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Mixed Value Problem for a One-Dimensional Nonlinear Nonstationary Twelve Moment Boltzmann's System Equations with the Maxwell-Auzhan Boundary Conditions

By Sh. Akimzhanova
Satbayev University

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Mixed Value Problem for a One-Dimensional Nonlinear Nonstationary Twelve Moment Boltzmann's System Equations with the Maxwell-Auzhan Boundary Conditions

Sh. Akimzhanova

Abstract- **В работе доказано существование и единственность решения начально-краевой задачи для нестационарной нелинейной одномерной двенадцати моментной системы уравнений Больцмана при граничных условиях Максвелла-Аужана в пространстве функций, непрерывных по времени и суммируемых в квадрате по пространственной переменной (рассмотрен случай чисто зеркального отражения от границы).**

Ключевые слова: однородное одномерное уравнение больцмана, система моментных уравнений больцмана, микроскопическое граничное условие максвелла, макроскопическое граничное условие.

Abstract- It is proved existence and uniqueness of solution of the problem with initial and boundary conditions of Maxwell-Auzhan (we consider pure specular reflection from the boundary) for the nonstationary nonlinear one-dimensional Boltzmann's twelve-moment system equations in space of functions continuous in time and summable in square by spatial variable.

Keywords: Boltzmann's homogeneous one-dimensional equation, Boltzmann's moment system equations, Microscopic Maxwell boundary condition, Macroscopic, boundary conditions.

I. Introduction

Many problems of the rarefied gas dynamics require the solution of one or another problem for the Boltzmann equation.The prediction of aerodynamic characteristics of aircraft at very high speeds and at high altitudes is an important problem in aerospace engineering. It is impossible to determine the aerodynamic characteristics of aircraft at high altitudes. The interaction of gas molecules with the surfaces of real bodies has been little studied. The aerodynamic characteristics of aircraft at very high speeds and at high altitudes can be determined by the methods of the theory of a rarefied gas [1]. To analyze the aerodynamic characteristics of aircraft in the transient regime, the complete integro-differential Boltzmann equation is used with appropriate boundary conditions. The determination of the boundary conditions on the surfaces that are streamlined with a rarefied gas is one of the most important questions in the kinetic theory of gases. In high-altitude aerodynamics, the interaction of gas with the surface of a streamlined body plays an important role [2]. The aerothermodynamic characteristics of bodies to the gas flow are determined by the transfer of momentum and energy to the surface of the body, that is, the connection between the velocities and the energies of the molecules incident on the surface and the molecules reflected from it,

Ref

1. Kogan M.N. Dynamic of rarefied gas. Moscow, Nauka, 1967, 440p.

Author: Satbayev University, 22a Satpaev str., 050013, Almaty, Republic of Kazakhstan. e-mail: shinar_a@mail.ru

which is the essence of the kinetic boundary conditions on the surface. Maxwell's boundary condition for solving specific problems more accurately describes the interaction of gas molecules with the surface. One of the approximate methods for solving the initial-boundary value problem for the Boltzmann equation is the moment method. With the help of the moment method, it is possible to determine the aerodynamic characteristics of aircraft, such as atmospheric parameters, flight speed, geometric parameters, and the like. We note that in work [3] two new models of boundary conditions were proposed: diffusive-moment and mirror-moment, generalizing the known boundary conditions of Cherchinyani, and in [4] aerodynamic characteristics of space vehicles were studied by the method of direct static modeling (Monte Carlo method) and various model of the interaction of gas molecules with the surface and their effect on aerodynamic characteristic. Moment methods are different from each other as sets of various systems of basic functions. For example, Grad in works [5] and [6] received moment system through decomposition of particles distribution function by Hermitte polynomials near the local Maxwell's distributions. Grad used Cartesian coordinates of velocities and Grad's moment system contained unknown hydrodynamic characteristics as density, temperature, average speed, etc. In [7] obtained the moment system which differs from the system of equations of Grad. In this case was used the spherical coordinates of velocity and distribution function is decomposed into a series by the eigenfunctions of the linearized collision operator [1], [8], which is the product of the Sonin polynomials and spherical functions. The expansion coefficients, the moments of the distribution function are defined differently from Grad. The resulting system of equations corresponding to the partial sum of the series, which was call the Boltzmann's moment system equations, is nonlinear hyperbolic system relative to the moments of the particles distribution function. Differential part of the resulting system is linear and quadratic nonlinearity is shaped as moments of the distribution function. Quadratic forms – the moments of the nonlinear collision integrals – are calculated in [9] and are expressed in terms of coefficients Talmi [10] and Klebsh-Gordon [11].

