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Fuzzy Foldness of BCI-Commutative Ideals in BCI -Algebras

By Mahasin. A. Ahmed & Esmat A. Amhed
Sudan University of Science and Technology

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FUZZYFOLDNESSOFBCICOMMUTATIVEIDEALSINBCIALGEBRAS

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Ref

3. K. Iséki and S. Tanaka, An Introduction to the theory of BCK-algebra, Math. Japonica, 23(1978), pp. 935-942.

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I. Introduction

The notions of BCK/BCI-algebras were introduced by Iséki [3] and were investigated by many types of research. The concept of fuzzy sets was introduced by Zadeh [12] In 1991, Xi [11] applied the concept to BCK-algebras. From then on, Jun, Meng et al. [10] applied the concept to the ideals.

The notions of *n*-fold implicative ideal and *n*-fold weak commutative ideals were introduced by Huang and Chen [1]. Y. B. Jun [4] discussed the fuzzification of *n*-fold positive implicative, commutative, and implicative ideal of BCK-algebra.

In this paper, we redefined a BCI – commutative ideals of BCI-algebra and studied the foldness theory of fuzzy BCI – commutative ideals, BCI – commutative weak ideals, fuzzy weak BCI – commutative ideals and weak BCI – commutative weak ideals in BCI-algebras. This theory can be considered as a natural generalization of BCI – commutative ideals. Indeed, given any BCI - algebra X, we use the concept of fuzzy point to characterize *n*-fold BCI - commutative ideals in X. Finally, we construct some algorithms for studying foldness theory of BCI – commutative ideals in BCI -algebra.

Author α: College of Science, Department of Mathematics, Sudan University of Science and Technology, Khartoum, Republic of Sudan. e-mail: mahasinaliahmed@gmail.com

Author σ: College of Electrical Engineering & IT Department of Electrical Engineering, Onaizah Colleges, KSA. e-mail: esmatphysics72@gmail.com

II. Preliminaries

Here we include some elementary aspects of BCI that are necessary for this paper, and for more detail, we refer to [1, 3].

An algebra $(X\ ;*\ ,0)$ of type (2,0) is called BCI-algebra 1f

$\forall x,y,z\in X$ the following conditions hold:

BCI-1. $((x*y)*(x*z))*(z*y)=0$;

BCI-2. $(x*(x*y))*y=0$;

BCI-3. $x*x=0$;

BCI-4. $x*y=0$ and $y*x=0\Rightarrow x=y$

A binary relation $\leq$ can be defined by

BCI-5. $x\leq y\Leftrightarrow x*y=0$,

then $(X,\leq)$ is a partially ordered set with least element 0.

The following properties also hold in any BCI-algebra ([5], [10]):

1. $x*0=x$;
2. $x*y=0$ and $y*z=0\Rightarrow x*z=0$;
3. $x*y=0\Rightarrow(x*z)*(y*z)=0$ and $(z*y)*(z*x)=0$;
4. $(x*y)*z=(x*z)*y$;
5. $(x*y)*x=0$;
6. $x*(x*(x*y))=x*y$; let $(X\ ,*\ ,0)$ be a BCI-algebra.

Definition 2.1 (Zadeh [12]). A fuzzy subset of a BCI-algebra X is a function $\mu:X\rightarrow[0,1]$.

Definition 2.2 (C. Lele [6]). Let ξ be the family of all fuzzy sets in X. For $x\in X$ and $\lambda\in(0,1],x_{\lambda}\in\xi$ is a fuzzy point iff

Notes

$$x_\lambda(y)=\begin{cases}\lambda & \text{if } x=y,\\ 0 & \text{otherwise.}\end{cases}$$

We denote by $\tilde{X}=\{x_\lambda : x\in X, \lambda\in(0,1]\}$ the set of all fuzzy points on X and we define a binary operation on $\tilde{X}$ as follows :

Notes

$$x_\lambda * y_\mu = (x*y)_{\min(\lambda,\mu)}$$

It is easy to verify $\forall x_\lambda, y_\mu, z_\alpha \in \tilde{X}$, the following conditions hold:
BCI-1'. $((x_\lambda * y_\mu)*(x_\lambda * z_\alpha))*(z_\alpha * y_\mu)=0_{\min(\lambda,\mu,\alpha)}$;

BCI-2'. $(x_\lambda *(x_\lambda * y_\mu))* y_\mu = 0_{\min(\lambda,\mu)}$;

BCI-3'. $x_\lambda * x_\mu = 0_{\min(\lambda,\mu)}$;

BCK-5'. $0_\mu * x_\lambda = 0_{\min(\lambda,\mu)}$;

Remark 2.3 (C. Lele [6]). The condition BCI-4 is not true $(\tilde{X},*)$. So the partial order $\leq (X,*)$ can not be extended to $(\tilde{X},*)$.
We can also establish the following conditions $\forall x_\lambda, y_\mu, z_\alpha \in \tilde{X}$:

1'. $x_\lambda * 0_\mu = x_{\min(\lambda,\mu)}$;

2'. $x_\lambda * y_\mu = 0_{\min(\lambda,\mu)}$ and $y_\mu * z_\alpha = 0_{\min(\mu,\alpha)} \Rightarrow x_\lambda * z_\alpha = 0_{\min(\lambda,\alpha)}$;

3'. $x_\lambda * y_\mu = 0_{\min(\lambda,\mu)} \Rightarrow (x_\lambda * z_\alpha)*(y_\mu * z_\alpha)=0_{\min(\lambda,\mu,\alpha)}$ and

$$(z_\alpha * y_\mu)*(z_\alpha * x_\lambda)=0_{\min(\lambda,\mu,\alpha)};$$

4'. $(x_\lambda * y_\mu)* z_\alpha = (x_\lambda * z_\alpha)* y_\mu$;

5'. $(x_\lambda * y_\mu)* x_\lambda = 0_{(\lambda,\mu)}$;

6'. $x_\lambda *(x_\lambda *(x_\lambda * y_\mu)) = x_\lambda * y_\mu$;

We recall that if A is a fuzzy subset of a BCI-algebra X, then we have the following:

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$$\tilde{A} = \{x_\lambda \in \tilde{X} : A(x) \geq \lambda \ , \ \lambda \in (0,1]\}. \qquad \text{(i)}$$

$$\forall \lambda \in (0,1], \tilde{X}_\lambda = \{x_\lambda : x \in X\}, \text{ and } \tilde{A}_\lambda = \{x_\lambda \in \tilde{X}_\lambda : A(x) \geq \lambda\} \qquad \text{(ii)}$$

also have $\tilde{X}_\lambda \subseteq \tilde{X}, \tilde{A} \subseteq \tilde{X}, \tilde{A}_\lambda \subseteq \tilde{A}, \tilde{A}_\lambda \subseteq \tilde{X}_\lambda$ and one can easily check that $(\tilde{X}_\lambda ; *, 0_\lambda)$ is a BCK-algebra.

