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Splitting Goldbach's Twin Prime Conjecture Asunder for Even Numbers

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Abstract- The Goldbach Conjecture remains one of the several unsolved mathematical problems today, along with the Twin Prime Conjecture. It is said to be one of the simplest mathematical problems to state yet the most difficult to prove. In this paper, the author explores a few even numbers about the problem, and, using them as his sample size, juxtaposes his ideas with those of others through literature review. The author explores a solution to the Goldbach conjecture which was put forward about 275 years ago, by a German mathematician, Christian Goldbach, who was a contemporary of the genius German mathematician, Leonhard Euler. Euler in a letter to Goldbach, had confessed at the time that unfortunately, he (Euler) could not prove Goldbach's mathematical poser. Through fundamental analysis and critical thinking, this author attempts to share his thoughts on how to resolve this long-standing mathematical debacle. The methodology used is basic, experimental, and exploratory, with the use of inferences through recognition of number patterns. We hope that this paper will make a small contribution to the discourse on Goldbach's conjecture, whose solution has eluded mathematicians for about 275 years.

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SPLITTINGGOLDBACHSTWINPRIMECONJECTUREASUNDERFOREVENNUMBERS

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Splitting Goldbach's Twin Prime Conjecture Asunder for Even Numbers

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Keywords: euler, goldbach, prime numbers, twin primes, even numbers, twin prime conjugates.

I. Introduction

Goldbach's twin prime conjecture states that an even number is the sum of two prime numbers (Kiersz, 2018; Linkletter, 2019). The Goldbach Conjecture dates back to a German/Prussian, Christian Goldbach, who was tutor to the young Csar II. In a letter to Leonhard Euler in 1742, Goldbach had made his conjecture known to the celebrated German mathematical prodigy, Euler, who in history is one of the most-gifted mathematicians ever to walk this earth (Linkletter, 2019).

Prime numbers are part of natural number integers, as whole or counting numbers, excluding zero. A prime number is one that cannot be divided by any other number except itself and one. All prime numbers tend to end with only four digits, namely 1, 3, 7, and 9, and this also applies to twin primes, which are a subset of prime numbers (Sakyi, 2020). Twin primes are prime numbers that are close to each other with a difference of two, of the general form p, and p+2. Examples are (3, 5), (5, 7), (11, 13), (17, 19) and (29, 31), among many others (Sakyi, 2020a). Even numbers are numbers that are divisible by two, and they always end with the digits 0, 2, 4, 6, and 8.

Goldbach's statement of the second twin prime conjecture is directly related to, and conjoined with, the Twin Prime Conjecture which, was posed in 1849 by the Frenchman, Polignac, and which, in ancient Greece, had been alluded to in Euclid and Eratosthenes' sieves of elimination (Murthy, n.d.; Sha, 2016). Goldbach's mathematical poser statement looked paradoxically simple yet complex, in that, every even number

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has many permutations and combinations of twin prime conjugates, some of which are twin prime number conjugates, while others are not so, because many large even numbers have several composite permutations of numbers that make them up, and some of these conjugates, are even and composite.

However, the hidden catch to the mathematical poser is that those even composite conjugates can be decomposed into factors of prime numbers, hence the poser. In a practical sense, the way to proceed on this problem is to break up an even number through the middle, and, through a listing process, isolate all twin conjugates below it that are truly prime, hence the title of this paper. The approach that we propose thus works for smaller numbers, and by extension, and deductive logic, it should work for all even numbers. However, for huge numbers, it becomes tedious, ludicrous, and cumbersome to proceed in that manner, unless an algorithm is made for the computer to automate the procedure.

We think that we would restate the Goldbach conjecture problem that every even number is, at least, a composite sum of, at least, two prime numbers. However, be that as it may, we will use a few examples to examine the problem in some detail, to bring clarity to the issue, and to show that sampling is the way to go in dealing with such an intractable, elusive, and humungous problem.