In [12] - [13], moment systems for the spatially homogeneous Boltzmann equation and the conditions for the representability of the solution of the spatially homogeneous Boltzmann equation in the form of the Poincare series were obtained. The method proposed in [12] (application of the Fourier transform with respect to the velocity variable in the isotropic case) greatly simplified the collision integral and, hence, the calculation of the moments from the collision integral. In [13] the result of [12] is generalized for the case of anisotropic scattering.

In work [14] presented a systematic nonperturbative derivation of a hierarchy of closed systems of moment equations corresponding to any classical theory. This paper is fundamental work where closed systems of moment equations describe a transition regime.

The Boltzmann equation is equivalent to an infinite system of differential equations relative to the moments of the particle distribution function in the complete system of eigenfunctions of linearized operator. As a rule, limited study to finite moment system equations as solving the infinite system of equations does not seem to be possible.

Finite system of moment equations for a specific task with a certain degree of accuracy replaces the Boltzmann equation. It's necessary, also roughly, to replace the boundary conditions for the particle distribution function by a number of macroscopic conditions for the moments, i.e. there arises the problem of boundary conditions for a

Ref

3. Latyshev A., Yushkanov A. Moment Boundary Conditions in Rarefied Gas Slip-Flow Problems// Fluid Dynamics, March, 2004, No2, p.p.193-208.

finite system of equations that approximate the microscopic boundary conditions for the Boltzmann equation. The question of boundary conditions for a finite system of moment equations can be divided into two parts: how many conditions must be imposed and how they should be prepared. From microscopic boundary conditions for the Boltzmann equation there can be obtained an infinite set of boundary conditions for each type of decomposition. However, the number of boundary conditions is determined not by the number of moment equations, i.e. it is impossible, for example, take as much boundary conditions as equations, although the number of moment equations affects the number of boundary conditions. In addition, the boundary conditions must be consistent with the moment equations and the resulting problem must be correct.

Grad in [5] described the construction of an infinite sequence of boundary conditions without the consent of the order of approximation for the decomposition of the boundary conditions and the expansion of the Boltzmann equation. Boundary conditions, even a one-dimensional Grad's moment system equations is a very difficult task, because Grad's moment system of equations is a hyperbolic system and this system of equations contains as coefficients unknown parameters like density, temperature, average speed, etc. In such case the characteristic equation is also dependent on the unknown parameters and thus appears to be very difficult to formulate the boundary conditions for the moment system. In work [15] had been discussed the boundary conditions for the 13-moment Grad system.

In work [7] had been shown approximation of the homogeneous boundary condition for the particle distribution function and proved the correctness of the initial-boundary value problem for nonstationary nonlinear Boltzmann's moment system equations in a three-dimensional space. More precisely, was proved the existence of a unique generalized solution of the initial-boundary value problem for the Boltzmann's moment system equations in the space of functions that are continuous in time and square summablein the space of variables. In addition, the same study shows the approximation of the microscopic boundary conditions for the Boltzmann equation. The boundary condition can be formulated as follows: determine the mirrored half of the distribution function from the known half, corresponding to the falling particles. The boundary condition is specified as an integral relation between particles falling to the boundary and particles reflected from the boundary (assuming that we know the probability of an event that a particle falling to the boundary with velocity v_i reflects with velocity v_r).

