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Availability and Profit Analysis of a Series-Parallel System

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Abstract- This paper presents the study of availability and profit analysis of a series-parallel system consisting of four units A, B, C and D arranged in series in which subsystem D consist of three units arranged in parallel. The system works if any of A, B, C or D work. The Markovian approach was adopted to model the system behaviour with the assumption that the failure and repair rates of each subsystem follow exponential distribution. The differential equations that described the system were formulated to analyse the probability for each state. These equations were solved recursively to obtain explicit expressions for steady-state availability, busy period of repairman and profit function. The computed results are demonstrated by graphs. The results show that increase in repair rate increases the availability and associated profit. The results of the designed model are beneficial to reliability and maintenance managers.

Keywords: *availability, profit, reliability, series-parallel.*

GJSFR-F Classification: *MSC 2010: 33E20*

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Ref

7. Sanusi, A., Yusuf, I., Mamuda. B.Y. (2020). Performance evaluation of an industrial configured as series-parallel system, Journal of Mathematical and Computational Science. 10, pp. 692-712.

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I. Introduction

The series-parallel systems consist of subsystems connected in series where each subsystem consists of units arranged in parallel. The failure of any of the subsystems will lead to the failure of the whole system. These systems can be seen in industries, power stations, manufacturing, production and telecommunications. Owing to their importance in industries and economy, reliability measures of such systems have become an area of interest. System are normally studied in terms their reliability measures such as steady-state availability, busy period, profit function and mean time to system failure (MTSF). Availability and profit of redundant systems can be enhanced using highly reliable structural system design. Improving the system reliability and availability will also increase the production and associated profit. The study of reliability of repairable systems is an important topic in Engineering and Operation Research. System reliability is a very meaningful measure and achieving required level of reliability and availability is an essential requisite. System reliability and availability depends on the system structure.

Due to their importance in promoting and sustaining industries and economy, reliability and economic analysis of such systems have received attention from different researchers, see for instance. Sanusi et al [7] have recently presented the performance evaluation of an industrial configured as series-parallel system. Gahlot et al.[1]have analyzed the performance assessment of serial system with different types of failure and repair policy. M. S. Rao et al. [6] have analyzed a system dynamics model for transient availability modeling of repairable redundant systems.

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Singh and Ayagi [8] presented the study of availability of standby complex system under waiting repair and human failure using Gumbel–Hougaard family copula distribution. Yang et al [10]have discussed the reliability assessment of system with inconsistent priors and multi-level data. Malik *et al*[4] have presented the study of performance modeling and maintenance priorities decision for water flow system of a coal based thermal power plant. Yusuf [12] has used Chapman Kolmogorov differential equation to present the availability and other relevant measures of a two-unit active parallel system connected with external supporting device for operation with online and offline preventive maintenance. Singh et al. [9] discussed the performance of the complex system in the series configuration under different failure and repair discipline. Kumar *et al*[3] have studied the availability and cost analysis of an engineering system involving subsystems in series configuration. Ram Niwas and Harish Garg [5] studied the availability, reliability and profit of an industrial system based on cost free warranty policy. Yusuf, I [11] presented the availability modelling and evaluation of repairable system subject to minor deterioration under imperfect repairs. M. S. Kadyan and Ramesh Kumar [2] have presented the study of the availability and profit analysis of feeding system in the sugar industry.

In this paper, the study of availability and profit analysis of a series-parallel system is presented. The primary focus of this study is to analyse the effects of system parameters on availability and profit and make comparison to know the most critical component in the system. The rest of this paper is organised as follows: Section 2 presents the notations and assumptions used throughout the study. System description is given in section 3. Section 4 deals with the formulation and solution of the model. The results of our numerical simulations and discussion are given in section 5. The paper is concluded in section 6.

II. Notations and Assumptions

a) Notations

$\alpha_1, \alpha_2, \alpha_3, \alpha_4$: Failure rates of subsystem A, B, C and D respectively.

$\beta_1, \beta_2, \beta_3, \beta_4$: Repair rates of subsystem A, B, C and D respectively.

A_V: Steady-state availability.

PF: Profit function.

C_0: Revenue generated when the system is in working state and has no income when in failed state.

