However, it also does not take into account the presence of the source and the influence of processes occurring in it, as well as the heat exchange between the air in the cavity and the surrounding liquid.
3. The Kirkwood-Bethe approximation was developed for underwater explosions of condensed explosives characterized by very high explosion product pressures and, consequently, high bubble expansion rates comparable to the speed of sound in water.

   In addition, calculations show that the solution of the Gilmore equation completely coincides with the solution of the Keller-Kolodner equation to describe the free vibrations of an air bubble in a compressible liquid for the initial parameters of the

problem corresponding to the typical parameters of an underwater pneumatic explosion, so the use of the Kirkwood-Bethe approximation for solving this class of problems is not justified.
4. The most adequate description of the process of underwater exhaust of compressed air can be built on the basis of the Keller-Kolodner equation, but with mandatory consideration of the dynamics of the moving element of the pneumatic source, which determines the mode of air flow from the operating chamber to the expanding cavity.

IV.   Annotation

The piece of work is devoted to the selection and justification of a theoretical model of a pneumatic source that adequately describes the process of pulsation in the water of an air cavity, as well as its acoustic radiation.

The classical solution of this problem given by Rayleigh for an incompressible liquid, the Herring and Keller-Kolodner differential equations obtained for a compressible fluid, and the Kirkwood-Bethe approximation developed for underwater explosions of condensed explosives are considered as possible approximations of this process.

It is shown that for those modes of expansion of the cavity that are typical for underwater exhaust of compressed air by a pneumatic source, the most adequate description of the pulsation process is the description of the pulsation process of the air cavity in a compressible liquid – the Keller-Kolodner differential equation. It is also shown that the influence of water viscosity and surface tension on the walls of the cavity can be ignored.

References Références Referencias

1. Akulichev V.A., Boguslavsky Yu.Ya., Ioffe A.I., Naugolnykh K.A. Radiation of spherical waves of finite amplitude // Journal of the Acoustical. 1967. V.13. №3. P. 321–328.
2. Levkovsky Yu.L. Dynamics of a spherical cavitation cavity (review) // Proceedings of the acoustic Institute. 1969. V.6. P. 102–123.
3. Maksakov A.A., Roy N.A. About the effectiveness of the radiation pulse compression at the exhaust of compressed air into the water // Journal of the Acoustical. 1980. T. 26. V.5. P. 764–768.
4. Gulenko V.I. Pneumatic sources of elastic waves for marine seismic exploration: Monograph. Krasnodar: The Kuban State University, 2003. 313 pp.
5. Gulenko V.I., Karpenko V.D. Theoretical study of dynamic and acoustic characteristics of pneumatic emitters for marine seismic exploration // Geophysics of the XXI century: 2001: Collection of works of the Fourth geophysical readings named

after V.V. Fedynsky. Moscow: Scientific world, 2002. P. 278–284.

6. Gulenko V.I., Shlykov V.A. Experimental study of dynamic and acoustic characteristics of the pneumatic transducers to the marine seismic exploration // Geophysics of the XXI century: 2001: Sat. proceedings of the Fourth geophysical readings named. V.V. Fedynsky. Moscow: Scientific world, 2002. P. 285–292.

7. Bath M. Spectral analysis in Geophysics: Translation with Engl. Moscow: Nedra, 1980. 535 pp.

8. Baum F.A., Orlenko L.P., Stanyukovich K.P., Chelyshev V.P., Shekhter B.I. Physics of explosion. Moscow: Nauka, 1975. 704 pp.

9. Cole R. Underwater explosions: Translation with Engl. Moscow: IL, 1950. 494 pp.

10. Naugolnykh K.A., Roy N.A. Hydrodynamic phenomena at electric discharges in water // Proceedings of the acoustic Institute. 1967. №3. P. 100–127.

11. Naugolnykh K.A., Roy N.A. Electric discharges in water. Moscow: Nauka, 1971. 155 pp.

12. Gribanov A.M. Estimation of some characteristics of a pneumatic elastic pulse Generator // Izvestiya vuzov. Geology and exploration. 1972. №.1. P. 128–132.

13. Gribanov A.M., Akentyev L.G. On the use of a spherical layer of compressed gas for generating elastic pulses // Geological and geophysical research on oil and gas on the shelves of the seas. Moscow: Vniiegazprom, 1980. P. 49–52.

14. Physical acoustics / under the editorship W. Mason. Vol. 1. Methods and devices of ultrasound research/ Moscow: Mir, 1967, 362 pp.

15. Heuckroth L.E., Glass I.I. Low-energy underwater explosions // The physics of fluids. 1968. V.11. №10. P. 2095–2107.

16. Safar M.H. The Radiation of acoustic waves from an air-gun // Geophysical Prospecting. 1976. V.24. №4. P. 756–772.

17. Keller J.B., Kolodner I.I. Dampind of underwater explosion bubble oscillations // Journal of Applied Physics. 1956. P. 1152–1161.

18. Kramer F.S., Peterson R.A., Walter W.C. Seismic energy sources // Offshore Technology Conference, 1-st. Houston, Texas, Proceeding. V.2. 1969. P. 387–416.

19. Miksis M.J., Lu T. Nonlinear radial oscillations of a gas bubble including thermal effects // Journal of the Acoustical Society of America. 1984. V.76. №3. P. 897–905.

20. Ziolkowsky A. A method for calculating the output pressure waveform from an air-gun // Geophysics, J. R. Astr. Soc. 1970. V.21. №2. P. 137–161.

21. Ziolkowsky A., Parkes G., Hatton L., Haughland T. The signature of an air gun array: computation from near-field measurements including interactions // Geophysics. 1982. V.47. №10. P. 1413–1421.

22. Schulze-Gattermann R. Physical aspects of the «Airpulser» as a seismic energy source // Geophysical Prospecting. 1972. V.20. №1. P. 155–192.