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Triple Reflections- A Discourse on twin Prime Conjecture, Pascal's Triangle, and Euler's E

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Abstract- The twin prime conjecture has attracted a lot of attention worldwide. It is still an unresolved problem, even though the work of Yitang Zhang has partially resolved it. The author of this paper aims to contribute to the discourse by employing basic mathematics and logic to arrive at some conclusions on the topic, and also to help in breaking new grounds. The researcher used secondary data to build his arguments in an exploratory manner, relying on the existing literature. The paper traces the background of the problem, and points to some of the breakthroughs that were made in the past. The paper examines Pascal's triangle and, it makes some revealing discoveries on the coefficients. The author also examines Euler's E, and links it to Pascal's triangle, and the twin prime problem. Furthermore the author derives new arithmetic terms that he can use to produce infinite numbers of twin primes. The author also discusses how numbers so obtained can thoroughly be checked to be non-composite, thus extending the field of twin primes. The author finally points to the application of twin primes in industry, academia, and other areas of practical knowledge.

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I. Introduction

We can observe from the expansion of binomial expressions of the form $(x+y)^n$ that the numerical coefficients of the terms in the expansion can be derived by using Pascal's triangle, named after the French mathematician(Wong, 2013). We can use Pascal's triangle to generate the binomial coefficients in the binomial expansion where a binomial is of the form $(x+y)^n$ and, the expansion takes the form of ${}^nCr$(Wong, 2013). Below are a few rows of the triangle showing the binomial coefficients of the first six terms, which are as follows

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

These coefficients when, summed up line by line horizontally, form the geometric series to infinity of the form, 1, 2, 4, 8, 16, 32, 64, 128,....., 2^{n-1}. They form the series of the form 2^{n-1} , where n is an integer from n=1 to n = infinity. If we sum the reciprocals of the series to infinity using a geometric progression, we obtain the sum to infinity as

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$$a / r \tag{1}$$

where a is the first term 2^0 and r is the ratio ½ where, r is less than 1.
The sum of the reciprocals of the series 1/1, 1/2, 1/4, 1/8, 1/16, 1/32, 1/64……. $1/2^{n-1}$ to infinity is, therefore, $2^0/1/2 = 1/1/2 = 2$.

$$1 \text{ to } \infty \sum 1/2^{n-1} = 2. \text{ (Author's derivation)} \tag{2}$$

(cf. Math Stack Exchange, 2018; Math Stack Exchange, n.d.)

Thus, the reciprocals of Pascal's triangle numbers form an infinite series that sum up to 2 in the limit. Brothers (2012) dealt with the products of the Pascal triangle and their ratios, which in the limit, summed up to *E* or Euler's constant, 2.7183. When we use the expressions S_{n+1}/S_n divided by S_n/S_{n-1}, we can take one line in the triangle and hold it as our baseline for computation. We use the product of the line just below it, and divide it by the multiplication of the numbers on the line we have taken as our reference point. We divide the resultant by the product of the baseline. We again divide the result by the multiplication of the numbers on the line above it. An example here will suffice.

						1						
					1		1					
				1		2		1				
			1		3		3		1			
		1		4		6		4		1	S_{n-1} 1	Product = 96
	1		5		10		10		5		1	S_n.....2 Product= 2,500 (Line held constant)
1		6		15		20		15		6		1	Sn+1……3 Product= 162,000

162,000/2,500 divided by 2,500/96 = 2.48832

If we continue this process to infinity, in the limit, we will find that the result that we obtain each time will approach the terminal value of 2.7183 or *E* (Euler's *E*), which is the sum of the reciprocals of the natural numbers to infinity (Brothers, 2012).

When we examine Euler's *E*, we find that *E* is the sum to infinity of the reciprocals of the natural counting numbers, 1/1, 1/2, 1/3, 1/4,…..1/n that gives us the sum to infinity of the series of reciprocals of the natural counting numbers of the common term, 1/n. Euler's *E* is the same as the expansion of the expression $(1+ 1/n)^n$ (Brothers, 2012). In the limit, this binomial term, gives us the approximate value 2.7183 or Euler's *E* (Brothers, 2012).

