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On Lorentzian α–Sasakian Manifolds

By Archana Srivastava & Amit Prakash
Sr Institute of Management and Technology

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ONLORENTZIANSASAKIAN MANIFOLDS

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20. PRASAD, B., (2002), 'A pseudo projective curvature tensor on a Riemannian manifold', *Bull Calcutta Mathematical Society*, pp163–166.

On Lorentzian α–Sasakian Manifolds

Archana Srivastava [α] & Amit Prakash [σ]

Abstract- The object of this paper to study various curvature tensors in a Lorentzian $\boldsymbol{\alpha}$– Sasakian manifold, we also study $\boldsymbol{\varphi}$–pseudo projectively flat, $\boldsymbol{\varphi}$–quasi conformally flat, $\boldsymbol{\varphi}$–quasi concircularly flat, $\boldsymbol{\varphi}$–$\boldsymbol{m}$–projectively flat Lorentzian $\boldsymbol{\alpha}$–Sasakian manifolds are an $\boldsymbol{\eta}$–Einstein Manifold.

Keywords: φ–pseudo projectively flat, φ–quasi conformally flat, φ–quasi concircularly flat, φ–m–projectively flat, Lorentzian α–Sasakian manifold, η–Einstein Manifold.

I. Introduction

Let $(M^n, g), n > 3$, be a connected semi Riemannian manifold of class C^∞ and ∇ be its Levi–Civita connection. Riemannian curvature tensor R is defined by

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z \tag{1.1}$$

Pseudo–projective curvature tensor $\tilde{P}$on a Riemannian manifold $(M^n, g), n > 2$ of type$(1,3)$is defined as follows (Prasad [20]).

$$\tilde{P}(X,Y)Z = a\,R(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y] - \frac{r}{n}\left[\frac{a}{n-1} + b\right][g(Y,Z)X - g(X,Z)Y] \tag{1.2}$$

where a and b are constants such that $a, b \neq 0$.

If $a = 1$ and $b = -\frac{1}{n-1}$, then (2) takes the form

$$\tilde{P}(X,Y)Z = R(X,Y)Z - \frac{1}{n-1}[S(Y,Z)X - S(X,Z)Y] = P(X,Y)Z$$

where P is Projective curvature tensor. Thus the Projective curvature tensor P is a particular case of the tensor $\tilde{P}$, for this reason $\tilde{P}$ is called Pseudoprojective curvature tensor.

Quasi–conformal curvature tensor $\tilde{C}$ on a Riemannian manifold (M^n, g), $n > 2$ of type $(1,3)$is defined as follows (Yano and Swaki [24]).

$$\tilde{C}(X,Y)Z = a\,R(X,Y) + b[S(Y,Z)X - S(X,Z) + g(Y,Z)QX - g(X,Z)QY]$$

$$-\frac{r}{n}\left[\frac{a}{n-1} + 2b\right][g(Y,Z)X - g(X,Z)Y] \tag{1.3}$$

where a and b are constants such that $a, b \neq 0$.

Author α: Department of Mathematics, SR Institute of Management and Technology, BKT, Lucknow - 227 202, India.
e-mail: archana.srivastava13@gmail.com
Author σ: Department of Mathematics, National Institute of Technology, Kurukshetra, Haryana-136119, India.

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If $a = 1$ and $b = -\frac{1}{n-2}$, then (3) takes the form

$$\tilde{C}(X,Y)Z = R(X,Y) - \frac{1}{n-2}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] + \frac{r}{(n-1)(n-2)}$$

$$[g(Y,Z)X - g(X,Z)Y] = C(X,Y)Z$$

where C is Conformal curvature tensor. Thus the Conformal curvature tensor C is a particular case of the tensor $\tilde{C}$. For this reason $\tilde{C}$ is calledQuasi–conformal curvature tensor.

