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Improved Average Penalty Cost (IAPC) Method to Obtain Initial basic Feasible Solution of Transportation Problem

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Abstract- Operational planning, scheduling and synchronization of all production activities are the key responsibilities of the management of a manufacturing plant. Transport modeling is one such activity which is directly involved in the production cost. Therefore, it is necessary for the management of the plant to design the transportation process in such way so that the total production cost is minimized, subject to the constraint that cannot be compromised. In the solution procedure of these transportation problems, an initial basic feasible solution (IBFS) is always require to reach at the optimal solution. In this study, a new algorithm is developed to find IBFS. The result of the proposed method is compared with more classical method naming Vogel's Approximation Method (VAM) and cost cell based method named Least Cost Method (LCM). Here the number of numerical problems is established and found in 58.3% cases the proposed method provides optimal where the rest of the cases it offers very near to optimal solution. For finding the degree of effectiveness of proposed method a study is carried out and simulation results show that Improved Average Penalty Cost (IAPC) yields better IBFS than VAM and LCM.

Keywords: transportation table (TT), IBFS, VAM, LCM.

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Strictly as per the compliance and regulations of:

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Abstract- Operational planning, scheduling and synchronization of all production activities are the key responsibilities of the management of a manufacturing plant. Transport modeling is one such activity which is directly involved in the production cost. Therefore, it is necessary for the management of the plant to design the transportation process in such way so that the total production cost is minimized, subject to the constraint that cannot be compromised. In the solution procedure of these transportation problems, an initial basic feasible solution (IBFS) is always require to reach at the optimal solution. In this study, a new algorithm is developed to find IBFS. The result of the proposed method is compared with more classical method naming Vogel's Approximation Method (VAM) and cost cell based method named Least Cost Method (LCM). Here the number of numerical problems is established and found in 58.3% cases the proposed method provides optimal where the rest of the cases it offers very near to optimal solution. For finding the degree of effectiveness of proposed method a study is carried out and simulation results show that Improved Average Penalty Cost (IAPC) yields better IBFS than VAM and LCM.

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I. Introduction

With each passing year, ecommerce business transactions are reaching new heights of success. With the entry of large ecommerce marketplace players into the logistics industry, this supply chain management business has become more competitive than conventional logistics service providers. In this competitive global market, each and every company must have a very good planning to deliver their product to the customer in the right place at right time. This type of planning is known as a transportation networking in which the main objective is to decide how to shift goods from various sending locations (known as origins) to various receiving locations (known as destinations) with minimal costs.

In The Mathematical Method of Production Planning and Organization (1939), Kantorovich [1] showed that all problems of economic allocation can be seen as maximizing a function subject to constraints. F.L. Hitchcock [2] in 1941 formally introduced the transportation problem by presenting a paper entitled 'The Distribution of a Product from Several Sources to Numerous Localities'. This presentation is considered as the origin, and first important contribution to the solution of transportation problems. Continuation of the improvement of transportation problems, Koopmans [3] in 1947 presented his historic paper titled 'Optimum Utilization of the Transportation Systems', which was based on his war time experience.

Ref

1. Kantorovich, L.V. (1939), The Mathematical Method of Production Planning and Organization. Management Science, 6, 363-422.

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G. B. Dantzig [4] in 1951 first introduced the logical solution procedure for the transportation problem. In the solution procedure of the transportation problem it is always required to find an initial basic feasible solution (IBFS) to obtain the optimal solution. It was again developed by Charnes et al. [5] in 1953 and referred as North West Corner Method (NWCM) in which the north-west-corner cost cell is considered at every stage of allocation. And then the next developed method is Least Cost Method (LCM) consists in allocating as much as possible in the lowest cost cell of the Transportation Table in making allocation in every stage. Reinfeld and Vogel [6] in 1958 developed a method known as Vogel's Approximation Method (VAM). Including the above mentioned methods, Row Minima Method (RMM) and Column Minima Method (CMM) are also considered as the well reputed methods for solving transportation problems which are discussed in most of the Operation Research books [7-12]. Among these methods, VAM is considered as the most efficient solution procedures for obtaining an initial basic feasible solution for the transportation problems.