In this article it is proved existence and uniqueness of solution of the problem with initial and boundary conditions of Maxwell-Auzhan (we consider pure specular reflection from the boundary) for the nonstationary nonlinear one-dimensional Boltzmann's twelve-moment system equations in space of functions continuous in time and summable in square by spatial variable.

Existence and uniqueness of the solutions of initial and boundary value problem for twelve-moment one-dimensional Boltzmann's system of equations with boundary conditions of Maxwell-Auzhan

In this section we prove existence and uniqueness of solutions of the initial and boundary value problem for twelve-moment one-dimensional Boltzmann's system of equations with boundary conditions of Maxwell-Auzhan in space of functionscontinuous in time and summable in square by spatial variable. Theorem of existence of global in time solution of the initial and boundary value problem for 3-dimensional nonlinear Boltzmann equation with boundary conditions of Maxwell proved in work [16].

Ref

5. G. Grad,Kinetic theory of rarefied gases.Comm. Pure Appl. Math, 2,331, 1949.

Statement of the problem: Find the solution of initial-boundary value problem for a homogeneous one-dimensional Boltzmann equation

$$\frac{\partial f}{\partial t}+|v|\cos\theta\frac{\partial f}{\partial x}=J(f,\ f),\ t\in(0,\ \mathrm{T}],\ x\in(-a,\ a),\ v\in R_3^v, \tag{1}$$

$$f\,|_{t=0}=f^0(x,v),\ (x,v)\in[-a,a]\times R_3^v, \tag{2}$$

$$f^+(t,x,v_1,v_2,v_3)=\beta f^-(t,x,v_1,v_2,-v_3)+(1-\beta)\eta\exp\left(-\frac{|v|^2}{2RT_0}\right),$$

$$v_3=|v|\cos\theta,(n,\ v)=(n,\ |v|\cos\theta)>0,\ \ x=-a\ \text{ or }\ x=a\,, \tag{3}$$

where $f\equiv f(t,x,v)$ is a particle distribution function in space of velocity and time; $f^0(x,v)$ is distribution of the particles at the initial time (fixed function); $J(f,\ f)\equiv\int[f(v')f(w')-f(v)f(w)]\sigma(\cos x)dwdv$ nonlinear collision operator, recorded for Maxwell molecules, n is external unit normal vector of the boundary. Condition (3) is a natural boundary condition for the Boltzmann equation, which makes it possible to determine the reflected half of the distribution function *f*, if we know the half corresponding to the falling particles. According to (3), some part of falling particles reflected specularly, and other particles are absorbed into the wall and emitted with the Maxwell distribution with corresponding wall temperature T_0.

Formula (3) refers to the case of a wall at rest; otherwise v must be replaced by $v-u_0$, u_0 being the velocity of the wall. β, T_0, u_0 may vary from point to point and with time [8].

For one-dimensional problems of the Eigen functions of linearized operator are [1], [8]:

$$g_{nl}(\alpha v)=\left(\frac{\sqrt{\pi}n!(2l+1)}{2\Gamma(n+l+3/2)}\right)^{1/2}\left(\frac{\alpha|v|}{\sqrt{2}}\right)^l S_n^{l+\frac{1}{2}}\left(\frac{\alpha^2|v|^2}{2}\right)P_l(\cos\theta),$$

$$2n+l=0,1,2,\ldots$$

where $S_n^{l+\frac{1}{2}}\left(\frac{\alpha^2|v|^2}{2}\right)$ is Sonin polynomials, $P_l(\cos\theta)$ is Legendre polynomials, Γ is Gamma function.