Notes

Definition 2.4(Isèki [2]). A nonempty subset of BCI-algebra X is called an ideal of X if it satisfies

1. $0 \in I$;

2. $\forall x, y \in X, (x * y \in I \text{ and } y \in I) \Rightarrow x \in I$

Definition 2.5 (Liu and Meng [7]). A nonempty subset I of BCI-algebra X is BCI-commutative ideal if it satisfies:

1. $0 \in I$;

2. $\forall x, y, z \in X$

$$((x * y) * z) \in I \text{ and } z \in I) \Rightarrow \Big(x * \big((y * (y * x))\big) * \big(0 * \big(0 * (x * y)\big)\big)\Big) \in I$$

Definition 2.6Xi [11]). A fuzzy subset A of a BCI-algebra X is a fuzzy ideal if

1. $\forall x \in X, A(0) \geq A(x)$;

2. $\forall x, y \in X, A(x) \geq \min(A(x * y), A(y))$.

Definition 2.7 (Xi [11]). A fuzzy subset A of a BCI-algebra X is called a fuzzy BCI- commutative ideal of X if.

1. $\forall x \in X, A(0) \geq A(x)$;

2. $\forall x, y, z \in X$

$$A\Big(x * \big((y * (y * x))\big) * \big(0 * \big(0 * (x * y)\big)\big)\Big) \geq \big(A((x * y) * z), A(z)\big)$$

Definition 2.8(C . Lele, [6]). $\tilde{A}$ is a weak ideal of $\tilde{X}$ if

1. $\forall \nu \in \text{Im}(A)\,;\, 0_\nu \in \tilde{A}$;

2. $\forall x_\lambda, y_\mu \in X$. Such that $x_\lambda * y_\mu \in \tilde{A}$ and $y_\mu \in \tilde{A}$, we have $x_{\min(\lambda,\mu)} \in \tilde{A}$.

Notes

Theorem 2.9 (Lele, [6]). Suppose that A is a fuzzy subset of a BCK - algebra X , then the following conditions are equivalent:

1. A is a fuzzy ideal ;

2. $\forall x_\lambda, y_\mu \in \tilde{A}$, $(z_\alpha * y_\mu) * x_\lambda = 0_{\min(\lambda,\mu,\alpha)} \Rightarrow z_{\min(\lambda,\mu,\alpha)} \in \tilde{A}$;

3. $\forall t \in (0,1]$, the t-level subset $A^t = \{x \in X : A(x) \geq t\}$ is an ideal when $A^t \neq \phi$;

4. $\tilde{A}$ is a weak ideal.

III. Fuzzy N-Fold BCI-Commutative Ideals in BCI - Algebras

Throughout this paper $\tilde{X}$ is the set of fuzzy points on BCI-algebra X and $n \in \mathrm{N}$ (where N the set of all the natural numbers).

Let us denote $(\cdots((x * y) * y) * \cdots) * y$ by $x * y^n$

and $(\cdots((x_{\min(\lambda,\mu)} * 0_\mu) * 0_\mu) * \cdots) * 0_\mu$ by $x_\lambda * y_\mu^n$ (where y and y_μ occurs respectively n times) with $x, y \in X$, $x_\lambda, y_\lambda \in \tilde{X}$.

Definition 3.1. A nonempty subset I of a BCI -algebra X is an n-fold BCI-commutative ideal of X if it satisfies :

1. $0 \in I$;

2. $\forall x, y, z \in X$;

$$((x * y) * z) \in I \text{ and } z \in I) \Rightarrow \Big(x * \big((y * (y * x))\big) * \Big(0 * \big(0 * (x * y^n)\big)\Big)\Big) \in I$$

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Definition 3.2 A fuzzy subset A of X is called a fuzzy n-fold BCI-commutative ideal of X if it satisfies :

1. $\forall x \in X \ , A(0) \geq A(x)$;

2. $\forall x, y, z \in X$,

$$A\Big(\big(x*(y*(y*x))*\big(0*\big(0*\big(x*y^{n}\big)\big)\big)\big)\Big) \geq \min\big(A\big((x*y)*z\big), A(z)\big).$$

Definition 3.3. $\tilde{A}$ is BCI-commutative weak ideal of $\tilde{X}$ if

1. $\forall v \in \mathrm{Im}(A)$, $0_v \in \tilde{A}$;

2. $\forall x_\lambda, y_\mu, z_\alpha \in \tilde{X}$

$$((x_\lambda * y_\mu) * z_\alpha) \in I \text{ and } z_\alpha \in I) \Rightarrow \Big(\big(x_\lambda * \big(y_\mu * \big(y_\mu * x_\lambda\big)\big)\big) * \big(0_\alpha * \big(0_\alpha * \big(x_\lambda * y_\mu\big)\big)\big)\Big) \in I$$

Definition 3.4. $\tilde{A}$ is n-fold BCI- commutative weak ideal of $\tilde{X}$ if

1. $\forall v \in \mathrm{Im}(A)$, $0_v \in \tilde{A}$;

2. $\forall x_\lambda, y_\mu, z_\alpha \in \tilde{X}$

$$((x_\lambda * y_\mu) * z_\alpha) \in I \text{ and } z_\alpha \in I) \Rightarrow \Big(x_\lambda * \big(\big(y_\mu * \big(y_\mu * x_\lambda\big)\big)\big) * \big(0_\alpha * \big(0_\alpha * \big(x_\lambda * y_\mu^{\,n}\big)\big)\big)\Big) \in I$$

Example 3.5. Let $X = \{0, a, b, c, d\}$ with $*$ defined by the following table

*	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	a	0	a	0
c	c	c	c	0	0
d	d	d	d	d	0

By simple computations, one can prove that $(X, *, 0)$ is BCI – algebra. let $t_1, t_2 \in (0,1]$ and define a fuzzy subset $t_1 = A(0) = A(a) = A(b) = A(c) \geq A(d) = t_2$.

Notes

One can easily check that for any $n \geq 3$

$$\tilde{A}=\{0_\lambda:\lambda\in(0,t_1]\}\cup\{a_\lambda:\lambda\in(0,t_1]\}\cup\{b_\lambda:\lambda\in(0,t_1]\}\cup\{c_\lambda:\lambda\in[0,t_1)\}\cup\{d_\lambda:\lambda\in(0,t_2]\}$$

It is an n-fold BCI- commutative weak ideal.

Notes

Remark 3.6. $\tilde{A}$ Is a 1-fold BCI- commutative weak ideal of a BCK-algebra $\tilde{X}$ if $\tilde{A}$ is BCI- commutative weak ideal of $\tilde{X}$.