II. Literature Review

Sampling is a method of choosing a representative size from a given population of discourse to be able to predict characteristics of the given phenomenon of inquiry (Spiegel, 1975; Saunders et al., 2016; Bryman & Bell, 2015). In 1966, Chen provided a proof of Goldbach's conjecture (Plus Magazine, n.d.; Curiosity, 2017). Further progress was made in 1998 when the problem of proving Goldbach's conjecture was subjected to a computer programme that solved an even number to the tune of 4×10^{14} (Plus Magazine, n.d.). Terence Tao had a crack at proving Goldbach's conjecture in 1996 (Curiosity, 2017).

An even number can be expressed in the form 2^{n-1} where $n > 1$. An odd number on the other hand, can be expressed in the form 2^{n-1} -1. We can also derive even numbers from the expression $(-2)^n$ where n is an even integer. Where n is odd, the expression $(-2)^n$ becomes a negative number and not applicable to this discussion.

In the past several mathematicians have made many propositions towards the solution of Goldbach's conjecture. We make the submission and observation that the mathematical complexity of the solutions which these mathematicians have contributed are beyond the bounds of this paper, hence we shall not discuss them. We rather propose a fundamental arithmetic approach that is a novelty and also basic. Some of the great mathematicians who have made attempts to solve Goldbach's conjecture include Wang (1984), Richstein (2001), Wu (2007), Chen (1995), Ramare (1995), Marshall (2017), Helfgott (2013), Hardy-Littlewood (1966), Zhou (2019), Melfi (1996) and Vinogradov (n.d.). We have admired the presentations of Wu and Ramare considerably, though we submit that their presentations are extremely intricate and hard for us to follow through the formidable mathematical arguments. We have included in the references many claims to the solution of the problem for the convenience of those who will be interested in looking at hard mathematical arguments.

We recognize what the famous English mathematician and physicist, Isaac Newton, said many years ago that if he had seen further than other men, it was because he had been standing on the shoulders of titans. We are also aware of what Archimedes

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said when he figured out how to catch the thieving goldsmith who had stolen part of gold that was given to him by his king to make a crown. Archimedes ran naked from his bath to the town square shouting, 'Eureka!' meaning, 'I have found out'. Finally, we are told an anecdote that Einstein said if you cannot explain a complex concept to a five year old, then you do not understand it yourself.

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III. Methodology

A paper of this nature does not require primary or field research. Therefore, the approach we have chosen here is by secondary research or desk analysis, logical reasoning, and both deductive and inductive methods of validating results through the use of basic arguments.

IV. Findings and Analysis

For purposes of building arguments from the ground up, using an exploratory method, let us take the even number 32, and find its twin conjugates. When we split 32 into two, we obtain 16 + 16.

Table 1: Sample 1 Even number 32

N+1	N-1
17	15
18	14
19	13
20	12
21	11
22	10
23	9
24	8
25	7
26	6
27	5
28	4
29	3
30	2
31	1
32	0

We can see that the even number 32 has three sets of twin prime conjugates, one of which is trivial (31+1). These pairs are (19, 13) (29, 3) and (31, 1). The set of prime sets for a small number such as 32 proves that every even number has pairs of twin prime conjugates. We see arithmetic series patterns emerging from these three pairs, namely, 1, 31, 61, 91, 121, 151 …….., whereby we find out that 91 and 121 are not prime numbers. The general term is 30n-29. We also see the pattern 3, 13, 23, 33, 43……., of the general form 10n-7. We see another series as 19, 29, 39, 49, 59, …….., with the general form 10n+9. Some of these patterns have the common difference between them as 10 and 30, respectively.

We line up the terms generated so far as 30n-29, 10n-7, and 10n+9.

