Global Journal of Science Frontier Research: a
Physics and Space Science
Volume 20 Issue 3 Version 1.0 Year 2020
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Platonic Quantum Set Theory Proposal and Fractal-Cantorian Heterotic Kaluza-Klein Spacetime

By Mohamed S. El Naschie
University of Alexandria

Abstract- The four basic building blocks of the cosmos hypothesized by the Athenian philosopher Plato and his corresponding theory are replaced by the backbone golden mean based scaling pertinent to each of the four kinds of blocks. In the course of doing this we stretch and generalize Plato's philosophical ideas to ultimately find out that it is essentially the deep philosophical origin of the golden mean number system of E-infinity Cantorian spacetime theory of high energy physics and quantum cosmology. Subsequently we use this new platonic form of E-infinity quantum set theory to uncover a remarkably rich Kaluza-Klein fractal version of Gross et al's ingenious Heterotic superstring theory.

Keywords: platonic building blocks of nature, e-infinity theory, heterotic superstrings, fractal heterotic kaluza-klein spacetime, the golden mean number system of cantorian spacetime.

GJSFR-A Classification: FOR Code: 020699

Strictly as per the compliance and regulations of:

Platonic Quantum Set Theory Proposal and Fractal-Cantorian Heterotic Kaluza-Klein Spacetime*

Mohamed S. El Naschie

Abstract- **The four basic building blocks of the cosmos hypothesized by the Athenian philosopher Plato and his corresponding theory are replaced by the backbone golden mean based scaling pertinent to each of the four kinds of blocks. In the course of doing this we stretch and generalize Plato's philosophical ideas to ultimately find out that it is essentially the deep philosophical origin of the golden mean number system of E-infinity Cantorian spacetime theory of high energy physics and quantum cosmology. Subsequently we use this new platonic form of E-infinity quantum set theory to uncover a remarkably rich Kaluza-Klein fractal version of Gross et al's ingenious Heterotic superstring theory.**

Keywords: platonic building blocks of nature, e-infinity theory, heterotic superstrings, fractal heterotic kaluza-klein spacetime, the golden mean number system of cantorian spacetime.

I. Introduction

The present author first encountered the ideas of Plato speculations on what the universe as we know it is made of ultimately not through the study of philosophy in general nor the writings of Plato in particular, but rather from the powerful recollections of Werner Heisenberg and Karl Friedrich Wizecker [1] and attending various informal talks, general lectures, philosophical seminars and subsequent reading, discussion and correspondence [1-6]. The effect on the writer who at the time, was nothing more than a diligent student of structural engineering at the Technical High School of Hannover, Germany where he later on obtained a so called Vordiplom in Civil Engineering and two years later his Diplom in 1969 (see Ref. [7]) was far more than profound, in fact mind blowing. The young man of that time was both elated and bewildered to observe Nobel Laureates in Physics and world famous scientists who were suspected to be capable of designing an atom bomb [8-11] not only engage in ideal philosophical discourses but considering philosophy to be in a sense the quasi foundation of their ground breaking work on high energy particle physics [12].

Author: Distinguished Professor, Dept. of Physics, Faculty of Science, University of Alexandria, Alexandria, Egypt. e-mail: chaossf@aol.com

* With deep appreciation and respect, this paper is dedicated to Prof. Gerard 'tHooft and Prof. David Gross for marking the way.

The author was exceedingly impressed but never the less not totally convinced and thought that the reference to philosophy as a precursor to the standard model and the Work at institutions such as CERN [13] is a consequence of the learnedness of those eminent scientists and an interesting look back in history to motivate the general non-specialized reader to endure what is otherwise a highly difficult mathematical and technical subject [2], [12-13]. All the same, as time has gone by and the author graduated from Hannover with a Dipl. Ing [7] and then got his Ph.D. from University College, London in Applied Mechanics [14] and finally as a young professor decided to change career altogether and moved to physics first in Cambridge, UK and then to many other universities world wide, it was time after time the early ideas on Plato's thesis that kept him captive and then via nonlinear dynamics, chaos, fractals and M. Feigenbaum's golden mean renormalization group [15-17] that gave him the feeling of a golden mean connection between Plato's philosophy, high energy physics and his relatively new discovery of the golden mean number system [18-25] which was then linked to the associated transfinite Turing golden mean computer as well as von Neumann's cellular automaton [26-28].

There are a great number of publications that emanated from pondering these subjects [29-87] from that period. In addition to that and with the passing of time, we gained an increased intuition that the golden mean number system holds the secret for solving the mystery implicit in Plato's speculation as well as supports the results of sophisticated experiments of various laboratories and powerful accelerators all around the world [84-85]. That is how our work on unification of all fundamental forces [2],[19],[34],[39] andthe empty set nature of the Aether, Casimir-dark energy reactors proposal, fuelless space travel and the like came about and can be viewed and understood with the benefit of hindsight [52],[57],[64].