Proposition 3.7 An ideal I of BCI -algebra X Is an n-fold BCI - commutative ideal if

$$\forall x,y\in X, x*y\in I \Rightarrow \left(x*\left(\left(y*\left(y*x\right)\right)\right)*\left(0*\left(0*\left(x*y^n\right)\right)\right)\right)\in I$$

Proof. If an ideal I is an n-fold BCI - commutative and $x*y\in I$ then $(x*y)*0\in I$ and $0\in I$, then we have

$$\left(x*\left(\left(y*\left(y*x\right)\right)\right)*\left(0*\left(0*\left(x*y^n\right)\right)\right)\right)\in I ,$$

thus this means that the condition satisfies.

Conversely, let an I an ideal satisfies the condition. If $(x*y)*z\in I$ and $z\in I$, then by the definition of ideas we have $x*y\in I$. It follows from the given condition that $\left(x*\left(\left(y*\left(y*x\right)\right)\right)*\left(0*\left(0*\left(x*y^n\right)\right)\right)\right)\in I$; this means that I is an n-fold BCI - commutative ideal .this finishes the proof.

Proposition 3.8. An n-fold BCI - commutative weak ideal is a weak ideal.

Proof. $\forall x_\lambda, y_\mu\in\tilde{X}$ let $x_\lambda*y_\mu=(x_\lambda*0_\lambda)*y_\mu\in\tilde{A}$ and $y_\mu\in\tilde{A}$, since $\tilde{A}$ n-fold BCI - commutative ideal we have

$$x_{\min(\lambda,\mu)}=\left(\left(x_\lambda*\left(0_\mu*\left(0_\mu*x_\lambda\right)\right)*\left(0_\mu*\left(0_\mu*\left(x_\lambda*0^n{}_\mu\right)\right)\right)\right)\right)=x_{\min(\lambda,\mu)}\in\tilde{A} ,$$

Thus $\tilde{A}$ is a weak ideal.

Proposition 3.9. Any fuzzy n-fold BCI - commutative ideal of BCI – algebras X is the fuzzy ideal of X .

Proof. let A be a fuzzy n-fold BCI - commutative ideal of X and let $x, z \in X$.

Then
$$\min\left(A(x*z), A(z)\right)$$
$$= \min\left(A\left((x*0)*z\right), A(z)\right)$$
$$\leq A\left(\left(x*\left(0*\left(0*x\right)\right)*\left(0*\left(0*\left(x*0^n\right)\right)\right)\right)\right)$$
$$= A\left(\left(x*\left(0*\left(0*x\right)\right)*\left(0*\left(0*\left(x*0\right)\right)\right)\right)\right)$$
$$= A(x*0)$$
$$= A(x)$$

Thus A is a fuzzy ideal of X.

Theorem 3.10. If A is a fuzzy subset of X, then A is a fuzzy n-fold BCI - commutative ideal if $\tilde{A}$ is an n-fold BCI -commutative weak ideal.

Proof. $\Rightarrow$ - Let $\lambda \in \operatorname{Im}(A)$, it is easy to prove that $0_\lambda \in \tilde{A}$;

- Let $(x_\lambda * y_\mu) * z_\alpha \in \tilde{A}$ and $z_\alpha \in \tilde{A}$

$A((x*y)*z) \geq \min(\lambda, \mu, \alpha)$ and $A(z) \geq \alpha$.

Since A is a fuzzy n-fold BCI -commutative ideal, we have

$$A\left(\left(x*\left(y*\left(y*x\right)\right)*\left(0*\left(0*\left(x*y^n\right)\right)\right)\right)\right) \geq \min\left(A\left((x*y)*z\right), A(z)\right)$$

$$\geq \min\left(\min(\lambda, \mu, \alpha), \alpha\right) = \min(\lambda, \mu, \alpha).$$

Therefore $\left(x*\left(\left(y*\left(y*x\right)\right)\right)*\left(0*\left(0*\left(x*y^n\right)\right)\right)\right)_{\min(\lambda,\mu,\alpha)} =$

$$\left(x_\lambda*\left(\left(y_\mu*\left(y_\mu*x_\lambda\right)\right)\right)*\left(0_\alpha*\left(0_\alpha*\left(x_\lambda*y_\mu{}^n\right)\right)\right)\right) \in \tilde{A}$$

$\Leftarrow$ - Let $x \in X$, it is easy to prove that $A(0) \geq A(x)$;

Notes

Notes

- Let $x,y,z \in X$ and let $A((x*y)*z)=\beta$ and $A(z)=\alpha$, then

$$((x*y)*z)_{\min(\beta,\alpha)} = (x_\beta * y_\alpha)*z_\alpha \in \tilde{A} \text{ and } z_\alpha \in \tilde{A}.$$

Since $\tilde{A}$ is n-fold BCI -commutative weak ideal, we have

$$\left(x_\lambda * \left(\left(y_\mu * \left(y_\mu * x_\lambda\right)\right)\right) * \left(0_\alpha * \left(0_\alpha * \left(x_\lambda * y_\mu{}^n\right)\right)\right)\right) =$$

$$\left(x * \left(\left(y * \left(y * x\right)\right)\right) * \left(0 * \left(0 * \left(x * y^n\right)\right)\right)\right)_{\min(\lambda,\mu,\alpha)} \in \tilde{A}$$

Thus $\left(x * \left(\left(y * \left(y * x\right)\right)\right) * \left(0 * \left(0 * \left(x * y^n\right)\right)\right)\right) \geq \min(\beta,\alpha)$

$$= \min\left(A\left((x*y)*z\right), A(z)\right)$$

Theorem 3.11. Suppose that $\tilde{A}$ is a weak ideal (namely A is a fuzzy ideal by Theorem 2.12), then the following conditions are equivalent:

1. A is a fuzzy n-fold BCI - commutative ideal ;

2. $\forall x_\lambda, y_\mu \in \tilde{X}$ such that $x_\lambda * y_\mu \in \tilde{A}$, we have

$$\left(x_\lambda * \left(\left(y_\mu * \left(y_\mu * x_\lambda\right)\right)\right) * \left(0_\alpha * \left(0_\alpha * \left(x_\lambda * y_\mu{}^n\right)\right)\right)\right) \in \tilde{A} \;;$$

3. $\forall t \in (0,1]$, the t-level subset $A^t = \{x \in X : A(x) \geq t\}$ is an n-fold BCI – commutative ideal when $A^t \neq \phi$;

4. $A\left(\left(x * \left(y * \left(y * x\right)\right) * \left(0 * \left(0 * \left(x * y^n\right)\right)\right)\right)\right) \geq A(x*y)$

5. $\tilde{A}$ is an n-fold BCI – commutative weak ideal.

