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Highlights

- One-Equation Model
- Solution of Integral Equation
- Twin Prime Conjecture
- Space of Monetary Risk Measures

Discovering Thoughts, Inventing Future

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Solution of Integral Equation in Two-Dimensional using Spectral Relationships

By F. M. Alharbi
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Abstract- This paper concerned using spectral relationships in the solution of the integral equation (IE) in two-dimensional. To discuss that, the (IE) in two-dimensional under certain conditions was considered. The existence of at least one solution of the (IE) was discussed by proving the continuity and compactness of an integral operators. Chebyshev polynomials of the first kind were used to transform the (IE) to a linear algebraic system. Many numerical results and estimating errors were calculated and plotted by the Maple program in different cases.

Keywords: *Integral equation (IE) in two-dimensional. Schauder fixed point theorem. Continuity and compactness of an integral operators. Spectral relationships .Linear algebraic system.*

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SOLUTIONOFINTEGRALEQUATIONINTWODIMENSIONALUSINGSPECTRALRELATIONSHIPS

Solution of Integral Equation in Two-Dimensional using Spectral Relationships

F. M. Alharbi

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Keywords: Integral equation (IE) in two-dimensional. Schauder fixed point theorem. Continuity and compactness of an integral operators. Spectral relationships .Linear algebraic system.

I. Introduction

In recent years, the main theorem of spectral relationships plays an important role in many applications in various areas, including models of nanotech nology particles, genetic engineering, medicine, mathematical chemistry, heat condition and physical phenomena, see [1-4].

On the other side, many problems of mathematical physics, contact problems in elastic media, many applications in electronic engineering applications, and mathematical biology models lead to an **(IEs)** in smooth or singular forms. The references [5-7] discuss the methods for solving the **(IEs)** in smooth form antically. While the singular forms take a large aria in types of researches , see [8-11]. In[10], a spectral technique for solving two-dimensional fractional integral equations with singular kernel was discussed by using Legendre and Chebyshev polynomials. Abdou and Salama in [12] obtained the spectral relationships for the Volterra-Fredholm integral equation **(V-FIE)** of the first kind. Abdou [13] discussed the spectral relationships that have many important applications in astrophysics, for the **(F-VIE)** of the first kind, when the kernel of position takes a generalized potential form. The relation between the contact problems and the **(F-VIE)** in three dimensions were obtained by Abdou and Moustafa in [14]. Abdou and Nasr in [15] used Chebyshev polynomial to obtain the solution of **(F-VIE)** when the kernel takes a logarithmic form. The application of Orthogonal polynomials in spectral relationships of some kinds of singular contact problems is discussed by Alharbi in [16].Abdou and Basseem in [17] used Chyebyshev polynomial, the main theorem of spectral relationships of **(FIE)** to obtain the solution of **(F-VIE)** of the second

Ref

10. F.M.Alharbi, *Boundary and Initial Value problems and Integral operator,* Computational and Applied Mathematics, 19(4)(2018), 391-404.

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kind numerically. Abdou and Alharbi in [18] derived a general main theorem of spectral relationships for a mixed integral equation of the first kind **(MIE)** in the space $L_2[-1,1] \times C[0,T]$.

Consider the integral equation,

$$\phi(x,y) = f(x,y) + \lambda \int_{-1}^{1}\int_{-1}^{1} k(x,u,y,v)\phi(u,v)dudv, \tag{1}$$

Under the following conditions:

i. $\left[\int_{-1}^{1}\int_{-1}^{1} k^2(x,u,y,v)\, dxdudydv\right]^{1/2} \le c,\ \ c \text{ is constant}.$

ii. The function $f(x,y)$ with its partial derivatives with respect to x and y are continuous in $L_2[-1,1] \times [-1,1]$ and its norm can be defined as,

$$\|f(x,y)\| = \left[\int_{-1}^{1}\int_{-1}^{1} f^2(x,y)\, dxdy\right]^{1/2} \le M \ \ ,\ \ M \text{ is constant}.$$

iii. The unknown function $\phi(x,y)$ in the space $L_2[-1,1] \times [-1,1]$ behaves as the given function $f(x,y)$.

II. The Existence of at Least one Solution of a Two Dimensional IE

Theorem 1

The integral equation (1) has at least one solution under the previous condition(i)-(iii).
Define the integral operator forms:

$$W\phi = \lambda \int_{-1}^{1}\int_{-1}^{1} k(x,u,y,v)\phi(u,v)du\, dv \tag{2}$$

Eq. (1) can be written in operator form as:

$$\bar{W}\phi = f + W\phi. \tag{3}$$

The proof of **theorem1** can be obtained automatically from the proofs of following lemmas.

Lemma1

The integral operator (3) under the conditions (i)-(iii) is bounded in the space $L_2[-1,1] \times [-1,1]$.

Ref

18. M.A. Abdou, F.M. Alharbi, *Generalized main theorem of spectral relationships for Logarithmic kernel and its applications*, Journal of Computational and Theoretical Nanoscience, 16(2019),1-8.

Notes

Proof:

Taking the norm of Eq.(3) we get

$$\|\overline{W}\phi\| \leq \|f(x,y)\| + |\lambda| \left\| \int_{-1}^{1}\int_{-1}^{1} k(x,u,y,v)\, \phi(u,v)dudv \right\| \tag{4}$$

Applying the Cauchy-Schwarz inequality, then using conditions (i)-(iii) to have

$$\|\overline{W}\phi(x,y)\| \leq M + |\lambda|C. \tag{5}$$

Eq.(5) means that the integral operator $\overline{W}$ maps the ball S_α into itself, where $\alpha = M + |\lambda|C$. Also, in the second term of inequality (5), we deduce that the integral operator $W\phi(x,y)$ is bounded in the space $L_2([-1,1]\times[-1,1])$. Therefore, $\overline{W}\phi(x,y)$ is also bounded.

Lemma 2

The integral operator (3) under the conditions (i)-(iii) is continuous in the space $L_2[-1,1]\times[-1,1]$.

Proof:

Let $\phi_1(x,y), \phi_2(x,y)$ be any two functions in the space $L_2[-1,1]\times[-1,1]$ then,

$$\|\overline{W}\phi_1(x,y) - \overline{W}\phi_2(x,y)\| \leq |\lambda| \left\| \int_{-1}^{1}\int_{-1}^{1} |k(x,u,y,v)|\,|\phi_1(x,y) - \phi_2(x,y)| \right\|$$

After applying Cauchy-Shwarz inequality then using the conditions(i)-(iii), the previous inequality becomes,

$$\|\overline{W}\phi_1(x,y) - \overline{W}\phi_2(x,y)\| \leq |\lambda| c \|\phi_1(x,y) - \phi_2(x,y)\|$$

So,

$$\|\overline{W}\| \leq |\lambda| c \tag{6}$$

Inequality (6) implies the continuity of $\overline{W}$ in the space $L_2[-1,1]\times[-1,1]$.

Lemma 3

Suppose that a sequence of continuous functions $\{k_{n,m}(x,u,y,v)\}$ such that,

$$\lim_{n,m\to\infty}\left[\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\left|k_{n,m}(x,u,y,v) - k(x,u,y,v)\right|^2 dxdudydv\right]^{1/2} = 0. \tag{7}$$

Then, there exist positive integers n_0, m_0, such that, for $n > n_0, m > m_0$, in general $n \neq m$, after neglecting the very small constants, we have

$$\left[\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\left|k_{n,m}(x,u,y,v)\right|^{2} dxdudydv\right]^{1/2} \leq c \qquad (8)$$

Proof:

$$\left[\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\left|k_{n,m}(x,u,y,v)\right|^{2} dxdudydv\right]^{1/2}$$

$$\leq \left[\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\left|k_{n,m}(x,u,y,v)-k(x,u,y,v)\right|^{2}\right.$$

$$+2\left|k_{n,m}(x,u,y,v)-k(x,u,y,v)\right|\left|k(x,u,y,v)\right|$$

$$\left.+\left|k(x,u,y,v)\right|^{2}\right)dxdudydv\Bigg]^{1/2}$$

Hence, for each $n > n_0, m > m_0$ using Eq.(7) and conditions(i), formula (8) is verified after neglected a small constant.

Lemma 4

Suppose that $\{\overline{W}_{n,m}(x,y)\}$ is a sequence of integral operators where the conditions(i)-(iii) are satisfied, then the sequence of operators

$$\overline{W}_{n,m}(x,y) = f(x,y) + \lambda\int_{-1}^{1}\int_{-1}^{1} k_{n,m}(x,u,y,v)\phi(u,v)dudv \qquad (9)$$

is the bounded and continuous sequence.

Proof:

Taking the norm of Eq.(9) then using the conditions (i)-(iii) to get

$$\left\|\overline{W}_{n,m}(x,y)\right\| \leq \alpha\,, \qquad \alpha = M + |\lambda|c \qquad (10)$$

Therefore $\overline{W}_{n,m}$ maps the largest ball S_α into itself.

Also to prove the continuity of $\overline{W}_{n,m}$ we choose any two functions $\phi_1(x,y), \phi_2(x,y)$ in S_α then applying Cauchy-Schwarz inequality and the conditions (i)-(iii) we get

$$\left\|\overline{W}_{n,m}\phi_1(x,y) - \overline{W}_{n,m}\phi_2(x,y)\right\| \leq |\lambda|c \quad \forall\, n > n_0\,, m > m_0 \qquad (11)$$

Notes

Notes

Lemma 5

If conditions (i)-(iii) are verified then the $\bar{W}(S_\alpha)$ is compact.

