Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 20 Issue 7 Version 1.0 Year 2020
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Response to Simply Supported Orthotropic Rectangular Plate Resting on a Variable Elastic Bi-Parametric Foundation under the Action of Moving Distributed Masses

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GJSFR-F Classification: *MSC 2010: 35A17*

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I. Introduction

The problems connected with the analysis of thin structural bodies (rods, beams, plates,and shells) with other bodies have widespread application in various fields of science and technology. The physical phenomena involved in the impact event include structural responses, contact effects and wave propagation. These problems are topical issues of research in the field of applied mechanics. Since these problems belong to the problems of dynamic contact interaction, their solution is connected with severe mathematical and calculation difficulties. To this end , several researchers had worked and some are still working on the dynamic behavior of orthotropic rectangular plates. Analytical investigation of the low-velocity impact response of circular orthotropic and transversely isotropic plates possessing curvilinear anisotropy under compressive preloading has been carried out recently by Rossikhin and Shitikova in [1] and [2], respectively. The equations of plate motion take the rotatory inertia and transverse shear deformations into account. In the case of the orthotropic target [1], the changes in the geometrical dimensions of the contact domain have been ignored and the contact interaction is modeled by a linear spring, and a force arising in it is the linear approximation of Herts?z contact force. Ambartsumian [3] examined the five fundamental differential equations describing the equilibrium of an orthotropic plate with a cylindrical anisotropy for the case when all radial planes crossing

Ref

1. Yu. A. Rossikhin and Shitikova M. V. (2006): Dynamic stability of a circular pre-stressed elastic orthotropic plate subjected to shock excitation, Shock Vibr. 13, pp. 197?214.

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the axis of anisotropy are the planes of elastic symmetry. Sveklo [4] suggested the contact theory for two anisotropic bodies under compression according to which the contact pressure is distributed over an elliptical contact region. The same structural effects are also true of the concrete slab in a composite girder bridge, but the steel orthotropic deck is considerably lighter, and therefore allows longer span bridges to be more efficiently designed. Awodola [5] studied the effect of plate parameters on the vibrations under moving masses of elastically supported plate resting on bi-parametric foundation with stiffness variation.Szekrenyes [6] investigated the interface fracture in orthotropic composite plates using second order shear deformation theory. Kadari [7] analyzed buckling in orthotropic nanoscale plates resting on elastic foundations. Yan [8] proposed elastic orthotropic models and used these in the nonlinear analysis of concrete structures subjected to monotonic or pseudo dynamic loading. Since these models can appropriately describe the strain softening behavior of concrete beyond the peak stress and show good agreement with the strength envelope obtained from experimental results Hu and Yao [9] studied the vibration solutions of rectangular orthotropic plates by symplectic geometry method. In the same vien, Alshaya, Hunt and Rowlands [10] investigated stresses and strains in thick perforated orthotropic plates. Gbadeyan and Dada [11] found the natural frequency of rectangular plates traversed by moving concentrated masses. Awodola and Adeoye [13] investigated the behavior of simply supported orthotropic rectangular plate by applying the technique of variable separable. Adeoye and Awodola [14] studied the dynamic behavior of orthotropic rectangular plate with clamped-clamped boundary conditions by making use of the technique of Shadnam Due to inability of researchers to solve orthotropic plates problems by analytical methods, this work aims at solving the governing equation by analytical solution and also considers the effect of the flexural rigidities in both x and y directions.

II. Governing Equation

The dynamic transverse displacement $W(x,y,t)$ of orthotropic rectangular plates when it is resting on a bi-parametric elastic foundation and traversed by distributed mass M_r moving with constant velocity c_r along a straight line parallel to the x-axis issuing from point y=s on the y-axis with flexural rigidities D_x and D_y is governed by the fourth order partial differential equation given as

$$D_x\frac{\partial^4}{\partial x^4}W(x,y,t)+2B\frac{\partial^4}{\partial x^2\partial y^2}W(x,y,t)+D_y\frac{\partial^4}{\partial y^4}W(x,y,t)+\mu\frac{\partial^2}{\partial t^2}W(x,y,t)-\rho hR_0$$

$$\left[\frac{\partial^4}{\partial x^2\partial t^2}W(x,y,t)+\frac{\partial^4}{\partial y^2\partial t^2}W(x,y,t)\right]+K_0(4x-3x^2+x^3)W(x,y,t)+S_0(-13+$$

$$12x+3x^2)\frac{\partial}{\partial x}W(x,y,t)-S_0(12-13x+6x^2+x^3)\left[\frac{\partial^2}{\partial x^2}W(x,y,t)+\frac{\partial^2}{\partial y^2}W(x,y,t)\right] \quad (2.1)$$

$$-\sum_{r=1}^{N}\left[M_r gH(x-ct)H(y-s)-M_r\left(\frac{\partial^2}{\partial t^2}W(x,y,t)+2c_r\frac{\partial^2}{\partial x\partial t}W(x,y,t)+c_r^2\right.\right.$$

Ref

10. Alshaya, A., John H., Rowlands R. (2016): Stresses and Strains in Thick Perforated Orthotropic Plates. Journal of Engineering Mechanics. vol. 142(11), 10p.

$$\frac{\partial^2}{\partial x^2}W(x,y,t)\Big)H(x-c_rt)H(y-s)\Big]=0$$

where D_x and D_y are the flexural rigidities of the plate along x and y axes respectively.

Notes

$$D_x=\frac{E_xh^3}{12(1-\nu_x\nu_y)},\quad D_y=\frac{E_yh^3}{12(1-\nu_x\nu_y)},\quad B=D_xD_y+\frac{G_oh^3}{6} \tag{2.2}$$

E_x and E_y are the Young's moduli along x and y axes respectively, G_o is the rigidity modulus, ν_x and ν_y are Poisson's ratios for the material such that $E_x\nu_y=E_y\nu_x$, ρ is the mass density per unit volume of the plate, h is the plate thickness, t is the time, x and y are the spatial coordinates in x and y directions respectively, R_o is the rotatory inertia correction factor, K_o is the foundation constant, S_o shear modulus and g is the acceleration due to gravity, H(.) is the Heaviside function.

Rewriting equation (2.1), one obtains

$$\mu\frac{\partial^2}{\partial t^2}W(x,y,t)+\mu\omega_n^2W(x,y,t)=\rho hR_0\left[\frac{\partial^4}{\partial x^2\partial t^2}W(x,y,t)+\frac{\partial^4}{\partial y^2\partial t^2}W(x,y,t)\right]-2B$$

$$\frac{\partial^4}{\partial x^2\partial y^2}W(x,y,t)-D_y\frac{\partial^4}{\partial y^4}W(x,y,t)-D_x\frac{\partial^4}{\partial x^4}W(x,y,t)+\mu\omega_n^2W(x,y,t)-K_0(4x-$$

$$3x^2+x^3)W(x,y,t)+G_o(-13+12x+3x^2)\frac{\partial}{\partial x}W(x,y,t)-G_0(12-13x+6x^2+x^3)\Big[\tag{2.3}$$

$$\frac{\partial^2}{\partial x^2}W(x,y,t)+\frac{\partial^2}{\partial y^2}W(x,y,t)\Big]+\sum_{r=1}^{N}\Big[M_rgH(x-c_rt)H(y-s)-M_r\Big(\frac{\partial^2}{\partial t^2}W(x,y,t)$$

$$+2c_r\frac{\partial^2}{\partial x\partial t}W(x,y,t)+c_r^2\frac{\partial^2}{\partial x^2}W(x,y,t)\Big)H(x-c_rt)H(y-s)\Big]$$

Simplifying equation (2.3) further, one obtains

$$\frac{\partial^2}{\partial t^2}W(x,y,t)+\omega_n^2W(x,y,t)=\sum_{r=1}^{N}\Big[R_0\Big(\frac{\partial^4}{\partial x^2\partial t^2}W(x,y,t)+\frac{\partial^4}{\partial y^2\partial t^2}W(x,y,t)\Big)-\frac{2B}{\mu}$$

$$\frac{\partial^4}{\partial x^2\partial y^2}W(x,y,t)-\frac{D_y}{\mu}\frac{\partial^4}{\partial y^4}W(x,y,t)-\frac{D_x}{\mu}\frac{\partial^4}{\partial x^4}W(x,y,t)+\omega_n^2W(x,y,t)-\frac{K_0}{\mu}(4x-3x^2$$

$$+x^3)W(x,y,t)+\frac{G_o}{\mu}(-13+12x+3x^2)\frac{\partial}{\partial x}W(x,y,t)-\frac{G_0}{\mu}(12-13x+6x^2+x^3)\Big(\frac{\partial^2}{\partial x^2}$$

$$W(x,y,t)+\frac{\partial^2}{\partial y^2}W(x,y,t)\Bigg)+\sum_{r=1}^{N}\ \frac{M_r}{\mu}gH(x-c_rt)H(y-s)-\frac{M_r}{\mu}\Bigg(\frac{\partial^2}{\partial t^2}W(x,y,t)+2c_r$$

$$\frac{\partial^2}{\partial x\partial t}W(x,y,t)+c_r^2\frac{\partial^2}{\partial x^2}W(x,y,t)\Bigg)H(x-c_rt)H(y-s)\Bigg)\Bigg] \tag{2.4}$$

where ω_n^2 is the natural frequencies, $n=1,2,3,...$

The initial conditions, without any loss of generality, is taken as

$$W(x,y,t)=0=\frac{\partial}{\partial t}W(x,y,t) \tag{2.5}$$

III. Analytical Approximate Solution

In order to solve equation (2.4), one applies technique of Shadnam et al which requires that the deflection of the plates be in series form as

$$W(x,y,t)=\sum_{n=1}^{N}\Psi_n(x,y)Q_n(t) \tag{3.1}$$

where

$$\Psi_n(x,y)=\Psi_{jm}(x)\Psi_{hm}(y)$$

$$\Psi_{jm}x=\sin\zeta_{jm}x+A_{jm}\cos\zeta_{jm}x+B_{jm}\sinh\zeta_{jm}x+C_{jm}\cosh\zeta_{jm}x$$

$$\Psi_{hm}y=\sin\varphi_{hm}y+A_{hm}\cos\varphi_{hm}y+B_{hm}\sinh\varphi_{hm}y+C_{hm}\cosh\varphi_{hm}y$$

$$\zeta_{jm}=\frac{\phi_{jm}}{L_x},\qquad \varphi_{hm}=\frac{\phi_{hm}}{L_y}$$

The right hand side of equation (2.4), taken into account equation (3.1), written in the form of series takes the form

$$\sum_{n=1}^{\infty}\Bigg[R_0\Bigg(\frac{\partial^2}{\partial x^2}\Psi_n(x,y)\ddot{Q}_n(t)+\frac{\partial^4}{\partial y^2}\Psi_n(x,y)\ddot{Q}_n(t)\Bigg)-\frac{2B}{\mu}\frac{\partial^4}{\partial x^2\partial y^2}\Psi_n(x,y)Q_n(t)-\frac{D_y}{\mu}\frac{\partial^4}{\partial y^4}$$

$$\Psi_n(x,y)Q_n(t)-\frac{D_x}{\mu}\frac{\partial^4}{\partial x^4}\Psi_n(x,y)Q_n(t)+\omega_n^2\Psi_n(x,y)Q_n(t)-\frac{K_0}{\mu}(4x-3x^2+x^3)\Psi_n(x,y)$$

$$Q_n(t)+\frac{G_o}{\mu}(-13+12x+3x^2)\frac{\partial}{\partial x}\Psi_n(x,y)Q_n(t)-\frac{G_0}{\mu}(12-13x+6x^2+x^3)\Bigg(\frac{\partial^2}{\partial x^2}\Psi_n(x,y)$$

$$Q_n(t) + \frac{\partial^2}{\partial y^2}\Psi_n(x,y)Q_n(t)\Bigg) + \sum_{r=1}^{N}\left(\frac{M_r}{\mu}gH(x-c_rt)H(y-s) - \frac{M_r}{\mu}\Bigg(\Psi_n(x,y)\ddot{Q}_n(t) + \right.$$

$$\left. 2c\frac{\partial}{\partial x}\Psi_n(x,y)\dot{Q}_n(t) + c_r^2\frac{\partial^2}{\partial x^2}\Psi_n(x,y)Q_n(t)\Bigg)H(x-c_rt)H(y-s)\right)\Bigg] = \sum_{n=1}^{N}\Psi_n(x,y)\Theta_n(t) \quad (3.2)$$