Proof. $1 \Rightarrow 2$ Let $x_\lambda, y_\mu \in \tilde{A}$ such that $x_\lambda * y_\mu \in \tilde{A}$. Since A is a fuzzy n-fold BCI - commutative ideal, we have

$$A\left(\left(x * \left(y * \left(y * x\right)\right) * \left(0 * \left(0 * \left(x * y^n\right)\right)\right)\right)\right) \geq \min\left(A\left((x*y)*(x*y)\right), A(x*y)\right)$$

$$= \min(A(0), A(x*y)) = A(x*y) \geq \min(\lambda, \mu).$$

Therefore $\left(x*\left(\left(y*\left(y*x\right)\right)\right)*\left(0*\left(0*\left(x*y^{n}\right)\right)\right)\right)_{\min(\lambda,\mu,\alpha)} =$

$$\left(x_{\lambda}*\left(\left(y_{\mu}*\left(y_{\mu}*x_{\lambda}\right)\right)\right)*\left(0_{\alpha}*\left(0_{\alpha}*\left(x_{\lambda}*y_{\mu}{}^{n}\right)\right)\right)\right) \in \tilde{A}$$

Notes

$2 \Rightarrow 3$ $\forall t \in (0,1], 0 \in A^{t}$.

Let $(x*y)*z \in A^{t}$ and $z \in A_{t}$ then we have $((x*y)*z)_{t} = (x_{t}*y_{t})*z_{t} \in \tilde{A}$ and $z_{t} \in \tilde{A}$.

Since $\tilde{A}$ it is a weak ideal, we have $x_{t}*y_{t} = (x*y)_{t} \in \tilde{A}$.

Using the hypothesis, we obtain

$$\left(x_{t}*\left(\left(y_{t}*\left(y_{t}*x_{t}\right)\right)\right)*\left(0_{t}*\left(0_{t}*\left(x_{t}*y_{t}{}^{n}\right)\right)\right)\right) =$$

$$\left(x*\left(\left(y*\left(y*x\right)\right)\right)*\left(0*\left(0*\left(x*y^{n}\right)\right)\right)\right)_{t} \in \tilde{A} \text{ hence}$$

$\left(x*\left(\left(y*\left(y*x\right)\right)\right)*\left(0*\left(0*\left(x*y^{n}\right)\right)\right)\right) \in A^{t}$. By Proposition 3.7, we obtain that $A^{t} = \{x \in X : A(x) \geq t\}$ is an n-fold BCI - commutative ideal.

$3 \Rightarrow 4$ Let $x, y \in X$ and $t = A(x*y)$, then $x*y \in A^{t}$.

Since A^{t} is an n-fold BCI – commutative ideal, we have

$\left(x*\left(\left(y*\left(y*x\right)\right)\right)*\left(0*\left(0*\left(x*y^{n}\right)\right)\right)\right) \in A^{t}$. Hence

$A\left(x*\left(\left(y*\left(y*x\right)\right)\right)*\left(0*\left(0*\left(x*y^{n}\right)\right)\right)\right) \geq t = A(x*y)$ 4. $\Rightarrow$ 5. Let $\lambda \in \mathrm{Im}(A)$. Obviously $0_{\lambda} \in \tilde{A}$.

- Let $(x_{\lambda}*y_{\mu})*z_{\alpha} \in \tilde{A}$ and $z_{\alpha} \in \tilde{A}$. Since $\tilde{A}$ is a weak ideal, we obtain $(x*y)_{\min(\lambda,\mu,\alpha)} \in \tilde{A}$. According to the hypothesis, we obtain

$A\left(x*\left(\left(y*\left(y*x\right)\right)\right)*\left(0*\left(0*\left(x*y^{n}\right)\right)\right)\right)\geq A\left(x*y\right)\geq \min\left(\lambda,\mu,\alpha\right)$, hence

$$\left(x*\left(\left(y*\left(y*x\right)\right)\right)*\left(0*\left(0*\left(x*y^{n}\right)\right)\right)\right)_{\min(\lambda,\mu,\alpha)}=$$

$$\left(x_{\lambda}*\left(\left(y_{\mu}*\left(y_{\mu}*x_{\lambda}\right)\right)\right)*\left(0_{\alpha}*\left(0_{\alpha}*\left(x_{\lambda}*y_{\mu}{}^{n}\right)\right)\right)\right)\in\tilde{A}.$$

5. $\Rightarrow$ 1. Follows from Theorem 3.10

Theorem 3.12 Let $\{\tilde{A}_{i\in I}\}$ be a family of *n*-fold BCI – commutative weak ideals and $\{A_{i\in I}\}$ be a family of fuzzy *n*-fold BCI – commutative ideals. then: (1) $\bigcap_{i\in I}\tilde{A}_i$ is an *n*-fold BCI – commutative weak ideal.

(2))$\bigcup_{i\in I}\tilde{A}_i$ is an *n*-fold BCI – commutative weak ideal.

(3))$\bigcap_{i\in I}A_i$ is a fuzzy *n*-fold BCI – commutative ideal.

(4))$\bigcup_{i\in I}A_i$ is a fuzzy *n*-fold BCI – commutative ideal.

Proof. (1) $\forall\lambda\in \operatorname{Im}\left(\bigcap_{i\in I}\tilde{A}_i\right)$, then $\lambda\in \operatorname{Im}(\tilde{A}_i),\forall i$, so, $0_{\lambda}\in\tilde{A}_i,\forall i$, i.e. $0_{\lambda}\in\bigcap_{i\in I}\tilde{A}_i$. For every $x_{\mu},y_{\lambda},z_{\alpha}\in\tilde{X}$, if $((x_{\lambda}*y_{\mu})*z_{\alpha})\in\bigcap_{i\in I}\tilde{A}_i$ *and* $z_{\alpha}\in\bigcap_{i\in I}\tilde{A}_i$, then $((x_{\lambda}*y_{\mu})*z_{\alpha})\in\tilde{A}_i$ *and* $z_{\alpha}\in\tilde{A}_i\ \forall i$, thus

$$\left(x_{\lambda}*\left(\left(y_{\mu}*\left(y_{\mu}*x_{\lambda}\right)\right)\right)*\left(0_{\alpha}*\left(0_{\alpha}*\left(x_{\lambda}*y_{\mu}{}^{n}\right)\right)\right)\right)\in\tilde{A}_i$$

So $\left(x_{\lambda}*\left(\left(y_{\mu}*\left(y_{\mu}*x_{\lambda}\right)\right)\right)*\left(0_{\alpha}*\left(0_{\alpha}*\left(x_{\lambda}*y_{\mu}{}^{n}\right)\right)\right)\right)\in\bigcap_{i\in I}\tilde{A}_i$. Thus $\bigcap_{i\in I}\tilde{A}_i$ is an

n-fold BCI - commutative weak ideals.