Proof:

$$\left\|\bar{W}_{n,m}\phi(x,y)-\bar{W}\phi(x,y)\right\|=$$

$$|\lambda|\left\|\int_{-1}^{1}\int_{-1}^{1}\left(k_{n,m}(x,u,y,v)-k(x,u,y,v)\right)\phi(u,v)dudv\right\|.$$

Hence, using condition(iii) yields

$$\left\|\bar{W}_{n,m}\phi(x,y)-\bar{W}\phi(x,y)\right\|$$

$$\leq|\lambda|E\left[\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\left|k_{n,m}(x,u,y,v)-k(x,u,y,v)\right|^2dxdudydv\right]^{1/2}$$

Also, from Eq.(7) we have the following condition:

$$\left\|\bar{W}_{n,m}\phi(x,y)-\bar{W}\phi(x,y)\right\|=0,\qquad as\ n,m\to\infty\,. \tag{12}$$

To prove the compactness of $\bar{W}$, we let $\{\phi_{n,m}(x,y)\}$ be any sequence in S_α. Then we can choose a subsequence$\{\phi_{n1,m}(x,y)\}$ such that $\{\bar{W}_{n1,m}\phi_{n1,m}(x,y)\}$ converges. From that subsequence, we can extract a new subsequence $\{\phi_{n1,m1}(x,y)\}$ in which $\{\bar{W}_{n1,m1}\phi_{n1,m1}(x,y)\}$converges, and so on.Thus, we obtain a chain of subsequences,

$$\{\phi_{n,m}(x,y)\}\supset\{\phi_{n1,m}(x,y)\}\supset\{\phi_{n1,m1}(x,y)\}\supset\cdots\supset\{\phi_{ni,mj}(x,y)\}\supset\cdots$$

Such that the sequence $\{\bar{W}_{ni,mk}\phi_{nj,ml}(x,y)\}$ converges for all $i=1,2,\dots,j\ and\ k=1,2,\dots,l$. Finally, we pick the sequence $\{\phi_{n_n,m_m}(x,y)\}$ which is a subsequence of every ϕ_{n_i,m_k} except for a finite number of elements, and clearly $\{\bar{W}_{ni,mk}\phi_{nn,mm}(x,y)\}$ converges for every i,k. Now, since

$$\left\|\bar{W}_{n_i,m_k}\phi_{n_n,m_m}-\bar{W}_{n_i,m_k}\phi_{p_p,q_q}\right\|\to 0\ \ as\ m,n,p,q\to 0\,.$$

For large j,k, and from( ),we get

$$\left\|\bar{W}\phi_{n_n,m_m}-\bar{W}\phi_{p_p,q_q}\right\|\leq 2\sigma,\quad \forall n,p>n_0(\sigma)\,,m,q>m_0(\sigma).$$

Hence, $\{\bar{W}\phi_{n,m}\}$ is a Cauchy sequence, so $\bar{W}(S_\alpha)$ is compact.

According to the previous lemmas, by Schauder fixed point theorem, see [19-20], $\bar{W}$ has at least one fixed point in S_α , and Theorem 1 is proved.

III. Chebyshev Polynomials and the System of the Integral Equation

Suppose the approximate kernel $k_{n,m}(x,u,y,v)$ in the continuous case as,

$$k_{n,m}(x,u,y,v)=\sum_{n=0}^{N}\sum_{m=0}^{M}\psi_n(x)\chi_n(u)\omega_m(y)\eta_m(v) \tag{12}$$

where it satisfies the condition in Eq.(7).

Therefore Eq.(1) reduce to the algebraic system form as,

$$\phi_{n,m}(x,y)=f_{n,m}(x,y)-\lambda\int_{-1}^{1}\int_{-1}^{1}k_{n,m}(x,u,y,v)\phi_{n,m}(u,v)dudv+R_{n,m} \tag{13}$$

where,

$$R_{n,m}=\left|\phi-\phi_{n,m}\right|\to 0 \quad as\ n,m\to\infty \tag{14}$$

is the approximate error.

To use the spectral relationships, we write the kernel of Eq.(12) in the form

$$k_{n,m}(x,u,y,v)=\sum_{n=0}^{N}\sum_{m=0}^{M}T_n(x)T_n(u)T_m(y)T_m(v) \tag{15}$$

where $T_l(z)$ is the Chybeshev polynomials of first kind and degree l .

Then Eq.(13) reduces to

$$\phi_{n,m}(x,y)-\lambda\sum_{n=0}^{N}\sum_{m=0}^{M}T_n(x)T_m(y)\int_{-1}^{1}\int_{-1}^{1}T_n(u)T_m(v)\phi(u,v)dudv=f_{n,m}(x,y) \tag{16}$$

Such that,

$$\phi_{n,m}(x,y)=\sum_{n=0}^{N}\sum_{m=0}^{M}a_{n,m}T_n(x)T_m(y) \tag{17}$$

Since,

$$\int_{-1}^{1}\int_{-1}^{1}k_{n,m}(x,u,y,v)\phi_{n,m}(u,v)dudv=T_n(x)T_m(y)\int_{-1}^{1}\int_{-1}^{1}T_n(u)T_m(v)T_i(u)T_j(v)dudv \tag{18}$$

then by using the relation, see [21]

$$T_n(u)T_i(u)=\frac{1}{2}\left[T_{n+i}(u)+T_{|n-i|}(u)\right] \tag{19}$$

Ref

21. I. M. Gradshteyn, I. M, Ryzhik, Tables of Integrals, Series and Products Academic Press, Inc. New York, 1994.

and,

$$\int_{-1}^{1} T_n(u)du = \begin{cases} \frac{2}{1-n^2} & , \ n=0,2,4,... \\ 0 \ , & n=1,3,5 \ ,... \end{cases} \tag{20}$$

Eq.(18) reduce to

Notes

$$T_n(x)T_m(y) \int_{-1}^{1} \int_{-1}^{1} T_n(u)T_m(v)T_i(u)T_j(v)dudv$$

$$= \left[\frac{1}{1-(n+i)^2} + \frac{1}{1-|n-i|^2}\right]\left[\frac{1}{1-(m+j)^2} + \frac{1}{1-|m-j|^2}\right] T_n(x)T_m(y) \tag{21}$$

Then, Eq.(13) reduce to obtaining the following algebraic system,

$$(I - \lambda\mu_{n,m,i,j})a_{n,m} = b_{n,m} \tag{22}$$

where,

$$\mu_{n,m,i,j} = \left[\frac{1}{1-(n+i)^2} + \frac{1}{1-|n-i|^2}\right]\left[\frac{1}{1-(m+j)^2} + \frac{1}{1-|m-j|^2}\right] \tag{23}$$

$$b_{n,m} = \int_{-1}^{1} \int_{-1}^{1} f(x,y)T_n(x)T_m(y)dxdy \tag{24}$$

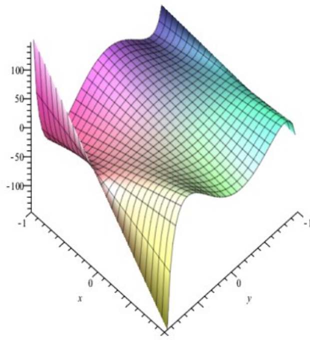
IV. Application and Numerical Discussion

Consider the IE

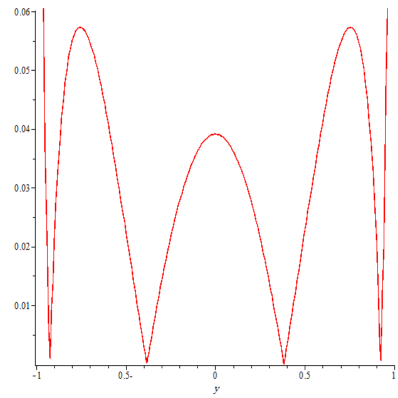
$$\phi(x,y) = f(x,y) + \lambda \int_{-1}^{1} \int_{-1}^{1} (x+u^2)y^2 v \, \phi(u,v)dudv \tag{25}$$

For fixed values of $N = M = 100$, we can graph the solution $\phi(x,y)$ at different values of λ, and different shapes of surfaces $f(x,y)$, then we can graph the estimating errors

$$f(x,y) = xy, \ \lambda = 0.04$$



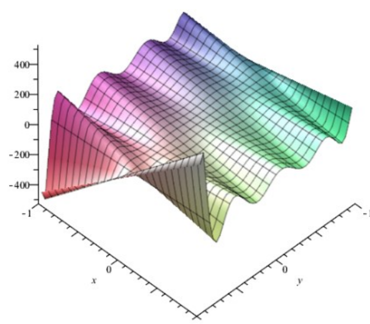
The solution $\phi_{n,m}(x,y)$



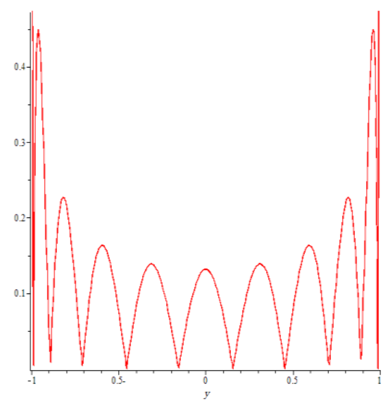
The estimating errorE_{100}

Figure(1)

$$f(x,y) = xy \ , \lambda = 0.1$$



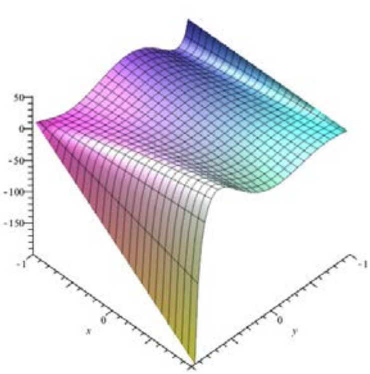
The solution $\phi_{n,m}(x,y)$



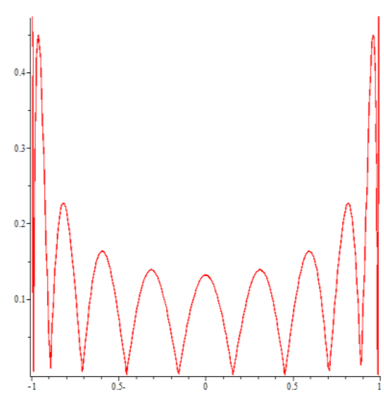
The estimating error E_{100}

Figure(2)

$$f(x,y) = x + y, \lambda = 0.04$$



The solution $\phi_{n,m}(x,y)$



The estimating errorE_{100}

Figure(3)