Notes

Multiplying both sides of equation (3.2) by $\Psi_m(x,y)$ and integrating on area A of the plate and considering the orthogonality of $\Psi_n(x,y)$, one obtains

$$\Theta_n(t) = \frac{1}{\theta^*}\sum_{n=1}^{\infty}\int_A\Bigg[R_0\left(\frac{\partial^2}{\partial x^2}\Psi_n(x,y)\ddot{Q}_n(t) + \frac{\partial^4}{\partial y^2}\Psi_n(x,y)\ddot{Q}_n(t)\right) - \frac{2B}{\mu}\frac{\partial^4}{\partial x^2\partial y^2}\Psi_n(x,y)$$

$$Q_n(t) - \frac{D_y}{\mu}\frac{\partial^4}{\partial y^4}\Psi_n(x,y)Q_n(t) - \frac{D_x}{\mu}\frac{\partial^4}{\partial x^4}\Psi_n(x,y)Q_n(t) + \omega_n^2\Psi_n(x,y)Q_n(t) - \frac{K_0}{\mu}(4x-$$

$$3x^2 + x^3)\Psi_n(x,y)Q_n(t) + \frac{G_o}{\mu}(-13+12x+3x^2)\frac{\partial}{\partial x}\Psi_n(x,y)Q_n(t) - \frac{G_0}{\mu}(12-13x$$

$$+6x^2+x^3)\left(\frac{\partial^2}{\partial x^2}\Psi_n(x,y)Q_n(t) + \frac{\partial^2}{\partial y^2}\Psi_n(x,y)Q_n(t)\right) + \sum_{r=1}^{N}\Bigg(\frac{M_r}{\mu}gH(x-c_rt)H(y-s)$$

$$-\frac{M_r}{\mu}\left(\Psi_n(x,y)\ddot{Q}_n(t) + 2c\frac{\partial}{\partial x}\Psi_n(x,y)\dot{Q}_n(t) + c_r^2\frac{\partial^2}{\partial x^2}\Psi_n(x,y)Q_n(t)\right)H(x-c_rt)H(y-s)$$

$$\Bigg)\Bigg]\Psi_m(x,y)dA \quad (3.3)$$

and zero when $n \neq m$
where

$$\theta^* = \int_A \Psi_n^2(x,y)dA \quad (3.4)$$

Making use of equation (3.3) and taking into account equation (3.2), equation (2.4) can be written as

$$\Psi_n(x,y)\left[\omega_n^2 Q_n(t) + \ddot{Q}_n(t)\right] = \frac{\Psi_n(x,y)}{\theta^*}\sum_{q=1}^{\infty}\int_A\Bigg[R_0\quad\frac{\partial^2\Psi_q(x,y)}{\partial x^2}\Psi_m(x,y)\ddot{Q}_q(t) + \frac{\partial^2\Psi_q(x,y)}{\partial y^2}$$

$$\Psi_m(x,y)\ddot{Q}_q(t)\Bigg) - \frac{2B}{\mu}\frac{\partial^2\Psi_q(x,y)}{\partial x^2\partial y^2}\Psi_m(x,y)Q_q(t) - \frac{D_y}{\mu}\frac{\partial^4\Psi_q(x,y)}{\partial y^4}\Psi_m(x,y)Q_q(t) + \frac{D_x}{\mu}\frac{\partial^4\Psi_q(x,y)}{\partial x^4}$$

$$\Psi_m(x,y)Q_q(t)-\frac{K_0}{\mu}(4x-3x^2+x^3)\Psi_q(x,y)\Psi_m(x,y)Q_q(t)+\frac{G_o}{\mu}(-13+12x+3x^2)\frac{\partial\Psi_q(x,y)}{\partial x}$$

$$\Psi_m(x,y)Q_q(t)-\frac{G_o}{\mu}(12-13x+6x^2+x^3)\Big(\frac{\partial^2\Psi_q(x,y)}{\partial x^2}\Psi_m(x,y)Q_q(t)+\frac{\partial^2\Psi_q(x,y)}{\partial y^2}\Psi_m(x,y)$$

$$Q_q(t)\Bigg)+\sum_{r=1}^{N}\Bigg(\frac{M_r}{\mu}g\Psi_m(x,y)H(x-c_rt)H(y-s)-\frac{M_r}{\mu}\Bigg(\Psi_q(x,y)\Psi_m(x,y)\ddot{Q}_q(t)+2c_r$$

$$\frac{\partial\Phi_q(x,y)}{\partial x}\Phi_m(x,y)\dot{Q}_q(t)+c_r^2\frac{\partial^2\Phi_q(x,y)}{\partial x^2}\Phi_m(x,y)Q_q(t)\Bigg)H(x-c_rt)H(y-s)\Bigg)\Bigg]dA \qquad (3.5)$$

On further simplification of equation (3.5), one obtains

$$\ddot{Q}_n(t)+\omega_n^2Q_n(t)=\frac{1}{\theta^*}\sum_{q=1}^{\infty}\int_A\Bigg[R_0\Bigg(\frac{\partial^2\Psi_q(x,y)}{\partial x^2}\Psi_m(x,y)\ddot{Q}_q(t)+\frac{\partial^2\Psi_q(x,y)}{\partial y^2}\Psi_m(x,y)\ddot{Q}_q(t)\Bigg)$$

$$-\frac{2B}{\mu}\frac{\partial^2\Psi_q(x,y)}{\partial x^2\partial y^2}\Psi_m(x,y)Q_q(t)-\frac{D_y}{\mu}\frac{\partial^4\Psi_q(x,y)}{\partial y^4}\Psi_m(x,y)Q_q(t)-\frac{D_x}{\mu}\frac{\partial^4\Psi_q(x,y)}{\partial x^4}\Psi_m(x,y)$$

$$Q_q(t)+\omega_q^2\Psi_q(x,y)\Psi_m(x,y)Q_n(t)-\frac{K_0}{\mu}(4x-3x^2+x^3)\Psi_q(x,y)\Psi_m(x,y)Q_q(t)+\frac{G_o}{\mu}(-13$$

$$+12x+3x^2)\frac{\partial\Psi_q(x,y)}{\partial x}\Psi_m(x,y)Q_q(t)-\frac{G_o}{\mu}(12-13x+6x^2+x^3)\Big(\frac{\partial^2\Psi_q(x,y)}{\partial x^2}\Psi_m(x,y)Q_q(t)$$

$$+\frac{\partial^2\Psi_q(x,y)}{\partial y^2}\Psi_m(x,y)Q_q(t)\Bigg)+\sum_{r=1}^{N}\Bigg(\frac{M_r}{\mu}g\Psi_m(x,y)H(x-c_rt)H(y-s)-\frac{M_r}{\mu}\Bigg(\Psi_q(x,y)\Psi_m(x,y)$$

$$\ddot{Q}_q(t)+2c_r\frac{\partial\Phi_q(x,y)}{\partial x}\Phi_m(x,y)\dot{Q}_q(t)+c_r^2\frac{\partial^2\Phi_q(x,y)}{\partial x^2}\Phi_m(x,y)Q_q(t)\Bigg)H(x-c_rt)H(y-s)\Bigg)\Bigg]dA$$

(3.6)

The system of equations in equation (3.6) is a set of coupled ordinary differential equations where $H(x-c_rt)$ and $H(y-s)$ are the Heaviside functions which are defined as

$$H\ (x-c_rt)=\begin{cases}1, for\ \ x\geq c_rt\\ 0, for\ \ x<c_rt\end{cases},\qquad H\ (y-s)=\begin{cases}1, for\ \ y\geq s\\ 0, for\ \ y<s\end{cases} \qquad (3.7)$$

With the properties

$$(i)\frac{d}{dx}[H(x-c_rt)]=\delta(x-c_rt),\qquad \frac{d}{dy}[H(y-s)]=\delta(y-s) \qquad (3.8)$$

$$(ii) f(x)H(x-c_r t) = \begin{cases} f(x), for \ x \geq c_r t \\ 0, for \ x < c_r t \end{cases}, \qquad f(y)H(y-s) = \begin{cases} f(y), for \ y \geq s \\ 0, for \ y < s \end{cases} \tag{3.9}$$

Using the Fourier series representation, the Heaviside functions take the form

Notes

$$H(x-c_r t) = \frac{1}{4} + \frac{1}{\pi}\sum_{r=1}^{N} \frac{\sin(2n+1)\pi(x-c_r t)}{2n+1}, 0 < x < 1 \tag{3.10}$$

$$H(y-s) = \frac{1}{4} + \frac{1}{\pi}\sum_{r=1}^{N} \frac{\sin(2n+1)\pi(y-s)}{2n+1}, 0 < y < 1 \tag{3.11}$$

On putting equations (3.7) to (3.11) into equation (3.6) and simplifying, one obtains

$$\ddot{Q}_n(t) + \omega_n^2 Q_n(t) - \frac{1}{\theta^*}\sum_{q=1}^{\infty}\Bigg[R_0 T_1 \ddot{Q}_q(t) - \frac{2B}{\mu}T_2 Q_q(t) - \frac{D_y}{\mu}T_3 Q_q(t) - \frac{D_x}{\mu}T_4 Q_q(t) + (\omega_q^2 F_4^* -$$

$$\frac{K_0}{\mu}F_5^*)Q_q(t) + \frac{G_0}{\mu}(T_6+T_7)Q_q(t) - \sum_{r=1}^{N}\frac{M_r}{\mu}\Bigg(\Bigg(T_8 + \frac{1}{\pi^2}\Bigg(\sum_{j=1}^{\infty}E_1^*\frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty}E_2^*$$

$$\frac{\sin(2j+1)\pi c_r t}{2j+1}\Bigg)\Bigg(\sum_{k=1}^{\infty}E_3^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_4^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{j=1}^{\infty}E_5^*\frac{\cos(2j+1)\pi c_r t}{2j+1}$$

$$-\sum_{j=1}^{\infty}E_6^*\frac{\sin(2j+1)\pi c_r t}{2j+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty}E_7^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_8^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)\ddot{Q}_q(t) +$$

$$2c_r\Bigg(T_9 + \frac{1}{\pi^2}\Bigg(\sum_{j=1}^{\infty}E_9^*\frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty}E_{10}^*\frac{\sin(2j+1)\pi c_r t}{2j+1}\Bigg)\Bigg(\sum_{k=1}^{\infty}E_{11}^*\frac{\cos(2k+1)\pi s}{2k+1}$$

$$-\sum_{k=1}^{\infty}E_{12}^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{j=1}^{\infty}E_{13}^*\frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty}E_{14}^*\frac{\sin(2j+1)\pi c_r t}{2j+1}\Bigg)$$

$$+\frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty}E_{15}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_{16}^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)\dot{Q}_q(t) + c_r^2\Bigg(T_{10} + \frac{1}{\pi^2}\Bigg(\sum_{j=1}^{\infty}E_{17}^*$$

$$\frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty}E_{18}^*\frac{\sin(2j+1)\pi c_r t}{2j+1}\Bigg)\Bigg(\sum_{k=1}^{\infty}E_{19}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_{20}^*\frac{\sin(2k+1)\pi s}{2k+1}$$

$$\Bigg) + \frac{1}{4\pi}\left(\sum_{j=1}^{\infty} E_{21}^{*}\frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty} E_{22}^{*}\frac{\sin(2j+1)\pi c_r t}{2j+1}\right) + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty} E_{23}^{*}\frac{\cos(2k+1)\pi s}{2k+1}$$

$$-\sum_{k=1}^{\infty} E_{24}^{*}\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)Q_q(t)\Bigg)\Bigg] = \sum_{q=1}^{\infty}\sum_{r=1}^{N}\frac{M_r g}{\mu\theta^{*}}\Psi_m(ct)\Psi_m(s) \tag{3.12}$$

which is the transformed equation governing the problem of an orthotropic rectangular plate resting on bi-parametric elastic foundation.