Quasi–concircular curvature tensor $\tilde{V}$ on a Riemannian manifold $(M^n, g), n > 2$ of type$(1,3)$is defined as follows (Prasad and Maurya[19]).

$$\tilde{V}(X,Y)Z = a\, R(X,Y)Z + \frac{r}{n}\left[\frac{a}{n-1} + 2b\right][g(Y,Z)X - g(X,Z)Y] \tag{1.4}$$

where a and b are constants such that $a, b \neq 0$. If $a = 1$ and$b = -\frac{1}{n-1}$, then from (4)

$$\tilde{V}(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y] = V(X,Y)Z$$

where V is the Concircular curvature tensor. Thus the Concircular curvature tensor $V$is a particular case of the tensor $\tilde{V}$. For this reason $\tilde{V}$ is called Quasi–concircular curvature tensor.

m–projective curvature tensor W on a Riemannian manifold $(M^n, g), n > 3$ of type $(1,3)$ is defined as follows (Pokhariyal and Mishra [18]).

$$W(X,Y)Z = R(X,Y)Z - \frac{1}{2(n-1)}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] \tag{1.5}$$

where Q is the Ricci operator defined by $S(X,Y) = g(QX,Y)$, S is the Ricci tensor, $r = \text{trace}(S)$ is the scalar curvature and $X,Y,Z \in \chi(M)$. $\chi(M)$ is being Lie algebra of vector fields of M.

In [21], Tanno classified connected almost contact metric manifolds whose automorphism groups possess the maximum dimension. For such a manifold, the sectional curvature of plane sections containing ξ is a constant, say c. He showed that they can be divided into three classes:

(1) Homogeneous normal contact Riemannian manifolds with $c > 0$,

(2) Global Riemannian products of a line or a circle with a Kaehler manifold of constant holomorphic sectional curvature if $c = 0$,

(3) A warped product space $\mathbb{R} \times_f C$ if $c < 0$.It is known that the manifolds of class (1) are characterized by admitting a Sasakian structure, (2) Kenmotsu [11]characterized the differential geometric properties of the manifolds of class (3); the structure so obtained is now known as Kenmotsu structure.

In general, these structures are not Sasakian[14]. In the Gray–Hervella classification of almost Hermitian manifolds[8], there appears a class, W_4 of Hermitian manifolds which are closely related to locally conformal Kaehler manifolds[10]. An almost contact metric structure on a manifold M is called a trans–Sasakian structure[13] if the product manifold $M \times \mathbb{R}$ belongs to the class W_4. The class $C_6 \oplus C_5$ ([13], [14])

Ref

18. POKHARIYAL,G. P., MISHRA, R. S., (1971), 'Curvature tensor and their relativistic significance 2nd', Yakohama Mathematical Journal, pp 97–103.

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coincides with the class of the trans–Sasakian structures of the type (α,β). In fact, in [13], local nature of the two subclasses, namely, C_5 and C_6 structures, of trans–Sasakian structures are characterized completely.

Also, in [16], Özgür and De studied quasi–conformally flat and quasi–conformally semisymmetric Kenmotsu manifolds. Then, in [25], Yildiz and Murathan studied Lorentzian α–Sasakain manifolds.

We note that trans–Sasakian structures of the type$(0,0)$, $(0,\beta)$and $(\alpha,0)$are cosymplectic [2], β–Kenmotsu[11] and α–Sasakain [11] respectively. In [22], it is proved that trans–Sasakian structures are generalised quasi–Sasakian.Thus, trans–Sasakian structures also provide a large class of generalized quasi–Sasakian structures.

An almost contact metric structure (φ,ξ,η,g) on M is calledtrans–Sasakian structure [13], if $(M\times\mathbb{R},J,G)$ belongs to the class W_4 [8], where J is the almost complex structure on $M\times\mathbb{R}$ defined by

$$J\left(X,f\,{}^{d}/_{dt}\right)=\left(\varphi X-f\xi,\eta(X)\,{}^{d}/_{dt}\right)$$

for all vector fields X on M and a smooth function f on $M\times\mathbb{R}$ and G is the product metric on $M\times\mathbb{R}$. This may be expressed by the condition [3]

$$(\nabla_X\varphi)Y=\alpha(g(X,Y)\xi-\eta(Y)X)+\beta\big(g(\varphi X,Y)\xi-\eta(Y)\varphi(X)\big) \tag{1.6}$$

