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Effects of Multicollinearity and Correlation between the Error Terms on Some Estimators in a System of Regression Equations

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Abstract- One of the assumptions of a single equation model is that there is one -way causation between the dependent variable Y and the independent variables X. When the assumption is not valid, as, in many econometric models, of lack of correlation between the independent variables and the error terms (U) is further violated, Ordinary Least Square estimator would no longer efficient, that was why this study examined the effects of multicollinearity and a correlation between the error terms on the performance of seven estimators and identified the estimator that yields the most preferred estimates under the separate or joint influence of the two correlation effects under consideration. A two-equation model in which the two correlation problems were introduced was used in this study. The error terms of the two equations were also correlated. The levels of correlation between the error terms and multicollinearity were specified between -1 and +1 at an interval of 0.2 except when the correlation value approached unity. A Monte Carlo experiment of 1000 trials was carried out at five levels of sample sizes 20, 30, 50, 100, and 250 at two runs.

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Effects of Multicollinearity and Correlation between the Error Terms on Some Estimators in a System of Regression Equations

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Abstract- One of the assumptions of a single equation model is that there is one -way causation between the dependent variable Y and the independent variables X. When the assumption is not valid, as, in many econometric models, of lack of correlation between the independent variables and the error terms (U) is further violated, Ordinary Least Square estimator would no longer efficient, that was why this study examined the effects of multicollinearity and a correlation between the error terms on the performance of seven estimators and identified the estimator that yields the most preferred estimates under the separate or joint influence of the two correlation effects under consideration. A two-equation model in which the two correlation problems were introduced was used in this study. The error terms of the two equations were also correlated. The levels of correlation between the error terms and multicollinearity were specified between -1 and +1 at an interval of 0.2 except when the correlation value approached unity. A Monte Carlo experiment of 1000 trials was carried out at five levels of sample sizes 20, 30, 50, 100, and 250 at two runs. The seven estimation methods namely; Ordinary Least Squares (OLS), Cochran – Orcutt (CORC), Maximum Likelihood Estimator (MLE), Multivariate Regression (MR), Full Information Maximum Likelihood (FIML), Seemingly Unrelated Regression Model (SUR) and Three-Stage Least Squares (3SLS) and their performances were thoroughly checked by subjecting the results obtained from each finite properties of the estimators into a multi-factor ANOVA model. The significant factors of the results were further examined using their estimated marginal means and the Least Significant Difference (LSD) methodology to determine the best estimator. The results when there is no correlation show that the OLS, CORC, and MLE estimators are generally preferred. Furthermore, the estimators of MR, FIML, SUR, and 3SLS are preferred for computing all the parameters of the model in the presence of multicollinearity and correlation between the error terms at all the sample sizes chosen.

I. General Introduction

Sometimes we may want to estimate more than one equations, which are closely related, The OLS and GLS estimation methods can be used to estimate the equations simultaneously, which has some advantages over estimation done one by one (Philip et al., 1990). As we will see later, estimating the system of equations is closely related to estimating models based on panel data (data from the same people/firms/countries for two or more periods).

One of the assumptions of a single equation model is that, there is one-way causation between the dependent variable Y and the independent variables Xs. When this assumption is not valid, as we have it in many econometric models, that is, the assumption of lack of correlation between the independent variables and the error terms

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(U) is further violated (i.e. E(XU) ≠ 0). Thus, the function no longer belongs to a one-way causation model but rather a wider system of regression equations (multi-equation model), which describe the relationship among all the relevant variables. In a multi-equation model, the dependent variable Y and independent variables now appear as well as explanatory variables in other equation(s) of the model.

Moreover, in a multi-equation model, there are problems of Autocorrelation and Multicollinearity, together with the presence of correlation between the error terms, which may eventually lead to a seemingly unrelated regression model (Lang et al., 2003; Olanrewaju et al., 2017). Consequently, some degree of Autocorrelation and Multicollinearity may have to be allowed in the system of regression equations. Therefore, this study examined and compared the effect of correlation between the error terms (ƛ) and Multicollinearity (δ) on the performances of seven methods of parameter estimation of a multi-equation model using the Monte Carlo approach.

Notes

a) *Aim and Objectives of the Study*

Consequently, the study examines the performances of some estimators of a single-equation and that of a system of Regression equation in the presence of correlation between the error terms, multicollinearity, and autocorrelation, study their effects on those estimators, and then, identify the preferred estimator(s) of the model parameters.

Very specifically, the study aims at the following:

(i) Examine the effect of sample size on the performance of the estimators

(ii) Examine the effect of multicollinearity (ƛ) and the correlation between the error terms (δ) jointly on the performance of seven estimators.

(iii) Identify the estimator that yields the most preferred estimates under the joint influence of the two correlation effects under consideration.

II. Literature Review and Theoretical Frame Work

a) *Estimation Methods under Multicollinearity in Single Equation*

Olanrewaju et al. (2017) stated that, one of the major assumptions of the explanatory variables in the classical linear regression model is that they are independent (orthogonal). Orthogonal variables can be set up in experimental designs, but such variables are not often in business and economic data. Thus when the explanatory variables are strongly interrelated, we have the problem of multicollinearity. When multicollinearity is not exact (i.e., the linear relationship between two explanatory variables is not perfect) but strong, the regression analysis is not affected; however, its results become ambiguous. Consequently, interpreting a regression coefficient as measuring the change in the response variable when the corresponding independent variable is increased by one unit, while other predictor variables are held constant is incorrect. This is because the OLS estimator of β given as;

$$\hat{\beta}_{(OLS)} = (X'X)^{-1}X'Y \tag{2.1}$$

and

$$V(\hat{\beta}_{(OLS)}) = \sigma^2(X'X)^{-1} \tag{2.2}$$

are affected by the sample value of the explanatory variables. Precisely, in this case

$$|X'X| \to 0$$

When multicollinearity is exact (perfect), the assumption that X has a full column rank break down and therefore $|X'X| = 0$. Consequently, the OLS estimate of equations (2.1) and (2.2) cannot be obtained. The concept of estimable function in which equations (2.1) and (2.2) now have an infinite solution of vectors is used. (Olanrewaju et al., 2017)

Notes

As long as multicollinearity is not perfect, the OLS estimates are still unbiased and BLUE (Johnson, 1984). Multicollinearity is associated with unstable estimated regression coefficients from the presence of strong linear relationships among the predictors. It is not a problem of model misspecification but rather that of data: and therefore, empirical study of this problem should only begin after the model has been satisfactorily specified (Charterjee, 2000). However, there may be some indications of the problem resulting from the process of adding, deleting, and transformation of variables or data points in search of a good model. Indications of multicollinearity that appear as a result of instability in regression coefficients are as follows.

i. Large changes in estimated OLS coefficients when a variable is dropped or added.
ii. Large changes in the estimated OLS coefficients when a data point is dropped or altered.