C_1: Cost of each repair for supporting device.

C_2: Cost of each repair for failed unit.

B_T: Busy period of repairman.

$\frac{d}{dt}$: Derivative with respect to time t.

b) Assumptions

1. The system may be in operating state or in a failed state.
2. Failure and repair rates are constant overtime and statistically independent.
3. A repair unit is as good as new performance wise for a specified duration.
4. Sufficient repair facilities are provided, i.e. no waiting time to start the repairs.
5. Standby units (if any) are of same nature and capacity as active units.
6. System failure/repair follow exponential distribution.

Ref

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III. Description of the Model

Notes

In this paper, a series-parallel system is considered. Figure 1 and figure 2 depict the block and transition diagrams of the system respectively. The system consists of four subsystems in which at least two units must be in operation in subsystem D for the system to work. The system failed when two units failed. The Markovian approach was adopted to model the system behaviour with the assumption that the failure and repair rates of each subsystem follow exponential distribution. The differential equations that described the system were formulated to analyse the probability for each state. These equations were solved recursively to obtain steady-state availability of the system and profit function. The model consists of four subsystems with the following description:

i) *Subsystem (A):* is a single unit arranged in series. Failure of this unit causes the complete failure of the system.

ii) *Subsystem (B):* a single unit arranged in series. Failure of this unit causes the complete failure of the system.

iii) *Subsystem (C):* is a single unit arranged in series. Failure of this unit causes the complete failure of the system.

iv) *Subsystem (D):* consists of four units arranged in parallel, two are in operative mode and one in cold standby mode. Complete failure of the system will occur only when two units failed at the same time.