Notes

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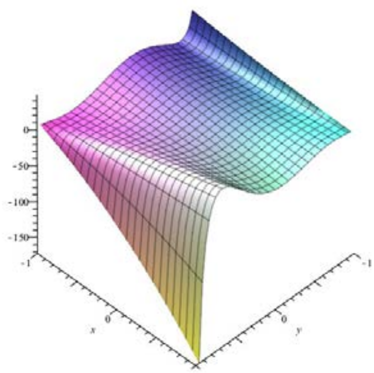
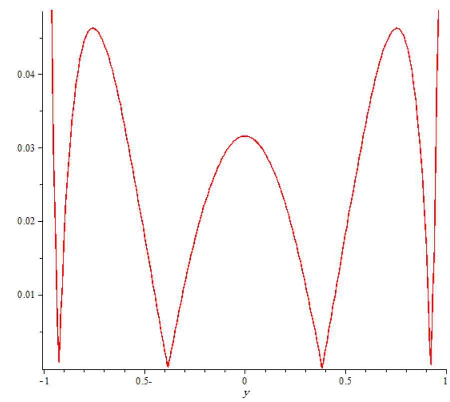
$$f(x,y) = x + y, \lambda = 0.1$$



The solution $\phi_{n,m}(x,y)$



The estimating error E_{100}

Figure(4)

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Notes

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On Lorentzian $\alpha-$Sasakian Manifolds

By Archana Srivastava & Amit Prakash

Sr Institute of Management and Technology

Abstract- The object of this paper to study various curvature tensors in a Lorentzian $\alpha-$ Sasakian manifold, we also study $\varphi-$pseudo projectively flat, $\varphi-$quasi conformally flat, $\varphi-$quasi concircularly flat, $\varphi-m-$projectively flat Lorentzian $\alpha-$Sasakian manifolds are an $\eta-$Einstein Manifold.

Keywords: *$\varphi-$pseudo projectively flat, $\varphi-$quasi conformally flat, $\varphi-$quasi concircularly flat, $\varphi-m-$projectively flat, Lorentzian $\alpha-$Sasakian manifold, $\eta-$Einstein Manifold.*

GJSFR-F Classification: *MSC 2010: 53C25*

ONLORENTZIANSASAKIAN MANIFOLDS

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20. PRASAD, B., (2002), 'A pseudo projective curvature tensor on a Riemannian manifold', *Bull Calcutta Mathematical Society*, pp163–166.

On Lorentzian α–Sasakian Manifolds

Archana Srivastava [α] & Amit Prakash [σ]

Abstract- The object of this paper to study various curvature tensors in a Lorentzian $\boldsymbol{\alpha}$– Sasakian manifold, we also study $\boldsymbol{\varphi}$–pseudo projectively flat, $\boldsymbol{\varphi}$–quasi conformally flat, $\boldsymbol{\varphi}$–quasi concircularly flat, $\boldsymbol{\varphi}$–$\boldsymbol{m}$–projectively flat Lorentzian $\boldsymbol{\alpha}$–Sasakian manifolds are an $\boldsymbol{\eta}$–Einstein Manifold.

Keywords: φ–pseudo projectively flat, φ–quasi conformally flat, φ–quasi concircularly flat, φ–m–projectively flat, Lorentzian α–Sasakian manifold, η–Einstein Manifold.

I. Introduction

Let $(M^n, g), n > 3$, be a connected semi Riemannian manifold of class C^∞ and ∇ be its Levi–Civita connection. Riemannian curvature tensor R is defined by

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z \tag{1.1}$$

Pseudo–projective curvature tensor $\tilde{P}$on a Riemannian manifold $(M^n, g), n > 2$ of type$(1,3)$is defined as follows (Prasad [20]).

$$\tilde{P}(X,Y)Z = a\,R(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y] - \frac{r}{n}\left[\frac{a}{n-1} + b\right][g(Y,Z)X - g(X,Z)Y] \tag{1.2}$$

where a and b are constants such that $a, b \neq 0$.

If $a = 1$ and $b = -\frac{1}{n-1}$, then (2) takes the form

$$\tilde{P}(X,Y)Z = R(X,Y)Z - \frac{1}{n-1}[S(Y,Z)X - S(X,Z)Y] = P(X,Y)Z$$

where P is Projective curvature tensor. Thus the Projective curvature tensor P is a particular case of the tensor $\tilde{P}$, for this reason $\tilde{P}$ is called Pseudoprojective curvature tensor.

Quasi–conformal curvature tensor $\tilde{C}$ on a Riemannian manifold (M^n, g), $n > 2$ of type $(1,3)$is defined as follows (Yano and Swaki [24]).

$$\tilde{C}(X,Y)Z = a\,R(X,Y) + b[S(Y,Z)X - S(X,Z) + g(Y,Z)QX - g(X,Z)QY]$$
$$-\frac{r}{n}\left[\frac{a}{n-1} + 2b\right][g(Y,Z)X - g(X,Z)Y] \tag{1.3}$$

where a and b are constants such that $a, b \neq 0$.

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If $a = 1$ and $b = -\frac{1}{n-2}$, then (3) takes the form

$$\tilde{C}(X,Y)Z = R(X,Y) - \frac{1}{n-2}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] + \frac{r}{(n-1)(n-2)}$$

$$[g(Y,Z)X - g(X,Z)Y] = C(X,Y)Z$$

where C is Conformal curvature tensor. Thus the Conformal curvature tensor C is a particular case of the tensor $\tilde{C}$. For this reason $\tilde{C}$ is calledQuasi–conformal curvature tensor.

Quasi–concircular curvature tensor $\tilde{V}$ on a Riemannian manifold $(M^n, g), n > 2$ of type$(1,3)$is defined as follows (Prasad and Maurya[19]).

$$\tilde{V}(X,Y)Z = a\,R(X,Y)Z + \frac{r}{n}\left[\frac{a}{n-1} + 2b\right][g(Y,Z)X - g(X,Z)Y] \tag{1.4}$$

where a and b are constants such that $a, b \neq 0$. If $a = 1$ and$b = -\frac{1}{n-1}$, then from (4)

$$\tilde{V}(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y] = V(X,Y)Z$$

where V is the Concircular curvature tensor. Thus the Concircular curvature tensor $V$is a particular case of the tensor $\tilde{V}$. For this reason $\tilde{V}$ is called Quasi–concircular curvature tensor.

m–projective curvature tensor W on a Riemannian manifold $(M^n, g), n > 3$ of type $(1,3)$ is defined as follows (Pokhariyal and Mishra [18]).

$$W(X,Y)Z = R(X,Y)Z - \frac{1}{2(n-1)}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] \tag{1.5}$$

where Q is the Ricci operator defined by $S(X,Y) = g(QX,Y)$, S is the Ricci tensor, $r = \text{trace}(S)$ is the scalar curvature and $X, Y, Z \in \chi(M)$. $\chi(M)$ is being Lie algebra of vector fields of M.

In [21], Tanno classified connected almost contact metric manifolds whose automorphism groups possess the maximum dimension. For such a manifold, the sectional curvature of plane sections containing ξ is a constant, say c. He showed that they can be divided into three classes:

(1) Homogeneous normal contact Riemannian manifolds with $c > 0$,

(2) Global Riemannian products of a line or a circle with a Kaehler manifold of constant holomorphic sectional curvature if $c = 0$,

(3) A warped product space $\mathbb{R} \times_f C$ if $c < 0$.It is known that the manifolds of class (1) are characterized by admitting a Sasakian structure, (2) Kenmotsu [11]characterized the differential geometric properties of the manifolds of class (3); the structure so obtained is now known as Kenmotsu structure.

In general, these structures are not Sasakian[14]. In the Gray–Hervella classification of almost Hermitian manifolds[8], there appears a class, W_4 of Hermitian manifolds which are closely related to locally conformal Kaehler manifolds[10]. An almost contact metric structure on a manifold M is called a trans–Sasakian structure[13] if the product manifold $M \times \mathbb{R}$ belongs to the class W_4. The class $C_6 \oplus C_5$ ([13], [14])

Ref

18. POKHARIYAL,G. P., MISHRA, R. S., (1971), 'Curvature tensor and their relativistic significance 2nd', *Yakohama Mathematical Journal*, pp 97–103.

coincides with the class of the trans–Sasakian structures of the type (α, β). In fact, in [13], local nature of the two subclasses, namely, C_5 and C_6 structures, of trans–Sasakian structures are characterized completely.

Also, in [16], Özgür and De studied quasi–conformally flat and quasi–conformally semisymmetric Kenmotsu manifolds. Then, in [25], Yildiz and Murathan studied Lorentzian α–Sasakain manifolds.

We note that trans–Sasakian structures of the type $(0,0)$, $(0,\beta)$ and $(\alpha,0)$ are cosymplectic [2], β–Kenmotsu[11] and α–Sasakain [11] respectively. In [22], it is proved that trans–Sasakian structures are generalised quasi–Sasakian. Thus, trans–Sasakian structures also provide a large class of generalized quasi–Sasakian structures.