Once the residual plot indicates that the regression model has been satisfactorily specified, multicollinearity may be present if:

i. The algebraic signs of the estimated coefficients do not conform to prior expectations. This may be because greater covariance between the explanatory variable produces greater sampling covariance for the OLS coefficients. Comparing the off–diagonal terms in $X'X$ and $(X'X)^{-1}$ show that a positive covariance for the X's gives a negative covariance for the $\hat{\beta}$'s, and vice versa. In a specific application, if $\hat{\beta}_2$ is below $\hat{\beta}_2, \hat{\beta}_3$ is most likely to exceed $\hat{\beta}_3$ and vice versa (provided that X's are positively correlated).
ii. Coefficients of variables X's that are expected to be important have large standard error (small t – value).

A thorough investigation of the presence of multicollinearity in a system of regression equations can be accomplished by several methods which include:

i. The use of variance inflation factor (VIF): Charterjee (2000).
ii. Principal component analysis approach: Seber (1984), Johnson and Wichern (1992), Charterjee (2000).
iii. The use of two-step procedure: Besley, et al. (1980).
iv. The Farrar – Glauber test: Farrar and Glauber (1967).

The assumption that the regressors X are treated as fixed variables in repeated samples is often violated by economists and other social scientists. The reason for this violation is because their X is often being generated by stochastic processes beyond the scientists' control. For instance, consider regressing daily bathing suit sales by a department store on the mean daily temperature. Surely, the department store cannot control daily temperature, so it would not be meaningful to think of repeated sampling when temperature levels are the same from sample to sample. Fomby et al. (1988) demonstrated that under general conditions, the essential results of the classical linear regression model remain intact even with stochastic regressors. Neter and Wasserman (1974) pointed out that all results on estimations, testing, and prediction obtained using the classical linear regression model still applies if the following conditions hold:

- The conditioned distribution of the dependent variable given the regressors are normal and independent, with means $X\beta$ and conditional variance σ^2
- The regressors are independent random variables, whose probability distribution does not involve the parameter of the classical linear regression model and the conditional variance σ^2.

However, they pointed out that modification would occur in the area of confidence interval calculated for each sample and the power of the test. This problem was also discussed and supported by Chartterjee et al. (2000).

The assumption that the values of X variables in a regression model are measured without error is hardly ever satisfied. Measurement errors may enter the value observed for the independent variable, 'for instance, when it is temperature, pressure, production line speed, or person's age. Consequently, the independent variable becomes correlated with the error terms (Neter and Wasserman, 1974). These measurement errors affect the residual variance, the multiple correlation coefficients, and the estimated regression coefficients. Its effects increase the residual variance and reduce the magnitude of the observed multiple correlations. The effects of these errors on the estimated regression coefficients are more difficult to assess (Chartterjee et al. 2000). A more extensive discussion of the aforementioned problem can be found in Cochran (1970), Fuller (1987), Chartterjee and Hadi (1988), and Chi – Lu and Van Ness (1999).

Dhrymes and Schwarz (1987) stated that "the heart of the problem is that the conditions on the parameters force the singularity of the covariance matrix-and to a certain degree the converse is true, i.e. the singularity of the covariance matrix implies certain restrictions." It is important to note that, as stated by Bewley (1986), "a necessary and sufficient condition for the OLS estimates to satisfy the adding-up criterion is that some linear combination of the regressors must be identically equal to the sum of regressands if the model is to be logically consistent."

Since the constraints in (2) depend on the values of the regressors, we postulate that the constraints are identically valid in the regressors, which induces restrictions on the parameters that are independent from the regressors. Thus, let Z be a T x P matrix of T-vectors $z_1,\ldots,z_p$, which constitute a base of the vector space containing the $\sum_i k_i$ regressors of all n equations. The obvious consequence is the existence of n matrices c_i of order P x k_i with $X_i = z.c_i$, for all i.

III. Research Methodology

a) *The Monte - Carlo Approach*

In a Monte-Carlo experiment, the experimenter artificially sets up a system (model) and specifies the distribution of the independent variables alongside with the values of the model parameters. More so, the values are then generated for the error terms and the independent variables as specified for a specified sample size. We then, make use of the generated and the parameter values to formulate data for the dependent variable. Next is to treat the generated data as if they are real-life data by estimating the parameters of the model via the estimation methods (estimators). This process of generating values for the disturbance term, independent variables, and

Notes

estimating the parameters of the model which is then, replicated a large number of times. The experimenter then builds up empirical distributions of the parameter estimates, which are then used to evaluate the performance of those estimators in relation to the parameter values. (Olanrewaju et al. 2017)

The Monte – Carlo studies as stated in Olanrewaju et al. 2017, can be designed generally by using the following summarized five steps as given below:

Notes

(a) The researcher specifies a model and assigns specific numeric values as in parameters. The assigned values are assumed to be the real values of the parameters.
(b) There is need to specify the distribution of error terms.
(c) He also uses the distribution of the error terms (U's) with the random drawings to get new different values for it.
(d) The experimenter now selects or generates values for the regressors (X's) depending on the specifications of the model.
(e) The researcher obtains or generates values for the dependent variable using the computed values of the regressors and the error terms.

The five steps mentioned above are repeated several times, say R, to have R replications.

Thus, the experimenter obtains an estimate of the model parameters for each replication, treating the generated data as real-life data.

b) The Model Formulation

The system of regression equation used in this research work as stated in Olanrewaju, 2013, is given as

$$y_{1t} = \beta_{01} + \beta_{11}x_{1t} + \beta_{12}x_{2t} + u_{1t} \tag{3.1}$$

where- $u_{1t} = \rho u_{1(t-1)} + e_{1t}$, $e_{1t} \approx (0, \sigma^2)$.

$$y_{2t} = \beta_{02} + \beta_{21}x_{1t} + \beta_{22}x_{3t} + u_{2t} \quad , \quad u_{2t} \approx N(0, \sigma^2) \tag{3.2}$$

Note: (1) Multicollinearity exists between X_1 and X_2 in equation (3.1)
(2) Autocorrelation exists in equation (3.1)
(3) There is a correlation between u_1 and u_2 of the two equations
(4) There is no correlation between x_1 and x_3 in equation (3.2), thus, equation (3.2) appears as a control equation.

