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By AMADI, Godpower Dike, BIU, Oyinebifun Emmanuel & ARIMIE, Christopher Onyema

University of Port Harcourt

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GJSFR-F Classification: *MSC 2010: 37M10*

Notes

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AMADI, Godpower Dike [α], BIU, Oyinebifun Emmanuel [σ] & ARIMIE, Christopher Onyema [ρ]

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Keywords: gross domestic product (GDP), vector autoregressive (VAR) model, ARIMA, forecast accuracy measure, model selection criteria.

I. Introduction

The analysis of time dependent variables is one of the methods designed for prediction of future events. Most variables are economical in nature and the economy of any nation partly depends on the interplay of these variables with respect to time. Indeed, time series plays a vital role in planning and predicting of future economy of any nation(Sani and Abdullahi, 2012). Nigeria Economy is not stable and for decades now, the country has been facing some economic crises, challenges or shocks both internally and externally. Due to the present unstable state of the economy resulting from fall in the oil price which Nigeria so much depend on for generating its internal Revenue (provides 75% of Nigeria's IGR) there is need to go back to Agriculture. However, focusing attention only to Agricultural sector may not solve the problem. There is need for Government to diversify by considering other sectors ofthe economy involved, in order to capture these variables along with Agricultural sector with the sole

Author α: Department of Mathematics/statistics, Faculty of Natural and Applied Science Ignatus Ajuru University of Education, Port Harcourt, Rivers State, Nigeria.
Author σ: Department of Mathematics/statistics, Faculty of Science, University of Port Harcourt, Rivers State, Nigeria.
Author ρ: Department of Radiology, University of Port Harcourt Teaching Hospital, Port Harcourt, Nigeria. e-mail: codarimie@yahoo.com

aim of studying their inter-relationship with respect to time and possibly develop a model for predicting the future of the various sectors under consideration.

The aim of this work is to build a suitable multivariate time series model (VAR Model) and univariate (ARIMA) Model for the sectors in Nigeria's Gross Domestic products (GDP). This would be achieved via the following specific objectives:(1) Obtain the correlation between the variables and stationarity of each variables, (2) Fit suitable ARIMA model of each variables and their forecasts, (3) Fit suitable VAR Model by using each variable as the dependent variable that can be used to predict future time series of the sectors in Nigeria's Gross Domestic products (GDP) and (4) To compare the forecasts of the two models for each variable. This work would help to improve macroeconomic policy formulation in Nigeria especially, by predicting future trend of output from major sectors(Agriculture, Manufacturing industries, Oil and Gas, Solid minerals, Transportation and General services).

Data for the five sectors under consideration in Nigeria's Gross Domestic products (GDP)were obtained from the National Bureau of Statistics. The data were quarterly data running from 1986-2018, making a total of 32 years.This work is limited to the sectors in Nigeria's Gross Domestic Product such as Agriculture, Industry, Building & Construction, and wholesale & Retail and Services. It did not consider other Nigeria's Economic variable such as Consumer Price Index, producer price index, crude oil production, etc.

II. Literature Review

Time series find application in Statistics, Signal Processing, Econometrics, Mathematical finance, Pattern recognition, Weather forecasting etc. Time Series analysis comprises methods for analyzing time series data in order to extract meaningful statistics and other characteristics of the data while time series forecasting involves use of model to predict future values based on previous observed values. In forecasting events that will occur in the future, a forecaster must rely on information concerning events that have occurred in the past. This is carried out by identifying a pattern that can be used to describe it. Believing that the identified pattern will continue to repeat itself in the future, this pattern is extended into the future in order to prepare a forecast. Methods of analysis of time series may be divided into Frequency Domain and Time domain. Frequency domain includes spectral analysis and Wavelet analysis. The latter include auto-correlation and cross correlation analysis.

Works on univariate and multivariate time series have been reported in the literatures.

Abdul and Marwan (2013), studied the effect of interest rate, inflation rate, GDP, on real economic growth rate in Jordan. Unit root test was employed to check the integration order of the variables. The results showed that inflation causes interest rate while all the other variables are independent of each other. Regression was also conducted to test the growth rate and interest rate which suggested that current interest rate has an influence on growth rate. Finally, it was shown that current GDP and one lag GDP have influence on growth rate.

Omoke (2010) investigated Inflation and Economic growth in Nigeria. Co-integration and Granger Causality was used to carry out the test. Consumer price index was used as proxy for inflation and GDP as perfect proxy for Economic growth. The results showed that there was a co- integration relationship between inflation and economic growth for the Nigerian data used. Also, Fadli (2011) noted that from

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empirical findings, Causality that runs from inflation to Economic growth indicates that inflation has an impact on growth.

Okororie (2012) used Buy-Balloutto model Nigeria Domestic Crude Oil production applying inverse square root transformation to stabilize the Variance.Quadratic trend was fitted and the error component was discovered to be random and normally distributed with mean zero and some constant Variance. Aminu *et al.*, (2013) examined the effect of unemployment and inflation on economic growth in Nigeria. Augmented Dickey-fuller technique was used in testing the unit root property of the series. Also, Granger Casualty test of causation between GDP, Unemployment and inflation implied that all the variables in the model are stationary. The results revealed that unemployment and inflation impacted positively on economic growth. However, Nasiru and Solomon (2012) showed that unemployment does not affect economic growth.

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Amos (2010) applied Time Series Modeling to South Africa Inflation data. Autoregressive integrated moving average (ARIMA) and Conditional heteroscedastic (ARCH) models were fitted to financial time series data. Box and Jenkins (1976) strategies were employed and the best fitted model for each family of model was selected. Onwukwe, and Nwafor (2014)used Multivariate Time Series model to study Major Economic Indicators in Nigeria. They obtained a stable Vector Autoregressive Model for the six economic variables. The result of the Granger causality analysis is unique enough, and there exist causality between variables. The Exchange rate as at the period of study reveals weak correlation which signifies the weak and devaluation of the Nigeria currency. Gross Domestic Product was seen as a good predictor to other economic indicators and the External Reverse. The relationship between these economic indicators is however significant and positive in either direction. This implies that the connection among these economic indicators and economic activities in Nigeria over some period of time is not automatic and the study also provides forecast value for the next two years from the last period of investigation.Other works reviewed which discussed the relationship among economic variables under consideration using univariate and multivariate times modeling techniques include Mphumuzi (2013), Francis and Charles (2012), Basutor (2014), Ruey (2013), Rokas (2012) and Abdulrasheed (2005). Whereas these works discussed co-integration, causality effects and relationships among the economic variables, our study highlights the most suitable model to forecast future performance of the five economic sectors with respect to the GDP.

III. Methods

This section explains the methods used in conducting this study and the reason(s)for choosing each method. Statistical tools used include: Autoregressive Integrated Moving Average (ARIMA), Stationary and Non-Stationary Time Series, Pre-whitening, Akaike Information Criterion (AIC), Schwartz-Bayesian Information Criteria (BIC) and Vector Auto regression (VAR).

a) *Vector Autoregressive (VAR) Model*

The objectives of univariate time series analysis is to find the dynamic dependence of Y_t that is the dependence of $Y_t$on its past values$Y_{t-1,}Y_{t-2,}Y_{t-3\ldots}$.A linear model implies that Y_t depends on its past values. The Vector Autoregressive (VAR) model is an approach in modeling dynamics among a set of variables. The approach usually focuses on the dynamic of multiple time series. Vector Autoregressive(VAR)

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model is also an independent reduced form dynamic model which involves constructing an equation that makes each endogenous variable a function of their own past values and past values of all other endogenous variables. The basic p-lag Vector autoregressive VAR(p) model has the form.

$$Y_t = C + \Pi_1 Y_{t-1} + \Pi_2 Y_{t-2} + \dots + \Pi_p Y_{t-p} + \varepsilon_t \quad t = 1, \dots, T. \tag{3.1}$$

where

$Y_t = (y_{1t}, y_{2t}, \dots y_{nt})$ is an $(n \times n)$ vector of time series variable

$\Pi = (n \times n)$ coefficient matrices

ε_t is an $(n \times 1)$ unobserved zero mean with white noise vector process(serially uncorrelated and independent) with invariant covariance matrix Σ

The model can be written in the matrix form as

$$\begin{pmatrix} y_{1t} \\ y_{2t} \\ . \\ . \\ y_{nt} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ . \\ . \\ c_n \end{pmatrix} + \begin{pmatrix} \pi_{11}^1 \pi_{12}^1 \cdots \pi_{1n}^1 \\ \pi_{21}^1 \pi_{22}^1 \cdots \pi_{2n}^1 \\ . \quad . \quad \cdots \quad . \\ . \quad . \quad \cdots \quad . \\ \pi_{n1}^1 \pi_{n2}^1 \cdots \pi_{nn}^1 \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \\ . \\ . \\ y_{nt-1} \end{pmatrix} + \begin{pmatrix} \pi_{11}^2 \pi_{12}^2 \cdots \pi_{1n}^2 \\ \pi_{21}^2 \pi_{22}^2 \cdots \pi_{2n}^2 \\ . \quad . \quad \cdots \quad . \\ . \quad . \quad \cdots \quad . \\ \pi_{n1}^2 \pi_{n2}^2 \cdots \pi_{nn}^2 \end{pmatrix} \begin{pmatrix} y_{1t-2} \\ y_{2t-2} \\ . \\ . \\ y_{nt-2} \end{pmatrix} + \cdots +$$

$$\begin{pmatrix} \pi_{11}^p \pi_{12}^p \cdots \pi_{1n}^p \\ \pi_{21}^p \pi_{22}^p \cdots \pi_{2n}^p \\ . \quad . \quad \cdots \quad . \\ . \quad . \quad \cdots \quad . \\ \pi_{n1}^p \pi_{n2}^p \cdots \pi_{nn}^p \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \\ . \\ . \\ y_{nt-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ . \\ . \\ \varepsilon_{nt} \end{pmatrix} \tag{3.2}$$