(2) $\forall\lambda\in \operatorname{Im}\left(\bigcup_{i\in I}\tilde{A}_i\right)$, then $\exists i_0\in I$, *such,that* $\lambda\in\tilde{A}_{i_0}$, so, $0_{\lambda}\in\tilde{A}_{i_0}$, i.e.

$0_{\lambda}\in\bigcup_{i\in I}\tilde{A}_i$. For every $x_{\mu},y_{\lambda},z_{\alpha}\in\tilde{X}$, if

$((x_\lambda * y_\mu) * z_\alpha) \in \bigcup_{i\in I} \tilde{A}_i$ *and* $z_\alpha \in \bigcup_{i\in I} \tilde{A}_i$, then $\exists i_0 \in I$ such that

$$((x_\lambda * y_\mu) * z_\alpha) \in \tilde{A}_{i_0} \text{ and } z_\alpha \in \tilde{A}_{i_0} \ \forall i \text{ , thus}$$

$$\left(x_\lambda * \left(\left(y_\mu * \left(y_\mu * x_\lambda\right)\right)\right) * \left(0_\alpha * \left(0_\alpha * \left(x_\lambda * y_\mu{}^n\right)\right)\right)\right) \in \tilde{A}_{i_0}$$

So $\left(x_\lambda * \left(\left(y_\mu * \left(y_\mu * x_\lambda\right)\right)\right) * \left(0_\alpha * \left(0_\alpha * \left(x_\lambda * y_\mu{}^n\right)\right)\right)\right) \in \bigcup_{i\in I} \tilde{A}_i$. Thus $\bigcup_{i\in I} \tilde{A}_i$ is an

n-fold BCI – commutative weak ideals.

(3) Follows from (1) and Theorem 3.10.

(4) Follows from (2) and Theorem 3.10.

IV. Fuzzy N - Fold Weak BCI – Commutative Ideals in BCI - Algebras

In this section, we define and give some characterizations of (fuzzy) n-fold weak BCI - commutative(weak) ideals in BCK-algebras.

Definition 4.1. A nonempty subset I of X is called an n-fold weak BCI – a commutative ideal of X if it satisfies

1. $0 \in I$;

2. $\forall x, y, z \in X$,$\left(x * y^n\right) * z \in I$,*and* ,$z \in I$

$$\Rightarrow x * \left(\left(y * \left(y * x\right)\right) * \left(0 * \left(0 * \left(\left(x * y\right) * y\right)\right)\right)\right) \in I$$

Lemma 4.2. An ideal I of X is called an n-fold weak BCI - commutative ideal if

$$\forall x, y, z \in X, \left(x * y^n\right) * z \in I \Rightarrow x * \left(\left(y * \left(y * x\right)\right) * \left(0 * \left(0 * \left(\left(x * y\right) * y\right)\right)\right)\right) \in I$$

Definition 4.3. A fuzzy subset A of X is called a fuzzy n-fold weak BCI – commutative ideal of X if it satisfies

1. $\forall x \in X$, $A(0) \geq A(x)$;

2. $\forall x, y, z, A\left(x * \left(\left(y * \left(y * x\right)\right) * \left(0 * \left(0 * \left(\left(x * y\right) * y\right)\right)\right)\right)\right) \geq \min\left(A\left(\left(x * y^n\right) * z\right), A(z)\right)$

Notes

Definition 4.4. $\tilde{A}$ is a weak BCI – commutative weak ideal of $\tilde{X}$ if

1. $\forall \nu \in \mathrm{Im}(A)$, $0_\nu \in \tilde{A}$;

2. $\forall x_\lambda, y_\mu, z_\alpha \in \tilde{X}$

Notes

$$\left(x_\lambda * y_\mu\right) * z_\alpha \in I, z_\alpha \in I \Rightarrow x_\lambda * \left(\left(y_\mu * \left(y_\mu * x_\lambda\right)\right)\right.$$
$$\left. * \left(0_\alpha * \left(0_\alpha * \left(\left(x_\lambda * y_\mu\right) * y_\mu\right)\right)\right)\right) \in I$$

Definition 4.5. $\tilde{A}$ is an n-fold a weak BCI – commutative weak ideal of $\tilde{X}$ if

1. $\forall \nu \in \mathrm{Im}(A)$, $0_\nu \in \tilde{A}$;

2. $\forall x_\lambda, y_\mu, z_\alpha \in \tilde{X}$;

$$\left(x_\lambda * y_\mu{}^n\right) * z_\alpha \in I,\ z_\alpha \in I \Rightarrow x_\lambda * \left(\left(y_\mu * \left(y_\mu * x_\lambda\right)\right)\right.$$
$$\left. * \left(0_\alpha * \left(0_\lambda * \left(\left(x_\lambda * y_\mu\right) * y_\mu\right)\right)\right)\right) \in I$$

Example 4.6 Let $X = \{0,1,2,3\}$ in which $*$ is given by the following table

*	0	a	b	c
0	0	0	0	0
a	a	0	0	0
b	b	b	0	0
c	c	c	c	0

Then $(X;*,0)$ it is a BCI-algebra. Let $t_1, t_2 \in (0,1]$ and let us define a fuzzy subset $A : X \to [0,1]$ by

$$t_1 = A(0) = A(a) = A(b) > A(c) = t_2$$

It is easy to check that for any $n > 2$

$$\tilde{A} = \{0_\lambda : \lambda \in (0,t_1]\} \cup \{a_\lambda : \lambda \in (0,t_2]\} \cup \{b_\lambda : \lambda \in (0,t_1]\} \cup \{c_\lambda : \lambda \in (0,t_2]\}$$

It is an n-fold weak BCI – commutative weak ideal.

Remark 4.7 $\tilde{A}$ is a 1-fold weak BCI – commutative weak ideal of a BCK-algebra X if $\tilde{A}$ is a weak BCI – commutative weak ideal.

Theorem 4.8 If A it is a fuzzy subset of X, then A is a fuzzy n-fold weak BCI – commutative ideal if $\tilde{A}$ is an n-fold weak BCI – commutative weak ideal.

