Global Journal of Science Frontier Research: A
Physics and Space Science
Volume 20 Issue 2 Version 1.0 Year 2020
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Dirac Generalized Relativistic Quantum Wave Function Which Gives Right Electrons Number in Each Energy Level by Controlling Quantized Atomic Radius

By Mubarak Dirar Abd-Alla Yagoub & Mohammed Idriss

Abstract- Using generalized special relativistic energy-momentum relation the linear energy-momentum Dirac relation was obtained. This equation was used to find Dirac generalized relativistic quantum equation. This equation was used to find the probability and the number of particles for each atomic energy level. The wave function found using this equation can give an expression for the number of electrons in each energy levels if the atomic radius is quantized and satisfy a certain relation.

Keywords: generalized special relativity, dirac equation, atomic energy levels, number of electrons in the energy level, atomic radius.

GJSFR-A Classification: FOR Code: 020699

Dirac Generalized Relativistic Quantum Wave Function Which Gives Right Electrons Number in Each Energy Level by Controlling Quantized Atomic Radius

Mubarak Dirar Abd-Alla Yagoub [α] & Mohammed Idriss [σ]

Abstract- Using generalized special relativistic energy-momentum relation the linear energy-momentum Dirac relation was obtained. This equation was used to find Dirac generalized relativistic quantum equation. This equation was used to find the probability and the number of particles for each atomic energy level. The wave function found using this equation can give an expression for the number of electrons in each energy levels if the atomic radius is quantized and satisfy a certain relation.

Keywords: generalized special relativity, dirac equation, atomic energy levels, number of electrons in the energy level, atomic radius.

I. Introduction

Quantum mechanics, as formulated by Bohr, Heisenberg, Schrödinger, Pauli, Dirac, and many others, is based on wave particle dual nature of the atomic world. Schrödinger equation is based in addition on the Newtonian energy-momentum relation [1, 2].

According to quantum mechanics, particles do not have definite values of position and momentum at the same moment. The square of the absolute value of the wave function correspond to regions where the particle is more likely to be found if a location measurement is done [3,4].

The Schrödinger equation is the key equation of Quantum mechanics. The first step in the development of a logically consistent theory of non relativistic Quantum mechanics is to drive a wave equation which can describe the particle, wave like behavior of a quantum particle. This equation can describe successfully the behavior of atoms including Hydrogen atom [5, 6].

Hydrogen atom consists of a positively charged proton and a negatively charged electron, moving in orbit under the action of centrifugal force and the influence of their mutual attraction [6,7]. The fast electrons can be described by relativistic Klein- Gordon or Dirac equation [8]. Despite the successes of quantum laws, they suffer from some setbacks, for example the wave function cannot give the correct number of electrons in each energy level [9,10]. It cannot also explain quantum gravity [11, 12].

Fatma Osman, Mubarak Dirar and other studies Quantum Equation for Generalized Special Relativistic Linear Hamiltonian, to solve some of these problems. They use generalized special relativistic energy – momentum relation a useful linear equation was obtained. They found that the coefficients and matrixes resembles that of Dirac relativistic quantum equation Anew quantum linear relativistic equation sensitive to the potential and the effects of fields was also obtained. This equation reduces to that of Dirac in the absence of fields [13].

The solution of this equation predicts the propagation of travelling wave inside fields without attenuation. Thus it can describe the electromagnetic wave propagation inside fields. It also predicts the existence of biophotons as stationary waves that spreads themselves, instantaneously through the surrounding media. It also shows that particles behave as harmonic oscillator inside atoms with rest mass energy equal to the zero point energy. These results agree with observations [14].