We can also take, for example, the number 64, that is an even and composite number that is 2^6. We can start our method by dividing 64 by 2 that, is 32 + 32. We line up 32 in two columns, and subtract 1 from one column, and add 1 to the other column, thus

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Table 2: Sample 2 Even number 64

N+1	N -1
33	31
34	30
35	29
36	28
37	27
38	26
39	25
40	24
41	23
42	22
43	21
44	20
45	19
46	18
47	17
48	16
49	15
50	14
51	13
52	12
53	11
54	10
55	9
56	8
57	7
58	6
59	5
60	4
61	3
62	2
63	1
64	0

(Source: Author)

We group the highlighted twin prime conjugate pairs as
(41, 23), (47, 17), (53, 11), (59, 5) and (61, 3)

We see a pattern of a difference of 6 emerging from 41, 47, 53, and 59 on the one hand, and 5, 11, 17, and 23 on the other hand, giving us the arithmetic series of the form 6n-1, and 6n+1, respectively. The composite sums that are not highlighted can be seen as trivial cases, because they can further be factorized into prime factors. The example of the even number, 64, shows us that it has five pairs or sets of additive or conjugate composite prime numbers, which when rearranged, form patterns of the general form 6n+1, and 6n-1 that can be used to generate twin primes.

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Table 3: Sample 3 Even number 144 (abridged)

N+1	N-1
73	71
83	61
97	47
101	43
103	41
113	31
127	17
131	13
137	7
139	5
143	1

(Source: Author)

From Table 3 above, we can see that the even number 144 has eleven pairs of twin prime conjugates. We can see that the larger the even number, the greater the number of twin prime conjugates that we can derive. The evidence from this analysis is that, so far as even numbers are infinite, so far will twin prime conjugate numbers and twin prime numbers be found (proof by fractal geometry, self-similarity, set theory, and by induction). From Table 3 above, we see arithmetic series patterns emerge as (7, 37, 67, 97, 127……..) (11, 41, 71, 101, 131…..) and (13, 43, 73, 103,……….) of the general forms 30n-23, 30n-19 and 30n-17, respectively (cf. Sakyi, 2020a)

We can extend the same analysis to a higher even number, such as 4096, that is 2^{12}. We divide 4096 into two and, add 1 continuously to one half of it till it reaches 2048 (half of 4096), and continuously subtract 1 from the other half until it reaches zero. The two lists should be scanned for prime numbers below 2048 on the one hand, and those from 2048 up to 4096 on the other hand. In effect we list all prime numbers below 4096, and then find their conjugates by subtracting each of these prime numbers from 4096, and checking whether the resultant is composite or prime. Table 4a below is a list of all twin prime numbers below the even number 4096, while Table 4b is a list of all the prime numbers below the even number 4096.

Table 4a: Twin Prime numbers below 4096

3, 5 7, 11, 17, 19, 29, 31, 41, 43, 59, 61, 71, 73, 101, 103, 107, 109, 137, 139, 149, 151, 179, 181, 191, 193, 197, 199, 227, 229, 239, 241, 269, 271, 281, 283, 311, 313, 347, 349, 419, 421, 431, 433, 461, 463, 521, 523, 569, 571, 599, 601, 617, 619, 641, 643, 659, 661, 821, 823, 827, 829, 857, 859, 881, 883, 1019, 1021, 1031, 1033, 1049, 1051, 1061, 1063, 1091, 1093, 1151, 1153, 1277, 1279, 1289, 1291, 1301, 1303, 1319, 1321, 1427, 1429, 1451, 1453, 1481, 1483, 1487, 1489, 1607, 1609, 1619, 1621, 1667, 1669, 1697, 1699, 1721, 1723, 1787, 1789, 1871, 1873, 1877, 1879, 1931, 1933, 1949, 1951, 1997, 1999, 2027, 2029, 2081, 2083, 2087, 2089, 2111, 2113, 2129, 2131, 2141, 2143, 2237, 2239, 2267, 2269, 2309, 2311, 2339, 2341, 2381, 2383, 2549, 2551, 2591, 2593, 2657, 2659, 2687, 2689, 2711, 2713, 2729, 2731, 2789, 2791, 2801, 2803, 2969, 2971, 2999, 3001, 3119, 3121, 3167, 3169, 3251, 3253, 3257, 3259, 3299, 3301, 3329, 3331, 3359, 3361, 3371, 3373, 3389, 3391, 3461, 3463, 3467, 3469, 3527, 3529, 3539, 3541, 3557, 3559, 3581, 3583, 3671, 3673, 3767, 3769, 3821, 3823, 3851, 3853, 3917, 3919, 3929, 3931, 4001, 4003, 4019, 4021, 4049, 4051, 4091, 4093.