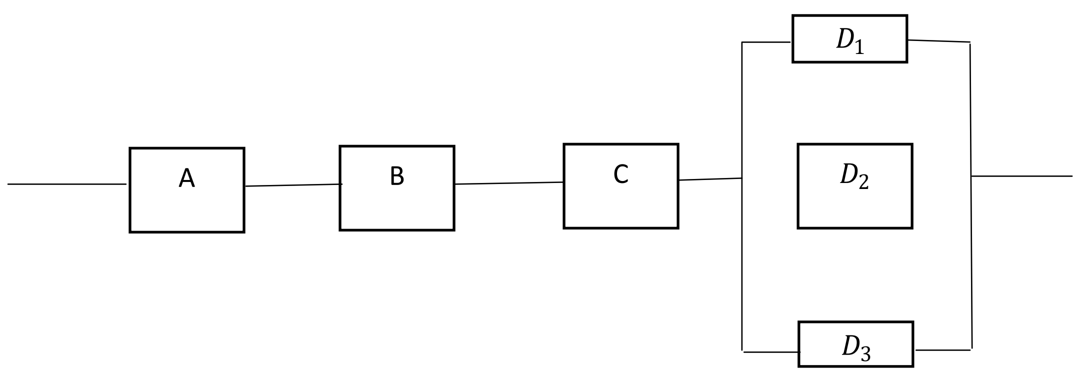

Figure 3.1: Reliability Block Diagram

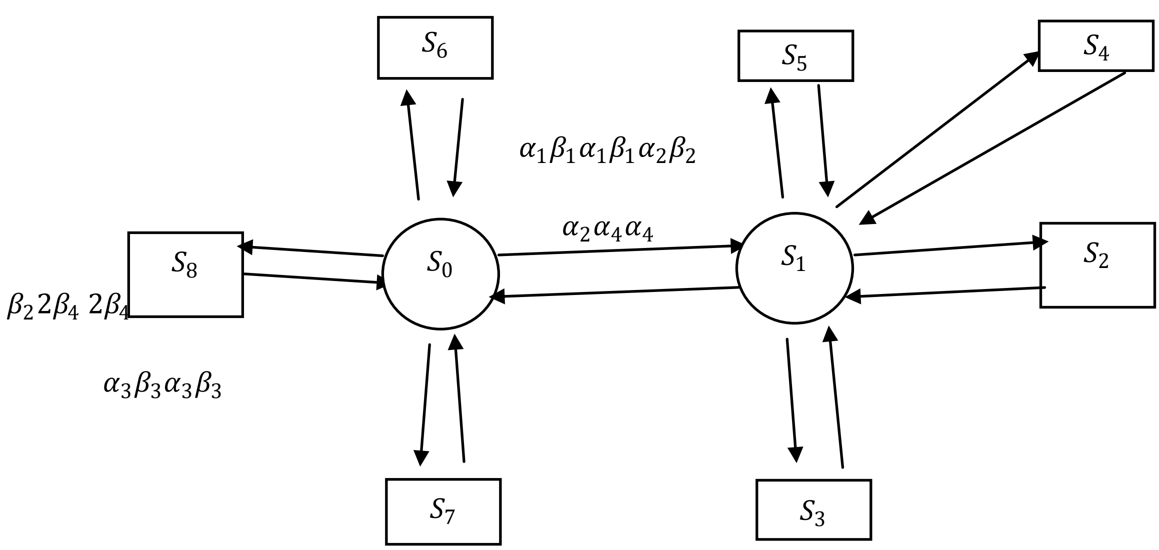

Figure 2: Transition Diagram of the System

IV. Formulation and Solution of Mathematical Model

The differential equations associated with the transition diagram (Figure 2), are derived on the basis of Markov birth-death process. Various probability considerations generate the following sets of differential equations:

$$\left[\frac{d}{dt}+\alpha_1+\alpha_2+\alpha_3+\alpha_4\right]P_0=2\beta_4P_1+\beta_1P_6+\beta_3P_7+\beta_2P_8 \quad (1)$$

$$\left[\frac{d}{dt}+\alpha_1+\alpha_2+\alpha_3+\alpha_4+2\beta_4\right]P_1=\alpha_4P_0+2\beta_4P_2+\beta_3P_3+\beta_2P_4+\beta_1P_5 \quad (2)$$

$$\left[\frac{d}{dt}+2\beta_1\right]P_2=\alpha_4P_1 \quad (3)$$

$$\left[\frac{d}{dt}+\beta_3\right]P_3=\alpha_3P_1 \quad (4)$$

$$\left[\frac{d}{dt}+\beta_2\right]P_4=\alpha_2P_1 \quad (5)$$

$$\left[\frac{d}{dt}+\beta_1\right]P_5=\alpha_1P_1 \quad (6)$$

$$\left[\frac{d}{dt}+\beta_1\right]P_6=\alpha_1P_0 \quad (7)$$

$$\left[\frac{d}{dt}+\beta_3\right]P_7=\alpha_3P_0 \quad (8)$$

$$\left[\frac{d}{dt}+\beta_2\right]P_8=\alpha_2P_0 \quad (9)$$

Where, the initial conditions at time t = 0 are:

$$P(t)_i=\begin{cases}0, if\ i=0\\0, if\ i\neq 0\end{cases} \quad (10)$$

a) Availability equations

To get the steady state availability of the system (ASS), (i.e., time independent performance behaviour) which mean d/dt =0 and t $\rightarrow\infty$, the above equations (eq.1 to 10) become:

$$[\alpha_1+\alpha_2+\alpha_3+\alpha_4]P_0=2\beta_4P_1+\beta_1P_6+\beta_3P_7+\beta_2P_8 \quad (11)$$

Notes

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$$[\alpha_1+\alpha_2+\alpha_3+\alpha_4+2\beta_4]P_1=\alpha_4P_0+2\beta_4P_2+\beta_3P_3+\beta_2P_4+\beta_1P_5 \quad (12)$$

$$2\beta_1P_2=\alpha_4P_1 \quad (13)$$

$$\beta_3P_3=\alpha_3P_1 \quad (14)$$

$$\beta_2P_4=\alpha_2P_1 \quad (15)$$

$$\beta_1P_5=\alpha_1P_1 \quad (16)$$

$$\beta_1P_6=\alpha_1P_0 \quad (17)$$

$$\beta_3P_7=\alpha_3P_0 \quad (18)$$

$$\beta_2P_8=\alpha_2P_0 \quad (19)$$

These equations were solved recursively, the values of steady state probabilities are as follows:

$$P_1=\frac{1}{4}\left(\frac{\alpha_4}{\beta_4}\right)P_0, P_2=\frac{1}{8}\left(\frac{\alpha_4}{\beta_4}\right)^2P_0, P_3=\frac{1}{4}\left(\frac{\alpha_3\alpha_4}{\beta_3\beta_4}\right)P_0, P_4=\frac{1}{4}\left(\frac{\alpha_2\alpha_4}{\beta_2\beta_4}\right)P_0, P_5=\frac{1}{4}\left(\frac{\alpha_1\alpha_4}{\beta_1\beta_4}\right)P_0,$$

$$P_6=\frac{\alpha_1}{\beta_1}P_0, P_7=\frac{\alpha_3}{\beta_3}P_0, P_8=\frac{\alpha_2}{\beta_2}P_0 \quad (20)$$

The probability of full working capacity, namely, P0 determined by using normalizing condition: (i.e. sum of the probabilities of all working states, reduced capacity and failed states is equal to 1).
$\sum_{I=0}^{8}P_i=1$, hence,

$$P_0=\frac{1}{\left(1+\frac{1\alpha_4}{4\beta_4}\right)\left(1+\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}\right)+\frac{1}{8}\left(\frac{\alpha_4}{\beta_4}\right)^2} \quad (21)$$

Now, the steady state availability of the system may be obtained as summation of all working state probabilities as
Hence $Av.=\sum_{i=0}^{1}P_i$ =P0 + P1

$$Av.=\frac{\alpha_4+4\beta_4}{4\beta_4\left[\left(1+\frac{1\alpha_4}{4\beta_4}\right)\left(1+\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}\right)+\frac{1}{8}\left(\frac{\alpha_4}{\beta_4}\right)^2\right]} \quad (22)$$

b) Profit Analysis

Let C_0, $C_1$and C_2 be the revenue generated when the system is in working state and has no income when in failed state, cost of each repair for supporting units and accumulated cost of each repair for failed units (corrective maintenance) respectively. Then the expected total profit per unit time incurred to the system in the steady-state is:

$$P_F = C_0 A_V - C_1 B_{T1} + C_2 B_{T2} \tag{23}$$

Where $B_{T1} = P_1$

Thus,

$$B_{T1} = \frac{\alpha_4}{4\beta_4\left[\left(1+\frac{1\alpha_4}{4\beta_4}\right)\left(1+\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}\right)+\frac{1}{8}\left(\frac{\alpha_4}{\beta_4}\right)^2\right]} \tag{24}$$

Also

$$B_{T2} = P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8$$

$$B_{T2} = \frac{\left(1+\frac{1\alpha_4}{4\beta_4}\right)\left(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}\right)+\frac{1}{8}\left(\frac{\alpha_4}{\beta_4}\right)^2}{\left(1+\frac{1\alpha_4}{4\beta_4}\right)\left(1+\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}\right)+\frac{1}{8}\left(\frac{\alpha_4}{\beta_4}\right)^2} \tag{25}$$

Combining eq.22, 23, 24 and 25 we obtain:

$$P_F = \frac{1}{\left(1+\frac{1}{4}\frac{\alpha_4}{\beta_4}\right)\left(1+\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}\right)+\frac{1}{8}\left(\frac{\alpha_4}{\beta_4}\right)^2}\left[\frac{C_0(\alpha_4+4\beta_4)}{4\beta_4} - \frac{C_1\alpha_4}{4\beta_4} + \left[\left(1+\frac{1}{4}\frac{\alpha_4}{\beta_4}\right)\left(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}\right)+\frac{1}{8}\left(\frac{\alpha_4}{\beta_4}\right)^2\right]\right] \tag{26}$$

V. Discussion of Results

In this section, we numerically obtained and compare the results for system availability and profit function for the developed models. The objectives here are to analyze graphically the effects of system parameters on availability and profit and make comparison. The following set of parameters values are fixed throughout the simulations for consistency.

Figure 4.1: $\alpha_2 = 0.2, \alpha_3 = 0.3, \alpha_4 = 0.4, \beta_2 = 0.01, \beta_3 = 0.02, \beta_4 = 0.04$.

Figure 4.2: $\alpha_1 = 0.15, \alpha_3 = 0.2, \alpha_4 = 0.3, \beta_1 = 0.02, \beta_3 = 0.03, \beta_4 = 0.04$.

