Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 20 Issue 1 Version 1.0 Year 2020
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

The Conceptualization Process based on Student Perspective in Solving Graph Function Problem

By Eva Dwi Minarti & Fiki Alghadari
Universitas Pendidikan Indonesia

Abstract- The objective of this study was to analyze the conceptualization process in solving graph function problems based on the student perspective. Samples in this study were three students from the sains program in one of the best high school in West Jakarta region by the reason of testing results. This study has explored the completion of the problem and its interpretation of written interviews. There was a graph function problem that is solved by students and interpreted its completion according to their conceptions. The analyze result stated that students view the problem proposed as a function problem and not a geometric problem, so students consequences of converting from graphic to algebraic representation. The findings of this study were a conceptualization process where there are four steps of completion and it is estimating the algebraic function of the graph, finding the function of the graph, determining the function formula of the graph, and determining the derivative of the function. However, on the completion of all students, there was neglected a concept of domain function to look for as a background by students, and this is a complexity factor of the conceptualization process in solving function problems by the way to determine algebraic representation.

Keywords: *conceptualization, algebraic, gradient function.*

GJSFR-F Classification: *MSC 2010: 05C25*

Strictly as per the compliance and regulations of:

1. C. Granberg, "Discovering and addressing errors during mathematics problem-solving-A productive struggle?," *J. Math. Behav.*, vol. 42, pp. 33–48, 2016.

The Conceptualization Process based on Student Perspective in Solving Graph Function Problem

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Abstract- The objective of this study was to analyze the conceptualization process in solving graph function problems based on the student perspective. Samples in this study were three students from the sains program in one of the best high school in West Jakarta region by the reason of testing results. This study has explored the completion of the problem and its interpretation of written interviews. There was a graph function problem that is solved by students and interpreted its completion according to their conceptions. The analyze result stated that students view the problem proposed as a function problem and not a geometric problem, so students consequences of converting from graphic to algebraic representation. The findings of this study were a conceptualization process where there are four steps of completion and it is estimating the algebraic function of the graph, finding the function of the graph, determining the function formula of the graph, and determining the derivative of the function. However, on the completion of all students, there was neglected a concept of domain function to look for as a background by students, and this is a complexity factor of the conceptualization process in solving function problems by the way to determine algebraic representation.

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I. Introduction

There are four general steps of problem-solving, it is understanding, planning, solving, and looking back [1]; [2]. This step was many created citations in all research related to problem-solving. Especially in the mathematics education field, since emersion of the steps that have been posted by Poly a and up to right now, it was much research about problem-solving done with the essence is including all of its general steps. Even though in every step there are many processes happened and in facts, it is a determinant factor of emersion solution for every solved problem. However, kind of process happened be look totality as a just appearance one of all general steps. While, for every process, the different type of problems so it will show a difference in completion process and finding a solution because who are thinking and solving a problem cannot be controlled the other. The new approach focused on the interaction between problem representation and solution generated [3]. There has been a lot of research about problem-solving but was not yet known as a suitable framework strategy to formulate a graphic[4]. This condition was made our to look that this is interesting to follow up in a study related to occur process in particular general steps of problem-solving, and we present to make this study with the focus on conceptualization process in solving function problem.

Author α: Universitas Pendidikan Indonesia, IKIP Siliwangi.
Author σ: Universitas Pendidikan Indonesia, STKIP Kusuma Negara Jakarta. e-mail: alghar6450@gmail.com

Conceptualization process related to what is to be conceptualized. Here, we use a definition of the concept that is an abstract idea, so its conceptualization process means is not out of the related to emerge abstract ideas as an object in students' interpretation in conception form. Therefore, in the solving process, there are conceptions from every solver who are a contribution to solving the problem[5]. [4]stated that problem-solving revers the process that be used in completion conceptualization and moved from the beginning to the end. This study is about problem-solving so that its idea was meant as the ways that opened to be the solving process. The object that is conceptualized in its process was a problem-solving or a solution. In the problem-solving process, there is a solution after the solving steps. In other words, what is anything will be conceptualized is not yet formed, so not for all general problem-solving steps will be a part of the conceptualization process. At four of the general problem-solving steps that be cited, and that be showing on the progress of process before a solution fixed, so the looking back step is not a part of the conceptualization process. However, what is the general steps of an understanding problem as a part of the conceptualization process? This question will be answered in the next section of the result research discussion.