The data is a secondary source obtained from National Bureau of Statistics for the five sectors under consideration in Nigeria's Gross Domestic products (GDP). The data is quarterly data running from 1986-2018 making a total of 32 years.

b) Stationary and non-stationary time series

A time series is said to be stationary if the statistical property e.g. the mean and variance are constant through time. If we have n values of observations $x_1, x_2, x_3, \dots x_n$ of a time series, we can plot this values against time to help us determine if the time series is stationary. If the n-values fluctuate with constant variation around a constant mean μ, then we can say that the time series is stationary and all processes which do not possess, this property are called "non-stationary". A non-stationary time series can be made stationary by transforming the time series into series of stationary time series value (differencing).

The general linear process allows us to represent X_t as the weighted sum of present and paste values of the white noise ε_t. ε_t can be represented as

$$X_t = \varepsilon_t + \psi_1 \varepsilon_{t-1} + \psi_2 \varepsilon_{t-2} + \dots$$

$$= \varepsilon_t + \sum_{j=1}^{\infty} \psi_j \varepsilon_{t-j}$$

$$= \sum_{j=o}^{\infty} \psi_j \varepsilon_{t-j} \quad where\ \psi_0 = 1 \tag{3.3}$$

Notes

Thus, white noise ε_t consist of a sequence of uncorrelated random variable with zero mean and constant variance. That is

$$E[\varepsilon_t] = 0 \qquad Var[\varepsilon_t] = \delta^2$$

c) Mixed Autoregressive moving average (ARMA) model

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According to Box and Jenkins (1970), mixed autoregressive moving average model is the combination of MA(q) and AR(p). Let $X_t$be the deviation from the mean μ, the ARMA(p, q) model can be written as

$$x_t - \phi x_{t-1} - \phi_2 x_{t-2} - \cdots - \phi_p x_{t-p} = \varepsilon_t - \theta \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q}.$$

Thus,

$$\phi(B) x_t = \theta(B) \varepsilon_t \qquad (3.4)$$

The equation (2.9) can be written as

$$x_t = \phi^{-1}(B)\theta(B)\varepsilon_t$$

$$= \frac{\theta(B)}{\phi(B)} \varepsilon_t = \frac{1-\theta_1 B - \cdots - \theta_q B^q}{1-\phi_1 B - \cdots \phi_p B^P} \varepsilon_t \qquad (3.5)$$

ARIMA model are resultant time series obtained if a non stationary time series which has variation in mean is differenced to get rid of the variation. The resultant series becomes stationary after differencing. The word "integrated" is used whenever differencing is applied to achieve stationarity. The ARIMA model is based on prior values in autoregressive term and the error made by previous prediction. The order of ARIMA model is given by p, d, q where,p represents the autoregressive component, $d$stands for the differencing to achieve stationarity and$q$ is the order of the moving average.

d) Seasonal Autoregressive integrated moving average.

Seasonal autoregressive integrated moving average (SARIMA) model is used for time series with seasonal and non-seasonal behavior. For ARIMA model, the autoregressive part deals with past observations while the moving average is concern with the errors associated with the series. The order of ARIMA model is $p, d, and\ q$ where,$p, d, and\ q$ are whole numbers ≥ 0.A process y_t is said to be an ARIMA model with parameter $p, d, and\ q$ if $\nabla^d y_t$ is shown by a stationary ARMA(p, q)model where ∇ stands for the difference operator. Thus, we write

$$\phi(B)\nabla^d z_t = \theta(B)\varepsilon_t \qquad (3.6)$$

SARIMA model has multiplicative and additive part. The multiplicative is so applied because of the assumption that there exist a significant parameter resulting from the multiplication between non seasonal parameters. By the use of ∇ and B notation, ARIMA(p, d, q) model can be written as

$$\phi(B) w_t = \theta(B) \varepsilon_t \qquad (3.7)$$

where the polynomial in B is given as

$$\phi(B) = 1 - \phi_1(B) - \cdots - \phi_p B^p$$

$$\theta(B) = 1 - \theta_1(B) - \cdots - \theta_q B^p$$

For further purpose of this study, we shall be focusing on multiplication model because of the assumptions that there is a major parameter because of the multiplication between the non-seasonal and seasonal model. This is denoted by $ARIMA\ (p,d,q) \times (P,D,Q)$ written as

$$\phi_p(B)\phi_P(B^s)\nabla^d\nabla_s^D z_t = \theta_q(B)\theta_Q(B^s)\varepsilon_t \quad (3.8)$$

Notes

where

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots \phi_p B^p$$

$$\Phi(B) = 1 - \Phi_1, sB^s - \Phi_2, sB^{2s} - \cdots - \Phi_p, sB^p$$

$$\nabla^d = 1 - B - B^2 - \cdots - B^d$$

$$\nabla_s^D = 1 - B^s - B^{2s} - \cdots - B^{2D}$$

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q$$

$$\Theta(B^s) = 1 - \Theta_1, sB^s - \Theta_2, sB^{2s} - \cdots - \Theta_Q, sB^{QS}$$

where z_t is the time series at period t, ε_t stands for the white noise, B represents the backshift operator, S is the duration of the seasonal model which could be weekly, quarterly or yearly, p=Autoregressive parameter, P=Seasonal autoregressive parameter, d=order of the monthly difference(quarterly difference), D=order of seasonal difference, q=moving average parameter and Q=seasonal moving average parameter.

e) Pre-whitening

This refers to fitting an ARIMA model for the input series sufficient to reduce the residual to white noise, then fitter the input series with this model to get white noise residual series, then filter the response series with the same model and cross-correlate the filtered response with filtered input series.

f) Model selection criteria

Before an ARMA(p, q) may be estimated for a time series $Y_{t,}$ the AR and MA order p and q have to be determined by examining the SACF and SPACF for $Y_{t..}$ The idea is to fit all ARMA(p q) models with order p ≤ $p_{max}$and q ≤ q_{max} and choose the value of p and q which minimizes some model selection criteria. For ARMA(p, q), the model selection criteria (MSC) is given by

$$MSC(p,q) = Ln\left(\sigma^2(p,q)\right) + c_T.\varphi(p,q) \quad (3.9)$$

where $\sigma^2(p,q)$ is the MLE of $\text{var}(\varepsilon_t) = \sigma^2$ without a degree of freedom correction from the ARMA(p q) model, c_T is a sequence indexed by the sample size T, and $\varphi(p,q)$ is a penalty function that penalizes large ARMA(p, q) models.

g) Information Criteria

The two most common information criteria are the AkaikeInformation Criteria (AIC) and the Schwarz-Bayesian(BIC).

i. *Akaike Information Criteria*

This was developed by Hirotugu Akaike under the name "Information criteria". The AIC is a measure of the relative goodness of fit of a statistical model. It was first published by Akaike in 1974.It tends to describe the exchange between bias and variance in model specification for a data set. Individual model could be ranked based on their AIC value. Thus

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$$AIC = -2InL(\theta) + 2K \tag{3.10}$$

where, $L(\theta)$ is the value of the likelihood function and$K$ is the number of parameters in the model.

ii. *Schwartz-Bayesian Information Criteria (SBIC or BIC)*

This is model selection criteria that involves selections among finite set of models. Whenever models are fitted, it is necessary sometimes to increase the likelihood by adding parameters but sometimes adding parameters may result to over fitting which can be resolved by introducing a penalty term for the number of parameter in the model, BIC is given by

$$BIC = -2InL(\theta) + KIn(N) \tag{3.11}$$

where, $L(\theta)$ is the value of the likelihood function evaluated,

K is the sum number of parameters estimated and N is the number of usable observation

Procedures for developing linear time series by Box and Jenkins (1976) proposed four steps in developing a linear time series which are Model identification, Estimation of parameters, Diagnostic Checking and Forecasting.

h) Accuracy Measures of the Estimated Values

To gauge the accuracy of our estimates, the estimated errors will be used to compare the two methods forecasts enumerated in the objective. This is done by subtracting the estimated forecast values (EFV) from the original values or [actual values (AV)] to obtain the estimate errors. The estimate error is denoted by

$$e_i = \mathrm{AV}_i - EFV_i \ , \ i = 1 \ , \ 2, \ldots v \tag{3.12}$$

where, v is the number of forecast values.

