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Cliffordian Hamiltonian Formulation

Discovering Thoughts, Inventing Future

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On the Standard Cliffordian Hamiltonian Formulation of Symplectic Mechanics using Frame and Co-Frame Fields

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Abstract- In the standard Cliffordian Hamiltonian mechanics we use canonical local basis $\{J_1^*, J_2^*, J_3^*\}$. In this paper using the frame fields $i_X = X^{an+i}\frac{\partial}{\partial x_{an+i}}, a = 0,1,2,\ldots,7$ instead of the Hamiltonian vector field in the Cliffordian Hamiltonian formulation we verified the generalized form of Hamiltonian equation which is in conformity with the results that have been obtained previously.

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ONTHESTANDARDCLIFFORDIANHAMILTONIANFORMULATIONOFSYMPLECTICMECHANICSUSINGFRAMEANDCOFRAMEFIELDS

Strictly as per the compliance and regulations of:

On the Standard Cliffordian Hamiltonian Formulation of Symplectic Mechanics using Frame and Co-Frame Fields

Gebreel Mohammed Khur Baba Gebreel [α] & Mohammed Ali Bashir [σ]

Abstract- In the standard Cliffordian Hamiltonian mechanics we use canonical local basis $\{J_1^*, J_2^*, J_3^*\}$. In this paper using the frame fields $i_X = X^{an+i}\frac{\partial}{\partial x_{an+i}}$, $a = 0, 1, 2, \dots, 7$ instead of the Hamiltonian vector field in the Cliffordian Hamiltonian formulation we verified the generalized form of Hamiltonian equation which is in conformity with the results that have been obtained previously.

I. Introduction

It is well-known that Modern differential geometry explains explicitly the dynamics of Hamiltonian's. Therefore, if Q is an $m-dimensional$ configuration manifold and $H: T^*Q \to R$ is a regular Hamiltonian function, then there is a unique vector field X on T^*Q such that dynamic equations are given by

$$i_X\Phi = \mathrm{dH} \qquad \to \qquad (1)$$

Where Φ indicates the symplectic form. The triple (T^*Q, Φ, X) is called Hamiltonian system on the cotangent bundle T^*Q.

Nowadays, there are a lot of studies about Hamiltonian mechanics, formalisms, systems and equations [1,2] and there in. There are real, complex, paracomplex and other analogues. We say that in order to obtain different analogous in different spaces is possible.

Quaternions were invented by Sir William Rowan Hamiltonian as an extension to the complex numbers. Hamiltonian's defining relation is most succinctly written as:

$$i^2 = j^2 = k^2 = ijk = -1 \qquad \to \qquad (2)$$

If it is compared to the calculus of vectors, quaternions have slipped into the realm of obscurity. They do however still find use in the computation of rotations. A lot of physical laws in classical, relativistic, and quantum mechanics can be written pleasantly by means of quaternions. Some physicists hope they will find deeper understanding of the universe by restating basic principles in terms of quaternion algebra. It is well-known that quaternions are useful for representing rotations in both

Ref

1. M. De Leon, P.R Rodrigues, Methods of Differential Geometry in Analytical Mechanics, North-Holland Mathematics Studies, vol.152, Elsevier, Amsterdam, 1989.

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quantum and classical mechanics [3]. We say that Cliffordian manifold is a quaternion manifold. Therefore, all properties defined on quaternion manifold of *dimension* $8n$ also is valid for Cliffordian manifold. Thus, it is possible to construct mechanical equations on Cliffordian *Kähler* manifold.

II. Preliminaries

Hereafter, all mappings and manifolds are assumed to be smooth, i.e. infinitely differentiable and the sum is taken over repeated indices. By $\mathcal{F}(M), \mathcal{X}(M)$ and $\Lambda^1(M)$ we denote the set of functions on M, the set of vector fields on M and the set of $1-forms\ on\ M$, respectively.

Theorem

Let f be differentiable ϕ , ψ are 1-form,then [4]:

- $d(f\phi) = df\wedge\phi + fd\phi$
- $d(\phi\wedge\psi) = d\phi\wedge\psi - \phi\wedge d\psi$

III. Frame Fields [5]

If U, x is a chart on a smooth $n - manifold$ then written $\mathrm{x} = (x^1, \ldots, x^n)$ we have vector fields defined on U by

$$\frac{\partial}{\partial x^i} : p \to \left.\frac{\partial}{\partial x^i}\right|_p$$

Such that the together the $\frac{\partial}{\partial x^i}$ form a basis at each tangent space at point in U. We call the set of fields $\frac{\partial}{\partial x^1}, \ldots, \frac{\partial}{\partial x^n}$ a holonomic frame field over U. If X is a vector field defined on some set including this local chart domain U then for some smooth functions X^i defined on U we have

$$X(p) = \sum X^i(p) \left.\frac{\partial}{\partial x^i}\right|_p$$

Or in other words

$$X|_U = \sum X^i \frac{\partial}{\partial x^i}$$

Notice also that $dx^i : p \to \left. dx^i \right|_p$ defines a field of co-vectors such that $dx^1|_p, \ldots dx^n|_p$ forms a basis of $T_p^* M\ for\ each\ p \in U$. The fields form what is called a holonomic co-frame over U. In fact, the functions X^i are given by $dx^i(X) : p \to \left. dx^i \right|_p (X_p)$.Type equation here.

IV. Cliffordian *Kähler* Manifolds

Here, we recalled the main concepts and structures given in [6,7]. Let Let M be a real smooth manifold of dimension m. Suppose that there is a $6 - dimensional$ vector bundle $V\ consisting\ of\ F_i (i = 1,2, \ldots, 6)$ tensors of $type(1,1)\ over\ M$. Such a local basis $\{F_1, F_2, \ldots, F_6\}$ is called a canonical local basis of the bundle V in a neighborhood $U\ of\ M$.

Ref

3. D. Stahlke, Quaternions in Classical Mechanics, Phys 621. http://www.stahlke.org/dan/phys-papers/quaternion-paper.pdf

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Notes

Then V is called an almost Cliffordian structure in M. The pair (M,V) is named an almost Cliffordian manifold with V. Hence, an almost Cliffordian manifold $M\ is\ of\ dimension\ m = 8n$. If there exists on (M,V) a global basis $\{F_1, F_2, \ldots, F_6\}$, then (M,V) is said to be an almost Cliffordian manifold; the basis $\{F_1, F_2, \ldots, F_6\}$ is called a global basis for V.

An almost Cliffordian connection on the almost Cliffordian manifold (M,V) is a linear connection $\nabla\ on\ M$ which preserves by parallel transport the vector bundle V. This means that if Φ is a cross-section (local-global) of the bundle V. Then $\nabla_x \Phi$ is also a cross-section (local-global, respectively) of V, X being an arbitrary vector field of M.

If for any canonical basis $\{J_i\}, i = \overline{1,6}\ of\ V$ in a coordinate neighborhood U, the identities

$$g(J_i X, J_i Y) = g(X,Y),\ \forall X,Y \in \chi(M), i = 1,2,\ldots,6 \quad \rightarrow \tag{3}$$

Hold, the triple$(M,g,V)$is named an almost Cliffordian Hermitian manifold or metric Cliffordian manifold denoting by V an almost Cliffordian structure V and by g a Riemannian metric and by (g,V) an almost Cliffordian metric structure.

Since each $J_i (i = 1,2,\ldots,6)$ is almost Hermitian structure with respect to g, setting

$$\Phi_i(X,Y) = g(J_i X, Y), \quad i = 1,2,\ldots,6 \quad \rightarrow \tag{4}$$

For any vector fields $X\ and\ Y$, we see that Φ_i are 6-local 2-forms.

If the Levi-Civita connection $\nabla = \nabla^g\ on\ (M,g,V)$ preserves the vector bundle V by parallel transport, then (M,g,V) is called a Cliffordian $K\ddot{a}hler$ manifold, and an almost Cliffordian structure Φ_i of M is called a Cliffordian $K\ddot{a}hler$ structure. A Clifford $K\ddot{a}hler$ manifold is Riemannian manifold (M^{8n}, g). For example, we say that R^{8n} is the simplest example of Clifford $K\ddot{a}hler$ manifold. Suppose that let

$$\{x_i, x_{i+n}, x_{i+2n}, x_{i+3n}, x_{i+4n}, x_{i+5n}, x_{i+6n}, x_{i+7n}\}, i = \overline{1,n}$$

be a real coordinate system on R^{8n}.

The frame field represents the natural bases over R of the tangent space $T(R^{8n}) of\ R^{8n}$ and can be written:

$$\left\{\frac{\partial}{\partial x_{an+i}}\right\}, \quad a = 0,1,2,3,\ldots 7 \quad \rightarrow \tag{5}$$

The co-frame field represents the natural bases over R of the cotangent space $T^*(R^{8n})\ of\ R^{8n}$ and can be written:

$$\{dx_{an+i}\}, \quad a = 0,1,2,3,\ldots,7 \quad \rightarrow \tag{6}$$

By structures $\{J_1, J_2, J_3\}$ the following expressions are obtained

$$J_1\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial x_{n+i}} \qquad J_2\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial x_{2n+i}} \qquad J_3\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial x_{3n+i}}$$

$$J_1\left(\frac{\partial}{\partial x_{n+i}}\right) = -\frac{\partial}{\partial x_i} \quad J_2\left(\frac{\partial}{\partial x_{n+i}}\right) = -\frac{\partial}{\partial x_{4n+i}} \quad J_3\left(\frac{\partial}{\partial x_{n+i}}\right) = -\frac{\partial}{\partial x_{5n+i}}$$

$$J_1\left(\frac{\partial}{\partial x_{2n+i}}\right) = \frac{\partial}{\partial x_{4n+i}} \quad J_2\left(\frac{\partial}{\partial x_{2n+i}}\right) = -\frac{\partial}{\partial x_i} \quad J_3\left(\frac{\partial}{\partial x_{2n+i}}\right) = -\frac{\partial}{\partial x_{6n+i}}$$
$$J_1\left(\frac{\partial}{\partial x_{3n+i}}\right) = \frac{\partial}{\partial x_{5n+i}} \quad J_2\left(\frac{\partial}{\partial x_{3n+i}}\right) = \frac{\partial}{\partial x_{6n+i}} \quad J_3\left(\frac{\partial}{\partial x_{3n+i}}\right) = -\frac{\partial}{\partial x_i}$$
$$J_1\left(\frac{\partial}{\partial x_{4n+i}}\right) = -\frac{\partial}{\partial x_{2n+i}} \quad J_2\left(\frac{\partial}{\partial x_{4n+i}}\right) = \frac{\partial}{\partial x_{n+i}} \quad J_3\left(\frac{\partial}{\partial x_{4n+i}}\right) = \frac{\partial}{\partial x_{7n+i}} \quad \rightarrow \quad (7)$$
$$J_1\left(\frac{\partial}{\partial x_{5n+i}}\right) = -\frac{\partial}{\partial x_{3n+i}} \quad J_2\left(\frac{\partial}{\partial x_{5n+i}}\right) = -\frac{\partial}{\partial x_{7n+i}} \quad J_3\left(\frac{\partial}{\partial x_{5n+i}}\right) = \frac{\partial}{\partial x_{n+i}}$$
$$J_1\left(\frac{\partial}{\partial x_{6n+i}}\right) = \frac{\partial}{\partial x_{7n+i}} \quad J_2\left(\frac{\partial}{\partial x_{6n+i}}\right) = -\frac{\partial}{\partial x_{3n+i}} \quad J_3\left(\frac{\partial}{\partial x_{6n+i}}\right) = \frac{\partial}{\partial x_{2n+i}}$$
$$J_1\left(\frac{\partial}{\partial x_{7n+i}}\right) = -\frac{\partial}{\partial x_{6n+i}} \quad J_2\left(\frac{\partial}{\partial x_{7n+i}}\right) = \frac{\partial}{\partial x_{5n+i}} \quad J_3\left(\frac{\partial}{\partial x_{7n+i}}\right) = -\frac{\partial}{\partial x_{4n+i}}$$

A canonical local basis $\{J_1^*, J_2^*, J_3^*\}$ *of* V^* of the cotangent space $T^*(M)$ of manifold M satisfies the following condition as follows:

$$J_1^{*2} = J_2^{*2} = J_3^{*2} = J_1^* J_2^* J_3^{*2} J_2^* J_1^* = -I\,, \quad \rightarrow \quad (8)$$

Defining by

$$J_1^*(dx_i) = dx_{n+i} \quad J_2^*(dx_i) = dx_{2n+i} \quad J_3^*(dx_i) = dx_{3n+i}$$
$$J_1^*(dx_{n+i}) = -dx_i \quad J_2^*(dx_{n+i}) = -dx_{4n+i} \quad J_3^*(dx_{n+i}) = -dx_{5n+i}$$
$$J_1^*(dx_{2n+i}) = dx_{4n+i} \quad J_2^*(dx_{2n+i}) = -dx_i \quad J_3^*(dx_{2n+i}) = -dx_{6n+i}$$
$$J_1^*(dx_{3n+i}) = dx_{5n+i} \quad J_2^*(dx_{3n+i}) = dx_{6n+i} \quad J_3^*(dx_{3n+i}) = -dx_i$$
$$J_1^*(dx_{4n+i}) = -dx_{2n+i} \quad J_2^*(dx_{4n+i}) = dx_{n+i} \quad J_3^*(dx_{4n+i}) = dx_{7n+i} \quad \rightarrow \quad (9)$$
$$J_1^*(dx_{5n+i}) = -dx_{3n+i} \quad J_2^*(dx_{5n+i}) = -dx_{7n+i} \quad J_3^*(dx_{5n+i}) = dx_{n+i}$$
$$J_1^*(dx_{6n+i}) = dx_{7n+i} \quad J_2^*(dx_{6n+i}) = -dx_{3n+i} \quad J_3^*(dx_{6n+i}) = dx_{2n+i}$$
$$J_1^*(dx_{7n+i}) = -dx_{6n+i} \quad J_2^*(dx_{7n+i}) = dx_{5n+i} \quad J_3^*(dx_{7n+i}) = -dx_{4n+i}$$

V. Hamiltonian Mechanics

Here, we obtain Hamiltonian equations and Hamiltonian mechanical system for quantum and classical mechanics structured on the standard Cliffordian *Kähler* manifold (R^{8n}, V).

Firstly, let (R^{8n}, V) be a standard Cliffordian *Kähler* manifold. Assume that a component of almost Cliffordian structure V^*, a Liouville form and a 1-form on the standard Cliffordian *Kähler* manifold (R^{8n}, V) are shown by $J_1^*, \lambda_{J_1^*}$ *and* $\omega_{J_1^*}$, respectively.

Then

$$\omega_{J_1^*} = \frac{1}{2}(x_i dx_i + x_{n+i}dx_{n+i} + x_{2n+i}dx_{2n+i} + x_{3n+i}dx_{3n+i} + x_{4n+i}dx_{4n+i} + x_{5n+i}dx_{5n+i} + x_{6n+i}dx_{6n+i} + x_{7n+i}dx_{7n+i}) \quad \rightarrow \quad (10)$$

Notes

Notes

In this equation can be concise manner

$$\omega_{J_1^*} = \frac{1}{2}\sum_{a=0}^{7} x_{an+i}\, dx_{an+i} \qquad \rightarrow \qquad (11)$$

And

$$\lambda_{J_1^*} = J_1^*(\omega_{J_1^*}) = \frac{1}{2}(x_i dx_{n+i} - x_{n+i} dx_i + x_{2n+i} dx_{4n+i} + x_{3n+i} dx_{5n+i} - x_{4n+i} dx_{2n+i}$$

$$- x_{5n+i} dx_{3n+i} + x_{6n+i} dx_{7n+i} - x_{7n+i} dx_{6n+i})$$

It is well-known that if $\Phi_{J_1^*}$ is closed *Kähler* form on the standard Cliffordian *Kähler* manifold (R^{8n}, V), then $\Phi_{J_1^*}$ is also a symplectic structure on Cliffordian *Kähler* manifold (R^{8n}, V).

Can be written Hamilton vector field X associated with Hamilton energy H by using frame fields formula:

$$X = \sum_{a=0}^{7} X^{an+i} \frac{\partial}{\partial x_{an+i}} \qquad \rightarrow \qquad (12)$$

Then

$$\Phi_{J_1^*} = -d\lambda_{J_1^*} = dx_{n+i} \wedge dx_i + dx_{4n+i} \wedge dx_{2n+i} + dx_{5n+i} \wedge dx_{3n+i} + dx_{7n+i} \wedge dx_{6n+i} \rightarrow \qquad (13)$$

Can be written i_X by using frame fields

$$i_X = X^{an+i} \frac{\partial}{\partial x_{an+i}} \quad a = 0,1,2,\ldots,7 \qquad \rightarrow \qquad (14)$$

If: $a = 0 \quad \Rightarrow \quad i_X = X^i \frac{\partial}{\partial x_i}$

$$i_X \Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^i \frac{\partial}{\partial x_i} dx_{n+i}.dx_i - X^i \frac{\partial}{\partial x_i} dx_i.dx_{n+i} + X^i \frac{\partial}{\partial x_i} dx_{4n+i}.dx_{2n+i}$$

$$- X^i \frac{\partial}{\partial x_i} dx_{2n+i}.dx_{4n+i} + X^i \frac{\partial}{\partial x_i} dx_{5n+i}.dx_{3n+i} - X^i \frac{\partial}{\partial x_i} dx_{3n+i}.dx_{5n+i} +$$

$$X^i \frac{\partial}{\partial x_i} dx_{7n+i}.dx_{6n+i} - X^i \frac{\partial}{\partial x_i} dx_{6n+i}.dx_{7n+i} \quad \Rightarrow$$

$$i_X \Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^{n+i} dx_i - X^i dx_{n+i} + X^{4n+i} dx_{2n+i} - X^{2n+i} dx_{4n+i}$$

$$+ X^{5n+i} dx_{3n+i} - X^{3n+i} dx_{5n+i} + X^{7n+i} dx_{6n+i} - X^{6n+i} dx_{7n+i}$$

If: $\quad a = 1 \quad \Rightarrow \quad i_X = X^{n+i} \frac{\partial}{\partial x_{n+i}}$

$$i_X \Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{n+i}.dx_i - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_i.dx_{n+i} + X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{4n+i}.dx_{2n+i} -$$

$$X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{2n+i}.dx_{4n+i} + X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{5n+i}.dx_{3n+i} - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{3n+i}.dx_{5n+i} +$$

$$X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{7n+i}.dx_{6n+i} - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{6n+i}.dx_{7n+i} \Rightarrow$$

$$i_X \Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^{n+i}dx_i - X^i dx_{n+i} + X^{4n+i}dx_{2n+i} - X^{2n+i}dx_{4n+i}$$
$$+X^{5n+i}dx_{3n+i} - X^{3n+i}dx_{5n+i} + X^{7n+i}dx_{6n+i} - X^{6n+i}dx_{7n+i}$$

If: $a = 2 \Rightarrow \quad i_X = X^{2n+i}\frac{\partial}{\partial x_{2n+i}}$

$$i_X \Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{n+i}.dx_i - X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_i.dx_{n+i} +$$
$$X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{4n+i}.dx_{2n+i} - X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{2n+i}.dx_{4n+i} + X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{5n+i}.dx_{3n+i} -$$
$$X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{3n+i}.dx_{5n+i} + X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{7n+i}.dx_{6n+i} - X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{6n+i}.dx_{7n+i} \Rightarrow$$
$$i_X \Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^{n+i}dx_i - X^i dx_{n+i} + X^{4n+i}dx_{2n+i} - X^{2n+i}dx_{4n+i}$$
$$+X^{5n+i}dx_{3n+i} - X^{3n+i}dx_{5n+i} + X^{7n+i}dx_{6n+i} - X^{6n+i}dx_{7n+i}$$

⋮

If: $a = 7 \quad \Rightarrow \quad i_X = X^{7n+i}\frac{\partial}{\partial x_{7n+i}}$

$$i_X \Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{n+i}.dx_i - X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_i.dx_{n+i} +$$
$$X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{4n+i}.dx_{2n+i} - X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{2n+i}.dx_{4n+i} + X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{5n+i}.dx_{3n+i} -$$
$$X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{3n+i}.dx_{5n+i} + X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{7n+i}.dx_{6n+i} - X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{6n+i}.dx_{7n+i} \Rightarrow$$
$$i_X \Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^{n+i}dx_i - X^i dx_{n+i} + X^{4n+i}dx_{2n+i} - X^{2n+i}dx_{4n+i}$$
$$+X^{5n+i}dx_{3n+i} - X^{3n+i}dx_{5n+i} + X^{7n+i}dx_{6n+i} - X^{6n+i}dx_{7n+i} \quad \rightarrow \quad (15)$$

For all $a = 0,1,2,3, \ldots ,7$ we obtain equation (15).

Furthermore, the differential of Hamilton energy is obtained as follows:

$$dH = \frac{\partial H}{\partial x_i}dx_i + \frac{\partial H}{\partial x_{n+i}}dx_{n+i} + \frac{\partial H}{\partial x_{2n+i}}dx_{2n+i} + \frac{\partial H}{\partial x_{3n+i}}dx_{3n+i} + \frac{\partial H}{\partial x_{4n+i}}dx_{4n+i}$$
$$+\frac{\partial H}{\partial x_{5n+i}}dx_{5n+i} + \frac{\partial H}{\partial x_{6n+i}}dx_{6n+i} + \frac{\partial H}{\partial x_{7n+i}}dx_{7n+i} \quad \rightarrow \quad (16)$$

According to Eq (1) if equaled Eq(15) and Eq(16), the Hamiltonian vector field is calculated as follows:

$$X = -\frac{\partial H}{\partial x_{n+i}}\frac{\partial}{\partial x_i} + \frac{\partial H}{\partial x_i}\frac{\partial}{\partial x_{n+i}} - \frac{\partial H}{\partial x_{4n+i}}\frac{\partial}{\partial x_{2n+i}} - \frac{\partial H}{\partial x_{5n+i}}\frac{\partial}{\partial x_{3n+i}} +$$
$$\frac{\partial H}{\partial x_{2n+i}}\frac{\partial}{\partial x_{4n+i}} + \frac{\partial H}{\partial x_{3n+i}}\frac{\partial}{\partial x_{5n+i}} - \frac{\partial H}{\partial x_{7n+i}}\frac{\partial}{\partial x_{6n+i}} + \frac{\partial H}{\partial x_{6n+i}}\frac{\partial}{\partial x_{7n+i}} \quad \rightarrow \quad (17)$$

Suppose that a curve

$$\alpha : R \rightarrow R^{8n} \quad \rightarrow \quad (18)$$

Be an integral curve of the Hamiltonian vector field, i.e. s

Notes

$$X\big(\alpha(t)\big) = \dot{\alpha} \ , \ \ t \in R \qquad \rightarrow \qquad (19)$$

In the local coordinates , it is obtained that

$$\alpha(t) = (x_i, x_{n+i}, x_{2n+i}, x_{3n+i}, x_{4n+i}, x_{5n+i}, x_{6n+i}, x_{7n+i}) \quad \rightarrow \qquad (20)$$

And

$$\dot{\alpha}(t) = \frac{dx_i}{dt}\frac{\partial}{\partial x_i} + \frac{dx_{n+i}}{dt}\frac{\partial}{\partial x_{n+i}} + \frac{dx_{2n+i}}{dt}\frac{\partial}{\partial x_{2n+i}} + \frac{dx_{3n+i}}{dt}\frac{\partial}{\partial x_{3n+i}} +$$

$$\frac{dx_{4n+i}}{dt}\frac{\partial}{\partial x_{4n+i}} + \frac{dx_{5n+i}}{dt}\frac{\partial}{\partial x_{5n+i}} + \frac{dx_{6n+i}}{dt}\frac{\partial}{\partial x_{6n+i}} + \frac{dx_{7n+i}}{dt}\frac{\partial}{\partial x_{7n+i}} \quad \rightarrow \qquad (21)$$

Considering Eq (19) if equaled Eq (17) and Eq (21), it follows:

$$\frac{dx_i}{dt} = -\frac{\partial H}{\partial x_{n+i}}, \quad \frac{dx_{n+i}}{dt} = \frac{\partial H}{\partial x_i}, \quad \frac{dx_{2n+i}}{dt} = -\frac{\partial H}{\partial x_{4n+i}}, \quad \frac{dx_{3n+i}}{dt} = -\frac{\partial H}{\partial x_{5n+i}},$$

$$\frac{dx_{4n+i}}{dt} = \frac{\partial H}{\partial x_{2n+i}}, \quad \frac{dx_{5n+i}}{dt} = \frac{\partial H}{\partial x_{3n+i}}, \quad \frac{dx_{6n+i}}{dt} = -\frac{\partial H}{\partial x_{7n+i}}, \quad \frac{dx_{7n+i}}{dt} = \frac{\partial H}{\partial x_{6n+i}} \quad \rightarrow \qquad (22)$$

Thus, the equations obtained in Eq. (22) are seen to be Hamiltonian equation with respect to component J_1^* of almost Cliffordian structure V^* on Cliffordian *Kähler* manifold (R^{8n}, V), and then the triple $(R^{8n}, \Phi_{J_1^*}, X)$ is seen to be a Hamiltonian mechanical system on Cliffordian *Kähler* manifold (R^{8n}, V).

Secondly, let (R^{8n}, V) be a cliffordian *Kähler* manifold.

Suppose that an element of almost cliffordian structure V^*, a Liouville form and a 1-form on cliffordian *Kähler* manifold (R^{8n}, V) are denoted by $J_2^*, \lambda_{J_2^*}$ *and* ωJ_2^*, respectively.

Putting:

$$\omega J_2^* = \frac{1}{2}(x_i dx_i + x_{n+i} dx_{n+i} + x_{2n+i} dx_{2n+i} + x_{3n+i} dx_{3n+i}$$

$$+ x_{4n+i} dx_{4n+i} + x_{5n+i} dx_{5n+i} + x_{6n+i} dx_{6n+i} + x_{7n+i} dx_{7n+i})$$

In this equation can be concise manner

$$\omega_{J_2^*} = \frac{1}{2}\sum_{a=0}^{7} x_{an+i}\, dx_{an+i}$$

We have:

$$\lambda_{J_2^*} = J_2^*(\omega J_2^*) = \frac{1}{2}(x_i dx_{2n+i} - x_{n+i} dx_{4n+i} - x_{2n+i} dx_i + x_{3n+i} dx_{6n+i}$$

$$+ x_{4n+i} dx_{n+i} - x_{5n+i} dx_{7n+i} - x_{6n+i} dx_{3n+i} + x_{7n+i} dx_{5n+i})$$

Assume that X is a Hamiltonian vector field related to Hamiltonian energy H and given by Eq(12). Take into consideration.

$$\Phi_{J_2^*} = -d\lambda_{J_2^*} = dx_{n+i} \wedge dx_{4n+i} + dx_{2n+i} \wedge dx_i + dx_{5n+i} \wedge dx_{7n+i} + dx_{6n+i} \wedge dx_{3n+i} \rightarrow \quad (23)$$

Then from Eq(14) we obtained

Notes

If: $a = 0 \quad \Rightarrow \quad i_X = X^i \frac{\partial}{\partial x_i}$

$$i_X \Phi_{J_2^*} = \Phi_{J_2^*}(X) = X^i \frac{\partial}{\partial x_i} dx_{n+i}.dx_{4n+i} - X^i \frac{\partial}{\partial x_i} dx_{4n+i}.dx_{n+i} + X^i \frac{\partial}{\partial x_i} dx_{2n+i}.dx_i - X^i \frac{\partial}{\partial x_i} dx_i.dx_{2n+i} + X^i \frac{\partial}{\partial x_i} dx_{5n+i}.dx_{7n+i} - X^i \frac{\partial}{\partial x_i} dx_{7n+i}.dx_{5n+i} + X^i \frac{\partial}{\partial x_i} dx_{6n+i}.dx_{3n+i} - X^i \frac{\partial}{\partial x_i} dx_{3n+i}.dx_{6n+i} \quad \Rightarrow$$

$$i_X \Phi_{J_2^*} = \Phi_{J_2^*}(X) = X^{n+i} dx_{4n+i} - X^{4n+i} dx_{n+i} + X^{2n+i} dx_i - X^i dx_{2n+i} + X^{5n+i} dx_{7n+i} - X^{7n+i} dx_{5n+i} + X^{6n+i} dx_{3n+i} - X^{3n+i} dx_{6n+i}$$

If: $a = 1 \quad \Rightarrow \quad i_X = X^{n+i} \frac{\partial}{\partial x_{n+i}}$

$$i_X \Phi_{J_2^*} = \Phi_{J_2^*}(X) = X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{n+i}.dx_{4n+i} - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{4n+i}.dx_{n+i} + X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{2n+i}.dx_i - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_i.dx_{2n+i} + X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{5n+i}.dx_{7n+i} - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{7n+i}.dx_{5n+i} + X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{6n+i}.dx_{3n+i} - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{3n+i}.dx_{6n+i} \Rightarrow$$

$$i_X \Phi_{J_2^*} = \Phi_{J_2^*}(X) = X^{n+i} dx_{4n+i} - X^{4n+i} dx_{n+i} + X^{2n+i} dx_i - X^i dx_{2n+i} + X^{5n+i} dx_{7n+i} - X^{7n+i} dx_{5n+i} + X^{6n+i} dx_{3n+i} - X^{3n+i} dx_{6n+i}$$

If: $a = 2 \quad \Rightarrow \quad i_X = X^{2n+i} \frac{\partial}{\partial x_{2n+i}}$

$$i_X \Phi_{J_2^*} = \Phi_{J_2^*}(X) = X^{2n+i} \frac{\partial}{\partial x_{2n+i}} dx_{n+i}.dx_{4n+i} - X^{2n+i} \frac{\partial}{\partial x_{2n+i}} dx_{4n+i}.dx_{n+i} + X^{2n+i} \frac{\partial}{\partial x_{2n+i}} dx_{2n+i}.dx_i - X^{2n+i} \frac{\partial}{\partial x_{2n+i}} dx_i.dx_{2n+i} + X^{2n+i} \frac{\partial}{\partial x_{2n+i}} dx_{5n+i}.dx_{7n+i} - X^{2n+i} \frac{\partial}{\partial x_{2n+i}} dx_{7n+i}.dx_{5n+i} + X^{2n+i} \frac{\partial}{\partial x_{2n+i}} dx_{6n+i}.dx_{3n+i} - X^{2n+i} \frac{\partial}{\partial x_{2n+i}} dx_{3n+i}.dx_{6n+i} \Rightarrow$$

$$i_X \Phi_{J_2^*} = \Phi_{J_2^*}(X) = X^{n+i} dx_{4n+i} - X^{4n+i} dx_{n+i} + X^{2n+i} dx_i - X^i dx_{2n+i} + X^{5n+i} dx_{7n+i} - X^{7n+i} dx_{5n+i} + X^{6n+i} dx_{3n+i} - X^{3n+i} dx_{6n+i}$$

$\vdots$

If: $a = 7 \quad \Rightarrow \quad i_X = X^{7n+i} \frac{\partial}{\partial x_{7n+i}}$

$$i_X \Phi_{J_2^*} = \Phi_{J_2^*}(X) = X^{7n+i} \frac{\partial}{\partial x_{7n+i}} dx_{n+i}.dx_{4n+i} - X^{7n+i} \frac{\partial}{\partial x_{7n+i}} dx_{4n+i}.dx_{n+i} + X^{7n+i} \frac{\partial}{\partial x_{7n+i}} dx_{2n+i}.dx_i - X^{7n+i} \frac{\partial}{\partial x_{7n+i}} dx_i.dx_{2n+i} + X^{7n+i} \frac{\partial}{\partial x_{7n+i}} dx_{5n+i}.dx_{7n+i} - X^{7n+i} \frac{\partial}{\partial x_{7n+i}} dx_{7n+i}.dx_{5n+i} + X^{7n+i} \frac{\partial}{\partial x_{7n+i}} dx_{6n+i}.dx_{3n+i} - X^{7n+i} \frac{\partial}{\partial x_{7n+i}} dx_{3n+i}.dx_{6n+i} \Rightarrow$$

Notes

$$i_X \Phi_{J_2^*} = \Phi_{J_2^*}(X) = X^{n+i}dx_{4n+i} - X^{4n+i}dx_{n+i} + X^{2n+i}dx_i - X^i dx_{2n+i}$$
$$+X^{5n+i}dx_{7n+i} - X^{7n+i}dx_{5n+i} + X^{6n+i}dx_{3n+i} - X^{3n+i}dx_{6n+i} \quad \rightarrow \quad (24)$$

For all $a = 0,1,2,3,\ldots,7$ we obtain equation (24).
Furthermore, the differential of Hamiltonian energy is obtained as follows:

$$dH = \frac{\partial H}{\partial x_i}dx_i + \frac{\partial H}{\partial x_{n+i}}dx_{n+i} + \frac{\partial H}{\partial x_{2n+i}}dx_{2n+i} + \frac{\partial H}{\partial x_{3n+i}}dx_{3n+i} + \frac{\partial H}{\partial x_{4n+i}}dx_{4n+i}$$
$$+\frac{\partial H}{\partial x_{5n+i}}dx_{5n+i} + \frac{\partial H}{\partial x_{6n+i}}dx_{6n+i} + \frac{\partial H}{\partial x_{7n+i}}dx_{7n+i} \quad \rightarrow \quad (25)$$

According to Eq(1), if equaled Eq(24) and Eq(25), the Hamiltonian vector field is calculated as follows:

$$X = -\frac{\partial H}{\partial x_{2n+i}}\frac{\partial}{\partial x_i} + \frac{\partial H}{\partial x_{4n+i}}\frac{\partial}{\partial x_{n+i}} + \frac{\partial H}{\partial x_i}\frac{\partial}{\partial x_{2n+i}} - \frac{\partial H}{\partial x_{6n+i}}\frac{\partial}{\partial x_{3n+i}} -$$
$$\frac{\partial H}{\partial x_{n+i}}\frac{\partial}{\partial x_{4n+i}} + \frac{\partial H}{\partial x_{7n+i}}\frac{\partial}{\partial x_{5n+i}} + \frac{\partial H}{\partial x_{3n+i}}\frac{\partial}{\partial x_{6n+i}} - \frac{\partial H}{\partial x_{5n+i}}\frac{\partial}{\partial x_{7n+i}} \quad \rightarrow \quad (26)$$

Assume that a curve

$$\alpha : R \rightarrow R^{8n} \quad \rightarrow \quad (27)$$

Be an integral curve of the Hamiltonian vector field X, i.e.

$$X(\alpha(t)) = \dot{\alpha}\ ,\ \ t \in R \quad \rightarrow \quad (28)$$

In the local coordinates, it is obtained that

$$\alpha(t) = (x_i, x_{n+i}, x_{2n+i}, x_{3n+i}, x_{4n+i}, x_{5n+i}, x_{6n+i}, x_{7n+i}) \quad \rightarrow \quad (29)$$

And

$$\dot{\alpha}(t) = \frac{dx_i}{dt}\frac{\partial}{\partial x_i} + \frac{dx_{n+i}}{dt}\frac{\partial}{\partial x_{n+i}} + \frac{dx_{2n+i}}{dt}\frac{\partial}{\partial x_{2n+i}} + \frac{dx_{3n+i}}{dt}\frac{\partial}{\partial x_{3n+i}} +$$
$$\frac{dx_{4n+i}}{dt}\frac{\partial}{\partial x_{4n+i}} + \frac{dx_{5n+i}}{dt}\frac{\partial}{\partial x_{5n+i}} + \frac{dx_{6n+i}}{dt}\frac{\partial}{\partial x_{6n+i}} + \frac{dx_{7n+i}}{dt}\frac{\partial}{\partial x_{7n+i}} \quad \rightarrow \quad (30)$$

Considering Eq(28) if equaled Eq(26) and Eq(30) it follows:

$$\frac{dx_i}{dt} = -\frac{\partial H}{\partial x_{2n+i}}, \frac{dx_{n+i}}{dt} = \frac{\partial H}{\partial x_{4n+i}}, \frac{dx_{2n+i}}{dt} = \frac{\partial H}{\partial x_i}, \frac{dx_{3n+i}}{dt} = -\frac{\partial H}{\partial x_{6n+i}},$$
$$\frac{dx_{4n+i}}{dt} = -\frac{\partial H}{\partial x_{n+i}}, \frac{dx_{5n+i}}{dt} = \frac{\partial H}{\partial x_{7n+i}}, \frac{dx_{6n+i}}{dt} = \frac{\partial H}{\partial x_{3n+i}}, \frac{dx_{7n+i}}{dt} = -\frac{\partial H}{\partial x_{5n+i}} \quad \rightarrow \quad (31)$$

The equations obtained in Eq(31) are known to be Hamiltonian equations with respect to component J_2^* of standard almost Cliffordian structure V^* on the standard Cliffordian *Kähler* manifold (R^{8n} , V) and then the triple $(R^{8n}, \Phi_{J_2^*}, X)$ is a Hamiltonian mechanical system on the standard Cliffordian *Kähler* manifold (R^{8n} , V).