Table 4b: Listing of prime numbers below the even number 4096

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541, 547, 557, 563, 569, 571, 577, 587, 593, 599, 601, 607, 613, 617, 619, 631, 641, 643, 647, 653, 659, 661, 673, 677, 683, 691, 701, 709, 719, 727, 733, 739, 743, 751, 757, 761, 769, 773, 787, 797, 809, 811, 821, 823, 827, 829, 839, 853, 857, 859, 863, 877, 881, 883, 887, 907, 911, 919, 929, 937, 941, 947, 953, 967, 971, 977, 983, 991, 997, 1009, 1013, 1019, 1021, 1031, 1033, 1039, 1049, 1051, 1061, 1063, 1069, 1087, 1091, 1093, 1097, 1103, 1109, 1117, 1123, 1129, 1151, 1153, 1163, 1171, 1181, 1187, 1193, 1201, 1213, 1217, 1223, 1229, 1231, 1237, 1249, 1259, 1277, 1279, 1283, 1289, 1291, 1297, 1301, 1303, 1307, 1319, 1321, 1327, 1361, 1367, 1373, 1381, 1399, 1409, 1423, 1427, 1429, 1433, 1439, 1447, 1451, 1453, 1459, 1471, 1481, 1483, 1487, 1489, 1493, 1499, 1511, 1523, 1531, 1543, 1549, 1553, 1559, 1567, 1571, 1579, 1583, 1597, 1601, 1607, 1609, 1613, 1619, 1621, 1627, 1637, 1657, 1663, 1667, 1669, 1693, 1697, 1699, 1709, 1721, 1723, 1733, 1741, 1747, 1753, 1759, 1777, 1783, 1787, 1789, 1801, 1811, 1823, 1831, 1847, 1861, 1867, 1871, 1873, 1877, 1879, 1889, 1901, 1907, 1913, 1931, 1933, 1949, 1951, 1973, 1979, 1987, 1993, 1997, 1999, 2003, 2011, 2017, 2027, 2029, 2039, 2053, 2063, 2069, 2081, 2083, 2087, 2089, 2099, 2111, 2113, 2129, 2131, 2137, 2141, 2143, 2153, 2161, 2179, 2203, 2207, 2213, 2221, 2237, 2239, 2243, 2251, 2267, 2269, 2273, 2281, 2287, 2293, 2297, 2309, 2311, 2333, 2339, 2341, 2347, 2351, 2357, 2371, 2377, 2381, 2383, 2389, 2393, 2399, 2411, 2417, 2423, 2437, 2441, 2447, 2459, 2467, 2473, 2477, 2503, 2521, 2531, 2539, 2543, 2549, 2551, 2557, 2579, 2591, 2593, 2609, 2617, 2621, 2633, 2647, 2657, 2659, 2663, 2671, 2677, 2683, 2687, 2689, 2693, 2699, 2707, 2711, 2713, 2719, 2729, 2731, 2741, 2749, 2753, 2767, 2777, 2789, 2791, 2797, 2801, 2803, 2819, 2833, 2837, 2843, 2851, 2857, 2861, 2879, 2887, 2897, 2903, 2909, 2917, 2927, 2939, 2953, 2957, 2963, 2969, 2971, 2999, 3001, 3011, 3019, 3023, 3037, 3041, 3049, 3061, 3067, 3079, 3083, 3089, 3109, 3119, 3121, 3137, 3163, 3167, 3169, 3181, 3187, 3191, 3203, 3209, 3217, 3221, 3229, 3251, 3253, 3257, 3259, 3271, 3299, 3301, 3307, 3313, 3319, 3323, 3329, 3331, 3343, 3347, 3359, 3361, 3371, 3373, 3389, 3391, 3407, 3413, 3433, 3449, 3457, 3461, 3463, 3467, 3469, 3491, 3499, 3511, 3517, 3527, 3529, 3533, 3539, 3541, 3547, 3557, 3559, 3571, 3581, 3583, 3593, 3607, 3613, 3617, 3623, 3631, 3637, 3643, 3659, 3671, 3673, 3677, 3691, 3697, 3701, 3709, 3719, 3727, 3733, 3739, 3761, 3767, 3769, 3779, 3793, 3797, 3803, 3821, 3823, 3833, 3847, 3851, 3853, 3863, 3877, 3881, 3889, 3907, 3911, 3917, 3919, 3923, 3929, 3931, 3943, 3947, 3967, 3989, 4001, 4003, 4007, 4013, 4019, 4021, 4027, 4049, 4051, 4057, 4073, 4079, 4091, 4093.