where

$$T_1 = \int_A \left[\frac{\partial^2}{\partial x^2}\Psi_q(x,y)\Psi_m(x,y) + \frac{\partial^2}{\partial y^2}\Psi_q(x,y)\Psi_m(x,y)\right] dA \tag{3.13}$$

$$T_2 = \int_A \frac{\partial^2}{\partial x^2}\left[\frac{\partial^2}{\partial x^2}\Psi_q(x,y)\right]\Psi_m(x,y)dA \tag{3.14}$$

$$T_3 = \int_A \frac{\partial^4}{\partial y^4}\left[\Psi_q(x,y)\right]\Psi_m(x,y)dA \tag{3.15}$$

$$T_4 = \int_A \frac{\partial^4}{\partial x^4}\left[\Psi_q(x,y)\right]\Psi_m(x,y)dA \tag{3.16}$$

$$F_4^{*} = \int_A \Psi_q(x,y)\Psi_m(x,y)dA \tag{3.17}$$

$$T_5 = 4U_1 - 3U_2 + U_3, \qquad T_6 = -13A_1 + 12A_2 + 3A_3 \tag{3.18}$$

$$T_7 = 12f_1 - 13f_2 + 6f_3 + f_4 + 12f_5 - 13f_6 + 6f_7 + f_8 \tag{3.19}$$

$$T_8 = \frac{1}{16}\int_A \Psi_q(x,y)\Psi_m(x,y)dA \tag{3.20}$$

$$E_1^{*} = \int_A \Psi_q(x,y)\Psi_m(x,y)\sin(2j+1)\pi x dA \tag{3.21}$$

$$E_2^{*} = \int_A \Psi_q(x,y)\Psi_m(x,y)\cos(2j+1)\pi x dA \tag{3.22}$$

$$E_3^{*} = \int_A \Psi_q(x,y)\Psi_m(x,y)\sin(2k+1)\pi y dA \tag{3.23}$$

$$E_4^{*} = \int_A \Psi_q(x,y)\Psi_m(x,y)\cos(2k+1)\pi y dA \tag{3.24}$$

$$E_5^{*} = E_1^{*}, \quad E_6^{*} = E_2^{*}, \quad E_7^{*} = E_3^{*}, \quad E_8^{*} = E_4^{*} \tag{3.25}$$

Notes

$$T_9 = \frac{1}{16}\int_A \frac{\partial}{\partial x}\Psi_q(x,y)\Psi_m(x,y)dA \tag{3.26}$$

$$E_9^* = \int_A \frac{\partial}{\partial x}\left(\Psi_q(x,y)\right)\Psi_m(x,y)\sin(2j+1)\pi x dA \tag{3.27}$$

Notes

$$E_{10}^* = \int_A \frac{\partial}{\partial x}\left(\Psi_q(x,y)\right)\Psi_m(x,y)\cos(2j+1)\pi x dA \tag{3.28}$$

$$E_{11}^* = \int_A \frac{\partial}{\partial x}\left(\Psi_q(x,y)\right)\Psi_m(x,y)\sin(2k+1)\pi y dA \tag{3.29}$$

$$E_{12}^* = \int_A \frac{\partial}{\partial x}\Psi_q(x,y)\Psi_m(x,y)\cos(2k+1)\pi y dA \tag{3.30}$$

$$E_{13}^* = E_9^*, \quad E_{14}^* = E_{10}^*, \quad E_{15}^* = E_{11}^*, \quad E_{16}^* = E_{12}^* \tag{3.31}$$

$$T_{10} = \frac{1}{16}\int_A \frac{\partial^2}{\partial x^2}\left(\Psi_q(x,y)\right)\Psi_m(x,y)dA \tag{3.32}$$

$$E_{17}^* = \int_A \frac{\partial^2}{\partial x^2}\left(\Psi_q(x,y)\right)\Psi_m(x,y)\sin(2j+1)\pi x dA \tag{3.33}$$

$$E_{18}^* = \int_A \frac{\partial^2}{\partial x^2}\left(\Psi_q(x,y)\right)\Psi_m(x,y)\cos(2j+1)\pi x dA \tag{3.34}$$

$$E_{19}^* = \int_A \Psi_q(x,y)\Psi_m(x,y)\sin(2k+1)\pi y dA \tag{3.35}$$

$$E_{20}^* = \int_A \frac{\partial^2}{\partial x^2}\left(\Psi_q(x,y)\right)\Psi_m(x,y)\cos(2k+1)\pi y dA \tag{3.36}$$

$$E_{21}^* = E_{17}^*, \quad E_{22}^* = E_{18}^*, \quad E_{23}^* = E_{19}^*, \quad E_{24}^* = E_{20}^* \tag{3.37}$$

$\Psi_m(x,y)$ is assumed to be the products of functions $\Psi_{pm}(x)\Psi_{bm}(y)$ which are the beam functions in the directions of x and y axes respectively. That is

$$\Psi_m(x,y) = \Psi_{pm}(x)\Psi_{bm}(y) \tag{3.38}$$

where

$$\Phi_m(x) = \sin\frac{\Gamma_m x}{L_x} + A_m\cos\frac{\Gamma_m x}{L_x} + B_m\sinh\frac{\Gamma_m x}{L_x} + C_m\cosh\frac{\Gamma_m x}{L_x} \tag{3.39}$$

$$\Phi_m(y) = \sin\frac{\Gamma_m y}{L_y} + A_m\cos\frac{\Gamma_m y}{L_y} + B_m\sinh\frac{\Gamma_m y}{L_y} + C_m\cosh\frac{\Gamma_m y}{L_y} \tag{3.40}$$

Notes

where A_{pm}, B_{pm}, C_{pm}, A_{bm}, B_{bm} and C_{bm} are constants determined by the boundary conditions. And Ψ_{pm} and Ψ_{bm} are called the mode frequencies
where

$$\lambda_{pm} = \frac{\xi_{pm}}{L_x}, \qquad \lambda_{bm} = \frac{\xi_{bm}}{L_y} \tag{3.41}$$

Considering a unit mass, equation (3.12) can be re-written as

$$\ddot{Q}_n(t) + \omega_n^2 Q_n(t) - \frac{1}{\theta^*}\sum_{q=1}^{\infty}\Bigg[R_0 T_1 \ddot{Q}_q(t) - \frac{2B}{\mu}T_2 Q_q(t) - \frac{D_y}{\mu}T_3 Q_q(t) - \frac{D_x}{\mu}T_4 Q_q(t) +$$

$$(\omega_q^2 F_4^* - \frac{K_0}{\mu}T_5)Q_q(t) + \frac{G_0}{\mu}(T_6 + T_7)Q_q(t) - \alpha\varrho\Bigg(\Bigg(T_8 + \frac{1}{\pi^2}\sum_{j=1}^{\infty}E_1^*\frac{\cos(2j+1)\pi ct}{2j+1}$$

$$-\sum_{j=1}^{\infty}E_2^*\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)\Bigg(\sum_{k=1}^{\infty}E_3^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_4^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg) + \frac{1}{4\pi}\Bigg($$

$$\sum_{j=1}^{\infty}E_5^*\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty}E_6^*\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty}E_7^*\frac{\cos(2k+1)\pi s}{2k+1} -$$

$$\sum_{k=1}^{\infty}E_8^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)\ddot{Q}_q(t) + 2c\Bigg(T_9 + \frac{1}{\pi^2}\Bigg(\sum_{j=1}^{\infty}E_9^*\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty}E_{10}^*$$

$$\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)\Bigg(\sum_{k=1}^{\infty}E_{11}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_{12}^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg) + \frac{1}{4\pi}\sum_{j=1}^{\infty}E_{13}^*$$

$$\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty}E_{14}^*\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty}E_{15}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_{16}^*$$

$$\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)\dot{Q}_q(t) + c^2\Bigg(T_{10} + \frac{1}{\pi^2}\Bigg(\sum_{j=1}^{\infty}E_{17}^*\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty}E_{18}^*\frac{\sin(2j+1)\pi ct}{2j+1}$$

$$\Bigg)\left(\sum_{k=1}^{\infty} E_{19}^{*}\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty} B_{20}^{*}\frac{\sin(2k+1)\pi s}{2k+1}\right)+\frac{1}{4\pi}\left(\sum_{j=1}^{\infty} E_{21}^{*}\frac{\cos(2j+1)\pi ct}{2j+1}\right.$$

$$\left.-\sum_{j=1}^{\infty} E_{22}^{*}\frac{\sin(2j+1)\pi ct}{2j+1}\right)+\frac{1}{4\pi}\left(\sum_{k=1}^{\infty} E_{23}^{*}\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty} E_{24}^{*}\frac{\sin(2k+1)\pi s}{2k+1}\right)$$

$$\Bigg)Q_q(t)\Bigg)\Bigg]=\sum_{q=1}^{\infty}\sum_{r=1}^{N}\frac{Mg}{\mu\theta^{*}}\Psi_m(ct)\Psi_m(s) \tag{3.42}$$

equation (3.42) is the fundamental equation of the problem. where

$$\alpha=\frac{M}{\mu\varrho},\quad \varrho=L_xL_y \tag{3.43}$$

$$\Psi_m(ct)=\sin\chi_m t+A_m\cos\chi_m t+B_m\sinh\chi_m t+C_m\cosh\chi_m t \tag{3.44}$$

$$\Psi_m(s)=\sin\nu_m+A_m\cos\nu_m+B_m\sinh\nu_m+C_m\cosh\nu_m \tag{3.45}$$

$$\chi_m=\frac{\phi_m c}{L_x},\ \nu_m=\frac{\phi_m s}{L_y} \tag{3.46}$$

We shall consider the situation where the orthotropic rectangular plate is simply supported at all its edges. The boundary conditions for an orthotropic rectangular plate having simple supports at all its edges are given by

$$W(0,y,t)=0=W(L_x,y,t)=0 \tag{3.47}$$

$$W(x,0,t)=0=W(x,L_y,t) \tag{3.48}$$

$$\frac{\partial^2}{\partial x^2}W(0,y,t)=0=\frac{\partial^2}{\partial x^2}W(L_x,y,t)=0 \tag{3.49}$$

$$\frac{\partial^2}{\partial y^2}W(0,y,t)=0=\frac{\partial^2}{\partial y^2}W(x,L_y,t)=0 \tag{3.50}$$

$$\Psi_m(0)=\Psi_m(L_x) \tag{3.51}$$

$$\Psi_m(0)=\Psi_m(L_y) \tag{3.52}$$

$$\frac{\partial^2}{\partial x^2}\Psi_m(0)=\frac{\partial^2}{\partial x^2}\Psi_m(L_x) \tag{3.53}$$