This study aims to analyze the conceptualization process in solving graph function based on the student perspective. Framework this research analyzed student conception in the process to solve, so the collection of student conception would construct a schematized systematics and this is called the conceptualization process. There is a factor related to truth conception because if it is not so the solution will not be found, or solution was be found but there is a doubt with its truth. Because of related to conception, this was showing there is a solvers point of view as a determinant of preference when they are beginning the solving perform. Therefore, there is a truth of conception as a key point that was created by solvers from their perspective. Here, presenting the problem was a function to create a different perspective in solving the context that is made by solvers, and its different is at least because of the problem which is required in open-ended type. With the requirement, the function problem satisfies a criterion because the function concept is a coherency of some representation both geometric and algebraic. In this study, function problem means is about the sketch of a gradient function graph, some times we use the term of gradient function or the first derivative function but its all the same means, and gradient function was stated in [6] as the first of derivative function.

II. Method

This is an explorative study toward the completion process that interpretation by a sample of this study in writing the interview. The completion process has been obtained from one item test which was developed for needed this study and was used to measuring students' ability to solve the function problem. The test has been adapted from [6], it was about a gradient function graph problem. Its problem that was proposed is to determine a graph of gradient function from Figure 1 below.

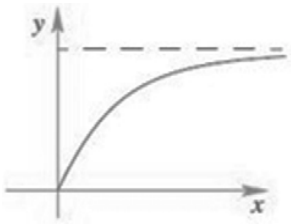

Figure 1: Sketch of the graphics function

Ref

6. P. Tobin and F. Cirrito, *International Baccalaureate Mathematics Standard Level.* Australia: IBID Press, 1997.

Then, this function problem was given to seven students from a high school in the region of West Jakarta. However, from seven on its students, just three students whose detail gave an answer related to the written interview question, and its students are initial as AN, NI, and PR. The interview question is how the way to solve and describes. Students who are a respondent in this study have been classified as good students and it was according to the recommendation of a mathematics teacher at the school whom concerned. Here, we used a literature [4]. In its article mentioned that the conceptualization process has been done by the ways to see in solver mind, it was seeing how they represented of the problem, how they produced a solution, and how its factors might be related with their knowledge, with the problem characteristic, and their success probably.

III. Result and Discussion

Before to analyze of conceptualization process completion, the problem context would be fixed by the student point of view who has solved it. Function problem can look like a problem of geometric or algebraicre presentation[5]. However, students had the freedom to look at the problem from the relativity of according to their more than familiar in solution finding, although the problem was showing about there is one of complexity.

a) Complexity of Problem

Aside from geometric or function point of view, the item has been proposed above was considered to be a problem. Here is not to show that function has a complex system, but in this study, that function was being used to be anything that including form and variable problem because the function is satisfied between geometric and algebraic. Completion as a student task was showing a figure of the sketch a derivative function graph. However, at the test just have a figure of function and not be complete sets coordinate pairs not clearly defined. We called this problem as a competitive problem because based on [7] views that stated an ability to read given information just in the graphic form needed to think about complexity properties. Whereas on the other side, in [8]report have been stated some of typical difficulties student have on graphic representation from differential was about basic calculus concepts.

The problem in the test was to draw the first derivative function of graph function that given in Figure 1. The completion of students made was a function graph figure. However, given information was not detailed, but completion can be made by identifying ways form and properties of function graph as soon as analyzing relevant concepts. For example, a function has domain, codomain, and range [9]. Needed knowledge of students about function limit, derivative, asymptote, property, and interval[10]. Domain, codomain, range, intervals of increase or decrease, extreme values as the element and properties of function which was represented graphically [3]. Drawing a function graph in the coordinate plane can be started from its concepts because every function would be drawing and needed domain and codomain to be an axis in rectangular Cartesian coordinate. By knowing coordinate pairs between domain and range, so it can be made a function graph figure. In other words, to draw derivative function in the problem so it is necessary to be known a concept about domain and codomain its graph. Furthermore, making the first function derivative graph figure by identifying gradient value along the curve when its gradient value as positive or negative gradient value relatively, and increase or decrease [8]; [6]. Thus, there are some elements to be necessary knew to sketch the gradient function graph, it is domain and codomain, the relative of gradient value, and concept about increasing and decreasing