Thirdly, let (R^{8n} , V) be a standard Cliffordian *Kähler* manifold. By J_3^*, $\lambda_{J_3^*}$ *and* ωJ_3^* we denote a component of almost Cliffordian structure V^*, a Liouville form and a 1-form on Cliffordian *Kähler* manifold (R^{8n} , V) respectively.
Let ωJ_3^* be given by:

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$$\omega J_3^* = \frac{1}{2}(x_i dx_i + x_{n+i}dx_{n+i} + x_{2n+i}dx_{2n+i} + x_{3n+i}dx_{3n+i}$$

$$+x_{4n+i}dx_{4n+i} + x_{5n+i}dx_{5n+i} + x_{6n+i}dx_{6n+i} + x_{7n+i}dx_{7n+i})$$

In this equation can be concise manner

$$\omega_{J_3^*} = \frac{1}{2}\sum_{a=0}^{7} x_{an+i}\, dx_{an+i}$$

Then it holds

$$\lambda_{J_3^*} = J_3^*(\omega J_3^*) = \frac{1}{2}(x_i dx_{3n+i} - x_{n+i}dx_{5n+i} - x_{2n+i}dx_{6n+i} - x_{3n+i}dx_i$$

$$+x_{4n+i}dx_{7n+i} + x_{5n+i}dx_{n+i} + x_{6n+i}dx_{2n+i} - x_{7n+i}dx_{4n+i})$$

It is well-known that if $\Phi_{J_3^*}$ is a closed *Kähler* form on the standard Cliffordian *Kähler* manifold (R^{8n}, V), then $\Phi_{J_3^*}$ is also a symplectic structure on Cliffordian *Kähler* manifold (R^{8n}, V).

Consider X. It is Hamiltonian vector field connected with Hamiltonian energy H and given by Eq(12) taking into:

$$\Phi_{J_3^*} = -d\lambda_{J_3^*} = dx_{3n+i}\wedge dx_i + dx_{n+i}\wedge dx_{5n+i} + dx_{2n+i}\wedge dx_{6n+i} + dx_{7n+i}\wedge dx_{4n+i} \rightarrow \quad (32)$$

Then from Eq(14) we obtained

If: $a = 0 \quad \Rightarrow \quad i_X = X^i \frac{\partial}{\partial x_i}$

$$i_X\Phi_{J_3^*} = \Phi_{J_3^*}(X) = X^i \frac{\partial}{\partial x_i} dx_{3n+i}.dx_i - X^i \frac{\partial}{\partial x_i} dx_i.dx_{3n+i} + X^i \frac{\partial}{\partial x_i} dx_{n+i}.dx_{5n+i}$$

$$-X^i \frac{\partial}{\partial x_i} dx_{5n+i}.dx_{n+i} + X^i \frac{\partial}{\partial x_i} dx_{2n+i}.dx_{6n+i} - X^i \frac{\partial}{\partial x_i} dx_{6n+i}.dx_{2n+i}$$

$$+X^i \frac{\partial}{\partial x_i} dx_{7n+i}.dx_{4n+i} - X^i \frac{\partial}{\partial x_i} dx_{4n+i}.dx_{7n+i} \;\Rightarrow$$

$$i_X\Phi_{J_3^*} = \Phi_{J_3^*}(X) = X^{3n+i}dx_i - X^i dx_{3n+i} + X^{n+i}dx_{5n+i} - X^{5n+i}dx_{n+i}$$

$$+X^{2n+i}dx_{6n+i} - X^{6n+i}dx_{2n+i} + X^{7n+i}dx_{4n+i} - X^{4n+i}dx_{7n+i}$$

If: $a = 1 \Rightarrow i_X = X^{n+i}\frac{\partial}{\partial x_{n+i}}$

$$i_X\Phi_{J_3^*} = \Phi_{J_3^*}(X) = X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{3n+i}.dx_i - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_i.dx_{3n+i} + X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{n+i}.dx_{5n+i}$$

$$-X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{5n+i}.dx_{n+i} + X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{2n+i}.dx_{6n+i} - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{6n+i}.dx_{2n+i} +$$

$$X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{7n+i}.dx_{4n+i} - X^{n+i} \frac{\partial}{\partial x_{n+i}} dx_{4n+i}.dx_{7n+i} \;\Rightarrow$$

$$i_X\Phi_{J_3^*} = \Phi_{J_3^*}(X) = X^{3n+i}dx_i - X^i dx_{3n+i} + X^{n+i}dx_{5n+i} - X^{5n+i}dx_{n+i}$$

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$$+X^{2n+i}dx_{6n+i} - X^{6n+i}dx_{2n+i} + X^{7n+i}dx_{4n+i} - X^{4n+i}dx_{7n+i}$$

If: $a = 2 \quad \Rightarrow \quad i_X = X^{2n+i}\frac{\partial}{\partial x_{2n+i}}$

$$i_X\Phi_{J_3^*} = \Phi_{J_3^*}(X) = X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{3n+i}.dx_i - X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_i.dx_{3n+i} +$$

$$X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{n+i}.dx_{5n+i} - X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{5n+i}.dx_{n+i} + X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{2n+i}.dx_{6n+i}$$

$$-X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{6n+i}.dx_{2n+i} + X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{7n+i}.dx_{4n+i} - X^{2n+i}\frac{\partial}{\partial x_{2n+i}}dx_{4n+i}.dx_{6n+i} \Rightarrow$$

$$i_X\Phi_{J_3^*} = \Phi_{J_3^*}(X) = X^{3n+i}dx_i - X^i dx_{3n+i} + X^{n+i}dx_{5n+i} - X^{5n+i}dx_{n+i}$$

$$+X^{2n+i}dx_{6n+i} - X^{6n+i}dx_{2n+i} + X^{7n+i}dx_{4n+i} - X^{4n+i}dx_{7n+i}$$

⋮

If: $a = 7 \quad \Rightarrow \quad i_X = X^{7n+i}\frac{\partial}{\partial x_{7n+i}}$

$$i_X\Phi_{J_3^*} = \Phi_{J_3^*}(X) = X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{3n+i}.dx_i - X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_i.dx_{3n+i} +$$

$$X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{n+i}.dx_{5n+i} - X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{5n+i}.dx_{n+i} + X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{2n+i}.dx_{6n+i}$$

$$-X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{6n+i}.dx_{2n+i} + X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{7n+i}.dx_{4n+i} - X^{7n+i}\frac{\partial}{\partial x_{7n+i}}dx_{4n+i}.dx_{7n+i} \Rightarrow$$

$$i_X\Phi_{J_3^*} = \Phi_{J_3^*}(X) = X^{3n+i}dx_i - X^i dx_{3n+i} + X^{n+i}dx_{5n+i} - X^{5n+i}dx_{n+i}$$

$$+X^{2n+i}dx_{6n+i} - X^{6n+i}dx_{2n+i} + X^{7n+i}dx_{4n+i} - X^{4n+i}dx_{7n+i} \quad \rightarrow \quad (33)$$

For all $a = 0,1,2,3,\dots,7$ we obtain equation (33).

Furthermore, the differential of Hamiltonian energy is obtained as follows:

$$dH = \frac{\partial H}{\partial x_i}dx_i + \frac{\partial H}{\partial x_{n+i}}dx_{n+i} + \frac{\partial H}{\partial x_{2n+i}}dx_{2n+i} + \frac{\partial H}{\partial x_{3n+i}}dx_{3n+i} + \frac{\partial H}{\partial x_{4n+i}}dx_{4n+i}$$

$$+\frac{\partial H}{\partial x_{5n+i}}dx_{5n+i} + \frac{\partial H}{\partial x_{6n+i}}dx_{6n+i} + \frac{\partial H}{\partial x_{7n+i}}dx_{7n+i} \quad \rightarrow$$

According to Eq(1) if equaled Eq(33) and Eq(34), the Hamiltonian vector field given by

$$X = -\frac{\partial H}{\partial x_{3n+i}}\frac{\partial}{\partial x_i} + \frac{\partial H}{\partial x_{5n+i}}\frac{\partial}{\partial x_{n+i}} + \frac{\partial H}{\partial x_{6n+i}}\frac{\partial}{\partial x_{2n+i}} + \frac{\partial H}{\partial x_i}\frac{\partial}{\partial x_{3n+i}} -$$

$$\frac{\partial H}{\partial x_{7n+i}}\frac{\partial}{\partial x_{4n+i}} - \frac{\partial H}{\partial x_{n+i}}\frac{\partial}{\partial x_{5n+i}} - \frac{\partial H}{\partial x_{2n+i}}\frac{\partial}{\partial x_{6n+i}} + \frac{\partial H}{\partial x_{4n+i}}\frac{\partial}{\partial x_{7n+i}} \quad \rightarrow \quad (35)$$

Assume that a curve

$$\alpha : R \rightarrow R^{8n} \quad \rightarrow \quad (36)$$

Be an integral curve of the Hamiltonian vector field, i.e.

$$X\big(\alpha(t)\big) = \dot{\alpha}, \;\; t \in R \quad \rightarrow \quad (37)$$

Notes

In the local coordinates, it is obtained that

$$\alpha(t) = (x_i, x_{n+i}, x_{2n+i}, x_{3n+i}, x_{4n+i}, x_{5n+i}, x_{6n+i}, x_{7n+i}) \quad \rightarrow \quad (38)$$

And

$$\dot{\alpha}(t) = \frac{dx_i}{dt}\frac{\partial}{\partial x_i} + \frac{dx_{n+i}}{dt}\frac{\partial}{\partial x_{n+i}} + \frac{dx_{2n+i}}{dt}\frac{\partial}{\partial x_{2n+i}} + \frac{dx_{3n+i}}{dt}\frac{\partial}{\partial x_{3n+i}} +$$

$$\frac{dx_{4n+i}}{dt}\frac{\partial}{\partial x_{4n+i}} + \frac{dx_{5n+i}}{dt}\frac{\partial}{\partial x_{5n+i}} + \frac{dx_{6n+i}}{dt}\frac{\partial}{\partial x_{6n+i}} + \frac{dx_{7n+i}}{dt}\frac{\partial}{\partial x_{7n+i}} \quad \rightarrow \quad (39)$$

Thinking out Eq(37) if equaled Eq(35) and Eq(39), it follows:

$$\frac{dx_i}{dt} = -\frac{\partial H}{\partial x_{3n+i}}, \frac{dx_{n+i}}{dt} = \frac{\partial H}{\partial x_{5n+i}}, \frac{dx_{2n+i}}{dt} = \frac{\partial H}{\partial x_{6n+i}}, \frac{dx_{3n+i}}{dt} = \frac{\partial H}{\partial x_i},$$

$$\frac{dx_{4n+i}}{dt} = -\frac{\partial H}{\partial x_{7n+i}}, \frac{dx_{5n+i}}{dt} = -\frac{\partial H}{\partial x_{n+i}}, \frac{dx_{6n+i}}{dt} = -\frac{\partial H}{\partial x_{2n+i}}, \frac{dx_{7n+i}}{dt} = \frac{\partial H}{\partial x_{4n+i}} \quad \rightarrow \quad (40)$$

Finally, the equations obtained in Eq(40) are obtained to be Hamiltonian equations with respect to component J_3^* of almost Cliffordian structure V^* on the standard Cliffordian *Kähler* manifold (R^{8n}, V), and then the triple $(R^{8n}, \Phi_{J_2^*}, X)$ is a Hamiltonian mechanical system on the standard Cliffordian *Kähler* manifold (R^{8n}, V).

Notes

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The Conceptualization Process based on Student Perspective in Solving Graph Function Problem

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Abstract- The objective of this study was to analyze the conceptualization process in solving graph function problems based on the student perspective. Samples in this study were three students from the sains program in one of the best high school in West Jakarta region by the reason of testing results. This study has explored the completion of the problem and its interpretation of written interviews. There was a graph function problem that is solved by students and interpreted its completion according to their conceptions. The analyze result stated that students view the problem proposed as a function problem and not a geometric problem, so students consequences of converting from graphic to algebraic representation. The findings of this study were a conceptualization process where there are four steps of completion and it is estimating the algebraic function of the graph, finding the function of the graph, determining the function formula of the graph, and determining the derivative of the function. However, on the completion of all students, there was neglected a concept of domain function to look for as a background by students, and this is a complexity factor of the conceptualization process in solving function problems by the way to determine algebraic representation.

Keywords: conceptualization, algebraic, gradient function.

GJSFR-F Classification: MSC 2010: 05C25

The Conceptualization Process based on Student Perspective in Solving Graph Function Problem

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Abstract- The objective of this study was to analyze the conceptualization process in solving graph function problems based on the student perspective. Samples in this study were three students from the sains program in one of the best high school in West Jakarta region by the reason of testing results. This study has explored the completion of the problem and its interpretation of written interviews. There was a graph function problem that is solved by students and interpreted its completion according to their conceptions. The analyze result stated that students view the problem proposed as a function problem and not a geometric problem, so students consequences of converting from graphic to algebraic representation. The findings of this study were a conceptualization process where there are four steps of completion and it is estimating the algebraic function of the graph, finding the function of the graph, determining the function formula of the graph, and determining the derivative of the function. However, on the completion of all students, there was neglected a concept of domain function to look for as a background by students, and this is a complexity factor of the conceptualization process in solving function problems by the way to determine algebraic representation.

Keywords: conceptualization, algebraic, gradient function.

I. Introduction

There are four general steps of problem-solving, it is understanding, planning, solving, and looking back [1]; [2]. This step was many created citations in all research related to problem-solving. Especially in the mathematics education field, since emersion of the steps that have been posted by Poly a and up to right now, it was much research about problem-solving done with the essence is including all of its general steps. Even though in every step there are many processes happened and in facts, it is a determinant factor of emersion solution for every solved problem. However, kind of process happened be look totality as a just appearance one of all general steps. While, for every process, the different type of problems so it will show a difference in completion process and finding a solution because who are thinking and solving a problem cannot be controlled the other. The new approach focused on the interaction between problem representation and solution generated [3]. There has been a lot of research about problem-solving but was not yet known as a suitable framework strategy to formulate a graphic[4]. This condition was made our to look that this is interesting to follow up in a study related to occur process in particular general steps of problem-solving, and we present to make this study with the focus on conceptualization process in solving function problem.

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1. C. Granberg, "Discovering and addressing errors during mathematics problem-solving-A productive struggle?," *J. Math. Behav.*, vol. 42, pp. 33–48, 2016.

Conceptualization process related to what is to be conceptualized. Here, we use a definition of the concept that is an abstract idea, so its conceptualization process means is not out of the related to emerge abstract ideas as an object in students' interpretation in conception form. Therefore, in the solving process, there are conceptions from every solver who are a contribution to solving the problem[5]. [4]stated that problem-solving revers the process that be used in completion conceptualization and moved from the beginning to the end. This study is about problem-solving so that its idea was meant as the ways that opened to be the solving process. The object that is conceptualized in its process was a problem-solving or a solution. In the problem-solving process, there is a solution after the solving steps. In other words, what is anything will be conceptualized is not yet formed, so not for all general problem-solving steps will be a part of the conceptualization process. At four of the general problem-solving steps that be cited, and that be showing on the progress of process before a solution fixed, so the looking back step is not a part of the conceptualization process. However, what is the general steps of an understanding problem as a part of the conceptualization process? This question will be answered in the next section of the result research discussion.

This study aims to analyze the conceptualization process in solving graph function based on the student perspective. Framework this research analyzed student conception in the process to solve, so the collection of student conception would construct a schematized systematics and this is called the conceptualization process. There is a factor related to truth conception because if it is not so the solution will not be found, or solution was be found but there is a doubt with its truth. Because of related to conception, this was showing there is a solvers point of view as a determinant of preference when they are beginning the solving perform. Therefore, there is a truth of conception as a key point that was created by solvers from their perspective. Here, presenting the problem was a function to create a different perspective in solving the context that is made by solvers, and its different is at least because of the problem which is required in open-ended type. With the requirement, the function problem satisfies a criterion because the function concept is a coherency of some representation both geometric and algebraic. In this study, function problem means is about the sketch of a gradient function graph, some times we use the term of gradient function or the first derivative function but its all the same means, and gradient function was stated in [6] as the first of derivative function.

II. Method

This is an explorative study toward the completion process that interpretation by a sample of this study in writing the interview. The completion process has been obtained from one item test which was developed for needed this study and was used to measuring students' ability to solve the function problem. The test has been adapted from [6], it was about a gradient function graph problem. Its problem that was proposed is to determine a graph of gradient function from Figure 1 below.

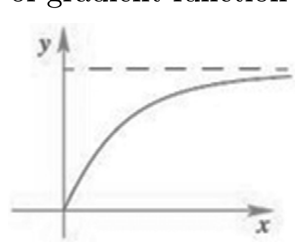

Figure 1: Sketch of the graphics function

Ref

6. P. Tobin and F. Cirrito, *International Baccalaureate Mathematics Standard Level.* Australia: IBID Press, 1997.

Then, this function problem was given to seven students from a high school in the region of West Jakarta. However, from seven on its students, just three students whose detail gave an answer related to the written interview question, and its students are initial as AN, NI, and PR. The interview question is how the way to solve and describes. Students who are a respondent in this study have been classified as good students and it was according to the recommendation of a mathematics teacher at the school whom concerned. Here, we used a literature [4]. In its article mentioned that the conceptualization process has been done by the ways to see in solver mind, it was seeing how they represented of the problem, how they produced a solution, and how its factors might be related with their knowledge, with the problem characteristic, and their success probably.

Ref

4. T. B. Ward, *Problem solving*. Elsevier Inc., 2012.

III. Result and Discussion

Before to analyze of conceptualization process completion, the problem context would be fixed by the student point of view who has solved it. Function problem can look like a problem of geometric or algebraicre presentation[5]. However, students had the freedom to look at the problem from the relativity of according to their more than familiar in solution finding, although the problem was showing about there is one of complexity.

a) *Complexity of Problem*

Aside from geometric or function point of view, the item has been proposed above was considered to be a problem. Here is not to show that function has a complex system, but in this study, that function was being used to be anything that including form and variable problem because the function is satisfied between geometric and algebraic. Completion as a student task was showing a figure of the sketch a derivative function graph. However, at the test just have a figure of function and not be complete sets coordinate pairs not clearly defined. We called this problem as a competitive problem because based on [7] views that stated an ability to read given information just in the graphic form needed to think about complexity properties. Whereas on the other side, in [8]report have been stated some of typical difficulties student have on graphic representation from differential was about basic calculus concepts.

The problem in the test was to draw the first derivative function of graph function that given in Figure 1. The completion of students made was a function graph figure. However, given information was not detailed, but completion can be made by identifying ways form and properties of function graph as soon as analyzing relevant concepts. For example, a function has domain, codomain, and range [9]. Needed knowledge of students about function limit, derivative, asymptote, property, and interval[10]. Domain, codomain, range, intervals of increase or decrease, extreme values as the element and properties of function which was represented graphically [3]. Drawing a function graph in the coordinate plane can be started from its concepts because every function would be drawing and needed domain and codomain to be an axis in rectangular Cartesian coordinate. By knowing coordinate pairs between domain and range, so it can be made a function graph figure. In other words, to draw derivative function in the problem so it is necessary to be known a concept about domain and codomain its graph. Furthermore, making the first function derivative graph figure by identifying gradient value along the curve when its gradient value as positive or negative gradient value relatively, and increase or decrease [8]; [6]. Thus, there are some elements to be necessary knew to sketch the gradient function graph, it is domain and codomain, the relative of gradient value, and concept about increasing and decreasing function. However, many students from high school not yet understand the relationship between calculus concepts, but they learn more about the segmental concept just on calculating big scale and typically problem-solving.

b) The Problem on Student Perspective

Different point of view will obtain a different figure. Views will determine the too different color. Every student probably has different views about function. The function was satisfied between geometric and algebra representation[5]. The function can be represented in an algebraic formula, graph, table, and context so that a combination of this representation will obtain rich in concept figure[3]; [9]. In [11]literature was stated that the chosen type of representation would be impacting success in problem-solving, and [3] added that graphic representation more accessible and give meaning to algebraic formulas. Related to this study, student views toward the problem guide theirs to the direction of the solving steps, and its look from their reasoning that students conceptualized the completion via constructing the algebraic function formula from graph figure. The following is a student finding of some algebraic representation of the graph in Figure 1 above. A student with the initials AN stated that the function algebraic formula of its figure was $y = -e^{-x} + a$ for $a \in \mathrm{N}$, while a student with the initials NI showed $y = -x^{-1} + a$ for $a \in \mathrm{N}$ as its algebraic function formula, whereas a student with the initials PR confirmed a graph on its figure by $y = 1 - a^{-\sqrt{x}} + a$, for $a > 1$ an a element real number as an algebraic function formula.

Based on its some algebraic expression, all three students who were respondents in this study view that this is a function problem. More precisely, students solved the problem by determining its function algebra formula to sketch a derivative function graph. All students solved in ways constructed the function first. This case like that is called in [10] that when students were given a function graph and then participated in sketching the derivative graph of its function gave, so many students tried to find the algebraic representation of given function first. After knowing its algebraic representation function, students would get the first derivative function, and then going to sketch the function graph figure on the rectangular coordinate system, and it happened as student techniques solved. We understood why students made these as a function problem. This is an effect of the question in its problem, because of a derivative function graph was being asked. While, learning about differential most identic with function, for example, algebraic function, trigonometry, or others. Therefore, students view that the sketch graph on its Figure 1 as the one of identical functions in their mind.

Finding the algebraic function formula of Figure 1 above was a skill of algebraic function analysis. We agree with [7]statement that students who learned differential actually skilled in the algebraic algorithm but they experience difficulty in understanding a concept so that the fact showed many students to learn a type of algorithmic formal problems by memorizing way as a simple mechanical calculation process. All three students viewed the problem as a function problem, however, its problem can be categorized too as a geometric problem. [12]stated that function is the organic combination of number and form, and the flexible conversion between symbolic language and graph. In[6], this problem viewed a geometric problem, and geometry of the curve was communicated to a negative or positive gradient concept at a particular point. Even though the Figure 1 was seen that coordinate pairs are not defined clearly, but the slope of the tangent can be imagined and it indicate for the positive or negative range of domain. However, this was about the tendency of student views toward the

Ref

9. H. Kashefi, Z. Ismail, and Y. M. Yusof, "Obstacles in the learning of two-variable functions through mathematical thinking approach," *Procedia - Soc. Behav. Sci.*, vol. 8, pp. 173–180, 2010.

problem. Nevertheless, the test was not viewed as a derivative function problem here because it is equally to be a base of completion when the view base is directed to one of two types of the problem.

Therefore, two perspectives can student made the views related to the problem that was proposed and completion conceptualization. The problem that was proposed can be categorized in geometric and function problems. When the conceptualization process in solving the problem was more dominant in using geometric context and was not a direction to determine the algebraic function formula. For example, by moving dynamic of the tangent along with graph figure so that the properties of the derivative function graph was known, or to solve ways with the technique in [6]explanation, we defined it be a geometric problem on student perspective. However, the trend of student performance in completion more to the utilization of function concept by formulating the algebraic function, so this was a function problem in judgment. Our investigation results toward student completion stated that it was a function problem. Its all depend on the point of view of student chose and started their completion construction analysis. Their chosen was the best way of course whose student knew at the time, student ways of thinking to the problem, and this writing is not to deny the result of their mind. The base of this claim is the student's point of view in solving. Some students in their explanation stated that solved the problem has been started by estimating the function of the graphics model. Student explanation about ways of estimating the function formula will be explaining in the section of step on the conceptualization process. In every completion of students make started indeed with their estimating result, and the completion of the key point is the first step chosen as a process conceptualization started here. Anyway, the first step is too important as a part and the rule in determining a lack of process to find the completion [13].

c) The consequence of the Solving Ways

Students who viewed the problem from a particular point of view, so they would face a consequence in the completion process, and every their point of view would distinguished complexity too. Students chose to solve the problem by determining the algebraic function formula so that all related consequences with the complexity function concept attached automatically in the conceptualizing process because the completion by different representations with the problem would emerge a conversion process. Switching a representation to the other are important tools in problem-solving [3]; [14]; [5]. In this study, students converted geometric to algebraic representation. [12]stated that to understand all of the function concepts, there are various internal attributes and internal-external relationships should be understood to investigate systematically and comprehensively. By completion process from student views, like a statement which was stated by [7]that student tends to depend on algebraic thinking style in the end than geometric thinking style which got information about function directly from graphics model without function expression. Even though, a student who used algebraic thinking are disadvantaged in terms of available time and perform less well in solving a problem[15]. In [10], [3], and [5] study was stated that student shows their difficulties usually in using of function properties, and in [14] study confirmed about student difficulties like to understanding a formal aspect including to identify and represented domain and range geometrically, especially when the domain in-bound. Therefore, the function was one of all difficult content to understand, and many students experienced their difficult when they faced the non-routine problem [9]; [5].

Ref

6. P. Tobin and F. Cirrito, *International Baccalaureate Mathematics Standard Level.* Australia: IBID Press, 1997.

d) Steps on Conceptualization Process

We summarized some occurred process when the student solved the problem that is proposed. It was estimating the algebraic function of a graph, finding the function of a graph, determining the function formula of the graph, and determining the derivative of the function. Among these processes, not for all is a conceptualization process in solving problems, because the conceptualization process is the occurred process before the completion has. Construction a graph was a goal that is an end product, In [3] has been stated that it was a learning model in the past. In other words, the sketching derivative function graph is the conceptualization process of completion.

i. *Estimating the algebraic function of the graph*

The first step that to be a finding in solving the problem phase of the conceptualization process was an estimation of the curve. Estimating an algebraic function formula was the fact student aware problems have so they took the step that by the particular domain function, but the codomain would be adjusted. Here, the student will see a geometry form of highlight a curve generally and match with various forms of the geometric representation of algebraic function formula in an analytical sketch that they ever learned, and this process might repeatedly occur. The repeated process in the problem-solving phase was drawing in [1] literature. In [13]study was stated student tendencies to pay the first attention to the figure, and this is a process of understanding in problem-solving. This literature was not stated about anything activity to match between the figure on seen and the other, but these activities were including a dynamic process. This is the answer to the question that has been in the introduction section, and it was an understanding phase in problem-solving steps as a dynamics when the conceptualization process occurred. In its context, there is a connection between student knowledge about the function of algebraic and geometric representation, and [16];[3]and [14]had stated that the connection was important in solving function problem. The student showed the knowledge connection means in some cut off their statement. AN has stated that Figure 1 similar to the upside-down of the exponential graph and the most frequently was e^x. While NI has written that the graph on the test directed to $y=-1/x$. Whereas PR has confirmed the approximate form of the curve was $y=\sqrt{x}$, and the other approximately form was an exponential function like $y=a^x$.

The student initials of AN has mentioned that the function graph in Figure 1 similar to the exponential graph. Because of in his perspective view that the problem was about function problems, so his student tends to use a formula that was similar to the problem before his found. AN's activities in solving the problem is an example which was [13]stated about the part in one of all general problem-solving steps, it is planning completion of the problem. Based on some cut-off students statement, there is a difference of learning trajectory for each related to conceptualization process in solving problem that is processed by AN, NI, and PR apparently, and we have seen the first step of completion process in general as a process that is said an estimation of the algebraic function of curve. AN has begun the completion process by finding a form of a similar curve with the graph in Figure 1. While NI has begun the completion process by investigating graphic form with the ways on function interval analyzed. Whereas PR has begun his completion by finding a suitable function algebraic formula with the graphic form model on Figure 1 displayed. All three students linked formula to a graph, imagined, and focused on key properties of the function. Its process called in [3] as a strategy.

Ref

1. C. Granberg, "Discovering and addressing errors during mathematics problem-solving-A productive struggle?," *J. Math. Behav.*, vol. 42, pp. 33–48, 2016.

ii. *Finding the function of a graph*

By looking at a curve form, all three students were not directly to find an algebraic function formula of its curve. This was there are on cut off statements each explanation that be written by all three students. There are some different process that is shown each student before to determine a function formula of graph figure on the display. AN has done translation $[0 \quad 1]$ toward a graph and crossed $(0,0)$, then its asymptotic was $\lim_{x\to\infty} -e^{-x}+1=1$. While NI has written that because of the bounds was a and not equally 0, so $y=-1/x$ must be added by a. Whereas PR has done exploration on some function formula of a curve. For example, when $y=a^x$ and tried to $a=2$ so $y=2^x$ but its formula was not suitable with the figure of the graph on the test. If $a=-2$, so $y=-2^x$, then the curve was reflected x-axis so that the curve form be similar. However, because of the curve crossed $f(c)=-1$, so the function must be added by 1 for suitability with the figure. The student has shown their process to scan graph, [3]stated this process as the ability to read algebraic expressions and make rough estimates of the patterns that would emerge in representation. Find the suitability between the graph and its algebraic function formula was a consequence in the way completion by this point of view.

Based on the information that is created by students, two of them showed a stead geometry transformation concept as a part of the finding process the ideal algebraic function formula with the graph. Transformation means is a translation by AN and reflection by PR. This happened are corresponding to the statement in [12]that the transformation of a figure was a frequent process to be used when students constructed a function as an algebraic representation, hence there is a requirement for the student to have mastery on specific properties of the function. Transformation as a different process between the completion by AN's or PR's and NI's. AN and PR found an algebraic formula of graph function globally. Incited of [15]has stated that student who solved a problem with a global approach, they would focus on the function behavior in a particular interval. However, NI has done to conceptualized by reading points at function graph, and this way was stated [3]as a pointwise approach when this approach distinguished with a global approach. Therefore, the global approach more powerful and gives a better understanding of the relationship between formula and graph function compared to pointwise approach.

iii. *Determining the function formula of the graph*

In finding an ideal function formula with a graphic form on the test, each student determined one function as a finding result. Here, AN has determined that a function of Figure 1 above $y=-e^{-x}+1$. While NI has shown his analysis result with an expression $y=-x^{-1}+a \; a\in \mathrm{N}$ as an algebraic representation of the graph. Whereas PR has determined by generalized way function formula to be $y=1-a^{-\sqrt{x}}$ for $a>1$ and a element real number, and for simplifying he used $e=a$ so that $y=1-e^{-\sqrt{x}}$. All three student has determined a function formula of the graph in Figure 1 based on their analysis result findings, and all of the algebraic function formula findings were different. AN and PR were the same to determine function formula by using the e number. This is a second of similarity in conceptualizing process completion of them after finding the process of curve function was similar to utilize a geometry transformation concept. Two of these students has shown an identical conceptualization process and the other student has chosen the different technique. Anyway, there was one function concept that has been missed on all three student views, and this discussion will be created on the missing concept function in the section.

Ref

15. Y. Y. Hong and M. O. J. Thomas, "Graphical construction of a local perspective on differentiation and integration," *Math. Educ. Res. J.*, vol. 27, no. 2, pp. 183–200, 2015.

iv. *Determining the derivative of the function*

For determining the derivative function of the graph in Figure 1, and because each student has determined the algebraic function formula, in this section was not many explained the ways of determining a derivative function. All three students likely can be said that they were fluently for determining a derivative function. A has written $y' = e^{-x}$. While NI has obtained a derivative function and written $y' = 1/x^2$.Whereas PR has shown $y' = x\sqrt{x}/2e^{-\sqrt{x}}$ as a derivative result of function formula that he constructed. We claim that to determine a derivative function as the end of the conceptualization process in solving the problem because after this section is student will sketch the graph of its derivative function, so the object that is obtained from its process is the solution of the problem. This claim is based on a statement at the introduction section that conceptualization process related to abstraction precess or emerged abstract ideas and object that be conceptualized is not yet fixed.

e) *Missing of the domain concept*

Looking at the completion has been created by the student, it has been clear that the problem which was given is not routine because each of student found ways of solving and different of the answer to this problem was explained its type as an open-ended non-routine problem. In general, the student who has answered the test classified in the smart student because of the three's be able to solve. Here, we are not to make sure that they are gifted, but based on completion that is created of them, they can be said a gifted from a side of function graph analysis to determine the algebraic function. The answer which has been given was based on the analysis process of form or geometry transformation toward constructed function. Knowledge of students has shown their mastery in-deep function materials and differential calculus. [9]have said that there are some terms for a relation to state a function, it is about a point of pairs, domain, codomain, and range. From the domain and function, it will obtain a range that is a part of the codomain. Related to those, there is no indicator in the problem to fault algebraic function formula that was student-constructed, but all three students have missed their identification toward the domain of a function that is one of all concepts, and it has been shown on the problem as terms of function graph. The student has shown domain the constant of function on determining the function formula of the graph at the discussion section, but they have not given domain for their function. This is probably a consequence of the completion process by algebraic representation ways. Because of the advantages of a graphic is to monitor and evaluate results[3]; [17], and this study analysis result was not looking for using its advantages by students. The student has been not understanding the domain concept, but they were missed to look for its background. Even though, in Figure 1, there was implicitly its definition related to domain function. Based on this condition, there is a probability like a statement in [14] that a factor of a student has lack flexibility in attaching the meaning of domain or range, and in [15], [3]dan[5]stated that flexibility thinking is necessary for mathematics and problem-solving.

A claim from [18]stated that neglecting a concept from the completion was because the student has its knowledge but they did not understand. However, we are more like than with statement in [8]that student has too late to realize the relationship between the original function and its derivative because of lack of appreciating the relation of them in the graphic representation. Therefore, the representation that has been including on the problem was served with an accurate level that is not detailed, so information and concept that students understood on the problem and problem-solving

Ref

9. H. Kashefi, Z. Ismail, and Y. M. Yusof, "Obstacles in the learning of two-variable functions through mathematical thinking approach," *Procedia - Soc. Behav. Sci.*, vol. 8, pp. 173–180, 2010.

strategy to find a solution are explained student knowledge and learning experiences. Placing of learning process emphasis on algebraic representations, it will be disregard graphical representation [8]. Even though, in [15]has stated that emphasis on the algebraic process tends to produce students with a dependence on the work form. All this time, learning about calculus placed the process on the accentuation of algebraic representation so it will be neglecting graphic representation. This was a reason as an effect of most students can easy solve routine differential problem,[8];[17]and [5]stated that with like this so they do not have a conceptual understanding about derivative graph of representation, and many researchers have explained about student difficulties when they were being given a graphic form. In[15], learning that emphasis on algebraic representations can hinder flexibility construction so it suggests the need to consider the construction of didactic situations.

IV. Conclusion

The problem of function that is proposed to students has been about to sketch a derivative function graph. Its problem in the student perspective was a function graph because students chose for solving it by determining the algebraic function formula from a graphic figure. When the problem as a function problem in the student view, there was some concept of function that attached to be viewed in the conceptualization process, and this was a consequence in solving by converting a graphic to an algebraic representation way. After the completion that students created and their interpretation of it was analyzed, there are four phases of the conceptualization process from identification result after the function problem in this study. The four are estimating the algebraic function of a graph, finding the function of a graph, determining the function formula of the graph, and determining the derivative of the function. However, the analysis study has the other finding, there is function concept missed on student views, it is a domain concept. All three students have determined a domain function of constant, but it was not a function domain. This condition cannot be claimed as a misunderstanding of the student on domain concept, and it is because of the complexity factor in the conceptualization process on solving function problem by algebraic representation way.

This study was limited to a function problem that it completion by determining algebraic representation, and when student progress in solving process on geometric problem perspective so that the phases in the conceptualization process will be different. Moreover, its phases are not the same too if the problem in another function context.

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(α, β) Pythagorean Fuzzy Numbers Descriptor Systems

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Abstract- By using pythagorean fuzzy sets and T-S fuzzy descriptor systems, the new (α, β)-pythagorean fuzzy descriptor systems are proposed in this paper. Their definition is given firstly, and the stability of this kind of systems is studied, the relation of (α, β)-pythagorean fuzzy descriptor systems and T-S fuzzy descriptor systems is discussed. The (α, β)-pythagorean fuzzy controller and the stability of (α, β)-pythagorean fuzzy descriptor systems are deeply researched. The (α, β)-pythagorean fuzzy descriptor systems can be better used to solve the problems of actual nonlinear control. The (α, β)-pythagorean fuzzy descriptor systems will be a new research direction, and will become a universal method to solve practical problems. Finally, an example is given to illustrate effectiveness of the proposed method.

Keywords: *pythagorean fuzzy sets; T-S fuzzy descriptor systems; stability.*

GJSFR-F Classification: *MSC 2010: 12D15 , 03B52*

PYTHAGOREANFUZZYNUMBERSDESCRIPTORSYSTEMS

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(α, β) Pythagorean Fuzzy Numbers Descriptor Systems

Chuan-qiang Fan [α], Wei-he Xie [σ] & Feng Liu [ρ]

Abstract- By using pythagorean fuzzy sets and T-S fuzzy descriptor systems, the new (α, β)-pythagorean fuzzy descriptor systems are proposed in this paper. Their definition is given firstly, and the stability of this kind of systems is studied, the relation of (α, β)-pythagorean fuzzy descriptor systems and T-S fuzzy descriptor systems is discussed. The (α, β)-pythagorean fuzzy controller and the stability of (α, β)-pythagorean fuzzy descriptor systems are deeply researched. The (α, β)-pythagorean fuzzy descriptor systems can be better used to solve the problems of actual nonlinear control. The (α, β)-pythagorean fuzzy descriptor systems will be a new research direction, and will become a universal method to solve practical problems. Finally, an example is given to illustrate effectiveness of the proposed method.