In addition, the accuracy measures are: Mean Error (ME), Mean Absolute Error (MAE), Mean Squared Error (MSE), Root Mean Squared Error (RMSE) and Mean Absolute Percentage Error (MAPE). However, this study will only consider three out of the five accuracy measures which are: Mean Error (ME), Mean Squared Error (MSE) and Mean Absolute Percentage Error (MAPE).

i. *Mean Error (ME)*

The first descriptive Statistics of Error used is called the Mean Error. It indicates the deviation between the actual values and estimates, Mean Error is given as

$$\mathrm{ME} = \left[\frac{1}{v}\sum_{i=1}^{v} e_i\right] \tag{3.13}$$

ii. *Mean Squared Error (MSE)*

MSE also indicates the fluctuations of the deviations and it can be calculated as

$$\text{MSE} = \left[\frac{1}{v}\sum_{i=1}^{v} e_i^2\right] \tag{3.14}$$

iii. *Mean Absolute Percentage Error (MAPE)*

This accounts for the percentage of deviation between the actual values and estimates. This can be obtained as

$$\text{MAPE} = 100 \times \left[\frac{1}{v}\sum_{i=1}^{v} \left|\frac{e_i}{AV_i}\right|\right] \quad (AV_i \neq 0) \tag{3.15}$$

Notes

i) Coefficient of Variation

To be able to identify the suitable model to use to forecast each of the variables, we will introduce the coefficient of variation. The coefficient of variation (CV) is a statistical measure of the dispersion of data points in a data series around the mean. The coefficient of variation represents the ratio of the standard deviation to the mean, and it is a useful statistic for comparing the degree of variation from one data series to another, even if the means are drastically different from one another. The coefficient of variation shows the extent of variability of data in a sample in relation to the mean of the population. In finance, the coefficient of variation allows investors to determine how much volatility, or risk, is assumed in comparison to the amount of return expected from investments. Ideally, the coefficient of variation formula should result in a lower ratio of the standard deviation to mean return, meaning a better risk-return trade-off. Note that if the expected return in the denominator is negative or zero, the coefficient of variation could be misleading.

This can be obtained by;

$$\text{CV} = \frac{\sigma}{\mu} \tag{3.16}$$

where, σ is the standard deviation and ` is the mean.

IV. Results Presentation and Analysis

The data used in this research were quarterly Gross Domestic Product (GDP) and some Nigeria Sectors Current Basic Prices [Quarterly (N' Billion)] collected from the 2018 Statistical Bulletin Real Sector Statistics (SBRS) for the period of 1981-2018 making a total of 37 years (Appendix A). Results of analysis of the data are presented in the following subsections. Minitab statistical software version 17 was used for the analysis.

a) Time Series Plots and Correlation Analysis

The five variables were plotted in the Figures 4.1 to 4.5 below. The correlation matrix in Table 4.1 shows that there is strong relationship among all the variables including the GDP.

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Table 4.1: Correlation Matrix between the variables

	Agriculture	Industry	Building Construction	Wholesale Retail Trade	Services	GDP
Agriculture	1					
Industry	0.953431	1				
Building Construction	0.926086	0.896342	1			
Wholesale Retail Trade	0.960794	0.921519	0.988916	1		
Services	0.942882	0.897907	0.993628	0.993051	1	
GDP	0.979138	0.951794	0.980361	0.994021	0.987319	1

Figures 4.1 to 4.5 show an upward trend in all the sectors examined with respect to the GDP series and the trend appear to be quadratic since they all appreciate from the first year to the last year (or curve shaped).

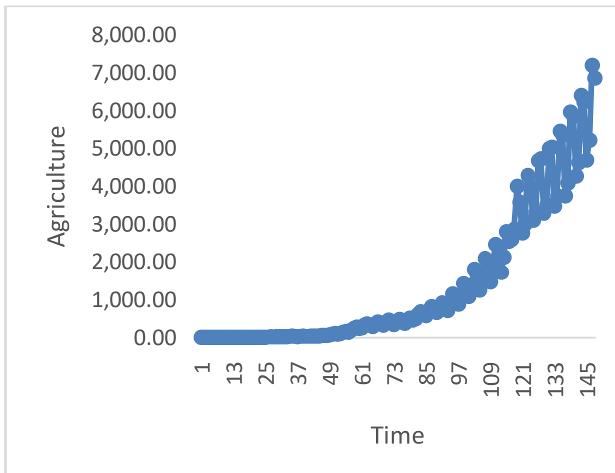

Figure 4.1: Agriculture Series Plot

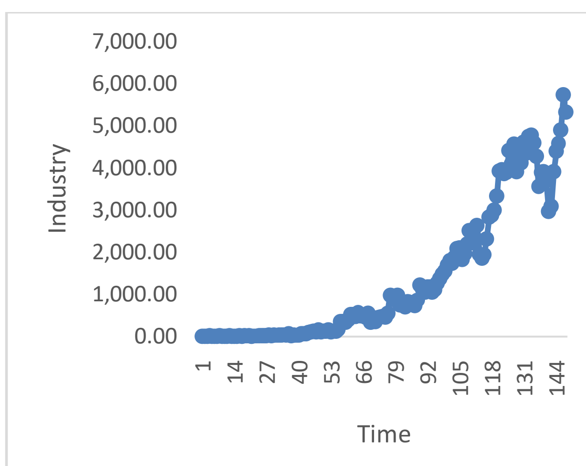

Figure 4.2: Industry Series Plot

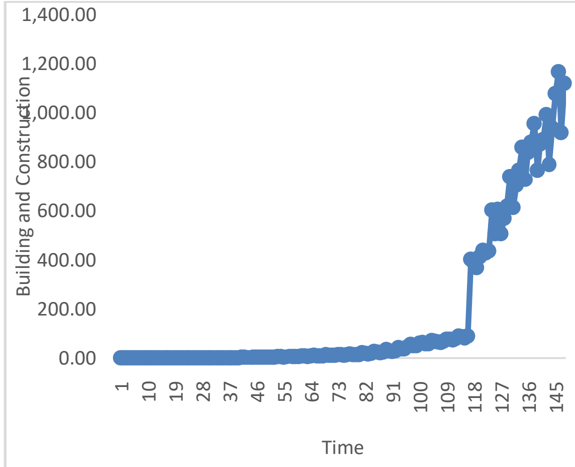

Figure 4.3: Building & Construction Series Plot

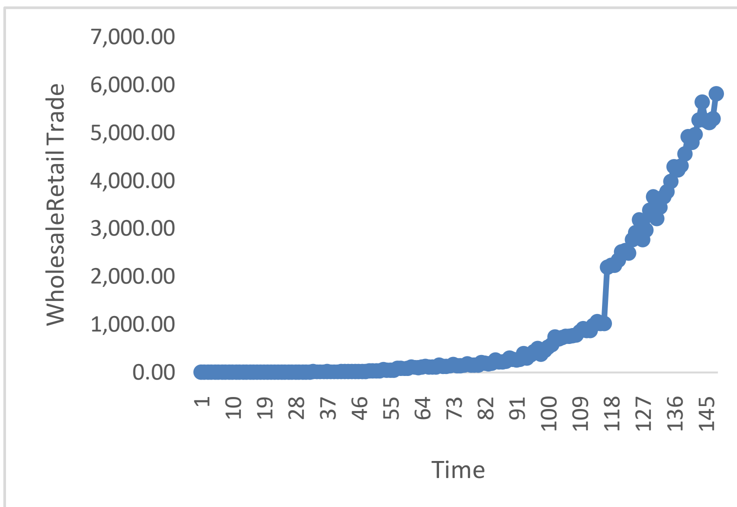

Figure 4.4: Wholesale Retail TradeSeries Plot

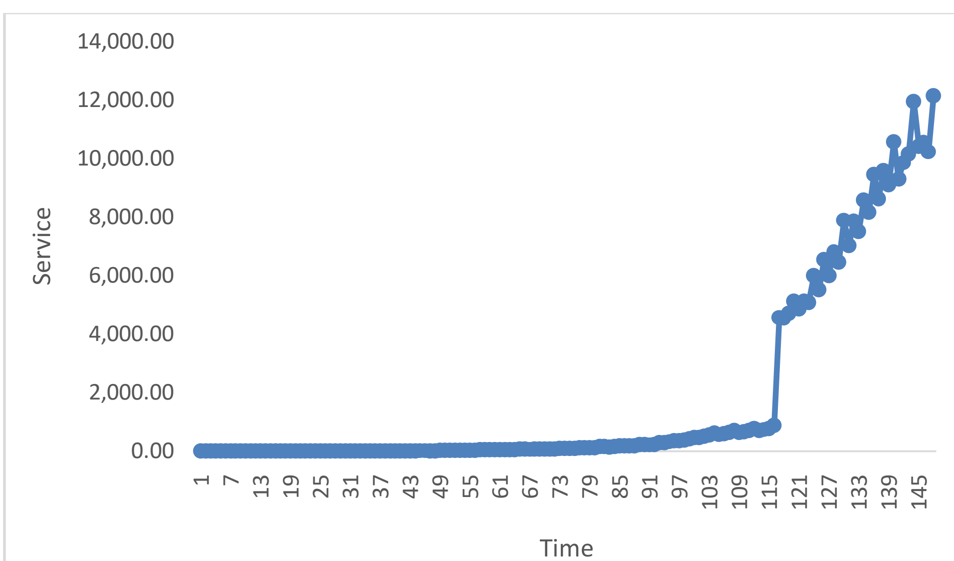

Figure 4.5: Service Series Plot

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b) Univariate Time Series Analysis

