Global Journal of Science Frontier Research: A
Physics and Space Science
Volume 20 Issue 10 Version 1.0 Year 2020
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Relation between Mass, Angular Momentum and Spin Parameter of Black Holes

By Dipo Mahto
Marwari College

Abstract- In the present research work, a well known relation between mass (M), angular momentum (J) and spin parameter (a*) has established by entirely new mathematical operation using the model for first law of black hole mechanics given by the equation $\delta M = \frac{k}{8\pi}\delta A + \Omega\delta J - \upsilon\delta Q$ (Bardeen et al. 1973) and the relation $\Omega = M/2J$. (Meinel, 2006), which concludes that the angular momentum may be regarded as additional characterizing parameter of black holes.

Keywords: *angular momentum, spin parameter and surface gravity.*

GJSFR-A Classification: FOR Code: 020199

RELATIONBETWEENMASSANGULARMOMENTUMANDSPINPARAMETEROFBLACKHOLES

Strictly as per the compliance and regulations of:

Relation between Mass, Angular Momentum and Spin Parameter of Black Holes

Dipo Mahto

Abstract- In the present research work, a well known relation between mass (M), angular momentum (J) and spin parameter (a*) has established by entirely new mathematical operation using the model for first law of black hole mechanics given by the equation $\delta M = \frac{k}{8\pi}\delta A + \Omega\delta J - \upsilon\delta Q$ (Bardeen et al. 1973) and the relation $\Omega = M/2J$.(Meinel, 2006), which concludes that the angular momentum may be regarded as additional characterizing parameter of black holes.

Keywords: angular momentum, spin parameter and surface gravity.

I. Introduction

The three quantities such as mass, spin and charge are treated as fundamental quantity of black hole physics to characterize the black holes [1]. There are so many physicists and researchers who had done their works in these fields whose contribution in these directions are in the years given as: Oppenheiner (1939), Bardeen et al. (1973), Hawking (1974. 1975), Strominger & Vafa (1996), Florov et al. (1997), Transchen (2000), t Hooft (2000) Wald(2001), Narayan (2005), Carlip(2009) , Mahto et al.(2014, 2016, 2017, 2018) [1-17]. The mass of the stellar bodies like black holes ranging from 5 solar masses to 20 solar masses are existing in XRBs[11] and from 10^6 solar masses to 10^{10} solar masses are existing in AGN [1]. The spin of black holes ranging from +1 to -1 including zero for co-rotating and counter rotating black holes respectively are used to charaterise the nature of the black holes. The zero spin is used to characterize the non-spinning black holes [11]. The -1/2 and +1/2 are also possible to charaterise the nature of the black holes. On the event horizon, the mass is equal to the charge of black hole (M=Q)[8], usually, the charge is neutralized by plasma which surrounding the black holes[11]. In the present research work, a well known relation ($J=a^*M^2$) between mass(M), angular momentum(J) and spin parameter(a*) have established by entirely new mathematical operation using the model for first law of black hole mechanics and the relation of mass, angular velocity and angular momentum ($\Omega = M/2J$).

II. Theoretical Discussion

The first law of black hole mechanics is simply an identity relating the change in mass, angular momentum, horizon area and charge of a black hole represented by the following relation [10, 11].

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega\delta J - \upsilon\delta Q \quad \ldots\ldots\ \ldots..(1)$$

Where $\Omega =$ Angular velocity of the horizon. $\upsilon =$ difference in the electrostatic potential between infinity and horizon.