Having said all that we must admit that for some strange or not fully understood reason from our side, it is only now that we just realized the utter depth and almost super advanced power of the ideas of Plato which when we probed further and more intense, turned out to lead to an even simpler theory than we could ever have guessed or imagined and this theory in turn lead to

a refinement of E-infinity Cantorian spacetime theory which we propose here to call the platonic quantum set theory [4-6]. It is the ultimate theory of unification which goes beyond unification of all fundamental interactions [43],[46],[60] and represents a first serious step to unify science and art [23],[27],[69],[85-86] all apart of giving a rational scientific explanation for artificial intelligence and even artificial life [18[,[28],[69],[81],[86].

In the first part of the present work we give an outline of this platonic quantum set theory proposal starting rom the main postulates of E-infinity Cantorian spacetime theory. In the second part we apply by way of illustration, the new proposal to D. Gross et al's ingenious Heterotic superstring theory [44-45],[67] to first converting it to a transfinite Heterotic string [44-45] and subsequently to a Heterotic fractal Kaluza-Klein spacetime theory [12],[40] which we use to determine accurately and with a minimal amount of computation, the density of the ordinary and the dark components of the energy of the cosmos [19],[21],[40],[47].

II. Plato's Quantum Set Theory Proposal

a) Review of previous concepts, the pre-quantum particle and the pre-quantum wave of E-infinity Cantorian spacetime theory

Clearly the best way to present our new proposal is to start where we left off in our transfinite set theory founded quantum mechanics, i.e. E-infinity theory Cantorian spacetime [2],[12],[20-21]. This is obvious from the simple self evident fact mentioned in the introduction that the present proposal is sharpening and simplifying E-infinity theory in the light of our new and deep understanding of the wealth, width and breadth of Plato's ideas that lay dormant in our subconscious from our very own early student days more than half a century ago. Let us summarize what we discovered and used extensively in the past and then we will reconnect and expand it in the light of our present new deeper understanding [20,21].

(a) A pre-quantum particle is modelled in E-infinity Cantorian-fractal spacetime theory by a zero set which has two dimensions, namely first a zero for being a point-like entity as a topological dimension and a second dimension which is a Hausdorff dimension $\phi = \left(\sqrt{5}-1\right)/2 = 0.618033$ for being not a simple point but really a pre-point belonging to a pre-geometry called Cantorian geometry or more accurately, it is a Mauldin-Williams one dimensional random Cantor set with ϕ as a Hausdorff dimension and a zero length, i.e. it is a thin Cantor set of a measure zero [20-21]. Needless to say that at this level of description all pre-quantum particles are the same. However as we move towards the experimentally observable standard model only the photon of all other quantum particles retain this ϕ value as explained in detail elsewhere in many previous publications.

(b) A pre-quantum wave is modelled in E-infinity using the empty set [20-21] of set theory and following the classical definition of empty set it was fixed again by two dimensions. The first is a topological dimension equal minus one as reasoned by the deductive Menger-Urysohn dimensional theory while the second is a Hausdorff dimension equal to ϕ^2. In other words, ϕ^2 is the intersection of two zero sets $(\phi)\otimes(\phi)=\phi^2$ exactly as what comes out of the dimensional function of Penrose fractal tiling universe deduced by Alain Connes, the creator of noncommutative geometry [20-30].

(c) From the preceding exposition it then turns out that there are higher dimensions implied by the above, namely ϕ^3 for spacetime fluctuation as well as Casimir topological pressure, ϕ^4 for the topological Unruh temperature and so on. However nothing has so much impact on mathematical physics in general and E-infinity in particular as the discovery of Hardy's quantum probability of entanglement which is found theoretically and experimentally to be exactly ϕ^5 as discussed in many relatively recent papers by the author and his associates and colleagues, notably Prof. Ji-Huan He, Prof. L. Marek-Crnjac and Prof. A. Helal [4-6],[15-21],[23]. Finally for the moment we must mention ϕ^6 which is the famous Barbero-Immirzi parameter which brings two fundamental theories together at least as far as black hole entropy is concerned [20-21]. That ends our quintessential minimal review of things that are retained in our new proposal and we may now direct our attention to the refinement we need to introduce to bring E-infinity to the level of the platonic E-infinity.

b) Deeper into the platonic quantum set theory proposal

From section 2.1 and following E-infinity golden mean quantum set theory, we have ϕ zero set from pre-quantum particles and ϕ^2 empty set pre-quantum wave. Consequently following the same thinking pattern of combinatoric basic to the platonic methodology we see that ϕ and ϕ^2 may be joined either inter-sectionally or via a simple intersection to give [20]

$$\phi\otimes\phi^2=\phi^3 \tag{1}$$

i.e. a quantum fluctuation equivalent to a Casimir topological pressure or a simple union

$$\phi+\phi^2=1 \tag{2}$$

which may be interpreted as a pre-classical particle. The vital point is that in the first case the inversion of ϕ^3 leads us to [12],[20]

$$1/\left(\phi\otimes\phi^2\right)=4+\phi^3 \tag{3}$$

which is a fractal dimension of a scale invariant Einstein four-dimensional space [20-30]. On the other hand, in the case of $\phi+\phi^2=1$, the inversion remains a neutral one dimension. Let us insist pedantically on looking inside this one dimension in terms of the zero set pre-quantum particles. This leads to the elementary expansion [12]

$$\begin{aligned}1/\phi&=1+\phi\\&=2\phi+\phi^2\end{aligned} \tag{4}$$

Similarly in terms of ϕ^2 we find that

$$\begin{aligned}1/\phi^2&=2+\phi\\&=2\phi+2\phi^2+\phi\\&=3\phi+2\phi^2\end{aligned} \tag{5}$$