Proof. $\Rightarrow$ - Let $\lambda \in \mathrm{Im}(A)$ obviously $0_\lambda \in \tilde{A}$;

- Let $(x_\lambda * y_\mu{}^n) * z_\alpha \in \tilde{A}$ and $z_\alpha \in \tilde{A}$, then

$$A((x * y^n) * z) \geq \min(\lambda, \mu, \alpha) \text{ and } A(z) \geq \alpha .$$

Since A is a fuzzy n-fold weak BCI - commutative ideal, we have

$$\forall x, y, z, A\Big(x * \big((y * (y * x)) * (0 * (0 * ((x * y) * y)))\big)\Big) \geq \min\Big(A\big((x * y^n) * z\big),$$

$$A(z)\Big) \geq \min(\min(\lambda, \mu, \alpha), \alpha) = \min(\lambda, \mu, \alpha).$$

$$\text{Therefore } \Big(x_\lambda * \big((y_\mu * (y_\mu * x_\lambda)) * (0_\alpha * (0_\alpha * ((x_\lambda * y_\mu) * y_\mu)))\big)\Big) \in \tilde{A} =$$

$$\Big(x_\lambda * \big((y_\mu * (y_\mu * x_\lambda)) * (0_\alpha * (0_\alpha * ((x_\lambda * y_\mu) * y_\mu)))\big)\Big) \in \tilde{A} .$$

$\Leftarrow$ - Let $x \in X$, it is easy to prove that $A(0) \geq A(x)$;

- Let $x, y, z \in X$, $A((x * y^n) * z) = \beta$ and $A(z) = \alpha$.

Then $((x * y^n) * z)_{\min(\beta, \alpha)} = ((x_\beta * y_\beta{}^n) * z_\alpha) \in \tilde{A}$ and $z_\alpha \in \tilde{A}$

Since $\tilde{A}$ is n-fold weak BCI - commutative weak ideal, we have

$$\Big(x_\lambda * \big((y_\mu * (y_\mu * x_\lambda)) * (0_\alpha * (0_\alpha * ((x_\lambda * y_\mu) * y_\mu)))\big)\Big) =$$

$$\Big(x_\lambda * \big((y_\mu * (y_\mu * x_\lambda)) * (0_\alpha * (0_\alpha * ((x_\lambda * y_\mu) * y_\mu)))\big)\Big) \in \tilde{A}$$

Hence

Notes

$$A\left(x*\left(\left(y*\left(y*x\right)\right)*\left(0*\left(0*\left(\left(x*y\right)*y\right)\right)\right)\right)\right)\geq \min\left(\beta,\alpha\right)$$

$$=\min\left(A\left(\left(x*y^{n}\right)*z\right),A\left(z\right)\right)$$

Notes

Proposition 4.9. Any fuzzy n-fold weak BCI – commutative ideal of X is the fuzzy ideal of X.

Proof. Let A be an n-fold weak BCI – commutative ideal of X and let

$x,z\in X$, then $\min\left\{A\left(x*z\right),A\left(z\right)\right\}$

$$\min\left\{A\left(\left(x*0\right)*z\right),A\left(z\right)\right\}$$

$$\leq A\left(x*\left(\left(0*\left(0*x\right)\right)*\left(0*\left(0*\left(\left(x*0\right)*0\right)\right)\right)\right)\right)$$

$$=A\left(x*\left(\left(0*\left(0*x\right)\right)*\left(0*\left(0*x\right)\right)\right)\right)$$

$$=A\left(x*0\right)$$

$$=A\left(x\right).$$

Thus A is a fuzzy ideal of X .

Corollary 4.10. An n-fold weak BCI – commutative weak ideal is a weak ideal.

Theorem 4.11. Suppose that $\tilde{A}$ is a weak ideal (namely A is a fuzzy ideal by Theorem 2.9), then the following conditions are equivalent:

1. A is a fuzzy n-fold weak BCI – commutative ideal ;

2. $\forall x_{\lambda},y_{\mu}\in\tilde{X}$ such that $(x_{\lambda}*y^{n}_{\min(\lambda,\mu)})\in\tilde{A}$, we have

$$\left(x_{\lambda}*\left(\left(y_{\mu}*\left(y_{\mu}*x_{\lambda}\right)\right)*\left(0_{\alpha}*\left(0_{\alpha}*\left(\left(x_{\lambda}*y_{\mu}\right)*y_{\mu}\right)\right)\right)\right)\right)\in\tilde{A}.$$

3. $\forall t\in(0,1]$, the t-level subset $A^{t}=\{x\in X:A(x)\geq t\}$,

is an n-fold weak IBCI – commutative ideal when $A^t \neq \phi$;

4. $,\forall x,y \in X, A\left(x * \left(\left(y*\left(y*x\right)\right)*\left(0*\left(0*\left(\left(x*y\right)*y\right)\right)\right)\right)\right) \geq A\left(x*y^n\right)$;

5. $\tilde{A}$ is an n-fold weak BCI – commutative weak ideal

Proof. 1 $\Rightarrow$2 - Let $(x_\lambda * y^n_{\min(\lambda,\mu)}) \in \tilde{A}$. Since A is a fuzzy n-fold weak BCI – commutative ideal, we have

$$A\left(x*\left(\left(y*\left(y*x\right)\right)*\left(0*\left(0*\left(\left(x*y\right)*y\right)\right)\right)\right)\right) \geq \min\left(A\left(\left(x*y^n\right)*0\right), A(0)\right)$$

$$= A(((x*y^n))) \geq \min(\lambda,\mu) \geq \min(\lambda,\mu,\alpha).$$

Therefore $\left(x*\left(\left(y*\left(y*x\right)\right)*\left(0*\left(0*\left(\left(x*y\right)*y\right)\right)\right)\right)\right)_{\min(\lambda,\mu,\alpha)}$

$$=\left(x_\lambda*\left(\left(y_\mu*\left(y_\mu*x_\lambda\right)\right)\left(0_\alpha*\left(0_\alpha*\left(\left(x_\lambda*y_\mu\right)*y_\mu\right)\right)\right)\right)\right)\in\tilde{A}$$

2 $\Rightarrow$3 – Obviously, $\forall t \in (0,1]$, $0 \in A^t$.

Let $(x*y^n) \in A^t$, we have

$$(x*y^n)_t = (x_t * y_t^n) \in \tilde{A}.$$

By the hypothesis, one obtains,

$$\left(x_t*\left(\left(y_t*\left(y_t*x_t\right)\right)\left(0_t*\left(0_t*\left(\left(x_t*y_t\right)*y_t\right)\right)\right)\right)\right)\in\tilde{A}$$

therefore $\left(x*\left(\left(y*\left(y*x\right)\right)\left(0*\left(0*\left(\left(x*y\right)*y\right)\right)\right)\right)\right)\in A^t$. Using Lemma 4.2. , we can conclude that

$A^t = \{x \in X : A(x) \geq t\}$ it is an n-fold weak BCI – commutative ideal.

3 $\Rightarrow$4- Let $x, y \in X$ and $t = A(x*y^n)$, then $(x*y^n) \in A^t$.