The models (3.1) and (3.2) were studied under two sub-divisions as given below:

1. There is no any form of correlation in the model i.e. δ=0, ρ =0 and ƛ=0
2. There is correlation between the error term and presence of multicollinearity in the model i.e. δ ≠ 0, ρ = 0 and ƛ ≠0.

c) Specifications and Choice of Parameters for Simulation Study

For the simulation study in this research work, the parameters of the model in equations 3.1 and 3.2 are fixed as β_{01} = 0.4; $ß_{11}$ = 1.8; $ß_{21}$ =2.5; $ß_{02}$ = 2.0; $ß_{12}$ = 4.5; $ß_{22}$ = -1.2. The Multicollinearity (δ) levels are -0.99, -0.9,-0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.99 and that of Correlation between error terms (ƛ) levels are -0.99, -0.9,-0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.99. The sample sizes (n) 20, 30, 50, 100 and 250 were used in the simulation. At a particular choice of sample size, multicollinearity level and correlation between the error terms, a Monte-Carlo experiment is performed 1000 times at two runs which were averaged at analysis stage.

d) The Data Generation for the Simulation Study

The generation of the data used in this simulation study is in three stages, which are: Generation of the

(i) independent variables

(ii) error terms

(iii) dependent variables

e) Computation of Data for the Independent Variables

The independent variables used in this study are fixed numbers at each trial of the simulation. They were computed using the equation provided by Ayinde (2007) to create normally distributed random variables with specified intercorrelations, i.e.

$$X_1 \sim N(\mu_1, \sigma_1^2), \qquad X_2 \sim N(\mu_2, \sigma_2^2), \qquad X_3 \sim N(\mu_3, \sigma_3^2)$$

$$Z_1 = \frac{X_1 - \mu_1}{\sigma_1}, \qquad Z_2 = \frac{X_2 - \mu_2}{\sigma_2}, \qquad Z_3 = \frac{X_3 - \mu_3}{\sigma_3}$$

$$Cor(X_1, X_2) = \rho_{12}, \qquad Cor(X_1, X_3) = 0, \qquad Cor(X_2, X_3) = 0$$

For the three normally distributed random variables given above, the newly derived equation is given as:

$$X_1 = \mu_1 + \sigma_1 Z_1$$

$$X_2 = \mu_2 + \rho_{12}\sigma_2 Z_1 + \sqrt{g_{22} Z_2} \qquad (3.3)$$

$$X_3 = \mu_3 + \rho_{13}\sigma_3 Z_1 + \frac{g_{23}}{\sqrt{g_{22}}} Z_2 + \sqrt{h_{33} Z_3}$$

Since $\rho_{13} = 0$ and $\rho_{23} = 0$, then X becomes

$$X_3 = \mu_3 + \frac{g_{23}}{\sqrt{g_{22}}} Z_2 + \sqrt{h_{33} Z_3}$$

where-, $g_{22} = \sigma_2^2[1 - \rho_{12}^2]$, $\quad g_{23} = 0$, $\quad g_{33} = \sigma_3^2$,

$$h_{33} = g_{33} - \frac{g_{23}^2}{g_{22}} \quad and\ Z_i \sim N(0,1), \quad for \quad i = 1,2,3.$$

f) Generation of the Error Terms

The two error terms, u1 and u2, assumed to be well behaved with a multivariate normal distribution u ~ NID (0, Σ) as expressed in equations 3.1 and 3.2 were also generated to exhibit correlation ƛ using the technique as provided by Ayinde (2007). Here is the equation in which the error terms values were generated

Suppose, $u_i \sim N(\mu, \sigma_i^2)$ $\quad i = 1,2$. If these variables are correlated, then, u_1 and u_2 can be gotten by equations

$$u_1 = \mu_1 + \sigma_1 z_1$$

$$u_2 = \mu_2 + \rho\sigma_2 z_1 + \sigma_2 z_2\sqrt{1 - \rho^2} \qquad (3.4)$$

Notes

Where $Z_i \sim N(0,1)$ for $i = 1,2$ and $|\rho| < 1$ is the value of the correlation between the two variables.

g) Generation of Data for the Dependent Variables

Considering the system of regression equation in 3.1 and 3.2, we have

$$y_{1t} - \beta_{01} - \beta_{11}x_{1t} - \beta_{12}x_{2t} - 0x_{3t} = u_{1t}$$

$$y_{2t} - \beta_{02} - \beta_{21}x_{1t} - 0x_{2t} - \beta_{22}x_{3t} = u_{2t} \tag{3.5}$$

Notes

We can write 3.4 in matrix form as:

$$Ay_t + \gamma x_t = u_t \tag{3.6}$$

Where

$$A = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \gamma = \begin{bmatrix} -\beta_{01} & -\beta_{11} & -\beta_{12} & 0 \\ -\beta_{02} & -\beta_{21} & 0 & -\beta_{22} \end{bmatrix}$$

$$x_t = \begin{bmatrix} x_{1t} \\ x_{2t} \\ x_{3t} \end{bmatrix} \quad \text{and} \quad u_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

Now, equation (3.6) becomes

$$y_t + A^{-1}\gamma x_t = A^{-1}u_t$$

$$y_t = -A^{-1}\gamma x_t + A^{-1}u_t$$

Then, equation 3.1 and 3.2 become,

$$y_{1t} = \beta_{01} + \beta_{11}x_{1t} + \beta_{12}x_{2t} + \rho u_{1(t-1)} + e_{1t} \quad \text{since,} \quad u_{1t} = \rho u_{1(t-1)} + e_{1t}$$

$$y_{2t} = \beta_{02} + \beta_{21}x_{1t} + \beta_{22}x_{3t} + u_{2t} \tag{3.7}$$

Therefore, equation (3.7) above was used to generate dependent variables y_1 and y_2 by substituting the values of model parameters, independent variables, and that of error terms as specified in the previous sections above.

h) The Evaluation, Comparative Analysis and Preference of Estimators

The evaluation and comparative analysis of the seven (7) estimation methods namely, Ordinary Least Squares (OLS), Cochran – Orcutt (CORC), Maximum Likelihood Estimator (MLE), Multivariate Regression (MR), Full Information Maximum Likelihood (FIML), Seemingly Unrelated Regression Model (SUR) and Three-Stage Least Squares (3SLS), were examined using the finite sampling properties of estimators, which include Bias (BB), Absolute Bias (AB), Variance (VAR), and the Mean Square Error (MSE) criteria.