Ulrich A. Wagener [13] in 1965 proposed a new procedure for the computation of a transport problem model which uses each column of the cost matrix. Kirca and Satir [14] in 1990, developed a heuristic to obtain an efficient IBFS to the transportation problems. This method is called Total Opportunity Cost Method (TOCM). The TOCM is formed by adding the row opportunity cost matrix (ROCM) and the column opportunity cost matrix (COCM) where, for each row in the initial transportation cost matrix, the ROCM is generated by subtracting the lowest cost in the row from the other cost elements in that row and, for each column in the initial transportation cost matrix, the COCM is generated by subtracting the lowest cost in the column from the other cost elements in that column. Kirca and Satir then essentially use the LCM with some tie-breaking rules on the TOCM to generate a feasible solution to the transportation problem. Mathirajan and Meenakshi [15] in 2004 analyzed some variants of VAM and extended TOCM using the VAM procedure. They coupled VAM with TOCM and achieved very efficient initial solutions. Kasana and Kumar [16] in 2005 proposed Extremum Difference Method (EDM) where they define the penalty as the differences of the highest and lowest unit transportation cost in each row and column and allocate as like as the VAM procedure. Koruko˘glu and Balli [17] in 2011 proposed an improved version of the well-known VAM by taking the total opportunity cost into account. They claimed through computational experiments that this improved VAM provided more efficient initial feasible solution to a large scale transportation problem. Rashid [18] in 2011 developed an effective approach for solving TPs by defining penalty as the differences of the highest and next to the highest cost in each row and column of a transportation table and allocate to the minimum cost cell corresponding to the highest penalty. This method is named as Highest Cost Difference Method (HCDM) for solving TPs. Khan A.R. [19] in 2011 presented a method by defining pointer cost as the difference of the highest and next to the highest cost in each row and column of a transportation table and allocate to the minimum cost cell corresponding to the highest three pointer cost. Again, Singh et al. [20] in 2012 modified the solution procedure of VAM using total opportunity cost and allocation costs.

Deshmukh [21] in 2012 proposed a new method called an innovative method (NMD) to obtain a better IBFS to the transportation problem. Sudhakar et al. [22] in 2012 proposed a new approach called "Zero Suffix Method (ZSM)" for obtaining a minimal total cost solution to the transportation problem. In this method, they proposed to obtain at least one zero in each row and each column of the transportation

Ref

5. Charnes A., Cooper W.W., Henderson A., 1953, 'An Introduction to Linear Programming', John Wiley & Sons, New York.

table by subtracting the least element of each row and then column from all the elements of the corresponding row and column. Get suffix value of each zero and assign the cell corresponding to the greatest suffix value. Delete the exhausted row / column to get the reduced table. Procedures are repeated for the reduced table until all the demands and supplies are exhausted. Islam M.A. et al. [23] in 2012, applied EDM on TOCM, and allocate to the minimum cost cell corresponding to the highest distribution indicator and again HCDM on TOCM for obtaining an IBFS. Md. AshrafulBabu et al. [24] in 2013, proposed a method for solving transportation problems, where first allocation was made in the lowest cost cell which appears along lowest demand/supply. They named the method "Lowest Allocation Method (LAM)". S. Aramuthakannanet al. [25] in 2013 proposed a new method to solve transportation problems in which the allocations are made basing on the minimum demand and supply. This method is known as Revised Distribution Method. Abdul Sattar Soomro et al. [26] in2014modified the VAM algorithm and proposed Minimum Transportation Cost Method (MTCM)by computing row penalty by the difference of two largest transportation costs and column penalty by the difference of two smallest costs. UtpalKanti Das et al. [27] in 2014 brought out a logical development in the solution procedure of VAM. Basically he proposed a different idea to calculation of penalty cost for improving VAM. Ahmed, M.M. et al. [28] in 2014 developed an algorithm for finding an IBFS for both the balanced and unbalanced TP. In this method the transportation matrix is transformed to a Modified Transportation Cost Matrix (MTCM). The MTCM is formed as the differences of the Row Modified Cost Matrix (RMCM) and the Column Modified Cost Matrix (CMCM) where, for each row in the initial transportation cost matrix, the RMCM is generated by subtracting each of the cost of the row from the largest cost of that row of the transportation table and, for each column in the initial transportation cost matrix, the CMCM is generated by subtracting each of the cost of the column from the largest cost of that column of the transportation table. Finally the penalty costs are defined as the differences of the highest and next to the highest cost in each row and column of the MTCM and allocate to the minimum cost cell corresponding to the highest penalty cost. Dr. Muwafaq Alkubaisi [29] in 2015 used the median cost as penalty instead of the difference of two smallest costs in a row and column and applied VAM algorithm in the rest of the procedure. Aminur Rahman Khan et al. [30] in 2015 used TOCM to define the pointer cost as the sum of all entries in the respective row or column of the TOCM and then allocate to the minimum cost cell corresponding to the highest pointer cost. Hossain Md. M. et al.[31] have used TOCM and defined the penalty cost as the difference between the highest and the lowest cell in the respective row and column of the TOCM. Then allocate to the minimum cost cell corresponding to the highest penalty cost where computing penalty cost for each and every allocation is avoided to ease up the computational complication. Z.A.M.S. Juman, M.A. Hoque [32] in 2015, proposed a better efficient heuristic solution technique by JHM (Juman & Hoque Method) to obtain a better IBFS for the TPs. Moreover, Uddin, M.S. et al. [33] in 2016 proposed a method to solve the transportation problems named as Improved Least Cost Mehtod (iLCM). This improvement is basically done by bringing changes in the existing solution procedure of the classical LCM. Using the TOCM, Azad and Hossain [34] in 2017 presented a method where penalties are the average of the row opportunity cost (row penalty) and the average of the column opportunity cost (column penalty). J. Ravi et al. [35] in 2019 used the result of difference from standard deviation (DFSD) method as the smallest unit cost element in the row/column (cell) from the immediate next smallest unit cost element in the same row/column is determining a

Ref

23. Islam, M.A., Haque, M.H., Uddin, M.S., 2012, '*Extremum Difference Formula on Total Opportunity Cost: A Transportation Cost Minimization Technique*', Prime University Journal of Multidisciplinary Quest, 6(1), 125-130.

penalty measure for the target row/column. Recently, Hossain Md. M. et al. [36] in 2020, a computationally easier solution procedure for TP is proposed which is known as 'Least Cost Mean Method (LCMM)'. They presented this LCMM by defining penalty cost as the average of the lowest cell cost and the next lowest cell cost for each row and column.