Since A^t is an n-fold weak is BCI – commutative ideal, we have

Notes

$\left(x * \left(\left(y * \left(y * x\right)\right)\left(0 * \left(0 * \left(\left(x * y\right) * y\right)\right)\right)\right)\right) \in A^t$, therefore

$A\left(x * \left(\left(y * \left(y * x\right)\right)\left(0 * \left(0 * \left(\left(x * y\right) * y\right)\right)\right)\right)\right) \geq t = A\left(x * y^n\right)$.

Notes

$4 \Rightarrow 5$ – Let $\lambda \in \operatorname{Im}(A)$, it is clear that $0_\lambda \in \tilde{A}$.

- Let $(x_\lambda * y_\mu^n) * z_\alpha \in \tilde{A}$ and $z_\alpha \in \tilde{A}$. Since $\tilde{A}$ it is a weak ideal, $(x * y^n)_{\min(\lambda,\mu)} \in \tilde{A}$. Using the hypothesis, we obtain

$A\left(x * \left(\left(y * \left(y * x\right)\right)\left(0 * \left(0 * \left(\left(x * y\right) * y\right)\right)\right)\right)\right) \geq A\left(x * y^n\right) \geq \min\left(\lambda, \mu, \alpha\right)$.

From this, one can deduce that

$\left(x * \left(\left(y * \left(y * x\right)\right) * \left(0 * \left(0 * \left(\left(x * y\right) * y\right)\right)\right)\right)\right)_{\min(\lambda,\mu,\alpha)}$

$= \left(x_\lambda * \left(\left(y_\mu * \left(y_\mu * x_\lambda\right)\right) * \left(0_\alpha * \left(0_\alpha * \left(\left(x_\lambda * y_\mu\right) * y_\mu\right)\right)\right)\right)\right) \in \tilde{A}$

$5 \Rightarrow 1$ Follows from Theorem 4.8

Theorem 4.12. Let $\{\tilde{A}_{i \in I}\}$ be a family of *n*-fold weak BCI – commutative weak ideals and $\{A_{i \in I}\}$ be a family of fuzzy *n*-fold weak BCI – commutative ideals. then (1) $\bigcap_{i \in I} \tilde{A}_i$ is an *n*-fold weak BCI – commutative weak ideal.

(2))$\bigcup_{i \in I} \tilde{A}_i$ is an *n*-fold weak BCI – commutative weak ideal.

(3))$\bigcap_{i \in I} A_i$ is a fuzzy *n*-fold weak BCI – commutative ideal.

(4))$\bigcup_{i \in I} A_i$ is a fuzzy *n*-fold weak BCI – commutative ideal.

Proof. (1) $\forall \lambda \in \operatorname{Im}\left(\bigcap_{i \in I} \tilde{A}_i\right)$, then $\lambda \in \operatorname{Im}(\tilde{A}_i), \forall i$, so, $0_\lambda \in \tilde{A}_i, \forall i$, i.e. $0_\lambda \in \bigcap_{i \in I} \tilde{A}_i$. For every $x_\mu, y_\lambda, z_\alpha \in \tilde{X}$, if $\left(x_\lambda * y^n{}_\mu\right) * z_\alpha \in \bigcap_{i \in I} \tilde{A}_i$ *and* $, z_\alpha \in \bigcap_{i \in I} \tilde{A}_i$, then $\left(x_\lambda * y^n{}_\mu\right) * \in \tilde{A}_i$ *and* $, z_\alpha \in \tilde{A}$ $\forall i$, thus

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$$\left(x_\lambda * \left(\left(y_\mu * \left(y_\mu * x_\lambda\right)\right)\left(0_\alpha * \left(0_\alpha * \left(\left(x_\lambda * y_\mu\right) * y_\mu\right)\right)\right)\right)\right) \in \tilde{A}_i \forall i$$

So $\left(x_\lambda * \left(\left(y_\mu * \left(y_\mu * x_\lambda\right)\right)\left(0_\alpha * \left(0_\alpha * \left(\left(x_\lambda * y_\mu\right) * y_\mu\right)\right)\right)\right)\right) \in \bigcap_{i\in I} \tilde{A}_i$. Thus $\bigcap_{i\in I} \tilde{A}_i$ is an *n*-fold weak BCI – commutative weak ideals

(2) $\forall \lambda \in \operatorname{Im}\left(\bigcup_{i\in I} \tilde{A}_i\right)$, then $\exists i_0 \in I$, *such, that* $\lambda \in \dot{\tilde{A}}_{i_0}$, so, $0_\lambda \in \tilde{A}_{i_0}$, i.e.

$0_\lambda \in \bigcup_{i\in I} \tilde{A}_i$. For every $x_\mu, y_\lambda, z_\alpha \in \tilde{X}$, if

$\left(x_\lambda * y^n{}_\mu\right) * z_\alpha \in \bigcup_{i\in I} \tilde{A}_i$ *and* $, z_\alpha \in \bigcup_{i\in I} \tilde{A}_i$, then $\exists i_0 \in I$ such that

$$\left(x_\lambda * y^n{}_\mu\right) * z_\alpha \in \tilde{A}_{i_0} \text{ and } , z_\alpha \in \tilde{A}_{i_0} \ \forall i \text{ , thus}$$

$$\left(x_\lambda * \left(\left(y_\mu * \left(y_\mu * x_\lambda\right)\right)\left(0_\alpha * \left(0_\alpha * \left(\left(x_\lambda * y_\mu\right) * y_\mu\right)\right)\right)\right)\right) \in \tilde{A}_{i_0}$$

So $\left(x_\lambda * \left(\left(y_\mu * \left(y_\mu * x_\lambda\right)\right)\left(0_\alpha * \left(0_\alpha * \left(\left(x_\lambda * y_\mu\right) * y_\mu\right)\right)\right)\right)\right) \in \bigcup_{i\in I} \tilde{A}_i$. Thus $\bigcup_{i\in I} \tilde{A}_i$ is an *n*-fold weak BCI – commutative weak ideals.

(3) Follows from (1) and Theorem 4.8.

(4) Follows from (2) and Theorem4.8.