Mathematically, for any estimator $\hat{\beta}_{ij}$ of Model (3.1) & (3.2)

(i) $$\bar{\hat{\beta}}_{ij} = \frac{1}{R}\sum_{l=1}^{R}\hat{\beta}_{ijl}$$

$$(ii)\quad Bias\left(\hat{\beta}_{ij}\right)=\frac{1}{R}\sum_{l=1}^{R}\left(\hat{\beta}_{ijl}-\beta_{ij}\right)=\bar{\hat{\beta}}_{ij}-\beta_{ij}$$

$$(iii)\quad AB\left(\hat{\beta}_{ij}\right)=\frac{1}{R}\sum_{l=1}^{R}\left|\hat{\beta}_{ijl}-\beta_{ij}\right|$$

$$(iv)\quad VAR\left(\hat{\beta}_{ij}\right)=\frac{1}{R}\sum_{l=1}^{R}\left(\hat{\beta}_{ijl}-\bar{\hat{\beta}}_{ij}\right)^2$$

$$(v)\quad MSE\left(\hat{\beta}_{ij}\right)=\frac{1}{R}\sum_{l=1}^{R}\left(\hat{\beta}_{ijl}-\beta_{ij}\right)^2$$, for $i = 0, 1, 2$; $j = 1,2$ and $l=1,2,\ldots,R$.

Using a computer program which was written with TSP software package to estimate all the model parameters and the criteria, the performances of the seven estimation methods considered in this work as mentioned above were examined by subjecting the results obtained from each finite properties of the estimators into a multi-factor analysis of variance model. Consequently, the highest order significant interaction effect, which has a "method" as a factor, is further examined using Duncan Multiple Range Test and the Least Significance Difference (LSD) test. The estimated marginal mean of the factor was investigated at a particular combination of levels of the correlations in which preferred estimators were chosen. An estimator is most preferred at a specific combination of levels of the correlation, if the marginal means is the smallest. Also, all estimators whose estimated marginal means are not significantly different from the most preferred are also preferred.

IV. Analysis of Results and Discussions

a) Results when we do not have any Form of Correlation in the Model

The performances of the estimators under various sample sizes base on finite sampling properties of estimators using the Analysis of Variance technique are presented and discussed in the table below.

Table 1: ANOVA Table showing the effect of estimators when there is no any form of correlation in the model

N	β_i	df	Value of F - Statistic							
			Equation One				Equation Two			
			BB	A B	VAR	MSE	BB	A B	VAR	MSE
20	β0	6, 7	0.0036	0.8051	1.3441	1.3538	7.38E-4	0.0020	2.4196	0.0039
	β1	6, 7	6.42E-5	0.0291	0.0283	0.0294	0.0777	0.3225	55.4948***	0.3018
	β2	6, 7	0.0851	0.1616	0.1830	0.1839	0.3852	0.2830	11.5274***	0.2496
30	β0	6, 7	0.0525	0.6623	0.3889	0.3818	0.0052	1.7846	2.9613	1.6736
	β1	6, 7	0.2897	0.0098	0.0129	0.0127	0.0080	0.0080	18.0066***	0.0033
	β2	6, 7	0.4588	0.0189	0.0164	0.0164	1.15E-4	0.0223	72.7679***	0.0293
50	β0	6, 7	0.2474	9.62E-5	2.86E-4	2.88E-4	0.0080	0.0080	0.4714	0.0289
	β1	6, 7	1.7866	3.33E-4	7.53E-4	7.57E-4	0.1259	0.1257	0.7172	0.1841
	β2	6, 7	0.0724	0.0010	0.0013	0.0013	0.0044	0.0160	0.6077	0.0326
100	β0	6, 7	2.56E-4	0.0297	0.0147	0.0140	3.26E-4	2.45E-5	0.7457	3.38E-4
	β1	6, 7	0.0017	0.0090	0.0048	0.0051	0.0041	0.0390	5.958**	0.0260
	β2	6, 7	0.0099	0.1326	0.1816	0.1780	5.09E-5	0.0257	1.267E3***	0.0237
250	β0	6, 7	0.0025	0.0012	3.06E-5	1.58E-5	2.88E-4	1.62E-4	0.3148	8.72E-4
	β1	6, 7	0.1215	0.0035	0.0078	0.0079	0.0023	0.0012	0.2603	1.72E-4
	β2	6, 7	2.15E-4	0.0058	0.0051	0.0053	0.0024	0.0067	61.5188***	0.0169

Notes

It was observed from Table1 above, that, all the estimators do not perform differently (P-value > 0.05) under all the criteria except under the variance criterion in some parameters of equation two. Thus, we concluded that all the estimators do not exhibit a significant difference in their performances under all the criteria in equation one. The results of the further test to identify those estimators that perform equivalently in equation2 are presented in Table 2. From the table, we observed that the performances of the OLS, CORC, MLE, MR, SUR, FIML, and 3SLS estimators are not significantly different. Meanwhile, the OLS, CORC, and MLE estimators are generally preferred.

Table 2: Results of a further test to identify Means that are not significantly different

n	β_i	Criterion	Equation	Estimated Marginal Means of the Estimators						
				OLS	**CORC**	**MLE**	**MR**	**FIML**	**SUR**	**3SLS**
20	β_1	VAR	Two	1.1020E-7^{a}	4.1333E-7^{a}	1.4404E-6^{a}	0.000247^{b}	0.000494^{c}	0.000247^{b}	0.000495^{c}
	β_2	VAR	Two	8.9795E-5^{a}	6.3520E-7^{a}	0.00001^{a}	0.005447^{b}	0.010333^{c}	0.005447^{b}	0.010344^{c}
30	β_1	VAR	Two	0.00001^{a}	0.00001^{a}	6.8997E-7^{a}	0.000002^{b}	2.7419E-6^{bc}	0.000002b	3.2419E-6^{c}
	β_2	VAR	Two	8.4995E-7^{a}	0.00001^{a}	0.00001^{a}	0.001206^{b}	0.001548^{c}	0.001206^{b}	0.001548^{c}
100	β_1	VAR	Two	4.2036E-8^{a}	0.00001^{a}	0.00001^{a}	9.0873E-7^{b}	9.2553E-7^{b}	9.0873E-7^{b}	9.2553E-7^{b}
	β_2	VAR	Two	2.3192E-8^{a}	1.1680E-7^{a}	3.0251E-7^{a}	0.00009^{b}	0.000096^{c}	0.00009^{b}	0.000097^{c}
250	β_2	VAR	Two	1.1307E-8^{a}	2.4719E-8^{a}	3.4559E-8^{a}	0.000016^{b}	0.000017^{b}	0.000016^{b}	0.000017^{b}