II. Formulation for a Transportation Problem

Transportation problem is a distribution type problem; to model this type of problems we use the following notations:

m Total number of sources/origins

n Total number of destinations

S_i Amount of supply at source i

d_j Amount of demand at destination j

c_{ij} Unit transportation cost from source i to destination j

x_{ij} Amount to be shipped from source i to destination j

Network representation of a transportation problem, is represented in Figure-2.1

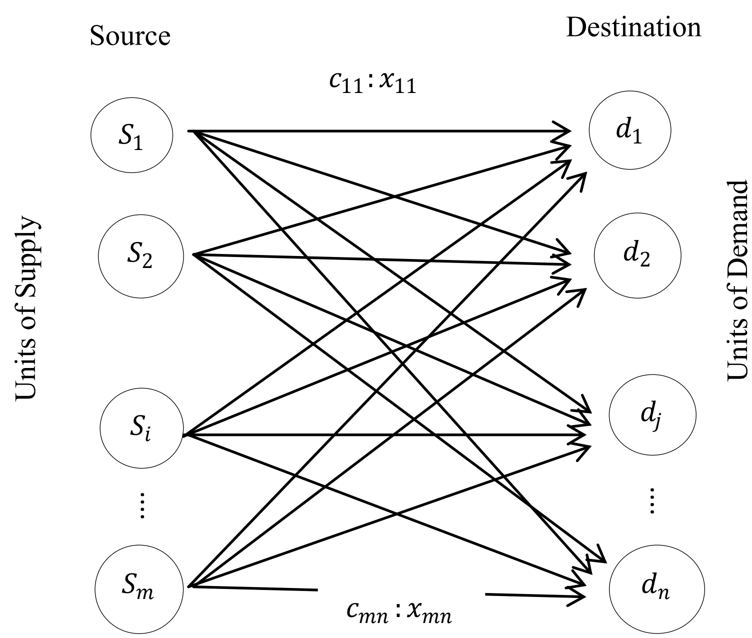

Figure 2.1: Network Diagram for transportation problem

The objective of the model is to determine the unknowns' x_{ij} that will minimize the total transportation cost while satisfying the supply and demand restrictions. Basing on this objective the Hitchcock-Koopman's transportation problem is mathematically formulated as:

Ref

36. Hossain, Md. M; Ahmed, M.M., 2020, '*A Comparative Study of Initial Basic Feasible Solution by a Least Cost Mean Method (LCMM) of Transportation Problem*', American Journal of Operations Research, 10(4), 122-131. DOI: 10.4236/ajor.2020.104008.

Notes

Minimize: $$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$ (Objective function)

Subjectto: $$\sum_{j=1}^{n} x_{ij} \le S_i \; ; i = 1,2,......,m$$ (Capacity constraints)

$$\sum_{i=1}^{m} x_{ij} \ge d_j \; ; \; j = 1,2,.......,n$$ (Requirement constraints)

$$x_{ij} \ge 0 \text{, for all } i \text{ and } j$$ (Non-negative condition)

There are two types of transportation problems, in a case where the supply of goods available for shipping at the origins is equal to the demand for goods at the destinations; the transportation problem is called balanced. In a case where the quantities are different, the problem is unbalanced. When a transportation problem is unbalanced, a dummy variable is used to even out demand and supply. A dummy variable is simply a fictional warehouse or store.

III. Algorithm

Step 1: Formulate the problem mathematically and if the problem is unbalance, balance the given Transportation Problem.

Step 2: Subtract the smallest cost (c_{ik} where $k = 1,2,.......n$) from each of the cost along the first row ($c_{i1}, c_{i2},.......c_{in}$, where $i = 1,2,........m$) of the TT and write those on the top right corner of the corresponding cost. Similar operation is applicable for rest of the rows.

Step 3: Applying the same process on each of the column and write the result on the right bottom corner of the corresponding cost.

Step 4: Magnitude of the subtraction of top and bottom element is put at the left bottom corner of the corresponding cost cell.

Step 5: Find the average of the left bottom elements of each row and put it at right side of the corresponding row as a row penalties. Similarly average column penalties place at the below of the corresponding column.

Step 6: Choose the highest average penalty costs and observe the row or column along which it appears. If a tie occurs in the highest average penalty, take the row/column along which lowest-cost appears. In case of tie for lowest cost cells, select the cell where maximum allocation can be put. If maximum allocation is in tie situation then select the cell for which sum of demand and supply is maximum in the TT. When the sum of demand and supply are same then choose any one of them.