The entropy and surface area of the black holes are related by the following equation

$$S = \frac{A}{4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

The above equation is differentiated partially, we have

$$\delta S = \frac{\delta A}{4} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

The use of above equation into the equation (1) gives the following equation

$$\delta M = \frac{\kappa}{2\pi}\delta S_{bh} + \Omega\delta J - \upsilon\delta Q \ldots\ldots\ldots\ldots.(4)$$

The equation(4) is also known as First law of black hole mechanics representing the change in mass due to change in entropy, angular momentum and charge of black holes, where K is the surface gravity given by the following equation called Kerr solution [13].

$$\kappa = \frac{(M^4 - J_H^2)^{1/2}}{2M\{M^2 + (M^4 - J_H^2)^{1/2}\}} \ldots\ldots\ldots\ldots\ldots..(5)$$

Actually an astronomical black hole is not likely to have any significant electric charge, because it will usually be rapidly neutralized by surrounding plasma [11] and hence due to this reason substituting $\delta Q = 0$ in the eqn (4), we have

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega\delta J \ldots\ldots\ldots\ldots\ldots..(6)$$

and

$$\delta M = \frac{\kappa}{2\pi}\delta S_{bh} + \Omega\delta J \ldots\ldots\ldots\ldots\ldots..(7)$$

Author: Associate Professor, Dept. of Physics, Marwari College, TMBU, Bhagalpur, India. e-mail: dipomahto@hotmail.com

The above relation shows the change in mass corresponding change in the angular momentum and entropy of black holes.

In two dimensional parameter space, the mass, angular momentum and angular velocity of spinning black hole is related by the following equation [18].

$$M = 2\Omega J \quad \text{.......................(8)}$$

The relation is satisfied for a continuous sequence of stationary anti-symmetry, uniformly rotating perfect fluid bodies with a cold matter and finite baryonic mass can reach a Kerr black hole [19]. From equation (8), we have

$$\Omega = \frac{M}{2J} \quad \text{.........................(9)}$$

With the substitution of equation (9) in eq^n (7), we get

$$\delta M = \frac{\kappa}{2\pi}\delta S + \frac{M}{2J}\delta J \quad \text{.............(10)}$$

The surface gravity (κ) of the black holes on the fluid side of transition line vanishes on the black hole side of the transition line as $V_0 \rightarrow -\infty$ [19]. This leads that the first term of the right side will be equal to zero and gives the following relation.

$$\delta M = \frac{M}{2J}\delta J \quad \text{.........................(11)}$$

or

$$\frac{\delta J}{J} = 2\frac{\delta M}{M} \quad \text{.......................(12)}$$

Now the both sides of the above equation is integrated, we have

$$\int \frac{\delta J}{J} = 2\int \frac{\delta M}{M} + constant \quad \text{.............(13)}$$

or

$$\log_e J = 2\log_e M + \log_e a*$$

or

$$\log_e J = \log_e M^2 + \log_e a*$$

or

$$\log_e J = \log_e (a*M^2)$$

or

$$J = a*M^2 \quad \text{...................(14)}$$

The equation (14) is the final model relating the mass, spin and angular momentum of black holes in which the term $a*$ is a proportionality constant having values 0, +1, -1, +1/2 & -1/2. This constant denotes the spin parameter of different test black holes. We know that the spinning parameters are $a*$=1 for co-rotation and $a*$=-1 for counter rotation in the bosonic field of black holes, while for non-spinning black holes $a*$=0. [11]. The relation as in equation (14) can be obtained as follows:

The angular momentum (J) of spinning black holes is given by the following equation [11].

$$J = \frac{a*GM^2}{c} \quad \text{.....................(15)}$$

In the above equation, the symbols a*, G, M and c are spin parameter, gravitational constant, mass and velocity of light respectively. The gravitational constant (G) and the velocity of light (c) are the universal constant and for convenience, it is assumed that G=c=1. Putting these values in equation (15), we obtain exactly the same result as in the equation 14). With the substitution of spin parameter (a*=0, +1 & -1), the equation (14) becomes to

$$J = 0 \quad \text{..........................(16)}$$

$$J = \pm M^2 \quad \text{..........................(17)}$$

Or ....

