Global Journal of Science Frontier Research: a
Physics and Space Science
Volume 20 Issue 12 Version 1.0 Year 2020
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Interaction of Electromagnetic Wave and Metamaterial with Inductive Type Chiral Inclusions

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GJSFR-A Classification: FOR Code: 260203

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Strictly as per the compliance and regulations of:

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I. Introduction

Now the materials (Greek. "meta" outside), i.e. composite materials with the various inclusions distributed both chaotically, and periodically are widely applied in particular in a radio engineering, at designing of space devices, in medicine, etc., [1, 2, 3, 4]. Due to these inclusions the received materials have many useful physical, electric, optical and other properties which are not present at natural substances. Among metamaterials are allocated the substances with chiral properties [5] which are capable to rotate a plane of polarization of electromagnetic waves. In optics as analogue of similar substances are the optical active substances, for example, quartz, a glucose solution etc.

However methods of calculation of metamaterials are enough limited [6]. Basically all calculations are based on the solving of the Maxwell's equations and the material equations selected according to a problem.

The existing method has restrictions since average characteristics of metamaterials are usually used only, for example, a chiral parameter.

In the present work attempt of more detailed approach to properties of chiral inclusions into metamaterials is made; the analysis of influence of these properties on interaction of chiral elements with the electromagnetic wave falling on a plate from a metamaterial is carried out.

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II. Standard Method of Calculation of a Metamaterial with an Electromagnetic Wave Interaction

At research of metamaterials with chiral inclusions on the basis of the Maxwell's equations usually use the material equations including so-called chiral parameter χ. In [5, 6, 7] the material equations in the following kind are offered:

$$\boldsymbol{D} = \varepsilon_a \boldsymbol{E} \mp i \frac{\chi}{V} \boldsymbol{H}, \quad (1)$$

$$\boldsymbol{B} = \mu_a \boldsymbol{H} \pm i \frac{\chi}{V} \boldsymbol{E}, \quad (2)$$

where D and B there are an induction of electric and magnetic fields in the electromagnetic wave propagating in a chiral medium, E and H – strength of the electric and magnetic components in wave, ε_a and μ_a - absolute electric and magnetic permeability of a chiral medium, V – velocity of an electromagnetic wave in a chiral medium, χ - a chiral parameter, in this case dimensionless size.

In [7] it is shown that the material equations (1) and (2) can be written down in more simple kind:

$$\boldsymbol{D} = (1 \pm \chi)\varepsilon_a \boldsymbol{E}, \quad (3)$$

$$\boldsymbol{B} = (1 \pm \chi)\mu_a \boldsymbol{H}. \quad (4)$$

In formulas (1) - (4) top signs define the right-turning chiral element, bottom signs – left-turning.

Using (3) and (4) it is possible to show [7] that if a chiral medium has only reactive resistance, the electromagnetic wave in it submits to the wave equations:

$$\Delta \boldsymbol{D} = \left(\frac{1 \pm \chi}{V}\right)^2 \frac{\partial^2 \boldsymbol{D}}{\partial t^2}, \quad (5)$$

$$\Delta \boldsymbol{B} = \left(\frac{1 \pm \chi}{V}\right)^2 \frac{\partial^2 \boldsymbol{B}}{\partial t^2}, \quad (6)$$

where t there is time.

III. Detailed Method of Calculation of Metamaterial with Electromagnetic Wave Interaction

Let's consider a plate of the metamaterial with chiral inclusions of the inductive type. The plate consist of the dielectric in which are included the current-carrying chiral elements as spirals which axis is directed across a plate. The chiral elements are distributed periodically.

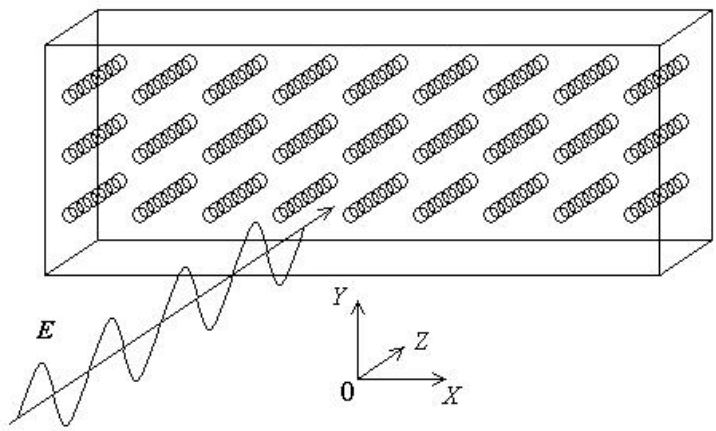

Figure 1: The plate of metamaterials irradiated by an electromagnetic wave

On fig. 1 the irradiation of a plate by an electromagnetic wave is shown. We assumed that chiral inclusions have no active resistance. The chiral element completely penetrates a plate.