Figure 4.3: $\alpha_1 = 0.4, \alpha_2 = 0.5, \alpha_4 = 0.6, \beta_1 = 0.05, \beta_2 = 0.06, \beta_4 = 0.07$.

Figure 4.4: $\alpha_1 = 0.25, \alpha_2 = 0.3, \alpha_3 = 0.35, \beta_1 = 0.03, \beta_2 = 0.04, \beta_3 = 0.05$.

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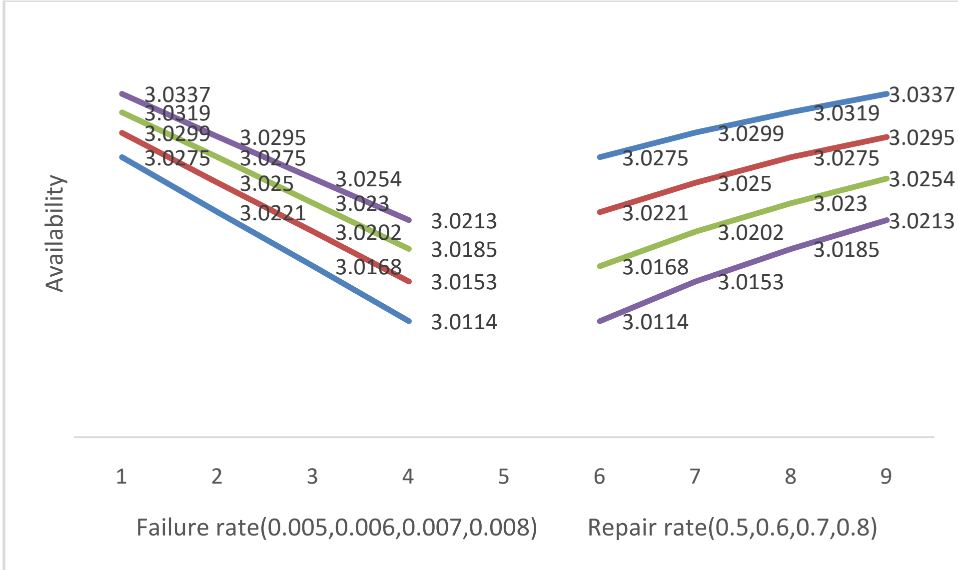

Figure 3: Effect of Failure rate α_1 and Repair rate β_1 on availability

Figure 3 shows the effect of failure rate α_1 and repair rate β_1 on the long run availability of the system. It is seen from this figure that as the failure rate α_1 increases, the system availability decreases considerably. Similarly, as the repair rate $\beta_1$increases, the system availability increases appreciably. This simulation suggests that regular repair will improve the system availability.Figure 4 presents the impact of failure rate α_2 and repair rate β_2 on system availability. We observe from this figure that as the failure rate α_2 grows, the system availability tends to decreases. It is also observed from this figure that the system availability increases with increase in the values of repair rate β_2.