In 2012, this author posted an article on *Ghanaweb.com* and *Academia. edu* to the effect that a great deal of the majority of the Twin Prime numbers (about 90 to 99 percent) can be generated by pairs of the terms 30n-1, 30n+1, on the one hand, and another two terms 30n-17, and 30n-19, on the other side. These arithmetic terms provide evidence of proof by induction (Stanford.edu, n.d.). These four terms were arrived at by sieving and using a formula for terms of arithmetic series. We can use the expressions 30n-1, 30n+1, 30n-17, and 30n-19 to produce an infinite number of twin

Notes

primes, thus, proving that we have unlimited amount of twin primes (cf. Sebah & Gourdon, 2002).

That was the author's contribution to the Twin Prime Conjecture conundrum. However, in this article, the author has elaborated more on this, as presented below. Caldwell (n.d.) stated that the Greek mathematicians, Euclid (c. 300) and Eratosthenes were the first people to develop sieves of elimination to produce twin primes. This author developed an interest in twin primes and thought of using simple patterns of recognition and critical thinking to contribute to the enlightened discourse on the twin prime conjecture, which is said to be one of the unresolved mathematical problems, which were first put forward by Polignac in 1849 in France (Sha, 2016; Kiersz, 2018).

Notes

II. Historical Background to Twin Prime Conjecture

The twin prime conjecture dates back from antiquity in ancient Greece, during the period of Euclid and Eratosthenes (Murty, n.d.; Brent, 1975). The problem was resurrected in 1849 by Alphonse Polignac. Viggo Brun, the Norwegian mathematician, in 1915, developed a new sieve based on those of Euclid and Eratosthenes (Murty, 2015).

In 1923, Hardy and Littlewood from Oxford University gave their proof of the twin prime conjecture: $2\pi\ (1- 1/(p\text{-}1)^2 * x/\log^2 x)$ with $p > 2$. Hardy and Littlewood's proof was based on a circular method, onethat was developed by an Indian genius called Srinivasa Ramanujan (Murty, n.d.). In 2015, Murty discussed Yitang Zhang's breakthrough theorem towards the solution of the twin prime conjecture, with the absolute value of $p\text{-}q< 70$ million. Zhang had given his proof from the University of New Hampshire in 2013 in a paper proving that, in fact, there are infinite numbers of twin primes (cf. Wolfram Math World, n.d. Agama, 2018).

Murty (n.d.) wrote that if there are infinitely p primes such that $p+2$ is also a prime, then generally if for an even number A here are infinitely many primes p, then it follows that $p + A$ is also prime (proof by induction, fractal geometry, and self-similarity as well as set theory). This author therefore humbly posits that he can prove the twin prime conjecture by simple logic and simple mathematics using arithmetic progression or series. Similar approaches had been made in the past including the work of two Frenchmen, Sebah and Gourdon(2002).

III. Discussion

Let us consider the first eight pairs of twin primes. These are:

(3,5), (5,7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61) and (71, 73)

(Source: en.wikipedia.org/wiki/Prime_pair)

Each pair of twin primes shows a difference of 2, hence the names twin primes or sexy primes or cousin primes or conjugate primes(Wolfram Math World, n.d.).
We can form their reciprocals and find the sum of these first eight pairs.

1/3, 1/5, 1/5, 1/7, 1/11, 1/13, 1/17, 1/19, 1/29, 1/31, 1/41, 1/43, 1/59, 1/61, 1/71, 1/73.