Keywords: pythagorean fuzzy sets; T-S fuzzy descriptor systems; stability.

I. Introduction

Pythagorean fuzzy sets[1-4] were proposed by Yager in 2013, are a new tool to deal with vagueness.Pythagorean fuzzy sets maintain the advantages of both membership and non-membership, but the value range of membership function and non-membership function is expanded from triangle to quarter circle. The expansion of the value area makes the amount of information of pythagorean fuzzy sets expand 1.57 times that of the intuitionistic fuzzy sets, and ensures that intuitionistic fuzzy sets are all pythagorean fuzzy sets. They can be used to characterize the uncertain information more sufficiently and accurately than intuitionistic fuzzy sets. Pythagorean fuzzy sets have attracted great attention of a great many scholars that have been extended to new fields and these extensions have been used in many areas such as decision making, aggregation operators, and information measures. Due to theirs wide scope of description cases are very common in diverse real-life issue, pythagorean fuzzy sets have given a boost to the management of vagueness caused by fuzzy scope. Pythagorean fuzzy sets have provided two novel algorithms in decision making problems under Pythagorean fuzzy environment.

Takagi-Sugeno (T-S) fuzzy systems [5-9] has been applied on intelligent computing research and complex nonlinear systems. T-S fuzzy systems have also been extended to new fields and these extensions have been used in many areas by a great many scholars. However, the membership functions of T-S fuzzy systems can't make full use of the all uncertain message in the premise conditions. So we decide to study the new (α, β)-pythagorean fuzzy descriptor systems in order to solve practical control problems more easily and feasible.

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The advantages of (α, β)-pythagorean fuzzy descriptor systems are the following:

1. Pythagorean fuzzy sets maintain the advantages of both membership and non-membership, but the value range of membership function and non-membership function is expanded from triangle to quarter circle. The expansion of the value area makes the amount of information of pythagorean fuzzy sets expand 1.57 times that of the intuitionistic fuzzy sets. They can be used to characterize the uncertain information more sufficiently and accurately than intuitionistic fuzzy sets.

2. The membership function and non-membership function of pythagorean fuzzy sets can be easy to be defined. The value ranges of membership function and non-membership function are also more consistent with objective reality and many hesitant problems and people's thinking.

3. Pythagorean fuzzy sets can ensure that intuitionistic fuzzy sets are all pythagorean fuzzy sets, i.e. intuitionistic fuzzy sets are the special examples of pythagorean fuzzy sets. So intuitionistic fuzzy control systems can be changed into (0,1)-pythagorean fuzzy control systems.

4. (α,β)-pythagorean fuzzy descriptor systems are a broader generalization of T-S fuzzy descriptor systems i.e. T-S fuzzy descriptor systems are the special examples of (α, β)-pythagorean fuzzy descriptor systems.

5. We can judge the degree of weight in the control process according to the value of membership function and non-membership function of the rules. By setting the values of α and β, we decide whether the rules will participate in the final calculation, thereby reducing the calculation process and improving the control efficiency and effectiveness.

6. In fact, (α, β)-pythagorean fuzzy descriptor systems are consistent with the control methods of human being. This method is to imitate the control process of people and also solves the most difficult problem for humans.

The rest of this paper is organized as follows: In Section 1, the basic concepts of T-S fuzzy descriptor systems are introduced. In Section 2, (α,β)-pythagorean fuzzy descriptor systems are firstly proposed. Then the relationship of T-S fuzzy descriptor systems and (α,β)-pythagorean fuzzy descriptor systems are discussed in Section 3. (α,β)-pythagorean fuzzy controller and the stability of (α,β)-pythagorean fuzzy descriptor systemsare deeply researched in Section 4. In Section 5, a numbers examples is given to show the corollaries are corrected. We discussed in detail the effects of controls in several cases. Through this practical example, we find that the selection of pythagorean fuzzy membership functions in the premise conditions of the rules has a great influence on the control effect. Therefore, the choice of pythagorean fuzzy membership functions must be determined after more tests, and we can not completely believe the original given functions. Finally, the conclusion is given in Section 6**.**

Notations:Throughout this paper, R^n and $R^{n\times m}$ denote respectively the n dimensional Euclidean space and $n\times m$ dimensional Euclidean space. PFS denotes pythagorean fuzzy set.

II. Preliminaries

This section will briefly introduce some baisc definitions and theorems on pythagorean fuzzy sets and T-S fuzzy descriptor systems.

Definition 1.1 [1-4] Let X be a universe of discourse. A PFS P in X is given by

$$P = \{< x, \mu_P(x), \nu_P(x) > \mid x \in X\},$$

where $\mu_P : X \to [0,1]$ denotes the degree of membership and $\nu_P : X \to [0,1]$ denotes the degree of non-membership of the element $x \in X$ to the set P, respectively, with the condition that $0 \le (\mu_P(x))^2 + (\nu_P(x))^2 \le 1$. The degree of indeterminacy $\pi_P(x) = 1 - (\mu_P(x))^2 - (\nu_P(x))^2$.

For convenience, a pythagorean fuzzy number $(\mu_P(x), \nu_P(x))$ denoted by $p = (\mu_P, \nu_P)$.

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Definition 1.2 [10,11] T-S fuzzy descriptor systems are as follows:

Rule i : if $x_1(t)$ is F_1^i and...and $x_n(t)$ is F_n^i, then

$$E\dot{x}(t) = A_i x(t) + B_i \mu(t)$$

$$y(t) = C_i x(t) + D_i \mu(t)$$

Where $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T \in R^n$ and $\mu(t) \in R^m$ are the state and control input, respectively; A_i, B_i, C_i and D_i are known real constant matrices with appropriate dimension; E is a singular matrix; $F_1^i, F_2^i, \ldots, F_n^i$ ($i = 1,2,\ldots,r$) are the fuzzy sets.

By fuzzy blending, the overall fuzzy model is inferred as follows

$$E\dot{x}(t) = A(t)x(t) + B(t)\mu(t)$$

$$y(t) = C(t)x(t) + D(t)\mu(t)$$

where

$$A(t) = \sum_{i=1}^{r} h_i(x(t))A_i, \; B(t) = \sum_{i=1}^{r} h_i(x(t))B_i, \; C(t) = \sum_{i=1}^{r} h_i(x(t))C_i, \; D(t) = \sum_{i=1}^{r} h_i(x(t))D_i,$$

and $h_i(x(t))$ is the normalized grade of membership, given as

$$h_i(x(t)) = \frac{\omega_i(x(t))}{\sum_{i=1}^{r} \omega_i(x(t))}, \; \omega_i(x(t)) = \prod_{i=1}^{n} \mu_{ij}(x_j(t)),$$

which is satisfying

$$0 \le h_i(x(t)) \le 1, \sum_{i=1}^{r} h_i(x(t)) = 1,$$

$\mu_{ij}(x_j(t))$ is the grade of membership function of $x_j(t)$ in F_j^i.

a) (α,β)-pythagorean fuzzy descriptor systems

As T-S fuzzy descriptor systems are very familiar to us, and pythagorean fuzzy sets are a new tool to deal with vagueness. So we decide to study the new (α, β)-pythagorean fuzzy descriptor systems in order to solve practical control problems more easily and feasible. Next, the related definitions of $(\alpha,\beta)-$pythagorean fuzzy descriptor systems are gradually given.

Definition 2.1: $(\alpha,\beta)-$pythagorean fuzzy descriptor systems are as follows:

Rule i : if $x_1(t)$ is P_1^i and...and $x_n(t)$ is P_n^i, then

$$E\dot{x}(t) = A_i x(t) + B_i \mu(t) \quad \text{(2.1.a)}$$

$$y(t) = C_i x(t) + D_i \mu(t) \quad \text{(2.1.b)}$$

where $x(t) = [x_1(t)), x_2(t), \cdots, x_n(t)]^T \in R^n$ and $\mu(t) \in R^m$ are the state vector and the control input vector, respectively; $y(t)$ is the measurable output vector; A_i, B_i, C_i and D_i are known real constant matrices with appropriate dimension; E is a singular matrix; $P_1^i, P_2^i, \ldots, P_n^i$ ($i = 1,2,\ldots,r$) are all pythagorean fuzzy sets.

By fuzzy blending, the overall fuzzy model is inferred as follows

$$E\dot{x}(t) = A(t)x(t) + B(t)\mu(t)$$

$$y(t) = C(t)x(t) + D(t)\mu(t)$$

where

$$A(t) = \sum_{i=1}^{r} h_i(x(t))A_i,\ B(t) = \sum_{i=1}^{r} h_i(x(t))B_i,$$

$$C(t) = \sum_{i=1}^{r} h_i(x(t))C_i,\ D(t) = \sum_{i=1}^{r} h_i(x(t))D_i,$$

and $h_i(x(t))$ is the normalized grade of membership, given as

$$h_i(x(t)) = \frac{h_i^{(\alpha,\beta)}(x(t))}{\sum_{i=1}^{r} h_i^{(\alpha,\beta)}(x(t))}, i = 1,2,3,\cdots,r;$$

where

$$h_i^{(\alpha,\beta)}(x(t)) = \begin{cases} h_i^1(x(t)) & when\ h_i^1(x(t)) \geq \alpha\ \ or\ \ h_i^2(x(t)) \leq \beta \\ 0 & else \end{cases}, \alpha+\beta \leq 1, i = 1,2,3,\cdots,r;$$

$$h_i^1(x(t)) = \frac{\mu_{P^i}(x(t))}{\sum_{i=1}^{r} \mu_{P^i}(x(t))},\ h_i^2(x(t)) = \frac{\nu_{p^i}(x(t))}{\sum_{i=1}^{r} \nu_{p^i}(x(t))},$$

where $h_i^1(x(t))$ and $h_i^2(x(t))$ are respectively positive and negative membership functions.

$$\sum_{i=1}^{r} h_i^1(x(t)) = 1, \sum_{i=1}^{r} h_i^2(x(t)) = 1;$$

$$\mu_{P^i}(x_j(t)) = \prod_{j=1}^{r} \mu_{P_j^i}(x_j(t)), \nu_{P^i}(x_j(t)) = \prod_{j=1}^{r} \nu_{P_j^i}(x_j(t));$$

$\mu_{P_j^i}(x_j(t))$ and $\nu_{P_j^i}(x_j(t))$ is the membership function value of $x_j(t)$ that belongs and doesn't belong to the intuitionistic fuzzy numbers set P_j^i.

Remark 2.1 1.We can judge the degree of weight in the control process according to the value of the positive and negative membership functions of the rules. By setting the values of α and β, we decide whether the rules will participate in the final calculation, thereby reducing the calculation process and improving the control efficiency and effectiveness.

2. In fact, (α,β)-pythagorean fuzzy descriptor systems are consistent with the control methods of human being. People generally proceed appropriate control at one point by the past experience, i.e. people's decisions are decided and implemented at roughly one point. This method is to imitate the control process of people.

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III. The Relations between (α,β)-Pythagorean Fuzzy Descriptor Systems and T-S Fuzzy Descriptor Systems

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Firstly, the relation of T-S fuzzy descriptor systems and (α,β)-pythagorean fuzzy descriptor systems is studied through an example.

When $\alpha=0,\beta=1$, then

$$h_i(x(t))=h_i^{(\alpha,\beta)}(x(t))=h_i^1(x(t))=\frac{\mu_i^M(x(t))}{\sum_{i=1}^r\mu_i^M(x(t))},\ h_i^2(x(t))=0,\ \mu_i^M(x(t))=\prod_{j=1}^n\mu_{ij}{}^M(x_j(t)).$$

Then the special (0,1)-pythagorean fuzzy descriptor systems are T-S fuzzy descriptor systems. In other words, T-S fuzzy descriptor systems are all the special (0,1)-pythagorean fuzzy descriptor systems. Therefore, it is easy to get the following Theorem 3.1.

Theorem 3.1 T-S fuzzy descriptor systems are all the (α,β)-pythagorean fuzzy descriptor systems.
Proof:It is so easy, so omit.

IV. (α,β)-Pythagorean Fuzzy Numbers Controller

Now we continue to study the feedback control and stability of pythagorean fuzzy descriptor systems according to the traditional research path of the control systems.
Suppose

Rule i : if $x_1(t)$ is $P_1^i(x_1(t))$ and ... and $x_n(t)$ is $P_n^i(x_n(t))$, then

$$u(x(t))=\sum_{i=1}^r h_i(x(t))G_ix(t) \tag{4.1}$$

where G_i (i=1,2,..., r) are the state feedback-gains matrices.

$$h_i(x(t))=\frac{h_i^{(\alpha,\beta)}(x(t))}{\sum_{i=1}^r h_i^{(\alpha,\beta)}(x(t))},i=1,2,3,\cdots,r;$$

where

$$h_i^{(\alpha,\beta)}(x(t))=\begin{cases}h_i^1(x(t)) & when\ \ h_i^1(x(t))\ge\alpha\ \ or\ \ h_i^2(x(t))\le\beta\\ 0 & else\end{cases},\alpha+\beta\le 1,i=1,2,3,\cdots,r;$$

$$h_i^1(x(t))=\frac{\mu_{P^i}(x(t))}{\sum_{i=1}^r\mu_{P^i}(x(t))},\ h_i^2(x(t))=\frac{\nu_{p^i}(x(t))}{\sum_{i=1}^r\nu_{p^i}(x(t))},$$

where $h_i^1(x(t))$ and $h_i^2(x(t))$ are respectively positive and negative membership functions.

$$\sum_{i=1}^r h_i^1(x(t))=1,\sum_{i=1}^r h_i^2(x(t))=1;$$

$$\mu_{P^i}(x_j(t))=\prod_{j=1}^r\mu_{P_j^i}(x_j(t)),\nu_{P^i}(x_j(t))=\prod_{j=1}^r\nu_{P_j^i}(x_j(t));$$

$\mu_{P_j^i}(x_j(t))$ and $\nu_{P_j^i}(x_j(t))$ is the membership function value of $x_j(t)$ that belongs and doesn't belong to the intuitionistic fuzzy numbers set P_j^i.

If we take (4.1) into (2.1) , we can get

$$E\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(x(t))h_j(x(t))(A_i+B_iG_j)x(t) \tag{4.2.a}$$

$$y(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(x(t))h_j(x(t))(C_i+D_iG_j)x(t) \tag{4.2.b}$$

The system stability is guaranteed by determining the feedback gains G_j.

Basic LMI-based stability conditions guaranteeing the stability of the above control system in the form of (4.2) are given in the following theorem.

Theorem 4.1: The system (4.1) is asymptotically stable, if there exist matrices $N_j\in R^{m\times n}$ (*j*=1,2, 3,..., *r*) and $K=K^T\in R^{n\times n}$ such that the following LMIs are satisfied:

$$K>0 \tag{4.3.a}$$

$$E^TK=K^TE\geq 0 \tag{4.3.b}$$

$$Q_{ij}=A_iK^{-1}+K^{-1}A_i^T+B_iN_j+N_j^TB_i^T<0 \qquad \forall i,j \tag{4.3.c}$$

where the feedback gains are defined as $G_j=N_jK$ for all j.

Proof: Considering the quadratic Lyapunov function

$$V(x(t))=x^T(t)E^TKx(t),$$

where $0<K=K^T\in R^{n\times n}$.

then

$$\dot{V}(x(t))=\dot{x}^T(t)E^TKx(t)+x^T(t)E^TK\dot{x}(t)=(E\dot{x}(t))^TKx(t)+x^T(t)K^T(E\dot{x}(t))=$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_jx^T(t)KK^{-1}\{A^T{}_iK+K^TN_j^TB_i^TK+K^TA_i+K^TB_iN_jK)\}K^{-1}Kx(t),$$

let *Z*=*Kx*(*t*), then

$$\dot{V}(x(t))=\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_jx^T(t)KK^{-1}\{A^T{}_iK+KN_j^TB_i^TK+KA_i+KB_iN_jK)\}K^{-1}Kx(t)$$

$$=\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_jZ(K^{-1}A^T{}_i+N_j^TB_i^T+A_iK^{-1}+B_iN_j)Z.$$

As $Q_{ij}=A_iK^{-1}+K^{-1}A_i^T+B_iN_j+N_j^TB_i^T<0$, so the system (4.1) is asymptotically stable.

V. Simulation Example

Example 5.1: Considering an inverted pendulum, as show in Fig.8, subject to parameter uncertainties[12] as the nonlinear plant to be controlled. The dynamic equation for the inverted pendulum is given by

$$\ddot{\theta}(t) = \frac{g\sin(\theta(t)) - am_p L\dot{\theta}(t)^2 \sin(2\theta(t))/2 - a\cos(\theta(t))\mu(t)}{4L/3 - am_p L\cos^2(\theta(t))}$$

Notes

Where $\theta(t)$ is the angular displacement of the pendulum, g =9.8m/s^2 is the acceleration due to gravity, $m_p \in [m_{p_{\min}}, m_{p_{\max}}]$=[2,3]*kg* is the mass of the pendulum, $M_c \in [M_{\min}, M_{\max}]$=[8,12] *Kg* is the mass of the cart, $a = 1/(m_p + M_c)$, 2*L*=1m is the length of the pendulum, and $u(t)$ is the force (in newtons) applied to the cart. The inverted pendulum is considered working in the operating domain characterized by $x_1 = \theta(t) \in [-5\pi/12, 5\pi/12]$ and $x_2 = \dot{\theta}(t) \in$ [-5,5].

Rule 1: If $x_1(t)$ is M_1^1, $x_2(t)$ is M_2^1, then

$$\begin{pmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{pmatrix} = \begin{pmatrix}0 & 1\\ 10.0078 & 0\end{pmatrix}\begin{pmatrix}x_1(t)\\ x_2(t)\end{pmatrix} + \begin{pmatrix}0\\ -0.1765\end{pmatrix}\mu(t);$$

Rule 2: If $x_1(t)$ is M_1^2, $x_2(t)$ is M_2^2, then

$$\begin{pmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{pmatrix} = \begin{pmatrix}0 & 1\\ 10.0078 & 0\end{pmatrix}\begin{pmatrix}x_1(t)\\ x_2(t)\end{pmatrix} + \begin{pmatrix}0\\ -0.0261\end{pmatrix}\mu(t);$$

Rule 3: If $x_1(t)$ is M_1^3, $x_2(t)$ is M_2^3, then

$$\begin{pmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{pmatrix} = \begin{pmatrix}0 & 1\\ 18.4800 & 0\end{pmatrix}\begin{pmatrix}x_1(t)\\ x_2(t)\end{pmatrix} + \begin{pmatrix}0\\ -0.1765\end{pmatrix}\mu(t);$$

Rule 4: If $x_1(t)$ is M_1^4, $x_2(t)$ is M_2^4, then

$$\begin{pmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{pmatrix} = \begin{pmatrix}0 & 1\\ 18.4800 & 0\end{pmatrix}\begin{pmatrix}x_1(t)\\ x_2(t)\end{pmatrix} + \begin{pmatrix}0\\ -0.0261\end{pmatrix}\mu(t);$$

Table 1: The membership functions of the IT-2 T-S fuzzy model of inverted pendulum

Left membership functions	Right membership functions
$M^1_{M_1^1}(x_1) = 1 - e^{-\frac{x_1^2}{1.2}}$	$M^3_{M_1^1}(x_1) = 1 - 0.23e^{-\frac{x_1^2}{0.25}}$
$M^1_{M_1^2}(x_1) = 1 - e^{-\frac{x_1^2}{1.2}}$	$M^3_{M_1^2}(x_1) = 1 - 0.23e^{-\frac{x_1^2}{0.25}}$
$M^1_{M_1^3}(x_1) = 0.23e^{-\frac{x_1^2}{0.25}}$	$M^3_{M_1^3}(x_1) = e^{-\frac{x_1^2}{1.2}}$
$M^1_{M_1^4}(x_1) = 0.23e^{-\frac{x_1^2}{0.25}}$	$M^3_{M_1^4}(x_1) = e^{-\frac{x_1^2}{1.2}}$
$M^1_{M_2^1}(x_2) = 0.5e^{-\frac{x_2^2}{0.25}}$	$M^3_{M_2^1}(x_2) = e^{-\frac{x_2^2}{1.5}}$
$M^1_{M_2^2}(x_2) = 1 - e^{-\frac{x_2^2}{1.5}}$	$M^3_{M_2^2}(x_2) = 1 - 0.5e^{-\frac{x_2^2}{0.25}}$
$M^1_{M_2^3}(x_2) = 0.5e^{-\frac{x_2^2}{0.25}}$	$M^3_{M_2^3}(x_2) = e^{-\frac{x_2^2}{1.5}}$
$M^1_{M_2^4}(x_2) = 1 - e^{-\frac{x_2^2}{1.5}}$	$M^3_{M_2^4}(x_2) = 1 - 0.5e^{-\frac{x_2^2}{0.25}}$

Next, according to the ideas based on the principles of interpolation and interval coverage, we firstly change the interval-valued T-S fuzzy model of inverted pendulum into the special (α,β)-pythagorean fuzzy descriptor systems of inverted pendulum as follows.

Rule 1: If $x_1(t)$ is $P_1^1(x_1(t))$, $x_2(t)$ is $P_2^1(x_2(t))$, then

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 10.0078 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ -0.1765 \end{pmatrix} \mu(t);$$

Rule 2: If $x_1(k)$ is $P_1^2(x_1(t))$, $x_2(t)$ is $P_2^2(x_2(t))$, then

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 10.0078 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ -0.0261 \end{pmatrix} \mu(t);$$

Rule 3: If $x_1(t)$ is $P_1^3(x_1(k))$, $x_2(t)$ is $P_2^3(x_2(t))$, then

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 18.4800 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ -0.1765 \end{pmatrix} \mu(t);$$

Rule 4: If $x_1(t)$ is $P_1^4(x_1(k))$, $x_2(t)$ is $P_2^4(x_2(t))$, then

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 18.4800 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ -0.0261 \end{pmatrix} \mu(t);$$

Table 2: The membership functions and non-membership functions of (α, β)-pythagorean fuzzy descriptor systems of inverted pendulum

Membership functions	Non-membership functions
$\mu_{P_1^1}(x_1) = 1 - e^{-\frac{x_1^2}{1.2}}$	$\nu_{P_1^1}(x_1) = \sqrt{1-(1-e^{-\frac{x_1^2}{1.2}})^2}$
$\mu_{P_1^2}(x_1) = 1 - e^{-\frac{x_1^2}{1.2}}$	$\nu_{P_1^2}(x_1) = \sqrt{1-(1-e^{-\frac{x_1^2}{1.2}})^2}$
$\mu_{P_1^3}(x_1) = 0.23e^{-\frac{x_1^2}{0.25}}$	$\nu_{P_1^3}(x_1) = \sqrt{1-(0.23e^{-\frac{x_1^2}{0.25}})^2}$
$\mu_{P_1^4}(x_1) = 0.23e^{-\frac{x_1^2}{0.25}}$	$\nu_{P_1^4}(x_1) = \sqrt{1-(0.23e^{-\frac{x_1^2}{0.25}})^2}$
$\mu_{P_2^1}(x_2) = 0.5e^{-\frac{x_2^2}{0.25}}$	$\nu_{P_2^1}(x_2) = \sqrt{1-(0.5e^{-\frac{x_2^2}{0.25}})^2}$
$\mu_{P_2^2}(x_2) = 1 - e^{-\frac{x_2^2}{1.5}}$	$\nu_{P_2^2}(x_2) = \sqrt{1-(1-e^{-\frac{x_2^2}{1.5}})^2}$
$\mu_{P_2^3}(x_2) = 0.5e^{-\frac{x_2^2}{0.25}}$	$\nu_{P_2^3}(x_2) = \sqrt{1-(0.5e^{-\frac{x_2^2}{0.25}})^2}$
$\mu_{P_2^4}(x_2) = 1 - e^{-\frac{x_2^2}{1.5}}$	$\nu_{P_2^4}(x_2) = \sqrt{1-(1-e^{-\frac{x_2^2}{1.5}})^2}$

According to the theorem 4.1, we can get

$$K = \begin{pmatrix} 1/7 & -1/7 \\ -1/7 & 8/7 \end{pmatrix}, K^{-1} = \begin{pmatrix} 8 & 1 \\ 1 & 1 \end{pmatrix}, N_1 = N_3 = (100 \quad 100), N_2 = N_4 = (1000 \;\; 1000),$$

Notes

Notes

So the above (α,β)-pythagorean fuzzy descriptor systems of inverted pendulum is asymptotically stable , and the state feedback-gains matrices $G_1=G_3=(0\quad 100)$, $G_2=G_4=(0\quad 1000)$.

The first case, suppose $x_1(0)=-\frac{11\pi}{29}$, $x_2(0)=-0.88$, $\alpha=0.3,\beta=0.25$, then take the variable $x_1(t)$ as the main factor of the control, and according to Table 2 we can control in three steps , i.e. $x_1(0)=-\frac{11\pi}{29}\rightarrow x_1(t_1)\rightarrow x_1(t_2)\approx 0$ and $0<t_1\le t_2$.

1. When $x_1(0)=-\frac{11\pi}{29}$, $x_2(0)=-0.88$, and $\alpha=0.30,\beta=0.25$, according to Table 2,

we can get $\mu_{P_1^1}(x_1(0))=\mu_{P_1^2}(x_1(0))$=0.69, $\nu_{P_1^1}(x_1(0))=\nu_{P_1^2}(x_1(0))$=0.72, $\mu_{P_1^3}(x_1(0))$ = $\mu_{P_1^4}(x_1(0))$ =0, $\nu_{P_1^3}(x_1(0))=\nu_{P_1^4}(x_1(0))$ =1, $\mu_{P_2^1}(x_2(0))=\mu_{P_2^2}(x_2(0))$ =0.02, $\nu_{P_2^1}(x_2(0))$ = $\nu_{P_2^2}(x_2(0))$ =1, $\mu_{P_2^3}(x_2(0))$ = $\mu_{P_2^4}(x_2(0))$ =0.40, $\nu_{P_2^3}(x_2(0))$ = $\nu_{P_2^4}(x_2(0))$ =0.92, noteworthy, $\mu_{P_1^1}(x_1(0))+\nu_{P_1^1}(x_1(0))=\mu_{P_1^2}(x_1(0))+\nu_{P_1^2}(x_1(0))$ =1.41>1, $\mu_{P_2^3}(x_2(0))+\nu_{P_2^3}(x_2(0))=\mu_{P_2^4}(x_2(0))+\nu_{P_2^4}(x_2(0))$ =1.32>1.Then according to Definition 2.1, taking it one step further, we can get h_1^1 =0.49, h_1^2 =0.22, h_2^1 =0.49, h_2^2 =0.22, h_3^1 =0.01, h_3^2 =0.28, h_4^1 =0.01, h_4^2 =0.28, so h_1 =0.5, h_2 =0.5, h_3 =0, h_4 =0, according to 4.2, so the overall fuzzy model of the (0.30,0.25)- pythagorean fuzzy descriptor systems is

$$\begin{pmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{pmatrix}=\begin{pmatrix}0 & 0.25\\ 2.5019 & -13.929\end{pmatrix}\begin{pmatrix}x_1(t)\\ x_2(t)\end{pmatrix},$$

The solution of the systems is $x(t)=\begin{pmatrix}-\frac{11}{29}\pi+0.25t\\ 0.2172-0.045t-1.0972e^{-13.93t}\end{pmatrix}$;

When $x_1(4)\approx-0.19103$, $x_2(4)\approx 0.0372$, and $\alpha=0.30$, $\beta=0.25$, according to Table 2, we can get $\mu_{P_1^1}(x_1(4))=\mu_{P_1^2}(x_1(4))$=0.03, $\nu_{P_1^1}(x_1(4))=\nu_{P_1^2}(x_1(4))$ =1, $\mu_{P_1^3}(x_1(4))=\mu_{P_1^4}(x_1(4))$ =0.20, $\nu_{P_1^3}(x_1(4))$ = $\nu_{P_1^4}(x_1(4))$ =0.98, $\mu_{P_2^1}(x_2(4))$ = $\mu_{P_2^2}(x_2(4))$ =0.50, $\nu_{P_2^1}(x_2(4))$ = $\nu_{P_2^2}(x_2(4))$ =0.87, $\mu_{P_2^3}(x_2(4))=\mu_{P_2^4}(x_2(4))$ =0, $\nu_{P_2^3}(x_2(4))=\nu_{P_2^4}(x_2(4))$ =1, noteworthy, $\mu_{P_1^1}(x_1(4))+\nu_{P_1^1}(x_1(4))=\mu_{P_1^2}(x_1(4))+\nu_{P_1^2}(x_1(4))$ =1.03>1, $\mu_{P_2^1}(x_2(4))+\nu_{P_2^1}(x_2(4))$ = $\mu_{P_2^2}(x_2(4))+\nu_{P_2^2}(x_2(4))$ =1.37>1.Then according to Definition 2.1, taking it one step further, we can get h_1^1 =0.49, h_1^2 =0.23, h_2^1 =0.49, h_2^2 =0.23, h_3^1 =0.01, h_3^2 =0.27, h_4^1 =0.01, h_4^2 =0.27, so h_1 =0.50, h_2 =0.50, h_3 =0, h_4 =0, then according to 4.2, so the overall fuzzy model of the (0.3,0.3)- pythagorean fuzzy descriptor systems is

$$\begin{pmatrix} \dot{x}_1(t-4) \\ \dot{x}_2(t-4) \end{pmatrix} = \begin{pmatrix} 0 & 0.25 \\ 2.5019 & -13.929 \end{pmatrix} \begin{pmatrix} x_1(t-4) \\ x_2(t-4) \end{pmatrix},$$

The solution of the systems is $x(t) = \begin{pmatrix} -0.19103 + 0.25t \\ 0.03754 - 0.045t - 0.00034e^{-13.93t} \end{pmatrix};$

When $x_1(4.764) \approx -0.0000344$, $x_2(4.764) \approx 0.00323$, so the overall fuzzy model of the(0.30, 0.25)-pythagorean fuzzy descriptor systems is *E*-asymptotic stability. But it takes a shorter time.

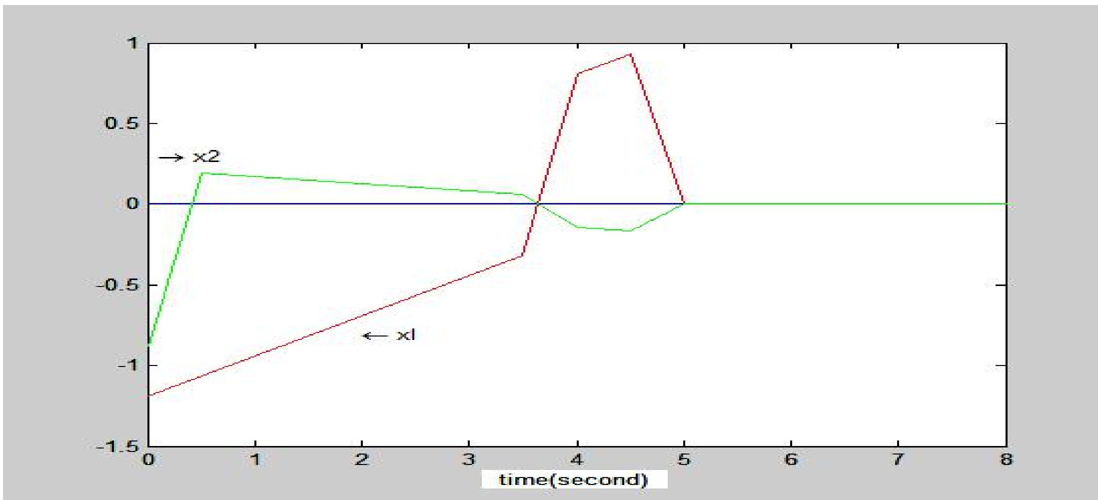

Fig. 1: $X_1(t)$ and $X_2(t)$ under the (0.30,0.25)-pythagorean fuzzy descriptor systems

The second case(interval-valued T-S fuzzy model of inverted pendulum), suppose $x_1(0) = -\frac{11\pi}{29}$, $x_2(0) = -0.88$, then take the variable $x_1(t)$ as the main factor of the control, and according to Tables 1 we can control in three step, i.e. $x_1(0) = -\frac{11\pi}{29} \to x_1(1) \to x_1(t) \approx 0$.

1. When $x_1(0) = -\frac{11\pi}{29} \in [-\frac{11\pi}{29}, 0)$, $x_2(0) = -0.88 \in [-0.88, 0)$, and $\lambda_1 = \lambda_2 = \frac{1}{2}$, according to table1, we can get h_1 =0.26, h_2 =0.58, h_3 =0.05, h_4 =0.11, according to theorem 4.1, so the overall interval-valued fuzzy model of the interval-valued fuzzy descriptor systems is

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -48.561 & -59.925 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix},$$

The solution of the systems is $x(t) = \begin{pmatrix} -1.2229e^{-0.82t} + 0.0319e^{-59.1t} \\ 1.0048e^{-0.82t} - 1.8848e^{-59.1t} \end{pmatrix};$

2. When $x_1(1) = -0.5386 \in [-0.5386, 0)$, $x_2(1) = 0.4425 \in [0, 0.4425)$, and $\lambda_1 = \lambda_2 = \frac{1}{2}$, according to table1, we can get h_1 =0.32, h_2 =0.25, h_3 =0.24, h_4 =0.19, according to 4.2, the overall interval-valued fuzzy model of the interval-valued fuzzy descriptor systems is

Notes

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -44.49 & -58.141 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix},$$

The solution of the systems is $x(t) = \begin{pmatrix} -0.4654\, e^{-0.78(t)} - 0.0732\, e^{-57.37(t)} \\ 0.4174\, e^{-0.78(t)} + 0.0251\, e^{-57.37(t)} \end{pmatrix};$

Notes

2. When $x_1(9.6) \approx -0.0006$, $x_2(9.6) \approx -0.0005$, so the IT2 T-S fuzzy descriptor system of the inverted pendulum will to be stable too.

Thus the stable control time of the (0.30,0.25)-pythagorean fuzzy descriptor systems of inverted pendulum is **4.836** second shorter than the interval-valued T-S fuzzy descriptor systems of the inverted pendulum.

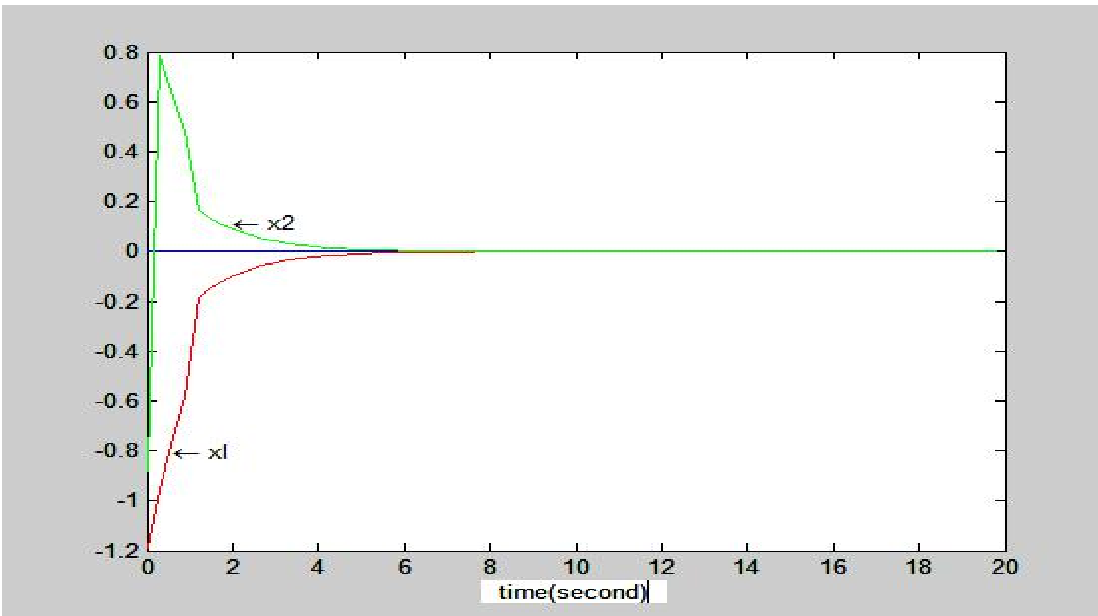

Fig. 2: $X_1(t)$ and $X_2(t)$ under the IT2 T-S fuzzy descriptor systems

Remark 5.1: In this way, the (0.30,0.25)-pythagorean fuzzy descriptor systems can get the better effect than the control effect of interval-valued T-S fuzzy model of inverted pendulum. It is easy to see that the (0.30,0.25)-pythagorean fuzzy descriptor systems has the best control, and can reduce the number of rules and thus reduce the amount of calculations.