Feature of a plate is the capacity distributed on its surfaces at dot inductive inclusions. Therefore to examine the interaction of separate chiral element having inductance and capacity with an electromagnetic wave is incorrectly.

Let's consider a plate consisting of chiral elements one lines, fig. 2.

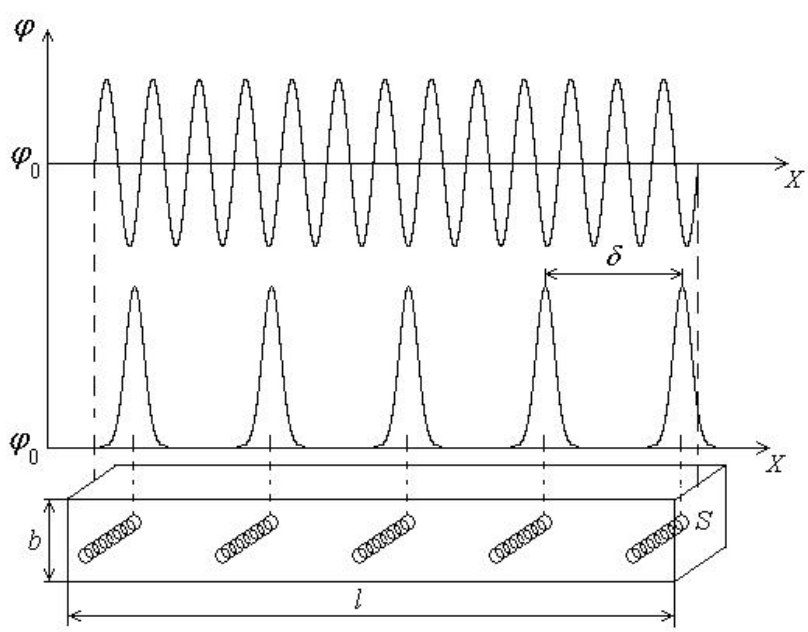

Figure 2: The single-row chiral plate

In [8] it has been shown that the potential φ on a plate submits to the nonlinear differential equation:

$$V^2(\varphi-\varphi_0)\frac{\partial^2\varphi}{\partial X^2}=\left(\frac{\partial\varphi}{\partial t}\right)^2-(\varphi-\varphi_0)^2\omega_0^2. \quad (7)$$

where φ_0 there is an origin of potential, V – a velocity of an electromagnetic field along a plate, ω_0 - natural frequency of the chiral system.

Let's notice that the nonlinear equation similar (7) arises at research of a self-induced transparency in substance [9].

IV. The Solution as Solitary Waves

The nonlinear equation (7) has solution as a solitary traveling wave:

$$\varphi - \varphi_0 = \varphi_{\max} \exp\left(-\frac{\left(k_0\left(X - X_0\right) \pm \omega_0\left(t - t_0\right)\right)^2}{2} \right). \quad (8)$$

where $k_0 = \dfrac{\omega_0}{V}$ there is a wave number of a natural traveling wave in the chiral medium, $\varphi_{\max}$ - a peak value of potential $\varphi - \varphi_0$, X_0 - coordinate of the chiral element center, and accordingly a maximum (center) of a wave impulse, t_0 - a time of achievement of this maximum. The sign a minus concerns to a wave spreading from left to right, and sign plus from right to left.

Growth of potential above chiral inclusions, fig. 2, is caused by proportionality of the chiral inclusions reactance their inductivities.

From the analysis of both curves it is possible to conclude that the top curve, fig. 2, concern to often enough inclusions of the chiral elements in a plate, and bottom to more rare. Therefore into the solution (8) to enter a chiral parameter it is irrational.

Obviously for the nonlinear equation (7) there should be a multiwave solution. Multiwave solutions are found for very much limited circle of the nonlinear wave equations [10, 11]. The multiwave solution should depend on concentration of the chiral elements in a plate. Only with its help it is possible to understand under what conditions it is possible is proved to enter the chiral parameter, i.e. to understand borders of the material equations (1) - (4) applicability.