Notes

$$\frac{\partial^2}{\partial y^2}\Psi_m(0) = \frac{\partial^2}{\partial y^2}\Psi_m(L_y) \tag{3.54}$$

$$\Psi_m(x) = \sin\frac{\Gamma_m x}{L_x} + A_m \cos\frac{\Gamma_m x}{L_x} + B_m \sinh\frac{\Gamma_m x}{L_x} + C_m \cosh\frac{\Gamma_m x}{L_x} \tag{3.55}$$

$$\Psi_m(y) = \sin\frac{\Gamma_m y}{L_y} + A_m \cos\frac{\Gamma_m y}{L_y} + B_m \sinh\frac{\Gamma_m y}{L_y} + C_m \cosh\frac{\Gamma_m y}{L_y} \tag{3.56}$$

Notes

On solving equations (3.51) and (3.53) simultaneously, one obtains

$$A_m = 0,\ B_m = 0,\ C_m = 0 \tag{3.57}$$

$$\Gamma_m = m\pi \tag{3.58}$$

On putting equations (3.55) to (3.58) into equations (3.13) to (3.37), the integrals become

$$T_1 = -\left[\frac{\pi^2 q^2}{L_x^2} + \frac{\pi^2 q^2}{L_y^2}\right]\int_0^{L_x} \sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx \int_0^{L_y} \sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy \tag{3.59}$$

$$T_2 = \frac{\pi^4 q^4}{L_x^2 L_y^2}\int_0^{L_x} \sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx \int_0^{L_y} \sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy \tag{3.60}$$

$$T_3 = \frac{\pi^4 q^4}{L_y^4}\int_0^{L_x} \sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx \int_0^{L_y} \sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy \tag{3.61}$$

$$T_4 = \frac{\pi^4 q^4}{L_x^4}\int_0^{L_y} \sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx \int_0^{L_y} \sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy \tag{3.62}$$

$$T_5 = 4\int_0^{L_x} x\sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx \int_0^{L_y} \sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy - 3\int_0^{L_x} x^2\sin\frac{q\pi x}{L_x}$$

$$\sin\frac{m\pi x}{L_x}dx \int_0^{L_y} \sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy + \int_0^{L_x} x^3\sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx \int_0^{L_y} \sin\frac{q\pi y}{L_y} \tag{3.63}$$

$$\sin\frac{m\pi y}{L_y}dy$$

$$T_6 = \frac{-13\pi q}{L_x}\int_0^{L_x} \cos\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx \int_0^{L_y} \sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy + \frac{12\pi q}{L_x}\int_0^{L_x} x\cos\frac{q\pi x}{L_x}$$

$$\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy+\frac{3\pi q}{L_x}\int_0^{L_x}x^2\cos\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy \tag{3.64}$$

Notes

$$T_7=\frac{12\pi^2q^2}{L_x^2}\int_0^{L_x}\sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy-\frac{13\pi^2q^2}{L_x^2}\int_0^{L_x}x\sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy+\frac{6\pi^2q^2}{L_x^2}\int_0^{L_x}x^2\sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy+\frac{\pi^2q^2}{L_x^2}\int_0^{L_x}x^3\sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy+\frac{12\pi^2q^2}{L_y^2}\int_0^{L_x}\sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy-\frac{13\pi^2q^2}{L_y^2}\int_0^{L_x}x\sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy+\frac{6\pi^2q^2}{L_y^2}\int_0^{L_x}x^2\sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy+\frac{\pi^2q^2}{L_y^2}\int_0^{L_x}x^3\sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy \tag{3.65}$$

$$T_8=\frac{1}{16}\int_0^{L_x}\sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy \tag{3.66}$$

$$E_1^*=\int_0^{L_x}\sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}\sin(2j+1)\pi x dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy \tag{3.67}$$

$$E_2^*=\int_0^{L_x}\sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}\cos(2j+1)\pi x dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy \tag{3.68}$$

$$E_3^*=\int_0^{L_x}\sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}\sin(2k+1)\pi y dy \tag{3.69}$$

$$E_4^*=\int_0^{L_x}\sin\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}\cos(2k+1)\pi y dy \tag{3.70}$$

$$E_5^*=E_1^*,\quad E_6^*=E_2^*,\quad E_7^*=E_3^*,\quad E_8^*=E_4^* \tag{3.71}$$

$$T_9 = \frac{\pi q}{16L_x}\int_0^{L_x}\cos\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy \tag{3.72}$$

$$E_9^* = \frac{q\pi}{L_x}\int_0^{L_x}\cos\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}\sin(2j+1)\pi x dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy \tag{3.73}$$

$$E_{10}^* = \frac{q\pi}{L_x}\int_0^{L_x}\cos\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}\cos(2j+1)\pi x dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy \tag{3.74}$$

Notes

$$E_{11}^* = \frac{q\pi}{L_x}\int_0^{L_x}\cos\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}\sin(2k+1)\pi y dy \tag{3.75}$$

$$E_{12}^* = \frac{q\pi}{L_x}\int_0^{L_x}\cos\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}\cos(2k+1)\pi y dy \tag{3.76}$$

$$E_{13}^* = E_9^*, \quad E_{14}^* = E_{10}^*, \quad E_{15}^* = E_{11}^*, \quad E_{16}^* = E_{12}^* \tag{3.77}$$

$$T_{10} = -\frac{\pi^2 q^2}{16L_x^2}\int_0^{L_x}\cos\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy \tag{3.78}$$

$$E_{17}^* = -\frac{q^2\pi^2}{L_x^2}\int_0^{L_x}\cos\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}\sin(2j+1)\pi x dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy \tag{3.79}$$

$$E_{18}^* = -\frac{q^2\pi^2}{L_x^2}\int_0^{L_x}\cos\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}\cos(2j+1)\pi x dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}dy \tag{3.80}$$

$$E_{19}^* = \frac{q^2\pi^2}{L_x^2}\int_0^{L_x}\cos\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}\sin(2k+1)\pi y dy \tag{3.81}$$

$$E_{20}^* = -\frac{q^2\pi^2}{L_x^2}\int_0^{L_x}\cos\frac{q\pi x}{L_x}\sin\frac{m\pi x}{L_x}dx\int_0^{L_y}\sin\frac{q\pi y}{L_y}\sin\frac{m\pi y}{L_y}\cos(2k+1)\pi y dy \tag{3.82}$$

$$E_{21}^* = E_{17}^* \quad E_{22}^* = E_{18}^* \quad E_{23}^* = E_{19}^* \quad E_{24}^* = E_{20}^* \tag{3.83}$$

On solving equations (3.59) to (3.66),(3.72) and (3.78), and substituting into equation (47), one obtains

$$\ddot{Q}_n(t) + \omega_n^2 Q_n(t) - \frac{1}{\theta^*}\sum_{q=1}^{\infty}\Bigg[-\frac{R_0 L_x L_y}{4}\quad\frac{\pi^2 q^2}{L_x^2} + \frac{\pi^2 q^2}{L_y^2}\Bigg)\ddot{Q}_q(t) - \frac{2B\pi^4 q^4}{4\mu L_x L_y}Q_q(t) - \frac{D_y \pi^4 q^4 L_x}{4\mu L_y^3}$$

$$Q_q(t) - \frac{D_x \pi^4 q^4 L_y}{4\mu L_x^3}Q_q(t) + \left(\frac{L_x L_y}{4}\omega_q^2 - \frac{K_0}{\mu}T_5\right)Q_q(t) + \frac{G_0}{\mu}(T_6+T_7)Q_q(t) - \varpi\varrho^*\Bigg(\Bigg($$

Notes

$$\frac{L_xL_y}{64}+\frac{1}{\pi^2}\left(\sum_{j=1}^{\infty}E_1^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_2^*\frac{\sin(2j+1)\pi ct}{2j+1}\right)\left(\sum_{k=1}^{\infty}E_3^*\frac{\cos(2k+1)\pi s}{2k+1}\right.$$

$$\left.-\sum_{k=1}^{\infty}E_4^*\frac{\sin(2k+1)\pi s}{2k+1}\right)+\frac{1}{4\pi}\left(\sum_{j=1}^{\infty}E_5^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_6^*\frac{\sin(2j+1)\pi ct}{2j+1}\right)+\frac{1}{4\pi}$$

$$\left(\sum_{k=1}^{\infty}E_7^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_8^*\frac{\sin(2k+1)\pi s}{2k+1}\right)\Bigg)\ddot{Q}_q(t)+2c\left(\frac{-4mL_x}{64(q^2-m^2)\pi}+\frac{1}{\pi^2}\left(\sum_{j=1}^{\infty}E_9^*\right.\right.$$

$$\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{10}^*\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)\left(\sum_{k=1}^{\infty}E_{11}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{12}^*\frac{\sin(2k+1)\pi s}{2k+1}\right.$$

$$\left.\right)+\frac{1}{4\pi}\left(\sum_{j=1}^{\infty}E_{13}^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{14}^*\frac{\sin(2j+1)\pi ct}{2j+1}\right)+\frac{1}{4\pi}\left(\sum_{k=1}^{\infty}E_{15}^*\frac{\cos(2k+1)\pi s}{2k+1}\right.$$

$$\left.-\sum_{k=1}^{\infty}E_{16}^*\frac{\sin(2k+1)\pi s}{2k+1}\right)\Bigg)\dot{Q}_q(t)+c^2\left(\frac{-\pi^2q^2L_y}{64L_x}+\frac{1}{\pi^2}\left(\sum_{j=1}^{\infty}E_{17}^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}\right.\right.$$

$$E_{18}^*\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)\left(\sum_{k=1}^{\infty}E_{19}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{20}^*\frac{\sin(2k+1)\pi s}{2k+1}\right)+\frac{1}{4\pi}\left(\sum_{j=1}^{\infty}E_{21}^*\right.$$

$$\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{22}^*\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)+\frac{1}{4\pi}\left(\sum_{k=1}^{\infty}E_{23}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{24}^*\right.$$

$$\left.\left.\frac{\sin(2k+1)\pi s}{2k+1}\right)\right)Q_q(t)\Bigg)\Bigg]=\sum_{q=1}^{\infty}\frac{Mg}{\mu\theta^*}\Phi_m(ct)\Phi_m(s) \tag{3.84}$$