To find an approximate solution of the problem (1) -(3) we apply the Galerkin method. We define the approximate solution of one-dimensional problem (1)-(3) as follows:

$$f_5(t,x,v)=\sum_{2n+l=0}^{5}f_{nl}(t,x)g_{nl}(\alpha v), \tag{4}$$

Ref
16. S. Mischler, Kinetic equations with Maxwell boundary conditions. Annalesscientifique de l ENS 43, fascicule 5, 719-760 (2010).

$$\int_{R_3^v}\left(\frac{\partial f_5}{\partial t}+|v|\cos\theta\frac{\partial f_5}{\partial x}-J(f_5,\ f_5)\right)f_0(\alpha|v|)g_{nl}(\alpha v)dv=0,$$

$$2n+l=0,1,\dots,5,\qquad (t,x)\in(0,\mathrm{T}]\times(-a,a), \tag{5}$$

$$\int_{R_3^v}\left[f_5(0,x,v)-f_5^0(x,v)\right]f_0(\alpha|v|)g_{nl}(\alpha v)dv=0,$$

$$2n+l=0,1,\dots,5,\qquad x\in(-a,a), \tag{6}$$

$$\int_{(n,v)>0}(n,v)f_0(\alpha|v|)f_5^+(t,x,v)g_{n,2l}(\alpha v)dv-\beta\int_{(n,v)<0}(n,v)f_0(\alpha|v|)\,f_5^-(t,x,v)g_{n,2l}(\alpha v)dv-$$

$$-(1-\beta)\eta\int_{(n,v)<0}(n,v)f_0(\alpha|v|)\exp\left(-\frac{|v|^2}{2R\mathrm{T}_0}\right)g_{n,2l}(\alpha v)dv=0$$

$$2(n+l)=0,2,4,\qquad x=-a\ \text{ or } x=-a\,, \tag{7}$$

where $n=(0,0,1)$ with $x=a$ and $n=(0,0-1)$ with $x=-a$;

$f_0(\alpha|v|)$ is the global Maxwell distribution;

$$f_{nl}(t,x)=\int_{R_3^v}f_5(t,x,v)f_0(\alpha|v|)g_{nl}(\alpha v)dv,$$

$$f_5^0(x,v)=\sum_{2n+l=0}^{5}f_{nl}^0(x)g_{nl}(\alpha v)dv,$$

$$f_{nl}^0(x)=\int_{R_3^v}f_5^0(x,v)f_0(\alpha|v|)g_{nl}(\alpha v)dv. \tag{8}$$

In the general case, the approximation of the boundary condition (3) depends on the parity or oddness of the approximation of the Boltzmann's moment system of equations [17]. When approximating a microscopic boundary condition, we took into account the approximation of the Boltzmann equation by moment equations corresponding to the fifth approximation (the twelve moment system of equations).

Thus, the approximation orders for the expansion of the boundary condition and the expansion of the Boltzmann equation are consistent. Macroscopic conditions (7) were called Maxwell – Auzhan boundary conditions [17].

Boltzmann's system of moment equations (5), corresponding to the decomposition (4) can be written in expanded form

$$\frac{\partial f_{nl}}{\partial t}+\frac{1}{\alpha}\frac{\partial}{\partial x}\left[l\left(\sqrt{\frac{2\left(n+l+\frac{1}{2}\right)}{(2l-1)(2l+1)}}f_{n,l-1}-\sqrt{\frac{2(n+1)}{(2l-1)(2l+1)}}f_{n+1,l-1}\right)+\right.$$

$$\left.+(l+1)\left(\sqrt{\frac{2\left(n+l+\frac{3}{2}\right)}{(2l+1)(2l+3)}}f_{n,l+1}-\sqrt{\frac{2n}{(2l+1)(2l+3)}}f_{n-1,l+1}\right)\right]=I_{nl},$$

$$2n+l=0,\ 1,\ \dots,\ 5, \tag{9}$$

Ref

17. A. Sakabekov, Auzhani Y. Boundary conditions for the one dimensional nonlinear nonstationary Boltzmann's moment system equations//Journal of mathematical physics,55,123507,(2014).

where the moments of the collision integral can be expressed in terms of coefficients Talmi and Klebsh-Gordon as follows [6]

$$I_{nl} = \sum \langle N_3 L_3 n_3 l_3 : l \,|\, nl00 : l \rangle \langle N_3 L_3 n_3 l_3 : l \,|\, n_1 l_1 n_2 l_2 : l \rangle (l_1 0 l_2 0 / l0)(\sigma_{l_3} - \sigma_0) f_{n_1 l_1} f_{n_2 l_2},$$

$\langle N_3 L_3 n_3 l_3 : l \,|\, n_1 l_1 n_2 l_2 : l \rangle$ is generalized Talmi coefficients, $(l_1 0 l_2 0 / l0)$is Klebsh-Gordon coefficients.