Their decimal representations are as follows

$$1/3 = 0.\ 3333333333$$

$$1/5 = 0.2000000000$$

1/5 =0.2000000000

1/7 =0.1428571429which is *pi* (π) minus 3)

1/11= 0.0909090909

1/13=0.0769230769

1/17 =0.0588235294

1/19=0.0526315790

1/29=0.0344827586

1/31=0.0322580645

1/41=0.0243902439

1/43= 0.0232558140

1/59=0.0169491525

1/61=0.0163934426

1/71=0.0140845070

1/73=0.0136986301

Notes

The sum of all the above reciprocals gives a total of 1.3, approximately. Thus the sum of the reciprocals of the first eight pairs of twin primes or first fifteen twin primes is 1.3, while the sum of the reciprocals of all-natural numbers to infinity, contained in Euler's *E,* is 2.71823. The sum of the reciprocals of the first fifteen twin prime numbers shows us that the probability density function or distribution of twin primes is about 50 percent or more of all-natural numbers if we consider the first fifteen twin prime numbers of significance.

Thus, twin primes are far in between as we approach infinity, but they never terminate as they become asymptotic to the reciprocals' distribution function of all-natural numbers (Brent, 1975). In a distribution of say p and q (where p and q are fractional), we shall assume q to have a very low probability of occurrence in an infinite series, where n is extremely huge. As we know, in the limit, this approximates the Poisson distribution in a case where q is very low in value and n is extraordinarily huge (Spiegel 1975: 108-130).

$$e^{-\lambda}\lambda^{x}/x!\text{for} \quad x = 0, 1. 2, 3... \tag{3}$$

(library 2. lincoln; Spiegel, 1975: 108-130)
The average or mean of the Poisson distribution is given as *np.*

Therefore, if we take Euler's E into consideration, we can subtract the sum of reciprocals of Pascal's coefficients derived from his triangle, which is the sum of 2from Euler's E and remain with 0.71823. This remainder or residual is significant because it represents the sum of all reciprocals of odd numbers (which are not part of Pascal's series), and also the total of all reciprocals of numbers which end with either 0 or 5 or those numbers which are squares such as 49, 121, 441, 169, among others. The process alluded to here is part of the elimination or sieving method, which uses both deductive and inductive logic. Euclid and Eratosthenes were the first mathematicians to devise means of sieving numbers that generated the twin prime numbers (Murty, 2013; Wong, 2012).

We subtract 1.3 (sum of reciprocals of the first 15 twin primes) from 2.71823. We end up with 1.41823 as the sum of all non-twin prime number reciprocals including primes, which do not twin as well as other numbers which are not of the form

2^{n-1}. Looking at the total of all reciprocals of natural numbers, and their sum to infinity to be Euler's *E* or 2.71823, and the sum of the reciprocals of the first fifteen twin primes to be 1.3 (as if these were all the significant reciprocals of twin primes), we would conclude that approximately, twin primes are distributed in probability density space to account for about 50 percent of all the natural numbers, which would be a false assumption to have.

Notes

None-the-less, despite their paucity or scarcity, as we go towards infinity, twin primes never end just as the natural numbers also never end because twin primes are a subset of the natural numbers, and they have all the characteristics and properties of natural numbers, but they become asymptotic as we approach infinity (proof by deduction according to theories of self-similarity and fractal geometry by Feigenbaum and Mandelbrot, respectively).

Thus, since natural numbers are to infinity, so are twin primes, since twin primes are a subset of all natural numbers (Wong, 2012). Wong (2012), in his paper, mentioned that Mitchell Feigenbaum came up with the concept of self-similarity, while Benoit Mandelbrot was the originator of the concept of fractal geometry. Self-similarity and fractal geometry, therefore, can be used to state that twin primes are infinite, since they are a subset of natural numbers, which are infinite. Thus, we can use the terms 30n-1, 30n+1, on the one hand, and the pair of terms, 30n-17, and 30n-19 on the other side as expressions to derive twin primes by making n as large as possible. These terms were derived first by using elimination methods and generating them by using the formula of arithmetic series(see Chen et al.). The scenario presents us with a unique binomial distribution, where both p and q are almost equal as n approaches infinity. Let us assume p=0.5 and q=0.5, and n is an exceptionally huge number say 50,000. We have $(p+q)^{50000}$