The solutions to equation (3.84) shall be obtained by considering two cases:

a) Simply Supported Orthotropic Rectangular Plate Tranversed by Moving Force

For moving force problem, one sets $\varpi=0$ in equation (3.84) which becomes

$$\ddot{Q}_n(t)+\left(1-\frac{L_xL_y}{4\theta^*}\right)\omega_n^2Q_n(t)-\frac{1}{\mu\theta^*}\left(-\frac{\mu R_0L_xL_y}{4}\left(\frac{\pi^2n^2}{L_x^2}+\frac{\pi^2n^2}{L_y^2}\right)\ddot{Q}_n(t)-\frac{2B\pi^4n^4}{4L_xL_y}\right.$$

$$\left.Q_n(t)-\frac{D_y\pi^4n^4L_x}{4L_y^3}Q_n(t)-\frac{D_x\pi^4n^4L_y}{4L_x^3}Q_n(t)-K_0T_5Q_n(t)+\frac{G_0}{\mu}(T_6+T_7)Q_n(t)\right)$$

$$-\frac{1}{\mu\theta^*}\sum_{q=1,q\neq=n}^{\infty}\Bigg(-\frac{\mu R_0 L_x L_y}{4}\quad\frac{\pi^2 q^2}{L_x^2}+\frac{\pi^2 q^2}{L_y^2}\Bigg)\ddot{Q}_q(t)-\frac{2B\pi^4 q^4}{4L_xL_y}Z_q(t)-\frac{D_y\pi^4 q^4 L_x}{4L_y^3}$$

$$Z_q(t)-\frac{D_x\pi^4 q^4 L_y}{4L_x^3}Q_q(t)+\Bigg(\frac{L_xL_y}{4}\mu\omega_q^2-K_0T_5\Bigg)Q_q(t)-G_0(T_6+T_7)Q_q(t)\Bigg)$$

$$=\sum_{q=1}^{\infty}\frac{Mg}{\mu\theta^*}\sin\frac{m\pi s}{L_y}\sin\frac{m\pi ct}{L_x} \tag{3.85}$$

On further simplification and re-arrangement, one obtains

$$\Bigg[1+\lambda\frac{\mu R_0 L_x L_y}{4}\quad\frac{\pi^2 n^2}{L_x^2}+\frac{\pi^2 n^2}{L_y^2}\Bigg)\Bigg]\ddot{Q}_n(t)+\Bigg(\epsilon_n^2-\lambda\Bigg(-\frac{2B\pi^4 n^4}{4L_xL_y}-\frac{D_y\pi^4 n^4 L_x}{4L_y^3}-\frac{D_x\pi^4 q^4 L_y}{4L_x^3}$$

$$-K_0T_5Q_n(t)+G_0(T_6+T_7)Q_n(t)\Bigg)\Bigg)Q_n(t)-\lambda\sum_{q=1,q\neq=n}^{\infty}\Bigg(-\frac{\mu R_0 L_x L_y}{4}\Bigg(\frac{\pi^2 q^2}{L_x^2}+\frac{\pi^2 q^2}{L_y^2}\Bigg)$$

$$\ddot{Q}_q(t)-\frac{2B\pi^4 q^4}{4L_xL_y}Q_q(t)-\frac{D_y\pi^4 q^4 L_x}{4L_y^3}Q_q(t)-\frac{D_x\pi^4 q^4 L_y}{4L_x^3}Q_q(t)+\Bigg(\frac{L_xL_y}{4}\mu\omega_q^2-K_0T_5\Bigg)$$

$$Q_q(t)+G_0(T_6+T_7)Q_q(t)\Bigg)=\sum_{q=1}^{\infty}Mg\lambda\sin\frac{m\pi s}{L_y}\sin\frac{m\pi ct}{L_x} \tag{3.86}$$

where

$$\epsilon_n^2=\Bigg(1-\frac{L_xL_y}{4\theta^*}\Bigg)\omega_n^2\qquad\lambda=\frac{1}{\mu\theta^*} \tag{3.87}$$

Consider a parameter $\lambda* < 1$ for any arbitrary mass ratio λ, defined as

$$\lambda=\frac{\lambda^*}{1+\lambda^*} \tag{3.88}$$

It can be shown that

$$\lambda=\lambda^*-o(\lambda^{*2}) \tag{3.89}$$

Retaining only $o(\lambda^*)$, one obtains

$$\lambda=\lambda^* \tag{3.90}$$

On putting equation (3.89) into equation (3.86),rewriting and simplifying further , one obtains

Notes

$$\ddot{Q}_n(t) + \left[\omega_n^2\left(1-\lambda^*\frac{\mu R_0 L_x L_y}{4}\quad\frac{\pi^2 n^2}{L_x^2}+\frac{\pi^2 n^2}{L_y^2}\right)+o(\lambda^{*2})+\ldots\right)-\lambda^*\quad\frac{2B\pi^4 n^4}{4L_xL_y}-\frac{D_x\pi^4 n^4 L_y}{4L_x^3}$$

$$-\frac{D_y\pi^4 n^4 L_x}{4L_y^3}+\left(\mu\omega_n^2-\frac{K_0L_y}{4}\varphi_1(x)\right)+\frac{G_0L_y}{4}\quad\frac{\pi n}{L_x}\varphi_2(x)+\left(\frac{\pi^2 n^2}{L_x^2}+\frac{\pi^2 n^2}{L_y^2}\right)\varphi_3(x)\Bigg)\Bigg(1-$$

$$\lambda^*\frac{\mu R_0 L_x L_y}{4}\left(\frac{\pi^2 n^2}{L_x^2}+\frac{\pi^2 n^2}{L_y^2}\right)+o(\lambda^{*2})+\ldots\Bigg)\Bigg)\Bigg]Q_n(t)-\eta^*\Bigg(1-\lambda^*\frac{\mu R_0 L_x L_y}{4}\left(\frac{\pi^2 n^2}{L_x^2}+\frac{\pi^2 n^2}{L_y^2}\right.$$

$$\left.\right)+o(\lambda^{*2})+\ldots\Bigg)\Bigg)\sum_{q=1,q\neq=n}^{\infty}\left(-\frac{\mu R_0 L_x L_y}{4}\quad\frac{\pi^2 q^2}{L_x^2}+\frac{\pi^2 q^2}{L_y^2}\right)\ddot{Q}_q(t)-\Bigg(\frac{2B\pi^4 q^4}{4L_xL_y}+\frac{D_x\pi^4 q^4 L_y}{4L_x^3}$$

$$+\frac{D_y\pi^4 q^4 L_x}{4L_y^3}+\frac{K_0L_y}{4}\varphi_1(x)-\frac{G_0L_y}{4}\left(\frac{\pi q}{L_x}\varphi_2(x)+\quad\frac{\pi^2 q^2}{L_x^2}+\frac{\pi^2 q^2}{L_y^2}\right)\varphi_3(x)\Bigg)\Bigg)\Bigg)Q_q(t)=$$

$$Mg\lambda^*\sin\frac{m\pi s}{L_y}\sin\frac{m\pi ct}{L_x} \tag{3.91}$$

Expanding equation (3.91), and retaining only $o(\lambda^*)$, one obtains

$$\ddot{Q}_n(t)+\left[\omega_n^2\left(1-\eta^*\frac{\mu R_0 L_x L_y}{4}\quad\frac{\pi^2 n^2}{L_x^2}+\frac{\pi^2 n^2}{L_y^2}\right)\right)-\eta^*\left(\frac{2B\pi^4 n^4}{4L_xL_y}-\frac{D_x\pi^4 n^4 L_y}{4L_x^3}-\frac{D_y\pi^4 n^4 L_x}{4L_y^3}\right.$$

$$+\left(\mu\omega_n^2-\frac{K_0L_y}{4}\varphi_1(x)\right)+\frac{G_0L_y}{4}\left(\frac{\pi n}{L_x}\varphi_2(x)+\left(\frac{\pi^2 n^2}{L_x^2}+\frac{\pi^2 n^2}{L_y^2}\right)\varphi_3(x)\right)\Bigg)\Bigg]Q_n(t)-\eta^*\Bigg(1$$

$$-\eta^*\frac{\mu R_0 L_x L_y}{4}\left(\frac{\pi^2 n^2}{L_x^2}+\frac{\pi^2 n^2}{L_y^2}\right)\Bigg)\sum_{q=1,q\neq=n}^{\infty}\left(-\frac{\mu R_0 L_x L_y}{4}\left(\frac{\pi^2 q^2}{L_x^2}+\frac{\pi^2 q^2}{L_y^2}\right)\ddot{Q}_q(t)-\Bigg(\frac{2B\pi^4 q^4}{4L_xL_y}\right.$$

$$+\frac{D_x\pi^4 q^4 L_y}{4L_x^3}+\frac{D_y\pi^4 q^4 L_x}{4L_y^3}+\frac{K_0L_y}{4}\varphi_1(x)+\frac{G_0L_y}{4}\left(\frac{\pi q}{L_x}\varphi_2(x)+\left(\frac{\pi^2 q^2}{L_x^2}+\frac{\pi^2 q^2}{L_y^2}\right)\varphi_3(x)\right)\Bigg)$$

$$Q_q(t)=Mg\eta^*\sin\frac{m\pi s}{L_y}\sin\frac{m\pi ct}{L_x} \tag{3.92}$$

Using Struble's technique, equation (3.92) can be rewritten as

$$\ddot{Q}_n(t)+\nu_n^2 Q_n(t)=0 \tag{3.93}$$

Hence, the entire equation (3.92) becomes

$$\ddot{Q}_n(t)+\nu_n^2 Q_n(t)=Mg\lambda^*\sin\frac{m\pi s}{L_y}\sin\frac{m\pi ct}{L_x} \tag{3.94}$$

where

$$\nu_n = \epsilon_n - \frac{\lambda^*}{2\epsilon_n}\left[\frac{\epsilon_n^2 R_0 L_x L_y}{4}\quad \frac{\pi^2 n^2}{L_x^2} + \frac{\pi^2 n^2}{L_y^2}\right) + \frac{2B\pi^4 n^4}{4L_x L_y} - \frac{D_y \pi^4 n^4 L_x}{4L_y^3} - \frac{D_x \pi^4 n^4 L_y}{4L_x^3} - \tag{3.95}$$

$$\frac{K_0 L_x L_y}{4} + \frac{G_0 L_x L_y}{4}\quad \frac{\pi^2 n^2}{L_x^2} + \frac{\pi^2 n^2}{L_y^2}\right)\Bigg]$$

is the modified frequency for simply supported orthotropic rectangular plate traversed by moving force.