The equation (7) supposes the multiwave solution as:

$$\varphi = \varphi_0 + \varphi_{\max} \sum_{n=1}^{N} \exp\left(-\frac{\left(k_0\left(X - X_{0n}\right) - \omega_0\left(t - t_{0n}\right)\right)^2}{2} \right), \quad (9)$$

where N there are quantity of the waves-impulses kept within a length l of a plate, fig. 2, equal to number of the chiral elements, n - current number of an impulse, X_{0n} - coordinates of waves-impulses maxima, t_{0n} - times of these maxima achievement.

Substituting (9) in (7) we shall find:

$$V^2 \left(\frac{\partial^2 \varphi}{\partial X^2} \right) \sum_{n=1}^{N} \varphi_n + \omega_0^2 \left(\sum_{n=1}^{N} \varphi_n \right)^2 = \left(\frac{\partial \varphi}{\partial t} \right)^2. \quad (10)$$

where it is designated:

$$\varphi_n = \exp\left(-\frac{\left(k_0\left(X - X_{0n}\right) - \omega_0\left(t - t_{0n}\right)\right)^2}{2} \right). \quad (11)$$

Finding the derivatives on coordinate X:

$$\frac{\partial^2 \varphi}{\partial X^2} = \sum_{n=1}^{N} \left(\varphi_n \left(k_0\left(X - X_{0n}\right) - \omega_0\left(t - t_{0n}\right)\right)^2 k_0^2 + \varphi_n k_0^2 \right) =$$

$$= k_0^2 \sum_{n=1}^{N} \varphi_n \left(k_0\left(X - X_{0n}\right) - \omega_0\left(t - t_{0n}\right)\right)^2 + k_0^2 \sum_{n=1}^{N} \varphi_n, \quad (12)$$

and on time t:

$$\frac{\partial \varphi}{\partial t} = -\omega_0 \sum_{n=1}^{N} \varphi_n \left(k_0\left(X - X_{0n}\right) - \omega_0\left(t - t_{0n}\right)\right), \quad (13)$$

we substitute (12) and (13) in the equation (10) and taking into account $k_0 V = \omega_0$ we have:

$$\left(\sum_{n=1}^{N}\varphi_n\left(k_0\left(X-X_{0n}\right)-\omega_0\left(t-t_{0n}\right)\right)^2+\sum_{n=1}^{N}\varphi_n\right)\sum_{n=1}^{N}\varphi_n=$$

$$=\left(\sum_{n=1}^{N}\varphi_n\right)^2+\left(\sum_{n=1}^{N}\varphi_n\left(k_0\left(X-X_{0n}\right)-\omega_0\left(t-t_{0n}\right)\right)\right)^2. \quad (14)$$

Reducing in the left and right parts (14) the identical addends $\left(\sum_{n=1}^{N}\varphi_n\right)^2$ we shall find:

$$\sum_{n=1}^{N}\varphi_n\sum_{n=1}^{N}\varphi_n\left(k_0\left(X-X_{0n}\right)-\omega_0\left(t-t_{0n}\right)\right)^2=$$

$$=\left(\sum_{n=1}^{N}\varphi_n\left(k_0\left(X-X_{0n}\right)-\omega_0\left(t-t_{0n}\right)\right)\right)^2, \quad (15)$$

Let's consider two one after the other going the identical impulses $n=1,2$. Writing down for this case the formula (15) we shall find:

$$\left(\varphi_1+\varphi_2\right)\left(\varphi_1\left(k_0\left(X-X_{01}\right)-\omega_0\left(t-t_{01}\right)\right)^2+\varphi_2\left(k_0\left(X-X_{02}\right)-\omega_0\left(t-t_{02}\right)\right)^2\right)=$$

$$=\left(\varphi_1\left(k_0\left(X-X_{01}\right)-\omega_0\left(t-t_{01}\right)\right)+\varphi_2\left(k_0\left(X-X_{02}\right)-\omega_0\left(t-t_{02}\right)\right)\right)^2. \quad (16)$$

Transforming the formula (16) we shall receive:

$$k_0\left(X_{02}-X_{01}\right)-\omega_0\left(t_{02}-t_{01}\right)=0. \quad (17)$$

The formula (17) shows that distance between chiral elements $\delta=\left(X_{02}-X_{01}\right)$, fig. 2, an electromagnetic impulse propagates in time $\left(t_{02}-t_{01}\right)$ with a speed $V=\frac{\omega_0}{k_0}$. The size $\frac{1}{\delta}$ characterizes linear concentration of the chiral elements in a plate.