In this way, it can get the better effect than the control effect of interval-valued T-S fuzzy model of inverted pendulum. Because the feedback more or less needs a little time, when the system carries out feedback instructions, but the time has gone, so the feedback that have been given are also lagging and out of date. $(\alpha,\beta)-$pythagorean fuzzy descriptor systems can be closer to the actual, and easy to control the error range. The new control method is more convenient and feasible!

VI. Conclusions

In this paper, the new $(\alpha,\beta)-$pythagorean fuzzy descriptor systemsare firstly introduced, and more consistent with the human way of thinking and more likely to be set up and more convenient for popularization. The new $(\alpha,\beta)-$pythagorean fuzzy descriptor systems is very simply and quickly. We can don't know the control principle, but we can directly achieve good control effect. The new theory can be studied in parallel to the basic framework of the original theories and easy to promote the old theories and achieve good results. In addition, we can judge the degree of weight in the control process according to the value of the positive and negative membership functions of the rules. By setting the values of α and β, we decide whether the rules will participate in the final calculation, thereby reducing the number of the rules and the calculation process, and improving the control efficiency and effectiveness. Otherwise, T-S fuzzy descriptor systems are the special examples of $(\alpha,\beta)-$ pythagorean fuzzy descriptor systems. $(\alpha,\beta)-$ pythagorean fuzzy controller and the stability of $(\alpha,\beta)-$ pythagorean fuzzy descriptor systems are deeply researched. At last, a numbers example is given to show the corollaries are corrected.

Notes

But the theoretical part of the new systems need to be in-depth studied, and specific applications are also to be further developed. For example, $(\alpha,\beta)-$pythagorean fuzzy descriptor systems can also be used as the model of autonomous learning in order to establish intelligent control, and can be used well in unmanned driving in the future. So $(\alpha,\beta)-$pythagorean fuzzy descriptor systems is just to meet the reality requirements.

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Two New Summation Formulae Resembling Contiguous Relation

By Salahuddin and Vinti
PDM University

Abstract- In this paper we have developed two summation formulae resembling contiguous relation and involving Hypergeometric function.

Keywords: *special functions, contiguous relation, summation formulae.*

GJSFR-F Classification: *MSC 2010: 33C05, 33C20, 33C45, 33C60*

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Two New Summation Formulae Resembling Contiguous Relation

Salahuddin and Vinti

Abstract- In this paper we have developed two summation formulae resembling contiguous relation and involving Hypergeometric function.

Keywords: special functions, contiguous relation, summation formulae.

I. Introduction and Results Required

Special functions are very useful according to their scope. So many problems related to the various areas of Science , Mathematical Analysis, Applied Mathematics can be solved by various types of special functions.
The discovery of a hypergeometric function has provided an intrinsic stimulation in the world of mathematics. It has also motivated the development of several domains such as complex functions, Riemann surfaces, differential equations, difference equations, arithmetic theory and so forth.
Summation formulae for hypergeometric function has an important role in applied mathematics.
Prudnikov et al[2,p.414] derived the following seven summation formulae

$$ {}_2F_1\left[\begin{matrix} a, & -a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^c}\left[\frac{1}{\Gamma(\frac{c+a+1}{2})\,\Gamma(\frac{c-a}{2})} + \frac{1}{\Gamma(\frac{c+a}{2})\,\Gamma(\frac{c-a+1}{2})}\right] \quad (1) $$

$$ {}_2F_1\left[\begin{matrix} a, & 1-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-1}}\left[\frac{1}{\Gamma(\frac{c+a}{2})\,\Gamma(\frac{c-a+1}{2})}\right] \quad (2) $$

$$ {}_2F_1\left[\begin{matrix} a, & 2-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{(a-1)\,2^{c-2}}\left[\frac{1}{\Gamma(\frac{c+a-2}{2})\,\Gamma(\frac{c-a+1}{2})} - \frac{1}{\Gamma(\frac{c+a-1}{2})\,\Gamma(\frac{c-a}{2})}\right] \quad (3) $$

$$ {}_2F_1\left[\begin{matrix} a, & 3-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{(a-1)(a-2)\,2^{c-3}}\left[\frac{(c-2)}{\Gamma(\frac{c+a-2}{2})\,\Gamma(\frac{c-a+1}{2})} - \frac{2}{\Gamma(\frac{c+a-3}{2})\,\Gamma(\frac{c-a}{2})}\right] \quad (4) $$

$$ {}_2F_1\left[\begin{matrix} a, & 4-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{(1-a)(2-a)(3-a)\,2^{c-4}}\left[\frac{(a-2c+3)}{\Gamma(\frac{c+a-4}{2})\,\Gamma(\frac{c-a+1}{2})} + \frac{(a+2c-7)}{\Gamma(\frac{c+a-3}{2})\,\Gamma(\frac{c-a}{2})}\right] \quad (5) $$

$$ {}_2F_1\left[\begin{matrix} a, & 5-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-5}\{\prod_{\gamma=1}^{4}(\gamma-a)\}}\left[\frac{\{2(c-2)(c-4)-(a-1)(a-4)\}}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-4}{2})} + \frac{(12-4c)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-5}{2})}\right] \quad (6) $$

$$ {}_2F_1\left[\begin{matrix} a, & 6-a\,; & \frac{1}{2} \\ c & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-6}\{\prod_{\delta=1}^{5}(\delta-a)\}}\left[\frac{(4c^2+2ac-a^2-a-34c+62)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-5}{2})} - \frac{(4c^2-2ac-a^2+13a-22c+20)}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-6}{2})}\right] \quad (7) $$

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The contiguous relation is defined as Abramowitz et al[1,p.558]

$$b\ {}_2F_1\left[\begin{matrix} a, & b+1 & ; & \\ c & & ; & z\end{matrix}\right] = (b-c+1)\ {}_2F_1\left[\begin{matrix} a, & b & ; & \\ c & & ; & z\end{matrix}\right] + (c-1)\ {}_2F_1\left[\begin{matrix} a, & b & ; & \\ c-1 & & ; & z\end{matrix}\right] \tag{8}$$

Salahuddin et al[2,3,5,6,7,8,9,10,11] derived the following summation formulae

$${}_2F_1\left[\begin{matrix} a, & 7-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-7}\{\prod_{\varsigma=1}^{6}(\varsigma-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-6}{2})}(-3a^2c+12a^2+21ac-84a+4c^3-48c^2+158c-120)+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-7}{2})}(2a^2-14a-8c^2+64c-108)\Big] \tag{9}$$

$${}_2F_1\left[\begin{matrix} a, & 8-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-8}\{\prod_{\xi=1}^{7}(\xi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-7}{2})}(-a^3-4a^2c+30a^2+4ac^2-4ac-107a+8c^3-124c^2+576c-762)+$$

$$+\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-8}{2})}(-a^3+4a^2c-6a^2+4ac^2-68ac+181a-8c^3+92c^2-288c+210)\Big] \tag{10}$$

$${}_2F_1\left[\begin{matrix} a, & 9-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-9}\{\prod_{\varpi=1}^{8}(\varpi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-8}{2})}(a^4-18a^3-8a^2c^2+80a^2c-85a^2+72ac^2-720ac+1494a+8c^4-$$

$$-160c^3+1056c^2-2560c+1680)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-9}{2})}(8a^2c-40a^2-72ac+360a-16c^3+240c^2-1072c+1360)\Big] \tag{11}$$

$${}_2F_1\left[\begin{matrix} a, & 10-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-10}\{\prod_{\upsilon=1}^{9}(\upsilon-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-10}{2})}(-a^4-4a^3c+42a^3+12a^2c^2-72a^2c-107a^2+8ac^3-252ac^2+$$

$$+1772ac-3054a-16c^4+312c^3-2000c^2+4704c-3024)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-9}{2})}(a^4-4a^3c+2a^3-12a^2c^2+192a^2c-$$

$$-553a^2+8ac^3-12ac^2-868ac+3406a+16c^4-392c^3+3320c^2-11224c+12264)\Big] \tag{12}$$

$${}_2F_1\left[\begin{matrix} a, & 11-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-11}\{\prod_{\varphi=1}^{10}(\varphi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-10}{2})}(5a^4c-30a^4-110a^3c+660a^3-20a^2c^3+360a^2c^2-1305a^2c-$$

$$-810a^2+220ac^3-3960ac^2+21010ac-31020a+16c^5-480c^4+5240c^3-25200c^2+50544c-30240)+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-11}{2})}(-2a^4+44a^3+24a^2c^2-288a^2c+530a^2-264ac^2+3168ac-8492a-32c^4+768c^3-6352c^2+$$

$$+20928c-22320)\Big] \tag{13}$$

Notes

Notes

$$ {}_2F_1\left[\begin{matrix} a, & 12-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = $$

$$ = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-12}\{\prod_{\chi=1}^{11}(\chi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-12}{2})}(a^5-6a^4c+9a^4-12a^3c^2+300a^3c-1103a^3+32a^2c^3- $$

$$ -408a^2c^2+46a^2c+6351a^2+16ac^4-800ac^3+10364ac^2-46852ac+62182a-32c^5+944c^4-10112c^3+47656c^2- $$

$$ -93776c+55440)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-11}{2})}(a^5+6a^4c-69a^4-12a^3c^2+12a^3c+769a^3-32a^2c^3+840a^2c^2- $$

$$ -5662a^2c+8301a^2+16ac^4-32ac^3-4612ac^2+42380ac-96002a+32c^5-1136c^4+15104c^3- $$

$$ -92536c^2+255392c-245640)\Big] \quad (14) $$

$$ {}_2F_1\left[\begin{matrix} a, & 13-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = $$

$$ = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-13}\{\prod_{\beta=1}^{12}(\beta-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-12}{2})}(-a^6+39a^5+18a^4c^2-252a^4c+275a^4-468a^3c^2+6552a^3c $$

$$ -18135a^3-48a^2c^4+1344a^2c^3-9834a^2c^2+5964a^2c+74246a^2+624ac^4-17472ac^3+167388ac^2-631176ac $$

$$ +752856a+32c^6-1344c^5+21824c^4-172032c^3+674384c^2-1187424c+665280)+ $$

$$ +\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-13}{2})}(-12a^4c+84a^4+312a^3c-2184a^3+64a^2c^3-1344a^2c^2 $$

$$ +6620a^2c-2436a^2-832ac^3+17472ac^2-112424ac+216216a-64c^5+2240c^4-29312c^3+176512c^2- $$

$$ -478752c+453600)\Big] \quad (15) $$

$$ {}_2F_1\left[\begin{matrix} a, & 14-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = $$

$$ = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-14}\{\prod_{\sigma=1}^{13}(\sigma-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-14}{2})}(a^6+6a^5c-87a^5-24a^4c^2+150a^4c+925a^4-32a^3c^3+1392a^3c^2 $$

$$ -12706a^3c+24615a^3+80a^2c^4-1728a^2c^3+5368a^2c^2+58986a^2c-242486a^2+32ac^5-2320ac^4+47328ac^3 $$

$$ -391568ac^2+1344076ac-1496568a-64c^6+2656c^5-42560c^4+330752c^3-1278144c^2+2222160c-1235520)+ $$

$$ +\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-13}{2})}(-a^6+6a^5c- $$

$$ -3a^5+24a^4c^2-570a^4c+2225a^4-32a^3c^3+48a^3c^2+7454a^3c-39225a^3-80a^2c^4+3072a^2c^3-35608a^2c^2+133626a^2c- $$

$$ -68104a^2+32ac^5-80ac^4-19872ac^3+313808ac^2-1676564ac+2856228a+64c^6-3104c^5+59360c^4-566848c^3+ $$

$$ +2810304c^2-6724560c+5897520)\Big] \quad (16) $$

$$ {}_2F_1\left[\begin{matrix} a, & 15-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = $$

$$ = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-15}\{\prod_{\varepsilon=1}^{14}(\varepsilon-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-14}{2})}(-7a^6c+56a^6+315a^5c-2520a^5+56a^4c^3-1344a^4c^2+5103a^4c+ $$

$$ +16520a^4-1680a^3c^3+40320a^3c^2-271215a^3c+449400a^3-112a^2c^5+4480a^2c^4-54040a^2c^3+150080a^2c^2+845824a^2c- $$

$$-3383296a^2+1680ac^5-67200ac^4+999600ac^3-6787200ac^2+20482140ac-21070560a+64c^7-3584c^6+80864c^5-$$

$$-940800c^4+5987520c^3-20296192c^2+32464368c-17297280)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-15}{2})}(2a^6-90a^5-48a^4c^2+768a^4c-$$

$$-1474a^4+1440a^3c^2-23040a^3c+77970a^3+160a^2c^4-5120a^2c^3+46640a^2c^2-90880a^2c-226192a^2-2400ac^4+$$

$$+76800ac^3-861600ac^2+3955200ac-6138120a-128c^6+6144c^5-116160c^4+1095680c^3-5363584c^2+$$

$$+12679168c-11009376)\Big] \tag{17}$$

$${}_2F_1\left[\begin{matrix} a, & 16-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-16}\{\prod_{\zeta=1}^{15}(\zeta-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-16}{2})}(-a^7+8a^6c-12a^6+24a^5c^2-792a^5c+3710a^5-80a^4c^3+1080a^4c^2+$$

$$+6280a^4c-66600a^4-80a^3c^4+5280a^3c^3-85480a^3c^2+435480a^3c-458929a^3+192a^2c^5-6240a^2c^4+45200a^2c^3+$$

$$+271560a^2c^2-3746640a^2c+8942052a^2+64ac^6-6336ac^5+186000ac^4-2408160ac^3+15005072ac^2-42553152ac+$$

$$+41722740a-128c^7+7104c^6-158720c^5+1827360c^4-11505152c^3+38596416c^2-61194240c+32432400)+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-15}{2})}(-a^7-8a^6c+124a^6+24a^5c^2-24a^5c-2818a^5+80a^4c^3-3000a^4c^2+26360a^4c-40760a^4-80a^3c^4+$$

$$+160a^3c^3+45080a^3c^2-534760a^3c+1499471a^3-192a^2c^5+10080a^2c^4-175760a^2c^3+1189560a^2c^2-2226480a^2c-$$

$$-2760884a^2+64ac^6-192ac^5-75120ac^4+1782560ac^3-16394608ac^2+65703616ac-93008652a+128c^7-8128c^6+$$

$$+210944c^5-2878240c^4+22080512c^3-94015552c^2+202146816c-165145680)\Big] \tag{18}$$

$${}_2F_1\left[\begin{matrix} a, & 17-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-17}\{\prod_{\vartheta=1}^{16}(\vartheta-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-16}{2})}(a^8-68a^7-32a^6c^2+576a^6c-638a^6+1632a^5c^2-29376a^5c+101320a^5+$$

$$+160a^4c^4-5760a^4c^3+44640a^4c^2+129600a^4c-1341071a^4-5440a^3c^4+195840a^3c^3-2303840a^3c^2+$$

$$+9743040a^3c-9832052a^3-256a^2c^6+13824a^2c^5-246560a^2c^4+1411200a^2c^3+4297408a^2c^2-64103040a^2c+$$

$$+143207628a^2+4352ac^6-235008ac^5+4977600ac^4-52289280ac^3+282566656ac^2-727036416ac+670152240a+$$

$$+128c^8-9216c^7+275456c^6-4423680c^5+41249792c^4-224907264c^3+683065344c^2-1014128640c+518918400+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-17}{2})}(16a^6c-144a^6-816a^5c+7344a^5-160a^4c^3+4320a^4c^2-22480a^4c-30960a^4+5440a^3c^3-$$

$$-146880a^3c^2+1157360a^3c-2484720a^3+384a^2c^5-17280a^2c^4+247840a^2c^3-1092960a^2c^2-1901760a^2c+$$

$$+15669504a^2-6528ac^5+293760ac^4-4999360ac^3+39804480ac^2-146267456ac+194890176a-256c^7+16128c^6-$$

$$-414976c^5+5610240c^4-42628864c^3+179788032c^2-383195904c+310867200)\Big] \tag{19}$$

$${}_2F_1\left[\begin{matrix} a, & 18-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-18}\{\prod_{\eta=1}^{17}(\eta-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-18}{2})}(-a^8-8a^7c+148a^7+40a^6c^2-256a^6c-3362a^6+80a^5c^3-4440a^5c^2+$$

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$$+49664a^5c-103400a^5-240a^4c^4+5520a^4c^3+18760a^4c^2-849520a^4c+3240271a^4-192a^3c^5+17760a^3c^4-440560a^3c^3+$$
$$+4091160a^3c^2-12923320a^3c+3622852a^3+448a^2c^6-20352a^2c^5+253360a^2c^4+576240a^2c^3-31091248a^2c^2+$$
$$+192701168a^2c-344444908a^2+128ac^7-16576ac^6+660032ac^5-12228640ac^4+118499872ac^3-604789504ac^2+$$
$$+1488844864ac-1324543920a-256c^8+18304c^7-542976c^6+8650240c^5-79993344c^4+432549376c^3-1303568384c^2+$$
$$+1923025920c-980179200)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-17}{2})}(a^8-8a^7c+4a^7-40a^6c^2+1264a^6c-6214a^6+80a^5c^3-120a^5c^2-$$
$$-32416a^5c+213904a^5+240a^4c^4-12720a^4c^3+186440a^4c^2-743120a^4c-456391a^4-192a^3c^5+480a^3c^4+216080a^3c^3-$$
$$-4278120a^3c^2+27569480a^3c-52277444a^3-448a^2c^6+30720a^2c^5-745840a^2c^4+7817520a^2c^3-30345632a^2c^2-$$
$$-19224224a^2c+253516684a^2+128ac^7-448ac^6-259264ac^5+8556320ac^4-118218848ac^3+813195488ac^2-$$
$$-2692403360ac+3335839536a+256c^8-20608c^7+696192c^6-12817024c^5+139638144c^4-913535872c^3+3463541888c^2-$$
$$-6848013696c+5284782720)\Big] \tag{20}$$

$${}_2F_1\left[\begin{matrix} a, & 19-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-19}\{\prod_{\lambda=1}^{18}(\lambda-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-18}{2})}(9a^8c-90a^8-684a^7c+6840a^7-120a^6c^3+3600a^6c^2-14046a^6c-99540a^6+$$
$$+6840a^5c^3-205200a^5c^2+1664856a^5c-2968560a^5+432a^4c^5-21600a^4c^4+277080a^4c^3+327600a^4c^2-20793831a^4c+$$
$$+70898310a^4-16416a^3c^5+820800a^3c^4-14644440a^3c^3+111013200a^3c^2-315518940a^3c+131909400a^3-576a^2c^7+$$
$$+40320a^2c^6-992880a^2c^5+9324000a^2c^4+4429536a^2c^3-636886080a^2c^2+3695816316a^2c-6211091160a^2+10944ac^7-$$
$$-766080ac^6+21827808ac^5-325310400ac^4+2707726176ac^3-12394025280ac^2+28254838896ac-23908836960a+256c^9-$$
$$-23040c^8+880512c^7-18627840c^6+238347264c^5-1891123200c^4+9158978048c^3-25507261440c^2+35661692160c-$$
$$-17643225600)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-19}{2})}(-2a^8+152a^7+80a^6c^2-1600a^6c+3148a^6-4560a^5c^2+91200a^5c-371488a^5-$$
$$-480a^4c^4+19200a^4c^3-185680a^4c^2-126400a^4c+4559182a^4+18240a^3c^4-729600a^3c^3+9799440a^3c^2-50068800a^3c+$$
$$+73373288a^3+896a^2c^6-53760a^2c^5+1107680a^2c^4-8467200a^2c^3-743936a^2c^2+274718720a^2c-822056088a^2-$$
$$-17024ac^6+1021440ac^5-24338240ac^4+292569600ac^3-1853708096ac^2+5798641920ac-6885423072a-$$
$$-512c^8+40960c^7-1374464c^6+25123840c^5-271685888c^4+1764075520c^3-6639757056c^2+13042437120c-$$
$$-10013310720)\Big] \tag{21}$$

$${}_2F_1\left[\begin{matrix} a, & 20-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-20}\{\prod_{\Upsilon=1}^{19}(\Upsilon-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-20}{2})}(33522128640+47215599696a+14182895460a^2+345040520a^3-$$
$$-140133105a^4+962073a^5+330750a^6-9330a^7+15a^8+a^9-67958134272c-57343402272ac-9605975576a^2c+$$
$$+295428296a^3c+58846422a^4c-2100880a^5c-32820a^6c+1640a^7c-10a^8c+48842214912c^2+$$
$$+25998562336ac^2+2187966784a^2c^2-168954152a^3c^2-6101120a^4c^2+380720a^5c^2-2240a^6c^2-40a^7c^2-17641896960c^3-$$
$$-5917427456ac^3-182014144a^2c^3+27821280a^3c^3-9440a^4c^3-19680a^5c^3+160a^6c^3+3666323456c^4+750095264ac^4-$$
$$-1895280a^2c^4-1926160a^3c^4+23280a^4c^4+240a^5c^4-465172736c^5-54369728ac^5+1155616a^2c^5+55104a^3c^5-672a^4c^5+$$
$$+36595328c^6+2174144ac^6-61824a^2c^6-448a^3c^6-1740800c^7-41984ac^7+1024a^2c^7+45824c^8+256ac^8-512c^9)+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-19}{2})}(-190253266560-131460917904a-15315714660a^2+1718684120a^3+100625805a^4-10839927a^5+$$
$$+135450a^6+7470a^7-195a^8+a^9+258458522112c+117489033888ac+5199265016a^2c-1259577944a^3c+961578a^4c+$$
$$+3256720a^5c-84780a^6c+40a^7c+10a^8c-139931759232c^2-40815588704ac^2+198370336a^2c^2+283436248a^3c^2-$$
$$-7330880a^4c^2-224080a^5c^2+7840a^6c^2-40a^7c^2+40472263680c^3+7213462784ac^3-274206656a^2c^3-26053920a^3c^3+$$
$$+1017440a^4c^3-480a^5c^3-160a^6c^3-6993636736c^4-700147936ac^4+42392880a^2c^4+896240a^3c^4-47280a^4c^4+240a^5c^4+$$
$$+757008896c^5+36475712ac^5-2849056a^2c^5+1344a^3c^5+672a^4c^5-51764608c^6-836416ac^6+88704a^2c^6-448a^3c^6+$$
$$+2170880c^7-1024ac^7-1024a^2c^7-50944c^8+256ac^8+512c^9)\Big] \quad (22)$$

$${}_2F_1\left[\begin{matrix} a, & 21-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-21}\{\prod_{\Psi=1}^{20}(\Psi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-20}{2})}(670442572800+946321185600a+284169369024a^2+4885689900a^3-$$
$$-3333875180a^4+41694345a^5+10037727a^6-381150a^7+1230a^8+105a^9-a^{10}-1394694005760c-1198379286720ac-$$
$$-203053089360a^2c+8433107760a^3c+1530533620a^4c-70408800a^5c-1146200a^6c+92400a^7c-1100a^8c+$$
$$+1048586614272c^2+578478838560ac^2+49539606520a^2c^2-4805882760a^3c^2-177714670a^4c^2+15397200a^5c^2-$$
$$-141500a^6c^2-4200a^7c^2+50a^8c^2-404078540800c^3-143591669760ac^3-4354528640a^2c^3+902932800a^3c^3-$$
$$-2094400a^4c^3-1108800a^5c^3+17600a^6c^3+91700259840c^4+20464187520ac^4-122473120a^2c^4-77439600a^3c^4+$$
$$+1402800a^4c^4+25200a^5c^4-400a^6c^4-13092907520c^5-1739633280ac^5+50240960a^2c^5+3104640a^3c^5-73920a^4c^5+$$
$$+1209103616c^6+87071040ac^6-3652320a^2c^6-47040a^3c^6+1120a^4c^6-72089600c^7-2365440ac^7+112640a^2c^7+$$
$$+2677760c^8+26880ac^8-1280a^2c^8-56320c^9+512c^{10})+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-21}{2})}(362387520000+268742591040a+$$
$$+41471452880a^2-1867829040a^3-305673060a^4+14303520a^5+225720a^6-18480a^7+220a^8-494250063360c-$$
$$-247867413696ac-19713479280a^2c+1984361232a^3c+69962284a^4c-6179040a^5c+56920a^6c+1680a^7c-20a^8c+$$
$$+268936121344c^2+89644203264ac^2+2511762176a^2c^2-547968960a^3c^2+1404480a^4c^2+665280a^5c^2-10560a^6c^2-$$
$$-78226625536c^3-16719935232ac^3+112602112a^2c^3+62139840a^3c^3-1126720a^4c^3-20160a^5c^3+320a^6c^3+$$
$$+13598953984c^4+1756191360ac^4-51029440a^2c^4-3104640a^3c^4+73920a^4c^4-1480941056c^5-104786304ac^5+$$
$$+4397120a^2c^5+56448a^3c^5-1344a^4c^5+101871616c^6+3311616ac^6-157696a^2c^6-4296704c^7-43008ac^7+2048a^2c^7+$$
$$+101376c^8-1024c^9)\Big] \quad (23)$$

$${}_2F_1\left[\begin{matrix} a, & 22-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-22}\{\prod_{\Xi=1}^{21}(\Xi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-22}{2})}(-1279935820800-1868233671360a-628352859744a^2-$$
$$-34417212780a^3+5753119700a^4+134236095a^5-20700687a^6+312270a^7+8730a^8-225a^9+a^{10}+2668809669120c+$$
$$+2417863186656ac+489345655848a^2c-219480864a^3c-3388493178a^4c+53042458a^5c+4970700a^6c-142820a^7c+$$
$$+390a^8c+10a^9c-2014029186048c^2-1197461040576ac^2-138256171792a^2c^2+5911683120a^3c^2+583619652a^4c^2-$$
$$-22165920a^5c^2-176120a^6c^2+10800a^7c^2-60a^8c^2+779711413248c^3+306554335232ac^3+17513420736a^2c^3-$$

Notes

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$$-1499242976a^3c^3 - 32128320a^4c^3 + 2199680a^5c^3 - 13440a^6c^3 - 160a^7c^3 - 177857647616c^4 - 45404661120ac^4 -$$
$$-823493664a^2c^4 + 154420560a^3c^4 - 467600a^4c^4 - 75600a^5c^4 + 560a^6c^4 + 25531683072c^5 + 4062167872ac^5 -$$
$$-27880608a^2c^5 - 7519456a^3c^5 + 86688a^4c^5 + 672a^5c^5 - 2370643968c^6 - 219093504ac^6 + 4625152a^2c^6 + 161280a^3c^6 -$$
$$-1792a^4c^6 + 142098432c^7 + 6765568ac^7 - 178176a^2c^7 - 1024a^3c^7 - 5305344c^8 - 103680ac^8 + 2304a^2c^8 + 112128c^9 +$$
$$+512ac^9 - 1024c^{10}) + \frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-21}{2})}(7610141548800 + 5664039006240a + 862754799384a^2 - 50618670580a^3 -$$
$$-7467040370a^4 + 438809595a^5 + 6355587a^6 - 793890a^7 + 14040a^8 - 5a^9 - a^{10} - 10762094073600c - 5490993903456ac -$$
$$-428082370072a^2c + 53697863232a^3c + 1803933278a^4c - 214732742a^5c + 2793980a^6c + 100060a^7c - 2370a^8c + 10a^9c +$$
$$+6167102701056c^2 + 2126343680320ac^2 + 53972779984a^2c^2 - 16286791024a^3c^2 + 92193948a^4c^2 + 28580160a^5c^2 -$$
$$-673960a^6c^2 + 240a^7c^2 + 60a^8c^2 - 1923629552128c^3 - 434298545536ac^3 + 5057543680a^2c^3 + 2145900064a^3c^3 -$$
$$-52633280a^4c^3 - 1200640a^5c^3 + 38080a^6c^3 - 160a^7c^3 + 366720941312c^4 + 51431687104ac^4 - 1928215296a^2c^4 -$$
$$-133374640a^3c^4 + 4718000a^4c^4 - 1680a^5c^4 - 560a^6c^4 - 45108419328c^5 - 3603513536ac^5 + 200868192a^2c^5 +$$
$$+3361568a^3c^5 - 160608a^4c^5 + 672a^5c^5 + 3654604800c^6 + 142266880ac^6 - 10065664a^2c^6 + 3584a^3c^6 + 1792a^4c^6 -$$
$$-193800192c^7 - 2561024ac^7 + 245760a^2c^7 - 1024a^3c^7 + 6471168c^8 - 2304ac^8 - 2304a^2c^8 - 123392c^9 + 512ac^9 + 1024c^{10})\Big] \quad (24)$$

Notes

$${}_2F_1\left[\begin{matrix} a, & 23-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-23}\{\prod_{\Omega=1}^{22}(\Omega-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-22}{2})}(-28158588057600 - 41169473009280a - 13846175136288a^2 -$$
$$-688985043120a^3 + 149373094200a^4 + 2886127860a^5 - 648260844a^6 + 12963720a^7 + 382800a^8 - 15180a^9 + 132a^{10} +$$
$$+60062080780800c + 55217638146528ac + 11293160726232a^2c - 82022959260a^3c - 93645815450a^4c + 1995539777a^5c +$$
$$+178029929a^6c - 6909430a^7c + 31460a^8c + 1265a^9c - 11a^{1}0c - 47111453908992c^2 - 28820579344128ac^2 -$$
$$-3392485386048a^2c^2 + 183098051136a^3c^2 + 17810133264a^4c^2 - 867081600a^5c^2 - 6985440a^6c^2 + 728640a^7c^2 - 7920a^8c^2 +$$
$$+19258856466432c^3 + 7922696165824ac^3 + 461791289168a^2c^3 - 49584004624a^3c^3 - 1091526436a^4c^3 + 105693280a^5c^3 -$$
$$-988680a^6c^3 - 20240a^7c^3 + 220a^8c^3 - 4723308327936c^4 - 1289002826496ac^4 - 22320173568a^2c^4 + 5914856640a^3c^4 -$$
$$-30824640a^4c^4 - 5100480a^5c^4 + 73920a^6c^4 + 745452131072c^5 + 130485126464ac^5 - 1364040832a^2c^5 - 359725520a^3c^5 +$$
$$+6190800a^4c^5 + 85008a^5c^5 - 1232a^6c^5 - 78371758080c^6 - 8302221312ac^6 + 235834368a^2c^6 + 10881024a^3c^6 -$$
$$-236544a^4c^6 + 5546010624c^7 + 322674176ac^7 - 12539648a^2c^7 - 129536a^3c^7 + 2816a^4c^7 - 260941824c^8 - 6994944ac^8 +$$
$$+304128a^2c^8 + 7822848c^9 + 64768ac^9 - 2816a^2c^9 - 135168c^{10} + 1024c^{11}) + \frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-23}{2})}(-14558535129600 -$$
$$-11503844032320a - 2137013714928a^2 + 23993967080a^3 + 17223845140a^4 - 382057830a^5 - 32189094a^6 + 1259940a^7 -$$
$$-5760a^8 - 230a^9 + 2a^{10} + 20652447375360c + 11426734414848ac + 1249606186752a^2c - 70894693632a^3c -$$
$$-6453736128a^4c + 319011840a^5c + 2486400a^6c - 264960a^7c + 2880a^8c - 11881425202176c^2 - 4559948772992ac^2 -$$
$$-249358186400a^2c^2 + 27700361696a^3c^2 + 584111304a^4c^2 - 57805440a^5c^2 + 541520a^6c^2 + 11040a^7c^2 - 120a^8c^2 +$$
$$+3722781351936c^3 + 967717718016ac^3 + 15372177408a^2c^3 - 4341281280a^3c^3 + 23278080a^4c^3 + 3709440a^5c^3 -$$
$$-53760a^6c^3 - 713155826176c^4 - 120677707136ac^4 + 1319899392a^2c^4 + 327847520a^3c^4 - 5645920a^4c^4 - 77280a^5c^4 +$$
$$+1120a^6c^4 + 88159518720c^5 + 9128189952ac^5 - 260370432a^2c^5 - 11870208a^3c^5 + 258048a^4c^5 - 7178121216c^6 -$$

$$-411665408ac^{6}+16002560a^{2}c^{6}+164864a^{3}c^{6}-3584a^{4}c^{6}+382500864c^{7}+10174464ac^{7}-442368a^{2}c^{7}-12831744c^{8}-$$
$$-105984ac^{8}+4608a^{2}c^{8}+245760c^{9}-2048c^{10})\Big] \quad (25)$$

$$ {}_2F_1\left[\begin{matrix} a, & 24-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = $$

$$=\frac{\sqrt{\pi}\ \Gamma(c)}{2^{c-24}\{\prod_{\nu=1}^{23}(\nu-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a-24}{2})}(53970627110400+81275748452640a+29965288299912a^{2}+$$
$$+2450226723044a^{3}-225124440810a^{4}-14621050415a^{5}+1046631306a^{6}+3307857a^{7}-1146390a^{8}+19675a^{9}-18a^{10}-$$
$$-a^{11}-115335576760320c-110963670941184ac-26002256223296a^{2}c-677875698640a^{3}c+179871679040a^{4}c+$$
$$+1164213764a^{5}c-418718356a^{6}c+7202120a^{7}c+106120a^{8}c-2940a^{9}c+12a^{10}c+90736096819200c^{2}+$$
$$+59116525645888ac^{2}+8571136553968a^{2}c^{2}-132498699168a^{3}c^{2}-43219364300a^{4}c^{2}+885926748a^{5}c^{2}+$$
$$+41948200a^{6}c^{2}-1232440a^{7}c^{2}+4020a^{8}c^{2}+60a^{9}c^{2}-37229387082752c^{3}-16647614537344ac^{3}-1382871809056a^{2}c^{3}+$$
$$+70301031136a^{3}c^{3}+4193488488a^{4}c^{3}-166658240a^{5}c^{3}-636720a^{6}c^{3}+54880a^{7}c^{3}-280a^{8}c^{3}+9168624153856c^{4}+$$
$$+2788555498944ac^{4}+110886499136a^{2}c^{4}-10435825456a^{3}c^{4}-136259200a^{4}c^{4}+10643360a^{5}c^{4}-64960a^{6}c^{4}-$$
$$-560a^{7}c^{4}-1453428736000c^{5}-292828010496ac^{5}-2923085312a^{2}c^{5}+743957760a^{3}c^{5}-3395840a^{4}c^{5}-263424a^{5}c^{5}+$$
$$+1792a^{6}c^{5}+153495164928c^{6}+19572034048ac^{6}-185488128a^{2}c^{6}-27007232a^{3}c^{6}+295680a^{4}c^{6}+1792a^{5}c^{6}-$$
$$-10911129600c^{7}-818236416ac^{7}+16882176a^{2}c^{7}+451584a^{3}c^{7}-4608a^{4}c^{7}+515633664c^{8}+20126976ac^{8}-$$
$$-493056a^{2}c^{8}-2304a^{3}c^{8}-15523840c^{9}-250880ac^{9}+5120a^{2}c^{9}+269312c^{10}+1024ac^{10}-2048c^{11})+$$
$$+\frac{1}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a-23}{2})}(-334846347897600-265325367396960a-48906562552488a^{2}+988440027044a^{3}+$$
$$+456572472690a^{4}-13283988815a^{5}-1027452594a^{6}+51835857a^{7}-355890a^{8}-16325a^{9}+282a^{10}-a^{11}+$$
$$+490301899960320c+275033962166784ac+29894874504896a^{2}c-2110031722960a^{3}c-182994379040a^{4}c+$$
$$+11313648836a^{5}c+75158356a^{6}c-13480120a^{7}c+217880a^{8}c-60a^{9}c-12a^{10}c-294885357427200c^{2}-$$
$$-116251083614912ac^{2}-6231457365232a^{2}c^{2}+863633176032a^{3}c^{2}+17220272300a^{4}c^{2}-2360841252a^{5}c^{2}+$$
$$+30594200a^{6}c^{2}+783560a^{7}c^{2}-16980a^{8}c^{2}+60a^{9}c^{2}+97997531697152c^{3}+26592490242944ac^{3}+357545095456a^{2}c^{3}-$$
$$-150694485536a^{3}c^{3}+1256431512a^{4}c^{3}+188722240a^{5}c^{3}-4067280a^{6}c^{3}+1120a^{7}c^{3}+280a^{8}c^{3}-20258796646144c^{4}-$$
$$-3653166613056ac^{4}+56989635136a^{2}c^{4}+13326094544a^{3}c^{4}-308739200a^{4}c^{4}-5484640a^{5}c^{4}+159040a^{6}c^{4}-560a^{7}c^{4}+$$
$$+2762273996800c^{5}+313271863296ac^{5}-11408255488a^{2}c^{5}-603912960a^{3}c^{5}+19523840a^{4}c^{5}-5376a^{5}c^{5}-1792a^{6}c^{5}-$$
$$-255403395072c^{6}-16677258752ac^{6}+860412672a^{2}c^{6}+11699968a^{3}c^{6}-510720a^{4}c^{6}+1792a^{5}c^{6}+16110796800c^{7}+$$
$$+517641216ac^{7}-33470976a^{2}c^{7}+9216a^{3}c^{7}+4608a^{4}c^{7}-682830336c^{8}-7521024ac^{8}+658944a^{2}c^{8}-2304a^{3}c^{8}+$$
$$+18595840c^{9}-5120ac^{9}-5120a^{2}c^{9}-293888c^{10}+1024ac^{10}+2048c^{11})\Big] \quad (26)$$

$$ {}_2F_1\left[\begin{matrix} a, & 25-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = $$

$$=\frac{\sqrt{\pi}\ \Gamma(c)}{2^{c-25}\{\prod_{\rho=1}^{24}(\rho-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a-24}{2})}(1295295050649600+1953217026028800a+720182075218848a^{2}+$$
$$+56242392874200a^{3}-6430140382484a^{4}-384225527550a^{5}+34503650159a^{6}+18877950a^{7}-48737217a^{8}+$$
$$+1122750a^{9}-2107a^{10}-150a^{11}+a^{12}-2824623532523520c-2750688728985600ac-650557782840576a^{2}c-$$