On solving equation (3.94) by Laplace transformation techniques one obtains

$$Q_n(t) = \frac{Mg\lambda^*}{\nu_n}\sin\frac{m\pi s}{L_y} \times \left[\frac{\nu_n \sin\frac{m\pi c}{L_x}t - (\frac{m\pi c}{L_x})\sin\nu_n t}{(\frac{m\pi c}{L_X})^2 - \nu_n^2}\right] \tag{3.96}$$

which on inversion becomes

$$W(x,y,t) = \frac{Mg\lambda^*}{\nu_n}\sin\frac{m\pi s}{L_y} \times \left[\frac{\nu_n \sin\frac{m\pi c}{L_x}t - (\frac{m\pi c}{L_x})\sin\nu_n t}{(\frac{m\pi c}{L_X})^2 - \nu_n^2}\right] \times \sin\frac{m\pi x}{L_x}\sin\frac{m\pi y}{L_y} \tag{3.97}$$

is the transverse displacement response to a moving force of a simply supported orthotropic rectangular plate.

b) Simply Supported Orthotropic Rectangular Plate Transversed by Moving Mass

Here, one seeks solution to the entire equation (3.84). To solve this problem, one makes use of the modified asymptotic method of Struble. The equation becomes,

$$\ddot{Q}_n(t) + \nu_n^2 Q_n(t) + \alpha\varrho^* \sum_{q=1}^{\infty}\Bigg[\Bigg(\frac{L_x L_y}{64} + \frac{1}{\pi^2}\Bigg(\sum_{j=1}^{\infty} E_1^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_2^*$$

$$\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)\Bigg(\sum_{k=1}^{\infty} E_3^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_4^* \frac{\sin(2k+1)\pi s}{2k+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{j=1}^{\infty} E_5^*$$

$$\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_6^* \frac{\sin(2j+1)\pi ct}{2j+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^*$$

$$\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)\ddot{Q}_q(t) + 2c\Bigg(\frac{-4mL_y}{64(q^2-m^2)\pi} + \frac{1}{\pi^2}\Bigg(\sum_{j=1}^{\infty} E_9^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{10}^*$$

Notes

Notes

$$\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)\Bigg(\sum_{k=1}^{\infty}E_{11}^{*}\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{12}^{*}\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)+\frac{1}{4\pi}\Bigg(\sum_{j=1}^{\infty}E_{13}^{*}$$

$$\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{14}^{*}\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)+\frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty}E_{15}^{*}\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{16}^{*}$$

$$\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)\dot{Q}_q(t)+c^2\Bigg(\frac{-\pi^2q^2L_y}{64L_x}+\frac{1}{\pi^2}\Bigg(\sum_{j=1}^{\infty}E_{17}^{*}\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{18}^{*} \qquad (3.98)$$

$$\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)\Bigg(\sum_{k=1}^{\infty}E_{19}^{*}\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{20}^{*}\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)+\frac{1}{4\pi}\Bigg(\sum_{j=1}^{\infty}E_{21}^{*}$$

$$-\sum_{j=1}^{\infty}E_{22}^{*}\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)+\frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty}E_{23}^{*}\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{24}^{*}\frac{\sin(2k+1)\pi s}{2k+1}$$

$$\Bigg)\Bigg)Q_q(t)\Bigg]=\frac{g\alpha}{\theta^*}\frac{m\pi s}{L_y}\sin\frac{m\pi ct}{L_x}$$

On further simplifications and rearrangements of equation (3.98), one obtains

$$\ddot{Q}_n(t)+\frac{2c\alpha\varrho^*}{1+\alpha\varrho^*\delta(i,j)\Bigg)}\Bigg(\frac{-4mL_y}{64(q^2-m^2)\pi}+\frac{1}{\pi^2}\Bigg(\sum_{j=1}^{\infty}E_{9}^{*}\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{10}^{*}$$

$$\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)\Bigg(\sum_{k=1}^{\infty}E_{11}^{*}\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{12}^{*}\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)+\frac{1}{4\pi}\Bigg(\sum_{j=1}^{\infty}E_{13}^{*}$$

$$\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{14}^{*}\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)+\frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty}E_{15}^{*}\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{16}^{*}$$

$$\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)\dot{Q}_n(t)+\frac{1}{1+\alpha\varrho^*\delta(i,j)\Bigg)}\Bigg[\nu_n^2+c^2\alpha\varrho^*\quad\frac{-\pi^2q^2L_y}{64L_x}+\frac{1}{\pi^2}\Bigg(\sum_{j=1}^{\infty}E_{17}^{*}$$

$$\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{18}^{*}\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)\Bigg(\sum_{k=1}^{\infty}E_{19}^{*}\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{20}^{*}$$

$$\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)+\frac{1}{4\pi}\Bigg(\sum_{j=1}^{\infty}E_{21}^{*}\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{22}^{*}\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)+\frac{1}{4\pi}$$

$$\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)\Bigg] Q_n(t) + \frac{\alpha \varrho^*}{\Big(1+\alpha\varrho^*\delta(i,j)\Big)}$$

$$\sum_{q=1,q\neq n}^{\infty} \Bigg[\Bigg(\frac{L_x L_y}{64} + \frac{1}{\pi^2}\Bigg(\sum_{j=1}^{\infty} E_1^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_2^* \frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)\Bigg(\sum_{k=1}^{\infty} E_3^*$$

$$\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_4^* \frac{\sin(2k+1)\pi s}{2k+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{j=1}^{\infty} E_5^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_6^*$$

$$\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)\ddot{Q}_q(t)$$

$$+\,2c\Bigg(\frac{-4mL_y}{64(q^2-m^2)\pi} + \frac{1}{\pi^2}\Bigg(\sum_{j=1}^{\infty} E_9^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{10}^* \frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)$$

$$\Bigg(\sum_{k=1}^{\infty} E_{11}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{12}^* \frac{\sin(2k+1)\pi s}{2k+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{j=1}^{\infty} E_{13}^* \frac{\cos(2j+1)\pi ct}{2j+1}$$

$$-\sum_{j=1}^{\infty} E_{14}^* \frac{\sin(2j+1)\pi ct}{2j+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty} E_{15}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{16}^* \frac{\sin(2k+1)\pi s}{2k+1}$$

$$\Bigg)\Bigg)\dot{Q}_q(t) + c^2\Bigg(\frac{-\pi^2 q^2 L_y}{64L_x} + \frac{1}{\pi^2}\Bigg(\sum_{j=1}^{\infty} E_{17}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)$$

$$\Bigg(\sum_{k=1}^{\infty} E_{19}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{20}^* \frac{\sin(2k+1)\pi s}{2k+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi ct}{2j+1}$$

$$-\sum_{j=1}^{\infty} E_{22}^* \frac{\sin(2j+1)\pi ct}{2j+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi s}{2k+1} \quad (3.99)$$

$$\Bigg)\Bigg)Q_q(t)\Bigg] = \frac{g\alpha}{\Big(\theta^* \quad 1+\alpha\varrho^*\delta(i,j)\Big)} \sin\frac{m\pi s}{L_y} \sin\frac{m\pi ct}{L_x}$$

We shall consider a parameter $\alpha^* < 1$ for any arbitrary mass ratio defined by

$$\alpha = \frac{\alpha^*}{1+\alpha^*} \quad (3.100)$$

Notes

By using binomial theorem and truncating after second terms, one obtains

$$\alpha = \alpha^* - o(\alpha^{*2}) \tag{3.101}$$

Considering only $o(\alpha^*)$, equation(3.101) becomes

$$\alpha = \alpha^* \tag{3.102}$$

Notes

Applying binomial expansion, one obtains

$$\frac{1}{(1+\alpha^*\varrho^*\delta(i,j))} = 1 - \alpha^*\varrho^*\delta(i,j) + (\alpha^*)^2 + ... \tag{3.103}$$

On putting equation (3.101) into equation (3.99), expanding and retaining only $o(\alpha^{*2})$, one obtains

$$\ddot{Q}_n(t) + 2c\alpha^*\varrho^*\Bigg(\frac{-4mL_y}{64(n^2-m^2)\pi} + \frac{1}{\pi^2}\Bigg(\sum_{j=1}^{\infty} E_9^*\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{10}^*\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)$$

$$\Bigg(\sum_{k=1}^{\infty} E_{11}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{12}^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{j=1}^{\infty} E_{13}^*\frac{\cos(2j+1)\pi ct}{2j+1}$$

$$-\sum_{j=1}^{\infty} E_{14}^*\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty} E_{15}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{16}^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)$$

$$\dot{Q}_n(t) + \Bigg[\nu_n^2\Bigg(1 - \alpha^*\varrho^*\Bigg(\frac{L_xL_y}{64} + \frac{1}{\pi^2}\Bigg(\sum_{j=1}^{\infty} E_1^*\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_2^*\frac{\sin(2j+1)\pi ct}{2j+1}$$

$$\Bigg)\Bigg(\sum_{k=1}^{\infty} E_3^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_4^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{j=1}^{\infty} E_5^*\frac{\cos(2j+1)\pi ct}{2j+1}$$

$$-\sum_{j=1}^{\infty} E_6^*\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty} E_7^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)$$

$$+c^2\alpha^*\varrho^*\Bigg(\frac{-\pi^2q^2L_y}{64L_x} + \frac{1}{\pi^2}\Bigg(\sum_{j=1}^{\infty} E_{17}^*\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{18}^*\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg)$$

$$\Bigg(\sum_{k=1}^{\infty} E_{19}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{20}^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{j=1}^{\infty} E_{21}^*\frac{\cos(2j+1)\pi ct}{2j+1}$$

$$-\sum_{j=1}^{\infty} E_{22}^*\frac{\sin(2j+1)\pi ct}{2j+1}\Bigg) + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty} E_{23}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{24}^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)$$

$$\Big)\Big)\Big]Q_n(t)+\alpha^*\varrho^*\Big[1-\alpha^*\varrho^*\Big(\frac{L_xL_y}{64}+\frac{1}{\pi^2}\Big(\sum_{j=1}^{\infty}E_1^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_2^*$$

$$\frac{\sin(2j+1)\pi ct}{2j+1}\Big)\Big(\sum_{k=1}^{\infty}E_3^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_4^*\frac{\sin(2k+1)\pi s}{2k+1}\Big)+\frac{1}{4\pi}\Big(\sum_{j=1}^{\infty}E_5^*$$

$$\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_6^*\frac{\sin(2j+1)\pi ct}{2j+1}\Big)+\frac{1}{4\pi}\Big(\sum_{j=1}^{\infty}E_7^*\frac{\cos(2j+1)\pi s}{2j+1}-\sum_{j=1}^{\infty}E_8^*$$

$$\frac{\sin(2j+1)\pi s}{2j+1}\Big)\Big)+o(\alpha^*)^2+\ldots\Big]\sum_{q=1,q\neq n}^{\infty}\Big[\Big(\frac{L_xL_y}{64}+\frac{1}{\pi^2}\Big(\sum_{j=1}^{\infty}E_1^*\frac{\cos(2j+1)\pi ct}{2j+1}$$

$$-\sum_{j=1}^{\infty}E_2^*\frac{\sin(2j+1)\pi ct}{2j+1}\Big)\Big(\sum_{k=1}^{\infty}E_3^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_4^*\frac{\sin(2k+1)\pi s}{2k+1}\Big)$$

$$+\frac{1}{4\pi}\Big(\sum_{j=1}^{\infty}E_5^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_6^*\frac{\sin(2j+1)\pi ct}{2j+1}\Big)+\frac{1}{4\pi}\Big(\sum_{j=1}^{\infty}E_7^*\frac{\cos(2j+1)\pi s}{2j+1}$$

$$-\sum_{j=1}^{\infty}E_8^*\frac{\sin(2j+1)\pi s}{2j+1}\Big)\Big)\ddot{Q}_q(t)+2c\Big(\frac{-4mL_y}{64(q^2-m^2)\pi}+\frac{1}{\pi^2}\Big(\sum_{j=1}^{\infty}E_9^*\frac{\cos(2j+1)\pi ct}{2j+1}$$

$$-\sum_{j=1}^{\infty}E_{10}^*\frac{\sin(2j+1)\pi ct}{2j+1}\Big)\Big(\sum_{j=1}^{\infty}E_{11}^*\frac{\cos(2j+1)\pi s}{2j+1}-\sum_{j=1}^{\infty}E_{12}^*\frac{\sin(2j+1)\pi s}{2j+1}\Big)+\frac{1}{4\pi}\Big($$

$$\sum_{j=1}^{\infty}E_{13}^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{14}^*\frac{\sin(2j+1)\pi ct}{2j+1}\Big)+\frac{1}{4\pi}\Big(\sum_{j=1}^{\infty}E_{15}^*\frac{\cos(2j+1)\pi s}{2j+1}-$$

$$\sum_{j=1}^{\infty}E_{16}^*\frac{\sin(2j+1)\pi s}{2j+1}\Big)\Big)\dot{Q}_q(t)+c^2\Big(\frac{-\pi^2q^2L_y}{64L_x}+\frac{1}{\pi^2}\Big(\sum_{j=1}^{\infty}E_{17}^*\frac{\cos(2j+1)\pi ct}{2j+1}-$$

$$\sum_{j=1}^{\infty}E_{18}^*\frac{\sin(2j+1)\pi ct}{2j+1}\Big)\Big(\sum_{j=1}^{\infty}E_{19}^*\frac{\cos(2j+1)\pi s}{2j+1}-\sum_{j=1}^{\infty}E_{20}^*\frac{\sin(2j+1)\pi s}{2j+1}\Big)+\frac{1}{4\pi}$$

$$\Big(\sum_{j=1}^{\infty}E_{21}^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{22}^*\frac{\sin(2j+1)\pi ct}{2j+1}\Big)+\frac{1}{4\pi}\Big(\sum_{j=1}^{\infty}E_{23}^*\frac{\cos(2j+1)\pi s}{2j+1}-$$

$$\sum_{j=1}^{\infty}E_{24}^*\frac{\sin(2j+1)\pi s}{2j+1}\Big)\Big)Q_q(t)\Big]=\frac{g\alpha^*}{\theta^*}\sin\frac{m\pi s}{L_y}\sin\frac{m\pi ct}{L_x} \tag{3.104}$$