Ref

4. T. B. Ward, *Problem solving*. Elsevier Inc., 2012.

function. However, many students from high school not yet understand the relationship between calculus concepts, but they learn more about the segmental concept just on calculating big scale and typically problem-solving.

b) The Problem on Student Perspective

Different point of view will obtain a different figure. Views will determine the too different color. Every student probably has different views about function. The function was satisfied between geometric and algebra representation[5]. The function can be represented in an algebraic formula, graph, table, and context so that a combination of this representation will obtain rich in concept figure[3]; [9]. In [11]literature was stated that the chosen type of representation would be impacting success in problem-solving, and [3] added that graphic representation more accessible and give meaning to algebraic formulas. Related to this study, student views toward the problem guide theirs to the direction of the solving steps, and its look from their reasoning that students conceptualized the completion via constructing the algebraic function formula from graph figure. The following is a student finding of some algebraic representation of the graph in Figure 1 above. A student with the initials AN stated that the function algebraic formula of its figure was $y = -e^{-x} + a$ for $a \in \mathrm{N}$, while a student with the initials NI showed $y = -x^{-1} + a$ for $a \in \mathrm{N}$ as its algebraic function formula, whereas a student with the initials PR confirmed a graph on its figure by $y = 1 - a^{-\sqrt{x}} + a$, for $a > 1$ an a element real number as an algebraic function formula.

Based on its some algebraic expression, all three students who were respondents in this study view that this is a function problem. More precisely, students solved the problem by determining its function algebra formula to sketch a derivative function graph. All students solved in ways constructed the function first. This case like that is called in [10] that when students were given a function graph and then participated in sketching the derivative graph of its function gave, so many students tried to find the algebraic representation of given function first. After knowing its algebraic representation function, students would get the first derivative function, and then going to sketch the function graph figure on the rectangular coordinate system, and it happened as student techniques solved. We understood why students made these as a function problem. This is an effect of the question in its problem, because of a derivative function graph was being asked. While, learning about differential most identic with function, for example, algebraic function, trigonometry, or others. Therefore, students view that the sketch graph on its Figure 1 as the one of identical functions in their mind.

Finding the algebraic function formula of Figure 1 above was a skill of algebraic function analysis. We agree with [7]statement that students who learned differential actually skilled in the algebraic algorithm but they experience difficulty in understanding a concept so that the fact showed many students to learn a type of algorithmic formal problems by memorizing way as a simple mechanical calculation process. All three students viewed the problem as a function problem, however, its problem can be categorized too as a geometric problem. [12]stated that function is the organic combination of number and form, and the flexible conversion between symbolic language and graph. In[6], this problem viewed a geometric problem, and geometry of the curve was communicated to a negative or positive gradient concept at a particular point. Even though the Figure 1 was seen that coordinate pairs are not defined clearly, but the slope of the tangent can be imagined and it indicate for the positive or negative range of domain. However, this was about the tendency of student views toward the

Ref

9. H. Kashefi, Z. Ismail, and Y. M. Yusof, "Obstacles in the learning of two-variable functions through mathematical thinking approach," *Procedia - Soc. Behav. Sci.*, vol. 8, pp. 173–180, 2010.

problem. Nevertheless, the test was not viewed as a derivative function problem here because it is equally to be a base of completion when the view base is directed to one of two types of the problem.

Therefore, two perspectives can student made the views related to the problem that was proposed and completion conceptualization. The problem that was proposed can be categorized in geometric and function problems. When the conceptualization process in solving the problem was more dominant in using geometric context and was not a direction to determine the algebraic function formula. For example, by moving dynamic of the tangent along with graph figure so that the properties of the derivative function graph was known, or to solve ways with the technique in [6]explanation, we defined it be a geometric problem on student perspective. However, the trend of student performance in completion more to the utilization of function concept by formulating the algebraic function, so this was a function problem in judgment. Our investigation results toward student completion stated that it was a function problem. Its all depend on the point of view of student chose and started their completion construction analysis. Their chosen was the best way of course whose student knew at the time, student ways of thinking to the problem, and this writing is not to deny the result of their mind. The base of this claim is the student's point of view in solving. Some students in their explanation stated that solved the problem has been started by estimating the function of the graphics model. Student explanation about ways of estimating the function formula will be explaining in the section of step on the conceptualization process. In every completion of students make started indeed with their estimating result, and the completion of the key point is the first step chosen as a process conceptualization started here. Anyway, the first step is too important as a part and the rule in determining a lack of process to find the completion [13].