An almost contact metric structure (φ, ξ, η, g) on M is calledtrans–Sasakian structure [13], if $(M \times \mathbb{R}, J, G)$ belongs to the class W_4 [8], where J is the almost complex structure on $M \times \mathbb{R}$ defined by

$$J\left(X, f\, {}^{d}/_{dt}\right) = \left(\varphi X - f\xi, \eta(X)\, {}^{d}/_{dt}\right)$$

for all vector fields X on M and a smooth function f on $M \times \mathbb{R}$ and G is the product metric on $M \times \mathbb{R}$. This may be expressed by the condition [3]

$$(\nabla_X \varphi)Y = \alpha(g(X,Y)\xi - \eta(Y)X) + \beta\big(g(\varphi X, Y)\xi - \eta(Y)\varphi(X)\big) \tag{1.6}$$

For some smooth functions α and β on M and we say that the trans–Sasakian structure is of type (α, β).
From (6) it follows that

$$\nabla_X \xi = -\alpha\varphi(X) + \beta(X - \eta(X)\xi) \tag{1.7}$$

$$(\nabla_X \eta)Y = -\alpha g(\varphi X, Y) + \beta g(\varphi X, \varphi Y) \tag{1.8}$$

Trans–Sasakian manifolds have been studied by De and Tripathi[7] and they obtained the following results:

$$\begin{aligned} R(X,Y)\xi = {} & (\alpha^2 - \beta^2)(\eta(Y)X - \eta(X)Y) + 2\alpha\beta(\eta(Y)\varphi X \eta(X)\varphi Y) + (Y\alpha)\varphi X \\ & -(X\alpha)\varphi Y + (Y\beta)\varphi^2 X - (X\beta)\varphi^2 Y \end{aligned} \tag{1.9}$$

$$\begin{aligned} R(\xi,Y)X = {} & (\alpha^2 - \beta^2)(g(X,Y)\xi - \eta(X)Y) + 2\alpha\beta(g(\varphi X, Y)\xi - \eta(X)\varphi Y) \\ & +(X\alpha)\varphi Y + g(\varphi X, Y)(grad\alpha) + (X\beta)(Y - \eta(Y)\xi) - g(\varphi X, \varphi Y)(grad\beta) \end{aligned} \tag{1.10}$$

$$R(\xi, X)\xi = (\alpha^2 - \beta^2 - \xi\beta)(\eta(X)\xi - X) \tag{1.11}$$

$$2\alpha\beta + \xi\alpha = 0 \tag{1.12}$$

$$S(X,\xi) = \big((n-1)(\alpha^2 - \beta^2) - \xi\beta\big)\eta(X) - (n-2)X\beta - (\varphi X)\alpha \tag{1.13}$$

$$Q\xi = ((n-1)(\alpha^2 - \beta^2) - \xi\beta)\xi - (n-2)(grad\beta) + \varphi(grad\alpha) \tag{1.14}$$

A trans–Sasakian structure of type (α, β) is α–Sasakian if $\beta = 0$ and α is a non–zero constant [10].
If $\alpha = 1$, then α–Sasakian manifold is a Sasakian manifold.

Ref

11. KENMOTSU, K., (1972), 'A class of almost contact Riemannian manifolds', *Tohoku Math. J.*, Japan, pp 93–103.

II. Lorentzian α–Sasakian Manifold

A differentiable manifold of dimension n is called Lorentzian α–Sasakian manifold if it admits a $(1,1)$–tensor field φ, a contravarient vector field ξ, a covariant vector field η and Lorentzian metric g which satisfy ([2], [5], [6], [7], [8], [12])

$$\eta(\xi) = -1, \tag{2.1}$$

$$\varphi^2 = I + \eta \otimes \xi \tag{2.2}$$

$$g(\varphi X, \varphi Y) = g(X,Y) + \eta(X)\eta(Y) \tag{2.3}$$

$$g(X,\xi) = \eta(X), \quad \varphi\xi = 0, \quad \eta(\varphi X) = 0 \tag{2.4}$$

for all $X, Y \in TM$.

From (1.7) and (1.8), a Lorentzian α–Sasakian manifold Msatisfying

$$\nabla_X \xi = -\alpha\varphi(X) \tag{2.5}$$

$$(\nabla_X \eta)Y = -\alpha g(\varphi X, Y) \tag{2.6}$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g.

A Lorentzian α–Sasakian manifold M is said to be η–Einstien if its Ricci tensor S is of the form

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y) \tag{2.7}$$

for any vector fields $X, Y,$ where a, b are functions on M.

Further, from equations (1.9)–(1.14) on a Lorentzian α–Sasakian manifold M the following relations holds:

$$R(\xi, X)Y = \alpha^2(g(X,Y)\xi + \eta(Y)X) \tag{2.8}$$

$$R(X,Y)\xi = \alpha^2(\eta(Y)X + \eta(X)Y) \tag{2.9}$$

$$R(\xi, X)\xi = \alpha^2(\eta(X)\xi + X) \tag{2.10}$$

$$S(X,\xi) = (n-1)\alpha^2\eta(X) \tag{2.11}$$

$$Q\xi = (n-1)\alpha^2\xi \tag{2.12}$$

$$S(\xi,\xi) = -(n-1)\alpha^2 \tag{2.13}$$

$$S(\varphi X, \varphi Y) = S(X,Y) + (n-1)\alpha^2\eta(X)\eta(Y) \tag{2.14}$$

III. φ-Pseudo Projectively Flat Lorentzian α- Sasakian Manifold

A differentiable manifold $(M^n, g), n > 2,$ satisfying the condition $\varphi^2\tilde{P}(\varphi X, \varphi Y)\varphi Z = 0,$ is called φ–pseudo projectively flat Lorentzian α– Sasakian manifold.

In this section we assume that Lorentzian α–Sasakian manifold $(M^n, g), n > 2,$ is φ–pseudo projectively flat. Then $\varphi^2\tilde{P}(\varphi X, \varphi Y)\varphi Z = 0,$ implies

Ref

2. BLAIR, D.E., (1976), 'Contact manifolds in Riemannian geometry', *Lectures Notes in Mathematics, Springer–Verlag*, Berlin.

$$g(\tilde{P}(\varphi X, \varphi Y)\varphi Z, \varphi W) = 0 \tag{3.1}$$

for any vector fields X, Y, Z, W.

By the use of (1.2), φ–pseudo projectively flat means

$$a\,'R(\varphi X, \varphi Y, \varphi Z, \varphi W) = -b[S(\varphi Y, \varphi Z)g(\varphi X, \varphi W) - S(\varphi X, \varphi Z)g(\varphi Y, \varphi W)] + \frac{r}{n}\left[\frac{a}{n-1} + b\right]$$
$$[g(\varphi Y, \varphi Z)g(\varphi X, \varphi W) - g(\varphi X, \varphi Z)g(\varphi Y, \varphi W)] \tag{3.2}$$

where $'R(X, Y, Z, W) = g(R(X, Y)Z, W)$.

Let $\{e_1, e_2, \dots, e_{n-1}, \xi\}$ be a local orthonormal basis of vector fields in M^n. By using the fact that $\{\varphi e_1, \varphi e_2, \dots, \varphi e_{n-1}, \xi\}$ is also a local orthonormal basis, if we put $X = W = e_i$ in (3.2) and sum up with respect to , then we have

$$a\sum_{i=1}^{n-1} {'R}(\varphi e_i, \varphi Y, \varphi Z, \varphi e_i) = -b\sum_{i=1}^{n-1}[S(\varphi Y, \varphi Z)g(\varphi e_i, \varphi e_i) - S(\varphi e_i, \varphi Z)g(\varphi Y, \varphi e_i) + \frac{r}{n}\left[\frac{a}{n-1} + b\right]$$
$$\textstyle\sum_{i=1}^{n-1}[g(\varphi Y, \varphi Z)g(\varphi e_i, \varphi e_i) - g(\varphi e_i, \varphi Z)g(\varphi Y, \varphi e_i) \tag{3.3}$$

It can be easily verified that ([2])

$$\textstyle\sum_{i=1}^{n-1} {'R}(\varphi e_i, \varphi Y, \varphi Z, \varphi e_i) = S(\varphi Y, \varphi Z) + g(\varphi Y, \varphi Z) \tag{3.4}$$

$$\textstyle\sum_{i=1}^{n-1} S(\varphi e_i, \varphi e_i) = r - (n-1)\alpha^2 \tag{3.5}$$

$$\textstyle\sum_{i=1}^{n-1} g(\varphi e_i, \varphi Z)S(\varphi Y, \varphi e_i) = S(\varphi Y, \varphi Z) \tag{3.6}$$

$$\textstyle\sum_{i=1}^{n-1} g(\varphi e_i, \varphi e_i) = (n-1) \tag{3.7}$$

$$\textstyle\sum_{i=1}^{n-1} g(\varphi e_i, \varphi Z)g(\varphi Y, \varphi e_i) = g(\varphi Y, \varphi Z) \tag{3.8}$$

So by the use of the (3.4)–(3.8), the equation (3.3) takes the form

$$[a + b(n-2)]S(\varphi Y, \varphi Z) = \left[\frac{r}{n}\left(\frac{a}{n-1} + b\right)(n-2) - a\right]g(\varphi Y, \varphi Z) \tag{3.9}$$

Then, by making use of (4.2.3) and (4.2.14), the equation (4.3.10) takes the form

$$S(Y, Z) = \frac{1}{[a + b(n-2)]}\left[\frac{r}{n}\left(\frac{a}{n-1} + b\right)(n-2) - a\right]g(Y, Z) + \frac{1}{[a + b(n-2)]}$$
$$[\tfrac{r}{n}\left(\tfrac{a}{n-1} + b\right)(n-2) - a - a\alpha^2 - b(n-1)(n-2)\alpha^2]\eta(Y)\eta(Z) \tag{3.10}$$

which shows that M^n is an η–Einstein manifold.Hence we can state the following theorem:

Theorem 3.1: Let M^n be an n–dimensional, $n > 2$, φ–pseudo projectively flat Lorentzian α–Sasakian manifold, then M^n is an η–Einstein Manifold.