Step 7: Place the first allocation then adjust the supply and demand requirements in the respective row and column. If the first allocation equals the demand, cross out the column. Now fulfill the row allocation along the basic cost cell by making the allocation(s) in the successive smallest cost cell/cells. Consider that the row is exhausted at some cell (i, j) with the allocation x_{ij}. Now follow the same procedure to fulfill the allocation along j-th column and continue the process until all the rim condition are satisfied. The process will reverse if first allocation equals the supply.

Step 8: If the allocation equals both the demand and the supply, cross out both the row and column. Find the next smallest cost cell along the crossed out row and column. Assign zero in that cell cost. Consider that the zero allocation x_{pq} is made in the cell (p, q) which is along the exhausted row. Now fulfill the allocation along q-th column following the procedure described in Step 7. In case of exhaust column, process will reverse.

Notes

Step 9: Finally calculate the total transportation cost from the cost table. This calculation is the sum of the product of cost and corresponding allocated value of the cost table.

IV. Numerical Examples of Transportation Problem

We consider the following numerical problems to solve by using the proposed and other methods.

Table 4.1: Numerical examples of balanced transportation problems

Problem Number	Type of the Problem	Data of the problems
P-1	5x6	$[c_{ij}]_{5x6}$=[5 3 7 3 8 5; 5 6 12 5 7 11; 2 8 3 4 8 2; 9 6 10 5 10 9; 5 3 7 3 8 5] $[s_i]_{5x1}$=[3, 4, 2, 8, 3] $[d_j]_{1x6}$=[3, 4, 6, 2, 1, 4]
P-2	3x4	$[c_{ij}]_{3x4}$=[9 8 5 7; 4 6 8 7; 5 8 9 5] $[s_i]_{3x1}$=[12, 14, 16] $[d_j]_{1x4}$=[8, 18, 13, 3]
P-3	3x5	$[c_{ij}]_{3x5}$=[5 7 10 5 3; 8 6 9 12 14; 10 9 8 10 15] $[s_i]_{3x1}$=[5, 10, 10] $[d_j]_{1x5}$=[3, 3, 10, 5, 4]
P-4	4x3	$[c_{ij}]_{4x3}$=[2 7 4; 3 3 1; 5 4 7; 1 6 2] $[s_i]_{4x1}$=[5, 8, 7, 14] $[d_j]_{1x3}$=[7, 9, 18]
P-5	3x4	$[c_{ij}]_{3x4}$=[10 2 20 11; 12 7 9 20; 4 14 16 18] $[s_i]_{3x1}$=[15, 25, 10] $[d_j]_{1x4}$=[5, 15, 15, 15]
P-6	3x4	$[c_{ij}]_{3x4}$=[4 6 8 8; 6 8 6 7; 5 7 6 8] $[s_i]_{3x1}$=[40, 60, 50] $[d_j]_{1x4}$=[20, 30, 50, 50]
P-7	3x3	$[c_{ij}]_{3x3}$=[4 3 5; 6 5 4 ; 8 10 7] $[s_i]_{3x1}$=[9, 8, 10] $[d_j]_{1x3}$=[7, 12, 8]
P-8	3x4	$[c_{ij}]_{3x4}$=[19 30 50 12; 70 30 40 60; 40 10 60 20] $[s_i]_{3x1}$=[7, 10, 18] $[d_j]_{1x4}$=[5, 8, 7, 15]
P-9	6x6	$[c_{ij}]_{6x6}$=[12 4 13 18 9 2; 9 16 10 7 15 11; 4 9 10 8 9 7; 9 3 12 6 4 5; 7 11 5 18 2 7; 16 8 4 5 1 10] $[s_i]_{6x1}$=[120, 80, 50, 90, 100, 60] $[d_j]_{1x6}$=[75, 85, 140, 40, 95, 65]
P-10	3x4	$[c_{ij}]_{3x4}$=[50 60 100 50; 80 40 70 50; 90 70 30 50] $[s_i]_{3x1}$=[20, 38, 16] $[d_j]_{1x4}$=[10, 18, 22, 24]
P-11	5x4	$[c_{ij}]_{5x4}$ = [10 20 5 7; 13 9 12 8; 4 15 7 9; 14 7 1 1; 3 12 5 19] $[s_i]_{5x1}$ = [200, 300, 200, 400, 400] $[d_j]_{1x4}$ = [500, 600, 200, 200]
P-12	3x4	$[c_{ij}]_{3x4}$=[13 18 30 8; 55 20 25 40; 30 6 50 10] $[s_i]_{3x1}$=[8, 10, 11] $[d_j]_{1x4}$=[4, 7, 6, 12]

Notes

V. Solution of a Problem with Illustration

Algorithm becomes more clear to the readers if goes through the illustrative solution of the related problems. Consider that the transportation problem is formulated and shown in Table-4.1 as an example 1.