Using in (15) $t_{0n}=\frac{X_{0n}}{V}=\frac{k_0X_{0n}}{\omega_0}$, we receive that expressions in brackets $\left(k_0\left(X-X_{0n}\right)-\omega_0\left(t-t_{0n}\right)\right)^2=\left(k_0\left(X\right)-\left(\omega_0t\right)\right)^2$ do not depend from *n* they can be taken out for a symbol of the sum and to reduce. In result (15) turns to identity.

Hence (9) is the multiwave solution of the nonlinear equation (7).

The most simple kind the multiwave solution (9) has in occasion of identical distance between all impulses and accordingly between of the chiral elements. In this case coordinates of maxima of impulses are $X_{0n}=n\delta$, and times of achievement of maxima $t_{0n}=\frac{k_0X_{0n}}{\omega_0}=\frac{k_0n\delta}{\omega_0}$.

On fig. 3 for an illustration the some impulses following one after another plotted under the formula (9) are shown under conditions of the dimensionless sizes: $V=0$ - absence of dependence on time (the figure fixed in time) $\varphi_0=0$, $\varphi_{\max}=1$, $k_0=2$, $\delta=4$.

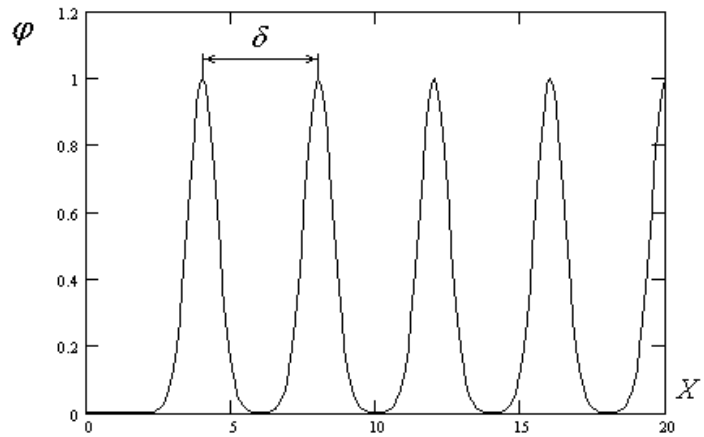

Figure 3: The impulses following one after another in the multiwave solution

Thus the formula (9) under condition of uniform distribution of identical impulses is the multiwave periodic solution of the nonlinear equation (7).

V. The Solution as Standing Waves

Let's consider in more detail another kind of the wave arising on single-row chiral plate at falling on it of an electromagnetic wave.

Standing waves are formed in linear systems as a result of superposition (interference) of the direct and reflected traveling waves more often. However it is known that standing waves can arise in nonlinear systems [12]. Many physical processes have essentially nonlinear character and process of standing waves occurrence in such systems is nontrivial. We shall examine an opportunity of standing waves occurrence in researched chiral medium.

The nonlinear equation (7) can be solved by a method of the Fourier variables division [13]. We search the solution of the equation (7) as:

$$\varphi - \varphi_0 = \varphi(X)T(t). \qquad (18)$$

where $\varphi(X)$ there is function only coordinates *X*, $T(t)$ - a function only time *t*.

Having substituted (18) in (7) we shall find:

$$V^2\varphi(X)T^2(t)\frac{\partial^2\varphi(X)}{\partial X^2} = \left(\varphi(X)\frac{\partial T(t)}{\partial t}\right)^2 - \varphi^2(X)T^2(t)\omega_0^2. \qquad (19)$$

Let's divide both parts of the equation on $\varphi^2(X)T^2(t)$. In result we shall receive:

$$V^2\frac{1}{\varphi(X)}\frac{\partial^2\varphi(X)}{\partial X^2} + \omega_0^2 = \left(\frac{1}{T(t)}\frac{\partial T(t)}{\partial t}\right)^2 = -\omega^2. \qquad (20)$$

where ω there is a constant.

The equation (20) breaks up to two independent equations. The equation dependent on *X* looks like:

$$\frac{\partial^2\varphi(X)}{\partial X^2} + \left(k_0^2 + \frac{\alpha^2}{V^2}\right)\varphi(X) = 0. \qquad (21)$$

Designating $k_S^2 = k_0^2 + \frac{\omega^2}{V^2}$ the solution of the equation (21) we shall write down as:

$$\varphi(X) = \varphi(0)\exp(ik_S X). \qquad (22)$$

where $\varphi(0)$ there is value of function $\varphi(X)$ in the beginning of coordinates.

The second equation of equality (20) looks like:

$$\frac{\partial T(t)}{\partial t} = i\omega T(t). \qquad (23)$$

Solving this equation we shall find:

$$T(t) = T(0)\exp(i\omega t), \qquad (24)$$

where $T(0)$ there is initial value of function $T(t)$.