Or $(0.5 +0.5)^{50000}= (1)^{50000}= 1$ which is the total area under the Gaussian or Normal curve (Spiegel 1975: 108-130; library2. lincoln)

The above discussion shows us that the distribution of twin primes is dense in about the first 300 natural number series but, as the series increases to infinity, the twin primes become scarce, yet they do not finish as they are like an asymptotic graph approaching zero but never becoming zero. Inherent here is the Central Limit Theorem, which states that the mean, as the distribution gets larger, stabilizes and approaches a fixed sum or parameter close to the center of the Gaussian Curve or Normal Curve(Spiegel 1975:112; Kwak & Jim, 2017).

Brun (1919) gave the sum of all reciprocals of twin primes as 1.90216054.Seventy-five years later, in 1994, Nicely,(1994) improved the total of twin prime reciprocals to 1.902160583209. However, we know that all twin primes tend to end with the digits 1, 3, 7, and 9, even though not all numbers that end with these digits are prime numbers. Examples of numerals that end with digits 1, 3, 7, and 9 and, which are non-twin primes, are 441, 443, 169,171, 837,839, 341, and 343, some of which, are squares of numbers.

In considering twin primes, we should eliminate all even numbers or numbers which end with 2, 4, 5, 6, 8, and 0.So, twin primes are distributed as approximately 4 out of every10 numbers with ending digits of all numbers, giving a ratio of 0.4 or 40 percent of all-natural numbers. This inference gives us an idea that number space is everywhere dense with twin primes. Euclid long ago created a sieve for generating prime numbers (Wong, 2012). However, our distribution of sums of reciprocals is as follows:

1. Sum to infinity of all reciprocals of natural counting numbers is (Euler's *E*) =2.71823

2. Sum to infinity of reciprocals of Pascal's Triangle coefficients, part of reciprocals of even numbers, is: 1 + ½ + ¼ + 1/8 +1/16 +....+ 1/n which adds up to2.0 in the limit (author's discovery)
3. Sum to infinity of reciprocals of all Twin Primes is approximately 1.9 (according to Brun, 1919, and Nicely, 1994)
4. Sum of reciprocals of all numbers which end with either 0 or 5 is 0.5438 (author's estimate)

$$(1/5 + 1/10 + 1/15 + 1/20 + \ldots\ldots\ldots + 1/n) = 1/5\ (1/1 + æ + 1/3 + \ldots\ldots + 1/n)$$
$$= 1/5(E) = 1/5\ (2.7183) = 0.5438$$

5. Sum of reciprocals of all numbers which are squares or end with the digits 1, 3, 7, and 9 but which include twin primes and primes that are non-twin primes------- (residual- ?) Residual= 2.71823- (0.5438 + 2) = 0.1745

Therefore, the addition of reciprocals of twin primes divided by addition of reciprocals of all-natural numbers is1.96/2.71823, which is approximately 72%. This author found a relationship among the three mathematicians, namely Euler, Pascal, and Brun, in terms of the numbers they are associated with, hence, the title of this article. The numbers are Euler (2.7183- sum of reciprocals of all-natural numbers from 1 to infinity), Pascal (2- sum of the series formed from the reciprocals of the coefficients of the binomial expansion to infinity), and Brun (1.96- sum of reciprocals of all twin primes to infinity). The relationship is

$$\text{Euler} - \text{Pascal} = \text{Brun/Euler} \quad \ldots\ldots\ldots\ldots \tag{4}$$

$$\text{Or Euler}^2 - \text{Euler x Pascal} = \text{Brun}\ldots\ldots\ldots\ldots 2 \tag{5}$$

From 1, $2.7183 - 2 = 1.96/2.7183$

$0.7183 = 0.72$

Thus $(1+1/n)^n - (1 + æ + ß + 1/8 + 1/16 + \ldots\ldots + 1/2^{n-1})$

$= (\text{Sum of reciprocals of twin primes}) / (1+1/n)^n$

When we consider Equation 2, we have $(2.7183)^2 - (2.7183 \text{ x } 2) = 1.96$

$7.38915489 - 5.4366 = 1.95255489$

From statement 5 above, we had a residual of 0.1745.