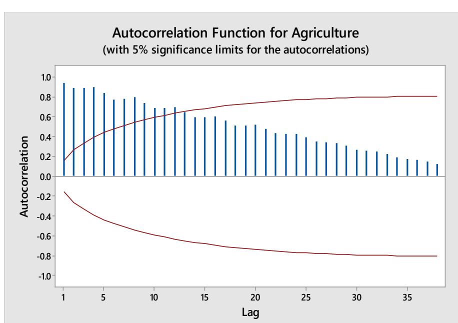
The ACF plot for Agriculture suggest an Autoregressive (AR) process while the PACF plot shows that the data is AR(5) with little sign of seasonality as shown in fig 4.8 and fig 4.9 respectively. The ACF plot for Industry suggest an Autoregressive (AR) process while the PACF plot shows that the data is AR(3) with little sign of seasonality as shown in fig 4.10 and fig 4.11 respectively. The ACF plot for Building & Construction suggest an Autoregressive (AR) process while the PACF plot shows that the data is AR(4) with little sign of seasonality in fig 4.12 and fig 4.13 respectively. The ACF plot for Wholesale and Retail suggest an Autoregressive (AR) process while the PACF plot shows that the data is AR(1) as shown in fig 4.15 and 4.16 respectively. The ACF plot for Service suggest an Autoregressive (AR) process while the PACF plot shows that the data is AR(2) as shown in fig 4.17 and 4.18 respectively. The ACF plot for Gross Domestic Product (GDP) suggest an Autoregressive (AR) process while the PACF plot shows that the data is AR(1) as shown in fig 4.19 and 4.20 respectively.

i. *Univariate Time Series Plots for the Differenced Data*

The plots of the ACF, PACF in figure 4.8 to 4.20 of the six variables shows that the series are not stationary and hence, need to be differenced to attain stationarity before fitting the ARIMA models.Figure 4.21 shows that the movement of the plot is sinusoidal (sine wave pattern) after first differencing indicating that the series is now stationary with mean zero and constant variance. The ACF and PACF plots in Figures4.22 and 4.23 respectively indicate the presence of seasonarity of order 2 and also have both AR(2) and MA(2) process in it.The difference series, ACF and PACF Plots show that the variables are now stationary with a seasonality of order 2.

ii. *ARIMA Model Parameter Estimate and Time Series Modelling*

In this section we show several fitted ARIMA models with the results of ACF and PACF correlogram. Also shown are the computed model residual sum of square, AIC and BIC as discussed in chapter three of this work. The parameter estimates and p-value of univariate time series model (ARIMA) are summarized for each variable as follows (Tables 4.2 to 4.6).



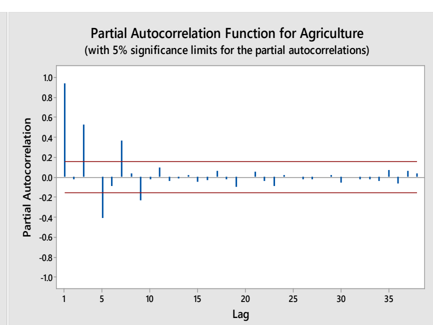


Figure 4.8: ACF Plot for Agriculture *Figure 4.9:* PACF Plot for Agriculture

Notes

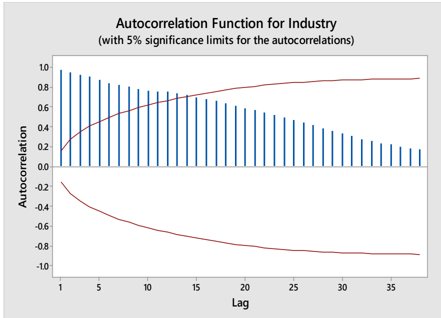


Figure 4.10: ACF Series Plot for Industry

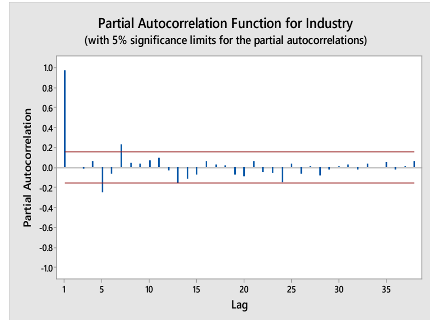


Figure 4.11: PACF Plot for Industry

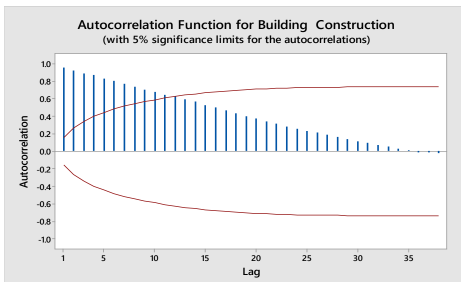


Figure 4.12: ACF Plot for Building & Construction

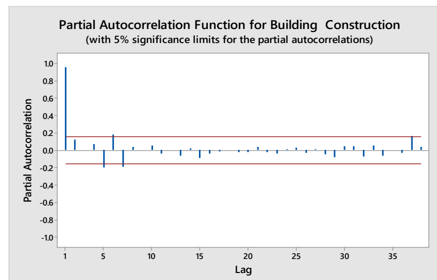


Figure 4.13: PACF Plot for Building & Construction

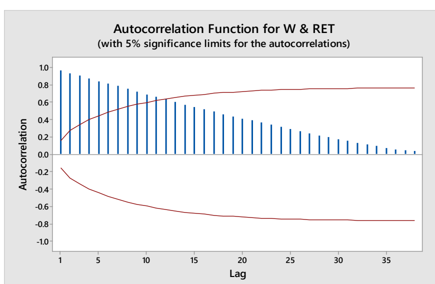


Figure 4.15: ACF Plot for Wholesale & Retail

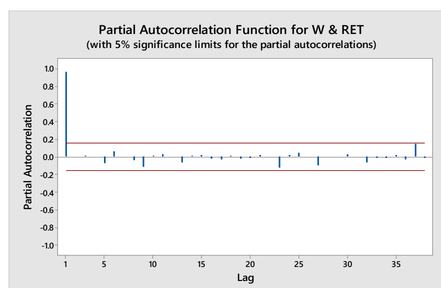


Figure 4.16: PACF Plot for Wholesale & Retail

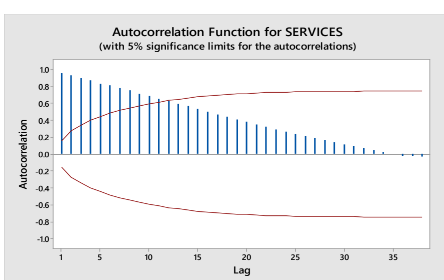


Figure 4.17: ACF Plot for Services

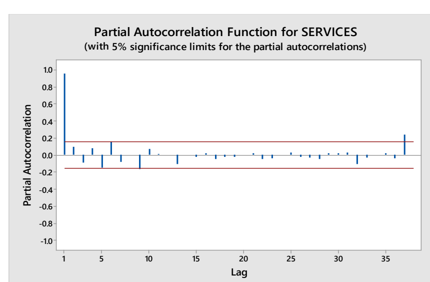


Figure 4.18: PACF Plot for Services

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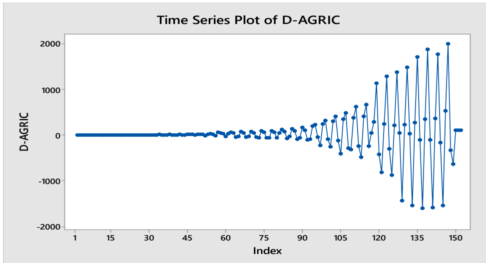

