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Conjecturing with Some Conjectures

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Abstract- This world has been seeing so many conjectures from the formation of this world. Particularly, mathematics world has been seeing so many conjectures from the civilization of this world, also venturing through on it. So many Indians, Chinese and Arabian scholars provided their participation in mathematics world. Since before Euclid so many mathematicians ventured on numbers, geometry, astronomy, etc… in number theory, there are so many conjectures like applications of GCD, Fermat Last theorem, Euler's totient function, Gold Bach conjecture, ABC conjecture, etc… some of this has been proved but so many of that still has not been proved. In this paper, I try to prove Gold Bach conjecture, stating abc conjecture for composite numbers, and try to deliver some conjectures.

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Abstract- This world has been seeing so many conjectures from the formation of this world. Particularly, mathematics world has been seeing so many conjectures from the civilization of this world, also venturing through on it. So many Indians, Chinese and Arabian scholars provided their participation in mathematics world. Since before Euclid so many mathematicians ventured on numbers, geometry, astronomy, etc… in number theory, there are so many conjectures like applications of GCD, Fermat Last theorem, Euler's totient function, Gold Bach conjecture, ABC conjecture, etc… some of this has been proved but so many of that still has not been proved. In this paper, I try to prove Gold Bach conjecture, stating abc conjecture for composite numbers, and try to deliver some conjectures.

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I. Introduction

In mathematics, the **greatest common divisor** (**gcd**) of two or more integers, which are not all zero, is the largest positive integer that divides each of the integers. For twoin tegers x, y, the greatest common divisor of x and y is denoted. For example, the gcd of 8 and 12 is 4, that is, In the name "greatest common divisor", the adjective "greatest" may be replaced by "highest", and the word "divisor" may be replaced by "factor", so that other names include **greatest common factor** (**gcf**), etc. Historically, other names for the same concept have included **greatest common measure**.

In number theory , **Euler's totient function** counts the positive integers up to a given integer n that are relatively prime to n. It is written using the Greek letter phi as $\varphi(n)$ or $\phi(n)$, and may also be called **Euler's phi function**. In other words, it is the number of integers k in the range $1 \le k \le n$ for which the greatest common divisor $\gcd(n, k)$ is equal to 1. The integers k of this form are sometimes referred to as totatives of n.

The ***abc*** **conjecture** (also known as the **Oesterlé–Masser conjecture**) is a conjecture in number theory, first proposed by Joseph Oesterle (1988) and David Masser (1985). It is stated in terms of three positive integers, a, b and c (hence the name) that are relatively prime and satisfy $a + b = c$. If d denotes the product of the distinct prime factors of abc, the conjecture essentially states that d is usually not much smaller than c. In other words: if a and b are composed from large powers of primes, then c is usually not divisible by large powers of primes. A number of famous conjectures and theorems in number theory would follow immediately from the *abc* conjecture or its versions. Goldfeld (1996) described the *abc* conjecture as "the most important unsolved problem in Diophantine analysis".

Various attempts to prove the abc conjecture have been made, but none are currently accepted by the mainstream mathematical community and as of 2020, the conjecture is still largely regarded as unproven.

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On 7 June 1742, the German mathematician Christian Goldbach wrote a letter to Leonhard Euler (letter XLIII), in which he proposed the following conjecture:

Every integer that can be written as the sum of two primes can also be written as the sum of as many primes as one wishes, until all terms are units.

A modern version of the marginal conjecture is:

Every even integer greater than 2 can be written as the sum of two primes.

[1]. **Wikipedia** gave some basic ideas about GCD, Euler's totient function, Gold Bach conjecture, ABC conjecture.[2]. **Balasubramani Prema Rangasamy** - Some Extensions on Numbers – Advances in Pure Mathematics – 2019. p. 944-958, gave some conjecturing ideas in Euler's totient function,

In this paper, I try to prove Gold Bach conjecture, stating abc conjecture for composite numbers, and try to deliver some conjectures.

Facts 1:

1. For any n > 2, we get $\varphi[x]$ is always even number.
2. $\varphi[E] \leq \frac{E}{2}$ is always even number, where E is even number.
3. We never find a number n ϵ I, which gives 2+12x|x ϵ I numbers when we find φ[n].
4. We cannot find a number n ϵ I, which gives $a^{\varphi(n)} \equiv 1$ mod n, where φ[n] = 2 + 12x|xϵI.