Notes

Applying the method of Struble technique to equation (3.104),its homogeneous part becomes

$$\ddot{Q}_n(t) + \delta_n^2 Q_n(t) = 0 \tag{3.105}$$

Hence, entire equation becomes

Notes

$$\ddot{Q}_n(t) + \delta_n^2 Q_n(t) = \frac{g\alpha^*}{\theta^*} \sin\frac{m\pi s}{L_y} \sin\frac{m\pi ct}{L_x} \tag{3.106}$$

where

$$\delta_n = \Bigg[\nu_n - \frac{1}{2\nu_n}\Bigg(\nu_n^2\alpha^*\varrho^*\Bigg(\frac{L_xL_y}{64} + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty} E_7^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg) -$$

$$c^2\alpha^*\varrho^*\Bigg(\frac{-\pi^2q^2L_y}{64L_x} + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty} E_{23}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{24}^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)\Bigg)\Bigg] \tag{3.107}$$

is the modified frequency for simply supported orthotropic rectangular plate.
On solving equation (3.106) by Laplace transformation techniques, one obtains

$$Q_n(t) = \frac{g\alpha^*}{\theta^*\delta_n} \sin\frac{m\pi s}{L_y} \times \left[\frac{\delta_n \sin\frac{m\pi c}{L_x}t - (\frac{m\pi c}{L_x})\sin\delta_n t}{(\frac{m\pi c}{L_X})^2 - \delta_n^2}\right] \tag{3.108}$$

which on inversion becomes

$$W(x,y,t) = \frac{g\alpha^*}{\theta^*\delta_n} \sin\frac{m\pi s}{L_y} \times \left[\frac{\sigma_n \sin\frac{m\pi c}{L_x}t - (\frac{m\pi c}{L_x})\sin\sigma_n t}{(\frac{m\pi c}{L_X})^2 - \delta_n^2}\right] \times \sin\frac{m\pi x}{L_x}\sin\frac{m\pi y}{L_y} \tag{3.109}$$

is the transverse displacement response to a moving mass of a simply supported orthotropic rectangular plate.

IV. Disscusion of the Analytical Solutions

For this undamped system, it is desirable to examine the phenomenon of resonance. From equation (3.96), it is clearly shown that the simply supported orthotropic rectangular plate on constant elastic foundation and traverse by moving distributed force with uniform speed reaches a state of resonance whenever

$$\nu_n = \frac{m\pi c}{L_x} \tag{4.1}$$

while equation (3.109) shows that the same simply supported orthotropic rectangular plate under the action of a moving mass experiences resonance when

$$\delta_n = \frac{m\pi c}{L_x} \tag{4.2}$$

where

$$\delta_n = \Bigg[\nu_n - \frac{1}{2\nu_n}\Bigg(\nu_n^2\alpha^*\varrho^*\Bigg(\frac{L_xL_y}{64} + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty}E_7^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_8^*\frac{\sin(2k+1)\pi s}{2k+1}$$

$$\Bigg)\Bigg) - c^2\alpha^*\varrho^*\Bigg(\frac{-\pi^2q^2L_y}{64L_x} + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty}E_{23}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_{24}^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)\Bigg)\Bigg] \tag{4.3}$$

Comparing equations (4.1) and (4.2), one obtains

$$\delta_n = \nu_n\Bigg[1 - \frac{1}{2\nu_n^2}\Bigg(\nu_n^2\alpha^*\varrho^*\Bigg(\frac{L_xL_y}{64} + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty}E_7^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_8^*\frac{\sin(2k+1)\pi s}{2k+1}$$

$$\Bigg)\Bigg) - c^2\alpha^*\varrho^*\Bigg(\frac{-\pi^2q^2L_y}{64L_x} + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty}E_{23}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_{24}^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)\Bigg)\Bigg] \tag{4.4}$$

$$= \frac{m\pi c}{L_x}$$

Obviously

$$1 - \frac{1}{2\nu_n^2}\Bigg(\nu_n^2\alpha^*\varrho^*\Bigg(\frac{L_xL_y}{64} + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty}E_7^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_8^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)$$

$$- c^2\alpha^*\varrho^*\Bigg(\frac{-\pi^2q^2L_y}{64L_x} + \frac{1}{4\pi}\Bigg(\sum_{k=1}^{\infty}E_{23}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_{24}^*\frac{\sin(2k+1)\pi s}{2k+1}\Bigg)\Bigg)\Bigg) < 1 \tag{4.5}$$

That is, $\delta_n < \nu_n$ implies that moving mass simply supported system researches the state of resonance earlier than the moving force system.

V. Graphs of the Numerical Solutions

To illustrate the analysis presented in this work, orthotropic rectangular plate is taken to be of length $L_y = 0.923m$, breadth $L_x = 0.432m$ the load velocity c=0.8123 m/s and $s = 0.4m$.The results are presented on the various graphs below for the simply supported boundary conditions.

a) Graphs for Simply Supported Boundary Conditions

Figures 5.1 and 5.2 display the effect of rotatory inertia R_o on the deflection profile of simply supported orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of rotatory inertia R_o increases.

Figures 5.3 and 5.4 display the effect of foundation modulus K_o on the deflection profile of

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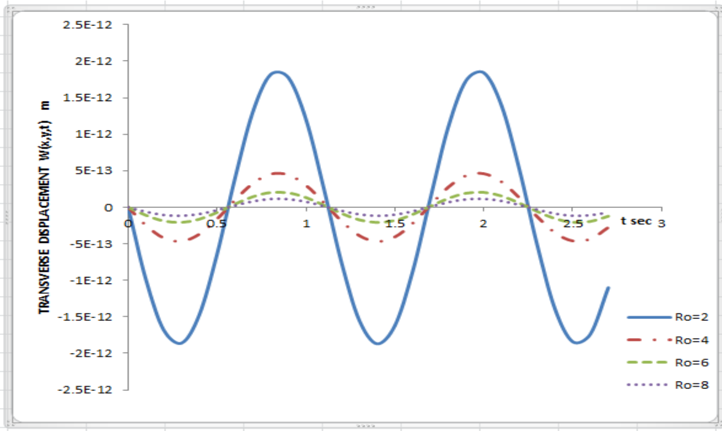

Figure 5.1: Displacement Profile of Simply Supported Orthotropic Rectangular Plate with Varying R_o and Traversed by Moving Force

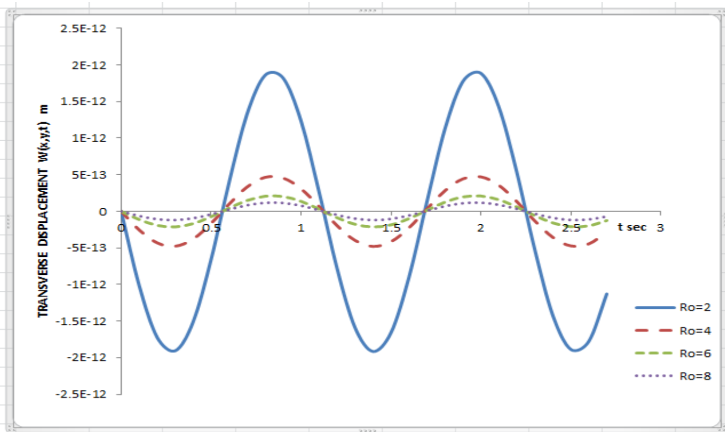

Figure 5.2: Displacement Profile of Simply Supported Orthotropic Rectangular Plate with Varying R_o and Traversed by Moving Mass

simply supported orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of foundation modulus K_o increases.

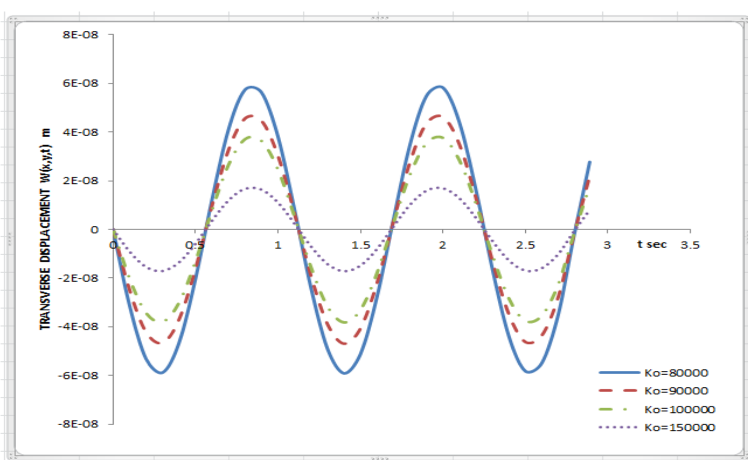

Figure 5.3: Displacement Profile of Simply Supported Orthotropic Rectangular Plate with Varying K_o and Traversed by Moving Force

Notes

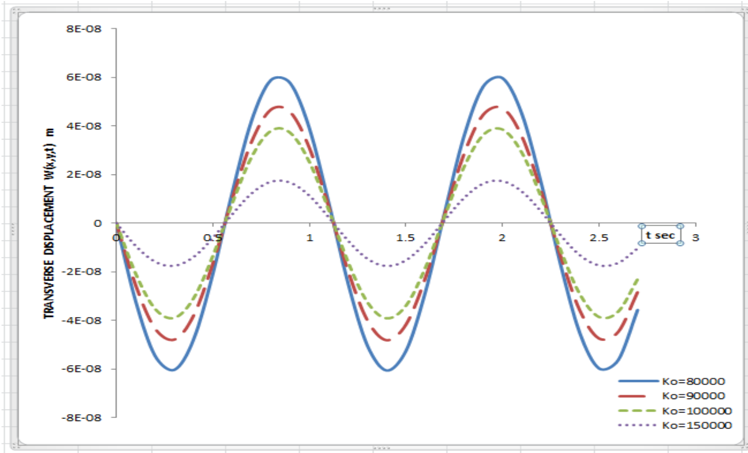

Figure 5.4: Displacement Profile of Simply Supported Orthotropic Rectangular Plate with Varying K_o and Traversed by Moving Mass

Figures 5.5 and 5.6 display the effect of shear modulus G_o on the deflection profile of simply supported orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of shear modulus G_o increases.