Figure 4.21: Series Plot for Differenced Agriculture

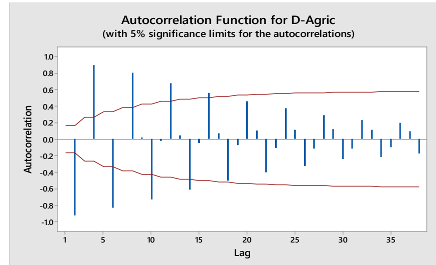

Figure 4.22: ACF for Differenced Agric Plot

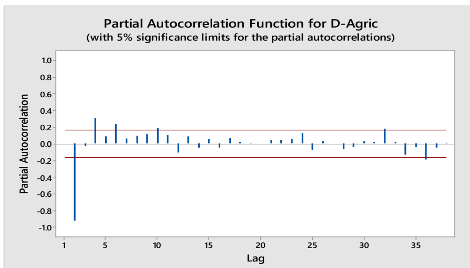

Figure 4.23: PACF for Differenced Agric Plot

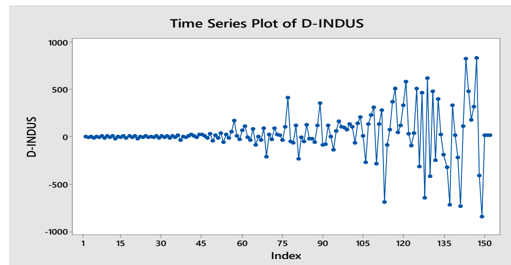

Figure 4.24: Series Plot for Differenced Industry

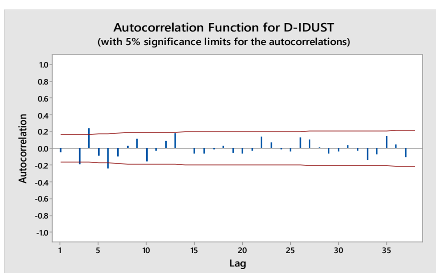

Figure 4.25: ACF for Differenced Industry Plot

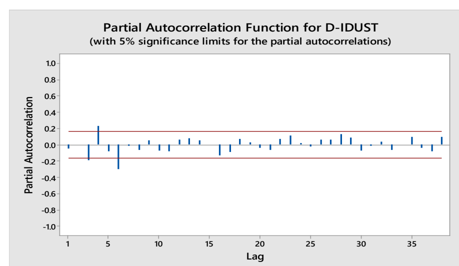

Figure 4.26: PACF for Differenced Industry Plot

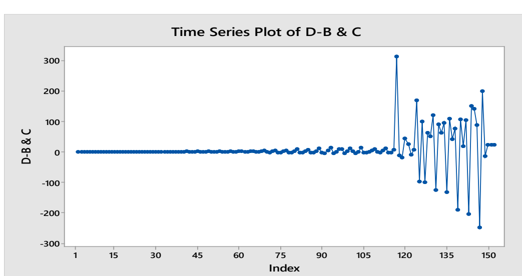

Figure 4.27: Series Plot for Differenced Building & Construction

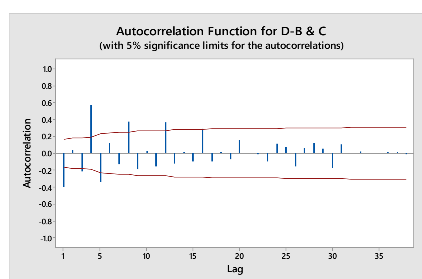

Figure 4.28: ACF for Differenced B & Con Plot

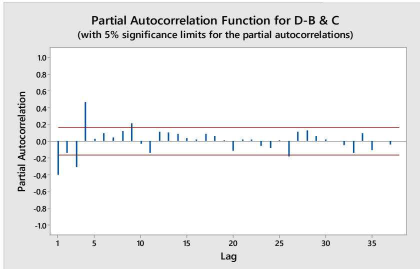

Figure 4.29: PACF for Differenced B & Con Plot

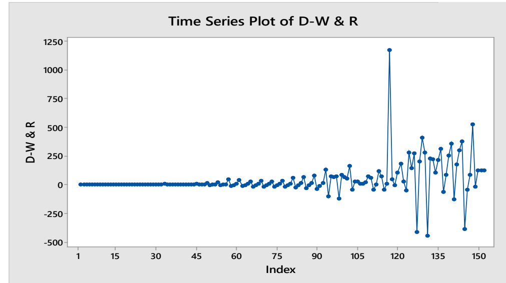

Figure 4.30: Series Plot for Differenced Wholesale & Retail

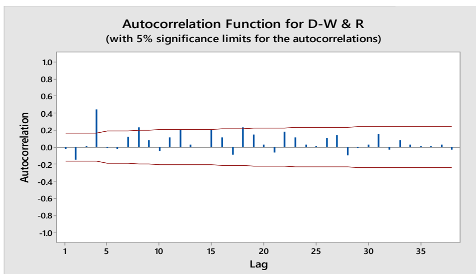

Figure 4.31: ACF for Differenced W & Retail Plot

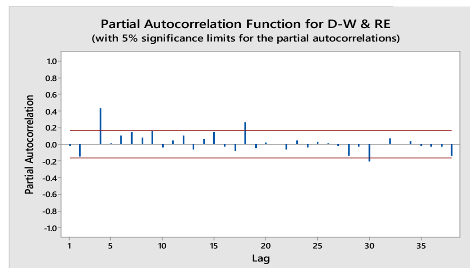

Figure 4.32: PACF for Differenced W & Retail Plot

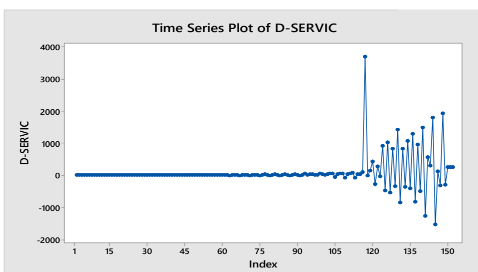

Figure 4.33: Series Plot for Differenced Services

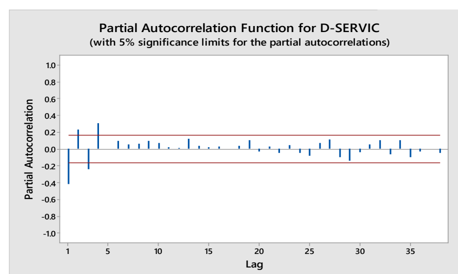

Figure 4.34: ACF for Differenced Services Plot

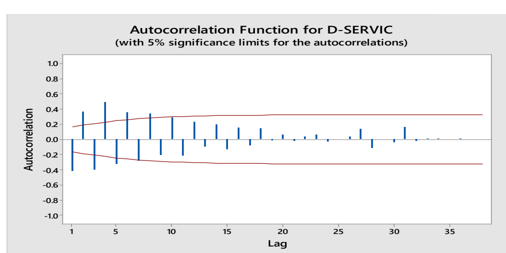

Figure 4.35: PACF for Differenced Services Plot

Notes

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Table 4.2: Univarite Model for Agricultural variable

Model	Regular AR(p) Estimates		Regular MA(q) Estimates		Seasonal AR(p) Estimates		Seasonal MA(q) Estimates		K	RSS	AIC	Rank	BIC	Rank
	ϕ_1	ϕ_2	θ_1	θ_2	$\phi_{1,4}$	$\phi_{2,4}$	$\theta_{1,4}$	$\theta_{2,4}$						
ARIMA $(1,1,1)(1,1,1)_4$	0.4886 (0.000***)		0.9269 (0.000***)		-0.8870 (0.000***)		-0.5997 (0.014**)		4	5262373	1549.39	1	1561.35	1
ARIMA $(0,1,1)(1,1,?\ 1)_4$			-0.4479 (0.000***)		-0.8812 (0.000***)		-0.4844 (0.029**)		3	6122286	1569.64	2	1578.61	2

*Footnote: ***-sig. at 1%, **-sig. at 5%,*-sig. at 10%,*

Table 4.3: Univariate Model for industry Variable

Model	Regular AR(p) Estimates		Regular MA(q) Estimates		Seasonal AR(p) Estimates		Seasonal MA(q) Estimates		k	RSS	AIC	Rank	BIC	Rank
	ϕ_1	ϕ_2	θ_1	θ_2	$\phi_{1,4}$	$\phi_{2,4}$	$\theta_{1,4}$	$\theta_{2,4}$						
ARIMA $(2,1,1)(0,1,1)_4$	-0.5247 (0.234)	0.1340 (0.205)	-0.5129 (0.233)				0.7943 (0.000***)		4	8305152	1616.47	1	1628.43	1
ARIMA $(2,1,1)(2,1,1)_4$	-0.4365 (0.255)	0.1875 (0.066*)	-0.4195 (0.268)		-0.3633 (0.082*)	-0.3981 (0.003**)	0.1999 (0.386)		6	8215884	1618.88	2	1636.82	2

*Footnote: ***-sig. at 1%, **-sig. at 5%,*-sig. at 10%,*

Table 4.4: Univariate Model for Building & Construction Variable

Model	Regular AR(p) Estimates		Regular MA(q) Estimates		Seasonal AR(p) Estimates		Seasonal MA(q) Estimates		k	RSS	AIC	Rank	BIC	Rank
	ϕ_1	ϕ_2	θ_1	θ_2	$\phi_{1,4}$	$\phi_{2,4}$	$\theta_{1,4}$	$\theta_{2,4}$						
ARIMA $(1,1,1)(2,1,1)_4$	0.3879 (0.003**)		0.8084 (0.000***)		-1.2593 (0.000***)	-0.4402 (0.000***)	-0.9285 (0.000***)		5	294377	1127.52	1	1142.47	1
ARIMA $(2,1,1)(2,1,1)_4$	-0.3794 (0.000***)	-0.4269 (0.000***)	-0.9870 (0.000***)		-1.3543 (0.000***)	-0.5217 (0.000***)	-0.9233 (0.000***)		6	297477	1131.06	2	1149.00	2

*Footnote: ***-sig. at 1%, **-sig. at 5%,*-sig. at 10%,*

Table 4.5: Univariate Model for Wholesale & Retail Variable

Model	Regular AR(p) Estimates		Regular MA(q) Estimates		Seasonal AR(p) Estimates		Seasonal MA(q) Estimates		K	RSS	AIC	Rank	BIC	Rank
	ϕ_1	ϕ_2	θ_1	θ_2	$\phi_{1,4}$	$\phi_{2,4}$	$\theta_{1,4}$	$\theta_{2,4}$						
ARIMA $(2,1,1)(1,1,1)_4$	0.4280 (0.229)	-0.1569 (0.146)	0.5335 (0.135)		0.2934 (0.042*)		0.8336 (0.000***)		5	2633258	1449.62	1	1464.57	1
ARIMA $(1,1,1)(2,1,1)_4$	0.6748 (0.000***)		0.9064 (0.000***)		-1.2722 (0.000***)	-0.2853 (0.007**)	-0.9506 (0.000***)		5	2990481	1468.32	2	1483.27	2