IV. φ–Quasi Conformally Flat Lorentzian α–Sasakian Manifold

A differentiable manifold $(M^n, g), n > 2$, satisfying the condition $\varphi^2\tilde{C}(\varphi X, \varphi Y)\varphi Z = 0$, is called φ–quasi conformally flat. (Cabrerizo, Fernandez,

Fernandez and Zhen [28]). In this section we assume that Lorentzian α–Sasakian manifold $(M^n, g), n > 2$, is φ–quasi conformally flat. Then $\varphi^2 \tilde{C}(\varphi X, \varphi Y)\varphi Z = 0$ implies

$$g\big(\tilde{C}(\varphi X, \varphi Y)\varphi Z, \varphi W\big) = 0 \tag{4.1}$$

for any vector fields $X, Y, Z, W \in \chi(M^n)$. So by the use of (4.1.3), φ–quasi conformally flat means

$$a'R(\varphi X, \varphi Y, \varphi Z, \varphi W) = -b[S(\varphi Y, \varphi Z)g(\varphi X, \varphi W) - S(\varphi X, \varphi Z)g(\varphi Y, \varphi W) + g(\varphi Y, \varphi Z)S(\varphi X, \varphi W)$$
$$-g(\varphi X, \varphi Z)S(\varphi Y, \varphi W)] + \frac{r}{n}\left[\frac{a}{n-1} + 2b\right][g(\varphi Y, \varphi Z)g(\varphi X, \varphi W) - g(\varphi X, \varphi Z)g(\varphi Y, \varphi W)] \tag{4.2}$$

where $'R(\varphi X, \varphi Y, \varphi Z, \varphi W) = g(R(\varphi X, \varphi Y)\varphi Z, \varphi W)$.

Let $\{e_1, e_2, \ldots, e_{n-1}, \xi\}$ be a local orthonormal basis of vector fields in M^n. By using the fact that $\{\varphi e_1, \varphi e_2, \ldots, \varphi e_{n-1}, \xi\}$ is also a local orthonormal basis, if we put $X = W = e_i$ in (4.2) and sum up with respect to i, then we have

$$a\sum_{i=1}^{n-1} {'R}(\varphi e_i, \varphi Y, \varphi Z, \varphi e_i) = -b\sum_{i=1}^{n-1}[S(\varphi Y, \varphi Z)g(\varphi e_i, \varphi e_i) - S(\varphi e_i, \varphi Z)g(\varphi Y, \varphi e_i)$$
$$+g(\varphi Y, \varphi Z)S(\varphi e_i, \varphi e_i) - g(\varphi e_i, \varphi Z)S(\varphi Y, \varphi e_i)] + \textstyle\sum_{i=1}^{n-1}[g(\varphi Y, \varphi Z)g(\varphi e_i, \varphi e_i)$$
$$-g(\varphi e_i, \varphi Z)g(\varphi Y, \varphi e_i)] \tag{4.3}$$

So by the virtue of (3.4)−(3.8), the equation (4.3) takes the form

$$[a + b(n-3)]S(\varphi Y, \varphi Z) = [-a - b\{r - (n-1)\alpha^2\} + \frac{r(n-2)}{n}\left\{\frac{a}{n-1} + 2b\right\}]g(\varphi Y, \varphi Z) \tag{4.4}$$

Then, by making use of (2.3) and (2.14), the equation (4.4) takes the for

$$S(Y, Z) = \frac{1}{[a + b(n-3)]}[\left\{\frac{r(n-2) - n(n-1)}{n(n-1)}\right\}a + \left\{\frac{n(n-1)\alpha^2 + (n-4)}{n}\right\}b]g(Y, Z) + \frac{1}{[a + b(n-3)]}[\left\{\frac{r(n-2) - n(n-1)^2\alpha^2 - n(n-1)}{n(n-1)}\right\} +$$
$$a\left\{\frac{(n-4) - n(n-1)(n-4)\alpha^2}{n}\right\}b]\eta(Y)\eta(Z) \tag{4.5}$$

which shows that M^n is an η–Einstein manifold. Hence we can state the following theorem:

Theorem 4.1: Let M^n be an n–dimensional, $n > 2, \varphi$–quasi conformally flat Lorentzian α–Sasakian manifold, then M^n is an η–Einstein Manifold.

V. φ –Quasi Concircularly Flat Lorentzian α–Sasakian Manifold

A differentiable manifold $(M^n, g), n > 2$, satisfying the condition $\varphi^2 \tilde{V}(\varphi X, \varphi Y)\varphi Z = 0$, is called φ–quasi concircularly flat Lorentzian α–Sasakian manifold.

In this section we assume that Lorentzian α–Sasakian manifold $(M^n, g), n > 2$, is φ–quasi concircularly flat. Then $\varphi^2 \tilde{V}(\varphi X, \varphi Y)\varphi Z = 0$, implies

$$g\big(\tilde{V}(\varphi X, \varphi Y)\varphi Z, \varphi W\big) = 0 \tag{5.1}$$

for any vector fields $X, Y, Z, W \in \chi(M^n)$.

Notes

So, by the use of (1.4), φ–quasi concircularly flat means

$$a'R(\varphi X,\varphi Y,\varphi Z,\varphi W) = -\frac{r}{n}\left[\frac{a}{n-1}+2b\right][g(\varphi Y,\varphi Z)g(\varphi X,\varphi W)-g(\varphi X,\varphi Z)g(\varphi Y,\varphi W)] \quad (5.2)$$

where $'R(\varphi X,\varphi Y,\varphi Z,\varphi W) = g(R(\varphi X,\varphi Y)\varphi Z,\ \varphi W)$.

Let $\{e_1,e_2,\dots,e_{n-1},\xi\}$ be a local orthonormal basis of vector fields in M^n by using the fact that $\{\varphi e_1,\varphi e_2,\dots,\varphi e_{n-1},\xi\}$ is also a local orthonormal basis, if we put $X = W = e_i$ in (5.2) and sum up with respect to i, then we have

$$a\sum_{i=1}^{n-1}{}'R(\varphi e_i\,,\varphi Y,\varphi Z,\varphi e_i)$$

$$= -\frac{r}{n}\left[\frac{a}{n-1}+2b\right]\sum_{i=1}^{n-1}[g(\varphi Y,\varphi Z)g(\varphi e_i,\varphi e_i)-g(\varphi e_i,\varphi Z)g(\varphi Y,\varphi e_i)]$$

$$-g(\varphi e_i,\varphi Z)g(\varphi Y,\varphi e_i)] \quad (5.3)$$

So, by virtue of (3.4)−(3.8), the equation (5.3) takes the form

$$aS(\varphi Y,\varphi Z) = -\left[a+\frac{r(n-2)}{n}\left\{\frac{a}{n-1}+2b\right\}\right]g(\varphi Y,\varphi Z) \quad (5.4)$$

Then, by making use of (2.3) and (2.14), the equation (5.4) takes the form

$$S(Y,Z) = -\left[1+\frac{r(n-2)}{na}\left\{\frac{a}{n-1}+2b\right\}\right]g(Y,Z)-\left[\frac{r(n-2)}{na}\left\{\frac{a}{n-1}+2b\right\}+1+(n-1)\alpha^2\right]\eta(Y)\eta(Z) \quad (5.5)$$

which shows that M^n is an η–Einstein manifold.
Hence we can state the following theorem:

Theorem 5.1: Let M^n be an n–dimensional, $n>2,\varphi$–quasi concircularly flat Lorentzian α–Sasakian manifold, then M^n is an η–Einstein manifold.

VI. φ–m–Projectively Flat Lorentzian α–Sasakian Manifold

A differentiable manifold $(M^n,g), n>3,$ satisfying the condition $\varphi^2 W(\varphi X,\varphi Y)\varphi Z = 0$ is called φ–m–projectively flat Lorentzian α–Sasakian manifold (Cabrerizo, Fernandez, Fernandez and Zhen [28]).

In this section we assume that Lorentzian α–Sasakian manifold $(M^n,g), n>3,$ is φ–m–projectively flat. Then $\varphi^2 W(\varphi X,\varphi Y)\varphi Z = 0$implies $\sum$

$$g(W(\varphi X,\varphi Y)\varphi Z,\varphi W) = 0 \quad (6.1)$$

for any vector fields $X,Y,Z,W \in \chi(M^n)$.

So, by the use of (1.5), φ–m–projectively flat means

$$R(\varphi X,\varphi Y,\varphi Z,\varphi W) = \frac{1}{2(n-1)}[S(\varphi Y,\varphi Z)g(\varphi X,\varphi W)-S(\varphi X,\varphi Z)g(\varphi Y,\varphi W)+g(\varphi Y,\varphi Z)$$

$$S(\varphi X,\varphi W)-g(\varphi X,\varphi Z)S(\varphi Y,\varphi W)] \quad (6.2)$$

where $'R(\varphi X,\varphi Y,\varphi Z,\varphi W) = g(R(\varphi X,\varphi Y)\varphi Z,\ \varphi W)$.

Ref

28. ZHEN, G., CABRERIZO, J. L., FERNANDEZ, M. (1997), 'On ξ – conformally flat contact metric manifold', *Indian J. Pure Appl. Math.*,pp725–734.

Let $\{e_1, e_2, \dots, e_{n-1}, \xi\}$ be a local orthonormal basis of vector fields in M^n. By using the fact that $\{\varphi e_1, \varphi e_2, \dots, \varphi e_{n-1}, \xi\}$ is also a local orthonormal basis, if we put $X = W = e_i$ in (6.2) and sum up with respect to i, then we have

$$\sum_{i=1}^{n-1} {}'R(\varphi e_i, \varphi Y, \varphi Z, \varphi e_i) = \frac{1}{2(n-1)} \sum_{i=1}^{n-1} [S(\varphi Y, \varphi Z) g(\varphi e_i, \varphi e_i) - S(\varphi e_i, \varphi Z) g(\varphi Y, \varphi e_i) +$$

$$g(\varphi Y, \varphi Z) S(\varphi e_i, \varphi e_i) - g(\varphi e_i, \varphi Z) S(\varphi Y, \varphi e_i)] \tag{6.3}$$

So, by the use of the (3.4)−(3.8), the equation (6.3) takes the form

$$S(Y,Z) = \frac{1}{(n+1)} [r - (n-1)\alpha^2 - 2(n-1)]\, g(Y,Z) + \frac{1}{(n+1)} [r - 2(n-1) - (n-1)$$

$$(n+2)\alpha^2] \eta(Y)\eta(Z) \tag{6.4}$$

which shows that M^n is an η−Einstein manifold. Hence we can state the following theorem:

Theorem 6.1: Let M^n be an n−dimensional, $n > 3, \varphi - m$−projectively flat Lorentzian α−Sasakian manifold, then M^n is an η−Einstein manifold.