Note: Means that have the same letter on top (superscript) are not different significantly.

b) Results when there is a Correlation between the Error Terms and Multicollinearity in the Model

The performances of the estimators under the influence of multicollinearity and a correlation between the error terms at various levels of sample sizes based on finite sampling properties of estimators using the Analysis of Variance technique are presented and discussed below.

i. *Effect on β_0*

The effect of estimators, multicollinearity and a correlation between the error terms on estimating β_0 based on the sampling properties of the estimators as revealed by the Analysis of Variance technique are shown in Table 3 below:

Table 3: ANOVA Table showing the effect of estimators, multicollinearity, and a correlation between the error terms on parameter β_0 in the model.

n	Factor	df	Value of F – Statistic							
			Equation One				Equation Two			
			BB	A B	VAR	MS	BB	A B	VAR	MS
20	E δ λ E* δ E* λ $\delta * \lambda$ E* $\delta * \lambda$	6,1183 12, 1183 12, 1183 72.1183 72,1183 144,1183 864,1183	.522 8.084E-5 102.30*** .00001 13.175*** 9.214E-5 .0001	126.179*** 0.0001 .335 0.0001 .044 0.001 0.001	240.379*** 0.0001 .558 0.0001 .071 0.0001 0.0001	211.902*** 0.0001 .530 0.001 0.068 0.001 0.001	.112 .002 7.270*** 7.229E-7 .003 .002 7.223E-7	.474 .001 1.011 4.4007E-6 .011 .001 4.4008E-6	103.035*** .728 9.985E3*** .001 5.035*** .728 .001	1.116 .001 3.950*** 6.9295E-6 .035 .001 6.9295E-6
30	E δ λ E* δ E* λ $\delta * \lambda$ E* $\delta * \lambda$	6,1183 12, 1183 12, 1183 72.1183 72,1183 144,1183 864,1183	2.938*** .003 50.267** * 3.2778E- 4 6.316*** .003 3.385E-4	208.454*** .425 7.641*** .378 1.114 .172 .170	156.891** * .018 6.468*** .002 .810 0.0001 0.0001	138.158*** .017 5.684*** .002 .711 0.0001 0.001	.615 1.102 .053 .005 .010 .030 1.8595E-4	198.853*** .766 43.910*** .583 6.519*** .041 .027	1.837 62,427*** 1.502E3*** .005 .071 2.404*** .002	180.426*** 5.507*** 223.773*** .539 4.454*** .205 .022
50	E δ λ E* δ E* λ $\delta * \lambda$ E* $\delta * \lambda$	6,1183 12, 1183 12, 1183 72.1183 72,1183 144,1183 864,1183	.104 71.543** * 57.528** * .018 6.529*** 1.188 .077	.299 72.758*** 2.567*** .008 .001 .287 7.833E-5	.488 63.348*** 2.802*** .021 .002 .328 1.8373E-4	.459 63.232*** 2.800*** .019 .001 .328 1.8E-4	1.542 1.323 2.234*** .061 .033 .056 .001	.944 36.402*** 86.860*** .120 .020 .683 .009	.390 40.609*** 283.525*** .015 .021 2.553*** .001	2.016 42.555*** 151.046*** .066 .010 2.111*** .001
100	E δ λ E* δ E* λ $\delta * \lambda$ E* $\delta * \lambda$	6,1183 12, 1183 12, 1183 72.1183 72,1183 144,1183 864,1183	.050 .106 .517 1.928E-4 .056 .002 .001	1.754 67.521*** .339 .061 .050 .322 .050	1.10 50.945*** .374 .062 .050 .346 .051	1.08 48.759*** .35 .059 .048 .333 .049	.042 1.028E-6 1.984** 1.3929E-4 .002 .006 .001	.002 .019 43.924*** .002 .005 .025 .004	.353 23.051*** 5.958E3*** .090 .358 2.564*** .245	.034 .041 26.839*** .002 .005 .020 .003
250	E δ λ E* δ E* λ $\delta * \lambda$ E* $\delta * \lambda$	6,1183 12, 1183 12, 1183 72.1183 72,1183 144,1183 864,1183	2.499** .253 27.426** * .076 3.318*** .114 .022	.656 6.806*** 1.84** .002 .016 1.813*** 0.0001	1.281 6.623*** 2.341*** 0.0001 0.001 1.852*** 0.0001	.786 6.545*** 1.844** 0.001 0.0001 1.773*** 0.0001	.038 .737 9.754*** 1.9645E-5 .001 .062 1.1272E-5	.040 .402 .359 7.3543E-5 .001 .1041.96E-5 .00001	0.0001 58.915*** 1.873E3*** 0.001 0.0001 17.846*** .0001	.106 .251 .433 .0001 .003 .074 .0001

From Table 3, the following points are observed:

- The effect of multicollinearity is generally significant under all the criteria when the sample sizes are moderate and high in equations one and two, but occasionally significant under variance and mean square error in equation two.
- The effect of correlation between the error terms is generally significant under all criteria in equations one and two.
- The effect of estimators is generally significant under all the criteria in both equations when the sample sizes are small (i.e., when n = 20 and 30).

Notes

- The interaction effect of estimators and multicollinearity is not significant under all the criteria in both equations.
- The interaction effect of estimators and the correlation between the error terms is occasionally significant under all the criteria in both equations.
- The interaction effect of estimators, correlation between the error terms and Multicollinearity is not significant under all the criteria in equations one and two.

More so, we can summarize that the performances of the estimators are affected by Multicollinearity and the correlation between the error terms under all criteria. The results of the LSD further test visa- vice their estimated marginal means, as shown in Tables 4 revealed that OLS and MLE estimators are preferred for the estimation of β_0.