14	134	254	374	494	614	734	854	974	1094
26	146	266	386	506	626	746	866	986	1106
38	158	278	398	518	638	758	878	998	1118
50	170	290	410	530	650	770	890	1010	1130
62	182	302	422	542	662	782	902	1022	1142
74	194	314	434	554	674	794	914	1034	1154
86	206	326	446	566	686	806	926	1046	1166
98	218	338	458	578	698	818	938	1058	1178
110	230	350	470	590	710	830	950	1070	1190
122	242	362	482	602	722	842	962	1082	1202

Above 100 numbers which are not the value of φ[n], where n is any positive integer.

Following numbers are also having the same character like above numbers i.e. 3(1+4x)-1| x ϵ I.

68	152	188	194	308	428	548	668	788	872
908	1028	1148	…						

All the above numbers having a common relation, that is, digit sum of above numbers would be 2 or 5 or 8. Digit sum of 12x would be 3, 6, and 9. These 3, 6 and 9 are rotational identities. They are stumping their existence in all ways.

38+30x | x ϵ I also not comes for φ[n].

i.e. 38 68 98 158 188 248 278 308 338 398 428
458 488 518 548 578 608 638 668 698 728
758 788 818 …

2. Balasubramani Prema Rangasamy - Some Extensions on Numbers – Advances in Pure Mathematics – 2019. p. 944-958.

In above numbers some numbers like 38 + 30 x 3 = 38 +90 = 128, 368 … would be value of φ[n], even though their digit sum would be 2 and 8.

114,318, 298, would not be a value of φ[n], contrarily their digit sum values are 3, 1 …

If digit sum 2,5 and 8 numbers mostly would not be a value of φ[n].

Notes

From the above we concluded that, there are so many numbers greater than equal to fourteen exist which would not be the value of φ[n].

Theorem 1: Let x ≥ 2 be the integer then $\varphi[\varphi[\varphi[\ldots\ldots \varphi[x]]]] = 1$. i.e. $\varphi^{n}[x] = 1$. where n is the totient order of x.

Proof: Let x be any number then $x > \varphi[x]$.

Let we take $\varphi[x] = x_1$ then $x_1 > \varphi[x_1]$

By this way we can obtain the totatives, $\varphi[x] > \varphi[x_1] > \varphi[x_2] > \cdots > \varphi[2] = 1$ ■

Ex:

1. Let x = 693 then

$693 > \varphi[693] = 360$

$360 > \varphi[360] = 96 = \varphi^{2}[693]$

$96 > \varphi[96] = 32 = \varphi^{3}[693]$

$32 > \varphi[32] = 16 = \varphi^{4}[693]$

$16 > \varphi[16] = 8 = \varphi^{5}[693]$

$8 > \varphi[8] = 4 = \varphi^{6}[693]$

$4 > \varphi[4] = 2 = \varphi^{7}[693]$

$2 > \varphi[2] = 1 = \varphi^{8}[693]$ Totient order of 693 is 8.■

Facts 2:

1. If GCD (a, b) = k then GCD (a^n, b^n) = k^n, where n ϵ Z.
2. If GCD $(a, b) = k$ and $\frac{a}{k} = c; \frac{b}{k} = d$ then GCD (a^n, b^n) = k^n and $\frac{a^n}{k^n} = c^n; \frac{b^n}{k^n} = d^n$, where n ϵ Z.
3. We can generalize above as If GCD $(a_1, a_2, a_3 \cdots a_i) = k$ and

   $\frac{a_1}{k} = b_1, \frac{a_2}{k} = b_2, \frac{a_3}{k} = b_3 \cdots \frac{a_i}{k} = b_i$ then GCD $(a_1^n, a_2^n, a_3^n \cdots a_i^n) = k^n$ and

   $\frac{a_1^n}{k^n} = b_1^n, \frac{a_2^n}{k^n} = b_2^n, \frac{a_3^n}{k^n} = b_3^n \cdots \frac{a_i^n}{k^n} = b_i^n$, where n ϵ Z.
4. If $p_1 p_2 p_3 \cdots p_i$ are distinct primes then $p_1^a \neq p_2^b \neq p_3^c \cdots \neq p_i^\alpha$
5. We can write any composite number as the product of prime numbers. i.e.