*Footnote: ***-sig. at 1%, **-sig. at 5%,*-sig. at 10%,*

Table 4.6: Univariate Model for Service Variable

Model	Regular AR(p) Estimates		Regular MA(q) Estimates		Seasonal AR(p) Estimates		Seasonal MA(q) Estimates		k	RSS	AIC	Rank	BIC	Rank
	ϕ_1	ϕ_2	θ_1	θ_2	$\phi_{1,4}$	$\phi_{2,4}$	$\theta_{1,4}$	$\theta_{2,4}$						
ARIMA $(1,1,0)(2,1,1)_4$	-0.2170 (0.013*)				0.3164 (0.056*)	0.1865 (0.153)	0.9099 (0.000***)		4	27398974	1791.93	2	1803.89	2
ARIMA $(1,1,1)(1,1,1)_4$	0.5163 (0.033*)		0.2458 (0.000***)		0.7332 (0.083*)		0.8101 (0.000***)		4	27214710	1790.94	1	1802.90	1

*Footnote: ***-sig. at 1%, **-sig. at 5%,*-sig. at 10%,*

Notes

Notes

From Tables 4.2 to 4.6, the identified model for the sectors are ARIMA$(1,1,1)(1,1,1)_4$ for Agricultural sector, ARIMA$(2,1,1)(0,1,1)_4$ for Industrial sector ARIMA$(1,1,1)(2,1,1)_4$ for Building and Construction sector ARIMA$(2,1,1)(1,1,1)_4$ for Wholesale and Retail sector and ARIMA$(1,1,1)(1,1,1)_4$ for Service sector.

Mathematically, the models can be expression as

$$\nabla\nabla_s^4(1-\phi_1 B)(1-\phi_{1,4}B^4)X_t = (1+\theta_1 B)(1+\theta_{1,4}B^4)e_t$$ for Agricultural sector

$$\nabla\nabla_s^4(1-\phi_1 B-\phi_{1,2}B^2)(1-\phi_{1,4}B^4)X_t = (1+\theta_1 B)(1+\theta_{1,4}B^4)e_t$$ for Industrial sector

$$\nabla\nabla_s^4(1-\phi_1 B)(1-\phi_{1,4}B^4-\phi_{2,4}B^8)X_t = (1+\theta_1 B)(1+\theta_{1,4}B^4)e_t$$ for Building and Construction sector

$$\nabla\nabla_s^4(1-\phi_1 B-\phi_{1,2}B^2)(1-\phi_{1,4}B^4)X_t = (1+\theta_1 B)(1+\theta_{1,4}B^4)e_t$$ for Wholesale and Retail sector

$$\nabla\nabla_s^4(1-\phi_1 B)(1-\phi_{1,4}B^4)X_t = (1+\theta_1 B)(1+\theta_{1,4}B^4)e_t$$ for Services sector.

Hence, substituting the values of the parameters becomes,

$$\nabla\nabla_s^4(1-0.4886B)(1+0.8870B^4)X_t = (1+0.9269B)(1-0.5997B^4)e_t$$ for Agricultural sector

$$\nabla\nabla_s^4(1+0.5247B-0.1340B^2)X_t = (1-0.5129B)(1+0.7943B^4)e_t$$ for Industrial sector

$$\nabla\nabla_s^4(1-0.3879B)(1+1.2593B^4+0.4402B^8)X_t = (1+0.8084B)(1-0.9285B^4)e_t$$ for Building and Construction sector

$$\nabla\nabla_s^4(1-0.4280B+0.1569B^2)(1-0.2934B^4)X_t = (1+0.5335B)(1+0.8336B^4)e_t$$ for Wholesale and Retail sector

$$\nabla\nabla_s^4(1-0.5163B)(1-0.7332B^4)X_t = (1+0.2458B)(1+0.8101B^4)e_t$$ for Services sector

c) Vector Autocorrelation (VAR) Model for each Variable

Multivariate analyses of the different sectors were done, with each variable placed as dependent against the others and VAR(2) model fitted to each of the variables. The results of the parameter estimate and p-values, in parenthesis, are summarized in Table 4.8

Table 4.8: VAR(2) Model and Its Parameter Estimates

VRA(2)	Agriculture	Industry	Building & Construction	Wholesale & Retail	Services
Constant	16.5611	22.2805	-3.9284	-10.7831	-46.7275
	(0.501)	(0.408)	(0.352)	(0.404)	(0.263)
Π_{1Agric}	0.3652	0.2035	0.0147	0.1339	0.2291
	(0.000)***	(0.011)**	(0.234)	(0.001)***	(0.062)*
Π_{2Agric}	-0.7765	-0.1662	0.0481	0.1695	0.1291
	(0.000)***	(0.036)**	(0.000)***	(0.000)***	(0.289)
$\Pi_{1Industry}$	0.2533	0.9674	0.0199	0.0800	0.3748
	(0.000)***	(0.000)***	(0.134)	(0.049)**	(0.005)***
$\Pi_{2Industry}$	0.3019	-0.0429	-0.0359	-0.1749	-0.4716
	(0.000)***	(0.628)	(0.010)**	(0.000)***	(0.001)***
$\Pi_{1B\ \&\ Con}$	2.9369	3.3954	0.1377	0.6971	-0.4844
	(0.000)***	(0.000)***	(0.304)	(0.091)*	(0.714)
$\Pi_{2B\ \&\ Con}$	0.0592	-2.5361	0.0579	2.0639	4.2928
	(0.943)	(0.006)***	(0.683)	(0.000)***	(0.003)***
$\Pi_{1W\ \&\ Retail}$	1.1688	0.2073	0.0710	0.8270	1.5163
	(0.000)***	(0.425)	(0.082)*	(0.000)***	(0.000)***
$\Pi_{2W\ \&\ Retail}$	1.1226	0.0169	-0.1302	-0.2959	-1.8169
	(0.000)***	(0.948)	(0.002)***	(0.019)**	(0.000)***
$\Pi_{1Services}$	-0.5402	-0.3347	0.0014	-0.1061	0.1506
	(0.000)***	(0.002)***	(0.932)	(0.041)**	(0.364)
$\Pi_{2Services}$	-0.3189	0.1586	0.0782	-0.0274	0.5247
	(0.000)***	(0.182)	(0.000)***	(0.630)	(0.005)***
No. of Significance	9	6	5	9	7
R^2	0.9898	0.9827	0.98998	0.9965	0.9917
AIC	13.5756	13.7553	10.0476	12.2868	14.6286
BIC	13.7964	13.9761	10.2683	12.5076	14.8494

*Footnote: ***-sig. at 1%, **-sig. at 5%,*-sig. at 10%,*

From Table 4.8, it is clear that VAR(2) models are very adequate, since the R^2 values are very close to one, the AIC and BIC have the least values when compared to VAR(1) and VAR(3).

d) Graphical Comparison of Univariate and Multivariate Fitted Value for each Sector

The fitted values for both models (ARIMA and VAR model) were obtained, so as to compare them graphically and also, using measurement of accuracy techniques.

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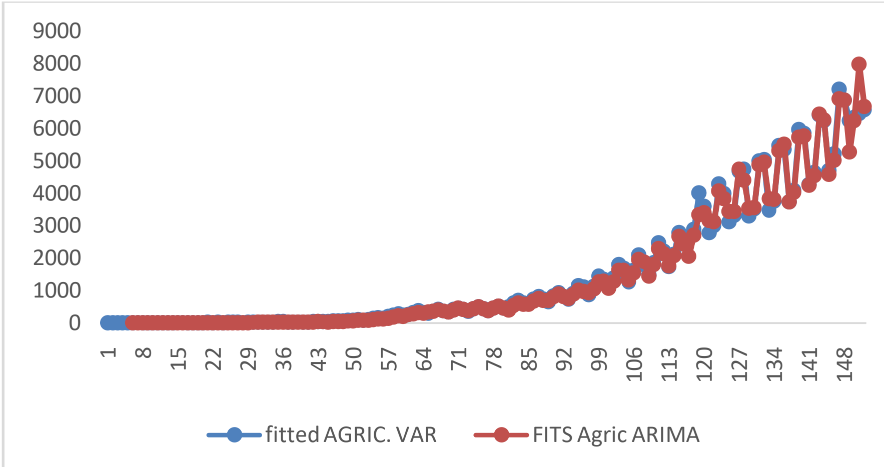