For some smooth functions α and β on M and we say that the trans–Sasakian structure is of type (α,β).
From (6) it follows that

$$\nabla_X\xi=-\alpha\varphi(X)+\beta(X-\eta(X)\xi) \tag{1.7}$$

$$(\nabla_X\eta)Y=-\alpha g(\varphi X,Y)+\beta g(\varphi X,\varphi Y) \tag{1.8}$$

Trans–Sasakian manifolds have been studied by De and Tripathi[7] and they obtained the following results:

$$\begin{aligned}R(X,Y)\xi=&(\alpha^2-\beta^2)(\eta(Y)X-\eta(X)Y)+2\alpha\beta(\eta(Y)\varphi X\eta(X)\varphi Y)+(Y\alpha)\varphi X\\&-(X\alpha)\varphi Y+(Y\beta)\varphi^2X-(X\beta)\varphi^2Y\end{aligned} \tag{1.9}$$

$$\begin{aligned}R(\xi,Y)X=&(\alpha^2-\beta^2)(g(X,Y)\xi-\eta(X)Y)+2\alpha\beta(g(\varphi X,Y)\xi-\eta(X)\varphi Y)\\&+(X\alpha)\varphi Y+g(\varphi X,Y)(grad\alpha)+(X\beta)(Y-\eta(Y)\xi)-g(\varphi X,\varphi Y)(grad\beta)\end{aligned} \tag{1.10}$$

$$R(\xi,X)\xi=(\alpha^2-\beta^2-\xi\beta)(\eta(X)\xi-X) \tag{1.11}$$

$$2\alpha\beta+\xi\alpha=0 \tag{1.12}$$

$$S(X,\xi)=\big((n-1)(\alpha^2-\beta^2)-\xi\beta\big)\eta(X)-(n-2)X\beta-(\varphi X)\alpha \tag{1.13}$$

$$Q\xi=((n-1)(\alpha^2-\beta^2)-\xi\beta)\xi-(n-2)(grad\beta)+\varphi(grad\alpha) \tag{1.14}$$

A trans–Sasakian structure of type (α,β) is α–Sasakian if $\beta=0$ and α is a non–zero constant [10].

If $\alpha=1$, then α–Sasakian manifold is a Sasakian manifold.

Ref

11. KENMOTSU, K., (1972), 'A class of almost contact Riemannian manifolds', *Tohoku Math. J.*, Japan, pp 93–103.

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II. Lorentzian α–Sasakian Manifold

A differentiable manifold of dimension n is called Lorentzian α–Sasakian manifold if it admits a $(1,1)$–tensor field φ, a contravarient vector field ξ, a covariant vector field η and Lorentzian metric g which satisfy ([2], [5], [6], [7], [8], [12])

$$\eta(\xi) = -1, \tag{2.1}$$

$$\varphi^2 = I + \eta\otimes\xi \tag{2.2}$$

$$g(\varphi X, \varphi Y) = g(X,Y) + \eta(X)\eta(Y) \tag{2.3}$$

$$g(X,\xi) = \eta(X), \;\; \varphi\xi = 0, \;\; \eta(\varphi X) = 0 \tag{2.4}$$

for all $X, Y \in TM$.

From (1.7) and (1.8), a Lorentzian α–Sasakian manifold Msatisfying

$$\nabla_X \xi = -\alpha\varphi(X) \tag{2.5}$$

$$(\nabla_X \eta)Y = -\alpha g(\varphi X, Y) \tag{2.6}$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g.

A Lorentzian α–Sasakian manifold M is said to be η–Einstien if its Ricci tensor S is of the form

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y) \tag{2.7}$$

for any vector fields X, Y, where a, b are functions on M.