V. Algorithms

Here We Give Some Algorithms For Studding The Structure Of The Foldness Of (Fuzzy BCI- commutative Ideals In BCI-Algebras)

Notes

Notes

Algorithm for ABCI- Commutative Ideals of BCI-Algebra

Input (X :*BCI-algebra*, $*$: binary operation, I : subset of X) ;
Output(" I is a BCI- commutative ideal of X or not");
Begin
If $I = \phi$ then
go to (1.);
End If
If $0 \notin I$ then
go to (1.);
End If
Stop:=false;
$i := 1$;
While $i \leq |X|$ and not (*Stop*) do
$j := 1$;
While $j \leq |X|$ and not (*Stop*) do
$k := 1$;
While $k \leq |X|$ and not (*Stop*) do
If $(x_i * y_j) * z_k \in I$ and $z_k \in I$ then
If $(x * ((y * (y * x))) * (0 * (0 * (x * y)))) \notin I$
Stop:=true;
EndIf
End If
Endwhile
Endwhile
Endwhile
If *Stop* then
Output (" I is a BCI- commutative ideal of X ")
Else
(1.) Output (" I is not a BCI- commutative ideal of X ")
End If
End

Algorithm for N-Fold BCI- Commutative Ideals of BCI - Algebra

Input (X :*BCI -algebra*, $*$: binary operation, I : subset of X) ;
Output(" I is n-fold BCI - commutative ideal of X or not");
Begin

If $I = \phi$ then
go to (1.);
End If
If $0 \notin I$ then
go to (1.);
End If
Stop:=false;
$i := 1$;
While $i \leq |X|$ and not (*Stop*) do
$j := 1$;
While $j \leq |X|$ and not (*Stop*) do
$k := 1$;
While $k \leq |X|$ and not (*Stop*) do
If $(x_i * y_j) * z_k \in I$ and $z_k \in I$ then
If $\left(x * \left(\left(y * \left(y * x\right)\right)\right) * \left(0 * \left(0 * \left(x * y^n\right)\right)\right)\right) \notin I$
Stop:=true;
EndIf
EndIf
Endwhile
Endwhile
Endwhile
If *Stop* then
Output ("I is an n-fold BCI - commutative ideal of X")
Else
(1.) Output ("I is not an n-fold BCI - commutative ideal of X")
End If
End

Algorithm for Fuzzy BCI- Commutative Ideals of Bcialgebra

Input (X : *BCI-algebra*, $*$: binary operation, A : the fuzzy subset of X);
Output("A is a fuzzy BCI- commutative ideal of X or not");
Begin
Stop:=false;
$i := 1$;
While $i \leq |X|$ and not (*Stop*) do
If $A(0) < A(x_i)$ then
Stop:=true;
End If

Notes

$j := 1$;
While $j \leq |X|$ and not (*Stop*) do
$k := 1$;
While $k \leq |X|$ and not (*Stop*) do
If $A\left(x*\left(\left(y*\left(y*x\right)\right)\right)*\left(0*\left(0*\left(x*y\right)\right)\right)\right) < \left(A\left(\left(x*y\right)*z\right), A\left(z\right)\right)$ then
Stop=true;
End If
Endwhile
Endwhile
Endwhile
If *Stop* then
Output (" A is not a fuzzy BCI- commutative ideal of X ")
Else
Output (" A is a fuzzy BCI- commutative ideal of X ")
End If
End

Algorithm for Fuzzy N–Fold BCI- Commutative Ideals of BCI-Algebra

Input (X : *BCI-algebra*, $*$: binary operation, A : the fuzzy subset of X);
Output(" A is a fuzzy n- fold BCI- commutative ideal of X or not");
Begin
Stop:=false;
$i := 1$;
While $i \leq |X|$ and not (*Stop*) do
If $A(0) < A(x_i)$ then
Stop:=true;
End If
$j := 1$;
While $j \leq |X|$ and not (*Stop*) do
$k := 1$;
While $k \leq |X|$ and not (*Stop*) do
$A\left(x*\left(\left(y*\left(y*x\right)\right)\right)*\left(0*\left(0*\left(x*y^n\right)\right)\right)\right) < \left(A\left(\left(x*y\right)*z\right), A\left(z\right)\right)$
Stop=true;
End If
Endwhile
Endwhile
Endwhile

Notes

If *Stop* then
Output (" A is not a fuzzy n- fold BCI- commutative ideal of X ")
Else
Output (" A is a fuzzy n- fold BCI- commutative ideal of X ")
End If
End

Algorithm for N–Fold Weak BCI- Commutative Ideals of BCI-Algebra

Input(X :*BCI-algebra*, I : subset of X, $n \in \mathbf{N}$);
Output(" I is an n-fold weak BCI - commutative ideal of X or not");
Begin
If $I = \phi$ then
go to (1.);
End If
If $0 \notin I$ then
go to (1.);
End If
Stop:=false;
$i := 1$;
While $i \le |X|$ and not (*Stop*) do
$j := 1$;
While $j \le |X|$ and not (*Stop*) do
$k := 1$;
While $k \le |X|$ and not (*Stop*) do
If $(x * y^n) * z \in I, and, z \in I$ then
If $x * ((y * (y * x)) * (0 * (0 * ((x * y) * y)))) \notin I$
Stop:=true;
EndIf
EndIf
Endwhile
Endwhile
Endwhile
If *Stop* then
Output (" I is an n-fold weak BCI - commutative ideal of X ")
Else
(1.) Output (" I is not an n-fold weak BCI - commutative ideal of X ")
End If
End

Notes

Notes

Algorithm for Fuzzy N-Fold Weak BCI- Commutative Ideals of BCI-Algebra

Input(X : *BCI-algebra*, $*$: binary operation, A fuzzy subset of X);
Output(" A is a fuzzy n-fold weak BCI -commutative ideal of X or not");
Begin
Stop:=false;
$i := 1$;
While $i \le |X|$ and not (*Stop*) do
If $A(0) < A(x_i)$ then
Stop:=true;
End If
$j := 1$;
While $j \le |X|$ and not (*Stop*) do
$k := 1$;
While $k \le |X|$ and not (*Stop*) do
If $\left(x*\left(\left(y*\left(y*x\right)\right)\left(0*\left(0*\left(\left(x*y\right)*y\right)\right)\right)\right)\right) < \min\left(A\left(\left(x*y^n\right)*z\right), A\left(z\right)\right)$
then
Stop=true;
End If
Endwhile
Endwhile
Endwhile
If *Stop* then
Output (" A is not a fuzzy n-fold weak BC I–a commutative ideal of X ")
Else
Output (" A is a fuzzy n-fold weak BCI –commutative ideal of X ")
End If
End

VI. Conclusion and Future Research

In this paper we introduce new notions of (fuzzy) n-fold BCI-commutative ideals, and (fuzzy) n-fold weak BCI - commutative ideals in BCI - algebras ., Then we studied relationships between different type of n- fold BCI – commutative ideals and investigate several properties of foldness theory of BCI – commutative ideals in BCI -algebras. Finally, we construct some algorithms for studying foldness theory of BCI – commutative ideals in BCI -algebras.

In our future study of foldness ideals in BCK/BCI algebras, maybe the following topics should be considered :

(1) developing the properties of foldness of positive implicative ideals of BCK/BCI algebras.

(2) finding useful results on other structures of foldness theory of ideals of BCK/BCI algebras.

(3) constructing the related logical properties of such structures.

(4) one may also apply this concept to study some applications in many fields like decision making knowledge base systems, medical diagnosis, data analysis, and graph theory.

Notes

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