We shall make the following propositions or assumptions:

p (v - the probability of drawing an even number from natural numbers is 0.5)

p(d - the probability of drawing an odd number is 0.5)

Therefore, the percentage probability distribution of twin prime numbers

$$= 0.5 \text{ x } 0.4 \text{ x } 0.1745$$

$$= 0.03490$$

Or 3.49%.

Notes

Notes

At the beginning of this discussion, we stated that all twin prime numbers end with digits 1, 3, 7, and 9 only, thus 4 out of 10 of all Arabic numerals. However, not all numerals that end with digits 1, 3, 7, and 9 are twin primes. We have, all the same, established here that twin primes, on average, in interval estimation, constitute only 3.49% of all-natural numbers up to infinity.

Table 1: Frequency table of Twin Primes occurrence at intervals of 100 up to 1000

Interval	Frequency of twin primes in interval
1-100	15
101-200	14
201-300	8
301-400	4
401-500	6
501-600	5
601-700	7
701-800	0
801-900	10
901-1000	0

Source: Adapted from: https://prime-numbers.info/list/twin-primes-up-to-10000

The average frequency is therefore 6.9 twin primes per 100 (6.9%) gap, which to infinity will approach 3.49 per 100 intervals or 3.49%.

IV. Analysis and Proof of Twin Prime Conjecture by Inference

However, subtracting (2) from (1) leaves us with 0.71823 as the residual, accounting for the sum of reciprocals other than the reciprocals of even numbers including, twin primes, which as we have shown above is 72% of all numbers. As we said before, twin prime numbers account for approximately 4 out of every ten numbers. So, we shall find 0.4 of this residual 0.1745, which is 0.0687292 or 6.9 percent, as we estimated in Table 1 above. Nevertheless, as we observed before, there are some numbers that end with the four digits 1, 3, 7, and 9, that are not twin prime numerals as some are squares of numbers and, therefore, they are composite. Thus, the value 0.0687292 could be slightly lower.

Thus, we can conclude that about 6.9% of all the natural numbers are prime numbers, including twin primes, which are statistically significant in the sense that twin prime numbers become rare as we approach infinity, but they do not terminate, just as all the natural numbers do not terminate. Twin primes are a subset of prime numbers, which in turn are also subsets of natural numbers and, as such, they have all the attributes of the natural numbers, in the sense that they are never-ending, and they are everywhere dense in probability density space.

In an article by Dubner (2005) he quoted that Hardy-Littlewood's C_2, which was derived by them in 1923 as Twin Prime Constant, was of the value 0.6601618158, and that their method could generate primes as large as those of about 600 digits by using the sieving method.

However, this author maintains that the pairs of terms derived by this author, namely (30n-17) and (30n-19) on the one hand, and (30n-1) and (30n+1) on the other hand, can generate an infinite series of twin primes with the proviso that for any numbers generated by those terms, we have to test them for compositeness by applying

the Fermat's test as cited by Dubner (2005).Below, we make a presentation on how arithmetic progression can be used to generate terms for twin prime numbers (cf. Sebah & Gourdon, 2002).