Further, from equations (1.9)–(1.14) on a Lorentzian α–Sasakian manifold M the following relations holds:

$$R(\xi,X)Y = \alpha^2(g(X,Y)\xi + \eta(Y)X) \tag{2.8}$$

$$R(X,Y)\xi = \alpha^2(\eta(Y)X + \eta(X)Y) \tag{2.9}$$

$$R(\xi,X)\xi = \alpha^2(\eta(X)\xi + X) \tag{2.10}$$

$$S(X,\xi) = (n-1)\alpha^2\eta(X) \tag{2.11}$$

$$Q\xi = (n-1)\alpha^2\xi \tag{2.12}$$

$$S(\xi,\xi) = -(n-1)\alpha^2 \tag{2.13}$$

$$S(\varphi X, \varphi Y) = S(X,Y) + (n-1)\alpha^2\eta(X)\eta(Y) \tag{2.14}$$

III. φ-Pseudo Projectively Flat Lorentzian α- Sasakian Manifold

A differentiable manifold $(M^n, g), n > 2$, satisfying the condition $\varphi^2\tilde{P}(\varphi X, \varphi Y)\varphi Z = 0$, is called φ–pseudo projectively flat Lorentzian α– Sasakian manifold.

In this section we assume that Lorentzian α–Sasakian manifold $(M^n, g), n > 2$, is φ–pseudo projectively flat. Then $\varphi^2\tilde{P}(\varphi X, \varphi Y)\varphi Z = 0$, implies

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Ref

2. BLAIR, D.E., (1976), 'Contact manifolds in Riemannian geometry', *Lectures Notes in Mathematics, Springer–Verlag*, Berlin.

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$$g(\tilde{P}(\varphi X,\varphi Y)\varphi Z,\varphi W)=0 \tag{3.1}$$

for any vector fields X,Y,Z,W.

By the use of (1.2), φ–pseudo projectively flat means

$$a\,{}'R(\varphi X,\varphi Y,\varphi Z,\varphi W)=-b[S(\varphi Y,\varphi Z)g(\varphi X,\varphi W)-S(\varphi X,\varphi Z)g(\varphi Y,\varphi W)]+\frac{r}{n}\Big[\frac{a}{n-1}+b\Big]$$
$$[g(\varphi Y,\varphi Z)g(\varphi X,\varphi W)-g(\varphi X,\varphi Z)g(\varphi Y,\varphi W)] \tag{3.2}$$

where ${}'R(X,Y,Z,W)=g(R(X,Y)Z,W)$.

Let $\{e_1,e_2,\ldots,e_{n-1},\xi\}$ be a local orthonormal basis of vector fields in M^n. By using the fact that $\{\varphi e_1,\varphi e_2,\ldots,\varphi e_{n-1},\xi\}$ is also a local orthonormal basis, if we put $X=W=e_i$ in (3.2) and sum up with respect to , then we have

$$a\sum_{i=1}^{n-1}{}'R(\varphi e_i\,,\varphi Y,\varphi Z,\varphi e_i)=-b\sum_{i=1}^{n-1}[S(\varphi Y,\varphi Z)g(\varphi e_i,\varphi e_i)-S(\varphi e_i,\varphi Z)g(\varphi Y,\varphi e_i)+\frac{r}{n}\Big[\frac{a}{n-1}+b\Big]$$
$$\textstyle\sum_{i=1}^{n-1}[g(\varphi Y,\varphi Z)g(\varphi e_i,\varphi e_i)-g(\varphi e_i,\varphi Z)g(\varphi Y,\varphi e_i) \tag{3.3}$$

It can be easily verified that ([2])

$$\textstyle\sum_{i=1}^{n-1}{}'R(\varphi e_i\,,\varphi Y,\varphi Z,\varphi e_i)=S(\varphi Y,\varphi Z)+g(\varphi Y,\varphi Z) \tag{3.4}$$

$$\textstyle\sum_{i=1}^{n-1}S(\ \varphi e_i,\varphi e_i)=r-(n-1)\alpha^2 \tag{3.5}$$

$$\textstyle\sum_{i=1}^{n-1}g(\varphi e_i,\varphi Z)S(\varphi Y,\varphi e_i)=S(\varphi Y,\varphi Z) \tag{3.6}$$

$$\textstyle\sum_{i=1}^{n-1}g(\ \varphi e_i,\varphi e_i)=(n-1) \tag{3.7}$$

$$\textstyle\sum_{i=1}^{n-1}g(\varphi e_i,\varphi Z)g(\varphi Y,\varphi e_i)=g(\varphi Y,\varphi Z) \tag{3.8}$$