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Triple Reflections- A Discourse on twin Prime Conjecture, Pascal's Triangle, and Euler's *E*

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Abstract- The twin prime conjecture has attracted a lot of attention worldwide. It is still an unresolved problem, even though the work of Yitang Zhang has partially resolved it. The author of this paper aims to contribute to the discourse by employing basic mathematics and logic to arrive at some conclusions on the topic, and also to help in breaking new grounds. The researcher used secondary data to build his arguments in an exploratory manner, relying on the existing literature. The paper traces the background of the problem, and points to some of the breakthroughs that were made in the past. The paper examines Pascal's triangle and, it makes some revealing discoveries on the coefficients. The author also examines Euler's *E*, and links it to Pascal's triangle, and the twin prime problem. Furthermore the author derives new arithmetic terms that he can use to produce infinite numbers of twin primes. The author also discusses how numbers so obtained can thoroughly be checked to be non-composite, thus extending the field of twin primes. The author finally points to the application of twin primes in industry, academia, and other areas of practical knowledge.

Keywords: arithmetic progression, binomial expansion, brun, cryptology, eratosthenes, euclid, euler, fractal geometry, hardy-littlewood, induction, natural numbers, probability density function, reciprocals, self-similarity, twin primes conjecture, pascal, data science.

GJSFR-F Classification: *MSC 2010: 35Q31*

TRIPLEREFLECTIONSADISCOURSEONTWINPRIMECONJECTUREPASCALSTRIANGLEANDEULERSE

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Triple Reflections- A Discourse on twin Prime Conjecture, Pascal's Triangle, and Euler's *E*

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Abstract- The twin prime conjecture has attracted a lot of attention worldwide. It is still an unresolved problem, even though the work of Yitang Zhang has partially resolved it. The author of this paper aims to contribute to the discourse by employing basic mathematics and logic to arrive at some conclusions on the topic, and also to help in breaking new grounds. The researcher used secondary data to build his arguments in an exploratory manner, relying on the existing literature. The paper traces the background of the problem, and points to some of the breakthroughs that were made in the past. The paper examines Pascal's triangle and, it makes some revealing discoveries on the coefficients. The author also examines Euler's *E*, and links it to Pascal's triangle, and the twin prime problem. Furthermore the author derives new arithmetic terms that he can use to produce infinite numbers of twin primes. The author also discusses how numbers so obtained can thoroughly be checked to be non-composite, thus extending the field of twin primes. The author finally points to the application of twin primes in industry, academia, and other areas of practical knowledge.

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I. Introduction

We can observe from the expansion of binomial expressions of the form $(x+y)^n$ that the numerical coefficients of the terms in the expansion can be derived by using Pascal's triangle, named after the French mathematician(Wong, 2013). We can use Pascal's triangle to generate the binomial coefficients in the binomial expansion where a binomial is of the form $(x+y)^n$ and, the expansion takes the form of ${}^nCr$(Wong, 2013). Below are a few rows of the triangle showing the binomial coefficients of the first six terms, which are as follows

$$
\begin{array}{c}
1\\
1\ 1\\
1\ 2\ 1\\
1\ 3\ 3\ 1\\
1\ 4\ 6\ 4\ 1\\
1\ 5\ 10\ 10\ 5\ 1
\end{array}
$$

These coefficients when, summed up line by line horizontally, form the geometric series to infinity of the form, 1, 2, 4, 8, 16, 32, 64, 128,……, 2^{n-1}. They form the series of the form 2^{n-1} , where n is an integer from n=1 to n = infinity. If we sum the reciprocals of the series to infinity using a geometric progression, we obtain the sum to infinity as

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$$a / r \tag{1}$$

where a is the first term 2^0 and r is the ratio ½ where, r is less than 1.
The sum of the reciprocals of the series 1/1, 1/2, 1/4, 1/8, 1/16, 1/32, 1/64……. $1/2^{n-1}$ to infinity is, therefore, $2^0/1/2 = 1/1/2 = 2$.

$$1 \text{ to } \infty \sum 1/2^{n-1} = 2. \text{ (Author's derivation)} \tag{2}$$

(cf. Math Stack Exchange, 2018; Math Stack Exchange, n.d.)

Thus, the reciprocals of Pascal's triangle numbers form an infinite series that sum up to 2 in the limit. Brothers (2012) dealt with the products of the Pascal triangle and their ratios, which in the limit, summed up to *E* or Euler's constant, 2.7183. When we use the expressions S_{n+1}/S_n divided by $S_n/S_{n-1,}$ we can take one line in the triangle and hold it as our baseline for computation. We use the product of the line just below it, and divide it by the multiplication of the numbers on the line we have taken as our reference point. We divide the resultant by the product of the baseline. We again divide the result by the multiplication of the numbers on the line above it. An example here will suffice.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1 S_{n-1} ………1 Product = 96

1 5 10 10 5 1 S_n…………2 Product= 2,500 (Line held constant)

1 6 15 20 15 6 1 Sn+1……3 Product= 162,000

162,000/2,500 divided by 2,500/96 = 2.48832

If we continue this process to infinity, in the limit, we will find that the result that we obtain each time will approach the terminal value of 2.7183 or *E* (Euler's *E*), which is the sum of the reciprocals of the natural numbers to infinity (Brothers, 2012).

When we examine Euler's *E*, we find that *E* is the sum to infinity of the reciprocals of the natural counting numbers, 1/1, 1/2, 1/3, 1/4,…..1/n that gives us the sum to infinity of the series of reciprocals of the natural counting numbers of the common term, 1/n. Euler's *E* is the same as the expansion of the expression $(1+ 1/n)^n$ (Brothers, 2012). In the limit, this binomial term, gives us the approximate value 2.7183 or Euler's *E* (Brothers, 2012).

In 2012, this author posted an article on *Ghanaweb.com* and *Academia. edu* to the effect that a great deal of the majority of the Twin Prime numbers (about 90 to 99 percent) can be generated by pairs of the terms 30n-1, 30n+1, on the one hand, and another two terms 30n-17, and 30n-19, on the other side. These arithmetic terms provide evidence of proof by induction (Stanford.edu, n.d.). These four terms were arrived at by sieving and using a formula for terms of arithmetic series. We can use the expressions 30n-1, 30n+1, 30n-17, and 30n-19 to produce an infinite number of twin

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primes, thus, proving that we have unlimited amount of twin primes (cf. Sebah & Gourdon, 2002).

That was the author's contribution to the Twin Prime Conjecture conundrum. However, in this article, the author has elaborated more on this, as presented below. Caldwell (n.d.) stated that the Greek mathematicians, Euclid (c. 300) and Eratosthenes were the first people to develop sieves of elimination to produce twin primes. This author developed an interest in twin primes and thought of using simple patterns of recognition and critical thinking to contribute to the enlightened discourse on the twin prime conjecture, which is said to be one of the unresolved mathematical problems, which were first put forward by Polignac in 1849 in France (Sha, 2016; Kiersz, 2018).

II. Historical Background to Twin Prime Conjecture

The twin prime conjecture dates back from antiquity in ancient Greece, during the period of Euclid and Eratosthenes (Murty, n.d.; Brent, 1975). The problem was resurrected in 1849 by Alphonse Polignac. Viggo Brun, the Norwegian mathematician, in 1915, developed a new sieve based on those of Euclid and Eratosthenes (Murty, 2015).

In 1923, Hardy and Littlewood from Oxford University gave their proof of the twin prime conjecture: $2\pi\ (1\text{-}\ 1/(p\text{-}1)^2\ *\ x/\log^2 x)$ with $p > 2$. Hardy and Littlewood's proof was based on a circular method, onethat was developed by an Indian genius called Srinivasa Ramanujan (Murty, n.d.). In 2015, Murty discussed Yitang Zhang's breakthrough theorem towards the solution of the twin prime conjecture, with the absolute value of $p\text{-}q< 70$ million. Zhang had given his proof from the University of New Hampshire in 2013 in a paper proving that, in fact, there are infinite numbers of twin primes (cf. Wolfram Math World, n.d. Agama, 2018).

Murty (n.d.) wrote that if there are infinitely p primes such that $p+2$ is also a prime, then generally if for an even number A here are infinitely many primes p, then it follows that $p + A$ is also prime (proof by induction, fractal geometry, and self-similarity as well as set theory). This author therefore humbly posits that he can prove the twin prime conjecture by simple logic and simple mathematics using arithmetic progression or series. Similar approaches had been made in the past including the work of two Frenchmen, Sebah and Gourdon(2002).

III. Discussion

Let us consider the first eight pairs of twin primes. These are:

(3,5), (5,7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61) and (71, 73)

(Source: en.wikipedia.org/wiki/Prime_pair)

Each pair of twin primes shows a difference of 2, hence the names twin primes or sexy primes or cousin primes or conjugate primes(Wolfram Math World, n.d.).
We can form their reciprocals and find the sum of these first eight pairs.

1/3, 1/5, 1/5, 1/7, 1/11, 1/13, 1/17, 1/19, 1/29, 1/31, 1/41, 1/43, 1/59, 1/61, 1/71, 1/73.

Their decimal representations are as follows

$$1/3 = 0.\ 3333333333$$

$$1/5 = 0.2000000000$$

1/5 =0.2000000000

1/7 =0.1428571429which is *pi* (π) minus 3)

1/11= 0.0909090909

1/13=0.0769230769

1/17 =0.0588235294

1/19=0.0526315790

1/29=0.0344827586

1/31=0.0322580645

1/41=0.0243902439

1/43= 0.0232558140

1/59=0.0169491525

1/61=0.0163934426

1/71=0.0140845070

1/73=0.0136986301

The sum of all the above reciprocals gives a total of 1.3, approximately. Thus the sum of the reciprocals of the first eight pairs of twin primes or first fifteen twin primes is 1.3, while the sum of the reciprocals of all-natural numbers to infinity, contained in Euler's *E,* is 2.71823. The sum of the reciprocals of the first fifteen twin prime numbers shows us that the probability density function or distribution of twin primes is about 50 percent or more of all-natural numbers if we consider the first fifteen twin prime numbers of significance.