(Source: factors-of.com)

We found five hundred and sixty eight (568) prime numbers below the number 4096 out of which 210 were twin prime numbers (36.9 %). We have highlighted the twin prime numbers in yellow in Table 4a above. This confirms that prime numbers and twin primes are everywhere dense in number space.

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Table 5: Sample of Twin Prime Conjugates of the even number 4096

4049	47
3929	167
3917	179
3853	241
3539	557
3527	569
3299	797
3119	977
2999	1097
2789	1307
2729	1367
2309	1787
2657	1439
2027	2069
1997	2099
1889	2207
1823	2273
1787	2309
1697	2399
1637	2459
1619	2477
1553	2543
1487	2609
1439	2657
1433	2663
1409	2687
4093	3
4091	5
4079	17
4073	23
4049	47
4013	83
4007	89

We take note from Table 5 above that majority of the twin prime conjugates of the number 4096 mostly end with the digits 3, 7, and 9 as discussed earlier. We also look at the lower numbers from Table 5 from the bottom upwards, and discern patterns of the series 17, 47, 77, ………, with the general form, 30n-13; then the series 23,53, 83,……..., with the general form, 30n-17; then the series 29, 59, 89,……..with the general form, 30n-1.

V. Proof

We can prove the Goldbach conjecture in the sense that, given any even number, to find its twin prime conjugates, we can recommend the following steps

1. List down all the prime numbers below that even number
2. After listing down all the prime numbers below it, eliminate all primes ending with certain digits according to the ending digit of the even number chosen, for even number ending with 0, maintain all the prime numbers generated from the list