Table 5.1: Transportation Problem of Example 1

Factories	Showrooms						Supply
	D_1	D_2	D_3	D_4	D_5	D_6	(a_i)
W_1	5	3	7	3	8	5	3
W_2	5	6	12	5	7	11	4
W_3	2	8	3	4	8	2	2
W_4	9	6	10	5	10	9	8
W_5	5	3	7	3	8	5	3
Demand (b_j)	3	4	6	2	1	4	

Before selecting the first cost cell as an allocation, we need to form a modified transportation table. In the 1st row of the given transportation cost matrix, 3 is the smallest cost. Subtract it from each of the cost along the first row [i.e.$(5-3)=2,\ (3-3)=0,\ (7-3)=4,\ (3-3)=0,\ (8-3)=5,\ (5-3)=2$]and put those results at the top right on the corresponding cost cell. Similarly, we subtract 5, 2, 5 and 3 from every element of 2nd, 3rd, 4th and 5throw respectively and place all the differences on the top right of the particular elements.

In the same way, we subtract 2, 3, 3, 3, 7 and 2 from each element of the 1st, 2nd, 3rd, 4th, 5th and 6th column respectively and place the result at the bottom left of the corresponding elements.

Forming modified transportation table (Table-5.2) whose elements remain same and place the magnitude of the subtraction of top and bottom element at the left bottom corner of the corresponding cost cell.

Table 5.2: Modified Transportation Table of Example 1

Factories	Showrooms						Supply
	D_1	D_2	D_3	D_4	D_5	D_6	(a_i)
W_1	${}_{1}5^{2}_{3}$	${}_{0}3^{0}_{0}$	${}_{0}7^{4}_{4}$	${}_{0}3^{0}_{0}$	${}_{4}8^{5}_{1}$	${}_{1}5^{2}_{3}$	3
W_2	${}_{3}5^{0}_{3}$	${}_{2}6^{1}_{3}$	${}_{2}12^{7}_{9}$	${}_{2}5^{0}_{2}$	${}_{2}7^{2}_{0}$	${}_{3}11^{6}_{9}$	4
W_3	${}_{0}2^{0}_{0}$	${}_{1}8^{6}_{5}$	${}_{1}3^{1}_{0}$	${}_{1}4^{2}_{1}$	${}_{5}8^{6}_{1}$	${}_{0}2^{0}_{0}$	2
W_4	${}_{3}9^{4}_{7}$	${}_{2}6^{1}_{3}$	${}_{2}10^{5}_{7}$	${}_{2}5^{0}_{2}$	${}_{2}10^{5}_{3}$	${}_{3}9^{4}_{7}$	8
W_5	${}_{1}5^{2}_{3}$	${}_{0}3^{0}_{0}$	${}_{0}7^{4}_{4}$	${}_{0}3^{0}_{0}$	${}_{4}8^{5}_{1}$	${}_{1}5^{2}_{3}$	3
Demand (b_j)	3	4	6	2	1	4	

Now determine the penalty cost for each row of the modified transportation table by taking the average of all left bottom entries in the respective row and place them along the right side of each corresponding rows.[i.e.$\frac{(1+0+0+0+4+1)}{6}=1,\ \frac{(3+2+2+2+2+3)}{6}=2.3,\ \frac{(0+1+1+1+5+0)}{6}=1.3,\ \frac{(3+2+2+2+2+3)}{6}=2.3,\ \frac{(1+0+0+0+4+1)}{6}=1$]

Do the same calculation for each column and place them in the bottom of the modified transportation table below the corresponding columns.

Table 5.3: Initial Basic Feasible Solution Using IAPC

Factories	Showrooms						Supply (a_i)	Row Penalty
	D_1	D_2	D_3	D_4	D_5	D_6		
W_1	${}_{1}5_{3}^{2}$	3 ${}_{0}3_{0}^{0}$	${}_{0}7_{4}^{4}$	0 ${}_{0}3_{0}^{0}$	${}_{4}8_{1}^{5}$	${}_{1}5_{3}^{2}$	3	(1)
W_2	3 ${}_{3}5_{3}^{0}$	${}_{2}6_{3}^{1}$	${}_{2}12_{9}^{7}$	${}_{2}5_{2}^{0}$	1 ${}_{2}7_{0}^{2}$	${}_{3}11_{9}^{6}$	4/3	(2.3)
W_3	0 ${}_{0}2_{0}^{0}$	${}_{1}8_{5}^{6}$	${}_{1}3_{0}^{1}$	${}_{1}4_{1}^{2}$	${}_{5}8_{1}^{6}$	2 ${}_{0}2_{0}^{0}$	2	(1.3)
W_4	${}_{3}9_{7}^{4}$	${}_{2}6_{3}^{1}$	6 ${}_{2}10_{7}^{5}$	2 ${}_{2}5_{2}^{0}$	${}_{2}10_{3}^{5}$	${}_{3}9_{7}^{4}$	8	(2.3)
W_5	${}_{1}5_{3}^{2}$	1 ${}_{0}3_{0}^{0}$	${}_{0}7_{4}^{4}$	${}_{0}3_{0}^{0}$	${}_{4}8_{1}^{5}$	2 ${}_{1}5_{3}^{2}$	3	(1)
Demand (b_j)	3	4	6	2	1	4/2		
Column Penalty	(1.6)	(1)	(1)	(1)	(3.4)	(1.6)		

a) Selecting first cost cell as a first allocation

From Table-3.3 it is observed that maximum penalty is (3.4) along with the D_5column/showroom and minimum transportation cost corresponding to this column is 7 in the cell (W_2, D_5). So we allocate 1 unit (min of 1 and 4) to the cell (W_2, D_5). Since the demand of the D_5 column is satisfied, we cross out this column and adjust the supply of the W_2 row/factory.