If in (9)$2n + l$ is from 0 to 5, we get the system of equations, corresponding fifth approximation of Boltzmann's moment system equations or twelve-moment Boltzmann's system equations.

The initial and boundary value problem for twelve-moment Boltzmann's system equations with boundary conditions of Maxwell-Auzhan (7) can be written in vector-matrix form [18] (we consider pure specular reflection from the boundary $\beta = 1$):

$$\frac{\partial u}{\partial t} + A\frac{\partial w}{\partial x} = J_1(u,\ w)$$

$$\frac{\partial u}{\partial t} + A'\frac{\partial u}{\partial x} = J_2(u,\ w),\ t \in (0,\ \mathrm{T}],\ x \in (-a,\ a), \tag{10}$$

$$u\big|_{t=0} = u_0(x),\quad w\big|_{t=0} = w_0(x),\quad x \in (-a,\ a), \tag{11}$$

$$\left(Aw^+ - Bu^+\right)\Big|_{x=-a} = \left(Aw^- + Bu^-\right)\Big|_{x=-a},\ t \in [0,\ \mathrm{T}], \tag{12}$$

$$\left(Aw^+ + Bu^+\right)\Big|_{x=a} = \left(Aw^- - Bu^-\right)\Big|_{x=a},\ t \in [0,\ \mathrm{T}], \tag{13}$$

where A' is transpose matrix; $u_0(x) = \left(f_{00}^0(x),\ f_{02}^0(x),\ f_{10}^0(x),\ f_{04}^0(x),\ f_{12}^0(x),\ f_{20}^0(x)\right)'$, $w_0(x) = \left(f_{01}^0(x),\ f_{03}^0(x),\ f_{05}^0(x),\ f_{11}^0(x),\ f_{13}^0(x),\ f_{21}^0(x)\right)'$ are given initial vector functions,

$$u = \left(f_{00},\ f_{02},\ f_{04},\ f_{10},\ f_{12},\ f_{20}\right),\ w = \left(f_{01},\ f_{03},\ f_{05},\ f_{11},\ f_{13},\ f_{21}\right),$$

$$J_1(u,v) = \left(0,\ I_{04},\ 0,\ I_{12},\ I_{20}\right),\ J_2(u,v) = \left(0,\ I_{03},\ I_{05},\ I_{11},\ I_{13},\ I_{21}\right),$$

$$A = \frac{1}{\alpha}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{\sqrt{3}} & \frac{3}{\sqrt{5}} & 0 & -\frac{2\sqrt{2}}{\sqrt{15}} & 0 & 0 \\ 0 & \frac{4}{\sqrt{7}} & \frac{5}{3} & 0 & -\frac{4\sqrt{2}}{3\sqrt{7}} & 0 \\ -\sqrt{\frac{2}{3}} & 0 & 0 & \sqrt{\frac{5}{3}} & 0 & 0 \\ 0 & -\frac{3\sqrt{2}}{\sqrt{35}} & 0 & \frac{2\sqrt{7}}{\sqrt{15}} & \frac{9}{\sqrt{35}} & -\frac{4}{\sqrt{15}} \\ 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & \sqrt{\frac{7}{3}} \end{bmatrix}$$