$$|J| = M^2 \quad \text{..........................(18)}$$

This also holds good $a*$=1/2 for co-rotation and $a*$=-1/2 for counter rotation in fermionic field of black holes. Hence the equation (14) becomes to

$$J = \pm\frac{1}{2}M^2 \quad \text{.......................(19)}$$

Or

$$|J| = \frac{1}{2}M^2 \quad \text{...................(20)}$$

In the equation (17), the quantity angular momentum is the characterizing parameter of the extreme Kerr black hole limit of rotating perfect fluid bodies in equilibrium to study quasi-stationary transitions leading the black holes [20]. This category belongs to bosonic character of extreme Kerr black holes. In my opinion, there should be same explanation of the equation (19) for fermionic character of extreme Kerr black holes.

It is well known that the angular momentum (J) is related to the moment of inertia as well as kinetic energy. Hence the variation in angular momentum of bosonic/Fermionic field of black holes affects the moment of inertia and their kinetic energy. The model as proposed in the present work shows that the angular momentum, moment of inertia and kinetic energy bosonic field of black holes (except zero spin) is always greater than to that of fermionic field of black holes.

The whole situations from equation (16) to (20) can be summarized in the following table.

Table 1: Angular Momentum of black holes (J)

S. No.	Spinning parameters (a*)	Angular Momentum (J)	Category of black holes
1.	0	0	Non-spinning black holes/black holes of bosonic character of zero spin
2.	$+\frac{1}{2}$	$+M^2/2$	Black holes of Fermionic character of half spin
3.	$-\frac{1}{2}$	$-M^2/2$	Black holes of Fermionic character of half spin
4.	$+1$	$+M^2$	Black holes of bosonic character of spin +1
5.	-1	$+M^2$	black holes of bosonic character of spin -1

III. Physical Significance of Proportionality Constant (A*)

In equation (14), the proportionality constant (a*) is the spin parameter and gives a lot of information regarding the black holes. This parameter has very important role to decide the black hole is either spinning or non-spinning. This also shows the Bosonic and Fermionic character of black holes depending on its values. This value is different for bosonic and fermionic character of black holes. The values of spin parameter (a*) equal to 0, +1 or -1 representing bosonic field of black holes and this category of black holes is non-spinning for a*=0 and spinning for a*=+1 or -1, while for a*=+1/2 or -1/2 representing bosonic field of spinning black holes. The black holes for which the spin parameter (a*) is positive known as co-rotating black holes and negative values of the spin parameter (a*) is known as counter rotating black holes.

IV. Result and Discussion

The research work is mainly concerned with the relation between the mass, angular momentum and spin parameter of black holes. In this work, the well known relation between the mass, angular momentum and spin parameter of black holes ($J=a^*M^2$) is established by entirely new method using the first law of black hole mechanics $\delta M = \frac{\kappa}{2\pi}\delta S + \Omega\delta J - \upsilon\delta Q$ (Bardeen et al. 1973) and the relation between angular velocity, mass and angular momentum of black holes given by $\Omega = M/2J$. (Meinel, 2006). With proper mathematical operation by applying some limit/condition on stationary anti-symmetry, uniformly rotating perfect fluid bodies of cold matter and finite baryonic mass to be a Kerr black hole, the required relation is obtained. From equation (18) and (20), it is clear that the magnitude of the angular momentum of bosonic field of black holes except zero spin (a*=0) is twice to that of the magnitude of the angular momentum of Fermionic field of spinning black holes. Hence the moment of inertia of bosonic field of black holes except zero spin (a*=0) is twice to that of the magnitude of the moment of inertia of the Fermionic field of spinning black holes, because the angular momentum and moment of inertia is related by $J = I\Omega$, where I represents moment of inertia of black holes. This also shows that the kinetic energy of of bosonic field of black holes except zero spin (a*=0) is greater than to that of the kinetic energy of the Fermionic field of spinning black holes.

We know that the mass, spin and charge are the fundamental quantities used to characterize the black holes. The angular momentum (J) is product of spin parameter (a*) and square of its mass(M), i.e. ($J=a^*M^2$). Hence the angular momentum may be used as characterizing parameter of black hole in addition to mass, spin and charge.