b) Second Allocation

Now according to Step 7 of our proposed algorithm, we need to fulfill the allocation of W_2 row by selecting the smallest cost cell along it. Here both (W_2, D_1)and (W_2, D_4) having the same cost (5). Since maximum allocation can be put in (W_2, D_1), so we allocate 3 unit in it.

c) Third Allocation

With the second allocation, both W_2 row and D_1 column is satisfied and we delete both of them. Here comes Step 8 to find the next allocation cell. In this case we find the smallest cost (2) along the both crossed out row and column. Assign zero (0) in the cost cell (W_3, D_1).

d) Fourth Allocation

Zero allocation is made in(W_3, D_1) which is along the exhausted columnD_1, therefore we fulfill the fourth allocation along W_3 row by choosing the smallest cost (2) and allocate 2 units (min of 2 and 4)in the cost cell(W_3, D_6). Since the supply of the W_3 row is satisfied, we cross out this row and adjust the demand of the D_6 column.

The above mentioned procedure is repeated for rest of the allocations. Thus we get the Fifth Allocation is 2 in the cell (W_5, D_6), Sixth Allocation is 1 in the cell (W_5, D_2), Seventh Allocation is 3 in the cell (W_1, D_2), Eighth Allocation is 0 in the cell (W_1, D_4), Ninth Allocation is 2 in the cell (W_4, D_4) and the Final Allocation is 6 in the cell (W_4, D_3).

Total transportation cost $z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$

$$z = 3\times3+0\times3+3\times5+1\times7+0\times2+2\times2+6\times10+2\times5+1\times3+2\times5$$

$$= 9+0+15+7+0+4+60+10+3+10$$

$$= 118$$

Notes

VI. Performance Evaluation of the Proposed Method

IBFS for the problems obtain by the well reputed methods and proposed method is tabulated below:

Table 6.1: IBFS of the problems

Method	Number of the problems											
	P-1	P-2	P-3	P-4	P-5	P-6	P-7	P-8	P-9	P-10	P-11	P-12
NWCM	129	320	234	102	520	980	150	975	4285	4160	16500	484
RMM	126	248	183	80	505	960	145	1064	2290	4120	9200	589
CMM	132	248	215	111	475	960	150	995	2915	3320	8900	476
LCM	134	248	191	83	475	960	145	894	2455	3500	10200	516
VAM	116	248	187	80	475	930	150	859	2310	3320	9800	476
IAPC	118	240	183	76	460	920	144	799	2290	3320	9200	412
Optimal Solution	116	240	183	76	435	920	139	799	2170	3320	8800	412

Now we calculate the Percentage of Correctness (PoC) of the IBFS obtained by various methods, just to justify the performance of the proposed method. This PoC do indicate the closeness between the obtained IBFS and the optimal solution (OS).To calculate this percentage, we have used the formula,

PoC= $100 - \{(\text{IBFS} - \text{OS}) \div \text{OS}\} \times 100$. This PoC is shown in Table-6.2

Table 6.2: PoC of IBFS

Method	Number of the problems											
	P-1	P-2	P-3	P-4	P-5	P-6	P-7	P-8	P-9	P-10	P-11	P-12
NWCM	88.79	66.67	72.13	65.79	80.46	93.48	92.09	77.97	02.53	74.70	12.50	82.52
RMM	91.38	96.67	100	94.74	83.91	95.65	95.68	66.83	94.47	75.90	95.45	57.04
CMM	86.21	96.67	82.51	53.95	90.80	95.65	92.09	75.47	65.67	100	98.86	84.47
LCM	84.48	96.67	95.63	90.79	90.80	95.65	95.68	88.11	86.87	94.58	84.09	74.76
VAM	100	96.67	97.81	94.74	90.80	98.91	92.09	92.49	93.55	100	88.64	84.47
IAPC	98.27	100	100	100	94.25	100	96.40	100	94.47	100	95.45	100
Optimal Solution	100	100	100	100	100	100	100	100	100	100	100	100

The calculation of PoC in the Table-6.2gives clear reflection that the IBFS obtained by IAPC are very proximate to optimal solution. The proximity between optimal solution and IBFS obtained by all methods discussed here is shown in the following Graph-1:

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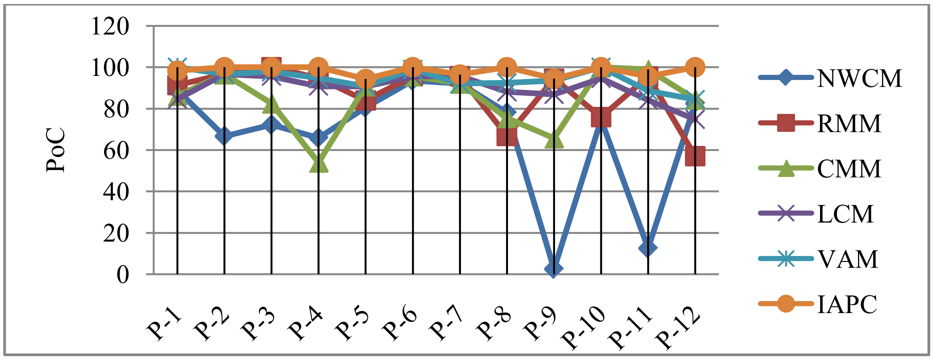


Graph 1: Graphical representation of PoC

From the data of Table-6.2, it is also observed that the performances of the proposed method are better than all other methods for solving the balanced transportation problems. In this study we also calculate the average of PoC (APoC) which is shown in the Table-6.3.