Ref

6. G. Grad, Principle of the kinetic theory of gases. Handuch der Physik, Volume 12, Springer, Berlin, p.p. 205-294.

Notes

$$B=\frac{1}{\alpha\sqrt{\pi}}\begin{bmatrix} \sqrt{2} & \sqrt{\frac{2}{3}} & -\sqrt{\frac{2}{105}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{21}} & -\frac{1}{2\sqrt{15}} \\ \sqrt{\frac{2}{3}} & 2\sqrt{2} & \frac{26}{3\sqrt{70}} & -1 & -\frac{2}{\sqrt{7}} & \frac{1}{2\sqrt{5}} \\ -\sqrt{\frac{2}{105}} & \frac{26}{3\sqrt{70}} & -\frac{4}{7\sqrt{2}} & \frac{5}{3\sqrt{35}} & -\frac{13}{\sqrt{245}} & \frac{1}{2\sqrt{7}} \\ -\frac{1}{\sqrt{3}} & -1 & \frac{5}{3\sqrt{35}} & \frac{6}{\sqrt{2}} & \frac{3}{\sqrt{14}} & -\frac{5}{2\sqrt{10}} \\ \frac{1}{\sqrt{21}} & -\frac{2}{\sqrt{7}} & -\frac{13}{\sqrt{245}} & \frac{3}{\sqrt{14}} & \frac{34}{7\sqrt{2}} & \frac{23\sqrt{2}}{4\sqrt{35}} \\ -\frac{1}{2\sqrt{15}} & \frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{7}} & -\frac{5}{2\sqrt{10}} & \frac{23\sqrt{2}}{4\sqrt{35}} & \frac{15}{\sqrt{2}} \end{bmatrix}$$

The matrices A and B are degenerated. Required to find a solution to the system of equations (10) satisfying the initial condition (11) and boundary conditions (12) and (13).Before we prove the existence and uniqueness of the solution of the problem (10) -(13).

For the problem (10) -(13) following theorem takes place.

Theorem: If $U_0=(u_0(x),w_0(x))\in L^2[-a,a]$, then problem (10)-(13) has unique solution in domain $[-a,a]\times[0,T]$, belonging to the space $C([0,T];L^2[-a,a])$, moreover

$$\|U\|_{C([0,T];L^2[-a,a])}\le C_1\|U_0\|_{L^2[-a,a]} \tag{14}$$

where C_1 is constant independent from $U=(u,w)'$ and $T\sim 0\left(\|U_0\|^{-1}_{L^2[-a,a]}\right)$.

Proof: Let $U_0\in L^2[-a,a]$. Let's prove estimation (14). We multiple first equation of system (10) by u and second equation by w, and integrate from $-a$ to a:

$$\frac{1}{2}\frac{d}{dt}\int_{-a}^{a}[(u,u)+(w,w)]dx+\int_{-a}^{a}\left[\left(A\frac{\partial w}{\partial x},u\right)+\left(A'\frac{\partial u}{\partial x},w\right)\right]dx=\int_{-a}^{a}[(J_1,u)+(J_2,w)]dx.$$

After integration by parts we receive

$$\frac{1}{2}\frac{d}{dt}\int_{-a}^{a}[(u,u)+(w,w)]dx+\left(u^-,Aw^-\right)\Big|_{x=a}-\left(u^-,Aw^-\right)\Big|_{x=-a}=\int_{-a}^{a}[(J_1,u)+(J_2,w)]dx. \tag{15}$$

Taking into account boundary conditions (12)-(13) we rewrite equality (15) in following form

$$\frac{1}{2}\frac{d}{dt}\int_{-a}^{a}[(u,u)+(w,w)]dx+\left(Bu^-,u^-\right)\Big|_{x=a}+\left(Bu^-,u^-\right)\Big|_{x=-a}-\left(\left(Aw^+-Bu^+\right),u^-\right)\Big|_{x=-a}+$$

$$+\left(\left(Aw^++Bu^+\right),u^-\right)_{x=a}=\int_{-a}^{a}[(J_1(u,w),u)+(J_2(u,w),w)]dx. \tag{16}$$

Let's use spherical representation [19] of vector $U(t,x)=r(t)\omega(t,x)$, where $\omega(t,x)=(\omega_1(t,x),\omega_2(t,x))', r(t)=\|U(t,.)\|_{L^2[-a,a]}, \|\omega\|_{L^2[-a,a]}=1.$