Consequently we have then the following elementary duality

$$\left(1+\phi\right)\left(\phi\right)=1 \tag{6}$$

and

$$\left(2+\phi\right)\left(\phi^2\right)=1 \tag{7}$$

which is a useful tautology that could be applied to $4+\phi^3$ of Einstein's scale invariant fractal spacetime to find the indistinguishability condition at the root of the two slit experiment with quantum particles, namely [21],[31],[50]

$$\left(1+\phi\right)+\left(2+\phi\right)=4+\phi^3 \tag{8}$$

and

$$\left(1+\phi\right)x\left(2+\phi\right)=4+\phi^3 \tag{9}$$

In other words in physical terms there is no difference between union and intersection in $4+\phi^3$ space and obtaining a which way information based on probability theory is fundamentally not possible and such a space is said to fundamentally and irreducibly nonlocal as discussed in numerous previous publications in the past twenty years or so [29-39]. The novel point however is the similarity to the platonic arguments applied many centuries ago using pure reason. For instance Plato's 8-sided platonic figure which represents air may be created by combining two five platonic figures which are each four sided. Thus in essence there is no fundamental difference between air and fire except the intrinsic topology of the geometric shape which is orchestrated to obey a single scaling law governed by number theoretical necessities dictated by the golden mean number system translating numeric to geometry as well as topology and visa versa. This is exactly what we conclude from the discussion of section 1.2 as well as this section because we can now get rid of any fundamental distinction between particles and fields or spacetime and quantum waves [20-21]. At the pre-geometry, pre-particle and pre-wave all the imagined fundamental problems evaporate and we are left with very simple golden mean combination [72-82]. The conventional theories by contrast are plagued with paradoxes and anomalies and require a huge super computer and the not yet invented quantum computer to tackle the highly complex time consuming computation if at all practically possible to calculate in the first place. The best one could do now is to apply our proposal to a concrete problem and that is our next task, which will be to develop a Heterotic Kaluza-Klein theory for determining the ordinary, and dark energy density sectors of the cosmos [19-28].

III. Fractal Heterotic Kaluza-Klein Spacetime from the Fractal Heterotic Superstring Theory

a) *Thefractal Heterotic superstring*

In numerous previous publications D. Gross et al's ingenious combination of the bosonic (old) string theory with D = 26 and superstring theory D = 10 resulting in the Heterotic superstring with 16 extra bosons was extended to a transfinite fractal version with the help of golden mean scaling and the 'tHooft's renormalon quasi particle $k=\phi^3\left(1-\phi^3\right)$[59-69]. That was how the new Heterotic superstring was used to accurately calculate the ordinary cosmic energy density as well as the density of the pure dark energy as well as the density of dark matter energy. Skipping the various details of the relatively exceedingly simple computation, we arrive at the following equations [74-76]:

(a) First we generate the main Heterotic spacetime [44-45],[67] from the inverse electromagnetic fine structure constant $\bar{\alpha}_2=137+k_o$ as follows

$$\frac{1}{2}\left(\frac{\bar{\alpha}_o}{2}\right)(\phi)^n \quad \underset{\rightarrow}{n=1} \quad 42+2k$$

$$\underset{\rightarrow}{2} \quad 26+k$$

$$\underset{\rightarrow}{3} \quad 16+k$$

$$\underset{\rightarrow}{4} \quad 10$$

$$\underset{\rightarrow}{5} \quad 6+k$$

$$\underset{\rightarrow}{6} \quad 4-k \qquad (10)$$

Note that while $4+2k$ is the inverse non-super symmetric quantum gravity coupling, $6+k$ is the compactified dimensions of superstring theory and $4-k$ is 'tHooft-Weltman-Wilson fractal spacetime, the $(26+k)-10=16+k$ are the extra gross bosons and k is the 'tHooft renormalon where $k=\phi^3(1-\phi^3)$ which is related to Hardy's quantum entanglement probability ϕ^5 by the simple relation $k=2\phi^5$ [74-76].

(b) As shown in many previous publications we can find the maximal energy density of the universe from $E=kmc^2$ of Umov-Lorentz-Poincare and Einstein by noting that k_{max} is equal one and in this case $k=\frac{(26-4)}{(26-4)}=\frac{22}{22}$ so that the 22 are divided into three parts, namely 1+5+16=22 leading to [75-76]

$$E=\left(\frac{1+5+16}{22}\right)kmc^2 \qquad (11)$$

where the ordinary density is

$$E(O)=\left(\frac{1}{22}\right)kmc^2 \qquad (12)$$

while the dark matter energy and the pure dark energy density of the cosmos are [74-76]

$$E(DM)=\left(\frac{5}{22}\right)mc^2 \qquad (13)$$

and

$$E(PD)=\left(\frac{16}{22}\right)mc^2 \qquad (14)$$

respectively.