c) The consequence of the Solving Ways

Students who viewed the problem from a particular point of view, so they would face a consequence in the completion process, and every their point of view would distinguished complexity too. Students chose to solve the problem by determining the algebraic function formula so that all related consequences with the complexity function concept attached automatically in the conceptualizing process because the completion by different representations with the problem would emerge a conversion process. Switching a representation to the other are important tools in problem-solving [3]; [14]; [5]. In this study, students converted geometric to algebraic representation. [12]stated that to understand all of the function concepts, there are various internal attributes and internal-external relationships should be understood to investigate systematically and comprehensively. By completion process from student views, like a statement which was stated by [7]that student tends to depend on algebraic thinking style in the end than geometric thinking style which got information about function directly from graphics model without function expression. Even though, a student who used algebraic thinking are disadvantaged in terms of available time and perform less well in solving a problem[15]. In [10], [3], and [5] study was stated that student shows their difficulties usually in using of function properties, and in [14] study confirmed about student difficulties like to understanding a formal aspect including to identify and represented domain and range geometrically, especially when the domain in-bound. Therefore, the function was one of all difficult content to understand, and many students experienced their difficult when they faced the non-routine problem [9]; [5].

Ref

6. P. Tobin and F. Cirrito, *International Baccalaureate Mathematics Standard Level.* Australia: IBID Press, 1997.

d) Steps on Conceptualization Process

We summarized some occurred process when the student solved the problem that is proposed. It was estimating the algebraic function of a graph, finding the function of a graph, determining the function formula of the graph, and determining the derivative of the function. Among these processes, not for all is a conceptualization process in solving problems, because the conceptualization process is the occurred process before the completion has. Construction a graph was a goal that is an end product, In [3] has been stated that it was a learning model in the past. In other words, the sketching derivative function graph is the conceptualization process of completion.

i. *Estimating the algebraic function of the graph*

The first step that to be a finding in solving the problem phase of the conceptualization process was an estimation of the curve. Estimating an algebraic function formula was the fact student aware problems have so they took the step that by the particular domain function, but the codomain would be adjusted. Here, the student will see a geometry form of highlight a curve generally and match with various forms of the geometric representation of algebraic function formula in an analytical sketch that they ever learned, and this process might repeatedly occur. The repeated process in the problem-solving phase was drawing in [1] literature. In [13]study was stated student tendencies to pay the first attention to the figure, and this is a process of understanding in problem-solving. This literature was not stated about anything activity to match between the figure on seen and the other, but these activities were including a dynamic process. This is the answer to the question that has been in the introduction section, and it was an understanding phase in problem-solving steps as a dynamics when the conceptualization process occurred. In its context, there is a connection between student knowledge about the function of algebraic and geometric representation, and [16];[3]and [14]had stated that the connection was important in solving function problem. The student showed the knowledge connection means in some cut off their statement. AN has stated that Figure 1 similar to the upside-down of the exponential graph and the most frequently was e^x. While NI has written that the graph on the test directed to $y = -1/x$. Whereas PR has confirmed the approximate form of the curve was $y = \sqrt{x}$, and the other approximately form was an exponential function like $y = a^x$.

The student initials of AN has mentioned that the function graph in Figure 1 similar to the exponential graph. Because of in his perspective view that the problem was about function problems, so his student tends to use a formula that was similar to the problem before his found. AN's activities in solving the problem is an example which was [13]stated about the part in one of all general problem-solving steps, it is planning completion of the problem. Based on some cut-off students statement, there is a difference of learning trajectory for each related to conceptualization process in solving problem that is processed by AN, NI, and PR apparently, and we have seen the first step of completion process in general as a process that is said an estimation of the algebraic function of curve. AN has begun the completion process by finding a form of a similar curve with the graph in Figure 1. While NI has begun the completion process by investigating graphic form with the ways on function interval analyzed. Whereas PR has begun his completion by finding a suitable function algebraic formula with the graphic form model on Figure 1 displayed. All three students linked formula to a graph, imagined, and focused on key properties of the function. Its process called in [3] as a strategy.