Table 4: Results of a further test on β_0 to identify Means that are not significantly different

n	Criterion	Equation	Means of the Estimators						
			OLS	CORC	MLE	MR	FIML	SUR	3SLS
20	AB	1	$.4165^{a}$	12.3996^{b}	$.3985^{a}$	$.4165^{a}$	$.4165^{a}$	$.4162^{a}$	$.4162^{a}$
	VAR	1	0.0514^{a}	0.0605^{b}	0.0525^{a}	0.0508^{a}	0.0508^{a}	0.0509^{a}	0.0509^{a}
	MS	1	0.0261^{a}	0.0362^{b}	0.0258^{a}	0.0262^{a}	0.0262^{a}	0.0262^{a}	0.0262^{a}
30	AB	1	$.1537^{b}$	$.1540^{b}$	$.1578^{c}$	$.1526^{a}$	$.1526^{a}$	$.1526^{a}$	$.1528^{a}$
	VAR	1	$.0369^{b}$	$.0388^{c}$	$.0368^{b}$	$.0363^{a}$	$.0363^{a}$	$.0364^{a}$	$.0364^{a}$
	MS	1	$.03695^{b}$	$.03886^{c}$	$.03704^{b}$	$.0364^{a}$	$.0364^{a}$	$.0364^{a}$	$.0364^{a}$

ii. *Effect on β_1*

The effects of estimators, multicollinearity and the correlation between the error terms on estimating β_1 based on the sampling properties of the estimators as revealed by Analysis of Variance technique are shown in Table 5 below:

Table 5: ANOVA Table showing the effect of estimators, multicollinearity and the correlation between the error terms on β_1 in the model

n	Factor	df	Value of F- Statistic							
			Equation One				Equation Two			
			BB	A B	VAR	MS	BB	A B	VAR	MS
20	E	6,1183	2.0628E-4	8.463***	60.534***	6.530***	5.773***	30.306***	19.129***	44.083***
	δ	12, 1183	.161	754.436***	748.809***	424.704***	.002	.010	.056	.028
	λ	12, 1183	3.4276E-4	10.665***	16.158***	5.522***	3.366***	33.571***	589.634***	135.683***
	E* δ	72.1183	.002	1.232	20.038***	2.124***	1.183E-4	1.1157E-4	0.0001	1.0709E-4
	E* λ	72,1183	4.3797E-5	1.352**	2.091***	.702	.282	1.330**	.810	.377
	$\delta * \lambda$	144,1183	1.269**	2.037***	5.463***	1.867***	.002	.010	.056	.028
	E* $\delta * \lambda$	864,1183	.169	.258	.707	.237	9.688E-5	1.116E-4	.0001	1.0712E-4
30	E	6,1183	.003	158.541***	147.575***	93.709***	.757	.546	1.706	.514
	δ	12, 1183	.010	2.808E3***	1.92E3***	1.641E3***	.095	1.403	21.173***	1.631
	λ	12, 1183	.360	65.244***	42.752***	31.059***	29.425***	2.290***	754.727***	6.762***
	E* δ	72.1183	.003	26.096***	48.948***	31.051***	.003	.001	.007	4.349E-4
	E* λ	72,1183	.045	8.167***	5.353***	3.885***	.020	.025	.110	.01
	$\delta * \lambda$	144,1183	1.810***	12.310***	14.444***	10.497***	.004	.067	.811	.061
	E* $\delta * \lambda$	864,1183	.228	1.541***	1.809***	1.313***	8.3324E-5	1.3466E-4	.001	1.2112E-4
	E	6,1183	.011	11.380***	53.193***	10.178***	1.055	11.985***	2.700**	15.502***

50	δ	12, 1183	.061	2.28E3***	757.932***	1.059E3***	24.371***	39.266***	29.383***	50.551***
	λ	12, 1183	5.712***	37.493***	18.736***	17.315***	6.055***	9.745***	482.516***	1.525
	E* δ	72.1183	.016	2.398***	18.731***	4.323***	.004	.058	.082	.036
	E* λ	72,1183	.718	3.747***	1.958***	1.602***	.053	.504	.137	.509
	$\delta * \lambda$	144,1183	5.041***	7.846***	6.925***	6.403***	.547	.267**	1.810***	.825
	E* $\delta * \lambda$	864,1183	.631	.775	.704	.582	.001	.006	.008	.007
100	E	6,1183	.011	290.024***	2.31E3***	488.385***	.974	1.935	.874	1.205
	δ	12, 1183	1.402	5.11E3***	2.52E4***	7.752E3***	.053	.131	.312	.282
	λ	12, 1183	.413	121.913***	616.392***	153.927***	8.505***	1.795E3***	2.296E3***	1.818E3***
	E* δ	72.1183	.045	60.79***	821.734***	172.583***	.002	.015	.023	.022
	E* λ	72,1183	.039	14.127***	73.903***	18.48***	.034	.147	.199	.132
	$\delta * \lambda$	144,1183	11.172***	25.653***	209.668***	52.298***	.031	.266	.451	.342
	E* $\delta * \lambda$	864,1183	1.385***	3.242***	26.401***	6.583***	.004	.044	.074	.056
250	E	6,1183	.031	294.712***	357.23***	228.023***	.182	.050	.127	.016
	δ	12, 1183	.689	5.22E3***	3.59E3***	3.651E3***	.115	.017	33.861***	.086
	λ	12, 1183	.045	119.336***	90.141***	72.591***	1.848**	14.942***	1.402E3***	10.687***
	E* δ	72.1183	.061	58.504***	120.792***	77.204***	.001	1.6106E-4	.0001	0.0001
	E* λ	72,1183	.007	14.923***	11.270***	9.076***	.006	.006	.0001	.001
	$\delta * \lambda$	144,1183	1.606***	26.177***	30.723***	24.824***	.097	.323	12.951	.295
	E* $\delta * \lambda$	864,1183	.203	3.372***	3.841***	3.103***	.000	1.8084E-4	.0001	2.6288E-4

From Table 5, the following are noticed:

- The effect of multicollinearity is generally significant under all criteria except under bias in equation one and occasionally significant under some criteria in equation two.
- The influence of correlation between the error terms is generally significant under all criteria in equations one and two but not significant under bias criterion in equation two.
- The effect of estimators is generally significant under all the criteria except under bias in equation one and occasionally significant in equation two. A further test as shown in Table 6 revealed that MR, FIML, SUR, and 3SLS are preferred to estimate β_1
- The interaction effect of estimators and multicollinearity is generally significant under all criteria except under bias in equation one, but not significant at all in two.
- The interaction effect of estimators and the correlation between the error terms is generally significant under all criteria except under bias in equation one, but not significant at all in equation two.
- The interaction effect of the correlation between the error terms and multicollinearity is generally significant under all criteria in equation one only.
- The interaction effect of estimators, the correlation between the error terms, and Multicollinearity is significant under all criteria in equation one except when the sample sizes are 20 and 50.