Using (18), (22) and (24) we shall find the solution of the equation (7) as:

$$\varphi - \varphi_0 = \varphi_A \exp(i\omega t)\exp(ik_S X), \qquad (25)$$

where it is designated $\varphi_A = T(0)\varphi(0)$ there is a peak value of potential $\varphi - \varphi_0$ on a plate.

The function $\varphi - \varphi_0$ should not have imaginary addends, the potential is real size. Use an exponents with imaginary parameters is entered for convenience of transformations. Really in these exponents it is necessary to take into account only real items. Therefore the formula (25) describes the solution of the equation (7) as standing waves:

$$\varphi - \varphi_0 = \varphi_A \cos\omega t \cos k_S X = \varphi_A \cos\omega t \cos\frac{2\pi X}{\delta}, \qquad (26)$$

where φ_A there is a peak value of standing waves, δ - length of a wave.

Condition of the nodes occurrence in a standing wave $X_{\text{ns}} = \pm(2n+1)\frac{\delta}{4}$, where $n = 0, 1, 2, \ldots$.

On the ends of the single-row chiral plate, fig. 2, should be nodes of a standing wave. If excitation of a wave occurs in the center of a plate the number of the maximal distant node from a center of a plate can be found under the formula $\pm\frac{l}{2} = \pm(2n_{\max}+1)\frac{\delta}{4}$ or $n_{\max} = \left(\frac{l}{\delta} - \frac{1}{2}\right)$.

It is necessary to note that running waves $\varphi - \varphi_0 = \frac{\varphi_A}{2}\cos(k_S X \pm \omega t)$ with account $k_S^2 = \frac{\omega_0^2 + \omega^2}{V^2}$ are not the solution of the equation (7) therefore the formula (26) from the physical point of view cannot be presented as a sum of the direct, and reflected from borders plate waves though mathematical this procedure is simple for making. It is consequence of the equation (7) nonlinearity.

It is interesting to track graphically a transition of the multiwave solution (9) in the solution as standing waves (26). This transition is carried out at rapprochement of impulses, fig. 2, 3, i.e. at reduction of size δ.

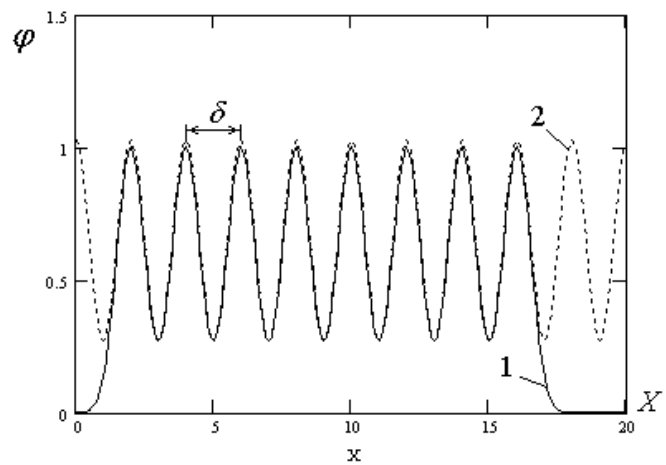

Figure 4: Transition of the multiwave solution in the solution as a standing wave: 1 - the multiwave solution, 2 - a standing wave

On fig. 4 the two curves are shown. Curve 1 is plotted under the formula (9) under conditions: $V = 0$, $\varphi_0 = 0$, $\varphi_{\max} = 1$, $k_0 = 2$, $\delta = 2$ for $N = 8$ impulses. Curve 2 is plotted (dotted line) under the formula (26) under conditions $\varphi_0 = 0{,}65$ and $\varphi_A \cos\omega t = 0{,}38$ for some moment of time *t*.

VI. Conclusion

Distribution of potential to a plate from a metamaterial with inductive chiral inclusions is investigated as with use of the material equations together with the Maxwell's equations, and on the basis of a detailed method of calculation of the chiral elements and an electromagnetic wave interaction. Comparison of two approaches has allowed to find out that introduction of the chiral parameter is correct only at enough high concentration of the chiral inclusions. At use of a detailed method of calculation the nonlinear equation for the potential having solutions as standing waves and solitary waves is received. Traveling waves are not the solution of this equation. Existence of the multiwave solution of the nonlinear equation is shown. At reduction of distance between chiral elements the process of transition of the multiwave solution of the nonlinear equation in the solution as a standing wave is investigated.

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