Ref

1. C. Granberg, "Discovering and addressing errors during mathematics problem-solving-A productive struggle?," *J. Math. Behav.*, vol. 42, pp. 33–48, 2016.

ii. *Finding the function of a graph*

By looking at a curve form, all three students were not directly to find an algebraic function formula of its curve. This was there are on cut off statements each explanation that be written by all three students. There are some different process that is shown each student before to determine a function formula of graph figure on the display. AN has done translation $[0 \quad 1]$ toward a graph and crossed $(0,0)$, then its asymptotic was $\lim_{x\to\infty} -e^{-x}+1=1$. While NI has written that because of the bounds was a and not equally 0, so $y=-1/x$ must be added by a. Whereas PR has done exploration on some function formula of a curve. For example, when $y=a^x$ and tried to $a=2$ so $y=2^x$ but its formula was not suitable with the figure of the graph on the test. If $a=-2$, so $y=-2^x$, then the curve was reflected x-axis so that the curve form be similar. However, because of the curve crossed $f(c)=-1$, so the function must be added by 1 for suitability with the figure. The student has shown their process to scan graph, [3]stated this process as the ability to read algebraic expressions and make rough estimates of the patterns that would emerge in representation. Find the suitability between the graph and its algebraic function formula was a consequence in the way completion by this point of view.

Based on the information that is created by students, two of them showed a stead geometry transformation concept as a part of the finding process the ideal algebraic function formula with the graph. Transformation means is a translation by AN and reflection by PR. This happened are corresponding to the statement in [12]that the transformation of a figure was a frequent process to be used when students constructed a function as an algebraic representation, hence there is a requirement for the student to have mastery on specific properties of the function. Transformation as a different process between the completion by AN's or PR's and NI's. AN and PR found an algebraic formula of graph function globally. Incited of [15]has stated that student who solved a problem with a global approach, they would focus on the function behavior in a particular interval. However, NI has done to conceptualized by reading points at function graph, and this way was stated [3]as a pointwise approach when this approach distinguished with a global approach. Therefore, the global approach more powerful and gives a better understanding of the relationship between formula and graph function compared to pointwise approach.

iii. *Determining the function formula of the graph*

In finding an ideal function formula with a graphic form on the test, each student determined one function as a finding result. Here, AN has determined that a function of Figure 1 above $y=-e^{-x}+1$. While NI has shown his analysis result with an expression $y=-x^{-1}+a \; a\in \mathrm{N}$ as an algebraic representation of the graph. Whereas PR has determined by generalized way function formula to be $y=1-a^{-\sqrt{x}}$ for $a>1$ and a element real number, and for simplifying he used $e=a$ so that $y=1-e^{-\sqrt{x}}$. All three student has determined a function formula of the graph in Figure 1 based on their analysis result findings, and all of the algebraic function formula findings were different. AN and PR were the same to determine function formula by using the e number. This is a second of similarity in conceptualizing process completion of them after finding the process of curve function was similar to utilize a geometry transformation concept. Two of these students has shown an identical conceptualization process and the other student has chosen the different technique. Anyway, there was one function concept that has been missed on all three student views, and this discussion will be created on the missing concept function in the section.

Ref

15. Y. Y. Hong and M. O. J. Thomas, "Graphical construction of a local perspective on differentiation and integration," *Math. Educ. Res. J.*, vol. 27, no. 2, pp. 183–200, 2015.

iv. *Determining the derivative of the function*

For determining the derivative function of the graph in Figure 1, and because each student has determined the algebraic function formula, in this section was not many explained the ways of determining a derivative function. All three students likely can be said that they were fluently for determining a derivative function. A has written $y' = e^{-x}$. While NI has obtained a derivative function and written $y' = 1/x^2$.Whereas PR has shown $y' = x\sqrt{x}/2e^{-\sqrt{x}}$ as a derivative result of function formula that he constructed. We claim that to determine a derivative function as the end of the conceptualization process in solving the problem because after this section is student will sketch the graph of its derivative function, so the object that is obtained from its process is the solution of the problem. This claim is based on a statement at the introduction section that conceptualization process related to abstraction precess or emerged abstract ideas and object that be conceptualized is not yet fixed.