Figure 4.39: Plot of Fitted values Comparison for Agriculture Sector

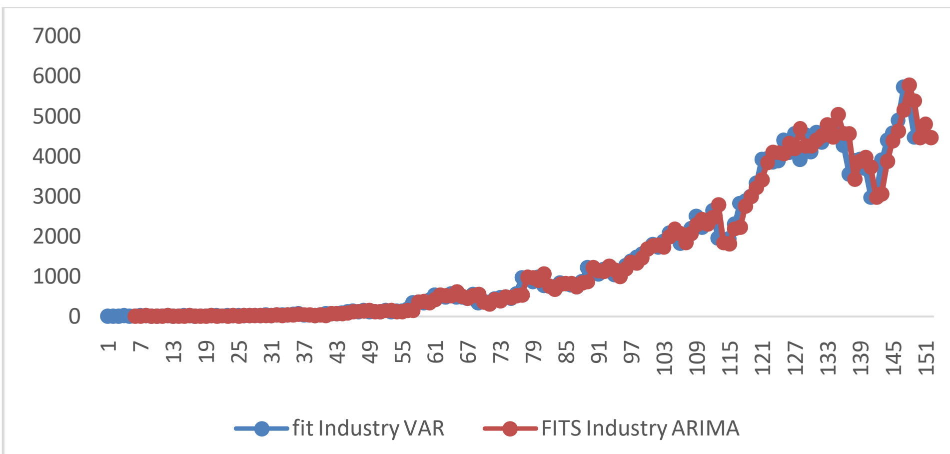

Figure 4.40: Plot of Fitted Values Comparison for Industry Sector

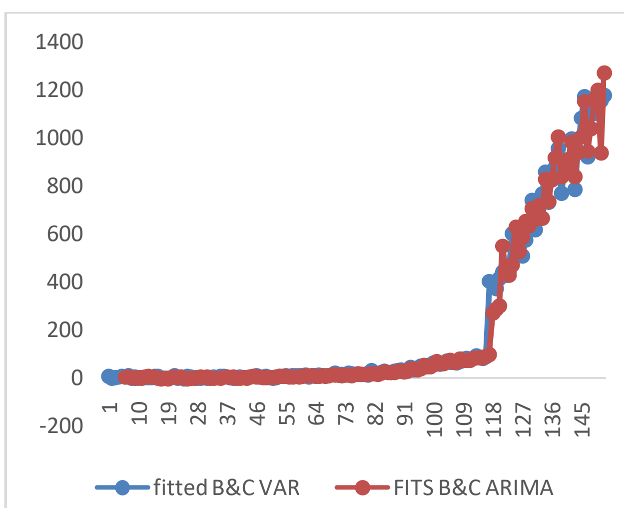

Figure 4.41: Plot of Fitted Values Comparison for Building and Construction Sector

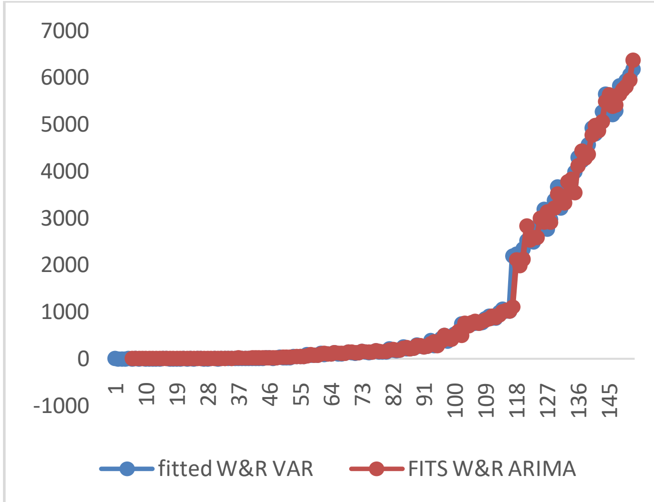

Figure 4.42: Plot of Forecast Comparison for Wholesale and Retail Sector

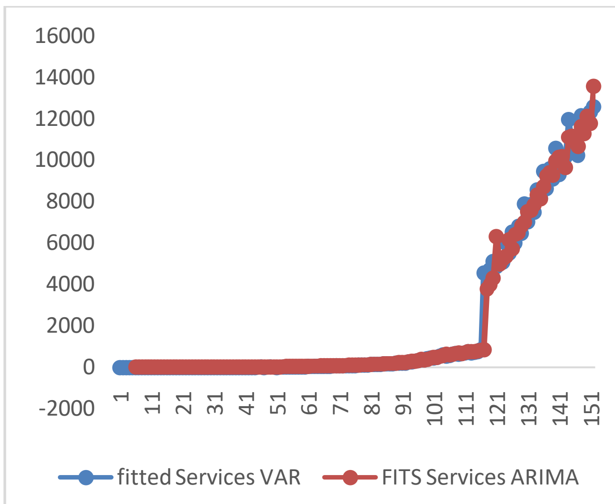

Figure 4.43: Plot of Fitted Values Comparison for Service Sector

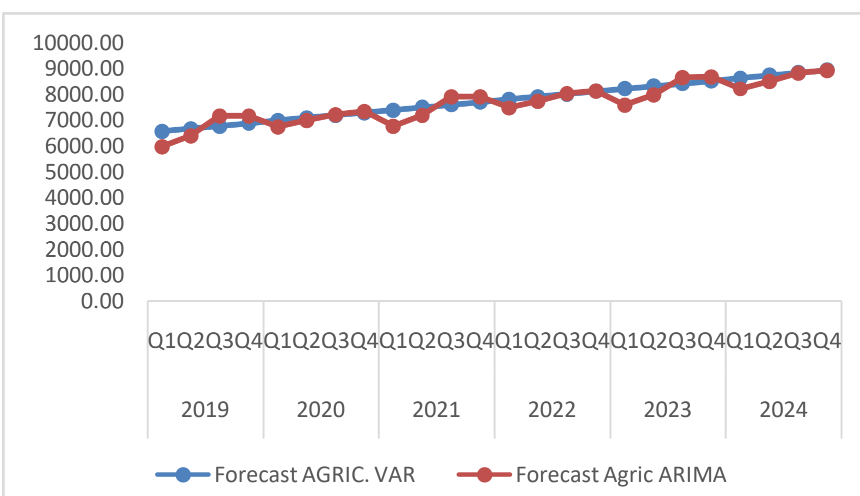

Figure 4.44: Plot of Forecast Comparison for Agriculture Sector

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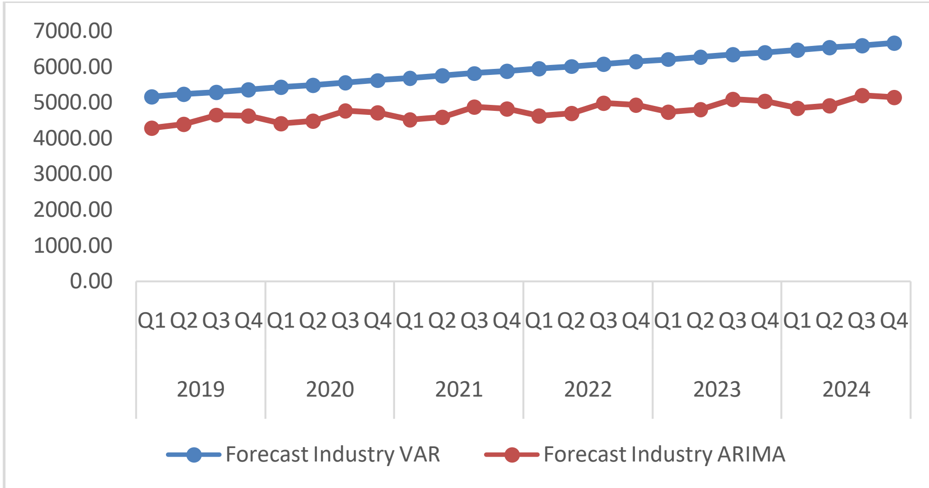