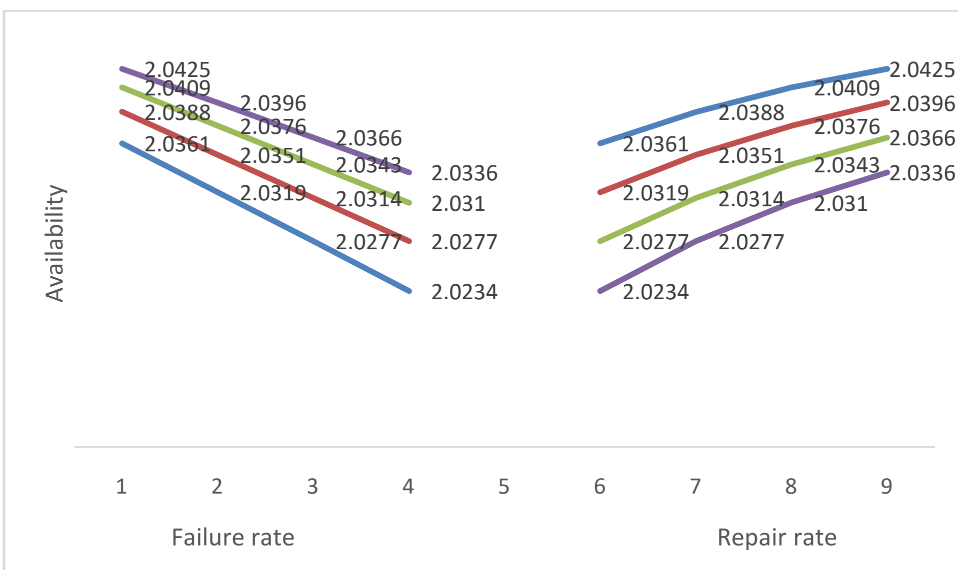

Figure 4: Effect of Failure rate α_2 and Repair rate β_2 on availability

The availability of system with different values of parameters $\alpha_3$and β_3 is correspondingly represented in Figure 5. From this figure, one can observe that these two parameters have significant effect on the system availability. Increasing parameter

$\alpha_3$leads to lower values for system availability, but increasing parameter β_3 leads to higher values for system availability.

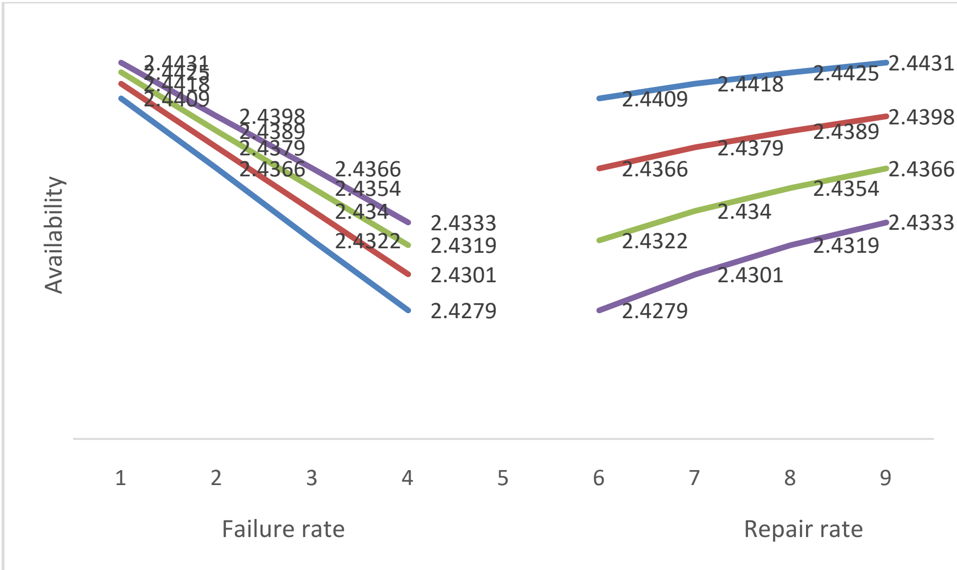

Figure 5: Effect of Failure rate α_3 and Repair rate β_3 on availability

In Figure 6, the availability of System is plotted with respect to parameters $\alpha_4$and β_4 respectively. From this figure, the behavior of availability with increasing parameter α_4 is descending. In the case of increasing parameter β_4 the system availability is ascending.