e) Missing of the domain concept

Looking at the completion has been created by the student, it has been clear that the problem which was given is not routine because each of student found ways of solving and different of the answer to this problem was explained its type as an open-ended non-routine problem. In general, the student who has answered the test classified in the smart student because of the three's be able to solve. Here, we are not to make sure that they are gifted, but based on completion that is created of them, they can be said a gifted from a side of function graph analysis to determine the algebraic function. The answer which has been given was based on the analysis process of form or geometry transformation toward constructed function. Knowledge of students has shown their mastery in-deep function materials and differential calculus. [9]have said that there are some terms for a relation to state a function, it is about a point of pairs, domain, codomain, and range. From the domain and function, it will obtain a range that is a part of the codomain. Related to those, there is no indicator in the problem to fault algebraic function formula that was student-constructed, but all three students have missed their identification toward the domain of a function that is one of all concepts, and it has been shown on the problem as terms of function graph. The student has shown domain the constant of function on determining the function formula of the graph at the discussion section, but they have not given domain for their function. This is probably a consequence of the completion process by algebraic representation ways. Because of the advantages of a graphic is to monitor and evaluate results[3]; [17], and this study analysis result was not looking for using its advantages by students. The student has been not understanding the domain concept, but they were missed to look for its background. Even though, in Figure 1, there was implicitly its definition related to domain function. Based on this condition, there is a probability like a statement in [14] that a factor of a student has lack flexibility in attaching the meaning of domain or range, and in [15], [3]dan[5]stated that flexibility thinking is necessary for mathematics and problem-solving.

A claim from [18]stated that neglecting a concept from the completion was because the student has its knowledge but they did not understand. However, we are more like than with statement in [8]that student has too late to realize the relationship between the original function and its derivative because of lack of appreciating the relation of them in the graphic representation. Therefore, the representation that has been including on the problem was served with an accurate level that is not detailed, so information and concept that students understood on the problem and problem-solving

Ref

9. H. Kashefi, Z. Ismail, and Y. M. Yusof, "Obstacles in the learning of two-variable functions through mathematical thinking approach," *Procedia - Soc. Behav. Sci.*, vol. 8, pp. 173–180, 2010.

strategy to find a solution are explained student knowledge and learning experiences. Placing of learning process emphasis on algebraic representations, it will be disregard graphical representation [8]. Even though, in [15]has stated that emphasis on the algebraic process tends to produce students with a dependence on the work form. All this time, learning about calculus placed the process on the accentuation of algebraic representation so it will be neglecting graphic representation. This was a reason as an effect of most students can easy solve routine differential problem,[8];[17]and [5]stated that with like this so they do not have a conceptual understanding about derivative graph of representation, and many researchers have explained about student difficulties when they were being given a graphic form. In[15], learning that emphasis on algebraic representations can hinder flexibility construction so it suggests the need to consider the construction of didactic situations.

IV. Conclusion

The problem of function that is proposed to students has been about to sketch a derivative function graph. Its problem in the student perspective was a function graph because students chose for solving it by determining the algebraic function formula from a graphic figure. When the problem as a function problem in the student view, there was some concept of function that attached to be viewed in the conceptualization process, and this was a consequence in solving by converting a graphic to an algebraic representation way. After the completion that students created and their interpretation of it was analyzed, there are four phases of the conceptualization process from identification result after the function problem in this study. The four are estimating the algebraic function of a graph, finding the function of a graph, determining the function formula of the graph, and determining the derivative of the function. However, the analysis study has the other finding, there is function concept missed on student views, it is a domain concept. All three students have determined a domain function of constant, but it was not a function domain. This condition cannot be claimed as a misunderstanding of the student on domain concept, and it is because of the complexity factor in the conceptualization process on solving function problem by algebraic representation way.

This study was limited to a function problem that it completion by determining algebraic representation, and when student progress in solving process on geometric problem perspective so that the phases in the conceptualization process will be different. Moreover, its phases are not the same too if the problem in another function context.

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Notes