Ebtisam A. Mohamed and other studied Derivation of Statistical Physical Laws from Quantum Mechanics. Their work mainly aims to derive Maxwell – Boltzmann distribution, Fermi – Dirac and Bose – Einstein distribution laws by using the quantum wave function in the energy and momentum space beside the relation between the number of particles and chemical potential in addition to some thermodynamic relations concerning probability. The ordinary quantum mechanical wave function for free particle was differentiated with respect to energy and momentum. The derivation of statistical laws from quantum wave function shows the general nature of quantum laws [15]. These successes motivate to try to use GSR Dirac equation to solve the problem of the number of electrons in the energy states. This is done in section (2). Sections (3) and sections (4) are devoted for discussion and conclusion.

Author α σ: Department of physics, Faculty of Science, Sudan University of Science and Technology, Khartoum, Sudan.
e-mail: garaof@yahoo.com

a) Time Independent Potential Dependent Special Relativistic Dirac Equation

The full Dirac potential (GSR) equation takes the from

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} = c\hbar^2\alpha.\nabla\left(\frac{\partial\psi}{\partial t}\right) - c\frac{\hbar}{i}V\alpha.\vec{\nabla}\psi + i\hbar\,\beta m_o c^2\left(\frac{\partial\psi}{\partial t}\right) + \beta m_o c^2 V\psi \quad (1)$$

To find time independent Dirac equation, substitute

$$\psi(r,t) = f(t)u(r) = e^{-i\omega t}u(r) \quad (2)$$

To get

$$\hbar^2\omega^2 = \frac{-i\omega\hbar^2 c\alpha}{u}\nabla u + \frac{ic\hbar v}{u}\alpha.\nabla u + \hbar\omega\beta m_o c^2 + \beta m_o c^2 v$$

From time dependent potential the above equation can be rewrite as

$$ic\hbar\, v\alpha\nabla u - i\hbar\omega(\hbar c)\alpha\nabla u = (\hbar^2\omega^2 - \beta m_o\hbar\omega c^2 - \beta m_o c^2 v)u$$

For simplify let

$$E = \hbar\omega \quad (3)$$

$$ic\hbar\, v\alpha\nabla u - ic\hbar E\alpha\nabla u = (E^2 - \beta E m_o c^2 - \beta m_o c^2 v)u = b_o u - \beta m_o c^2 v u \quad (4)$$

$$b_o = E^2 - \beta E m_o c^2 \quad (5)$$

For hydrogen atoms and hydrogen like atoms the potential is spherical up systemic, in the form

$$V = \frac{-c_0}{r} \quad (6)$$

Let

$$c_0 = \frac{qq_0}{4\pi\varepsilon} \quad (7)$$

Where q & $q_0 \equiv$ charge of particles and r $\equiv$ distance between the centers of particles.

A direct substitution of equation (6) in (4) yields

$$ic\hbar\alpha[v\nabla u - E\nabla u] = b_o u + \frac{\beta m_o c^2 c_0}{r}u \quad (8)$$

Try now a solution

$$c_1 e^{-c_2 r} \quad (9)$$

$$ic\hbar\alpha\left[\frac{c_0}{r}c_2 + c_2 E\right]u = b_o u + \frac{\beta m_o c^2 c_0}{r}u \quad (10)$$

Equating free terms and that having the power (r^{-1}), one gets

$$ic\hbar\alpha c_0 c_2 = \beta m_o c^2 c_0 = -i^2\beta m_o c^2 c_0$$

$$c_2 = -\frac{i\beta}{\alpha}\frac{m_o c}{\hbar} \quad (11)$$

$$ic\alpha\,\hbar c_2 E = b_o = (E^2 - \beta E m_o c^2) = i^2(\beta m_o c^2 - E)E$$

$$c_2 = -i\frac{(E - \beta m_o c^2)}{c\hbar\alpha} \quad (12)$$

Comparing (11) with (12) yields

$$E - \beta E m_o c^2 = \beta m_o c^2$$

$$E = 2\beta m_o c^2 \quad (13)$$

Let

$$c_3 = c\hbar\alpha c_0\,,\ c_4 = c\hbar\alpha c_0 E\,,\ c_5 = \beta m_o c^2 c_0 \quad (14)$$