3. For an even number that ends with the digit two, eliminate all listed prime numbers that end with seven, because seven subtracted from twelve is five, that gives us a non-prime number
4. For an even number that ends with the digit four, eliminate all primes that end with nine, because 9 taken from 14 leave us with five, like it applies to number three above
5. For an even number that ends with six, eliminate all primes that end with one, because one from six gives us five
6. For an even number that ends with eight, eliminate all primes that end with three because three from eight leave us with five
7. Examine for patterns of arithmetic series in the numbers so generated of the twin conjugates
8. Use the general arithmetic series of terms generated to produce twin prime numbers that are greater than the even number you chose to work on
9. Add one to all the numbers produced from those terms to obtain even numbers greater than the one you worked on. For example, for the terms 30n-1, 30n+1, 30n-13, 30n-17, 30n-19, 30n-29, 30n-23, 10n-7, 10n+9, 6n+1, and 6n-1 we can generate twin primes to infinity, as we derived these terms from the common patterns among all twin prime numbers (Sakyi, 2020). These general terms appear when we analyse any even number by listing all prime numbers below it, and we can use the sieve we have prescribed here above to reduce the list of prime numbers with which to deal
10. These general terms can generate twin primes to infinity. That, being the case, for any twin numbers that we generate, we can add 1 to each of them to obtain an even number, because twin primes always end with the digits 1, 3, 7, and 9. Therefore, there is an infinitude of twin primes, and also an infinitude of even numbers (Sakyi, 2020).
11. The probability of choosing an even number is 50 percent (0.5) and an odd number is also 50 percent (0.5).
12. The probability of choosing a prime number is 36.8 percent (see above)
13. The probability of choosing a twin prime number is 3.49 percent (Sakyi, 2020)
14. Finally, so far as we can generate an infinite series of twin primes from the terms we have produced herewith, so also can we state categorically that all even numbers have at least two prime conjugates. The larger the even number, the greater the number of twin prime conjugates that are there. Statistically speaking, from the samples referred to here, it is self-evident from the statistical point of view that what applies to small samples in number theory also apply to the infinitude of numbers. From our discussion so far, we have seen even numbers such as 32, 64, 144, and 4096 as numbers that already have sets of twin prime conjugates. We also submit that this elemental question can be solved in a common sense manner, which we have demonstrated here beyond all reasonable doubt. The Goldbach Conjecture can therefore be put to rest after 275 years. Table 6 below shows the distribution of twin prime conjugates for our convenient sample of seven chosen even numbers, namely 8, 16, 32, 64, 128, 144, and 4096. The evidence we adduce here is that as we choose higher even numbers, we observe that the number of twin prime conjugates rise exponentially to infinity by extrapolation.

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Table 6: Frequency of twin prime conjugates in a sample of even numbers

Number	Frequency of twin prime conjugates
8	1
16	2
32	3
64	5
128	4
144	10
4096	33

VI. Discussion

We are informed by sampling in statistics that to know the characteristics of a large population, we can choose a random sample, convenient sample, stratified sample, or any type of sampling method which is justified by the circumstance and purpose intended (Saunders et al., 2016; Bryman & Bell, 2015; Spiegel, 1974). We have here demonstrated that we do not have to break all the millions of eggs in a shopping mall to prove that the eggs which are being sold have salmonella or Corona virus (COVID-19). We can use a scientifically-selected sample size to analyse in order to predict, through sample statistics, the true population parameters or characteristics in an efficient and effective manner (Yamane, 1970; Spiegel, 1975; Kwak, 2017)

We found out that every even number has many prime numbers below it, and in fact, a few steps down from the even number lurk twin primes. Besides that, we made the following observations or discoveries.

1. If an even number ends with the digit two, we list all prime numbers below it and eliminate those that end with the digit seven, because seven subtracted from an even number that ends with two will always result in a number with five at the end, which is a composite number that can further, be factorized. Therefore, the prime numbers below it, that end with one, three, and nine are to be considered as twin prime conjugates for the even number under consideration. This applies to all even numbers to infinity
2. If an even number ends with the digit four, then consider all prime numbers that end with the digits one, three, and seven as prime candidates of prime conjugates of that even number. All prime numbers below it that end with the digit nine are to be eliminated because nine taken away from fourteen leave us with the number five at the end. It is the same as even numbers that end with the digit two discussed above. This also applies to all even numbers up to infinity
3. An even number minus an even number results in another even number. However, an even number minus an odd number can either give you an odd prime number or an odd composite number. For example, $36 - 17 = 19$, $72 - 29 = 43$, and $74 - 29 = 45$. So, in our discussion, we have to eliminate all odd numbers which are composite by concentrating on odd prime numbers whose subset is the set of twin primes.
4. If an even number ends with the digit six, then prime numbers below it that end with one should be eliminated because six minus one leave us with five. Therefore, we have to consider only all prime numbers below that even number that end with digits three, seven, and nine.
5. If an even number ends with the digit eight, then consider eliminating prime numbers below it which end with the digit three because eight minus three leave us

with five. Therefore, prime numbers that end with the digits one, seven, and nine are candidates for prime conjugates of that even number under consideration

6. However, if an even number ends with the digit zero, then all prime numbers below it that end with digits one, three, seven, and nine are all prime number conjugates. This shows us that zero is a neutral digit and operator, above all the other digits

We further our discussion of prime conjugates of even numbers with more examples.