Substituting the values $u=r(t)\omega_1(t,x)$, $w=r(t)\omega_2(t,x)$ into (17) we have that

$$\frac{dr}{dt}+rP(t)=r^2Q(t) \tag{17}$$

where

$$P(t)=\left(B\omega_1^-,\omega_1^-\right)\big|_{x=a}+\left(B\omega_1^-,\omega_1^-\right)\big|_{x=-a}+\left[\left(A\omega_2^+,\omega_1^-\right)\big|_{x=a}+\left(B\omega_1^+,\omega_1^-\right)\big|_{x=a}+\right.$$
$$\left.+\left(B\omega_1^+,\omega_1^-\right)\big|_{x=-a}-\left(A\omega_2^+,\omega_1^-\right)\big|_{x=-a}\right],$$

$$Q(t)=\int_{-a}^{a}\left[\left(J_1(\omega_1,\omega_2),\omega_1\right)+\left(J_2(\omega_1,\omega_2),\omega_2\right)\right]dx.$$

Let's study equation (17) with initial condition

$$r(0)=\|U_0\|=\|U_0\|_{L^2[-a,a]}. \tag{18}$$

Solution of the problem (17) -(18) has following form

$$r(t)=\left\{\exp\left(\int_0^t P(\tau)d\tau\right)\left[\frac{1}{\|U_0\|}-\int_0^t Q(\tau)\exp\left(-\int_0^\tau P(\xi)d\xi\right)d\tau\right]\right\}^{-1}.$$

If $R(t)\equiv\int_0^t Q(\tau)\exp\left(-\int_0^\tau P(\xi)d\xi\right)d\tau\le 0,\forall t$ then $r(t)$ is bounded for $\forall t\in[0,+\infty)$.

Let $R(t)>0$. We denote by T_1 the moment of time at which

$$\frac{1}{\|U_0\|}-\int_0^{T_1} Q(\tau)\exp\left(-\int_0^\tau P(\xi)d\xi\right)d\tau=0.$$

Then $r(t)$ is bounded for $\forall t\in[0,T]$, where $T<T_1$, moreover $T_1\sim 0\left(\|U_0\|^{-1}\right)$, since integrand $Q(\tau)\exp\left(-\int_0^t P(\xi)d\xi\right)$ is bounded. Hence $\forall t\in[0,T]$ takes place a priori estimation (14).

Now we prove the existence of a solution for (10) -(13) with help of Galerkin method. Let us $\{v_l(x)\}_{l=1}^{\infty}$ be a basis in space $L_2[-a,a]$, where dimension of vector $v_l(x)$ is

Ref

19. Pokhozhaev S.I. On an approach to nonlinear equation, Dokl.Akad. Nauk USSR 247, 1327-1331 (1979).

Notes

equal to dimension of vector U. For each m we define an approximate solution U_m of (10)-(13) as follows:

$$U_m = \sum_{j=1}^{m} c_{jm}(t) v_j(x), \tag{19}$$

$$\int_{-a}^{a}\left(\left(\frac{\partial U_m}{\partial t} + A_1 \frac{\partial U_m}{\partial x}\right), v_1(x)\right)dx = \int_{-a}^{a}(J(U_m), v_i(x))dx,\ i=\overline{1,m},\ t\in(0,T] \tag{20}$$

$$U_m\big|_{t=0} = U_{0m}(x), x\in R, \tag{21}$$

$$\left(Aw_m^- \pm Bu_m^-\right)\big|_{x=\pm a} = \left(Aw_m^+ \pm Bu_m^+\right)\big|_{x=\pm a} \tag{22}$$

where U_{0m} is the orthogonal projection in L^2 of function U_0 on the subspace, spanned by $v_1,...v_m$.

$$J(u_m) = (J_1(u_m, w_m), J_2(u_m, w_m))'$$

We represent $v_j(x)$ in the form $v_j^{(1)} = \left(v_j^1, v_j^2\right)'$, where

$$v_j^{(1)} = \left(v_{j1}, v_{j2}, v_{j3}\right)', v_j^{(2)} = \left(v_{j4}, v_{j5}, v_{j6}\right)'.$$