Sub in (8) to get

$$-i\frac{c_3}{r}\nabla u - ic_4\nabla u = b_o u + \frac{c_5}{r}u \tag{15}$$

Multiply by (r) and use the fact that

$$\nabla u = \frac{du}{dr} \tag{16}$$

One gets

$$-ic_3\frac{du}{dr} - ic_4 r\frac{du}{dr} = (b_o r + c_5)u$$

$$-i(c_3 + c_4 r)\frac{du}{dr} = (b_o r + c_5)u \tag{17}$$

Let

$$y = c_3 + c_4 r\,,\quad dy = c_4 r\,,\quad dr = \left(\frac{1}{c_4}\right)dy \tag{18}$$

Thus

$$-iy(c_4)\frac{du}{dy} = \left[\frac{b_o}{c_4}(y - c_3) + c_5\right]u$$

$$= \left[\frac{b_o}{c_4}y - \frac{b_o c_3}{c_4} + c_5\right]u$$

$$[b_1 y - b_2 + c_5]u = [b_1 y + b_3]u$$

$$b_1 = \frac{b_0}{c_4}\,,\qquad b_2 = \frac{b_o c_3}{c_4}\,,\qquad b_3 = c_5 - b_2 \;= c_5 - \frac{b_o c_3}{c_4} \tag{19}$$

$$-ic_4\frac{du}{dy} = \left[b_1 + \frac{b_3}{y}\right]u\,,\quad \frac{du}{u} = \frac{-1}{ic_4}\left[b_1 + \frac{b_3}{y}\right]dy = i\left[b_4 + \frac{b_5}{y}\right]dy \tag{20}$$

$$b_4 = \frac{b_1}{c_4}\,,\quad b_5 = \frac{b_3}{c_4} \tag{21}$$

$$\int\frac{du}{u} = i\int b_4 dy + ib_5\int\frac{dy}{y} + b_6$$

$$\ln u = ib_4 y + ib_5 \ln y + b_6 \tag{22}$$

$$\ln u - \ln y^{ib_5} = ib_4 y + b_6 \tag{23}$$

$$\ln\left(\frac{u}{y^{ib_5}}\right) = ib_4 y + b_6$$

$$\frac{u}{y^{ib_5}} = e^{ib_4 y}e^{b_6} \tag{24}$$

$$u = b_7 e^{y^{ib_5}}e^{ib_4 y} \tag{25}$$

$$where b_7 = e^{b_6} \tag{26}$$

Thus where in view b_7 of (18) yields

$$u = b_7(c_3 + c_4 r)^{ib_5}e^{ib_4(c_3 + c_4 r)} \tag{27}$$

In view of equations (21),(19) and (4)
Let $b_5 = 0$
Thus

$$b_3 = 0,\quad b_2 = c_5\,,\quad c_5 = \frac{b_o c_3}{c_4},\quad b_0 = \frac{c_4 c_5}{c_3} \;= \frac{E}{c_0}\beta m_o c^2 c_0$$

From (5)

$$E^2 - \beta m_o c^2 E = \beta E m_o c^2$$

$$E^2 = 2\beta m_o c^2 E$$

$$E = 2\beta m_o c^2 \tag{28}$$

Thus

$$u = b_7(c_3 + c_4 r)e^{ib_4(c_3+c_4 r)} \tag{29}$$

The probability of finding the particle at position

$$|u|^2 = {b_7}^2(c_3 + c_4 r)^2 \tag{30}$$

But experimentally it was found that the number of electrons in the energy level n is a nature number n_0

Thus

$$|u|^2 = {b_7}^2(c_3 + c_4 r)^2 = \mathrm{n}_0 \tag{31}$$

$$\mathrm{n}_0 = 2{,}8{,}18{,}32{,}50{,}72{,}98$$

Thus

$$\mathrm{r} = \frac{{\mathrm{n}_0}^{\frac{1}{2}}}{b_7 c_4} - \frac{c_3}{c_4} \tag{32}$$

II. Discussion

The GSR Dirac obtained by Fatma (Quantum Equation for Generalized Special Relativistic Linear Hamiltonian) has been exhibited in equation (1). The time independent part has been found in equation (4), by assuming the time dependent part to be time oscillating.