   $c = p_1^a p_2^b p_3^c \cdots$

Theorem 2: Let a, b, c are composite positive integers and a + b =c, also k = GCD (a, b, c) then x + y = z is relatively prime with each other, where $x = \frac{a}{k}; y = \frac{b}{k}; z = \frac{c}{k}$.

Proof:

Let a, b, c are composite positive integers and a + b =c, also k = GCD (a, b, c) then we can write a + b =c as $xk + yk = zk$

If GCD$(a_1, a_2, a_3 \cdots a_i) = 1$, we multiply each element of GCD $(a_1, a_2, a_3 \cdots a_i) = 1$ with k, we get GCD $(ka_1, ka_2, ka_3 \cdots ka_i) = k$. Which means all the relatively prime numbers changed into composite number of k.

So we divide, $xk + yk = zk$ by k

Hence we get $x + y = z$ ■

Theorem 3: Let a, b, a+b are composite positive integers with GCD k, then we can find

$a^n + b^n = (a + b)^n - \sum_{i=1}^{n-1} \binom{n}{i} a^{n-i} b^i$ with GCD k^n.

If we divide $a^n + b^n = (a + b)^n - \sum_{i=1}^{n-1} \binom{n}{i} a^{n-i} b^i$ by k^n, we get x + y = z with GCD 1,

where $x = \frac{a^n}{k^n}; y = \frac{b^n}{k^n}$ and $z = \frac{c^n - \sum_{i=1}^{n-1} \binom{n}{i} a^{n-i} b^i}{k^n} = \frac{(a^n + b^n)}{k^n}$.

Facts 3:

1. Except two, all prime numbers are odd number.
2. Two only the number stated as even prime.
3. Except 2, $p \pm o \subset E$ Are composite numbers and $p \pm e \subset O$ may be prime or composite.
4. Two and above digits Prime numbers ended with one, three, seven and nine.
5. If even integer ended with zero, we can express 0 = 1 + 9 = 3 + 7 = 5 + 5. These 1, 3, 5, 7 and 9 are ended digit in certain number. But other than 10, we cannot express zero ended number as sum of two five ended prime numbers.
6. If even integer ended with two, we can express 2 = 1 + 1 = 2 + 9 = 5 + 7.
7. If even integer ended with four, we can express 4 = 1 + 3 = 5 + 9 = 7 + 7.
8. If even integer ended with six, we can express 6 = 1 + 5 = 3 + 3 = 7 + 9.
9. If even integer ended with eight, we can express 8 = 1 + 7= 3 + 5 = 9 + 9.
10. Four is the only even number, expressed as sum of two even prime. i.e. 4 = 2 + 2.
11. Two, three, five and seven are base prime. Nine is not base prime but numbers which are ended with nine may be prime number. Here we considered single digit prime numbers are base prime numbers.
12. If digit sum of odd number is either three or six or nine, it would be a composite number.
13. All odd prime numbers having even integer relationship with each other.

Notes

Definition: Residue factors

Let A be a dividend, its factors are abcd and B be a divisor, its factors are abc then residue factor of A ÷ B is d.

Ex

1. Let A =48 and B =16 then factors of A = 2^4 x 3 and factors of B = 2^4 then residue factor $= \frac{2^4 \times 3}{2^4} = 3$. Residue factor is odd.

2. Let A =210 and B =14 then

   Factors of A = 2 x 3 x 5 x 7 and Factors of B = 2 x 7 then residue factor

   $= \frac{2 \times 3 \times 5 \times 7}{2 \times 7} = 3 \times 5 = 15$. Residue factor is odd.

3. Let A = 48 and A = 24 then

   Factors of A = 2^4 x 3 and Factors of B = 2^3 x 3 then residue factor $= \frac{2^4 \times 3}{2^3 \times 3} = 2$.

   Residue factor is even.

Arithmetic operations of odd and even integers

Addition

O + O = E

O + E = O

E + O = O

E + E = E

Subtraction

|O – O | = E

|O – E | = O

|E – O | = O

|E – E | = E

Multiplication

O x O = O

O x E = E

E x O = E

E x E = E

Division

O ÷ O = O

E ÷ E = O if residue factor is odd

E ÷ E = E if residue factor is even ■

Notes

Summations of prime numbers and composite numbers

$$\sum_{O} O_i = O ; \sum_{E} O_i = E ; \sum_{O} E_i = E ; \sum_{E} E_i = E$$

$$\sum_{O} p_i = O \text{ and } \sum_{E} p_i = E ;$$

Gold Bach conjecture

Every even integer greater than two can be expressed as the sum of two prime numbers.