(b) Clearly these are the approximate values which may be made more accurate by including the 'tHooft renormalon, i.e. transfinite correction k so that one finds [74-76]

$$E(O)=\left(\frac{1}{22+k}\right)mc^2 \qquad (15)$$

$$E(DM)=\left(\frac{5}{22+k}\right)mc^2 \qquad (16)$$

and

$$E(PD)=\left(\frac{16+k}{22+k}\right)mc^2 \qquad (17)$$

Again except for the exact result for E(O), the two other values given by equations (16) and (17) are not exact but only very good approximations because the ordinary energy density is decoupled from the dark energy sector but within the dark energy section, dark matter energy density and pure dark energy density are weakly coupled and the coupling constant Δ was determined in previous publications [75-76] and enters into the corresponding density with a minus and plus sign cancelling out in the final analysis so that at the end one finds

$$E=\left[\frac{1}{22+k}+\frac{5-\Delta}{22+k}+\frac{16+k+\Delta}{22+k}\right](mc^2) \qquad (18)$$

(c) Now we proceed further by inserting the exact value of the coupling namely $\Lambda=0.080325$ in equation (18) and find all the exact values for the corresponding cosmic energy density of the dark sector as explained in previous publications. However ignoring the minor effect of the Λ coupling one finds the rather satisfactory accurate approximation for the dark sector, namely [74-76]

$$E(DM)=0.2254248\,mc^2 \qquad (19)$$

and

$$E(PD)=0.72949016\,mc^2 \qquad (20)$$

in addition to the exact E(O) which is independent of coupling Δ. Now we are in a position to show how the previous result may be obtained from a fractal quasi-Heterotic version of the classical D = 5 Kaluza-Klein spacetime theory. The preceding calculations and conclusions can lead us to consider empty space to be a quasi highly advanced material which could be used in engineering [83].

b) The fractal Heterotic Kaluza-Klein spacetime theory

In what follows we will show a remarkable reduction of D. Gross et al's Heterotic superstring [74-76] to a fractal Heterotic Kaluza-Klein theory that we will

apply to find the cosmic energy density of the universe in the case of ignoring the coupling Λ.

Let us recall first that scale invariance converts D = 4 to $4+\phi^3$ as explained earlier on using continued fraction expansion D = 4 + 4 in previous publications. The fractal, self-similar Kaluza-Klein spacetime dimension is subsequently found from adding an extra-compactified dimension leading to our $D=5+\phi^3$ which was the subject of numerous previous publications. The next step is to go back to the three parts dissection characteristic for Gross et al's Heterotic theory by writing that

$$\left(5+\phi^3\right)-1=4+\phi^3 \tag{21}$$

and consequently we have the same opposite sign dimension and corresponding maximal energy density picture as in Gross Heterotic string case [74-76]. To show this more clearly we start from the Newtonian kinetic energy

$$E=\frac{1}{2}mv^2 \tag{22}$$

and let $v\to c$ while remembering that m must be replaced by $\left(5+\phi^3\right)m$ so that

$$\begin{aligned} E&=\frac{1}{2}\left(5+\phi^3\right)mc^2\\ &=\frac{1}{2}\left(\phi^3+1+4\right)mc^2 \end{aligned} \tag{23}$$

Setting m = 1 and c = ϕ as should be, we have

$$E=\frac{1}{2}\left(\phi^3+1+4\right)\phi^2 \tag{24}$$

Introducing the renormalon k one finds

$$E=\frac{1}{2}\left[\phi^3+\left(1+k\right)+\left(4-k\right)\right]\phi^2 \tag{25}$$

That way E$\to$ the energy density $\gamma_{\max}$. Consequently we have

$$\begin{aligned} \gamma_{\max}&=\frac{1}{2}\left(\phi^5+5\phi^2\right)\\ &=1 \end{aligned} \tag{26}$$

where $\gamma_{(c)}=\phi^5/2$ and $\gamma_{(b)}=5\phi^2/2$ exactly as should be. The reader must have noticed the versatility of the platonic golden mean theory equivalence to E-infinity quantum set theory and how we move from Newtonian mechanics to relativistic quantum mechanics and visa versa. What is also remarkable is the unit interval Cantor set geometry building blocks of the platonic thinking where the maximal average speed is ϕ, i.e. exactly equal to the Hausdorff dimension of the pre-quantum particle. Last but by no means least, the triality of the dissection of E into $\left(\phi^5/2\right)$, $\left(1-k\right)\phi^2/2$ and $\left(4-k\right)\phi^2/2$ emulating D. Gross et all's Heterotic theory gives us a clear logic for discriminating between the dark matter energy and the pure dark energy of the dark sector respectively and agree quantitatively completely with the result obtained in the earlier sections using D. Gross et al's Heterotic theory in its transfinite form [74-76].