For hydrogen like atoms the potential is given by equation (6). The GSR Dirac equation for hydrogen atom is given by (8). Rearranging for simplification is found by defining new variably in equation (19). The solution of this equation is a complex wave function given by equation (25) and (27). In its general form this solution is purely complex. To make the wave function (r) dependent the energy should be proportional to the rest mass energy as shown by equation (28). This is quite natural as well as the stable atom corresponds to minimum non existed atomic state [see equation (29)]. The probability or the square of the wave function can be made to be equal to the number of atoms in each level [see equation (31)] by adjusting the atomic radius(r) to be dependent on the number of electrons in a certain energy level. This expression [see equation (32)] shows that (r) increases and quantized. This conforms to observation.

III. Conclusion

The GSR Dirac equation can successfully explain the mystery of the electrons quantum number if the atomic radius is quantized and satisfies a certain relation. It shows also that the stable atom corresponds to the minimum relativistic Einstein energy.

References Références Referencias

1. Michele Maggiore (2005), A Modern Introduction to Quantum Field Theory, Oxford University Press.
2. David Tong (2007), Quantum Field Theory, Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, Wilberforce Road, Cambridge, UK.
3. David_J._Griffiths (2005), Introduction to Quantum Mechanics, Reed College.
4. B.H. Bransden and C. J. Joachain (2000), Quantum mechanics. Second edition, Essec CM 202 England. ISBN 0582-35691.
5. Phillips A. C (2003) Introduction to Quantum Mechanics, New York, ISBN 0-470-85323-9 (Hardback), 0-470-85324-7 (Paperback).
6. Andrew Igla (2017), Quantum Mechanics Schrodinger Schrodinger's Equation Transmission and Reflection at barrier the tunnel effect Compton Effect and university physics, ISBN: soft cover 798-1-5245-9260-8 e Book 978-1-5245-9259-2.
7. Raymond A. Serway and John W. Jewett (2014), Physics for Scientists and Engineers with Modern Physics, Compositor: Lachina Publishing Services 2014, 2010, 2008 by Raymond A. Serway, Library of Congress Control Number: 2012947242, ISBN-13: 978-1-133-95405-7, ISBN-10: 1-133-95405-7.
8. Greiner, W. (2000) Relativistic Quantum Mechanics, Springer, Berlin, 3rd edition.
9. Hilo, M.H.M., et al. (2011), Using of the generalized special relativity (GSR) in estimating the neutrino masses to explain the conversion of electron neutrinos. NS, 3, 334-338. doi: 10.4236/ns.2011. 32020.
10. P. Dirac (1928), The quantum theory of the electron. In Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, The Royal Society.
11. David McMahon (2008), Quantum Field Theory Demystified, McGraw-Hill Companies.

12. Hobson, E fstathion and Lasen by (2006), General relativity An Introduction for physicists, ISBN -13 978-0521-82951-9 published in New York.
13. Fatma Osman, Mubarak Dirar, Ahmed Alhassen, Musa Ibrahim and Sawsan Ahmed, (2019), Quantum Equation for Generalized Special Relativistic Linear Hamiltonian, IJRERD, ISSN: 2455-8761.
14. Fatma Osman, Mubarak Dirar, Ahmed Alhassen, Musa Ibrahim and Sawsan Ahmed, (2019), Time Independent Generalized Special Relativity Quantum.
15. Mubarak Dirar Abd Allah, Ebtisam Abd-Alla Mohamed, Ibrahim Adam Ibrahim Hammad, Ibrahim Alfaki (2017), Derivation of Statistical Physical Laws From Quantum Mechanics, American Journal of Innovative Research and Applied Sciences. ISSN 2429-5396, www.american-jiras.com

This page is intentionally left blank