The coefficients $c_{jm}(t)$ are determined from the equations

$$\sum_{j=1}^{m}\Big\{\frac{dc_{jm}}{dt}\int_{-a}^{a}(v_j, v_i)dx + c_{jm}\Big[\left(Bv_j^{-(1)}, v_i^{-(1)}\right)\big|_{x=a} + \left(Bv_j^{-(1)}, v_i^{-(1)}\right)\big|_{x=-a} +$$

$$+\left(Bv_i^{-(1)}, v_j^{-(1)}\right)\big|_{x=a} + \left(Bv_i^{-(1)} + v_j^{-(1)}\right)\big|_{x=-a} + \left(\left(Av_j^{+(2)} + Bv_i^{+(1)}\right), v_j^{-(1)}\right)\big|_{x=a} -$$

$$-\left(\left(Av_i^{+(2)} - Bv_i^{+(1)}\right), v_j^{-(1)}\right)\big|_{x=-a} + \left(\left(Av_j^{+(2)} + Bv_j^{+(1)}\right), v_i^{-(1)}\right)\big|_{x=a} -$$

$$-\left(\left(Av_j^{+(2)} - Bv_j^{+(1)}\right), v_j^{-(1)}\right)\big|_{x=-a}\Big] - \int_{-a}^{a}\left(\left(A\frac{\partial v_i^{(2)}}{\partial x}, v_j^1\right) + \left(A'\frac{\partial v_i^{(1)}}{\partial x}, v_j^{(2)}\right)\right)dx\Big\} =$$

$$= \int_{-a}^{a}\left(J\left(\sum_{j=1}^{m} c_{jm} v_j\right), v_i\right)dx,\ i=\overline{1,m}, t\in(0,T] \tag{23}$$

$$c_{im}(0) = d_{im}, i = \overline{1,m} \tag{24}$$

where d_{im} is *i*-th component of U_{0m}.

We multiply (21) by $c_{im}(t)$ and sum over *i* from *1* to *m*:

$$\int_{-a}^{a} \left(\left(\frac{\partial U_m}{\partial t} + A_1 \frac{\partial U_m}{\partial x} \right), U_m \right) dx = \int_{-a}^{a} (J(U_m), U_m) dx.$$

With help of above shown arguments now we prove that $r_m(t)$ is bounded in some time interval $[0, T_m]$, where

$$U_m(t,x) = r_m(t) w_m(t,x), T_m \approx 0\left(\|U_{0m}\|^{-1}\right), T_m \geq T, \forall m \text{ and}$$

$$\|U_m\|_{C([0,T];L^2[-a,a])} \leq C_2 \|U_0\|_{L^2[-a,a]}, \tag{25}$$

where C_2 is constant and independent from *m*.Then solvability of system equations (19) - (22) or (23) -(24) follows from estimation (25).

Thus, the sequence $\{U_m\}$ of approximate solutions of problem (10)-(13) is uniformly bounded in function space $C([0,T]; L^2[-a,a])$. Moreover, homogeneous system of equations $rE + \frac{1}{\alpha} A\xi$ with respect to τ, ξ has only trivial solution. Then it follows from results in [20] that $U_m \to U$ is week in $C([0,T]; L^2[-a,a])$ and $J(U_m) \to J(U)$ is week in $C([0,T]; L^2[-a,a])$ as $m \to \infty$. Further, it can be shown by standard methods that limit element is a weak solution of the problem (10) -(13).
The theorem is proved.

II. Conclusion

We prove the theorem of existence and uniqueness of the local solution of the initial and boundary value problem for twelve-moment one-dimensional Boltzmann's system of equations with boundary conditions of Maxwell-Auzhan in space of functionscontinuous in time and summable in square by spatial variable, because the solution existence time depends on the norm of the initial vector function at the power minus one. Therefore, the smaller the norm of the initial vector function, the longer the solution existence time of the initial and boundary value problem for six-moment one-dimensional Boltzmann's system of equationsand vice versa.

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