c) The interpretive power of the platonic theory and the dark section of cosmic energy

Without going into much detail within the present work, we would still like to briefly emphasize the interpretive power of our present theory and illustrate it using again the dark section of cosmic energy as an example. The point is that the distinction between dark matter energy and pure dark energy may be explained using the distinction between the Immirzi parameter ϕ^6 in eight dimensions and the Unruh temperature in ten dimensions. In other words, the maximal E could be rephrased as in the following equation

$$E\cong\frac{1}{2}mc^2\left(\phi^5+8\phi^6+10\phi^4\right) \tag{27}$$

Put in a different way we may write the exact equation

$$\gamma_{(\max)}=\frac{1}{2}\left(\phi^5+5\phi^2\right) \tag{28}$$

approximately as

$$\gamma_{(\max)}\simeq\frac{1}{2}\left(\phi^5+8\phi^6+10\phi^4\right) \tag{29}$$

We stress that here, as before, the approximation stems from ignoring the coupling Λ between dark matter energy and pure dark energy and nothing more than that [70-83].

IV. Conclusion

It would seem that great philosophers of antiquity of the stature of Plato and Pythagoras were asking the right questions, which as is well known, is normally half the right answer. In this way it seem they indeed made greater strides than we initially imagined and in fact we have just started now to realize. There is little doubt, if any as far as the present author is concerned, that these Platonic-Pythagorean ideas and

ideals are helping us in moving slowly but surely towards understanding nature, particularly cosmology and physics. We feel strongly that this is true even from our highly advance view point of modern sciences such asquantum cosmology, quantum physics and the general theories of unification. We just need to stress more the golden mean number systems and the Cantor sets geometry and topology and presto, we find Plato's theories all of a sudden becoming a highly advanced form of E-infinity Cantorian spacetime theory, string theories, loop quantum gravity, Brane theories and twistors. We conclude that from the viewpoint of deep understanding rather than pragmatic and engineering applications, natural science could not do or advance without a healthy portion of philosophical depth and pure mathematical contemplation. In fact in a forthcoming sequel to the present paper, we will show that given nothing more than an empty set Aether there is simply a probability of 27.316 percent of artificial life and all that it entails to spontaneously appear out of true insubstantial nothingness. It seems that in our universe there is after alla place for Einstein Gods, science and Spinoza's God. The Author willingly admits that he has moved much closer to the views of one of his teachers, Prof. K.F. Weizacher, namely that science and belief are not exclusive but as in quantum mechanics, complimentary, something which he did not embrace particularly in his youth and the roaring 60's of the last century in Germany.

Acknowledgement

The Author is deeply indebted to the fundamental work of G. 'tHooft, D. Gross as well as his colleagues and friends, O. Rössler, H. Otto, Ji-Huan He, A. Helal and the great golden mean philosopher. In addition he is very grateful to enduring interest of Prof. S. Nada and Prof. L. Marek-Crnjac.

References Références Referencias

1. Werner Heisenberg, Der Teil und das Ganze (The part and the whole). R. Pieper, Munich, Germany, 1969.
2. P. Weibel, G. Ord, O.E. Rossler (Editor), Spacetime Physics and Fractality. Festschrift in honour of Mohamed El Naschie on the occasion of his 60th birthday.Springer, Vienna-New York (2005).
3. Max Born, PhysikimWandelMeiner Zeit (The changing physics of my time). F. Vieweg and Sohn, Brauenschweig,/Wiesbaden, Germany, 1983.
4. M.S. El Naschie, Fuzzy platonic spaces as a model for quantum physics. Mathematical Methods, Physical Methods and Simulation Science & Technology, 2, 2018, pp. 23-33.
5. M.S. El Naschie, Hilbert space, Poincaré dodecahedron and golden mean transfiniteness. Chaos, Solitons & Fractals, 31(4), 2007, pp. 787-793.
6. J.P. Luninet, A cosmic hall of mirrors. Physics World, September, 2005, pp. 23-27.
7. M.S. El Naschie, Woelbkrafttorsion duennwandigem Querschnitt mit nichtlineare material gesetz. Diplom Arbeit (Master Thesis), Lehrstuhl fur Baumechanik (Prof. Th. Lehmann) in Technical University of Hannover, Germany. Student No. A/1573, 1969.
8. M.S. El Naschie, Heisenberg's critical mass calculations for an explosive nuclear reaction.Chaos, Solitons & Fractals, 11(6), 2000, 987-997.
9. M.S. El Naschie, From implosion to fractal spheres: a brief account of the historical development os scientific ideas leading to the trinity test and beyond. Chaos, Solitons & Fractals, 10(11), 1999, pp. 1955-1965.
10. M.S. El Naschie, On Nishinas estimate of the critical mass for fission and early nuclear research in Japan. Chaos, Solitons & Fractals, 11, 2000, pp. 1809-1818.
11. M.S. El Naschie, On the Zel'dovichkhariton critical mass for fast fission. Chaos, Solitons & Fractals, 1, 2000, pp. 819-824.
12. M.S. El Naschie, A review of E-infinity and the mass spectrum of high energy particle physics. Chaos, Solitons & Fractals, 19, 2004, pp. 209-236.
13. G. 't Hooft: In search of the ultimate building blocks. Cambridge University Press, Cambridge (1997).
14. El Naschie, M.S., The role of formulation in elastic buckling. Ph.D. Thesis, Civil Eng. Dept., University College, University of London, April (1974).
15. M.S. El Naschie, Feigenbaum scenario for turbulence and Cantorian E-infinity theory of high energy particle physics. Chaos, Solitons & Fractals, 32(3), 2007, pp.911-915.
16. E. Goldfain, Critical behaviour in continuous dimension, E-infinity theory and particle physics. Chaos, Solitons & Fractals, 38, 2008, pp. 928-935.
17. M.S. El Naschie, A fractal Rindler-Regge triangulation in the hyperbolic plane and cosmic de Sitter accelerated expansion. Journal of Quantum Information Science, 5(1), 2015, pp. 24-31.
18. M.S. El Naschie, Simulating the Quantum Universe Via the Golden mean Number Expert-Like System, International Journal of Artificial Intelligence and Mechatronics, 7(4), 2019, pp. 15-18.
19. Mohamed S. El Naschie, Golden mean unification via fractional statistics leading to the accurate cosmic dark energy density of a cosmos with pointless geometry. International Journal of Innovation in Science & Mathematics, 6(5), 2018, pp. 167-170.
20. M.S. El Naschie, Elements of a New Set Theory Based Quantum Mechanics with Applications in