Table 6.3: Average of PoC

	NWCM	RMM	CMM	LCM	VAM	IAPC
Limit	02.53 to 93.48	66.83 to 100	53.95 to 100	74.76 to 96.67	84.47 to 100	94.25 to 100
APoC	67.47	87.31	85.19	89.84	94.18	98.24

VII. Degree of Effectiveness

The performance of IAPC is measured using percentage decrease in the total cost associated with an IBFS of a numerical problem obtained by IAPC over the corresponding one obtained by LCM and VAM. Also the performance of IAPC is measured by comparing the percentage increases of each of the total costs associated with the IBFS (obtained by these methods) from the optimal solution. These performance measures are shown in table-6.1. Performance measure of IAPC over LCM and VAM for 12 numerical problems has chosen from the literature.

Table 7.1: Performance Measure of IAPC

Problem Chosen from	Initial Cost with an IBFS by			% decrease in IBFS by IAPC over LCM and VAM		Optimal Solution	% increase from the Optimal Solution		
	LCM	VAM	IAPC	LCM	VAM		LCM	VAM	IAPC
Wagener [13]	134	116	118	11.94	-1.72	116	15.52	0.00	1.72
Azad and Hossain [34]	248	248	240	3.22	3.22	240	3.33	3.33	0.00
Ray and Hossain [12]	191	187	183	4.19	2.14	183	4.37	2.18	0.00
Ahmed et al [28]	83	80	76	8.43	5.00	76	9.21	5.26	0.00
Seethalakshmy et at [37]	475	475	460	3.16	3.16	435	9.19	9.19	5.75
Juman and Hoque [32]	960	930	920	4.17	1.07	920	4.35	1.09	0.00
Azad and Hossain [34]	145	150	144	0.69	4.00	139	4.32	7.91	3.60
Babu et al [24]	894	859	799	10.63	6.98	799	11.89	7.51	0.00
Khan et al [30]	2455	2310	2290	6.72	0.86	2170	13.13	6.45	5.53
Hossain and Ahmed [36]	3500	3320	3320	5.14	0.00	3320	5.42	0.00	0.00
Alkubaisi [29]	10200	9800	9200	9.80	6.12	8800	15.91	11.36	4.54
Sudhakar et al [22]	516	476	412	20.15	13.44	412	25.24	15.53	0.00

Notes

It can easily be observed from Table 6.1 that the IAPC leads to better IBFS over VAM for 10 out of 12 problems considered. For the remaining 2 problems, in one problem both IAPC and VAM lead to an IBFS with the same total cost and in the other problem VAM leads better IBFS than IAPC. Also IAPC leads to better IBFS over LCM for 12 out of 12 problems considered. Besides, from the results in Table-6.1 it can easily be observed that IAPC led to the optimal solution in 7 out of 12 considered problems, whereas each of LCM and VAM led to the optimal solutions to 0 and 2 out of 12 respectively. In the remaining 5 problems, the percentage increases in the total cost from the optimal solution in case of IAPC is the least.

Comparative efficiencies of IAPC over LCM and VAM in respect of percentage decrease in the total cost for an IBFS and percentage increase of the total cost for an IBFS from the optimal solution, for each category of the considered problems are depicted by bar-charts in Fig. 1-2.