Meanwhile, we can now infer that the performances of the estimators are affected by Multicollinearity under all the criteria. The results of the LSD further test visa- vice their estimated marginal means revealed that MR, FIML, SUR, and 3SLS estimators are preferred to estimate β_1.

Notes

Notes

Table 6: The Results of a further test on β_1 to identify Means that are not significantly different

n	Criterion	Equation	Means of the Estimators						
			OLS	CORC	MLE	MR	FIML	SUR	3SLS
20	AB	2	-4.5023E-2[a]	-1.624E-2[b]	-1.554E-2[b]	-4.8626E-2[a]	-4.8625E-2[a]	-4.7201E-2[a]	-4.7201E-2[a]
	VAR	2	.0211[b]	.0243[c]	.023587[c]	.0194[a]	.0194[a]	.01937[a]	.01937[a]
	MS	2	.0289[a]	.0361[b]	.0355[b]	.0272[a]	.0272[a]	.0272[a]	.0272[a]

iii. *Effect on* β_2

The effects of estimators, multicollinearity, and the correlation between the error terms on estimating β_2 based on the sampling properties of the estimators as revealed by Analysis of Variance technique are shown in Table 7.

Table 7: ANOVA Table showing the effect of estimators, multicollinearity, and the correlation between the error terms on β_2 in the model

n	Factor	df	Value of F-Statistic							
			Equation One				Equation Two			
			BB	A B	VAR	MS	BB	A B	VAR	MS
20	E	6,1183	.016	12.631***	72.973***	7.606***	21.763***	826.562***	835.104***	696.964***
	δ	12, 1183	.076	658.856***	740.685**	413.919***	.008	.001	.007	.002
	λ	12, 1183	8.333***	20.211***	*	6.698***	437.321**	10.453***	190.904***	51.064***
	E* δ	72.1183	.001	.873	19.898***	2.064***	*	.001	.002	.001
	E* λ	72,1183	1.112	2.559***	19.804***	.852	.001	47.414***	56.989***	40.609***
	$\delta * \lambda$	144,1183	.576	1.398***	2.575***	1.819***	4.52	.001	.007	.002
	E* $\delta * \lambda$	864,1183	.077	.177	5.403***	.231	.008	.001	.002	.001
					.699		.001			
30	E	6,1183	.001	294.82***	187.7***	118.147***	.020	9.803***	1.161E3***	10.327
	δ	12, 1183	.005	2.497E3***	1.99E3***	1.68E3***	1.461	22.377***	8.295***	12.489***
	λ	12, 1183	12.32***	124.811***	54.313***	39.098***	.350	232.749***	249.605***	123.808***
	E* δ	72.1183	.002	19.011***	50.449***	31.715***	1.731E-4	.220	2.372	.063
	E* λ	72,1183	1.549	15.63***	6.801***	4.891***	.001	2.561	55.147***	1.306
	$\delta * \lambda$	144,1183	.813	9.306	14.939***	10.759***	.037	.637	1.105	.448
	E* $\delta * \lambda$	864,1183	.102	1.165***	1.871***	1.346***	4.2151	.006	.751	.012
50	E	6,1183	.112	16.819***	63.519***	12.201***	2.35**	149.153***	93.822***	79.377***
	δ	12, 1183	.040	2.085E3***	748.809**	1.05E3***	45.294***	65.760***	21.965***	39.80***
	λ	12, 1183	29.57***	70.712***	*	20.874***	19.278***	19.495***	20.868***	3.224***
	E* δ	72.1183	.014	1.967***	22.333***	4.496***	.021	.427	2.668***	1.846***
	E* λ	72,1183	3.698***	7.29***	18.590***	1.932***	.087	9.106***	5.732***	4.744***
	$\delta * \lambda$	144,1183	3.07***	5.862***	2.335***	6.422***	0.997	.473	.906	.465
	E* $\delta * \lambda$	864,1183	.384	.559	6.876***	.575	.002	.093	.249	.214
					.692					
100	E	6,1183	.128	479.091***	2.48E3***	586.269***	.042	162.452***	2.263E3***	234.664***
	δ	12, 1183	.576	3.90E3***	2.18E4***	7.508***	2.2067E-6	.003	.072	.006
	λ	12, 1183	70.852**	220.649***	662.388**	185.141***	.404	374.329***	547.88***	127.768***
		72.1183	*	36.496***	*	167.242***	1.4205E-4	.594	7.862***	.740
	E* δ	72,1183	.035	25.850***	711.661**	22.232***	.001	16.767***	116.846***	9.684***
	E* λ	144,1183	7.768***	15.603***	*	50.67***	.002	.121	0.0001	.028
	$\delta * \lambda$	864,1183	5.393***	2.015***	79.451***	6.389***	4.4135E-4	.070	.408	.035
			.752		181.451**					
					*					

	E* δ * λ				22.888***					
250	E	6,1183	.026	454.998***	414.92***	267.172***	.158	76.360***	2.259E3***	128.267***
	δ	12, 1183	.825	3.744E3***	3.37E3***	3.455E3***	.427	2.495***	8.747***	1.711
	λ	12, 1183	11.413***	191.494***	103.404***	83,962***	.916	54.825***	435.096***	18.869***
	E* δ	72.1183	.035	32.665***	113.669***	73.194***	.001	.052	1.976***	.074
	E* λ	72,1183	1.427	23.939***	12.927***	10.496***	.010	5.122***	98.201***	4.05***
	δ * λ	144,1183	.705	16.084***	29.035***	23.643***	.036	.394	4.352***	.134
	E* δ * λ	864,1183	.088	2.010***	3.629***	2.955***	0.003298	.069	.985	.047

From Table 7, the following points are observed:

- The influence or effect of multicollinearity is generally significant under all criteria, but not under bias in equation one and occasionally significant under some of the criteria in equation two.
- The effect of the correlation between the error terms is generally significant under all criteria in both equations, but occasionally significant under bias criterion in equation two.
- The effect of estimators is generally significant under all the criteria except under bias in equation one and occasionally significant under bias criterion again in equation two.
- The interaction effect of estimators and multicollinearity is generally significant under all criteria except under bias in equation 1but occasionally in equation two.
- The interaction effect of estimators and the correlation between the error terms is generally significant under all criteria except under bias in equation two.
- The interaction effect of estimators, Multicollinearity, and the correlation between the error terms is generally significant under all criteria except under bias in equation two.