V. Sum of Squares of Reciprocals of the Natural Numbers

We shall here, examine the sum of the squares of the reciprocals of the natural numbers. We shall concentrate on the first 20 terms. The distribution is 1/1, ¼, 1/9, 1/16, 1/25, 1/36, 1/49, 1/64, 1/81, 1/100, 1/121, 1/144, 1/169, 1/196, 1/225, 1/256, 1/289, 1/324, 1/361, and 1/400. Their decimal equivalents are

Notes

1.00000

0.25000

0.11110

0.06250

0.04000

0.02778

0.02041

0.01563

0.01235

0.01000

0.00826

0.00694

0.00592

0.00510

0.00444

0.00391

0.00309

0.00277

0.00250

Total = 1.59172

The total summation of the squares of reciprocals of the first twenty natural numbers is 1.59172. Eremenko (2013) pointed out in his paper that in 1735, Euler solved the problem of summing up squared reciprocals of all-natural numbers to infinity by obtaining a value 1.6462614966 by using the formula

$$1 \text{ to } \infty \sum 1/n^2 = \Pi^2/6 \text{(Eremenko, 2013).}$$

We can see that the total of the first twenty squared reciprocals of the natural numbers is 1.59172, compared with Euler's sum of all the squared reciprocals to infinity. If we divide 1.59172 by Euler's sum, 1.6462614966, we obtain approximately

97%. This result informs us that only the first twenty terms are remarkably significant, contributing 97% of the total value of the sum of reciprocals of squared numbers to infinity. This result leads to us to conclude that in examining number theory, the further away we get into extraordinarily gigantic numbers, the more abstract and elusive the analysis becomes, so it is like the saying, 'the harder you look, the less you see'. This also informs us to use the first few numbers within our reach, as our reliable sample, for rigorous analysis in order to arrive at valid, reliable, and verifiable truths through deductive and inductive logic.

Notes

VI. Findings and Analysis

Consider terms of an arithmetic progression (A.P.) or series, all of which have a common difference of 6. We derive the general term of an A.P. as

$$a + (n-1)\, d, \tag{7}$$

where, "a" is the first term of the series, and, "d" is a common difference.

[1] 5, 11, 17, 23, 29, 35, 41, 47,.....6n-1

[2] 7, 13, 19, 25, 31, 37, 43, 49.....6n+1 (cf. Wolfram Maths World, n.d.; Wikipedia.org)

[3] 11, 17, 23, 29, 35, 41, 47.....6n+5

[4] 13, 19, 25, 31, 37, 43, 49.....6n+7

[5] 17, 23, 29, 35, 41, 47, 53.....6n+11 (cf. Wikipedia.org)

[6] 19, 25, 31, 37, 43, 49, 55,.....6n+13

[7] 23, 29, 35, 41, 47, 53, 59..... 6n+17

[8] 29, 35, 41, 47, 53, 59, 65..... 6n+23

[9] 31, 37, 43, 49, 55, 61, 67..... 6n+25

We can generate an infinite number of twin primes by using the pairs of terms in [1] [2], [3] [4], [5] [6], and [8] [9].
These are

[(6n-1), (6n+1)]

[(6n+5), (6n+7)]

[(6n+11), (6n+13)]

[(6n+23), (6n+25)]

We can extend the same series by having a common difference of 30 instead of 6. Let us consider the following sets of arithmetic series

[1] 5, 35, 65, 95, 125, 155.....(This series collapses since all the terms are divisible by 5)

[2] 7, 37, 67, 97, 127..... 30n-23

[3] 11, 41, 71, 101, 131, 161,.... 30n-19

[4] 13, 43, 73, 103, 133, 163,.....30n-17

[5] 17, 47, 77, 107, 137, 167... 30n-13

[6] 19, 49, 79, 109, 139,.....30n-11

[7] 23, 53, 83, 113, 143, ...30n-7

[8] 29, 59, 89, 119, 149, 179 ...30n-1

[9] 31, 61, 91, 121, 151,.....30n+1

Notes

From the above sets, we can see the pairs of terms [3] [4], [5] [6], and [8] [9] that can be used to generate infinite numbers of twin prime pairs.
These are

[(30n-19), (30n-17)],

[(30n-13), (30n-11)],

[(30n-1), (30n+1)]

We can use the first set of terms by setting n=9,999,999,999,999. We can set up two twin primes with values of 299,999,999,999,953 and 299,999,999,999,951 or 2.99 x 10^{14}(cf. Kurzweg, 2016; Sebah & Gourdon, 2002). We can see that these two numbers are not divisible by any even number, nor by a number ending with 0 or 5. The test of divisibility, by 7, states that the last three digits should be taken off the number, and deducted from the remaining digits, whichever is larger. If the resultant number is not exactly divisible by 7, then the original number is not divisible by 7 (Maths Smart, 2013).