Table 7a below shows the frequency of prime conjugates found in the even numbers 12, 24, 48, 96, 192, and 384. We can see that each preceding number is twice the next number. We can clearly see the trend of prime conjugates as the numbers get bigger. The frequency shows a steady upward trend at first followed by an exponential growth later. The frequency intervals are 2, 2, 2, 4, and 9. Cumulatively, this shows an exponential growing curve. Table 7b shows the occurrence of twin conjugates in the even numbers 12, 24, 48, 96, 192, and 384.

Table 7a: Sample even numbers frequency

Even Number	Frequency of Twin Prime Conjugates below even number
12	1
24	3
48	5
96	7
192	11
384	20

Table 7b: Sample even numbers distribution of twin conjugates

Even Number	Distribution of Twin Prime Conjugates below even number
12	(5, 7)
24	(5,19) (11, 13) (7, 17)
48	(19, 29) (5, 43), (7, 41) (11, 37) (17, 31)
96	(7, 89) (13, 83) (17, 79) (23, 73) (29, 67) (37, 59) (43, 53)
192	(29, 163) (11, 181) (19, 173) (13, 179) (53, 139)(61, 131)(43, 149)(41, 151) (89, 103)(83, 109)(79,113)
384	(5, 379)(11, 373) (17, 367) (31, 353)(37, 347) (47, 337)(53, 331)(67, 317)(71, 313)(73, 311) (101, 283)(103, 281)(107, 277)(113, 271)(127, 257)(133,251)(151, 233)(157, 227)(173, 211)(191,193)

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VII. Flowchart Towards Solution of Goldbach's Twin Prime Conjecture

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1. We assume an even number, V
2. We eliminate all even numbers that end with digits 0, 2, 4, 6, and 8, and that are less than V, because an even number minus an even number produces another even number.
3. With the remaining odd numbers, we go to the list of prime numbers that are less than V by eliminating all odd numbers that are composite or divisible or are squares. For example, the number 1681 has root 41, and 2401 has root 49.
4. Depending on the last digit of V, we do the following. We let V_0, V_2, V_4, V_6, and V_8 represent even numbers that end with the digits 0, 2, 4, 6, and 8 respectively. For all even numbers that end with zero, V_0, we maintain all prime conjugates that we listed as R + p = V. For all even numbers that end with 2, V_2, we eliminate all R values that end with 7, because 12 minus 7 will give a number that ends with a digit 5. For all even numbers that end with 4, V_4, we eliminate all R values that end with 9. For all V_6, we eliminate all numbers that end with the digit 1. For all V_8, we eliminate all listed values of R which end with the digit 3.
5. From step 4, we subtract remaining prime numbers, p from V, and let (V-p) be equal to R.
   V-p = R. Therefore R + p = V (a constant, K)
6. We check all results for the expression R + p =V to ensure that we only remain with results where both R and p are prime conjugates that add up to V. To check all Rs, we apply divisibility rules for 3 and 7, and check that R is neither a square nor a composite number that can further be factorized
7. We list out all R + p that are prime conjugates, and look out for patterns of arithmetic series by observing the numbers vertically and horizontally for patterns in order to generate infinite series of twin primes of the general forms 6n-1, 6n+1, 30n-23, 30n-17, 30n-19, 30n-29, 10n-7, and 10n+9, among others (Sakyi, 2020) that are greater than V
8. We use our series derived from the arithmetic series to generate twin prime numbers that are greater than V by adding 1 to the terms we obtained in order to obtain even numbers greater than V
9. Having gone through this cycle to generate higher even numbers, we go back in a loop to the starting point in step 1 above to repeat the whole process