So by the use of the (3.4)–(3.8), the equation (3.3) takes the form

$$[a+b(n-2)]S(\varphi Y,\varphi Z)=\Big[\tfrac{r}{n}\Big(\tfrac{a}{n-1}+b\Big)(n-2)-a\Big]g(\varphi Y,\varphi Z) \tag{3.9}$$

Then, by making use of (4.2.3) and (4.2.14), the equation (4.3.10) takes the form

$$S(Y,Z)=\frac{1}{[a+b(n-2)]}\Big[\frac{r}{n}\Big(\frac{a}{n-1}+b\Big)(n-2)-a\Big]g(Y,Z)+\frac{1}{[a+b(n-2)]}$$
$$[\tfrac{r}{n}\Big(\tfrac{a}{n-1}+b\Big)(n-2)-a-a\alpha^2-b(n-1)(n-2)\alpha^2]\eta(Y)\eta(Z) \tag{3.10}$$

which shows that M^n is an η–Einstein manifold.Hence we can state the following theorem:

Theorem 3.1: Let M^n be an n–dimensional, $n>2,\varphi$–pseudo projectively flat Lorentzian α–Sasakian manifold, then M^n is an η–Einstein Manifold.

IV. φ–Quasi Conformally Flat Lorentzian α–Sasakian Manifold

A differentiable manifold $(M^n,g),n>2,$ satisfying the condition $\varphi^2\tilde{C}(\varphi X,\varphi Y)\varphi Z=0,$ is called φ–quasi conformally flat. (Cabrerizo, Fernandez,

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Fernandez and Zhen [28]). In this section we assume that Lorentzian α–Sasakian manifold $(M^n, g), n > 2$, is φ–quasi conformally flat. Then $\varphi^2 \tilde{C}(\varphi X, \varphi Y)\varphi Z = 0$ implies

$$g\big(\tilde{C}(\varphi X, \varphi Y)\varphi Z, \varphi W\big) = 0 \tag{4.1}$$

for any vector fields $X, Y, Z, W \in \chi(M^n)$. So by the use of (4.1.3), φ–quasi conformally flat means

$$a' R(\varphi X, \varphi Y, \varphi Z, \varphi W) = -b[S(\varphi Y, \varphi Z)g(\varphi X, \varphi W) - S(\varphi X, \varphi Z)g(\varphi Y, \varphi W) + g(\varphi Y, \varphi Z)S(\varphi X, \varphi W)$$
$$-g(\varphi X, \varphi Z)S(\varphi Y, \varphi W)] + \frac{r}{n}\left[\frac{a}{n-1} + 2b\right][g(\varphi Y, \varphi Z)g(\varphi X, \varphi W) - g(\varphi X, \varphi Z)g(\varphi Y, \varphi W)] \tag{4.2}$$

where $'R(\varphi X, \varphi Y, \varphi Z, \varphi W) = g(R(\varphi X, \varphi Y)\varphi Z,\ \varphi W)$.

Let $\{e_1, e_2, \dots, e_{n-1}, \xi\}$ be a local orthonormal basis of vector fields in M^n. By using the fact that $\{\varphi e_1, \varphi e_2, \dots, \varphi e_{n-1}, \xi\}$ is also a local orthonormal basis, if we put $X = W = e_i$ in (4.2) and sum up with respect to i, then we have

$$a\sum_{i=1}^{n-1}{}'R(\varphi e_i, \varphi Y, \varphi Z, \varphi e_i) = -b\sum_{i=1}^{n-1}[S(\varphi Y, \varphi Z)g(\varphi e_i, \varphi e_i) - S(\varphi e_i, \varphi Z)g(\varphi Y, \varphi e_i)$$
$$+g(\varphi Y, \varphi Z)S(\varphi e_i, \varphi e_i) - g(\varphi e_i, \varphi Z)S(\varphi Y, \varphi e_i)] + \textstyle\sum_{i=1}^{n-1}[g(\varphi Y, \varphi Z)g(\varphi e_i, \varphi e_i)$$
$$-g(\varphi e_i, \varphi Z)g(\varphi Y, \varphi e_i)] \tag{4.3}$$