Proof:

We know 4 = 2 + 2, 6 = 3 + 3, 8 = 3 + 5, 10 = 5 + 5 = 7 + 3, 12 = 7 + 5, 14 = 7 + 7, 16 = 5 + 11 …but is it true for all even numbers? So, we try to prove every even integer greater than two can be expressed as the sum of two prime numbers by some ideological concepts. We know the fact 4 only expressed by sum of two even prime. In other words, No even integer greater than four can be expressed by sum of two even prime. But it can be expressed by two odd prime.

Let E be an even number and O_1 and O_2 are odd numbers.

We can express E as sum of two odd numbers. i. e. $E = O_1 + O_2$.

Using above facts, we can say all odd prime numbers are the members of odd numbers. i.e. $p \subset O$

Here we recall one thing, every even integer can be expressed as the difference of two primes. i.e. $E = p_1 - p_2$

Every prime number can be expressed as the sum of odd number and even number. Also we can express prime number as the difference of odd number and even number.

i.e. $o_1 + e_1 = p_1 \in O$ and $o_2 - e_2 = p_2 \in O$.

Hence, we can express $E = (o_1 + o_2) = ((o_1 + e_1) + (o_2 - e_2)) = p_1 + p_2$.

More precisely we can express above as,

$$E = (o_1 + o_2) = ((o_1 + e) + (o_2 - e)) = p_1 + p_2 = ((o_1 - e) + (o_2 + e)) \blacksquare$$

Ex 1:

Let 94, we can express 94 = 63 + 41 = 25 + 69 = 37 + 57 = … for instance,

1. Let 63 and 41, its sum is 94. 94 is even number. Four ended number. Possibility of summation is 1+3, 5+9, and 7+7. 63 and 41 are odd numbers but 63 is composite number and 41 is prime number. We need one prime number instead of 63. We know prime number 61 is near to 63. But 61+2 is 63 so we subtract 2 from 63. Now we get 61. To balance equality of sum, we should add the same 2 with 41. Now we get 43 and 43 is a prime. Also we get 1 and 3 combination. So we can express 94 = (63-2) + (41+2) = 61 + 43.
2. Let 25 and 69, its sum is 94. 94 is even number. Four ended number. Possibility of summation is 1+3, 5+9, and 7+7. 25 and 69 are odd numbers but both are composite

Notes

numbers, we need two prime numbers instead of 25 and 69. We know prime number {…17, 19, 23, 29, 31, 37…} is near to 25. Also {…57, 61, 67, 71, 73 …} near to 69. Select

If e = 2 then

$$94 = (25 + 69) = ((25 - 2) + (69 + 2)) = 23 + 71$$

e = 2 is opted for this way of summation.

But $94 = (25 + 69) = ((25 + 2) + (69 - 2)) = 27 + 67$,

e=2 is not suit for this way of summation.

If e = 6 then $94 = (25 + 69) = ((25 - 6) + (69 + 6)) = 19 + 75$, e = 6 is not suit for this.

If e = 6 then $94 = (25 + 69) = ((25 + 6) + (69 - 6)) = 31 + 64$, e = 6 is not suit for this.

If e = 8 then $94 = (25 + 69) = ((25 + 8) + (69 - 8)) = 33 + 61$, e = 8 is not suit for this.

If e = 8 then $94 = (25 + 69) = ((25 - 8) + (69 + 8)) = 17 + 77$, e = 8 is not suit for this.

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If e = 16 then

$$94 = (25 + 69) = ((25 - 16) + (69 + 16)) = 9 + 85 ,$$

e = 16 is not suit for this way of summation.

But, $94 = (25 + 69) = ((25 + 16) + (69 - 16)) = 41 + 53$,

e=16 is opted for this way of summation.

From above we concluded that, until we expressed sum of two prime numbers equal to an even number, we do repeatedly the above.

References Références Referencias

1. Wikipedia
2. Balasubramani Prema Rangasamy - Some Extensions on Numbers – Advances in Pure Mathematics – 2019. p. 944-958.

Notes