In this case, the numbers are both not divisible by 7.We can, therefore, conclude that we can make 'n' as large as we can, and use any of the terms above to generate twin primes. We can also, therefore, generalize as [30 (10^z – 1) -17] and [30 (10^z-1)-19] where, the power Z, is astronomically a huge number to which 10 is raised, such as a zillion or quintillion or octillion. We subtract 1 from it to leave us with a number which is made up of digits of 9, for example, the gigantic number 999,999,999,999,999,999,999,999,999.

When we multiply this number by 30 and subtract 17 and 19 from the resulting number respectively, we obtain resultant numbers that always end up with the last three digits as 951 and 953 respectively. The sums of all the digits in these numbers give us even numbers, which means they are not divisible by either 3 or 9. We chose the power Z as the symbol of an exceptionally gigantic number, because Z is the last letter of the alphabet.

VII. Uses and Practical Applications of Twin Prime Numbers

We can use twin primes in many ways, in cryptology or encoding a secret language (cf. Alan Turing), or in medicine in naming unique drugs or viruses such as SARS, CORONAVIRUS, MERS, or in astronomy, in naming newly discovered planets, asteroids, stars, and constellations, as twin primes are peculiar, and they do not recur in number space, nor are they divisible by any number, except by themselves, and 1 (Wong, 2013).We can liken twin primes to the Kondratieff cycle in Economics, which is a long cycle with periodicity that lasts between 40 to 60 years(Ganti, 2020).

We can also, in line with demands for non-recurring numbers for use in the sciences and industry, envisage a new series distribution given by reciprocals of Euler's *e* thus

$$(1+1/e)^n \text{ for } n=0 \text{ to } n=\text{infinity. } 1/e = 0.367886456.$$

Therefore, the first term in the series is 1, followed by 1.367886456, then 1.367886456^2, 1.367886456^3, +.....+ 1.367886456^n.

Notes

These numbers also produce unique irrational numbers, which can be used in many applications just as the twin prime numerals.

VIII. A Test for Twin Prime Numbers

We can use the pairs of terms (2n+1) and (2n-1); or (30n-1) and (30n+1); and (30n-17) and (30n-19)to produce twin prime numbers with a difference of 2. We will need to test the numbers generated by having a computerized algorithm flowchart thus

1. Use (2n+1) and (2n-1) to generate pairs of twin primes. You can also use the terms derived by this author (30n-1) and (30n+1) on one side, or (30n-17) and (30n-19) on the other side.
2. Find out whether the numbers generated by these terms end with the digits 1, 3,7, and 9. If yes, proceed to the next stage.
3. We need to test using Fermat's composite test whether numbers generated by step 1 are divisible by 3,7, 11, and 13. If yes, discard them. If no, go to the next step.
4. Find the square root of the number, and if it is a perfect square, reject it. If it is not a perfect square, move on to the next stage.
5. List all numbers tested which have successfully passed these steps as twin primes, if they have a difference of 2, end with the digits 1, 3, 7, and 9 and they are not divisible by any number except by 1 and themselves.

IX. Conclusion

We can, therefore, assert that we can generate infinite series of twin prime numbers using the pair of terms (30n-1) and (30n+1) on the one hand, and on the other side, the dual terms (30n-17) and (30n-19) as well as the other duo terms tabulated above. However, numbers that are generated by all the terms of the arithmetic series produced in this article have to undergo the examination for compositeness by using Fermat's method. Twin prime numbers have numerous practical applications in medicine, pharmacology, astronomy, cryptology, data science, and machine learning, among many other uses.

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