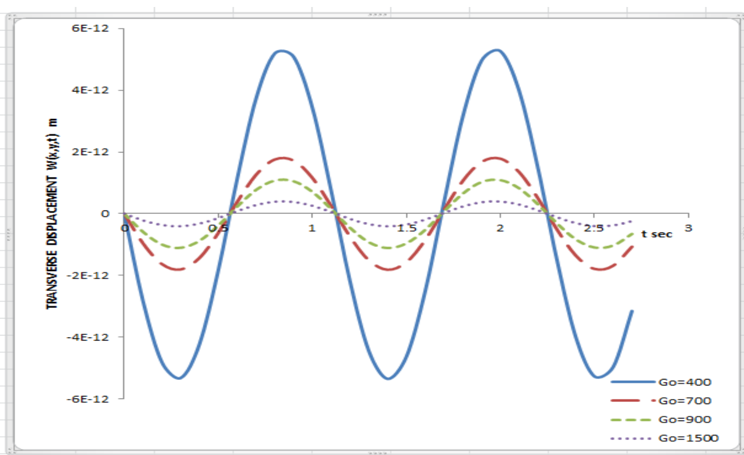

Figure 5.5: Displacement Profile of Simply Supported Orthotropic Rectangular Plate with Varying G_o and Traversed by Moving Force

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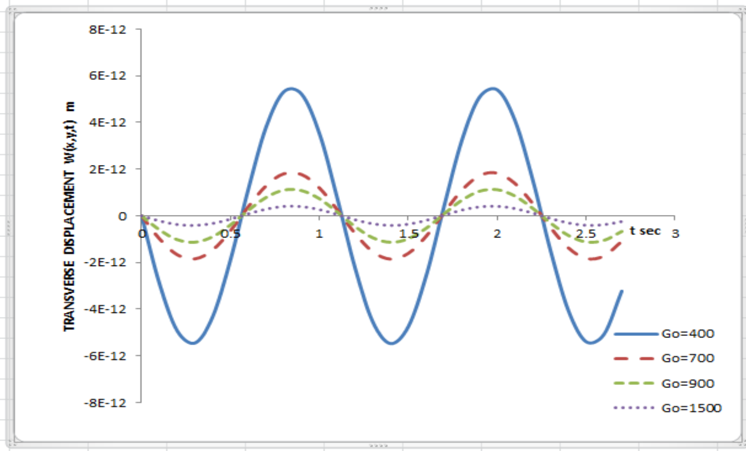

Figure 5.6: Displacement Profile of Simply Supported Orthotropic Rectangular Plate with Varying G_o and Traversed by Moving Mass

Figures 5.7 and 5.8 display the effect of flexural rigidity of the plate along x-axis D_x on the deflection profile of simply supported orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of flexural rigidity D_x increases.

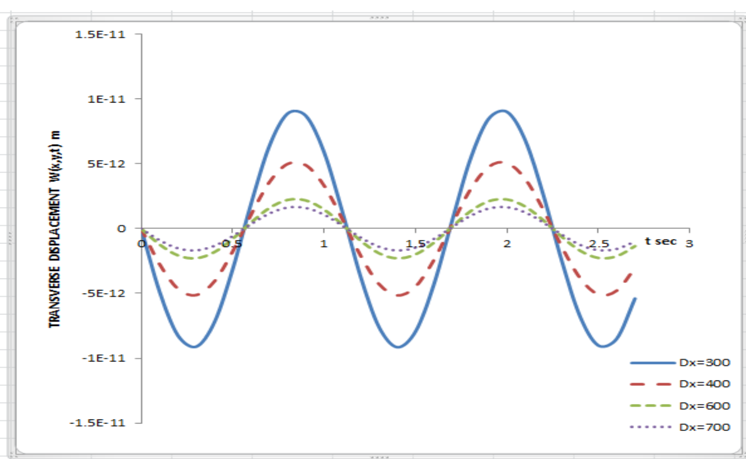

Figure 5.7: Displacement Profile of Simply Supported Orthotropic Rectangular Plate with Varying D_x and Traversed by Moving Force

Notes

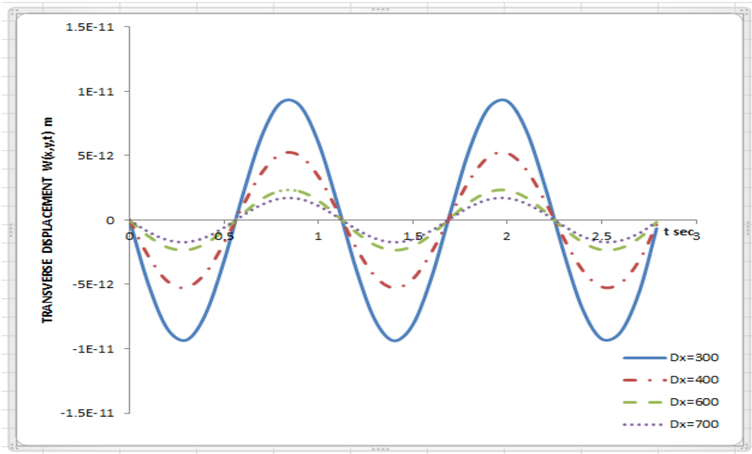

Figure 5.8: Displacement Profile of Simply Supported Orthotropic Rectangular Plate with Varying D_x and Traversed by Moving Mass

Figures 5.9 and 5.10 display the effect of flexural rigidity of the plate along y-axis D_y on the deflection profile of simply supported orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of flexural rigidity D_y increases.

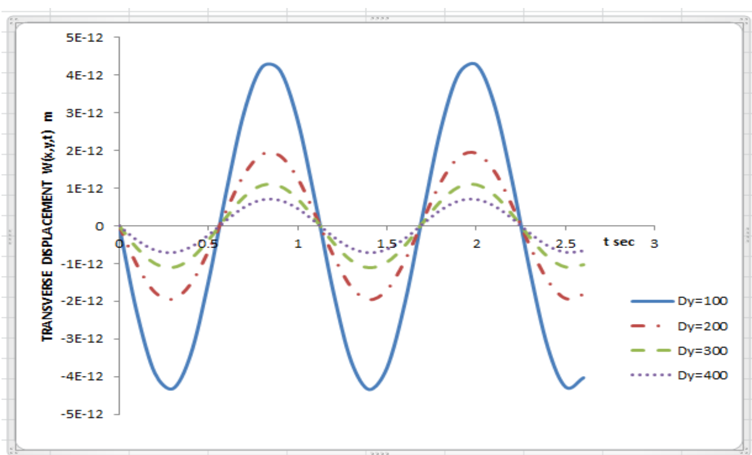

Figure 5.9: Displacement Profile of Simply Supported Orthotropic Rectangular Plate with Varying D_y and Traversed by Moving Force

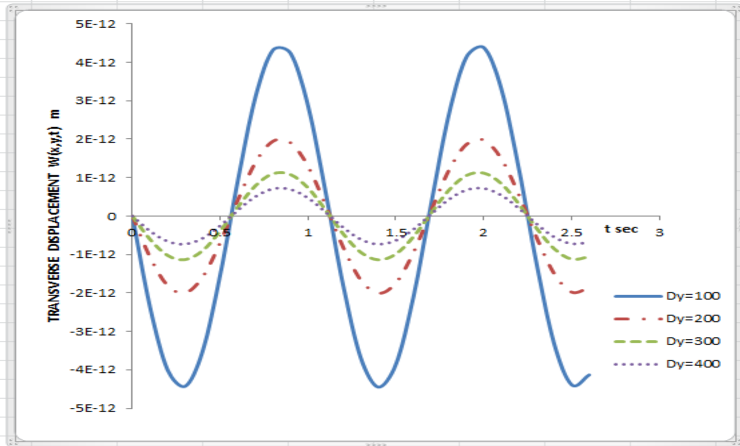

Figure 5.10: Displacement Profile of Simply Supported Orthotropic Rectangular Plate with Varying D_y and Traversed by Moving Mass

Figure 5.11 displays the comparison between moving force and moving mass for fixed values of R_o, G_o, K_o, D_x and D_y.

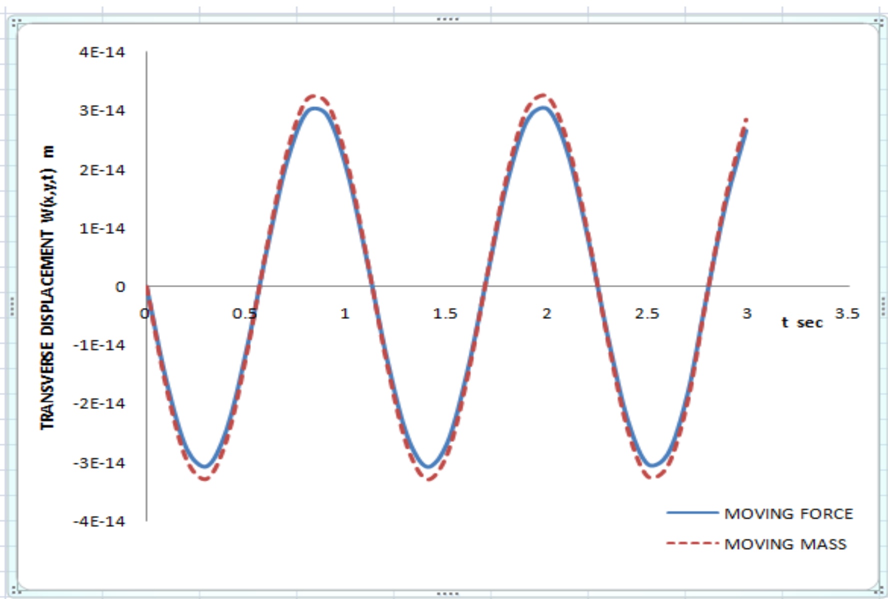

Figure 5.11: Displacement Profile of Comparison between Moving Force and Moving Mass

VI. Conclusion

In this work, the problem of response to simply supported orthotropic rectangular plate resting on a variable elastic bi-parametric foundation under the action of moving distributed masses has been studied. The closed form solutions of the fourth order partial differential equations with variable and singular coefficients governing the orthotropic rectangular plates is obtained for both cases of moving force and moving mass using a solution technique that is based on the separation of variables which was used to remove the singularity in the governing fourth order partial differential equation and thereby reducing it to a sequence of coupled second order differential equations. The modified Struble's asymptotic technique and Laplace transformation techniques are then employed to obtain the analytical solution to the two-dimensional dynamical problem.

The solutions are then analyzed. The analyses show that, for the same natural frequency and the critical speed for the moving mass problem is smaller than that of the moving force problem. Resonance is reached earlier in the moving mass system than in the moving force problem. That is to say the moving force solution is not an upper bound for the accurate solution of the moving mass problem.

Notes

The results in plotted curves show that as the rotatory inertia correction factor R_o increases, the amplitudes of plates decrease for both cases of moving force and moving mass problems. The flexural rigidities along both the x-axis D_x and y-axis D_y increase, the amplitudes of plates decrease for both cases of moving force and moving mass problems. As the shear modulus G_o and foundation modulus K_o increase, the amplitudes of plates decrease for both cases of moving force and moving mass problems.

It is shown further from the results that for fixed values of rotatory inertia correction factor, flexural rigidities along both x-axis and y-axis, shear modulus and foundation modulus, the amplitude for the moving mass problem is greater than that of the moving force problem which implies that resonance is reached earlier in moving mass problem than in moving force problem of simply supported orthotropic rectangular plates resting on bi-parametric foundation.

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