V. Conclusion

The following conclusions may be drawn from the present research work.

1. The angular momentum may be used as additional characterizing parameter of black holes.
2. The angular momentum, moment of inertia and kinetic energy of of bosonic field of black holes except zero spin (a*=0) is greater than to that of the Fermionic field of spinning black holes.

VI. Future Plan of Present Research Work

1. The calculation of angular momentum of different test black holes in both categories of black holes like XRBs and AGN.
2. The determination of angular momentum of black holes may be used to calculate moment of inertia and kinetic energy of the black holes and hence temperature of the black holes.

References Références Referencias

1. R. Ruffini & J. A. Wheeler: Introducing the Black Hole, Physics Today. 24 (1): 30–41, 1971.

2. J. R. oppenheimer, H. Synder: "on continued Gravitational Contraction" physical review 56, P-455, 1939.
3. J. M. Bardeen, B. Carter, and S. W. Hawking: The four laws of black hole mechanics. Comm. Math. Phy. 31 (2): 161-170. doi: 10.1007/BFO1645742., (1973). http://projecteuclid.org/uclid.com/110385-8973.
4. Hawking. S.W., (1974). "Black hole explosions?". Nature, 248:30-31. doi:10.3038/248030a0. http://www.nature.com/nature/journal/v248/n5443abs/248030a0.html.
5. Hawking, S.W.,: "Particle creation by Black hole" Comm. Math. Phys. 43,199-220, (1975). 3,3,1.
6. Strominger, A; Vafa, C: "Microscopic origin of Bekenstein- Hawking entropy". Physics Letters B379:99.doi:10.1016/0370-2693(96)00345-0. (1996)
7. Frolov, V.P., and Fursaev, D.V.: " Mechanism of Generation of Black hole . Entropy in Sakharov's Induced gravity." Phys. Rev. D, 56, 2212-2225,(1997).
8. Jennine Transchen : " An introduction to black hole Evoporation " arXiv: gr-qc/0010055V1,13 oct, 2000.
9. Genard't Hooft: "The Holographic Principle". arXiv:hep-th/0003004, (2000).
10. R. M. Wald: The Thermodynamics of black holes, Living reviews in relativity, 2001.
11. Ramesh Narayan : " Black holes in Astrophysics". arXiv: gr-qc/0506078V1,14 Jan, 2005.
12. Carlip,S: "Black Hole Thermodynamics and Statistical Mechanics. 769. pp.89. doi: 10.1007/978-3-540-88460-6 3, (2009).
13. The Kerr-Newman solution :- http://en.wikipedia.org/wiki/surface_ gravity
14. Dipo Mahto, B. K. Jha, M.P. Singh and P. Jha: Characteristics of Non-spinning Black Holes, British Journal of Applied Science & Technology 4(29):4136-4147, 2014.
15. Dipo Mahto and Alok Ranjan: Characteristics of Spining Black Holes, British Journal of Applied Science & Technology, 17(5):1-9, 2016 USA.
16. Dipo Mahto and Annuradha Kumari: Change in entropy of spinning black holes due to corresponding change in mass in XRBs. International Journal of Astronomy and Astrophysics, vol.8, p171-177, 2018.
17. Dipo Mahto and Annuradha Kumari: Change in mass of spinning black holes w.r.t. angular momentum. Journal of Modern Physics, vol.9. p1037-1042, 2018.
18. R. Meinel: Quasistationary collapse to the Extreme Kerr black hole, Ann. Phys (Leipzig), 13, 600 (2004).
19. R. Meinel: On the black hole limit of rotating fluid bodies in equilibrium, Classical and Quantum gravity, 23(4), Dec, 2006.
20. H. Labranche, D. Petroff, M. Ansorg: The Parametric Transition of Strange Matter Rings to a Black Hole, General Relativity and Gravitation, 2007.