Thus, twin primes are far in between as we approach infinity, but they never terminate as they become asymptotic to the reciprocals' distribution function of all-natural numbers (Brent, 1975). In a distribution of say p and q (where p and q are fractional), we shall assume q to have a very low probability of occurrence in an infinite series, where n is extremely huge. As we know, in the limit, this approximates the Poisson distribution in a case where q is very low in value and n is extraordinarily huge (Spiegel 1975: 108-130).

$$e^{-\lambda}\lambda^{x}/x! \text{ for } \quad x = 0, 1. 2, 3... \tag{3}$$

(library 2. lincoln; Spiegel, 1975: 108-130)

The average or mean of the Poisson distribution is given as *np.*

Therefore, if we take Euler's E into consideration, we can subtract the sum of reciprocals of Pascal's coefficients derived from his triangle, which is the sum of 2from Euler's E and remain with 0.71823. This remainder or residual is significant because it represents the sum of all reciprocals of odd numbers (which are not part of Pascal's series), and also the total of all reciprocals of numbers which end with either 0 or 5 or those numbers which are squares such as 49, 121, 441, 169, among others. The process alluded to here is part of the elimination or sieving method, which uses both deductive and inductive logic. Euclid and Eratosthenes were the first mathematicians to devise means of sieving numbers that generated the twin prime numbers (Murty, 2013; Wong, 2012).

We subtract 1.3 (sum of reciprocals of the first 15 twin primes) from 2.71823. We end up with 1.41823 as the sum of all non-twin prime number reciprocals including primes, which do not twin as well as other numbers which are not of the form

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2^{n-1}. Looking at the total of all reciprocals of natural numbers, and their sum to infinity to be Euler's E or 2.71823, and the sum of the reciprocals of the first fifteen twin primes to be 1.3 (as if these were all the significant reciprocals of twin primes), we would conclude that approximately, twin primes are distributed in probability density space to account for about 50 percent of all the natural numbers, which would be a false assumption to have.

None-the-less, despite their paucity or scarcity, as we go towards infinity, twin primes never end just as the natural numbers also never end because twin primes are a subset of the natural numbers, and they have all the characteristics and properties of natural numbers, but they become asymptotic as we approach infinity (proof by deduction according to theories of self-similarity and fractal geometry by Feigenbaum and Mandelbrot, respectively).

Thus, since natural numbers are to infinity, so are twin primes, since twin primes are a subset of all natural numbers (Wong, 2012). Wong (2012), in his paper, mentioned that Mitchell Feigenbaum came up with the concept of self-similarity, while Benoit Mandelbrot was the originator of the concept of fractal geometry. Self-similarity and fractal geometry, therefore, can be used to state that twin primes are infinite, since they are a subset of natural numbers, which are infinite. Thus, we can use the terms 30n-1, 30n+1, on the one hand, and the pair of terms, 30n-17, and 30n-19 on the other side as expressions to derive twin primes by making n as large as possible. These terms were derived first by using elimination methods and generating them by using the formula of arithmetic series(see Chen et al.). The scenario presents us with a unique binomial distribution, where both p and q are almost equal as n approaches infinity. Let us assume p=0.5 and q=0.5, and n is an exceptionally huge number say 50,000. We have $(p+q)^{50000}$

Or $(0.5+0.5)^{50000} = (1)^{50000} = 1$ which is the total area under the Gaussian or Normal curve (Spiegel 1975: 108-130; library2. lincoln)

The above discussion shows us that the distribution of twin primes is dense in about the first 300 natural number series but, as the series increases to infinity, the twin primes become scarce, yet they do not finish as they are like an asymptotic graph approaching zero but never becoming zero. Inherent here is the Central Limit Theorem, which states that the mean, as the distribution gets larger, stabilizes and approaches a fixed sum or parameter close to the center of the Gaussian Curve or Normal Curve(Spiegel 1975:112; Kwak & Jim, 2017).

Brun (1919) gave the sum of all reciprocals of twin primes as 1.90216054.Seventy-five years later, in 1994, Nicely,(1994) improved the total of twin prime reciprocals to 1.902160583209. However, we know that all twin primes tend to end with the digits 1, 3, 7, and 9, even though not all numbers that end with these digits are prime numbers. Examples of numerals that end with digits 1, 3, 7, and 9 and, which are non-twin primes, are 441, 443, 169,171, 837,839, 341, and 343, some of which, are squares of numbers.

In considering twin primes, we should eliminate all even numbers or numbers which end with 2, 4, 5, 6, 8, and 0.So, twin primes are distributed as approximately 4 out of every10 numbers with ending digits of all numbers, giving a ratio of 0.4 or 40 percent of all-natural numbers. This inference gives us an idea that number space is everywhere dense with twin primes. Euclid long ago created a sieve for generating prime numbers (Wong, 2012). However, our distribution of sums of reciprocals is as follows:

1. Sum to infinity of all reciprocals of natural counting numbers is (Euler's E) =2.71823

2. Sum to infinity of reciprocals of Pascal's Triangle coefficients, part of reciprocals of even numbers, is: 1 + ½ + ¼ + 1/8 +1/16 +….+ 1/n which adds up to2.0 in the limit (author's discovery)
3. Sum to infinity of reciprocals of all Twin Primes is approximately 1.9 (according to Brun, 1919, and Nicely, 1994)
4. Sum of reciprocals of all numbers which end with either 0 or 5 is 0.5438 (author's estimate)

(1/5 + 1/10 + 1/15 + 1/20 + ………+ 1/n) = 1/5 (1/1 + æ + 1/3 + ……. + 1/n)
= 1/5(*E*) = 1/5 (2.7183)= 0.5438

5. Sum of reciprocals of all numbers which are squares or end with the digits 1, 3, 7, and 9 but which include twin primes and primes that are non-twin primes------- (residual- ?) Residual= 2.71823- (0.5438 + 2) = 0.1745

Therefore, the addition of reciprocals of twin primes divided by addition of reciprocals of all-natural numbers is1.96/2.71823, which is approximately 72%. This author found a relationship among the three mathematicians, namely Euler, Pascal, and Brun, in terms of the numbers they are associated with, hence, the title of this article. The numbers are Euler (2.7183- sum of reciprocals of all-natural numbers from 1 to infinity), Pascal (2- sum of the series formed from the reciprocals of the coefficients of the binomial expansion to infinity), and Brun (1.96- sum of reciprocals of all twin primes to infinity). The relationship is

Euler – Pascal = Brun/Euler ………….. (4)

Or Euler^2 – Euler x Pascal = Brun………….2 (5)

From 1, 2.7183 – 2 = 1.96/2.7183

0.7183 = 0.72

Thus $(1+1/n)^n$ – (1+ æ + ß + 1/8 + 1/16 + ……+ $1/2^{n-1}$)

= (Sum of reciprocals of twin primes) / $(1+1/n)^n$

When we consider Equation 2, we have $(2.7183)^2$ – (2.7183 x 2)= 1.96

7.38915489 – 5.4366 = 1.95255489

From statement 5 above, we had a residual of 0.1745.

We shall make the following propositions or assumptions:

p (v - the probability of drawing an even number from natural numbers is 0.5)

p(d - the probability of drawing an odd number is 0.5)

Therefore, the percentage probability distribution of twin prime numbers

= 0.5 x 0.4 x 0.1745

= 0.03490

Or 3.49%.

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At the beginning of this discussion, we stated that all twin prime numbers end with digits 1, 3, 7, and 9 only, thus 4 out of 10 of all Arabic numerals. However, not all numerals that end with digits 1, 3, 7, and 9 are twin primes. We have, all the same, established here that twin primes, on average, in interval estimation, constitute only 3.49% of all-natural numbers up to infinity.

Table 1: Frequency table of Twin Primes occurrence at intervals of 100 up to 1000

Interval	Frequency of twin primes in interval
1-100	15
101-200	14
201-300	8
301-400	4
401-500	6
501-600	5
601-700	7
701-800	0
801-900	10
901-1000	0

Source: Adapted from: https://prime-numbers.info/list/twin-primes-up-to-10000

The average frequency is therefore 6.9 twin primes per 100 (6.9%) gap, which to infinity will approach 3.49 per 100 intervals or 3.49%.

IV. Analysis and Proof of Twin Prime Conjecture by Inference

However, subtracting (2) from (1) leaves us with 0.71823 as the residual, accounting for the sum of reciprocals other than the reciprocals of even numbers including, twin primes, which as we have shown above is 72% of all numbers. As we said before, twin prime numbers account for approximately 4 out of every ten numbers. So, we shall find 0.4 of this residual 0.1745, which is 0.0687292 or 6.9 percent, as we estimated in Table 1 above. Nevertheless, as we observed before, there are some numbers that end with the four digits 1, 3, 7, and 9, that are not twin prime numerals as some are squares of numbers and, therefore, they are composite. Thus, the value 0.0687292 could be slightly lower.

Thus, we can conclude that about 6.9% of all the natural numbers are prime numbers, including twin primes, which are statistically significant in the sense that twin prime numbers become rare as we approach infinity, but they do not terminate, just as all the natural numbers do not terminate. Twin primes are a subset of prime numbers, which in turn are also subsets of natural numbers and, as such, they have all the attributes of the natural numbers, in the sense that they are never-ending, and they are everywhere dense in probability density space.

In an article by Dubner (2005) he quoted that Hardy-Littlewood's C_2, which was derived by them in 1923 as Twin Prime Constant, was of the value 0.6601618158, and that their method could generate primes as large as those of about 600 digits by using the sieving method.

However, this author maintains that the pairs of terms derived by this author, namely (30n-17) and (30n-19) on the one hand, and (30n-1) and (30n+1) on the other hand, can generate an infinite series of twin primes with the proviso that for any numbers generated by those terms, we have to test them for compositeness by applying the Fermat's test as cited by Dubner (2005).Below, we make a presentation on how arithmetic progression can be used to generate terms for twin prime numbers (cf. Sebah & Gourdon, 2002).