In summary, it can be inferred that the performances of the estimators are affected by Multicollinearity under all the criteria. The results of the LSD further test visa- vice their estimated marginal means revealed that MR, FIML, SUR, and 3SLS estimators are preferred to estimate β_2.

Conclusively, the estimator of MR, FIML, SUR, and 3SLS is preferred to estimate all the parameters of the regression model in the presence of multicollinearity and the correlation between the error terms at all the levels of sample sizes.

V. Summary of the Findings and Conclusions

a) *When there is no any form of correlation*

The summary of the results from the Analysis of variance tables of the criteria showing the performances of the estimators and sample sizes on parameters of the two-equation model when there is no form of correlation are presented in Table 8 below:

Notes

Notes

Table 8: Summary of results when there is no form of correlation

N	Eqn	Parameters	Preferred	Overall Assessment	Most Preferred
20	1	β_{01}	All	All	OLS
		β_{11}	All		
		β_{21}	All		
	2	β_{02}	OLS, CORC	OLS,CORC,MLE	
		β_{12}	OLS, CORC,MLE		
		β_{22}	OLS, CORC,MLE		
30	1	β_{01}	All	All	OLS
		β_{11}	All		
		β_{21}	All		
	2	β_{02}	All	OLS, CORC,MLE	
		β_{12}	OLS, CORC,MLE		
		β_{22}	OLS, CORC,MLE		
50	1	β_{01}	All	All	OLS
		β_{11}	All		
		β_{21}	All		
	2	β_{02}	All	All	
		β_{12}	All		
		β_{22}	All		
100	1	β_{01}	All	All	OLS
		β_{11}	All		
		β_{21}	All		
	2	β_{02}	OLS, CORC	OLS,CORC,MLE	
		β_{12}	OLS, CORC,MLE		
		β_{22}	OLS, CORC,MLE		
250	1	β_{01}	All	l Al	OLS
		β_{11}	All		
		β_{21}	All		
	2	β_{02}	OLS, CORC	OLS,CORC,MLE	
		β_{12}	OLS, CORC,MLE		
		β_{22}	OLS, CORC,MLE		

From table 8, when there is no correlation in the model under the equation one in all the five sample sizes, all the methods are equally good in estimating all the parameters β_{01}, β_{11} and β_{21}, thus it can be concluded that all the estimation methods are preferred in estimating all the model parameters in equation one.

Under the second equation, it was observed that OLS, CORC AND MLE estimation methods can estimate all the parameters of the model in all the sample sizes except when the sample sizes are 30 and 50. However, observing the two equations together, we can deduce that OLS is most preferred in estimating all the parameters of the two equations among all the estimation methods used due to its simplicity and efficiency over others.

b) When there are Multicollinearity and correlation between the error terms

The summary of results from the Analysis of variance tables of the criteria showing the performances of the estimators and sample sizes on parameters of the two-equation model when there is the presence of correlation between the error terms and multicollinearity are presented in Table 9 below:

Table 9: Summary of results of the model in the presence of multicollinearity and the correlation between the error terms

n	Eqn	Parameters	Preferred	Overall Assessment	Most Preferred
20	1	β_{01}	OLS,CORC,MLE	MR,FIML,SUR,3SLS	SUR, 3SLS
		β_{11}	MR,FIML,SUR,3SLS		
		β_{21}	MR,FIML,SUR,3SLS		
	2	β_{02}	All Except CORC	MR,FIML,SUR,3SLS	
		β_{12}	All Except CORC,MLE		
		β_{22}	MR,FIML,SUR,3SLS		
30	1	β_{01}	OLS,CORC,MLE	All	SUR,3SLS,FIML
		β_{11}	All		
		β_{21}	MR,FIML,SUR,3SLS		
	2	β_{02}	All Except CORC	All Except CORC	
		β_{12}	All		
		β_{22}	All		
50	1	β_{01}	OLS,CORC,MLE	MR,FIML,SUR,3SLS	FIML,SUR,3SLS
		β_{11}	MR,FIML,SUR,3SLS		
		β_{21}	MR,FIML,SUR,3SLS		
	2	β_{02}	All	MR,FIML,SUR,3SLS	
		β_{12}	All		
		β_{22}	MR,FIML,SUR,3SLS		
100	1	β_{01}	All	MR,FIML,SUR,3SLS	FIML,SUR,3SLS
		β_{11}	MR,FIML,SUR,3SLS		
		β_{21}	MR,FIML,SUR,3SLS		
	2	β_{02}	All	All	
		β_{12}	All		
		β_{22}	All		
250	1	β_{01}	All	MR,FIML,SUR,3SLS	FIML,SUR,3SLS
		β_{11}	MR,FIML,SUR,3SLS		
		β_{21}	MR,FIML,SUR,3SLS		
	2	β_{02}	All	All	
		β_{12}	All		
		β_{22}	All		

Table 9 summarized the case when there is the presence of correlation between the error terms and multicollinearity in the model under the equation one in all the five sample sizes; we observed that all the estimating methods are equally good in estimating the parameters β_{01} when the sample sizes are 100 and 250, but when the sample sizes are 20, 30 and 50 OLS, CORC and MLE estimation methods are also okay. Meanwhile, for parameters β_{11} and β_{21}, MulReg, FIML, SUR, and 3SLS estimators are preferred for their estimation; thus, it can be concluded that MulReg, FIML, SUR, and 3SLS estimating method are preferred in estimating all the model parameters in equation one.

Under equation two, it was observed that all estimation methods except CORC are good in estimating all the parameters of the model at all level of the sample sizes.

However, critically looking at the two equations considered in this study together, we can conclude that FIML, SUR, and 3SLS are preferred in computing all the parameters of the two equations among all the estimation methods used.

Notes

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Recommendation

The research work has revealed that FIML, SUR, and 3SLS methods of estimation are the most preferred estimator in estimating all the parameters of the model based on the four criteria used, namely, Bias, Absolute Bias, Variance and Mean Square Error under the five-level of sample sizes considered. It can, therefore, be recommended that when the validity of other correlation assumptions cannot be authenticated in a system of the regression model, the most preferred estimators to use are FIML, SUR, and 3SLS. Meanwhile, for any model without a form of correlation, the OLS, CORC, and MLE estimation methods are most preferred.

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