High Energy Quantum Physics and Cosmology. International Journal of High Energy Physics, 4, 2017, pp.65-74.
21. Mohamed S. El Naschie: High energy physics and cosmology as computation. American Journal of Computational Mathematics, 6(3), 2016, pp. 185-199.
22. Alexey Stakov (assisted by Scott Olsen), The Mathematics of Harmony. World Scientific, Singapore, 2009.
23. M.S. El Naschie, From Pythagorean mathematical music theory to the density of the dark energy sector of the cosmos and unification of art with science. International Journal of Engineering Innovation and Research, 8(6), 2019, pp. 249-261.
24. Mae-Wan Ho, Mohamed El Naschie and Giuseppe Vitiello, Is spacetime fractal and quantum coherent in the golden mean. Global Journal of Science Frontier Research, 15(1), 2015. On line ISSN: 2249-4626 and print ISSN: 0975-5896.
25. S.A. Olsen, Golden ratio beauty as scientific function. Lebenswelt. Aesthetics and philosophy of experience, (11), 2017.Doi: https://doi.org/10.13130/2240-9599/9457.
26. M.S. El Naschie, Ji-Huan He, S. Nada, L. Marek-Crnjac, M. Helal: Golden mean computer for high energy physics. Fractal Spacetime and Noncommutative Geometry in Quantum and High Energy Physics. 2(2), 2012, pp. 80-92.
27. M.S. El Naschie, Spinoza's God, Leibniz's Monadology And The Universal Music Of Einstein's Cantorian Nature, International Journal of Innovation in Science and Mathematics, 7(1), 2019, pp. 33-39.
28. M.S. El Naschie and Scott Olsen. The Universe as a Golden super Computer, Basically dialogue and discussion between Mohamed El Naschie and Scott Olsen in the form of an informal lecture taking place in Florida, USA on 2nd October, 2017. Youtube., https://www.youtube.com/watch?v=3GiyWK_JQxg&t=1056s
29. M.S. El Naschie, Superstrings, Knots, and Noncommutative Geometry in E-infinity Space. International Journal of Theoretical Physics, 37(12), 1998, pp.2935-2951.
30. Mohamed S. El Naschie: Elementary number theory in superstrings, loop quantum mechanics, twistors and E-infinity high energy physics. Chaos, Solitons & Fractals, 27(2), 2006, pp. 297-330.
31. M.S. El Naschie, The idealized quantum two-slit gedanken experiment revisited – criticism and reinterpretation. Chaos, Solitons & Fractals, 27(4), 2005, pp. 843-849.
32. M.S. El Naschie, On two new fuzzy Kähler manifolds, Klein modular space and 't Hooft holographic principles. Chaos, Solitons & Fractals, 29(4), 2006, pp. 876-881.
33. M.S. El Naschie, Hilbert, Fock and Cantorian spaces in the quantum two-slit gedanken experiment. Chaos, Solitons & Fractals, 27(1), 2006, pp. 39-42.
34. L. Marek-Crnjac, M.S. El Naschie, and J.H. He, Chaotic fractals at the root of relativistic quantum physics and cosmology.International Journal of Modern Nonlinear Theory and Application, 2(1), 2013, pp.78-88.
35. M.S. El Naschie, Banach Tarski theorem and Cantorian spacetime, Chaos, Solitons & Fractals, 5(8), 1995, pp. 1503-1508.
36. M.S. El Naschie, Einstein's dream and fractal geometry. Chaos, Solitons & Fractals, 24(1), 2005, pp. 1-5.
37. M.S. El Naschie, On certain "empty" Cantor sets and their dimensions. Chaos, Solitons & Fractals, 4(2), 1994, pp. 293-296.
38. M.S. El Naschie, The golden mean in quantum geometry, knot theory and related topics. Chaos, Solitons & Fractals, 18(8), 1999, pp. 1303-1307.
39. M.S. El Naschie, Exceptional Lie groups hierarchy and the structure of the micro universe. International Journal of Nonlinear Science & Numerical Simulation, 8(3), 2007, p. 445-450.
40. Mohamed S. El Naschie, On a fractal version of Witten's M-theory. International Journal of Astronomy and Astrophysics, 6(2), 2016, pp. 135-144.
41. J. Argyris and C. Ciubotariu: On El Naschie's complex time and gravitation. Chaos, Solitons & Fractals, 8(5), 1997, pp. 743-751.
42. M.S. El Naschie, On 336 kissing spheres in 10 dimensions, 528 P-Brane states in 11 dimensions and 60 elementary particles in the standard model. Chaos, Solitons & Fractals, 24(2), 2005, pp. 447-457.
43. M.S. El Naschie, SU(5) grand unification in a transfinite form." Chaos, Solitons & Fractals, 32(2), (2007), pp. 370-374.
44. M.S. El Naschie, On a general theory for quantum gravity ineraction and an experimental confirmation of Heterotic strings. Chaos, Solitons & Fractals, 12(5), 2001, pp. 875-880.
45. M.S. El Naschie, A general theory for the topology of transfinite Hetoretic strings and quantum gravity. Chaos, Solitons & Fractals, 12, 2001, pp. 969-988.
46. Mohamed S. El Naschie: Experimentally Based Theoretical Arguments that Unruh's Temperature, Hawking's Vacuum Fluctuation and Rindler's Wedge Are Physically Real. American Journal of Modern Physics, 2(6), 2013, pp. 357-361.
47. M.S. El Naschie, On a non-perturbative quantum relativity theory leading to a Casimir dark energy nano reactor proposal. Open Journal of Applied Science, 5(7), 2015, pp. 313-324.