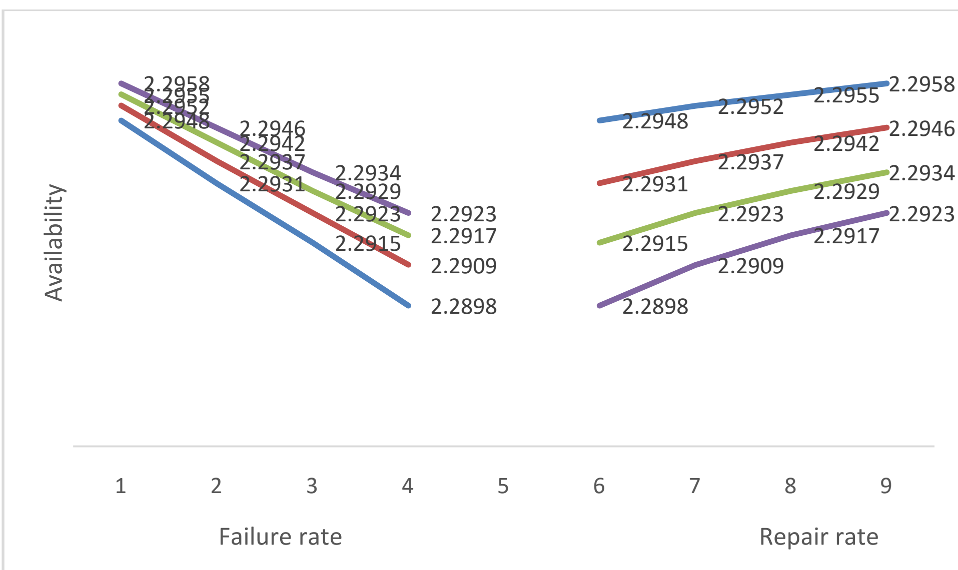

Figure 6: Effect of Failure rate α_4 and Repair rate β_4 on availability

The expected profit function of system versus parameters $\alpha_1$and β_1 respectively, is graphically illustrated in Figure 7. As shown, the behavior of profit function of system is descending with parameter α_1 and is ascending with parameter β_1.

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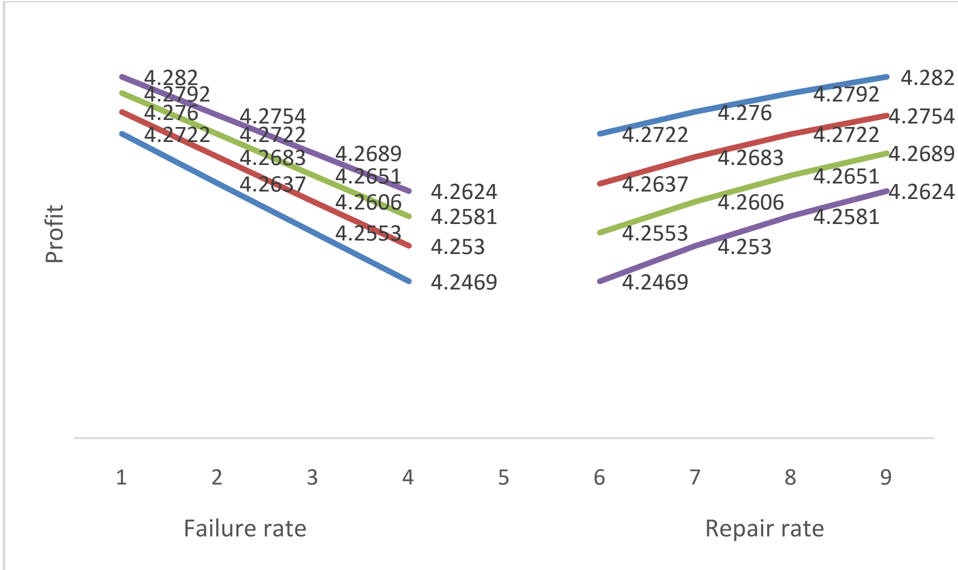

Figure 7: Effect of Failure rate α_1 and Repair rate β_1 on profit

In Figure 8, the profit functions of system are plotted with respect to parameter α_2 and β_2 respectively. From Figure 8, the behavior of profit function with parameter β_2 is rising while with respect to parameter α_2 the profit function is falling.