Figure 4.45: Plot of Forecast Comparison for Industry Sector

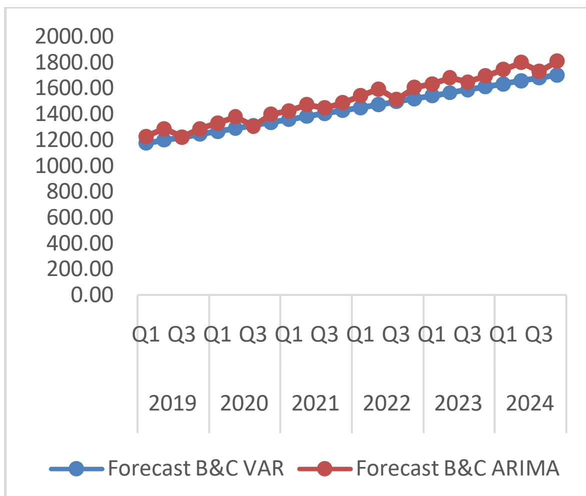

Figure 4.46: Plot of Forecast Comparison for Building and Construction Sector

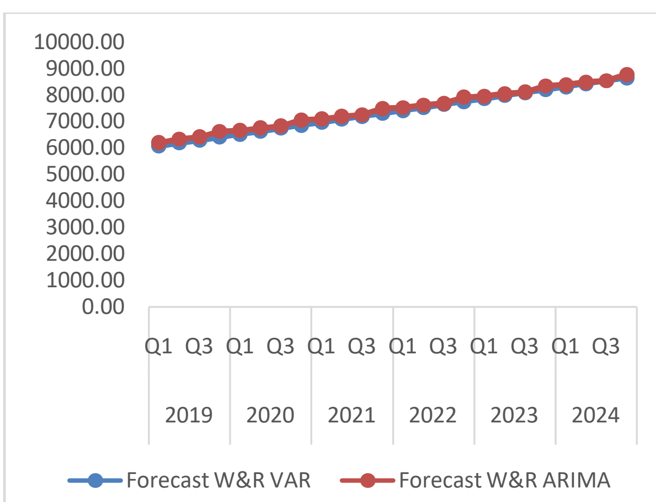

Figure 4.47: Plot of Forecast Comparison for Service Sector

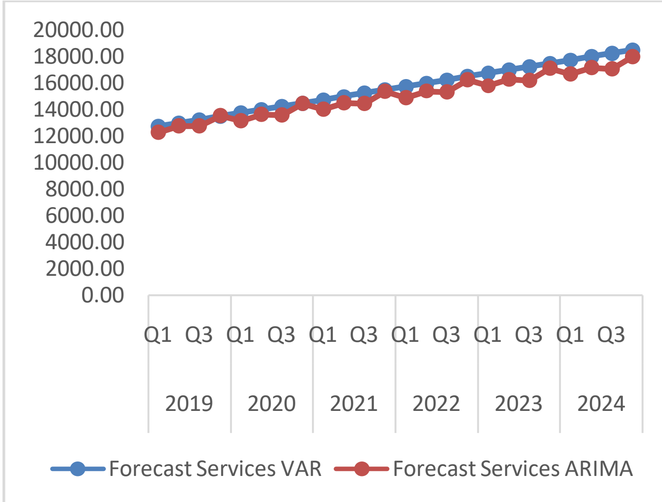

Figure 4.48: Plot of Forecast Comparison for Building and Construction Sector

Figures4.39 to 4.43 showed the comparison of the fitted values using adequate ARIMA and VAR models for each variable. The graphical representation shows no difference between both models in all the fitted graphs.

Table 4.9: Measure Accuracy of Fitted values for ARIMA and VAR Model

Variables	Models	ME	MSE	MAPE
Agriculture	ARIMA	1.9085×10^{-6}	2527.87	9.55%
	VAR	20.43415	10524747	**7.41%**
Industry	ARIMA	2.24×10^{-6}	2594.53	**4.92%**
	VAR	2.8877	16610305	11.80%
Building & Con	ARIMA	-2.032×10^{-6}	61530.14	78.89%
	VAR	2.9942	588754.3	**67.52%**
Wholesale & Ret	ARIMA	-2.359×10^{-6}	2525.96	25.01%
	VAR	18.39138	5266516	**10.91%**
Service	ARIMA	4.27×10^{-6}	2517.29	17.15%
	VAR	41.1486	54429421	**8.16%**

Footnote: The bold MAPE are best model for each variable.

Table 4.9 shows that for the variables (Agriculture, Building and Construction, Wholesale and Retail and Services) the VAR model have a better Mean Absolute Percentage Error (MAPE) values when compared to the ARIMA model. For Industry variable, the ARIMA model have better MAPE than the VAR model.

e) Graphical Comparison of Univariate and Multivariate Forecasts for Each Sector

The identified ARIMA and VAR models were used to obtained forecasts, so as to compare them using Coefficient of Variation (CV) and graphically. Figures 4.44 to 4.48 showed the forecasts comparison between the adequacy ARIMA and VAR models for each variable. The graphical representation shows no difference for thevariables, except the Industry variable.

f) FORECAST Comparison

Notes

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Table 4.10: Variables Forecasts Comparison using Coefficient of Variation (CV)

Years	Quarterly	Forecast AGRIC. VAR	Forecast Agric ARIMA	Forecast Industry VAR	Forecast Industry ARIMA	Forecast B&C VAR	Forecast B&C ARIMA	Forecast W&R VAR	Forecast W&R ARIMA	Forecast Services VAR	Forecast Services ARIMA
2019	Q1	6560.13	5963.30	5159.96	4283.53	1174.59	1229.17	6068.29	6185.7	12744.20	12278.7
	Q2	6662.83	6377.50	5225.20	4384.21	1197.63	1289.50	6180.47	6320.6	12994.94	12759.9
	Q3	6765.53	7154.20	5290.44	4649.02	1220.66	1223.79	6292.65	6413.1	13245.68	12783.4
	Q4	6868.23	7150.20	5355.67	4615.60	1243.69	1288.37	6404.83	6616.4	13496.43	13559
2020	Q1	6970.93	6730.70	5420.91	4405.14	1266.73	1333.91	6517.01	6642.4	13747.17	13152.7
	Q2	7073.63	6975.40	5486.14	4480.42	1289.76	1380.19	6629.19	6752.3	13997.91	13640.7
	Q3	7176.33	7209.30	5551.38	4763.33	1312.79	1308.95	6741.36	6822.4	14248.65	13583
	Q4	7279.03	7330.40	5616.61	4717.01	1335.83	1400.12	6853.54	7047.6	14499.40	14475.1
2021	Q1	7381.73	6767.20	5681.85	4515.74	1358.86	1426.21	6965.72	7079.5	14750.14	14037.5
	Q2	7484.43	7168.20	5747.09	4584.46	1381.89	1474.13	7077.90	7182.8	15000.88	14523.5
	Q3	7587.13	7886.80	5812.32	4872.04	1404.93	1449.23	7190.08	7246.5	15251.62	14444
	Q4	7689.83	7898.20	5877.56	4822.39	1427.96	1488.92	7302.26	7478.1	15502.37	15363.8
2022	Q1	7792.53	7463.30	5942.79	4623.50	1450.99	1543.02	7414.44	7511.7	15753.11	14918
	Q2	7895.23	7725.80	6008.03	4690.53	1474.03	1595.06	7526.61	7613.1	16003.85	15403.3
	Q3	7997.93	8014.20	6073.27	4979.32	1497.06	1514.24	7638.79	7674.8	16254.60	15318.3
	Q4	8100.63	8123.10	6138.50	4928.81	1520.09	1607.06	7750.97	7908.3	16505.34	16244.8
2023	Q1	8203.33	7574.40	6203.74	4730.52	1543.13	1634.44	7863.15	7942.4	16756.08	15797
	Q2	8306.03	7959.80	6268.97	4797.12	1566.16	1680.57	7975.33	8043.2	17006.82	16282.1
	Q3	8408.73	8630.20	6334.21	5086.22	1589.19	1649.77	8087.51	8104.4	17257.57	16195.8
	Q4	8511.43	8652.50	6399.44	5035.49	1612.23	1698.35	8199.69	8338.4	17508.31	17123.9
2024	Q1	8614.13	8204.80	6464.68	4837.36	1635.26	1747.05	8311.86	8372.7	17759.05	16675.6
	Q2	8716.83	8481.20	6529.92	4903.85	1658.29	1798.81	8424.04	8473.3	18009.79	17160.6
	Q3	8819.53	8812.40	6595.15	5193.03	1681.33	1729.64	8536.22	8534.4	18260.54	17074
	Q4	8922.24	8911.60	6660.39	5142.24	1704.36	1810.53	8648.40	8768.5	18511.28	18002.5
Coefficient of Variation (CV)		9.38	10.25	7.8	5.11	11.31	12.07	10.78	10.27	11.35	10.69

Footnote: The bold CV are best forecasts for each variable.

The results in Table 4.10, confirm that the best model to forecasts Agriculture and, Building and Construction variable is the VAR model. It also, confirm that the best model to forecasts Industry variable is ARIMA model. However, there is contradiction for both Wholesale/Retail and Services variables, where their VAR models have a better Mean Absolute Percentage Error (MAPE), but their forecast values show the ARIMA model as being better.

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V. Summary and Conclusion

This paper studied univariate and multivariate time series analysis for the sectors in Nigeria's Gross Domestic Product (GDP) which are Agriculture, Industry, Building & Construction, Wholesale & Retail, and Services. First, the plot for the five sectors was done to determine the trend component that exist among them. It was discovered that the trend component was Quadratic with appreciation and depreciation over time. The Correlation matrix was obtained to determine the degree of relationship among the variables. It was shown that a strong positive relationship, above 90% exists among all the variables. Modeling of the sectors performance was done using ARIMA and VAR models. For each of the economic variables ARIMA and VAR models were built and the best model selected for further analysis. VAR 2 of lag 4 was identified as the suitable model for the data set using the AIC and BIC selection criteria. Furthermore, the VAR (2) model was used to forecast for the next 6 years to come. Finally, coefficient of variation, CV was used to compare and identify the best model forecast for each of the variable which confirm that the best model to forecast Agriculture and Building/Construction variables is the VAR model while for Industry, Wholesale/Retail and Services variables the best model to forecast is the ARIMA model.

This study was able to identify

1. The correlation among the multivariate time series data and was considered as positive correlation.
2. Suitable model using the univariate and multivariate time series method.
3. Forecast for the next six years for all the sectors in Nigeria's GDP.
4. The best model for forecasting of each of the variable.

In conclusion, this study identified VAR model to be more suitable for forecasting Agriculture and Building/Construction variables and ARIMA model for forecasting Industry, Wholesale/Retail and Services variables.

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