Notes

Notes

$$-14031184440000a^3c+5306549788800a^4c+16204297200a^5c-15185557296a^6c+337740000a^7c+5397600a^8c-$$

$$-234000a^9c+1872a^{10}c+2298271622344704c^2+1532660626368000ac^2+226240061985280a^2c^2-$$

$$-5059473463200a^3c^2-1376735080736a^4c^2+36195503400a^5c^2+1749343288a^6c^2-69774000a^7c^2+360240a^8c^2+$$

$$+9000a^9c^2-72a^{10}c^2-988040333070336c^3-458238106560000ac^3-39008403657600a^2c^3+2443984233600a^3c^3+$$

$$+148362815328a^4c^3-7556640000a^5c^3-26644800a^6c^3+4368000a^7c^3-43680a^8c^3+258663765079040c^4+$$

$$+82943231664000ac^4+3361869113440a^2c^4-398515370400a^3c^4-5286817592a^4c^4+599592000a^5c^4-5544560a^6c^4-$$

$$-84000a^7c^4+840a^8c^4-44346659635200c^5-9621229363200ac^5-85147625472a^2c^5+33231744000a^3c^5-$$

$$-227834880a^4c^5-20966400a^5c^5+279552a^6c^5+5176704237568c^6+731346739200ac^6-9600109568a^2c^6-$$

$$-1516300800a^3c^6+24726016a^4c^6+268800a^5c^6-3584a^6c^6-418737586176c^7-36241920000ac^7+1000396800a^2c^7+$$

$$+35942400a^3c^7-718848a^4c^7+23472279552c^8+1127232000ac^8-40769280a^2c^8-345600a^3c^8+6912a^4c^8-$$

$$-894566400c^9-19968000ac^9+798720a^2c^9+22110208c^{10}+153600ac^{10}-6144a^2c^{10}-319488c^{11}+2048c^{12})+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-25}{2})}(642905026283520+536430721132800a+114941722154688a^2+1925655420000a^3-$$

$$-909114008400a^4-1437781800a^5+2535320424a^6-57018000a^7-892320a^8+39000a^9-312a^{10}-$$

$$-943741088299008c-566915213203200ac-77644945591872a^2c+1953684829600a^3c+462411503408a^4c-$$

$$-12465164600a^5c-579414472a^6c+23314000a^7c-120640a^8c-3000a^9c+24a^{10}c+569402217805824c^2+$$

$$+245281402080000ac^2+19642608356800a^2c^2-1277504155200a^3c^2-73799166896a^4c^2+3813264000a^5c^2+$$

$$+12856480a^6c^2-2184000a^7c^2+21840a^8c^2-189917707732992c^3-57743834144000ac^3-2208865074240a^2c^3+$$

$$+270734475200a^3c^3+3442060496a^4c^3-400624000a^5c^3+3708320a^6c^3+56000a^7c^3-560a^8c^3+39416989081600c^4+$$

$$+8229521664000ac^4+65103933440a^2c^4-27902784000a^3c^4+194055680a^4c^4+17472000a^5c^4-232960a^6c^4-$$

$$-5396707745792c^5-741874022400ac^5+9980880896a^2c^5+1519526400a^3c^5-24790528a^4c^5-268800a^5c^5+$$

$$+3584a^6c^5+501075714048c^6+42561792000ac^6-1178311680a^2c^6-41932800a^3c^6+838656a^4c^6-31738847232c^7-$$

$$-1506048000ac^7+54481920a^2c^7+460800a^3c^7-9216a^4c^7+1350635520c^8+29952000ac^8-1198080a^2c^8-$$

$$-36925440c^9-256000ac^9+10240a^2c^9+585728c^{10}-4096c^{11})\Big] \qquad (27)$$

$${}_2F_1\left[\begin{matrix} a, & 26-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-26}\{\prod_{\tau=1}^{25}(\tau-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-26}{2})}(-2490952020480000-3856567328006400a-1535121422480160a^2-$$

$$-162685541616312a^3+7957280277044a^4+1100869544310a^5-43694979215a^6-1539415206a^7+90397857a^8-$$

$$-769110a^9-18725a^{10}+318a^{11}-a^{12}+5440375458201600c+5512278582766080ac+1453485267708864a^2c+$$

$$+72388041958576a^3c-9060718762800a^4c-307607766540a^5c+27726305208a^6c-96424644a^7c-19614240a^8c+$$

$$+339500a^9c-552a^{10}c-12a^{11}c-4437289667266560c^2-3123375119467776ac^2-539380387024256a^2c^2-$$

$$-4354450180400a^3c^2+2864279730800a^4c^2-8089061876a^5c^2-4607346604a^6c^2+84726040a^7c^2+721000a^8c^2-$$

$$-22260a^9c^2+84a^{10}c^2+1913345384014848c^3+952103060350336ac^3+102944105640800a^2c^3-$$

$$-2607695699200a^3c^3-390111231160a^4c^3+9154149592a^5c^3+261055760a^6c^3-7910000a^7c^3+27720a^8c^3+$$

$$+280a^9c^3-502609149259776c^4-176315887270400ac^4-10847680617600a^2c^4+618019552640a^3c^4+$$

$$+24394548640a^4c^4-1011055360a^5c^4-1400000a^6c^4+237440a^7c^4-1120a^8c^4+86485139701760c^5+$$

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$$+21027657410560ac^5+589073089024a^2c^5-61326539008a^3c^5-482101760a^4c^5+45516800a^5c^5-272384a^6c^5-$$
$$-1792a^7c^5-10133908029440c^6-1655765594112ac^6-7453361664a^2c^6+3226325760a^3c^6-17606400a^4c^6-$$
$$-854784a^5c^6+5376a^6c^6+822856065024c^7+86073947136ac^7-952220160a^2c^7-90969600a^3c^7+944640a^4c^7+$$
$$+4608a^5c^7-46299721728c^8-2875799040ac^8+57504000a^2c^8+1221120a^3c^8-11520a^4c^8+1771069440c^9+$$
$$+57728000ac^9-1320960a^2c^9-5120a^3c^9-43929600c^{10}-596992ac^{10}+11264a^2c^{10}+636928c^{11}$$
$$+2048ac^{11}-4096c^{12})+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-25}{2})}(16072626136089600+13440284546567040a+2865451725805248a^2+$$
$$+29899479296856a^3-26048131417484a^4+79808986650a^5+85137424259a^6-2401982262a^7-36821217a^8+$$
$$+2318910a^9-27607a^{10}+6a^{11}+a^{12}-24265950238448640c-14741496935911296ac-2017268942695968a^2c+$$
$$+68446835425936a^3c+14014027182600a^4c-473895443004a^5c-21308046816a^6c+1112026236a^7c-$$
$$-8236440a^8c-250180a^9c+3984a^{10}c-12a^{11}c+15219075677764608c^2+6700424924066688ac^2+$$
$$+535657463741216a^2c^2-42544045416400a^3c^2-2402722747640a^4c^2+155381269148a^5c^2+343370524a^6c^2-$$
$$-129859240a^7c^2+1932560a^8c^2-420a^9c^2-84a^{10}c^2-5339375497061376c^3-1680535240613504ac^3-$$
$$-62742622448800a^2c^3+9664699301120a^3c^3+112990802120a^4c^3-19005822248a^5c^3+240492560a^6c^3+$$
$$+4669840a^7c^3-93240a^8c^3+280a^9c^3+1181843678785536c^4+259516705395200ac^4+1441658770560a^2c^4-$$
$$-1119356174720a^3c^4+11001976160a^4c^4+1038876160a^5c^4-20614720a^6c^4+4480a^7c^4+1120a^8c^4-$$
$$-175498953625600c^5-25896839035904ac^5+463983127552a^2c^5+71818487552a^3c^5-1570741760a^4c^5-$$
$$-22414336a^5c^5+598528a^6c^5-1792a^7c^5+18057365540864c^6+1692014693376ac^6-59402198016a^2c^6-$$
$$-2493308160a^3c^6+74215680a^4c^6-16128a^5c^6-5376a^6c^6-1304623362048c^7-70987152384ac^7+$$
$$+3406487040a^2c^7+38423040a^3c^7-1543680a^4c^7+4608a^5c^7+65999751168c^8+1780938240ac^8-$$
$$-106026240a^2c^8+23040a^3c^8+11520a^4c^8-2289039360c^9-21345280ac^9+1720320a^2c^9-5120a^3c^9+51836928c^{10}-$$
$$-11264ac^{10}-11264a^2c^{10}-690176c^{11}+2048ac^{11}+4096c^{12})\Big] \tag{28}$$

$$ {}_2F_1\left[\begin{matrix} a, & 27-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$
$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-27}\{\prod_{\omega=1}^{26}(\omega-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-26}{2})}(-64764752532480000-100380043171699200a-$$
$$-39961833294277440a^2-4123736953919856a^3+256023569984728a^4+31801134144780a^5-$$
$$-1558156952170a^6-50373915228a^7+3827623254a^8-43587180a^9-1136590a^{10}+29484a^{11}-182a^{12}+$$
$$+144050006577254400c+147452461905565440ac+39195512438330976a^2c+1832112082132488a^3c-$$
$$-287018424824820a^4c-8938296035370a^5c+1036785419267a^6c-6318393822a^7c-961728573a^8c+$$
$$+22376250a^9c-61503a^{10}c-2106a^{11}c+13a^{12}c-121037881922979840c^2-86868374537887488ac^2-$$
$$-15242604318923072a^2c^2-55558110535200a^3c^2+95763174889200a^4c^2-590522711688a^5c^2-$$
$$-1917856820 88a^6c^2+4514393520a^7c^2+41787200a^8c^2-2063880a^9c^2+15288a^{10}c^2+54356751317087232c^3+$$
$$+27874136758186368ac^3+3085125461785120a^2c^3-100181778231600a^3c^3-14205835897240a^4c^3+$$
$$+417933311796a^5c^3+12524891652a^6c^3-518427000a^7c^3+2810080a^8c^3+49140a^9c^3-364a^{10}c^3-$$
$$-15048508078170112c^4-5510896488115200ac^4-348584266284800a^2c^4+24233385496320a^3c^4+$$
$$+989824006080a^4c^4-52703239680a^5c^4-42806400a^6c^4+22014720a^7c^4-203840a^8c^4+2766863106748416c^5+$$
$$+713725312135680ac^5+20286412405376a^2c^5-2682886437504a^3c^5-20797594272a^4c^5+2971987200a^5c^5-$$
$$-26784576a^6c^5-314496a^7c^5+2912a^8c^5-352289946910720c^6-62361643309056ac^6-195338078208a^2c^6+$$

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$$+166299194880a^3c^6 - 1296422400a^4c^6 - 79252992a^5c^6 + 978432a^6c^6 + 31756854910976c^7 + 3702483053568ac^7-$$
$$-54505331712a^2c^7 - 5923756800a^3c^7 + 91503360a^4c^7 + 808704a^5c^7 - 9984a^6c^7 - 2041738223616c^8-$$
$$-147221867520ac^8 + 3924211200a^2c^8 + 113218560a^3c^8 - 2096640a^4c^8 + 93007269888c^9 + 3751488000ac^9-$$
$$-126813440a^2c^9 - 898560a^3c^9 + 16640a^4c^9 - 2931568640c^{10} - 55351296ac^{10} + 2050048a^2c^{10} + 60782592c^{11}+$$
$$+359424ac^{11} - 13312a^2c^{11} - 745472c^{12} + 4096c^{13}) + \frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-27}{2})}(-30954687834470400 - 27083696955505920a-$$
$$-6507826104671232a^2 - 266037152739696a^3 + 46345379507128a^4 + 1310367681180a^5 - 161428233286a^6+$$
$$+1051866972a^7 + 147525474a^8 - 3450060a^9 + 9518a^{10} + 324a^{11} - 2a^{12} + 46830753703157760c+$$
$$+30177797417929728ac + 4904720971886848a^2c + 2028304333440a^3c - 30073515460800a^4c + 214205215392a^5c+$$
$$+590364183336a^6c - 1402591680a^7c - 12732160a^8c + 635040a^9c - 4704a^10c - 29447558550300672c^2-$$
$$-13975007786673408ac^2 - 1454937127911872a^2c^2 + 50384404663200a^3c^2 + 6588240225360a^4c^2-$$
$$-197467198488a^5c^2 - 5739479928a^6c^2 + 239757840a^7c^2 - 1301440a^8c^2 - 22680a^9c^2 + 168a^{10}c^2+$$
$$+10362233929793536c^3 + 3584472290058240ac^3 + 215080588441600a^2c^3 - 15425517864960a^3c^3-$$
$$-605115409920a^4c^3 + 32676618240a^5c^3 + 23331840a^6c^3 - 13547520a^7c^3 + 125440a^8c^3 - 2301160681971712c^4-$$
$$-569028657484800ac^4 - 15308983466240a^2c^4 + 2095673368320a^3c^4 + 15505116480a^4c^4 - 2290498560a^5c^4+$$
$$+20657280a^6c^4 + 241920a^7c^4 - 2240a^8c^4 + 342894825799680c^5 + 58835112763392ac^5 + 146424004608a^2c^5-$$
$$-154482370560a^3c^5 + 1214760960a^4c^5 + 73156608a^5c^5 - 903168a^6c^5 - 35405951328256c^6 - 4035795757056ac^6+$$
$$+60337815552a^2c^6 + 6391042560a^3c^6 - 98757120a^4c^6 - 870912a^5c^6 + 10752a^6c^6 + 2567120683008c^7+$$
$$+182218014720ac^7 - 4867645440a^2c^7 - 139345920a^3c^7 + 2580480a^4c^7 - 130323050496c^8 - 5203491840ac^8+$$
$$+175925760a^2c^8 + 1244160a^3c^8 - 23040a^4c^8 + 4535336960c^9 + 85155840ac^9 - 3153920a^2c^9 - 103043072c^{10}-$$
$$-608256ac^{10} + 22528a^2c^{10} + 1376256c^{11} - 8192c^{12})\Big] \qquad (29)$$

$${}_2F_1\left[\begin{matrix} a, & 28-a & ; & \frac{1}{2} \\ c & & ; & 2 \end{matrix}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-28}\{\prod_{\varrho=1}^{27}(\varrho-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-28}{2})}(124903451312640000 + 198245272713580800a+$$
$$+84237341463908640a^2 + 10772713785416184a^3 - 197837366197044a^4 - 74046181005314a^5 + 1239432836775a^6+$$
$$+162122858357a^7 - 5261040477a^8 - 33773187a^9 + 3183285a^{10} - 36841a^{11} + 21a^{12} + a^{13} - 278170295082854400c-$$
$$-294899108770863360ac - 85743719979624576a^2c - 6115709513194896a^3c + 428497731401864a^4c+$$
$$+31572110867940a^5c - 1543488682402a^6c - 26614826532a^7c + 2007873630a^8c - 19898340a^9c - 268982a^{10}c+$$
$$+4788a^{11}c - 14a^{12}c + 234198965910666240c^2 + 176184125195674752ac^2 + 34997280444483552a^2c^2+$$
$$+1019255391215056a^3c^2 - 178004305027800a^4c^2 - 3137600392428a^5c^2 + 384865270776a^6c^2 - 2779008876a^7c^2-$$
$$-186739560a^8c^2 + 3261020a^9c^2 - 6552a^{10}c^2 - 84a^{11}c^2 - 105436987989000192c^3 - 57440774292713472ac^3-$$
$$-7587729681637376a^2c^3 + 30167244537600a^3c^3 + 31516664669440a^4c^3 - 267758688960a^5c^3 - 36622587456a^6c^3+$$
$$+700882560a^7c^3 + 3534720a^8c^3 - 127680a^9c^3 + 448a^{10}c^3 + 29272395221864448c^4 + 11567453274584576ac^4+$$
$$+959504446627200a^2c^4 - 30984268494080a^3c^4 - 2804671306080a^4c^4 + 71402698080a^5c^4 + 1330539840a^6c^4-$$
$$-42060480a^7c^4 + 151200a^8c^4 + 1120a^9c^4 - 5398575951060992c^5 - 1531129739443200ac^5 - 71188221133056a^2c^5+$$
$$+4429295426304a^3c^5 + 121707496512a^4c^5 - 5277242880a^5c^5 + 744576a^6c^5 + 919296a^7c^5 - 4032a^8c^5+$$

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$$+689572103331840c^6+137397211710464ac^6+2713645670400a^2c^6-317715584256a^3c^6-1414103040a^4c^6+$$
$$+177612288a^5c^6-1032192a^6c^6-5376a^7c^6-62363802566656c^7-8441027076096ac^7-5987837952a^2c^7+$$
$$+12894704640a^3c^7-76968960a^4c^7-2626560a^5c^7+15360a^6c^7+4022644322304c^8+351717355008ac^8-$$
$$-4223888640a^2c^8-291152640a^3c^8+2868480a^4c^8+11520a^5c^8-183833481216c^9-9617372160ac^9+$$
$$+185546240a^2c^9+3210240a^3c^9-28160a^4c^9+5812561920c^10+160635904ac^{10}-3446784a^2c^{10}-11264a^3c^{10}$$
$$-120881152c^{11}-1400832ac^{11}+24576a^2c^{11}+1486848c^{12}+4096ac^{12}-8192c^{13})+$$
$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-27}{2})}(-835776577757721600-7325602667888665920a-1754217999593309568a^2-$$
$$-63547438453734448a^3+1435874126626940a^4+34847676651046a^5-5714447085179a^6+56439882581a^7+$$
$$+63675453345a^8-190140027a^9+798847a^{10}+31367a^{11}-385a^{12}+a^{13}+1296685506612695040c+$$
$$+8434407685832570880ac+1372637324739749760a^2c-759650829872528a^3c-970433188681304a^4c+$$
$$+11103150302292a^5c+2258084186290a^6c-66672519396a^7c-586759950a^8c+43633100a^9c-481306a^{10}c+84a^{11}c+$$
$$+14a^{12}c-843760907368455168c^2-407726081879932800ac^2-42627997732439328a^2c^2+1851674567870416a^3c^2+$$
$$+2265860939024400a^4c^2-8357231234028a^5c^2-240471792456a^6c^2+13132553364a^7c^2-99628200a^8c^2-$$
$$-2195620a^9c^2+32424a^{10}c^2-84a^{11}c^2+310294269783343104c^3+110429360845348864ac^3+$$
$$+6630455726683136a^2c^3-576173662357760a^3c^3-22396834908160a^4c^3+1518843466560a^5c^3-733569984a^6c^3-$$
$$-930840960a^7c^3+12835200a^8c^3-2240a^9c^3-448a^{10}c^3-72831305901223936c^4-18763381574731264ac^4-$$
$$-488099338700160a^2c^4+85064353434880a^3c^4+542806813920a^4c^4-124938122400a^5c^4+1528927680a^6c^4+$$
$$+23419200a^7c^4-433440a^8c^4+1120a^9c^4+11625428332593152c^5+2111137037294592ac^5+$$
$$+364443241728a^2c^5-7095335831808a^3c^5+75517299648a^4c^5+5026549248a^5c^5-92416128a^6c^5+16128a^7c^5+$$
$$+4032a^8c^5-1307344350531584c^6-160943873108992ac^6+3096581440512a^2c^6+347557964544a^3c^6-$$
$$-7182551040a^4c^6-84306432a^5c^6+2085888a^6c^6-5376a^7c^6+105454131576832c^7+8307462070272ac^7-$$
$$-280139655168a^2c^7-9574394880a^3c^7+264053760a^4c^7-46080a^5c^7-15360a^6c^7-6129210697728c^8-$$
$$-282331740672ac^8+12658510080a^2c^8+120433920a^3c^8-4481280a^4c^8+11520a^5c^8+254489174016c^9+$$
$$+5851028480ac^9-322741760a^2c^9+56320a^3c^9+28160a^4c^9-7360821248c^{10}-58876928ac^{10}+4392960a^2c^{10}$$
$$-11264a^3c^{10}+140836864c^{11}-24576ac^{11}-24576a^2c^{11}-1601536c^{12}+4096ac^{12}+8192c^{13})\Big] \tag{30}$$

$$ {}_2F_1\left[\begin{matrix} a, & 29-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]= $$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-29}\{\prod_{\varphi=1}^{28}(\varphi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-28}{2})}(34972966367539200000+5555901589554355200a+$$
$$+23611336129967904000a^2+2968375730620124160a^3-80455098221845200a^4-23094951841018120a^5$$
$$+522309882964940a^6+5767788455233a^7-229366815305a^8-1161910491a^9+180370407a^{10}-2797949a^{11}+$$
$$+3325a^{12}+203a^{13}-a^{14}-79187056672066560000c-84686710811339750400ac-24797480979079065600a^2c-$$
$$-171617279580731040a^3c+147288420458676000a^4c+1017261537926760a^5c-607065660100200a^6c-$$
$$-943838942760a^7c+96558281820a^8c-1243943400a^9c-18619020a^{10}c+511560a^{11}c-2940a^{12}c+$$
$$+68465040514103808000c^2+52360251704785543680ac^2+10547634313792235520a^2c^2+27669690834216368a^3c^2-$$
$$-62827701012409200a^4c^2-970543104268920a^5c^2+16257739742734a^6c^2-161294509908a^7c^2-10458065394a^8c^2+$$

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$$+246088780a^9c^2 - 790566a^{10}c^2 - 17052a^{11}c^2 + 98a^12c^2 - 31949201758873190 40c^3 - 1785014716346695680ac^3 -$$

$$-240816902070528000a^2c^3 + 2045465086963200a^3c^3 + 1188161185612800a^4c^3 - 14134893897600a^5c^3 -$$

$$-1735016492160a^6c^3 + 42316243200a^7c^3 + 228614400a^8c^3 - 13641600a^9c^3 + 94080a^{10}c^3 + 928567869196713984c^4 +$$

$$+380255719502460928ac^4 + 32418584463820800a^2c^4 - 1288125293230720a^3c^4 - 115836728362880a^4c^4 +$$

$$+3658531836960a^5c^4 + 72650029536a^6c^4 - 3160758720a^7c^4 + 17357760a^8c^4 + 227360a^9c^4 - 1568a^{10}c^4 -$$

$$-181307281284218880c^5 - 53972606059223040ac^5 - 2584272171962880a^2c^5 + 195478551313920a^3c^5 +$$

$$+5608442125440a^4c^5 - 313713146880a^5c^5 + 282804480a^6c^5 + 98219520a^7c^5 - 846720a^8c^5 + 24848775724408832c^6 +$$

$$+5280766105517056ac^6 + 104851642776832a^2c^6 - 15801579295488a^3c^6 - 64686839616a^4c^6 + 13307653632a^5c^6 -$$

$$-116038272a^6c^6 - 1091328a^7c^6 + 9408a^8c^6 - 2451416728535040c^7 - 361346358804480ac^7 + 301765201920a^2c^7 +$$

$$+759844915200a^3c^7 - 6318950400a^4c^7 - 280627200a^5c^7 + 3225600a^6c^7 + 176187932065792c^8 +$$

$$+17257259464704ac^8 - 269957270016a^2c^8 - 21766536960a^3c^8 + 318769920a^4c^8 + 2338560a^5c^8 - 26880a^6c^8 -$$

$$-9232454246400c^9 - 563530598400ac^9 + 14458752000a^2c^9 + 342988800a^3c^9 - 5913600a^4c^9 + 348808728576c^{10} +$$

$$+11988601856ac^{10} - 380244480a^2c^{10} - 2286592a^3c^{10} + 39424a^4c^{10} - 9248440320c^{11} - 149667840ac^{11}$$

$$+5160960a^2c^{11} + 163201024c^{12} + 831488ac^{12} - 28672a^2c^{12} - 1720320c^{13} + 8192c^{14}) +$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-29}{2})}(1613913397544448000 + 1472574330511280640a + 389021279417326080a^2 +$$

$$+24196510822078560a^3 - 2274657906963120a^4 - 143258950036440a^5 + 8920185210300a^6 + 128774925720a^7 -$$

$$-13826578500a^8 + 179655000a^9 + 2646420a^{10} - 73080a^{11} + 420a^{12} - 2508255032493772800c -$$

$$-1717842284795966976ac - 319975449572777472a^2c - 7155985266367904a^3c + 1861324669488208a^4c +$$

$$+25627804125096a^5c - 4686483308420a^6c + 47891228952a^7c + 2978091900a^8c - 70441000a^9c + 226772a^{10}c +$$

$$+4872a^{11}c - 28a^{12}c + 1635663709850664960c^2 + 843196968357212160ac^2 + 106898081437578240a^2c^2 -$$

$$-1158164266406400a^3c^2 - 517205009606400a^4c^2 + 6506815190400a^5c^2 + 743164813440a^6c^2 - 18275846400a^7c^2 -$$

$$-96768000a^8c^2 + 5846400a^9c^2 - 40320a^{10}c^2 - 603022201203621888c^3 - 232515149352194048ac^3 -$$

$$-18822315352579072a^2c^3 + 781393137041920a^3c^3 + 66374684913920a^4c^3 - 2128630197120a^5c^3 -$$

$$-411825496 32a^6c^3 + 1809265920a^7c^3 - 9945600a^8c^3 - 129920a^9c^3 + 896a^{10}c^3 + 141928904910028800c^4 +$$

$$+40368743871436800ac^4 + 1844139367411200a^2c^4 - 143432935027200a^3c^4 - 3974328892800a^4c^4 +$$

$$+225483955200a^5c^4 - 218131200a^6c^4 - 70156800a^7c^4 + 604800a^8c^4 - 22721029659049984c^5 -$$

$$-4664860707047424ac^5 - 87699486460416a^2c^5 + 13718425437696a^3c^5 + 52894283904a^4c^5 - 11425268736a^5c^5 +$$

$$+99676416a^6c^5 + 935424a^7c^5 - 8064a^8c^5 + 2562816998277120c^6 + 368118604062720ac^6 - 475610849280a^2c^6 -$$

$$-763960780800a^3c^6 + 6389913600a^4c^6 + 280627200a^5c^6 - 3225600a^6c^6 - 207355683897344c^7 - 19928195211264ac^7 +$$

$$+315044315136a^2c^7 + 24915240960a^3c^7 - 364984320a^4c^7 - 2672640a^5c^7 + 30720a^6c^7 + 12088613560320c^8 +$$

$$+728067225600ac^8 - 18711475200a^2c^8 - 440985600a^3c^8 + 7603200a^4c^8 - 503429701632c^9 - 17152706560ac^9 +$$

$$+544107520a^2c^9 + 3266560a^3c^9 - 56320a^4c^9 + 14603550720c^{10} + 235192320ac^{10} - 8110080a^2c^{10} - 280199168c^{11} -$$

$$-1425408ac^{11} + 49152a^2c^{11} + 3194880c^{12} - 16384c^{13})\Big] \qquad (31)$$

$$
{}_2F_1\left[\begin{matrix} a, & 30-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] =
$$

$$
= \frac{\sqrt{\pi}\;\Gamma(c)}{2^{c-30}\{\prod_{\varkappa=1}^{29}(\varkappa-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\;\Gamma(\frac{c+a-30}{2})}(-6761440164390912000 - 10975820682116828160a-
$$

$$
-4935233255279884416a^2 - 729789149421755136a^3 - 42201978141149688a^4 + 4751293608022388a^5+
$$

$$
+20580722419698a^6 - 12993499610273a^7 + 207448509161a^8 + 9479314299a^9 - 309595783a^{10} + 1647709a^{11}+
$$

$$
+35427a^{12} - 427a^{13} + a^{14} + 15326298997068810240c + 16912364017713907968ac + 5341563944575762176a^2c+
$$

$$
+486193174327051888a^3c - 17721927335789288a^4c - 2655392142674852a^5c + 68415639214898a^6c+
$$

$$
+4076609731294a^7c - 151651399494a^8c - 156424450a^9c + 60914966a^{10}c - 708022a^{11}c + 742a^{12}c + 14a^{13}c-
$$

$$
-13273287874275557376c^2 - 10581609146505394176ac^2 - 2359216299514810368a^2c^2 - 113984108675034240a^3c^2+
$$

$$
+10464620334250304a^4c^2 + 462806131944480a^5c^2 - 27019467604880a^6c^2 - 213739449504a^7c^2 + 24126311664a^8c^2-
$$

$$
-258857760a^9c^2 - 2150960a^{10}c^2 + 40992a^{11}c^2 - 112a^{12}c^2 + 6206845674605838336c^3 + 3655758996318896128ac^3+
$$

$$
+566360717498711040a^2c^3 + 8471349113362176a^3c^3 - 2354033956865280a^4c^3 - 18437749885120a^5c^3+
$$

$$
+3762081665664a^6c^3 - 35808999744a^7c^3 - 1303626240a^8c^3 + 23047360a^9c^3 - 51072a^{10}c^3 - 448a^{11}c^3-
$$

$$
-1808232623781232640c^4 - 790682156498434048ac^4 - 82059994215443456a^2c^4 + 1023013859383680a^3c^4+
$$

$$
+271425411296640a^4c^4 - 3312431939040a^5c^4 - 235347504672a^6c^4 + 4630785600a^7c^4 + 13513920a^8c^4-
$$

$$
-614880a^9c^4 + 2016a^{10}c^4 + 353978861595516928c^5 + 114221183100068864ac^5 + 7421904568692480a^2c^5-
$$

$$
-278541991695360a^3c^5 - 17101748153280a^4c^5 + 460523722176a^5c^5 + 5856950400a^6c^5 - 195941760a^7c^5+
$$

$$
+705600a^8c^5 + 4032a^9c^5 - 48646217148710912c^6 - 11412178770864128ac^6 - 406527582805504a^2c^6+
$$

$$
+27328469113344a^3c^6 + 538828983168a^4c^6 - 24621112320a^5c^6 + 27928320a^6c^6 + 3279360a^7c^6 - 13440a^8c^6+
$$

$$
+4812572103507968c^7 + 801309941284864ac^7 + 11028903438336a^2c^7 - 1492276712448a^3c^7 - 3166279680a^4c^7+
$$

$$
+645550080a^5c^7 - 3624960a^6c^7 - 15360a^7c^7 - 346867926056960c^8 - 39565589673984ac^8 + 77974516224a^2c^8+
$$

$$
+48272843520a^3c^8 - 301551360a^4c^8 - 7729920a^5c^8 + 42240a^6c^8 + 18227487657984c^9 + 1352747703296ac^9-
$$

$$
-16971947520a^2c^9 - 893770240a^3c^9 + 8363520a^4c^9 + 28160a^5c^9 - 690554748928c^{10} - 30856196096ac^{10}+
$$

$$
+573089792a^2c^{10} + 8245248a^3c^{10} - 67584a^4c^{10} + 18358992896c^{11} + 435724288ac^{11} - 8798208a^2c^{11}-
$$

$$
-24576a^3c^{11} - 324812800c^{12} - 3248128ac^{12} + 53248a^2c^{12} + 3432448c^{13} + 8192ac^{13} - 16384c^{14})+
$$

$$
+\frac{1}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a-29}{2})}(46803488615967283200 + 42767159087365735680a + 11270601192070601856a^2+
$$

$$
+661015913631944304a^3 - 765849103840465 12a^4 - 4417245600019672a^5 + 336434811648432a^6+
$$

$$
+3976287855967a^7 - 608379703391a^8 + 9950104899a^9 + 147972013a^{10} - 5745971a^{11} + 49203a^{12} - 7a^{13} - a^{14}-
$$

$$
-74415813125868103680c - 51370802672923567872ac - 9590167310349136896a^2c - 174040376046710672a^3c+
$$

$$
+63985787842864088a^4c + 678065157135388a^5c - 186286806337838a^6c + 2516062112734a^7c + 145725587994a^8c-
$$

$$
-4413443650a^9c + 20549494a^{10}c + 541898a^{11}c - 6202a^{12}c + 14a^{13}c + 50034321004680953856c^2+
$$

$$
+26188677313214367744ac^2 + 3340987169121850368a^2c^2 - 556473561270777760a^3c^2 - 18708842259562304a^4c^2+
$$

$$
+314035097808480a^5c^2 + 32131358547920a^6c^2 - 977872074144a^7c^2 - 4833796464a^8c^2 + 459678240a^9c^2-
$$

$$
-4723600a^{10}c^2 + 672a^{11}c^2 + 112a^{12}c^2 - 19178889908274610176c^3 - 7572855930087606272ac^3-
$$

$$
-617930250420794880a^2c^3 + 31207762807608576a^3c^3 + 2572172951249280a^4c^3 - 100552461707200a^5c^3-
$$