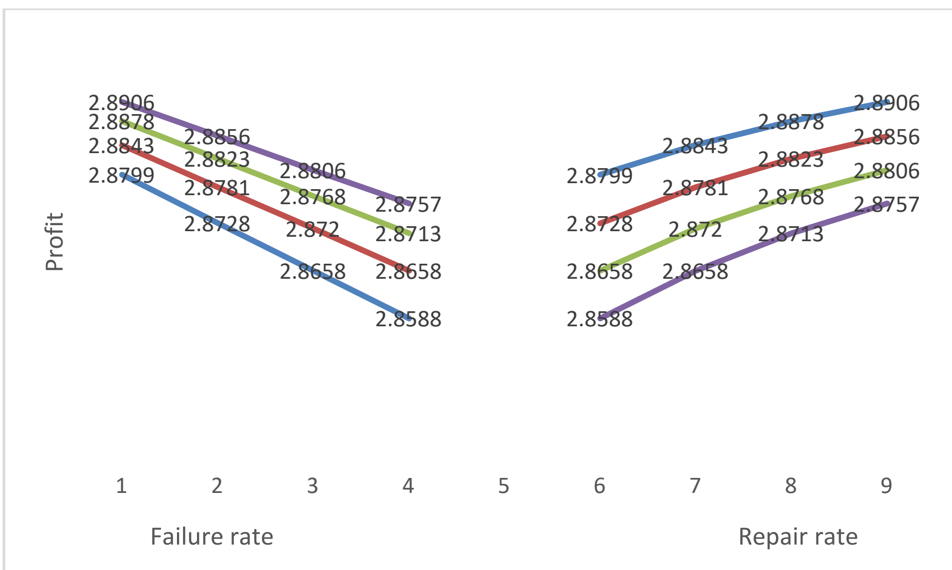

Figure 8: Effect of Failure rate α_2 and Repair rate β_2 on profit

The effect of failure rate α_3 and repair rate β_3 on profit function is depicted in figure 9. This figure indicates that as the values of failure rate increases, the profit function also decreases while the profit function increases with increase in the values of repair rate.

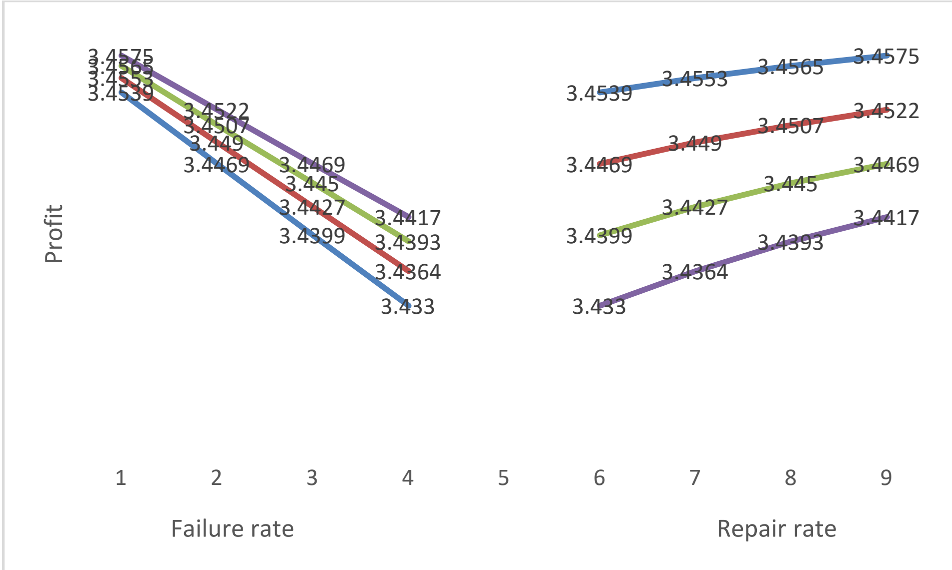

Figure 9: Effect of Failure rate α_3 and Repair rate β_3 on profit

Finally, in order to compare the system availability and profit function of the system, we only compare the availability and profit functions of subsystems A and B respectively. The profit function of subsystem A is drastically more than the profit function of subsystem B with increase in the values of repair rates as evident from figure 7 and figure 8. This simulation suggests that subsystem A gives the highest expected profit function. Thus, subsystem A is the most critical component in the system which needs special attention and careful observation.

VI. Conclusion

In this work, we constructed four different series-parallel systems consisting of subsystems *A*, *B, C* and *D.* Subsystems *A, B* and *C* consist of single unit each in series while subsystem *D* consists of four units arranged in parallel. We developed the explicit expressions for the availability, busy period and profit function for the four systems and performed a comparative analysis. Parametric investigation of various system parameters on system availability and profit function has been captured. It is interesting to see that as the repair rates of each subsystem increase; the availability and profit also increase. This implies that the major repair should be invoked to reduce the failure rate and maximize the system profit.

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