48. A.J. Babchin and Mohamed S. El Naschie: On the real Einstein beauty E = Kmc^2. World Journal of Condensed Matter Physics, 6(1), 2016, pp. 1-6.
49. M.S. El Naschie, The measure concentration of convex geometry in a quasi Banach spacetime behind the supposedly missing dark energy of the cosmos. American Journal of Astronomy and Astrophysics, 2(6), 2014, pp. 72-77.
50. M.S. El Naschie, A new solution for the two-slit experiment. Chaos, Solitons & Fractals, 2(5), 2005, pp. 935-939.
51. Mohamed S. El Naschie, Leila Marek-Crnjac, Mohamed Atef Helal, Ji-Huan He: A Topological Magueijo-Smolin Varying Speed of Light Theory, the Accelerated Cosmic Expansion and the Dark Energy of Pure Gravity. Applied Mathematics,5(12), 2014, pp. 1780-1790.
52. M.S. El Naschie, Kerr black hole geometry leading to dark matter and dark energy via E-infinity theory and then possibly to a nano spacetime singularities reactor. Natural Science, 7(4), 2015, pp. 210-225.
53. M.S. El Naschie, Banach spacetime-like Dvoretzky volume concentration as cosmic holographic dark energy. International Journal of High Energy Physics, 2(1), 2015, 13-21.
54. M.S. El Naschie: Determining the missing dark energy density of the cosmos from a light cone exact relativistic analysis. Journal of Physics, 2(2), 2013, pp. 19-25.
55. Mohamed S. El Naschie, From a dual Einstein-Kaluza spacetime to 'tHooft renormalon and the reality of accelerated cosmic expansion. Journal of Modern Physics, 8(8), 2017, pp. 1319-1329.
56. M.S. El Naschie, Application of Dvoretzky's Theorem of Measure Concentration in Physics and Cosmology. Open Journal of Microphysics, 5, 2015, pp.11-15.
57. Mohamed S. El Naschie: topological effect and a proposal for a Casimir-dark energy nano reactor. World Journal of Nanoscience and Engineering. 5(1), 2015, pp. 26-33.
58. A. Mejias, L.D.G. Sigalotti, E. Sira, F. de Felice, On El Naschie's complex time, Hawking imaginary time and special relativity. Chaos, Solitons & Fractals, 19(4), 2004, pp. 773-777.
59. M.S. El Naschie: 'tHooft's ultimate building blocks and spacetime as an infinite dimensional set of transfinite discrete points. Chaos, Solitons & Fractals, 25(3), 2005, pp. 521-524.
60. M.S. El Naschie.SO(10) grand unification in a fuzzy setting. Chaos, Solitons & Fractals, 32(3), 2007, pp. 958-961.
61. Mohamed S. El Naschie: To dark energy theory from a Cosserat-like model of spacetime. Problems in Nonlinear Analysis in Engineering, 1(41),20, 2014, pp. 79-98. (In Russian and English – printed in the University of Kazan, Russian Federation).
62. M.S. El Naschie: Dark energy and its cosmic density from Einstein's relativity and gauge fields renormalization leading to the possibility of a new 'tHooft quasi particle. The Open Journal of Astronomy, 8, 2015, pp. 1-17.
63. M.S. El Naschie, Quasi exceptional E12 Lie symmetry group with 685 dimensions, KAC-Moody algebra and E-infinity Cantorian spacetime. Chaos, Solitons & Fractals, 38, 2008, pp. 990-992.
64. M.S. El Naschie, The Casimir effect as a pure topological phenomenon and the possibility of a Casimir nano reactor- A preliminary conceptual design. American Journal of Nano Research and Applications, 3(3), 2013, pp. 33-40.
65. Mohamed S. El Naschie, Cosserat-Cartan Modification of Einstein-Riemann Relativity and Cosmic Dark Energy Density.American Journal of Modern Physics. 3(2), 2014, pp. 82-87.
66. M.S. El Naschie, Looped light on dark energy. Journal of Quantum information Science, 7(2), 2017, pp. 43-47.
67. Mohamed S. El Naschie: Heterotic string spacetime from probability theory. Chaos, Solitons & Fractal, 12(3), 2001, pp. 617-621.
68. M.S. El Naschie, A fundamentally fractal universe from Einstein, Kaluza-Klein, Witten and Vafa's spacetime. International Journal of Articial Intelligence & Mechatronics, 8(2), 2019, pp. 23-33.
69. M.S. El Naschie, On the fractal counterpart of C. Vafa's twelve-dimensional F-theory and the A. Schoenberg twelve-tone music implicit in the standard model of high energy elementary particles. International Journal of Innovation in Science and Mathematics. 7(5), 2019, pp. 222-230.
70. M.S. El Naschie, Gravity looks like electro-magnetism when seen through fractal logic glasses. International Journal of Innovation in Science & Mathematics, 7(4), 2019, pp. 180-185.
71. M.S. El Naschie, Electromagnetic gravity and the quanta. International Journal of Applied Science & Mathematics, 6(4), 2019, pp. 2349-2394.
72. Carlos Castro Perelman, Note on the golden mean nonlocality in quantum mechanics and fractal spacetime. Academiaedu [219].
73. B.G. Sidharth, A non-reductionist model for fundamental physics. Mathematical Methods, Physical Methods, Simulation Science & Technology, 3, 2019, pp. 89-98.
74. Mohamed S. El Naschie: From the heterotic string quartet to the cosmic dark matter, dark energy and ordinary energy symphony. American Journal of Astronomy and Astrophysics, 5(2), 2017, pp. 21-24.
75. M. S. El Naschie. A combined Heterotic String and Kähler Manifold Elucidation of Ordinary Energy, Dark Matter, Olber's Paradox and Pure Dark Energy Density of the Cosmos. Journal of Modern Physics, 8(7).2017, pp. 1101-1118.