Notes

Notes

$$-1945358951424a^6c^3+112831486656a^7c^3-854904960a^8c^3-14450240a^9c^3+198912a^{10}c^3-448a^{11}c^3+$$
$$+4737652999555604480c^4+1394083224658414592ac^4+637702340546562 56a^2c^4-5994963943259520a^3c^4-$$
$$-165690819368640a^4c^4+11870436900000a^5c^4-26042015328a^6c^4-5516145600a^7c^4+70855680a^8c^4-$$
$$-10080a^9c^4-2016a^{10}c^4-804781701766094848c^5-173082746257294336ac^5-3095138947918080a^2c^5+$$
$$+628850081495040a^3c^5+1766625026880a^4c^5-710151349824a^5c^5+8365123200a^6c^5+104039040a^7c^5-$$
$$-1794240a^8c^5+4032a^9c^5+97597927269220352c^6+14916118888978432ac^6-53460036102656a^2c^6-$$
$$-39840066986496a^3c^6+440358344832a^4c^6+22064609280a^5c^6-377905920a^6c^6+53760a^7c^6+13440a^8c^6-$$
$$-8630073563660288c^7-900484605464576ac^7+17956861353984a^2c^7+1544786045952a^3c^7-30214072320a^4c^7-$$
$$-297246720a^5c^7+6850560a^6c^7-15360a^7c^7+561592474234880c^8+37734427923456ac^8-1220290689024a^2c^8-$$
$$-34672999680a^3c^8+890799360a^4c^8-126720a^5c^8-42240a^6c^8-26862108708864c^9-1061440635904ac^9+$$
$$+44645061120a^2c^9+363292160a^3c^9-12587520a^4c^9+28160a^5c^9+932581163008c^{10}+18492289024ac^{10}-$$
$$-950208512a^2c^{10}+135168a^3c^{10}+67584a^4c^{10}-22848782336c^{11}-158523392ac^{11}+11010048a^2c^{11}-$$
$$-24576a^3c^{11}+374333440c^{12}-53248ac^{12}-53248a^2c^{12}-3678208c^{13}+8192ac^{13}+16384c^{14})\Big] \qquad (32)$$

$${}_2F_1\left[\begin{matrix} a, & 31-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-31}\{\prod_{\vartheta=1}^{30}(\vartheta-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-30}{2})}(-202843204931727360000-329526641070543974400a-$$
$$-148192702618125404160a^2-21658122462897527040a^3+9722889879655680a^4+1590985596583891 20a^5-$$
$$-967254435840a^6-490253327976720a^7+9909256423440a^8+414792045360a^9-17018472240a^{10}+$$
$$+122231760a^{11}+2840880a^{12}-52080a^{13}+240a^{14}+466802430683494348800c+519033146796495256320ac+$$
$$+164949222985279959168a^2c+14769765039630034320a^3c-680706007712528720a^4c-9175027013859300 0a^5c+$$
$$+2935763232708224a^6c+161360534973585a^7c-7364342102145a^8c+2500344525a^9c+3911185523a^{10}c-$$
$$-60969405a^{11}c+109165a^{12}c+3255a^{13}c-15a^{14}c-414075635848567848960c^2-334813261843713392640ac^2-$$
$$-75572573936717905920a^2c^2-3508604162375731200a^3c^2+392743927281884160a^4c^2+$$
$$+16485242844787200a^5c^2-1178569050412800a^6c^2-7610697884160a^7c^2+1309085487360a^8c^2-$$
$$-18023846400a^9c^2-167865600a^{10}c^2+4999680a^{11}c^2-26880a^{12}c^2+199928658130806030336c^3+$$
$$+120313411708083025920ac^3+18991014146939294720a^2c^3+228969490870081920a^3c^3-$$
$$-92456288210742080a^4c^3-536544028552800a^5c^3+178570308069840a^6c^3-2197559659680a^7c^3-$$
$$-82666918320a^8c^3+1975394400a^9c^3-6824720a^{10}c^3-104160a^{11}c^3+560a^{12}c^3-60648845538056601600c^4-$$
$$-27329375418592542720ac^4-2908495070746890240a^2c^4+49850050806835200a^3c^4+$$
$$+11474023776537600a^4c^4-180112947033600a^5c^4-12591968417280a^6c^4+315229824000a^7c^4+945100800a^8c^4-$$
$$-74995200a^9c^4+483840a^10c^4+12479370238337146880c^5+4192758239972766720ac^5+$$
$$+280853229778835712a^2c^5-12726157894300800a^3c^5-795295816175040a^4c^5+26431624205280a^5c^5+$$
$$+360301027296a^6c^5-16739553600a^7c^5+91405440a^8c^5+937440a^9c^5-6048a^{10}c^5-1822444137279651840c^6-$$
$$-450829569304657920ac^6-16543535489064960a^2c^6+1351753242255360a^3c^6+27946244229120a^4c^6-$$
$$-1656693964800a^5c^6+3348172800a^6c^6+399974400a^7c^6-3225600a^8c^6+194120876679053312c^7+$$
$$+34630926348380160ac^7+471615845905920a^2c^7-83776087272960a^3c^7-132129333120a^4c^7+$$
$$+55017907200a^5c^7-462432000a^6c^7-3571200a^7c^7+28800a^8c^7-15312178746163200c^8-1910620958146560ac^8+$$
$$+69595191705560a^2c^8+3225307852800a^3c^8-27665510400a^4c^8-942796800a^5c^8+10137600a^6c^8+$$
$$+899363697520640c^9+75101815557120ac^9-1211625451520a^2c^9-76032633600a^3c^9+1057196800a^4c^9+$$
$$+6547200a^5c^9-70400a^6c^9-39186998231040c^{10}-2051861053440ac^{10}+50601492480a^2c^{10}+$$
$$+1005649920a^3c^{10}-16220160a^4c^{10}+1248504659968c^{11}+37012869120ac^{11}-1105397760a^2c^{11}-5713920a^3c^{11}+$$
$$+92160a^4c^{11}-28242739200c^{12}-396165120ac^{12}+12779520a^2c^{12}+429383680c^{13}+1904640ac^{13}-$$

$$-61440a^2c^{13}-3932160c^{14}+16384c^{15})+\frac{1}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a-31}{2})}(-905873974055930880000-858098697079639680000a-$$

$$-245628747686817446400a^2-19981232693973593280a^3+1031943630415320000a^4+12301377002464816a^5-$$

$$-415292450573728a^6-21219497625214a^7+992922344830a^8-596890182a^9-520580634a^{10}+8143142a^{11}-$$

$$-14630a^{12}-434a^13+2a^{14}+1442405611082612736000c+1042219077691539456000ac+$$

$$+217171110916048158720a^2c+907956238606835712a^3c-110873520753201152a^4c-4298678159370240a^5c+$$

$$+320998107247616a^6c+1861076883456a^7c-349590936576a^8c+4846356480a^9c+44506112a^{10}c-1333248a^{11}c+$$

$$+7168a^{12}c-971584881307213824000c^2-538130363571345530880ac^2-79721913322736455680a^2c^2-$$

$$+901353268032a^7c^2+32944812768a^8c^2-791407680a^9c^2+2737952a^{10}c^2+41664a^{11}c^2-224a^{12}c^2+$$

$$+373208607416301649920c^3+157921778859469701120ac^3+15952606912939622400a^2c^3-$$

$$-302154674653593600a^3c^3-6193795836764160a^4c^3+100485128540160a^5c^3+6706712334336a^6c^3-$$

$$-169189171200a^7c^3-495452160a^8c^3+39997440a^9c^3-258048a^{10}c^3-92406418686546247680c^4-$$

$$-29580464888567951360ac^4-1894159565158405120a^2c^4+88763379364646400a^3c^4+530796049376640a^4c^4-$$

$$-178779430300800a^5c^4-238041486144a^6c^4+11176368000a^7c^4-61071360a^8c^4-624960a^9c^4+4032a^{10}c^4+$$

$$+15736059110851870720c^5+3749986096309862400ac^5+131811624217804800a^2c^5-1105518682275840a^3c^5-$$

$$-221498812416000a^4c^5+1332394721280a^5c^5-2754232320a^6c^5-319979520a^7c^5+2580480a^8c^5-$$

$$-1913293644536545280c^6-331656186879139840ac^6-4265364778956800a^2c^6+79042932261888a^3c^6+$$

$$+111474429696a^4c^6-51423375360a^5c^6+432391680a^6c^6+3333120a^7c^6-26880a^8c^6+169629668668866560c^7+$$

$$+20708673005813760ac^7-8234460512256a^2c^7-3456418775040a^3c^7+29769400320a^4c^7+1005649920a^5c^7-$$

$$-10813440a^6c^7-11067772358410240c^8-909367886254080ac^8+1478279835648a^2c^8+91364866560a^3c^8-$$

$$-1270663680a^4c^8-7856640a^5c^8+84480a^6c^8+530788342824960c^9+27474355814400ac^9-67843522560a^2c^9-$$

$$-1340866560a^3c^9+21626880a^4c^9-18475205591040c^{10}-543581716480ac^{10}+1623592960a^2c^{10}+8380416a^3c^{10}-$$

$$-135168a^4c^{10}+453792235520c^{11}+6338641920ac^{11}-20447232a^2c^{11}-7452590080c^{12}-33013760ac^{12}+$$

$$+106496a^2c^{12}+73400320c^{13}-327680c^{14})\Big] \tag{33}$$

$${}_2F_1\left[\begin{matrix} a, & 32-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=$$

$$=\frac{\sqrt{\pi}\ \Gamma(c)}{2^{c-32}\{\prod\limits_{\varpi=1}^{31}(\varpi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a-32}{2})}(393008709555221760000+651194829215986003200a+$$

$$+307720612018593866880a^2+51153440461235528688a^3+1330940239653764544a^4-296857314253591112a^5-$$

$$-7921227385233624a^6+908822265956143a^7+402064837152a^8-1099599340796a^9+20437153968a^{10}+$$

$$+154052570a^{11}-7588896a^{12}+63308a^{13}-24a^{14}-a^{15}-905281327552108953600c-1035422190413513441280ac-$$

$$-351184084010029910016a^2c-38338797951234352128a^3c+451749423064008064a^4c+206106543083095360a^5c-$$

$$-1447679061168672a^6c-415046253507664a^7c+8508403087952a^8c+199440969840a^9c-7487180848a^{10}c+$$

$$+47316752a^{11}c+584304a^{12}c-7280a^{13}c+16a^{14}c+804169092960729907200c^2+674782022139214399488ac^2+$$

$$+165841199803790742528a^2c^2+10946190747818876544a^3c^2-567565724112283456a^4c^2-$$

$$-485179956474779844a^5c^2+1590023100897232a^6c^2+53334001088048a^7c^2-2274073929456a^8c^2+$$

$$+4737986736a^9c^2+642792304a^{10}c^2-7496944a^{11}c^2+9968a^{12}c^2+112a^{13}c^2-388961945977802194944c^3-$$

Notes

$-245240098536550637568ac^3 - 43311426401359084544a^2c^3 - 1395255045945941760a^3c^3+$
$+165983593526283904a^4c^3 + 4591339102100160a^5c^3 - 322284615941984a^6c^3 - 765397859520a^7c^3+$
$+208379147424a^8c^3 - 2341805760a^9c^3 - 12518688a^{10}c^3 + 262080a^{11}c^3 - 672a^{12}c^3 + 118230509217761009664c^4+$
$+56420416163828755456ac^4 + 6990463455382573824a^2c^4 + 337094733942383336a^3c^4 - 23923420575560640a^4c^4-$
$-35063040521760a^5c^4 + 291853283998O8a^6c^4 - 321523472928a^7c^4 - 7433455680a^8c^4 + 133744800a^9c^4-$
$-310464a^{10}c^4 - 2016a^{11}c^4 - 24381290438326222848c^5 - 8782568098611445760ac^5 - 730717283214645248a^2c^5+$
$+13675686801784320a^3c^5 + 1956221896719360a^4c^5 - 28876551392640a^5c^5 - 1297457659008a^6c^5+$
$+26112871680a^7c^5 + 40024320a^8c^5 - 2620800a^9c^5 + 8064a^{10}c^5 + 3568887719037157376c^6+$
$+960485441528322048ac^6 + 49667323801539072a^2c^6 - 2072805728685056a^3c^6 - 91894140143232a^4c^6+$
$+2584449716352a^5c^6 + 22947859200a^6c^6 - 826855680a^7c^6 + 2943360a^8c^6 + 13440a^9c^6 - 381067609873448960c^7-$
$-75290787841798144ac^7 - 2072992543587328a^2c^7 + 150201995412480a^3c^7 + 2162904361728a^4c^7-$
$-105258362880a^5c^7 + 189150720a^6c^7 + 10982400a^7c^7 - 42240a^8c^7 + 30132576016863232c^8 + 4259476607648768ac^8+$
$+39713460593664a^2c^8 - 6478199887104a^3c^8 - 3138938880a^4c^8 + 2216670720a^5c^8 - 11996160a^6c^8 - 42240a^7c^8-$
$-1774218876289024c^9 - 172982726737920ac^9 + 673615990784a^2c^9 + 171263600640a^3c^9 - 1092044800a^4c^9-$
$-21964800a^5c^9 + 112640a^6c^9 + 77495587749888c^{10} + 4944943984640ac^{10} - 63368718336a^2c^{10}-$
$-2649946112a^3c^{10} + 23586816a^4c^{10} + 67584a^5c^{10} - 2474963632128c^{11} - 95577284608ac^{11} + 1707290624a^2c^{11}+$
$+20766720a^3c^{11} - 159744a^4c^{11} + 56117960704c^{12} + 1156386816ac^{12} - 22044672a^2c^{12} - 53248a^3c^{12}-$
$-855113728c^{13} - 7454720ac^{13} + 114688a^2c^{13} + 7847936c^{14} + 16384ac^{14} - 32768c^{15})$

$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-31}{2})}(-280820932088106009600 - 266336026000972663680Oa - 76101293036098483776Oa^2-$
$-5978998104140773684 8a^3 + 383809792567525766 4a^4 + 41288720995653829 6a^5 - 1710218600467053 6a^6-$
$-788560364968145a^7 + 45162707702112a^8 - 113794887932a^9 - 29121053808a^{10} + 578885594a^{11} - 1601376a^{12}-$
$-54964a^{13} + 504a^{14} - a^{15} + 456529909516369920000c + 332117975280352223232 0ac+$
$+69385241612160119808 0a^2c + 26869933301958220800a^3c - 4073155305339587968a^4c-$
$-144960223788782272a^5c + 13373713249687584a^6c + 48530135178160a^7c - 17254272251984a^8c+$
$+297414505584a^9c + 2773686832a^{10}c - 118937840a^{11}c + 953232a^{12}c - 112a^{13}c - 16a^{14}c-$
$-316108062904637693952 0c^2 - 177374361185343920332 8ac^2 - 264570033924753638400a^2c^2-$
$-14050588810090417 92a^3c^2 + 1448652525436875584a^4c^2 + 2719092197913376a^5c^2 - 319399998229755 2a^6c^2+$
$+50491558757936a^7c^2 + 1813029642480a^8c^2 - 55967482704a^9c^2 + 272830096a^{10}c^2 + 5276432a^{11}c^2 - 56560a^{12}c^2+$
$+112a^{13}c^2 + 125721181536828574924 8c^3 + 542821659317684920320ac^3 + 55394584008959679488a^2c^3-$
$-1371732146004729600a^3c^3 - 249881887807080064a^4c^3 + 5098386147168960a^5c^3 + 325475917621088a^6c^3-$
$-10145058512064a^7c^3 - 25272802464a^8c^3 + 3568138560a^9c^3 - 34317024a^{10}c^3 + 4032a^{11}c^3 + 672a^{12}c^3-$
$-324881117647402082304c^4 - 107015854008936485888ac^4 - 6926032383442477824a^2c^4+$
$+388761985674363776a^3c^4 + 23023713558513600a^4c^4 - 940801602446880a^5c^4 - 12549395769792a^6c^4+$
$+787489416672a^7c^4 - 5878918080a^8c^4 - 79144800a^9c^4 + 1020096a^10c^4 - 2016a^11c^4 + 58269865681132978176c^5+$
$+14434175239659732992ac^5 + 507569977317398528a^2c^5 - 51503477691302400a^3c^5 - 1031098175523840a^4c^5+$
$+78913013289600a^5c^5 - 261319992192a^6c^5 - 28545135360a^7c^5 + 343176960a^8c^5 - 40320a^9c^5 - 8064a^{10}c^5-$
$-75424835401035120 64c^6 - 1375984874765199360ac^6 - 163694099280737 28a^2c^6 + 4065809536857088a^3c^6+$

$$+1341327292032a^{4}c^{6}-3613476171648a^{5}c^{6}+40882033920a^{6}c^{6}+422096640a^{7}c^{6}-6814080a^{8}c^{6}+13440a^{9}c^{6}+$$
$$+7212051168540753 92c^{7}+94113516697784320ac^{7}-588042790421504a^{2}c^{7}-203145224764416a^{3}c^{7}+$$
$$+2278013244672a^{4}c^{7}+89713367040a^{5}c^{7}-1438103040a^{6}c^{7}+168960a^{7}c^{7}+42240a^{8}c^{7}-5158033410644787 2c^{8}-$$
$$-4622210372353024ac^{8}+93568112901120a^{2}c^{8}+6406705933056a^{3}c^{8}-118823147520a^{4}c^{8}-994920960a^{5}c^{8}+$$
$$+21457920a^{6}c^{8}-42240a^{7}c^{8}+2769079768711168c^{9}+160632598511616ac^{9}-4979646746624a^{2}c^{9}-$$
$$-119617935360a^{3}c^{9}+2876262400a^{4}c^{9}-337920a^{5}c^{9}-112640a^{6}c^{9}-111011105390592c^{10}-3805382496256ac^{10}+$$
$$+150700222464a^{2}c^{10}+1061226496a^{3}c^{10}-34400256a^{4}c^{10}+67584a^{5}c^{10}+3272190984192c^{11}+56546713600ac^{11}-$$
$$-2719428608a^{2}c^{11}+319488a^{3}c^{11}+159744a^{4}c^{11}-68803764224c^{12}-418050048ac^{12}+27156480a^{2}c^{12}$$
$$-53248a^{3}c^{12}+976224256c^{13}-114688ac^{13}-114688a^{2}c^{13}-8372224c^{14}+16384ac^{14}+32768c^{15})\Big] \qquad (34)$$

$$_2F_1\left[\begin{array}{ccc} a, & 33-a & ; \\ c & & ; \end{array} \frac{1}{2}\right]=$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-33}\{\prod_{\tau=1}^{32}(\tau-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-32}{2})}(12576278705767096320000+2085186042455697054 7200a+$$
$$+985494740758517605 2480a^{2}+162443543638901357 6064a^{3}+34691223091312022544a^{4}-$$
$$-10657758864295730880a^{5}-242490144783523000a^{6}+36347810238286008a^{7}-92082876669487a^{8}-$$
$$-50233536212640a^{9}+1138218046460a^{10}+7765026192a^{11}-552484058a^{12}+6172320a^{13}-4940a^{14}-264a^{15}+$$
$$+a^{16}-29375637080868126720000c-33822953521504663633920ac-11534741688339332628480a^{2}c-$$
$$-1247318164867203416064a^{3}c+228201787399345500 16a^{4}c+75746695925943060 48a^{5}c-$$
$$-84914980281410048a^{6}c-17316950021682432a^{7}c+433392396752128a^{8}c+9606215587584a^{9}c-$$
$$-460516397824a^{10}c+3803095296a^{11}c+52672256a^{12}c-1005312a^{13}c+4352a^{14}c+26669053078361541181440c^{2}+$$
$$+22655654678828527386624ac^{2}+5628843122258229460992a^{2}c^{2}+364720598945819922432a^{3}c^{2}-$$
$$-229574290201583503 36a^{4}c^{2}-1849106009143236096a^{5}c^{2}+73265900144129792a^{6}c^{2}+2327692620840576a^{7}c^{2}-$$
$$-122805604797568a^{8}c^{2}+434231597184a^{9}c^{2}+46423237504a^{10}c^{2}-727106688a^{11}c^{2}+1558144a^{12}c^{2}+29568a^{13}c^{2}-$$
$$-128a^{14}c^{2}-13276554097840248324096c^{3}-8527557701096075034624ac^{3}-1531141541685441052672a^{2}c^{3}-$$
$$-46499966481831751680a^{3}c^{3}+6834918833916766208a^{4}c^{3}+178742102347607040a^{5}c^{3}-$$
$$-15578800189494784a^{6}c^{3}-144831399229 44a^{7}c^{3}+12676556057088a^{8}c^{3}-181619665920a^{9}c^{3}-1088844288a^{10}c^{3}+$$
$$+36191232a^{11}c^{3}-182784a^{12}c^{3}+4183912176750576009216c^{4}+2048627772744257667072ac^{4}+$$
$$+259599284321332817920a^{2}c^{4}+4766929592455895 04a^{3}c^{4}-1042172568281602560a^{4}c^{4}+591530393836800a^{5}c^{4}+$$
$$+1548005345752448a^{6}c^{4}-21488964279552a^{7}c^{4}-529593246336a^{8}c^{4}+12925059840a^{9}c^{4}-46134144a^{10}c^{4}-$$
$$-532224a^{11}c^{4}+2688a^{12}c^{4}-901671972868968153088c^{5}-336106259258033307648ac^{5}-$$
$$-28768633067902959616a^{2}c^{5}+678076651803893760a^{3}c^{5}+92218372409180160a^{4}c^{5}-1692557156505600a^{5}c^{5}-$$
$$-77849036998656a^{6}c^{5}+1993171783680a^{7}c^{5}+2814873600a^{8}c^{5}-361912320a^{9}c^{5}+2193408a^{10}c^{5}+$$
$$+139192132696440569856c^{6}+39163709061769003008ac^{6}+2092243353303842816a^{2}c^{6}-$$
$$-104389799581224960a^{3}c^{6}-4779754566062080a^{4}c^{6}+165520520782848a^{5}c^{6}+1574992505856a^{6}c^{6}-$$
$$-79698769920a^{7}c^{6}+428144640a^{8}c^{6}+3548160a^{9}c^{6}-21504a^{1}0c^{6}-15842385815912054784c^{7}-$$
$$-3313937807940452352ac^{7}-93891013177229312a^{2}c^{7}+8286819500703744a^{3}c^{7}+125191632039936a^{4}c^{7}-$$
$$-7962071040000a^{5}c^{7}+22036439040a^{6}c^{7}+1516584960a^{7}c^{7}-11489280a^{8}c^{7}+1352753717551824896c^{8}+$$

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$$+205696018035695616ac^{8}+1846128096868352a^{2}c^{8}-407844026320896a^{3}c^{8}+81687783936a^{4}c^{8}+$$
$$+213236305920a^{5}c^{8}-1724574720a^{6}c^{8}-11151360a^{7}c^{8}+84480a^{8}c^{8}-87414807455596544c^{9}-$$
$$-9359636030029824ac^{9}+52940629147648a^{2}c^{9}+12879850536960a^{3}c^{9}-111737077760a^{4}c^{9}-3033169920a^{5}c^{9}+$$
$$+30638080a^{6}c^{9}+4279210143121408c^{10}+308423755825152ac^{10}-5039751135232a^{2}c^{10}-254518640640a^{3}c^{10}+$$
$$+3365683200a^{4}c^{10}+17842176a^{5}c^{10}-180224a^{6}c^{10}-157590197633024c^{11}-7158795730944ac^{11}+$$
$$+169615753216a^{2}c^{11}+2867724288a^{3}c^{11}-43450368a^{4}c^{11}+4293100830720c^{12}+110927511552ac^{12}-$$
$$-3129491456a^{2}c^{12}-14057472a^{3}c^{12}+212992a^{4}c^{12}-83852525568c^{13}-1029439488ac^{13}+31195136a^{2}c^{13}+$$
$$+1109917696c^{14}+4325376ac^{14}-131072a^{2}c^{14}-8912896c^{15}+32768c^{16})$$
$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-33}{2})}(-65536c^{15}+16711680c^{14}+229376a^{2}c^{13}-7569408ac^{13}-1944649728c^{13}-$$
$$-50692096a^{2}c^{12}+1672839168ac^{12}+136767275008c^{12}-319488a^{4}c^{11}+21086208a^{3}c^{11}+4700200960a^{2}c^{11}-$$
$$-166588071936ac^{11}-6490170392576c^{11}+59744256a^{4}c^{10}-3943120896a^{3}c^{10}-234336886784a^{2}c^{10}+$$
$$+9880146591744ac^{10}+219687595212800c^{10}+225280a^{6}c^{9}-22302720a^{5}c^{9}-4212961280a^{4}c^{9}+$$
$$+318534881280a^{3}c^{9}+6385782734848a^{2}c^{9}-388581283774464ac^{9}-5467350755508224c^{9}-34467840a^{6}c^{8}+$$
$$+3412316160a^{5}c^{8}+126600376320a^{4}c^{8}-14548978667520a^{3}c^{8}-63061551218688a^{2}c^{8}+$$
$$+10677406306811904ac^{8}+101605453158088704c^{8}-84480a^{8}c^{7}+11151360a^{7}c^{7}+1727278080a^{6}c^{7}-$$
$$-213503938560a^{5}c^{7}-132194118144a^{4}c^{7}+411663197620224a^{3}c^{7}-1770221953955840a^{2}c^{7}-$$
$$-210333358052302848ac^{7}-1417367362882437120c^{7}+10053120a^{8}c^{6}-1327011840a^{7}c^{6}-19603584000a^{6}c^{6}+$$
$$+6998660444160a^{5}c^{6}-108366929459712a^{4}c^{6}-7386371104831488a^{3}c^{6}+81588957922920448a^{2}c^{6}+$$
$$+2998385401531441152ac^{6}+14789061893062983680c^{6}+16128a^{10}c^{5}-2661120a^{9}c^{5}-321699840a^{8}c^{5}+$$
$$+59852136960a^{7}c^{5}-1169166679296a^{6}c^{5}-125633606757120a^{5}c^{5}+3584138069952000a^{4}c^{5}+$$
$$+81155276599034880a^{3}c^{5}-1581946349642745856a^{2}c^{5}-30822380253791182848ac^{5}-$$
$$-113998595959458562048c^{5}-1370880a^{10}c^{4}+226195200a^{9}c^{4}-1709030400a^{8}c^{4}-1252367424000a^{7}c^{4}+$$
$$+48558756153600a^{6}c^{4}+1092730056042240a^{5}c^{4}-58182434494609920a^{4}c^{4}-460255718632596480a^{3}c^{4}+$$
$$+18390328035742767104a^{2}c^{4}+224712599605813321728ac^{4}+634238712296638578688c^{4}-1344a^{12}c^{3}+$$
$$+266112a^{11}c^{3}+23120832a^{10}c^{3}-6471400320a^{9}c^{3}+263725670208a^{8}c^{3}+10943807124096a^{7}c^{3}-$$
$$-777928079414464a^{6}c^{3}-711059420649600a^{5}c^{3}+533035048250363648a^{4}c^{3}+25486745680704000a^{3}c^{3}-$$
$$-135043097758270087168a^{2}c^{3}-1123667660559841959936ac^{3}-2449470002402175418368c^{3}+68544a^{12}c^{2}-$$
$$-13571712a^{11}c^{2}+405574848a^{10}c^{2}+68559765120a^{9}c^{2}-4757196954048a^{8}c^{2}+2935731705984a^{7}c^{2}+$$
$$+5939303383785024a^{6}c^{2}-64891677848976000a^{5}c^{2}-2678695653148796160a^{4}c^{2}+$$
$$+16568080735699671552a^{3}c^{2}+610385093977265700864a^{2}c^{2}+3626708216318306279424ac^{2}+$$
$$+6147778466426910474240c^{2}+32a^{14}c-7392a^{13}c-390880a^{12}c+182042784a^{11}c-11582726176a^{10}c-$$
$$-115023090336a^{9}c+30965876629600a^{8}c-571118887841568a^{7}c-19091657228488064a^{6}c+$$
$$+461856383439694848a^{5}c+6264028269209728000a^{4}c-91339795704196148736a^{3}c-$$
$$-1538581445063915139072a^{2}c-6717806419589716746240ac-8864971566659828121600c-544a^{14}+$$
$$+125664a^{13}-6561184a^{12}-479910816a^{11}+57700381088a^{10}-1178029251168a^{9}-55758969524192a^{8}+$$
$$+2166179809154592a^{7}+12571450930103296a^{6}-968963896585149696a^{5}-3718430128264001024a^{4}+$$
$$+161641469374376538624a^{3}+1636843212624447221760a^{2}+5336242598307820953600a+)$$
$$+5446224138734346240000)\Big] \qquad (35)$$

II. Main Summation Formulae

$$ {}_2F_1\left[\begin{matrix} a, & 34-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = $$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-34}\{\prod_{\upsilon=1}^{33}(\upsilon-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-34}{2})}(-a^{16}-16ca^{15}+552a^{15}+144c^2a^{14}-960ca^{14}-$$

$$-61300a^{14}+672c^3a^{13}-69552c^2a^{13}+1341536ca^{13}-3188640a^{13}-3360c^4a^{12}+86688c^3a^{12}+5324592c^2a^{12}-$$

$$-160448736ca^{12}+893429978a^{12}-8064c^5a^{11}+1391040c^4a^{11}-57892128c^3a^{11}+675371088c^2a^{11}+$$

$$+1913491328ca^{11}-42259217616a^{11}+29568c^6a^{10}-1596672c^5a^{10}-58507680c^4a^{10}+4939770528c^3a^{10}-$$

$$-97425499248c^2a^{10}+637745413536ca^{10}-679177064060a^{10}+42240c^7a^9-10200960c^6a^9+674889600c^5a^9-$$

$$-16772817600c^4a^9+96429549216c^3a^9+2162481363504c^2a^9-30602158333472ca^9+96600982267680a^9-$$

$$-126720c^8a^8+11278080c^7a^8+77172480c^6a^8-36620559360c^5a^8+1449098814240c^4a^8-23877120095136c^3a^8+$$

$$+165235818684816c^2a^8-230483197183008ca^8-1226032082271953a^8-112640c^9a^7+34974720c^8a^7-$$

$$-3230261760c^7a^7+130764775680c^6a^7-2306500893312c^5a^7+3540470472000c^4a^7+478879645736096c^3a^7-$$

$$-6865999686024336c^2a^7+35702359184340112ca^7-56926821859747224a^7+292864c^{10}a^6-37847040c^9a^6+$$

$$+941867520c^8a^6+81421401600c^7a^6-6360613252224c^6a^6+191531039825664c^5a^6-2983525466664160c^4a^6+$$

$$+23601377861232864c^3a^6-66887688285102432c^2a^6-171349976392137312ca^6+970162781289673720a^6+$$

$$+159744c^{11}a^5-60622848c^{10}a^5+7264829440c^9a^5-419474626560c^8a^5+13030146108672c^7a^5-$$

$$-204202261194624c^6a^5+583540879363200c^5a^5+34105994432199360c^4a^5-604797950755286464c^3a^5+$$

$$+4500934007255040960c^2a^5-15349700918246080064ca^5+18032678411541111360a^5-372736c^{12}a^4+$$

$$+64696320c^{11}a^4-3723765760c^{10}a^4+18217267200c^9a^4+7981069992192c^8a^4-446305959348480c^7a^4+$$

$$+12294973420280320c^6a^4-200674790599472640c^5a^4+1979058220034118784c^4a^4-$$

$$-10989497453339129472c^3a^4+25626181659079681152c^2a^4+24868566942528322944ca^4-$$

$$-1625550508787452235024a^4-114688c^{13}a^3+51437568c^{12}a^3-7629000704c^{11}a^3+580894279680c^{10}a^3-$$

$$-26357259044864c^9a^3+752583428619264c^8a^3-13405245851942912c^7a^3+131414968911367680c^6a^3-$$

$$-189582728369132032c^5a^3-12350316429882216192c^4a^3+164535540220835095808c^3a^3-$$

$$-1003640035944714619392c^2a^3+3066084837270607896576ca^3-3729376960624172771712a^3+245760c^{14}a^2-$$

$$-54362112c^{13}a^2+4933906432c^{12}a^2-223377862656c^{11}a^2+3771371974656c^{10}a^2+125079308242944c^9a^2-$$

$$-9607037567407104c^8a^2+295699286442734592c^7a^2-5592425781466230784c^6a^2+$$

$$+70550631531440265216c^5a^2-602426562087707409408c^4a^2+3415017222324544668672c^3a^2-$$

$$-12175495593555803848704c^2a^2+24335792458878329358336ca^2-20356991448951985263360a^2+32768c^{15}a-$$

$$-16957440c^{14}a+3011592192c^{13}a-287238561792c^{12}a+17309632258048c^{11}a-713135217180672c^{10}a+$$

$$+20958229483012096c^9a-449430876321140736c^8a+7099495571552440320c^7a-82537581030322446336c^6a+$$

$$+698493539819345641472c^5a-4205722061372110774272c^4a+17317855875926642909184c^3a-$$

$$-45563112661811594821632c^2a+67419616553704566374400ca-41219947864700265830400a-65536c^{16}+$$

$$+17793024c^{15}-2211512320c^{14}+166744424448c^{13}-8519516422144c^{12}+312074416422912c^{11}-$$

$$-8455899290009600c^{10}+172360498514755584c^9-2661498857805185024c^8+31102264714320543744c^7-$$

$$-272691657845724938240c^6+1762901990601227501568c^5-8164662989246420484096c^4+$$

$$+25863672054363928657920c^3-51874886067231617187840c^2+57069837985455577497600c-$$

$$-24412776311194951680000)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-33}{2})}(a^{16}-16ca^{15}+8a^{15}-144c^2a^{14}+9120ca^{14}-81500a^{14}+672c^3a^{13}-$$

$$-1008c^2a^{13}-1057504ca^{13}+12624080a^{13}+3360c^4a^{12}-383712c^3a^{12}+10269168c^2a^{12}-45388896ca^{12}-$$

$$-474997418a^{12}-8064c^5a^{11}+20160c^4a^{11}+38069472c^3a^{11}-1363401648c^2a^{11}+14494057088ca^{11}-$$

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$$-33810573664a^{11}-29568c^6a^{10}+4612608c^5a^{10}-205386720c^4a^{10}+2544023328c^3a^{10}+27796961808c^2a^{10}-$$
$$-724678763424ca^{10}+3216119281100a^{10}+42240c^7a^9-147840c^6a^9-380688000c^5a^9+22723377600c^4a^9-$$
$$-513166514784c^3a^9+4661939328816c^2a^9-8709953919392ca^9-51357622524560a^9+126720c^8a^8-$$
$$-24203520c^7a^8+1506193920c^6a^8-34540450560c^5a^8-71570046240c^4a^8+16204139338464c^3a^8-$$
$$-261857467759536c^2a^8+1559021903289312ca^8-2692481789166367a^8-112640c^9a^7+506880c^8a^7+$$
$$+1595235840c^7a^7-133310580480c^6a^7+4732090543488c^5a^7-83745693662400c^4a^7+666593010344096c^3a^7-$$
$$-4131359139457744c^2a^7-23165656452810608ca^7+86522703948738904a^7-292864c^{10}a^6+64655360c^9a^6-$$
$$-5164177920c^8a^6+182875123200c^7a^6-1878169915776c^6a^6-68074153061376c^5a^6+2628077390788960c^4a^6-$$
$$-38418840677445536c^3a^6+273051760943387712c^2a^6-846545819401172352ca^6+631093555519267400a^6+$$
$$+159744c^{11}a^5-878592c^{10}a^5-3190415360c^9a^5+342798888960c^8a^5-16842096784128c^7a^5+$$
$$+463461663417984c^6a^5-7339417059680640c^5a^5+59854266852536640c^4a^5-90310567207056064c^3a^5-$$
$$-2356940358918858240c^2a^5+17001254751738871936ca^5-34870989959766941120a^5+372736c^{12}a^4-$$
$$-91852800c^{11}a^4+8951388160c^{10}a^4-442018508800c^9a^4+10738844759808c^8a^4-33798228414720c^7a^4-$$
$$-5584817036888320c^6a^4+172950328432738560c^5a^4-2604812988902886784c^4a^4+$$
$$+22413955532039260288c^3a^4-107665604140785748992c^2a^4+247916722085516649984ca^4-$$
$$-163094646060428607216a^4-114688c^{13}a^3+745472c^{12}a^3+3016339456c^{11}a^3-396123453440c^{10}a^3+$$
$$+25082545371136c^9a^3-957329613487104c^8a^3+23621316938948608c^7a^3-381894289103613440c^6a^3+$$
$$+3903706996488493568c^5a^3-21832056928714878208c^4a^3+20446598252654236928c^3a^3+$$
$$+520682847876528490752c^2a^3-2978394462127108894464ca^3+5212143058443128698752a^3-245760c^{14}a^2+$$
$$+66060288c^{13}a^2-7595241472c^{12}a^2+490016477184c^{11}a^2-19237815607296c^{10}a^2+447757785964544c^9a^2-$$
$$-4412341152992256c^8a^2-71812472582704128c^7a^2+3474378122362706944c^6a^2-$$
$$-64066228054770484224c^5a^2+706034553660500932608c^4a^2-4941545743482768232448c^3a^2+$$
$$+21433437906881583826944c^2a^2-52196720760610813753344ca^2+53998442915060528352000a^2+$$
$$+32768c^{15}a-245760c^{14}a-1082769408c^{13}a+168052498432c^{12}a-13098781908992c^{11}a+$$
$$+647199810363392c^{10}a-22073665714479104c^9a+540929119272554496c^8a-9702091844762972160c^7a+$$
$$+127964363284151668736c^6a-1232798084221039833088c^5a+8513076418112397340672c^4a-$$
$$-40685706128830676176896c^3a+126493369710655289536512c^2a-227290235806789959536640ca+$$
$$+176278465478072598681600a+65536c^{16}-18907136c^{15}+2503966720c^{14}-201788260352c^{13}+$$
$$+11058430050304c^{12}-436240918151168c^{11}+12789531346042880c^{10}-283628762956988416c^9+$$
$$+4796266602801496064c^8-61870182616413208576c^7+604697686382763868160c^6-$$
$$-4412533510493582098432c^5+23448090496368918429696c^4-87165765493549449707520c^3+$$
$$+212025425525307834531840c^2-298172745643157463859200c+179725396599156215808000)\Big] \qquad (36)$$

$$ {}_2F_1\left[\begin{matrix} a, & 35-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = $$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-35}\{\prod_{=1}^{34}(\quad -a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-34}{2})}(65536c^{17}-20054016c^{16}-278528a^2c^{15}+9748480ac^{15}+$$
$$+2824830976c^{15}+75202560a^2c^{14}-2632089600ac^{14}-242904268800c^{14}+487424a^4c^{13}-34119680a^3c^{13}-$$
$$-8659087360a^2c^{13}+323966361600ac^{13}+14256348725248c^{13}-114057216a^4c^{12}+7984005120a^3c^{12}+$$
$$+548044922880a^2c^{12}-24071775436800ac^{12}-604897426538496c^{12}-452608a^6c^{11}+47523840a^5c^{11}+$$
$$+10350692352a^4c^{11}-821576304640a^3c^{11}-19717159641088a^2c^{11}+1205201984430080ac^{11}+$$
$$+19171218005688320c^{11}+89616384a^6c^{10}-9409720320a^5c^{10}-423437414400a^4c^{10}+48852131328000a^3c^{10}+$$
$$+305660336357376a^2c^{10}-42973452470108160ac^{10}-462397427582238720c^{10}+239360a^8c^9-33510400a^7c^9-$$