So by the virtue of (3.4)−(3.8), the equation (4.3) takes the form

$$[a + b(n-3)]S(\varphi Y, \varphi Z) = [-a - b\{r - (n-1)\alpha^2\} + \frac{r(n-2)}{n}\left\{\frac{a}{n-1} + 2b\right\}]g(\varphi Y, \varphi Z) \tag{4.4}$$

Then, by making use of (2.3) and (2.14), the equation (4.4) takes the for

$$S(Y,Z) = \frac{1}{[a+b(n-3)]}[\left\{\frac{r(n-2)-n(n-1)}{n(n-1)}\right\}a + \left\{\frac{n(n-1)\alpha^2+(n-4)}{n}\right\}b]g(Y,Z) + \frac{1}{[a+b(n-3)]}[\left\{\frac{r(n-2)-n(n-1)^2\alpha^2-n(n-1)}{n(n-1)}\right\} +$$
$$a\left\{\frac{(n-4)-n(n-1)(n-4)\alpha^2}{n}\right\}b]\eta(Y)\eta(Z) \tag{4.5}$$

which shows that M^n is an η–Einstein manifold. Hence we can state the following theorem:

Theorem 4.1: Let M^n be an n–dimensional, $n > 2$, φ–quasi conformally flat Lorentzian α–Sasakian manifold, then M^n is an η–Einstein Manifold.

V. φ –Quasi Concircularly Flat Lorentzian α–Sasakian Manifold

A differentiable manifold $(M^n, g), n > 2$, satisfying the condition $\varphi^2\tilde{V}(\varphi X, \varphi Y)\varphi Z = 0$, is called φ–quasi concircularly flat Lorentzian α–Sasakian manifold.

In this section we assume that Lorentzian α–Sasakian manifold $(M^n, g), n > 2$, is φ–quasi concircularly flat. Then $\varphi^2\tilde{V}(\varphi X, \varphi Y)\varphi Z = 0$, implies

$$g\big(\tilde{V}(\varphi X, \varphi Y)\varphi Z, \varphi W\big) = 0 \tag{5.1}$$

for any vector fields $X, Y, Z, W \in \chi(M^n)$.

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So, by the use of (1.4), φ–quasi concircularly flat means

$$a' R(\varphi X, \varphi Y, \varphi Z, \varphi W) = -\frac{r}{n}\left[\frac{a}{n-1} + 2b\right][g(\varphi Y, \varphi Z)g(\varphi X, \varphi W) - g(\varphi X, \varphi Z)g(\varphi Y, \varphi W)] \quad (5.2)$$

where $'R(\varphi X, \varphi Y, \varphi Z, \varphi W) = g(R(\varphi X, \varphi Y)\varphi Z,\ \varphi W)$.

Let $\{e_1, e_2, \dots, e_{n-1}, \xi\}$ be a local orthonormal basis of vector fields in M^n by using the fact that $\{\varphi e_1, \varphi e_2, \dots, \varphi e_{n-1}, \xi\}$ is also a local orthonormal basis, if we put $X = W = e_i$ in (5.2) and sum up with respect to i, then we have

$$a \sum_{i=1}^{n-1} {}'R(\varphi e_i, \varphi Y, \varphi Z, \varphi e_i)$$

$$= -\frac{r}{n}\left[\frac{a}{n-1} + 2b\right] \sum_{i=1}^{n-1} [g(\varphi Y, \varphi Z)g(\varphi e_i, \varphi e_i) - g(\varphi e_i, \varphi Z)g(\varphi Y, \varphi e_i)]$$

$$-g(\varphi e_i, \varphi Z)g(\varphi Y, \varphi e_i)] \quad (5.3)$$

So, by virtue of (3.4)−(3.8), the equation (5.3) takes the form

$$aS(\varphi Y, \varphi Z) = -\left[a + \frac{r(n-2)}{n}\left\{\frac{a}{n-1} + 2b\right\}\right] g(\varphi Y, \varphi Z) \quad (5.4)$$