We have proved by the above flowchart and analysis of samples of even numbers that every even number is made up of two twin prime conjugates, a mathematical poser which was first put forward by Christian Goldbach in 1742 in a letter to his fellow celebrated German mathematician, Leonhard Euler (Curiosity, 2017)

VIII. Applications of Twin Prime Conjugates

While twin prime numbers have constant distance of two between them, twin prime conjugates have variable distances between them due to discrete accretion or discrete increment. All twin prime conjugates of a particular even number have a constant sum or total while twin prime numbers have a constant difference between them, but not their total. We can apply many uses and applications to the twin prime conjugates of even numbers that were referred to in Goldbach's conjecture.

According to Merriam-Webster's Dictionary (4th ed.) a dyad is a pair or two items that occur simultaneously and are regarded as one unit, always conjoined like Siamese twins. Dyads occur in governance as diarchy or governance by two rulers as we have in Libya or in some unstable countries in Africa such as South Sudan, where we have government in exile, and de facto government in power on the ground. In biology, Webster dictionary states that we can refer to the division of a tetrad in meiosis into a double chromosome as a dyad or as prime conjugates.

In sociology, we can refer to a social relationship as a dyad or a twin prime conjugate, and we can code the partners with twin prime conjugate numbers. In mathematics, we have binary number system based on two digits of zeros and ones, which is used in electrical alternating currents as well as in computer science as signals of zeros and ones for calculations, data processing, sending messages, storing information as electrical impulses, and in Boolean algebra matrices and calculations.

Furthermore we can apply the bi-numerals for coding pairs of human organs such as the eyes, ears, lungs, ovaries, brain lobes, testicles, vagina lobes, pair of limbs, kidneys, the buttocks, and the breasts, among others. All these organs that come in pairs need coding and we can code them by using prime number conjugates when doing surgery or recommending treatment and therapy. We can have many applications in optometry, ophthalmology, psychology, pharmacology, optics and waves in physics, musical chords and scales in music, use of transistors and chips in electronics, in cadastral mapping we can employ prime conjugates in triangulation, and taking cadastral readings and coordinates as prime conjugates. We can use them in random statistical selection of items to include in a survey; we can apply prime dyads in seed multiplication, in chromatography in differentiating colour hues in the spectrum; we can apply in molecular biology and physics; in nanotechnology; and in virology in naming viruses such as SARS, MERS, CORONA virus, and Ebola strains, among many other applications. We can apply prime conjugates to the study of astronomy where binary stars can be named by prime conjugates. For example, according to Webster, binary stars that occur in certain constellations are

Table 8: Binary stars and their constellations (adapted from Webster Dictionary)

Binary star	Constellation
Aldebaran	Taurus
Adhara	Canis Major
Spica	Virgo
Rigel	Orion
Procyon	Canis Minor
Capella	Auriga
Acrux	Crux
Sirius	Canis Major
Canopus	Carina

IX. Conclusion

We consider the proof of Goldbach's Conjecture a corollary of the Twin Prime Conjecture, in that we observed that you cannot deal with one and leave out the other. They are like conjoined Siamese twins. Our earlier paper on the Twin Prime Conjecture is therefore germane to have a look at it (Sakyi, 2020). We finally conclude that all even numbers end with the digits 0, 2, 4, 6, and 8 and that every even number apart from two, has at least two prime conjugates which form arithmetic series that can be used to generate twin primes that are higher than a chosen even number. When we add one to the twin primes that we have generated, we derive higher even numbers. Since in an earlier paper we proved twin prime numbers to be infinite, by the same argument,

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we stand by our earlier assertion that twin primes, twin prime conjugates, and in fact the natural numbers are all infinite (Sakyi, 2020) We also made the discovery that the bigger an even number that we analyze for twin prime conjugates, the higher the amount of twin prime conjugates that we find.

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