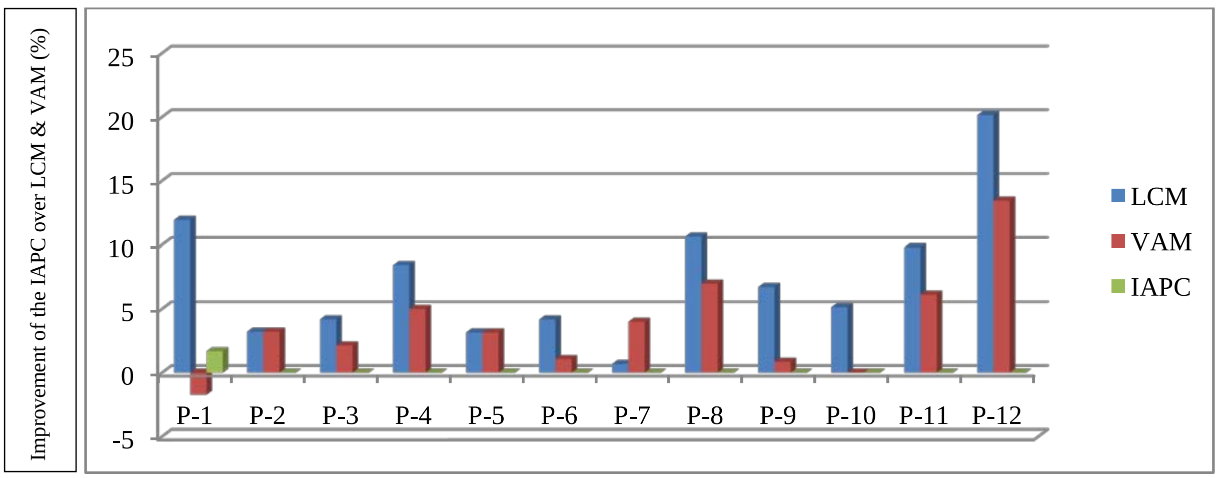


Fig. 1: Improvement of the IAPC over LCM & VAM (%)

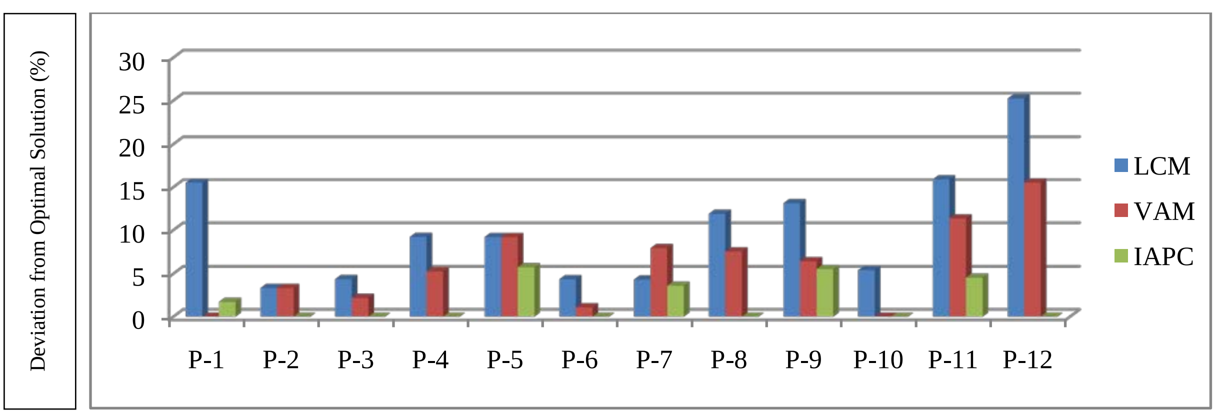
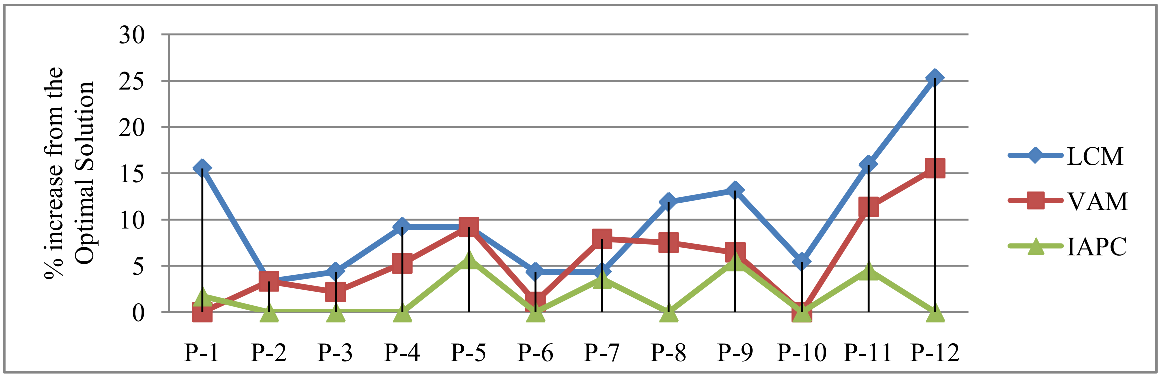


Fig. 2: Deviation from Optimal Solution (%)

According to the simulation results from Table-6.1 it is also perceived that in 41.67% cases IAPC provides increased IBFS than the optimal solution. The percentage increases in the total cost from the optimal solution in case of LCM and VAM is 100% and 83.33% cases respectively. The following Graph-2 represents the percentage increases of IBFS from optimal solution.

Notes



Graph-2: Graphical representation of percentage increases of IBFS

VIII. Conclusion

TP is an optimal path selecting procedure to transport goods by spending minimum transportation cost which is sometimes to blame for a company's inability to properly serve customers. To improve the company's position in the market, manager needs to build uptransportation networks in order to save transportation cost and time so that the market prices of daily commodities remain affordable. In this study developed IAPC perform promisingly in finding IBFS to the TP in comparison with other two best available methods VAM and LCM. The performance evaluation of IAPC has been carried out to justify its efficiency by solving twelve numerical examples chosen from the literature.IAPC provides better feasible solutions than existing method which are very close to optimal solution and sometimes it is equal to optimal solution. But it is not grantee that all time IAPC provides least feasible solution but most of the times it gives better approach.

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