Then, by making use of (2.3) and (2.14), the equation (5.4) takes the form

$$S(Y, Z) = -\left[1 + \frac{r(n-2)}{na}\left\{\frac{a}{n-1} + 2b\right\}\right] g(Y, Z) - \left[\frac{r(n-2)}{na}\left\{\frac{a}{n-1} + 2b\right\} + 1 + (n-1)\alpha^2\right] \eta(Y)\eta(Z) \quad (5.5)$$

which shows that M^n is an η–Einstein manifold.
Hence we can state the following theorem:

Theorem 5.1: Let M^n be an n–dimensional, $n > 2$, φ–quasi concircularly flat Lorentzian α–Sasakian manifold, then M^n is an η–Einstein manifold.

VI. φ–m–Projectively Flat Lorentzian α–Sasakian Manifold

A differentiable manifold $(M^n, g), n > 3$, satisfying the condition $\varphi^2 W(\varphi X, \varphi Y)\varphi Z = 0$ is called φ–m–projectively flat Lorentzian α–Sasakian manifold (Cabrerizo, Fernandez, Fernandez and Zhen [28]).

In this section we assume that Lorentzian α–Sasakian manifold $(M^n, g), n > 3$, is φ–m–projectively flat. Then $\varphi^2 W(\varphi X, \varphi Y)\varphi Z = 0$implies $\sum$

$$g(W(\varphi X, \varphi Y)\varphi Z, \varphi W) = 0 \quad (6.1)$$

for any vector fields $X, Y, Z, W \in \chi(M^n)$.

So, by the use of (1.5), φ–m–projectively flat means

$$R(\varphi X, \varphi Y, \varphi Z, \varphi W) = \frac{1}{2(n-1)}[S(\varphi Y, \varphi Z)g(\varphi X, \varphi W) - S(\varphi X, \varphi Z)g(\varphi Y, \varphi W) + g(\varphi Y, \varphi Z)$$

$$S(\varphi X, \varphi W) - g(\varphi X, \varphi Z)S(\varphi Y, \varphi W)] \quad (6.2)$$

where $'R(\varphi X, \varphi Y, \varphi Z, \varphi W) = g(R(\varphi X, \varphi Y)\varphi Z,\ \varphi W)$.

Ref

28. ZHEN, G., CABRERIZO, J. L., FERNANDEZ, M. (1997), 'On ξ − conformally flat contact metric manifold', *Indian J. Pure Appl. Math.*,pp725–734.

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Let $\{e_1, e_2, \ldots, e_{n-1}, \xi\}$ be a local orthonormal basis of vector fields in M^n. By using the fact that $\{\varphi e_1, \varphi e_2, \ldots, \varphi e_{n-1}, \xi\}$ is also a local orthonormal basis, if we put $X = W = e_i$ in (6.2) and sum up with respect to i, then we have

$$\sum_{i=1}^{n-1} {}'R(\varphi e_i, \varphi Y, \varphi Z, \varphi e_i) = \frac{1}{2(n-1)} \sum_{i=1}^{n-1} [S(\varphi Y, \varphi Z) g(\varphi e_i, \varphi e_i) - S(\varphi e_i, \varphi Z) g(\varphi Y, \varphi e_i) +$$

$$g(\varphi Y, \varphi Z) S(\varphi e_i, \varphi e_i) - g(\varphi e_i, \varphi Z) S(\varphi Y, \varphi e_i)] \tag{6.3}$$

So, by the use of the (3.4)−(3.8), the equation (6.3) takes the form

$$S(Y,Z) = \frac{1}{(n+1)} [r - (n-1)\alpha^2 - 2(n-1)]\, g(Y,Z) + \frac{1}{(n+1)} [r - 2(n-1) - (n-1)$$

$$(n+2)\alpha^2] \eta(Y) \eta(Z) \tag{6.4}$$

which shows that M^n is an η−Einstein manifold. Hence we can state the following theorem:

Theorem 6.1: Let M^n be an n−dimensional, $n > 3, \varphi - m$−projectively flat Lorentzian α−Sasakian manifold, then M^n is an η−Einstein manifold.

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