76. M.S. El Naschie, Cosmic accelerated expansion, dark matter and dark energy from a Heterotic superstrings scenario. International Journal of Innovation in Science and Mathematics, 5(2), 2017, pp53-56. ISSN (Online): 2347–9051.
77. L. Marek-Crnjac: NASA EM-drive via E-infinity Cantorian fractal spacetime. Sryawa Journal of Physics, 2(2), 2018, pp. 1-4.
78. Mohamed S. El Naschie: A theoretical justification of NASA electromagnetic drive based on cosmic dark matter. International Journal of Engineering Innovation and Research, 7(1), 2018, pp. 69-71.
79. Mohamed S. El Naschie: NASA's EM drive thrust from the forces of quantum vacuum. Advances in Aerospace Science and Technology, 3(1), 2018, pp. 1-9.
80. M.S. El Naschie, World formula interpretation of $E = mc^2$. International Journal of Applied Science and Mathematics. 5(1), 2018, pp. 67-75.
81. M.S. El Naschie, Symmetria massima of the fractal M-theory via the golden mean number system -A new language for a deep dialogue between man and nature. International Journal of Artificial Intelligence and Mechatronics, 7(3), 2018, pp.11-14.
82. M.S. El Naschie, From Nikolay Umov $E= kmc^2$ via Albert Einstein's $E= \gamma mc^2$ to the dark energy density of the cosmos $E=(21/22)\ mc^2$. World Journal of Mechanics, 8, 2018, pp.73-81.
83. Mohamed S. El Naschie: Spacetime as a new frontier advanced material with applications in physics, engineering, chemistry and cosmology. Advances in Material Physics and Chemistry, 7(9), 2017, pp. 347-352.
84. I. Affleck, Golden ratio seen in a magnet. Nature, 464, 2010, pp. 362-363. doi: 10.1038/464.
85. Scott Olsen et al, A grand unification of sciences, art and consciousness: Rediscovering the Pythagorean Plato's golden mean numbering system. In press. 2020.
86. M.S. El Naschie, Cellular automaton based on the golden mean number system as a foundation for artificial intelligence and artificial life. In press. 2020.
87. M.S. El Naschie, Massive geometry from a fractal-Cantorian perspective. In press. 2020.