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$$-6092669440a^6c^9+783406131200a^5c^9+2738095576832a^4c^9-1849788509578240a^3c^9+$$
$$+6134529596687360a^2c^9+1124486899904614400ac^9+8567195636023033856c^9-38776320a^8c^8+$$
$$+5428684800a^7c^8+115941196800a^6c^8-35449311744000a^5c^8+511296823521792a^4c^8+$$
$$+46089057313474560a^3c^8-481001116898396160a^2c^8-21923756528499302400ac^8-$$
$$-122197833783174758400c^8-71808a^{10}c^7+12566400a^9c^7+1832540160a^8c^7-348918662400a^7c^7+$$
$$+6172232153472a^6c^7+927105695725440a^5c^7-25806827138495744a^4c^7-743425737102737920a^3c^7+$$
$$+13789128191585139712a^2c^7+319992028619053486080ac^7+1336108426350974861312c^7+9047808a^{10}c^6-$$
$$-1583366400a^9c^6+3619123200a^8c^6+11131065792000a^7c^6-425265822584064a^6c^6-13050897868673280a^5c^6+$$
$$+642961365023577600a^4c^6+6947065184830848000a^3c^6-243346443758637778944a^2c^6-$$
$$-3482008248178128936960ac^6-11076836161750660546560c^6+11424a^{12}c^5-2399040a^{11}c^5-263517408a^{10}c^5+$$
$$+73054766400a^9c^5-2910527403168a^8c^5-169079349996480a^7c^5+11291606553959776a^6c^5+$$
$$+48256622440931520a^5c^5-9658760174539039872a^4c^5-16723871670952328960a^3c^5+$$
$$+2888756598389050934272a^2c^5+27876156018911379988480ac^5+68317489405233074012160c^5-$$
$$-1028160a^{12}c^4+215913600a^{11}c^4-5598331200a^{10}c^4-1444821840000a^9c^4+98253076299840a^8c^4+$$
$$+428068648022400a^7c^4-159374525077310400a^6c^4+1426838518041552000a^5c^4+89403395392422929664a^4c^4-$$
$$-435843532723349076480a^3c^4-23414901217565786449920a^2c^4-160224213692847944908800ac^4-$$
$$-304192382673162365042688c^4-816a^{14}c^3+199920a^{13}c^3+14765520a^{12}c^3-6284485200a^{11}c^3+$$
$$+398612051376a^{10}c^3+7962540709200a^9c^3-1429039099466000a^8c^3+22788340329370000a^7c^3+$$
$$+1182513401362330048a^6c^3-25292561498881255040a^5c^3-478573803999125343488a^4c^3+$$
$$+5976578012667753084160a^3c^3+127023404881657459338240a^2c^3+634777240241376226713600ac^3+$$
$$+932795875611063784046592c^3+44064a^{14}c^2-10795680a^{13}c^2+535157280a^{12}c^2+59538175200a^{11}c^2-$$
$$-6671191998240a^{10}c^2+112788136692000a^9c^2+9143232056972640a^8c^2-315432698542869600a^7c^2-$$
$$-3473975419503608448a^6c^2+181208084347605047040a^5c^2+1163390977004204551680a^4c^2-$$
$$-35669294433941421657600a^3c^2-436626786712719685994496a^2c^2-1618295924915884422512640ac^2-$$
$$-1822610275828337195089920c^2+17a^{16}c-4760a^{15}c-180268a^{14}c+146208160a^{13}c-13054717690a^{12}c+$$
$$+88120404400a^{11}c+39003094772092a^{10}c-1543215142453600a^9c-14612071043898911a^8c+$$
$$+1562552746219120040a^7c-4736174600923434200a^6c-598592074368941968000a^5c+$$
$$+365146648870832375184a^4c+106137153450268466037120a^3c+847239268104348903058176a^2c+$$
$$+2335768724989391800227840ac+1965578489900781222297600c-306a^{16}+85680a^{15}-$$
$$-6273000a^{14}-299880000a^{13}+61827146388a^{12}-2358015216480a^{11}-43444280788920a^{10}+$$
$$+4587058359576000a^9-44987359504501458a^8-2431462813489820880a^7+$$
$$+34101207520671720240a^6+695819842162977772800a^5-5040324061911608046624a^4-$$
$$-126089811549628554136320a^3-692625212006097143416320a^2-$$
$$-1402269449483905673932800a-830034394580628357120000)+$$
$$+\frac{1}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a-35}{2})}(-2a^{16}+560a^{15}+288c^2a^{14}-10368ca^{14}+21304a^{14}-70560c^2a^{13}+$$
$$+2540160ca^{13}-17224480a^{13}-6720c^4a^{12}+483840c^3a^{12}-5224800c^2a^{12}-125435520ca^{12}+$$
$$+1534110676a^{12}+1411200c^4a^{11}-101606400c^3a^{11}+2220876000c^2a^{11}-14110588800ca^{11}-9627472960a^{11}+$$
$$+59136c^6a^{10}-6386688c^5a^{10}+155305920c^4a^{10}+2613219840c^3a^{10}-140376408480c^2a^{10}+$$
$$+1572320356992ca^{10}-4635463909912a^{10}-10348800c^6a^9+1117670400c^5a^9-43025136000c^4a^9+$$
$$+683641728000c^3a^9-2896286316000c^2a^9-25857351273600ca^9+180609356989600a^9-253440c^8a^8+$$
$$+36495360c^7a^8-1511516160c^6a^8-2299207680c^5a^8+1704525641280c^4a^8-46245001904640c^3a^8+$$
$$+507786362408160c^2a^8-2197587979255680ca^8+1887531627998654a^8+35481600c^8a^7-5109350400c^7a^7+$$

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$$+287675942400c^6a^7-7892988364800c^5a^7+100895661820800c^4a^7-227670578150400c^3a^7-$$
$$-7843164587042400c^2a^7+74010152078755200ca^7-186490783323096560a^7+585728c^{10}a^6-105431040c^9a^6+$$
$$+6459847680c^8a^6-110386298880c^7a^6-5022780204288c^6a^6+299421134724096c^5a^6-6667446243465920c^4a^6+$$
$$+75999636858816000c^3a^6-430657137310684416c^2a^6+892705835979555840ca^6+416748638455072112a^6-$$
$$-61501440c^{10}a^5+11070259200c^9a^5-830411366400c^8a^5+33496901222400c^7a^5-771243287189760c^6a^5+$$
$$+9446225872849920c^5a^5-32215357224758400c^4a^5-64051174401984000 0c^3a^5+$$
$$+886772009969290368 0c^2a^5-4329940396371448320 0ca^5+7339240604190975424 0a^5-745472c^{12}a^4+$$
$$+161021952c^{11}a^4-13411414016c^{10}a^4+501113733120c^9a^4-3100159028736c^8a^4-$$
$$-475231972368384c^7a^4+21225059657027072c^6a^4-457211060670228480c^5a^4+$$
$$+5787846047814457088c^4a^4-43588398900552726528c^3a^4+180335395055518585344c^2a^4-$$
$$-316265575283186780160ca^4+144837316919439702 72a^4+52183040c^{12}a^3-11271536640c^{11}a^3+$$
$$+1064364421120c^{10}a^3-57679740518400c^9a^3+1974553010411520c^8a^3-44068357364613120c^7a^3+$$
$$+630557884861864960c^6a^3-5208905338701926400c^5a^3+12926188664722403840c^4a^3+$$
$$+18845246776176285696 0c^3a^3-208786079332222065408 0c^2a^3+859293075371666181120 0ca^3-$$
$$-1318779556646094635904 0a^3+491520c^{14}a^2-123863040c^{13}a^2+$$
$$+13258219520c^{12}a^2-776930918400c^{11}a^2+25807775465472c^{10}a^2-373802800250880c^9a^2-$$
$$-6111328116664320c^8a^2+44834063354707968 0c^7a^2-114153027810677166 08c^6a^2+$$
$$+1749337958224988651 52c^5a^2-175613751959124799488 0c^4a^2+115945215303699519897 60c^3a^2-$$
$$-482644202445685220843 52c^2a^2+113926717871059832487 936ca^2-$$
$$-114960614685575385429 504a^2-17203200c^{14}a+4335206400c^{13}a-495999795200c^{12}a+$$
$$+34096398336000c^{11}a-1570577115627520c^{10}a+51180657028300800c^9a-1214077345177804800c^8a+$$
$$+21243051812222174720 0c^7a-274700995202027806720c^6a+2603603321741504839680c^5a-$$
$$-17732068752225396531200c^4a+83737636730775930470400c^3a-257629842512237495623680c^2a+$$
$$+458653714607985911562240ca-352793322231849920655360a-131072c^{16}+37748736c^{15}-$$
$$-4990238720c^{14}+401398824960c^{13}-21955086712832c^{12}+864385160970240c^{11}-$$
$$-25290493887053824c^{10}+559710380402933760c^9-9445456211215974400c^8+$$
$$+121594149907523960832c^7-1186033529251163471872c^6+8637776397559556997120c^5-$$
$$-45816122976197136285696c^4+170024112173728535150592c^3-$$
$$-412932973224831943901184c^2+579933330967828133314560c-349180770555828500889600)\Big] \quad (37)$$

III. Evaluation of the Main Formulae

Using the contiguous relation (8) and the formula of Salahuddin et al(35), the result(36)can be established and on the same way result(37) can be established.

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The Scientific Graph Programming Paradigms of Systems from Modules to the Application Areas

By E. M. Lavrischeva

Abstract- The mathematical basics of graph paradigm programming of applied systems (AS) are presented. The vertices of graph are been the functional modules of the systems and the arcs define the connections between them. The graph is represented by an adjacency and reach ability matrix. A number of graph' program structures and their representation by mathematical operations (unions, connections, differences, etc.) are shown. Given the characteristics of graph structures, complexes, units, and systems created from the modules of the graph. The method of modelling the system on the graph of modules, which describe in the programming languages (LP) and the advanced operations of association (link, assembling, make, building, config etc.). The standard of configuration (2012) Assembly of heterogeneous software elements in AS of different fields of knowledge is made. A brief description of modern and future programming paradigms for formal theoretical creation of systems from intelligent and service-components of the Internet is given. The modules and AS are protected and quality assessment. There are giving the new direction of modern paradigms programming in the near future.

Keywords: *graph theory; adjacency matrix, reach ability; mathematical operations; configuration assembling; paradigm programming; future technologies.*

GJSFR-F Classification: *MSC 2010: 90C35*

THESCIENTIFICGRAPHPROGRAMMINGPARADIGMSOFSYSTEMSFROMMODULESTOTHEAPPLICATIONAREAS

Strictly as per the compliance and regulations of:

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E. M. Lavrischeva

Annotation- The mathematical basics of graph paradigm programming of applied systems (AS) are presented. The vertices of graph are been the functional modules of the systems and the arcs define the connections between them. The graph is represented by an adjacency and reach ability matrix. A number of graph' program structures and their representation by mathematical operations (unions, connections, differences, etc.) are shown. Given the characteristics of graph structures, complexes, units, and systems created from the modules of the graph. The method of modelling the system on the graph of modules, which describe in the programming languages (LP) and the advanced operations of association (link, assembling, make, building, config etc.). The standard of configuration (2012) Assembly of heterogeneous software elements in AS of different fields of knowledge is made. A brief description of modern and future programming paradigms for formal theoretical creation of systems from intelligent and service-components of the Internet is given. The modules and AS are protected and quality assessment. There are giving the new direction of modern paradigms programming in the near future.

Keywords: graph theory; adjacency matrix, reach ability; mathematical operations; configuration assembling; paradigm programming; future technologies.

I. Introduce. The Graph Theory and Paradigms of Programs

Programming theory is a mathematical science, the object of study of which is the mathematical abstraction of the functions of programs with a certain logical and information structure, focused on computer execution. With the advent of the LP began to develop new methods of analysis of algorithms of AS problems, the graph theory for the representation the structure AS by separate programs elements, displaying them in the vertices of the graph to create a complex structure of AS (programs, aggregate, large program, system, etc.). Programs elements of missile defense were first called modules, programs, then objects, components, services, etc. For the formal specification of these elements were formed the corresponding *programming paradigms*, allowing from the point of view of the theory and graphs to describe the problems of different AS (medicine, biology, chemistry, genetics, etc.).

II. Graph Theory of Programs from Modules

The basis for the creation of systems of modules was the method of assembling the graph (70-80 years of the last century) heterogeneous modules in specialized

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software packages (Lipaev V. V.) and in the system APROP of ES OS (IBM-360) [1, 2]. Formed Assembly programming [3-5], which "provided the building is already existing individual pieces of software (such reuses) in the complex structure" [6]. The interface of the modules was described initially in a special description language link, and then in equivalent operations: make BSD, Java (1996); config SPAROL, building, assembling Grid (2002), etc. [7-12]. And after 90-x there were standard languages of the description of these operations of IDL, API, WSDL and the standard statement of config of the IEEE 828-2012 standard (Configuration Management) for receiving a configuration file of any application system from ready modules, objects, components, services and other resources.

A module is a formally described program element that displays certain AS function that has the property of completeness and connectivity with other elements according to the data specified in the interface part of the description. From a mathematical point of view, a module is a mapping of a set of initial data X to a set of output Y in the form $M: X \to Y$.

A number of restrictions and conditions are imposed on X, Y and M to make the module an independent program element among other types of program objects [1-3].

Types of connections between modules via input and output parameters are as follows:

1) linking of control: $CP = K_1 + K_2$, where K_1 is the coefficient of the calling mechanism; K_2 is the coefficient of transition from the environment of the calling module to the environment of the called;
2) Linking of data: $CI = \sum_{i=1}^{n} K_i F(x_i)$, where Ki - the weight coefficient iron of the parameter; $F(x_i)$ - the element function for the parameter x_ii.

Coefficients $K_{id} = 1$ – for simple variables and $K_{id} > 1$ – for complex variables (array, record, etc.). $F(x_{.I}) = 1$ if x_i - a simple variable and $F(x_i) > 1$ if complex.

The program, modular structure is given by the graph $G = (X, E)$, where X - a finite set of vertices; E - a finite subset of the direct product of X z on the set of relations on the arcs of the graph. The program structure represents a pair $S = (T, \chi)$, where T - a model of a program, modular structure; χ - a characteristic function given on the set of vertices X of the graph G.

The value of the characteristic function χ is defined as:

X(x) = 1 if the module with vertex $x \in X$ is included in the modular system;

X(x) = 0 if the module with vertex $x \in X$ is not included in the modular system and is not accessed from other modules.

Definition 1: Two models of program structures $T_1 = (G_1, Y_1, F_1)$ and $T_2 = (G_2, Y_2, F_2)$ are identical if $G_1 = G_2$, $Y_1 = Y_2$, $F_1 = F_2$. The T_1 model is isomorphic to the T_2 model if $G_1 = G_2$ between sets Y_1 and Y_2 exists an isomorphism φ, and for any $x \in X$ $F_2(x)=\varphi(f_1(x))$.

Definition 2: Two program structure $S_1 = (T_1, \chi_1)$ and $S_2 = (T_2, \chi_2)$ are identical if $T_1 = T_2$, $\chi_1 = \chi_2$ and the structures S_1 and S_2 are isomorphic, then T_1 is isomorphic to T_2 and $\chi_1 = \chi_2$.

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The concept of isomorphism of program structures and their models is used in the specification of the abstraction level at which operations on these structures are defined. For isomorphic graph objects, operations will be interpreted in the same way without orientation to a specific composition of program elements, provided that such operations are defined over pairs (G, χ). The software module is described in the LP and has an interface section in which external and internal parameters are set for data exchange between related modules through interface (Call/RMI) operations, etc.

The interface defines the connection of heterogeneous software modules according to the data and the way they are displayed by programming systems with the *LP*. Its main functions are: data transfer between program elements (modules), data conversion to the equivalent form and transition from the environment and platform of the called module to the caller and back. Functions of conversion of different, non-equivalent data types is carried out with the help of a previously developed library of 64 primitive functions for heterogeneous data types of *LP* in the APROP system [1-5] and included in the common system environments of the OS (IBM, MS, Oberon, UNIX, etc.).

In practice, the assembly method of software modules is performed by operations (link, make, assembling, config. weaver) special programs [1] OS libraries (OS ES, IBM, MS.Net, etc.), a builder of complex applications in OS RV for SM computers, complication modules for ERM "Elbrus" are used. In these operations and interface modules and data type conversion library [1, 2].

Next, we consider the mathematical theory of graphs of software modular structures and mathematical operations (union, projection, difference, etc.) implementation of ways of linking the graph modules and the semantics of the data transformation by the vertices of the graph. Software modules are described in modern LP and with help of the new paradigms programming.

a) Definition of a modular structure graph

To represent modular structures, we use the mathematical apparatus of graph theory, in which the graph G is treated as a pair of objects $G = (X, E)$, where X - a finite set of vertices, and E is a finite subset of the direct product of $X \times X \times Z$ - arcs of the graph, corresponding to a finite vertex (Fig. 1).

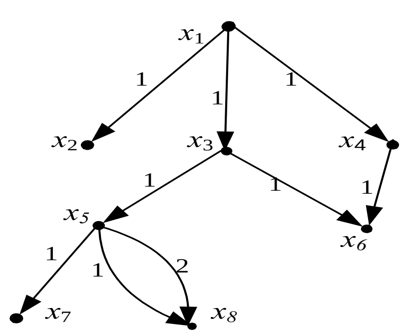

Fig. 1: Graph of program from modules

The set of arcs of the graph have the form: E = −(x_1, x_2, 1), (x_l, x_3, 1), (x_5, x_8, 1), (x_5, x_8, 2)″ [1-7]. Based on this definition, we can say that the graph G is a multi-graph, since its two vertices can be connected by several arcs. To distinguish these arcs introduced their numbering positive integers – 1, 2. (Fig.1) and vertices of the graph x_1,

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x_2, ..., x_8 form a set of X. From the module corresponding to the vertex x_5, there are two calling operators to the modules, with vertices x_7, x_8.

Definition 3: A program aggregate is a pair $S = (T, \chi)$, where T - a model of the program modular structure of the aggregate; χ - a characteristic function defined on the set of vertices X of the graph of the modular structure G. The value of the χ function is defined as follows:

$\chi(x) = 1$ if the module corresponding to the vertex $x \in X$, - included in the unit;

$\chi(x) = 0$ if the module corresponding to the vertex $x \in X$, - not included in the software unit, but it is accessed from other modules previously included.

Definition 4: The model of the program structure of the program unit is an object described by the triple $T = (G, Y, F)$, where $G = (X, E)$ - a directed graph of a modular structure;

Y is a set of modules included in the program aggregate;

F is a correspondence function that puts an element of the set y at each vertex X of the graph.

Function F maps X to Y, $$F: X \rightarrow Y. \quad (1)$$

In General, an element from Y can correspond to several vertices from the set X (which is typical for the dynamic structure of the aggregate) [5, 15, and 20].
The graph of software aggregates has the following properties:

1) Graph G has one or more connectivity elements, each of which represents an acyclic graph, i.e. does not contain oriented cycles;
2) In each graph G is allocated a single vertex, which is called the root and is characterized by the fact that there are no arcs included in it and the corresponding module of the software unit is performed first;
3) Cycles are allowed only for the case when some vertex has a recursive reference to itself. Typically, this feature is implemented by the compiler with the corresponding *LP* and this type of communication is not considered by the intermodule interface. Therefore, such arcs are not included in the graph. The exception to the consideration of other types of cycles is due to the fact that some modules will have to remember the history of their calls in order to return control correctly, which contradicts the properties of the modules;
4) An empty graph G_0 corresponds to an empty program structure.

Next, the graph G will be used to illustrate mathematical operations on modular structures. For Fig.2. three types of subgraphs are shown and their description is given.

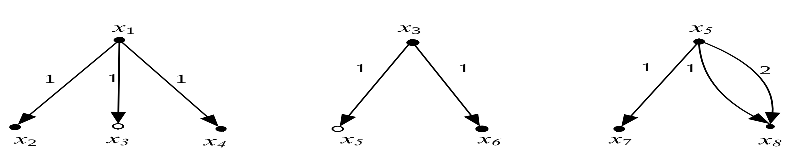

Fig. 2: The graphs of modules structures

A sub graph - a fragment of a software aggregate $G^s = (X^s, E^s)$ for whose functions one of two conditions is satisfied:

$C(S) = 1$, if $\chi(x) = 1$ for any x of X;

Notes

Notes

$C(S) = 0$, if there is x such that $\chi(x) = 0$;

$R\ (S^s) = 0$, if the modular structure is part of a higher-level structure and $R(S) = 1$ if the software assembly is ready to run.

Given these combinations C and R, the subgraph can be: open ($C = 0,\ R = 0$); closed at the top ($C = 0,\ R = 1$); closed at the bottom ($C = 1,\ R = 0$).

The graph of the module (m) is represented as: $G^m = (X^m,\ E^m)$. It contains a single vertex $x \in X^m$ for which $\chi(x_j) = 1$. This vertex is the root. An arc of the form $(x_j,\ x_e,\ k)$ means calling the module to the corresponding vertex x_j , i.e. to the module with the vertex x_l. The dark circle on the+ graph corresponds to the vertex for which $\chi(x) = 1$; light – $\chi(x) = 0$.

Program graph $G^p = (X^p,\ E^p)$ which is performed $C\ (S^p) = 1;\ R\ (S^p) = 1$. An example of a graph of such a program modular structure is shown in Fig. 1.

The graph of the complex $G^c = (X^c,\ E^c)$ consists of n connectivity components (n > 1), each of which is a graph and includes: $G^c = G_1^p \cup G_2^p \cup, \ldots, \cup G_n^p$,

where $X^c = X_1^p \cup X_2^p \cup, \ldots, \cup X_n^p$ и $E^c = E^p\ E_1^p \cup E_1^p\ E_2^p \cup, \ldots, \cup E_n^p$.

These definitions of the graph of the program module, program and complex are used for the process of assembling the modules. These concepts may differ from similar ones, which are considered in other contexts of the work.

b) Matrix representation of graphs from program elements of module type

To determine the main operations on software structures, we use the mathematical apparatus of the matrix representation of graphs in the form of an adjacency and reachability matrix. That is, the graph is represented by the matrix $M = m\ (i,\ j)$ of adjacency and is proved by the reach ability matrix [5, 11-13]. The element of the matrix m_{ij} determines the number of call operators with index *i*, to the module with index *j*.

In addition to the adjacency matrix (calls), the characteristic vector $Vi = \chi\ (xi)$ for i-elements is used. For a modular structure graph (Fig. 1) characteristic vector and adjacency matrix have the form:

$$V = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \qquad M = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{2}$$

We analyze adjacency matrices and characteristic vectors for subgraphs and graphs of modular structures corresponding to different types – program, complex, aggregate, etc. For subgraphs (Fig.2) vectors and matrices have the form:

$$V_3^s = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad M_3^s = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \qquad V_1^s = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix},$$

$$M_1^s = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; \quad V_5^s = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad M_5^s = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{3}$$

For the program graph (Fig. 1) the characteristic vector and the matrix of calls coincide with V and M, respectively, and determine the form (2), in which all elements of V are equal to one. In the case of the complex, the characteristic vector and the call matrix have the following form:

$$V^c = \begin{pmatrix} V_1^p \\ V_2^p \\ \dots \\ V_n^p \end{pmatrix}, \quad M^c = \begin{pmatrix} M_1^p & 0 & \dots & 0 \\ 0 & M_2^p & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & M_n^p \end{pmatrix} \tag{4}$$

Here V_i^p and M_i^p $(i = \overline{1, n})$ denote the characteristic vector and the adjacency matrix for the graph of the i-th program included in the graph of the complex. In the future, the matrix representation is used when performing mathematical operations on software structures.

c) The relation of the reach ability graph of program structures

Let $G = (X, E)$ - a graph of a program of modular structure; x_i, x_j - vertices belonging to X. If there is an oriented chain from x_i to x_j in the graph G, then the vertex x_j is reachable from the vertex x_i. The following statement is true: if the vertex x_j is reachable from x_l – из x_j , x_l – from x_j , then x_l is reachable from x_l. The proof of this fact is obvious.

Consider a binary relation on the set X that determines the reach ability of one vertex of a graph to another. We introduce the notation $x_i \rightarrow x_j$ - reach ability of the vertex x_j from x_i. The relation is transitive. Denote by $D(x_i)$) the set of vertices of graph G reachable from x_i.. Then the equality

$$\overline{x_i} = \{x_i\} \cup D(x_i) \tag{5}$$

of determines the transitive closure of x_i in relation to the achievability of tops. We prove the following theorems.

Theorem 1. For the selected element of connectivity of the graph of the program structure, any vertex is reachable from the root corresponding to the given vertex of the graph, i.e. the equality (x_1 – root vertex)

$$\overline{x_1} = \{x_1\} \cup D(x_1) = X. \tag{6}$$

Evidence. Suppose the vertex x_i $(x_i \in X)$ is unattainable from x_1. Then $x_i \notin \overline{x_1}$ and the set X' = $X \setminus \overline{x_1}$ - not empty. Since the selected component of the graph is connected, there is a vertex $x_j \in \overline{x_1}$ and a chain H (x_i, xj), leading from x_i to x_j. Based on the acyclicity of the graph G, in X' there should be a simple chain $H(x_i, x_j)$, where the vertex x_l does not include arcs (this chain can be empty if X' consists only of x_i). Consider the chain $H(x_l, x_j) = H(x_l, x_i) \cup H(x_i, x_j)$. This means that the module x_i is reachable from vertices x_1 and x_i and both vertices contain no incoming arcs. This

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contradicts the definition of a graph of a modular structure with a single root vertex. The theorem is proved.

Thus, the results of theorem1 reinforce the requirement that there are no oriented cycles in the graph of the program structure.

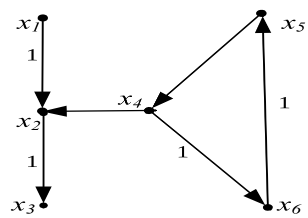

Fig. 3: A graph that contains directed cycle

The results of this theorem are important to substantiate the requirement of the absence of oriented cycles in the graph of the program structure with respect to the notion of reachability. Consider the graph shown in Fig. 3. From this figure it is clear that the graph contains a directed cycle and modules corresponding to vertices x_4, x_5, x_6 will never be executed.

Thus, the results of theorem1 reinforce the requirement that there are no oriented cycles in the graph of the program structure.

$$\mathbf{A}=\begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (7)$$

We analyze the matrix representation of the reach ability relation for the graph of the program structure Fig. 1 with the reach ability matrix A, which has the form (7). Coefficient a_{ij} = 1 if the module corresponding to the index l is reachable from the module corresponding to the index *i* the Following results are based on the following theorem from graph theory.

Theorem 2: The coefficient m_{ij} of the *l*-th degree of the adjacency matrix M^l determines the number of different routes containing *l* arcs and connecting vertex x_i to the vertex of the x_j –oriented graph. The proof of this theorem is given in [20]. Consider the following three consequences of this theorem.

Corollary 1.1: Matrix $\overline{M} = \sum_{l=1}^{n} M^i$, where *M* is the adjacency matrix of a directed graph with n vertices coincides up to the numerical values of the coefficients with the reachability matrix *A*.

Evidence. In a directed graph containing n vertices, the maximum path length without repeating arcs cannot exceed n. Therefore, the sequence of degrees of the adjacency matrix M^i, where i = 1,2, ..., n determines the number of all possible paths in the graph

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with the number of arcs ≤ p. Let the coefficient $\overline{m_{ij}}$ of the matrix M be different from zero. This means that there is a degree of matrix M^i in which the corresponding coefficient $\overline{m_{ij}}$ is also nonzero. Therefore, there is a path from vertex x_i to x_j, i.e. vertex x_j is reachable from x_i. This consequence determines the connection of the matrix of calls of the graph of the modular structure *M*, coinciding with the reachability matrix A, and determines the algorithm for constructing the latter.

Corollary 1.2. Let there be a coefficient $m_{ii} > 0$ for some *i* in the sequence of degrees of the adjacency matrix M^i. Then there is a cycle in the original graph.

Evidence. Let $m_{ii} > 0$ for some *l*. Therefore x_l reachable from x_i, i.e. there is a cycle. According to the theorem, this cycle has *l* arcs (generally repeated).

Corollary 1.3. Let the *n*-th degree of the adjacency matrix of the M^n of the acyclic graph coincide with the zero matrix (all coefficients are zero).

Evidence. If the graph is acyclic, then the simplest path cannot have more than $n-1$ arcs.

If M^n has a coefficient other than zero, then there must be a path consisting of *n* arcs. And this way can only be oriented cycle. Therefore, all coefficients of M^n for an acyclic graph are zero. This consequence provides a necessary and sufficient condition for the absence of cycles in the graph of a modular structure.

For acyclic graphs, the reachability ratio is equivalent to a partially strict order. The transitivity of the reachability ratio was considered above. Anti-symmetry follows from the absence of oriented cycles: if the vertex x_j is reachable from x_i, then the opposite is not true.

We introduce the notation $x_i¿x_j$ if vertex x_j is reachable from vertex x_i.

Let G = (X, E) be an acyclic graph corresponding to some program structure.

Consider the decreasing chain of elements of a partially ordered set X: $x_{i1} > x_{i2} > \ldots > x_{in}$. …, where " > " denotes the reachability ratio.

Since X is finite, the chain breaks. The verte x_{in} has no outgoing arcs, i.e. the element x_{in} is minimal (it corresponds to a module that does not contain access to other modules). The maximum element in the set *X* is the root vertex.

d) Mathematical operations on the graph elements

Mathematical operations (U, ∩, /, +, - , *P, C, R*) on graphs are performed at the level of abstractions of elements of program structures that lead to changes in graph elements and characteristic functions of systems: S = (G, χ) [20].

Let $S_1 = (G_1, \chi_1)$ and $S_2 = (G_2, \chi_2)$ be two graphs of program structures $G_1 = (X_1, E_1)$ and $G_2 = (X_2, E_2)$ respectively.

We introduce the following notations:

D (x) – the set of vertices reachable from the vertex x;

D(x)* – the set of vertices from which vertex x is reachable.

The same symbols are used for the same vertices included in the graphs G_1 and G_2. The main operations on the program structures are discussed below

Merge (join) operation $$S = S_1 \cup S_2 \quad (9)$$

is intended to form a graph of the structure of the complex and is formally defined as follows S_1 and S_2 – any program structures that satisfy the definitions of claim 1:

$$G = G_1 \oplus G_2, \quad X = X_1 \oplus X_2, \quad E_1 \oplus E_2, \tag{10}$$

where the symbol denotes a direct sum provided:

$$\chi(x) = \chi_1(x), \text{ if } \chi \in X_{1.}$$

$$\chi(x) = \chi_2(x), \text{ if } \chi \in X_{2.}$$

Notes

The same vertices included in G_1 and G_2 are represented by different objects in the operations of combining program structures. The characteristic vector and adjacency matrix of the program structure S are defined as follows:

$$V_{1,2}=\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}, \; M_{1,2}=\begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}, \tag{11}$$

where $V_{1,2}$ and $M_{1,2}$ are characteristic vectors and adjacency matrices of modular structures S_1 and S_2 respectively. This operation is associative, but not commutative – the order of the operands determines the order of the components of the complex.

It should be noted that if the operands S_1 and S_2 satisfy the conditions for defining program structures, the result S will also satisfy the same requirements. The join operation increases the number of connected graph elements. In addition, the column structures may themselves have multiple items of connectedness. For the rest of the operation counts of the operands and result are the only element of connection.

The connection operation. We denote by x_i and x_j the root vertices of graphs G_1 and G_2 of program structures S_1 and S_2, respectively. This operation

$$S=S_1+S_2, \tag{12}$$

which is execute if these structures meet the following conditions:

set X' = $X_2 \cap X_2$ not empty;

vertex $x_j \in$ X' and $\chi(x_j) = 0$;

$$D^*(x) \cap D(x) = 0 \text{ for every } x \in X', \text{ where } D^*(x) \in X_1 \text{ и } D(x) \in X_2;$$

$$G = G_1 \cup G_2, \quad X = X_1 \cup X_2, \quad E = E_1 \cup E_2, \tag{13}$$

The characteristic function χ is satisfied under the condition:

$$\chi(x) = \chi_1(x), \text{ if } x \in X_1 \text{ “ } X';$$

$$X(x) = \max(\chi_1(x), \chi_2(x)) > \text{ if } x \in X',$$

$$\chi(x) = \chi_2(x), \text{ if } x \in X_2 \text{ “ } X'.$$

First condition means that there are common vertices in graphs G_1 and G_2. According to the second condition, the root vertex G_2 belongs to the common part and for S_1 the object corresponding to x_j is not included in the program structure yet.

The third condition prohibits the existence of cycles in the result graph. Indeed, if there is $x_n \in D^*(x) \cap D(x)$, then $x_n > x$ and $x > x_n$, and $x > x_n$, then this means the existence of a cycle.

If S_1 and S_2 satisfy the above conditions, the connection operation is partial.

Let us determine whether the result of the connection operation belongs to the class of program structures. Since X' is not empty, the graph G has one connected component. The root vertex of the graph G is x_i. The graph G itself has no oriented cycles, i.e. is acyclic.

Thus, S belongs to the class of program structures under consideration.

This connection operation is not commutative and is generally not associative. To show that this operation is not associative, consider the result $S = (S_1 + S_2) + S_3$, where the root vertices of graphs G_2 and G_3 are part of the vertices of graph G_1 and $X_2 \cap X_3 \neq 0$.

Then the result of the $S_2 + S_3$ join operation is undefined.

The operation of projection. Let $S_1 = (G_1, \chi_1)$ be a program structure and $x_i \in X_1$. The operation of projection of this structure to the top of the graph S_1 is denoted as $S = P_{rxi}(S_1)$ and is defined as

$$G(X, E), \quad X = \overline{x}_i, \quad E = \{(x_i, x_j, K) | x_i, x_j \in X\}, \tag{14}$$

for the characteristic function is $\chi(x) = \chi_1(x)$, if $x \in X$. The projection operation defines the program structure S_1 in the structure S. let's check the belonging of the structure S to the class of the considered program structures. If the graph of the structure S_1 is connected acyclically, then the same properties will be possessed by the graph G. There is a single root vertex x_i in the graph G. Thus, the program structure S belongs to the class under consideration.

The difference operation for program structures is defined as follows. Let $S_1 = (G_1, \chi_1)$ be a program structure and $x_i \in X_1$. The difference operation is performed on this structure and its projection to the vertex x_i of the corresponding graph (x_i is not the cortical vertex of the graph G_1). Formally, the difference operation of the program structure has the form:

$$S = S_1 - P_{r_{xi}}(S_1), \tag{15}$$

and defined as follows

$$G = -X, E), \quad X = (X_1 \setminus \overline{x_i}) \cup X'$$

$$\Gamma = \{(x_i, x_i, K) \mid x_i, x_i \in X\}, \tag{16}$$

where the set X' consists of such elements for which

$$X' = \{x'_j / (x_l \in X_1 \setminus x_i) \& (x'_j \in \overline{x_i}) \& (x_l, x'_j, K) \in E\} \tag{17}$$

Here, the characteristic function χ is defined as:

$$\chi(x) = \chi_1(x), \text{ если } x \in X_1 \setminus \overline{x_i};$$

$$\chi(x) = 0), \text{ если } x \in X'.$$

The set X includes vertices that are not included in the set $\overline{x_i}$, and those vertices $\overline{x_i}$ that include arcs from vertex $X_1 \setminus \overline{x_i}$ (sets X'). The characteristic function for elements $x' \in X'$ is zero. The difference operation is the inverse of the join operation, i.e. the equality is performed:

$$S - P_{r_{xi}}(S) + P_{r_{xi}}(S) = S. \tag{18}$$

Notes

Let us check that S, defined in (15), belongs to the class of program structures. If the graph is G, connected and acyclic, then the graph G_1 will have the same properties. The root vertex G is the same as the root vertex G_1. Thus, S satisfies the conditions for determining the program structure given in paragraph 1.

Let S* be the set of program structures given by the direct product G* X χ*, where G^* and χ^* are the set of graphs and the set of characteristic functions. Denote by $\Omega = \{U, \cap, /, +, -\}$ - set of mathematical operations on program structures and P, C and R - predicates of:

$$\Omega = \{U, \cap, /, +, - , P, C, R\}. \tag{19}$$

Notes

Thus, an algebraic system $\Sigma = (S, \Omega)$ over a set of program structures and operations on them (union, connection, differences and projections) is defined.

e) Characteristics of simple and complex graph structures

Among the variety of program structures there are three main ones – a simple, complex structure with a call of modules from the external environment and a dynamic structure. The main purpose of various structures is the most optimal use of the main memory during the execution of the unit [15-20].

Simple structure. An aggregate with a simple structure is created in the process of building modules based on the operations of link calls. The amount of main memory occupied by an aggregate with a simple structure is constant and equal to the sum of the volumes of individual modules: $V_s = \sum_{i=1}^{n} v_i$, where v_i is the amount of memory occupied by the i-th module. The corresponding graph of a modular structure is always connected.

Complex structure. Assembly of complex structures with dynamic invocation of modules in the shared memory is created in the Assembly process of the modules. In such an aggregate, the connections between the modules are not so rigid and their sequence is determined by the modules included in the chain. The modules are loaded into the main memory at the time of processing. When finished, the memory is freed and used to load another module. As in the case of a simple structure, the graph of a complex program structure is also connected (Fig.4) and is reflected in the adjacency matrix (2).

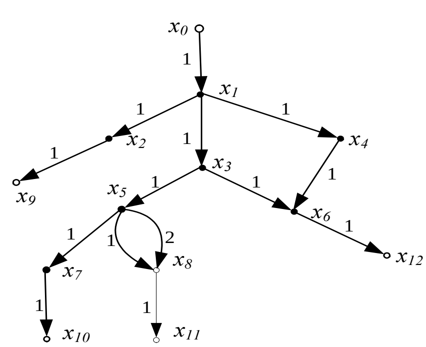

Fig. 4: Modification graph of program structure

The amount of main memory required depends on the number and composition of modules and the maximum amount of memory is equal to the sum of individual modules:

$$v_0^{\max} = V_s = \sum_{i=1}^{n} v_i \,.$$

The minimum amount of memory required when performing the aggregate is calculated by Floyd's algorithm, which determines the shortest path in the graph, in which each arc corresponds to a weight coefficient, called the arc length. The following transformations are performed to apply the Floyd algorithm.

1) Let's add new vertices and arcs to the graph. The vertices are x_0, $x_{n+1}, \ldots , x_{n+m}$, where m is the number of end vertices. New arcs include $(x_0, x_1, 1)$, $(x_{r1}, x_{n+1}, 1)$, ..., $(x_{rn}, x_{n+rn}, 1)$. In them x_1 corresponds to the main module and all x_i – to the end vertices. After performing operations, the graph of the modular structure (Fig. 1) is given to the graph on Fig. 5 with vertices x_0, x_9, x_{10}, x_{11}, x_{12}. It vertices correspond to the weight coefficients:

$$v_0 = v_9 = v_{10} = v_{11} = v_{12} = 0$$

2) Each arc of the form (x_i , x_j, k) is assigned a coefficient $v_{ij} = \dfrac{v_i + v_j}{2}$.

Consider all routes leading from x_0 to one of the other additional vertices. The length of the shortest route path is calculated as follows:

$$l_{0,n+p} = v_{01} + \ldots + v_{rp,n+p} = \frac{v_0 + v_1}{2} + \ldots + \frac{v_{2p} + v_{n+p}}{2} = \frac{v_0}{2} + v_1 + \ldots + v_{rp} + \frac{v_{n+p}}{2} = v_{1+} \ldots {}_{+}\, v_{rp}.$$

This length $l_{0,\, n+p}$ will be equal to the sum of the memory modules for path $x_1, \ldots , x_{rp}$.

Thus, applying Floyd's algorithm to the graph in Fig. 2, we solve the problem of calculating the amount of memory for the maximum chain.

3) We replace the adjacency matrix with the path matrix. For each $m_{ij}>0$, the corresponding location will be v_{ij}. The values $m_{ij} = \varnothing$ are replaced by $-\infty$. The program implementing Floyd's algorithm has the following form (it is assumed that the path matrix is described as a two-dimensional matrix $(n \times n)$: this length $l_{0,\, n+p}$ will be equal to the sum of the memory modules for path $x_1, \ldots , x_{rp}$.

```
for k = 1 to n do
for i = 1 to n do
for j = 1  to n do
if M[i, j ]<M [ i , k] + M[k, j] then
M [i, j]: = M [ i , k] + M[k, j].
```

As a result of this algorithm, a matrix of maximum paths will be constructed. The maximum of $l_{0,n+p}$ will determine the minimum amount of $l_{0,n+p}$ memory for the memory-overlapping aggregate.

The most complex structure for the values $V_0^{min} \leq V_0 \leq V_0^{max}$ can be constructed by following the algorithms proposed in [2-6]. The qualitative dependence of V_0 on the number of dynamic sites is shown in Fig.5. Here n is the number of modules in the unit. Despite the different kind of curves, they have a common pattern – any V_0 is enclosed between the values of v_0^{max} и v_0^{min}.

Notes

Dynamic structure. The mechanism of dynamic links between modules is different from the call mechanism. Dynamic objects are loaded into the main memory when they are accessed. By analogy, we call the volume loaded with a single treatment of a dynamic element, has its own program structure, for which the adjacency matrix is composed. If the same modules are found in different dynamic structures, they are different objects.

The original graph is used for illustration (Fig.1). Let the module corresponding to the vertex x_1, be dynamically called from the module corresponding to the vertex x_3. The resulting modified graph is shown in Fig. 6. A dashed arrow indicates a dynamic call. The module corresponding to the vertex x_6, occurs twice.

We construct an adjacency matrix for this aggregate. Each dynamic element will have its own *CALL* . To distinguish a dynamic call, the corresponding matrix elements will contain negative numbers whose absolute values specify the number of dynamic calls between the data of the module pair [20].

The adjacency matrix will look like:

$$M=\begin{pmatrix} x_1 & x_2 & x_4 & x_6 & x_3 & x_5 & x_6 & x_7 & x_8 \\ 0 & 1 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad (20)$$

We investigate the qualitative dependence of the amount of the number of dynamic segments (Fig.5. and 6). With one component in the software unit of a simple structure we have $V^1_d = V_s$. If each dynamic component consists of one module, then the modified Floyd algorithm finds the maximum path and $V_d^n = V_0^{min}$.

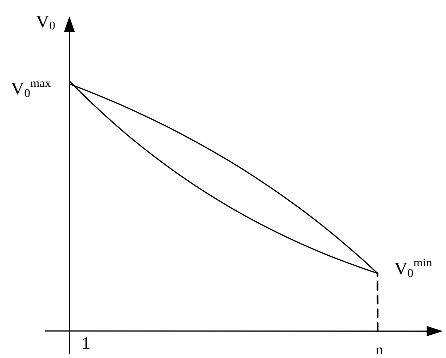

Fig. 5: Grafics of qualitative dependence V_a from the number of sub graphs

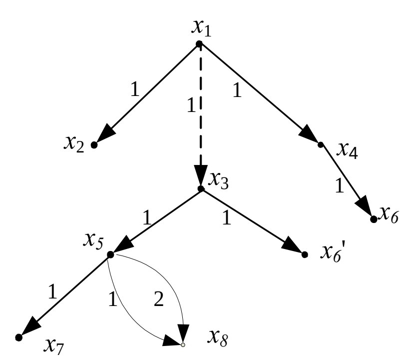

Fig. 6: Graph programs structure with dynamic Calls

For intermediate values, the dependence is more complex. On fig.7 presents two curves (1, 2), and n is the number of modules in the program unit.

Curve 1 defines a relationship in which different segments do not have the same modules. Curve 2 describes the dependence for the case when different segments have the same modules. For them, the required memory increases due to the duplication of

Ref

20. Lavrischeva E. M. Software Engineering of computer systems. Paradigms, technologies, CASE- means – Science Dumka.- 2014.-284p.

such modules. However, dependence 2 is typical for the case when there are no identical modules in dynamic structures and they are written in high-level *LP*. These modules are handled by utility tools – memory management, I/O, emergency handling, etc.

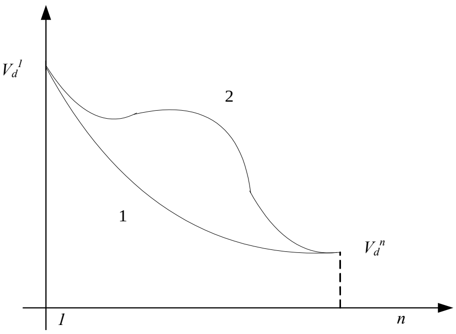

Fig. 7: Grafic dependence V_a from the number of dynamic elements

Due to the duplication of modules there is an increase in the main memory of the *OS*. Thus, curve 1 is characteristic of software aggregates of graphs in the form of a tree, which ensures that there are no identical modules in the graph. Despite the lack of dynamic structure in terms of memory savings, there is a significant advantage – independence from editing links. Each dynamic object can be modified, and editing relationships in the *OS* is not required.

III. Assembling Nodules of Graph G to Programs

Let the graph G be represented by the set of modules X= $\{x_1, x_2, ..., x_m\}$, as described in *LP*, and located at the vertices of the graph. The modules are assembled into a software unit. In this case, each pair of modules x_i, x_j (*i, j* – languages from the set of *LP* are connected by the relation of call on the basis of which the module of communication x'_{ij} is formed. In General, for simple program structures, the aggregate contains link communication (call) operators and forward and reverse transformations of data types passed from the calling module (in *i*-language) to the calling module (in *j*-language) and back [23].

LP allows you to describe the information part - passport modules with a description of the transmitted data [8-14] and operations call modules. Taking into account the passports of the modules, the software structure of the unit is built (program - *Prog*, complex - *Comp*, package - *Pac*). The passport describes the special language WSDL containing: - a subset of the operations associate link elements of the graph in the language *L'* that contains a description of the parameters from the list of actual and formal parameters of the invocation; - mathematical operations on the graph and operations of binding modules in a complex structure (*Prog*, *Comp*, *Agr*, *Pac* and so on).

The operator modules link (make, config, assembling, etc. since 1994) takes the form:

Link <aggregate type> <aggregate name> (<main module name>, <additional list of module names>) <execution mode>, when constructing specific program structures, the vertices of the graph – modules can be marked with special symbols ρ, denoting:

ρ = Λ – formation of a fragment with the name of the module;

ρ = * – the beginning of the dynamic fragment with the vertex marked by this symbol;

ρ = + the module in the graph G is marked as the main program of the complex;

ρ = / – means enabling debugging or testing of the unit.

Ref
23. Ekaterina M.Lavrischeva. Assemblling Paradigms of Programming in Software Engineering.- 2016, 9, p.296-317, http://www.scrip.org/journal/jsea, http:// dx.do.org/10.4236/jsea.96021.

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Using these designations, the graph G will take the form shown in figure 8 and has a representation: $E = \{(x_5, x_7, 1),\ (x_5, x_8, 1),\ (x_5, x_8, 2)\}$.

The aggregate is given a unique name corresponding to the generated root module. For the graph $\Gamma = \{(x_4,\ x_6,\ 1)\}$ a fragment of operators providing a dynamic call will be formed in the communication module x'_{46}. For a pair of modules specified in Fig.8 vertices x_4, x_6, the structure of the corresponding part of the unit, including the communication module, is shown in Fig. 9. Similarly links of heterogeneous modules and other types of calls are implemented.

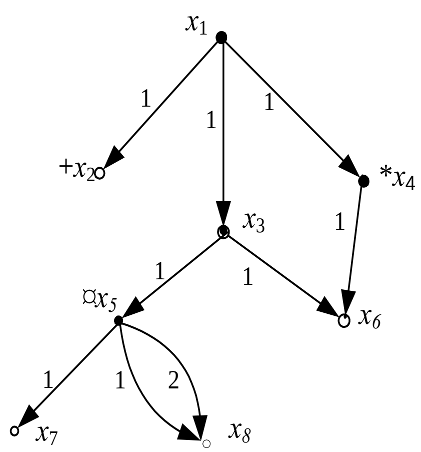

Fig. 8: Graph of software unit with control marks on the graph

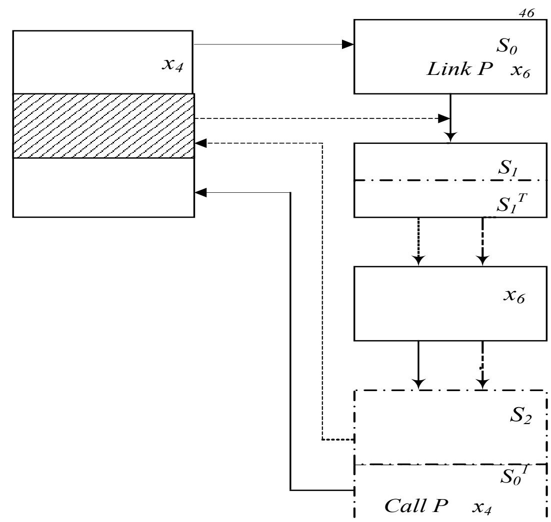

Fig. 9: Graph of modular structure with dynamic call [1]

Thus, for a pair of modules x_i, x_j, a module of connection x_{ij} of the form:

$$x'_{ij} = S_0 * (S_1 \times S_1^T) * (S_2 \times S_2^T) * S_0^1,$$

where S_0 is a fragment of the aggregate that defines the environment of x*j* module functioning;

S_1 – a fragment of the aggregate, including a sequence of calls to functions from the set $\{P,\ C,\ S\}$, each of which performs the necessary conversion of the actual parameters when referring to the x_j -module;

S2 – a system with a fragment of operators for the inverse transformation of data types transmitted from x_j to x_i after its execution;

S_0^1 – a piece of software structures with operators epilogue for the vertex x_i, for the restoration of the environment.

For the described program structures, set the link operations to build the individual programs in Fig.8:

Link Prog P_1 (x_1, x_2);

Link Prog P_2 (x_1, x_3) (x_3, x_6);

Link Comp P_3 $((x_1, x_3)$ $(x_3, x_5/\ x'_{58}) + (x_5, x_7)$; (20)

Link Prog P_4 $(x_1, x_4), (x_4, x_6)$;

Link Comp $(P_1 \cup P_2 \cup P_3 \cup P_4)$.

Programs of the complex (aggregate) are given unique names (P_1, P_2, P_3, P_4) corresponding to the root names of the modules in the chains of the graph.

Thus, the process of constructing the program structure on the graph includes:

1. Enter the module description in the *LP* (L') and perform syntax checking.
2. Select the required modules and interfaces from the repositories and place them in the graph.
3. Translation of the unit modules in the *LP*.
4. Generation of communication modules for each interconnected pair of graph modules.
5. Assembly of the elements of the graph in the finished structure, linking modules in the operating system (IBM, MS, Oberon, Unix и др.) [1-5].
6. Test the system on data sets and assess the reliability of the unit.

After the modules are built, the name of the software Assembly is entered into the boot library. If you create a fragment that is later included in another aggregate, its name must match the name of the main module. In connection with the transition to the Internet environment to work with various software and system services in the configuration assembly of such tools provides security, data protection and quality assessment of ready-made modules, service resources and web systems in Internet.

a) Ready-made software elements configurate to the system

Under *the configuration of the system* is understood the structure of some of its version, including software elements, combined with each other by link operations with parameters that specify the options for the functioning of the system [1, 2, 16-22]. Version or variant of system configuration according to the IEEE Standard 828-2012 (Configuration) includes:

– configuration basis – BC (Configuration Baseline);

– configuration items (Configuration Item);

– program elements (modules, components, services, etc.) included in the graph;

Configuration Management is to monitor the modification of configuration parameters and components of the system, as well as to conduct system monitoring, accounting and auditing of the system, maintaining the integrity and its performance. According to the standard, the configuration includes the following tasks:

1. Configuration identification.
2. Configuration Control.
3. Configuration Status Accounting.
4. Configuration audit.
5. Trace configuration changes during system maintenance and operation;
6. Verification of the configuration components and testing of the system.

A configuration build uses a system model and a set of out-of-the-box components that accumulate in the operating environment repositories or libraries, and selects their operating environment Configurator (for example, in http://7dragons.ru/ru). The Configurator assembles the components according to their interfaces and generates a system configuration file.

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The Configurator assembly of components and reuses with operation config, which is equivalent to the operations link (20) for figure 8, taking into account their interfaces. The *config* statement generates a program variant or system configuration file of Comp.

IV. THE MODERN AND FUTURE PARADIGMS PROGRAMMING

Notes

The theory of system programming is represented by numerous paradigms - mathematical, object-component, ontological, technical, service, aspect, etc. They replace on the *LP* and realized by different paradigms [15-17, 23-25], some of this paradigms are presented below.

a) Mathematical programming paradigms

Theory of graphs for design software modular structures with mathematical operations (union, projection, difference, etc.) implementation of linking the graph modules (objects) and the semantics of the transformation of data transmitted by the vertices of the graph *G*.

Object-component methods - OCM

OCM are the mathematical design of systems from ready-made resources (objects, components, services, etc.) to OM (Object Model). It is the formal method which transform the elements *OM* to a component model or a service model [15, 26].

Graph objects is designed on four levels:

Generalizing for determining *SD* base notions without considering of their essences and properties;

Structuring for ordering objects in the *OM* taking into account relationships between them;

Characterization for forming concepts of objects on the base of them properties and descriptions;

Behavioral level for descriptions of conduct depending on events (such as time).

That is, vertices of the graph *G* are objects of two types: $O = (O_0, O_1, O_2, O_n)$ with *the* object relations hold $\forall i\ (i > 0) \Rightarrow (O_i \in O_0)$ and interface objects *I* (Fig.10).

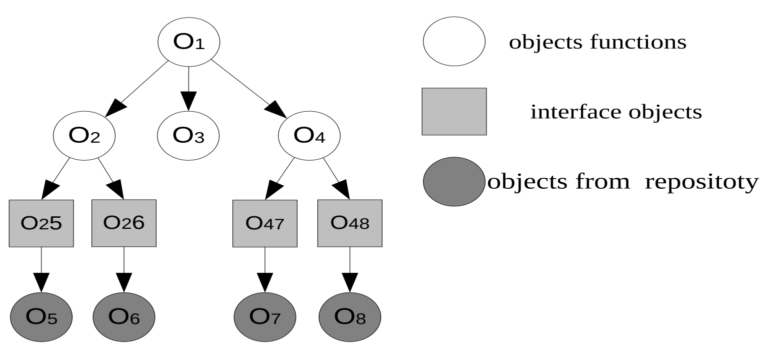

Figure 10: Object-interface graph *G*

At the vertices of a graph *G* contains the functional objects $O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8$ and interface objects —$0'_{25}, O'_{26}, O'_{47}, O'_{48}$, which are placed in the repository of system, and arcs correspond to relationships between all kinds of objects. The parameters of the external characteristics of the interface objects are passed between objects through specified interfaces and are designated in language IDL *in* (input interface), *out* (output) and *inout* (intermediate). Based on the graph *G* we can construct a program P_0— P_5 using mathematical operation ∪ Assembly link:

1) $P_0 = (P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_5)$.
2) $P_1 = O_2 \cup O_5$, $link\ P_1 = In\ O'_5\ (O_2 \cup O_5)$;
3) $P_2 = O_2 \cup O_6$, $link\ P_2 = In\ O'_6\ (O_2 \cup O_8)$;
4) P_3;
5) $P_4 = O_4 \cup O_7$, $link\ P_4 = In\ O'_7\ (O_4 \cup O_7)$;
6) $P_5 = O_4 \cup O_8$, $link\ P_4 = In\ O'_8\ (O_4 \cup O_8)$;

The set of objects and interfaces of the graph is reflected by general or individual properties and descriptions of the object model. Verification of properties of objects is provided by the specific operations (classification, specialization, aggregation, etc.) [15, 20, 29].

Component paradigm. The basis of this paradigm - OCM graph in which vertexes are the components of the CRP (reuses), interfaces and arcs specify the subject classification and the relationship between the vertices. Components are described by the formalisms of the triangle of Frege [15]:

- sign – identifier of the real function entity;
- denotation – the designation of this entity;
- concept – a set of properties defined by logical connections and must be true. Operations of OCM and component algebra represented on the website http://7dragons/ru/ru (in the VS environment.MS IBMSphere, Java, Linux, Intel etc.) [20].

Service-component paradigm. System and service-components - web resources implement intellectual knowledge of specialists about applied fields in the Internet environment [22 - 26]. Each implements some function and communicates with the technological interface to interact with other services through protocols and provide Assembly and solution of applications of different nature. The means of describing the application systems include:

XML for description and construction of SSA components;

WSDL to describe web services and their interfaces;

SOAP to determine the formats of requests to the web services;

UDDI for integration of services and their storage in libraries;

building configuration (config) of the service resources in some high-quality and secure systems.

The theory of graphs develop in the school of A.P. Ershov (V.I. Kasyanov, V.E. Itkin, A. A. Evstigneev et al.) for programming Systems [13]. The graph theory has been actively developing in the Russian Academy of Sciences (I.B.Burdonov, A.S. Kosachev, V. V. Kulyamin [19]. The theory of conformity for systems with blocking and destruction for the schematic organization of memory in Linux.

Methods of production of factories (Product Line/Product Family) programs and Appfab and certificate them of the quality are discussed [23].

Application of the ontology language OWL (www.semantic˙web.com), resource language (RDF) and intelligent agents of ISO 15926 standard for networking.

Ontology of Life Cycle and *Computational geometry* is a part of computer graphics and algebra. Used in the practice of computing and control machines, numerical control etc. is also used in robotics (motion planning and pattern recognition tasks), geographic information systems (geometric search, route planning), design chips, etc.[25-30].

Ref

19. Burdonov I. B., Kosachev A. S., Kulyamin V. V. Theory for systems with locks and destructions.-Moscow, 2008.- 411p.

Cloud technologies (PaaS, SaaS) are related to the Internet and are used to create adaptive applications that interact through agents of web pages.

Device configuring Big Data Processing Devices (Big Data) in Smart Data Internet 4.0.

b) Intellectualization of systems

The intelligent system implements creative tasks, the knowledge of which is stored in its memory. It includes — knowledge base, output mechanism and intelligent interface. The main tasks of artificial intelligence: symbolic modeling of thought processes, work with natural languages, presentation and use of knowledge, - machine learning, biological modeling of artificial intelligence, robotics [30].

c) Application technical programming

Event management paradigm based on the processing of external events (event-driven programming) in the Window environment. Features of the event paradigm are the use of testing methods based on operational (scenario) profiles of programs [20, 25, and 28].

Coordinated and parallel programming provides a division of the computational process into several subtasks (processes) for TRAN's computers and supercomputers, the results of which are sent via communication channels. Languages for parallel programming - PVM, LAM. CHMP and MPI (Message Passing Interface) interface descriptions and OpenMP. The POSIX standard provides messaging between programs in YAP C, C+ and Fortran.

Programming on classes and on a prototype in OOP. The principles of the OOP are: inheritance – the mechanism of establishing relations "descendant-ancestor" (the ability to generate one class from another with the preservation of all the properties and methods of the class-ancestor); encapsulation (the hiding of class implementation); abstraction (description of interaction only in terms of messages/events in the subject area); polymorphism (the possibility of replacing the interaction of objects of one object with another object with a similar structure). Many modern languages are specially created for programming on classes, for example, Smalltalk, C++, Java, Python, PHP, Object Pascal (Delphi), VB.NET, Xbase++, etc.

Programming by prototype. Creating a new object is done by one of two methods: cloning an existing object, or by creating an object from scratch. Reuse (inheritance) is made by cloning an existing instance of the object —a prototype Clone, a sample. An example of a prototype language is the Self language and it is the basis of such programming languages as JavaScript, Squeak, Cecil, Newton Script, Io, MOO, REBOL, Keno and etc.

The Agile methodology is focused on the close collaboration of a team of developers and users. It is based on a waterfall model lifecycle incremental and rapid response to changing demands on PP. The team works according to the schedule and financing of the project.

eXtreme Programming (XP) implements the principle of "collective code ownership". It any member of the group can change not only your code but also code another programmer. Each module is supplied with the Autonomous test (unit test) for regression testing of modules. Tests written by the programmers and they have the right to write tests for any module. Thus, most of the errors are corrected at the stage of encoding, or when you view the code, or by dynamic testing.

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30. Lavrischeva E.M. Scientific Basis of System Programming.- Journal of Software Engineering and Applications (JSEA), Vol. 11 No. 8 of August issue, 2018.-N 11.- p.408-434, ISSN 1945-3124, ISSN online 1945-3116. ISSN Print http:// www.scirp.org/journal/jsea

SCRUM is agile methodology project management firm Advanced Development Methods, Inc., used in organizations (Fuji-Xerox, Canon, Honda, NEC, Epson, Brother, 3M, Xerox and Hewlett - Packard etc.) are based on an iterative lifecycle model with well-defined development process, including requirements analysis, design, programming, testing (http://agile.csc.ncsu.edu).

DSDM (Dynamic Systems Development Method) for rapid development of RAD (Rapid Application.

d) Ensuring the quality of AS, Web systems

According to the ISO / IEC 12207 standard, the software LC standard regulates the planning, quality management and cost estimation for the creation of the system. In these processes, lifecycle analyses quality assurance; verification and validation (V&V) resources and evaluating the degree of achievement of individual quality indicators; test of the finished system; data collection about the failures, defects etc. and failures; assessment of the reliability on the respective reliability models based on the results of testing [33, 35, 36].

The ISO/IEC 9000 (1-4) Quality model defines characteristics q_1 — q_6: q_1 (functionality), q_2 (reliability), q_3 (usability), q_4 (efficiency), q_5 (maintainability), $q_{6(}$portability).

Each q_i characteristic is calculated according to special formulas and metrics of the standard. Reliability is evaluated according to the errors, defects and failures in the SOFTWARE obtained during the testing process and according to the corresponding reliability models (evaluation, measurement, etc.). Data on all quality indicators q_1 — q_6 are evaluated by the formula:

$$q_i = \sum_{j=1}^{6} a_{ij} m_{ij} w_{ij}$$

where a_{ij}-attributes of each quality indicator (i = 1..6); m_i - metrics of each quality attribute; w_i - weight of each attribute of the system quality indicator. The obtained values for the indicators of the quality model are included in the product quality certificate. Options for assessing the quality of configured systems are given on the website http://7dragons.ru/ru.

e) Perspective directions for the development of the Internet

Promising areas of development for the Internet1 include [28]:

The information objects (IO) that specifies the digital projection of real or abstract objects that use Semantic Web Ontology interoperability interfaces. IO through Web services began more than 10 years ago. Interaction semantics IO is based on RDF and OWL language of ISO 15926 Internet 3.0.

The next step of the development of the Internet is Web 4.0, which allows network participants to communicate, using intelligent agents. A new stage in the development of enterprise solutions-cloud (PaaS, SaaS) who spliced with Internet space and used to create Adaptive applications. Cloud services interact through the Web page by using agents.

Internet of Things Smart IoT to support competitive APPS using: distributed microservices; Hypercat Mobile; GSM-R traffic control. Industrial Internet develops concepts - “smart energy”, “smart transportation”, “smart appliances”, “smart industry”, “smart homes and cities”, etc. Internet stuff (Internet of Things, Smart IoT) indicates the Smart support competing APPS using distributed micro services such as

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Hyper cat (mobile communications); industrial Internet (Industrial), covering the new automation - transportation, appliances, industry¸, and another.

f) Computer nanotechnology

Today computer nanotechnology is actually already working with the smallest elements, "atoms" similar to the thickness of the thread (transistors, chips, crystals, etc.). For example, a video card from 3.5 million particles on single crystal, multi-touch maps for retinal embedded in the eyeglasses, etc.

In the future, ready-made software elements will be developed in the direction of nanotechnology by "reducing" to look even smaller particles with predetermined functionality. Automation of communication, synthesis of such particles will give a new small element, which will be used like a chip in a small device for use in medicine, genetics, physics, etc.[28].

V. Conclusion

In the early stages of the emergence of the method of assembling large programs and complexes of spent modules in the LP used the theoretical apparatus of graphs to create modular program structures. Graph theory allows to establish the shortest path of program elements and prove the correctness of binding graph modules using adjacency matrices, reach ability and mathematical operations (association, connection, difference, etc.) in complex program structures (complex, aggregate, system, etc.). Initially, the method of Assembly on the basis of graph theory was widely implemented in the complex systems (Juza, Ruza, Prometheus) under the leadership of Lipaev V.V. [1-5], and was supported by A. P. Ershov in the IPI SO Academy of Sciences SSSR and his researcher and scientist, who formulated the theoretical aspects of the application of graph theory in programming [6-14]. Since 2013, graph theory has been used in the modeling of complex systems of objects, components, services, etc. (OCM) [15] and has been used in the world practice in the transition to the Internet environment [22, 23]. The paper describes the features of modeling systems using graph theory and mathematical operations on elements of software structures. The new Assembly operations – *config* of the IEEE Standard 828-2012 (Configuration) are implemented in different environments of Internet. Elements of the graph set transition labels to obtain reactions at the time of exposure to test sets, proof of completeness of testing systems of AS and the measurement of quality ready AS.

Programming paradigms on the graph and ontology of mathematical modeling of applied problems for vital areas of society (medicine, biology, physics, mathematics, economics, etc.) will become the main tools of smart machines and AS of the 21st century [15, 24-31].

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We accept the manuscript submissions in any standard (generic) format.

We typeset manuscripts using advanced typesetting tools like Adobe In Design, CorelDraw, TeXnicCenter, and TeXStudio. We usually recommend authors submit their research using any standard format they are comfortable with, and let Global Journals do the rest.

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2. Drafting the paper and revising it critically regarding important academic content.
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Unless specified in the notification, the Editorial Board's decision on publication of the paper is final and cannot be appealed before making the major change in the manuscript.

Acknowledgments

Contributors to the research other than authors credited should be mentioned in Acknowledgments. The source of funding for the research can be included. Suppliers of resources may be mentioned along with their addresses.

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Authors can submit papers and articles in an acceptable file format: MS Word (doc, docx), LaTeX (.tex, .zip or .rar including all of your files), Adobe PDF (.pdf), rich text format (.rtf), simple text document (.txt), Open Document Text (.odt), and Apple Pages (.pages). Our professional layout editors will format the entire paper according to our official guidelines. This is one of the highlights of publishing with Global Journals—authors should not be concerned about the formatting of their paper. Global Journals accepts articles and manuscripts in every major language, be it Spanish, Chinese, Japanese, Portuguese, Russian, French, German, Dutch, Italian, Greek, or any other national language, but the title, subtitle, and abstract should be in English. This will facilitate indexing and the pre-peer review process.

The following is the official style and template developed for publication of a research paper. Authors are not required to follow this style during the submission of the paper. It is just for reference purposes.

Manuscript Style Instruction (Optional)

- Microsoft Word Document Setting Instructions.
- Font type of all text should be Swis721 Lt BT.
- Page size: 8.27" x 11'", left margin: 0.65, right margin: 0.65, bottom margin: 0.75.
- Paper title should be in one column of font size 24.
- Author name in font size of 11 in one column.
- Abstract: font size 9 with the word "Abstract" in bold italics.
- Main text: font size 10 with two justified columns.
- Two columns with equal column width of 3.38 and spacing of 0.2.
- First character must be three lines drop-capped.
- The paragraph before spacing of 1 pt and after of 0 pt.
- Line spacing of 1 pt.
- Large images must be in one column.
- The names of first main headings (Heading 1) must be in Roman font, capital letters, and font size of 10.
- The names of second main headings (Heading 2) must not include numbers and must be in italics with a font size of 10.

Structure and Format of Manuscript

The recommended size of an original research paper is under 15,000 words and review papers under 7,000 words. Research articles should be less than 10,000 words. Research papers are usually longer than review papers. Review papers are reports of significant research (typically less than 7,000 words, including tables, figures, and references)

A research paper must include:

a) A title which should be relevant to the theme of the paper.
b) A summary, known as an abstract (less than 150 words), containing the major results and conclusions.
c) Up to 10 keywords that precisely identify the paper's subject, purpose, and focus.
d) An introduction, giving fundamental background objectives.
e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition, sources of information must be given, and numerical methods must be specified by reference.
f) Results which should be presented concisely by well-designed tables and figures.
g) Suitable statistical data should also be given.
h) All data must have been gathered with attention to numerical detail in the planning stage.

Design has been recognized to be essential to experiments for a considerable time, and the editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned unrefereed.

i) Discussion should cover implications and consequences and not just recapitulate the results; conclusions should also be summarized.
j) There should be brief acknowledgments.
k) There ought to be references in the conventional format. Global Journals recommends APA format.

Authors should carefully consider the preparation of papers to ensure that they communicate effectively. Papers are much more likely to be accepted if they are carefully designed and laid out, contain few or no errors, are summarizing, and follow instructions. They will also be published with much fewer delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and suggestions to improve brevity.

Format Structure

It is necessary that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

All manuscripts submitted to Global Journals should include:

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The title page must carry an informative title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) where the work was carried out.

Author details

The full postal address of any related author(s) must be specified.

Abstract

The abstract is the foundation of the research paper. It should be clear and concise and must contain the objective of the paper and inferences drawn. It is advised to not include big mathematical equations or complicated jargon.

Many researchers searching for information online will use search engines such as Google, Yahoo or others. By optimizing your paper for search engines, you will amplify the chance of someone finding it. In turn, this will make it more likely to be viewed and cited in further works. Global Journals has compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Keywords

A major lynchpin of research work for the writing of research papers is the keyword search, which one will employ to find both library and internet resources. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining, and indexing.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy: planning of a list of possible keywords and phrases to try.

Choice of the main keywords is the first tool of writing a research paper. Research paper writing is an art. Keyword search should be as strategic as possible.

One should start brainstorming lists of potential keywords before even beginning searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in a research paper?" Then consider synonyms for the important words.

It may take the discovery of only one important paper to steer in the right keyword direction because, in most databases, the keywords under which a research paper is abstracted are listed with the paper.

Numerical Methods

Numerical methods used should be transparent and, where appropriate, supported by references.

Abbreviations

Authors must list all the abbreviations used in the paper at the end of the paper or in a separate table before using them.

Formulas and equations

Authors are advised to submit any mathematical equation using either MathJax, KaTeX, or LaTeX, or in a very high-quality image.

Tables, Figures, and Figure Legends

Tables: Tables should be cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g., Table 4, a self-explanatory caption, and be on a separate sheet. Authors must submit tables in an editable format and not as images. References to these tables (if any) must be mentioned accurately.

Figures

Figures are supposed to be submitted as separate files. Always include a citation in the text for each figure using Arabic numbers, e.g., Fig. 4. Artwork must be submitted online in vector electronic form or by emailing it.

Preparation of Eletronic Figures for Publication

Although low-quality images are sufficient for review purposes, print publication requires high-quality images to prevent the final product being blurred or fuzzy. Submit (possibly by e-mail) EPS (line art) or TIFF (halftone/ photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Avoid using pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings). Please give the data for figures in black and white or submit a Color Work Agreement form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

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Tips for Writing a Good Quality Science Frontier Research Paper

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1. Choosing the topic: In most cases, the topic is selected by the interests of the author, but it can also be suggested by the guides. You can have several topics, and then judge which you are most comfortable with. This may be done by asking several questions of yourself, like "Will I be able to carry out a search in this area? Will I find all necessary resources to accomplish the search? Will I be able to find all information in this field area?" If the answer to this type of question is "yes," then you ought to choose that topic. In most cases, you may have to conduct surveys and visit several places. Also, you might have to do a lot of work to find all the rises and falls of the various data on that subject. Sometimes, detailed information plays a vital role, instead of short information. Evaluators are human: The first thing to remember is that evaluators are also human beings. They are not only meant for rejecting a paper. They are here to evaluate your paper. So present your best aspect.

2. Think like evaluators: If you are in confusion or getting demotivated because your paper may not be accepted by the evaluators, then think, and try to evaluate your paper like an evaluator. Try to understand what an evaluator wants in your research paper, and you will automatically have your answer. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

3. Ask your guides: If you are having any difficulty with your research, then do not hesitate to share your difficulty with your guide (if you have one). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work, then ask your supervisor to help you with an alternative. He or she might also provide you with a list of essential readings.

4. Use of computer is recommended: As you are doing research in the field of science frontier then this point is quite obvious. Use right software: Always use good quality software packages. If you are not capable of judging good software, then you can lose the quality of your paper unknowingly. There are various programs available to help you which you can get through the internet.

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6. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right? It is a good habit which helps to not lose your continuity. You should always use bookmarks while searching on the internet also, which will make your search easier.

7. Revise what you wrote: When you write anything, always read it, summarize it, and then finalize it.

8. Make every effort: Make every effort to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in the introduction—what is the need for a particular research paper. Polish your work with good writing skills and always give an evaluator what he wants. Make backups: When you are going to do any important thing like making a research paper, you should always have backup copies of it either on your computer or on paper. This protects you from losing any portion of your important data.

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Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

14. Arrangement of information: Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

15. Never start at the last minute: Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

16. Multitasking in research is not good: Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

17. Never copy others' work: Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

18. Go to seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

19. Refresh your mind after intervals: Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.

20. Think technically: Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

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22. Report concluded results: Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

23. Upon conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

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The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

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- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
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- Avoid use of extra pictures—include only those figures essential to presenting results.

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Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

Abstract: This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

Approach:

- o Single section and succinct.
- o An outline of the job done is always written in past tense.
- o Concentrate on shortening results—limit background information to a verdict or two.
- o Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

Introduction:

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.

The following approach can create a valuable beginning:

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

Approach:

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

Procedures (methods and materials):

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

Materials may be reported in part of a section or else they may be recognized along with your measures.

Methods:

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

What to stay away from:

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

Figures and tables:

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

Discussion:

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Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- Give details of all of your remarks as much as possible, focusing on mechanisms.
- Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

Describe generally acknowledged facts and main beliefs in present tense.

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Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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