V. Sum of Squares of Reciprocals of the Natural Numbers

Notes

We shall here, examine the sum of the squares of the reciprocals of the natural numbers. We shall concentrate on the first 20 terms. The distribution is 1/1, ¼, 1/9, 1/16, 1/25, 1/36, 1/49, 1/64, 1/81, 1/100, 1/121, 1/144, 1/169, 1/196, 1/225, 1/256, 1/289, 1/324, 1/361, and 1/400. Their decimal equivalents are

1.00000

0.25000

0.11110

0.06250

0.04000

0.02778

0.02041

0.01563

0.01235

0.01000

0.00826

0.00694

0.00592

0.00510

0.00444

0.00391

0.00309

0.00277

0.00250

Total = 1.59172

The total summation of the squares of reciprocals of the first twenty natural numbers is 1.59172. Eremenko (2013) pointed out in his paper that in 1735, Euler solved the problem of summing up squared reciprocals of all-natural numbers to infinity by obtaining a value 1.6462614966 by using the formula

$$1 \text{ to } \infty \sum 1/n^2 = \Pi^2/6 \text{(Eremenko, 2013).}$$

We can see that the total of the first twenty squared reciprocals of the natural numbers is 1.59172, compared with Euler's sum of all the squared reciprocals to infinity. If we divide 1.59172 by Euler's sum, 1.6462614966, we obtain approximately

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97%. This result informs us that only the first twenty terms are remarkably significant, contributing 97% of the total value of the sum of reciprocals of squared numbers to infinity. This result leads to us to conclude that in examining number theory, the further away we get into extraordinarily gigantic numbers, the more abstract and elusive the analysis becomes, so it is like the saying, 'the harder you look, the less you see'. This also informs us to use the first few numbers within our reach, as our reliable sample, for rigorous analysis in order to arrive at valid, reliable, and verifiable truths through deductive and inductive logic.

VI. Findings and Analysis

Consider terms of an arithmetic progression (A.P.) or series, all of which have a common difference of 6. We derive the general term of an A.P. as

$$a + (n-1)\, d, \tag{7}$$

where, "a" is the first term of the series, and, "d" is a common difference.

[1] 5, 11, 17, 23, 29, 35, 41, 47,…………….6n-1

[2] 7, 13, 19, 25, 31, 37, 43, 49…………….6n+1 (cf. Wolfram Maths World, n.d.; Wikipedia.org)

[3] 11, 17, 23, 29, 35, 41, 47………………6n+5

[4] 13, 19, 25, 31, 37, 43, 49………………6n+7

[5] 17, 23, 29, 35, 41, 47, 53………………6n+11 (cf. Wikipedia.org)

[6] 19, 25, 31, 37, 43, 49, 55,…………….6n+13

[7] 23, 29, 35, 41, 47, 53, 59………….. 6n+17

[8] 29, 35, 41, 47, 53, 59, 65………….. 6n+23

[9] 31, 37, 43, 49, 55, 61, 67………….. 6n+25

We can generate an infinite number of twin primes by using the pairs of terms in [1] [2], [3] [4], [5] [6], and [8] [9].

These are

[(6n-1), (6n+1)]

[(6n+5), (6n+7)]

[(6n+11), (6n+13)]

[(6n+23), (6n+25)]

We can extend the same series by having a common difference of 30 instead of 6. Let us consider the following sets of arithmetic series

[1] 5, 35, 65, 95, 125, 155……(This series collapses since all the terms are divisible by 5)

[2] 7, 37, 67, 97, 127………. 30n-23

[3] 11, 41, 71, 101, 131, 161,…. 30n-19

[4] 13, 43, 73, 103, 133, 163,…..30n-17

[5] 17, 47, 77, 107, 137, 167… 30n-13

[6] 19, 49, 79, 109, 139,…..30n-11

[7] 23, 53, 83, 113, 143, …..30n-7

[8] 29, 59, 89, 119, 149, 179 …30n-1

[9] 31, 61, 91, 121, 151,………30n+1

Notes

From the above sets, we can see the pairs of terms [3] [4], [5] [6], and [8] [9] that can be used to generate infinite numbers of twin prime pairs.
These are

[(30n-19), (30n-17)],

[(30n-13), (30n-11)],

[(30n-1), (30n+1)]

We can use the first set of terms by setting n=9,999,999,999,999. We can set up two twin primes with values of 299,999,999,999,953 and 299,999,999,999,951 or 2.99 x 10^{14}(cf. Kurzweg, 2016; Sebah & Gourdon, 2002). We can see that these two numbers are not divisible by any even number, nor by a number ending with 0 or 5. The test of divisibility, by 7, states that the last three digits should be taken off the number, and deducted from the remaining digits, whichever is larger. If the resultant number is not exactly divisible by 7, then the original number is not divisible by 7 (Maths Smart, 2013).

In this case, the numbers are both not divisible by 7.We can, therefore, conclude that we can make 'n' as large as we can, and use any of the terms above to generate twin primes. We can also, therefore, generalize as [30 (10^z – 1) -17] and [30 (10^z-1)-19] where, the power Z, is astronomically a huge number to which 10 is raised, such as a zillion or quintillion or octillion. We subtract 1 from it to leave us with a number which is made up of digits of 9, for example, the gigantic number 999,999,999,999,999,999,999,999,999.

When we multiply this number by 30 and subtract 17 and 19 from the resulting number respectively, we obtain resultant numbers that always end up with the last three digits as 951 and 953 respectively. The sums of all the digits in these numbers give us even numbers, which means they are not divisible by either 3 or 9. We chose the power Z as the symbol of an exceptionally gigantic number, because Z is the last letter of the alphabet.

VII. Uses and Practical Applications of Twin Prime Numbers

We can use twin primes in many ways, in cryptology or encoding a secret language (cf. Alan Turing), or in medicine in naming unique drugs or viruses such as SARS, CORONAVIRUS, MERS, or in astronomy, in naming newly discovered planets, asteroids, stars, and constellations, as twin primes are peculiar, and they do not recur in number space, nor are they divisible by any number, except by themselves, and 1 (Wong, 2013).We can liken twin primes to the Kondratieff cycle in Economics, which is a long cycle with periodicity that lasts between 40 to 60 years(Ganti, 2020).

We can also, in line with demands for non-recurring numbers for use in the sciences and industry, envisage a new series distribution given by reciprocals of Euler's *e* thus

$$(1+1/e)^n \text{ for n=0 to n=infinity. } 1/e = 0.367886456.$$

Therefore, the first term in the series is 1, followed by 1.367886456, then 1.367886456^2, 1.367886456^3, +..........+ 1.367886456^n.

These numbers also produce unique irrational numbers, which can be used in many applications just as the twin prime numerals.

Notes

VIII. A Test for Twin Prime Numbers

We can use the pairs of terms (2n+1) and (2n-1); or (30n-1) and (30n+1); and (30n-17) and (30n-19)to produce twin prime numbers with a difference of 2. We will need to test the numbers generated by having a computerized algorithm flowchart thus

1. Use (2n+1) and (2n-1) to generate pairs of twin primes. You can also use the terms derived by this author (30n-1) and (30n+1) on one side, or (30n-17) and (30n-19) on the other side.
2. Find out whether the numbers generated by these terms end with the digits 1, 3,7, and 9. If yes, proceed to the next stage.
3. We need to test using Fermat's composite test whether numbers generated by step 1 are divisible by 3,7, 11, and 13. If yes, discard them. If no, go to the next step.
4. Find the square root of the number, and if it is a perfect square, reject it. If it is not a perfect square, move on to the next stage.
5. List all numbers tested which have successfully passed these steps as twin primes, if they have a difference of 2, end with the digits 1, 3, 7, and 9 and they are not divisible by any number except by 1 and themselves.

IX. Conclusion

We can, therefore, assert that we can generate infinite series of twin prime numbers using the pair of terms (30n-1) and (30n+1) on the one hand, and on the other side, the dual terms (30n-17) and (30n-19) as well as the other duo terms tabulated above. However, numbers that are generated by all the terms of the arithmetic series produced in this article have to undergo the examination for compositeness by using Fermat's method. Twin prime numbers have numerous practical applications in medicine, pharmacology, astronomy, cryptology, data science, and machine learning, among many other uses.

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Notes

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Analysis and Calculation of Energy Indicators based on the Formation of the Predicted Value of the Total and Specific Energy Consumption

By I. U. Rakhmonov
Tashkent State Technical University

Abstract- Accurate forecasting of electricity consumption determines the success of industrial enterprises. Each enterprise, having an accurate prediction of the amount of power consumed, strictly controls it, since deviations entail disruptions in work, is subject to fines. Power consumption forecasting for a certain period is the most urgent task in the today's electricity market. Existing forecasting methods have individual characteristics and have their own advantages and disadvantages. The choice of forecasting methods depends on such major factors as the time for which the forecasting is performed, as well as the amount of information.

Keywords: forecasting method, power consumption, specific energy consumption, products, raw materials, technological factors, production factors, calculation accuracy, energy indicators, components of the technological process.

GJSFR-F Classification: MSC 2010: 83C40

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Notes

Analysis and Calculation of Energy Indicators based on the Formation of the Predicted Value of the Total and Specific Energy Consumption

I. U. Rakhmonov

Abstract- Accurate forecasting of electricity consumption determines the success of industrial enterprises. Each enterprise, having an accurate prediction of the amount of power consumed, strictly controls it, since deviations entail disruptions in work, is subject to fines. Power consumption forecasting for a certain period is the most urgent task in the today's electricity market. Existing forecasting methods have individual characteristics and have their own advantages and disadvantages. The choice of forecasting methods depends on such major factors as the time for which the forecasting is performed, as well as the amount of information. When forecasting power consumption, it is necessary to take into account various factors related to the technological features of production, organization of equipment operation, etc.The present article describes the proposed method for determining the forecasted values of power consumption parameters in terms of total and specific power consumption, which differs from the existing methods in that it takes into account, when forecasting power consumption parameters, the features of production that characterize the production process, the power consumption modes of process equipment, and the impact of technological and operational factors on energy performance that affect the forecast of readings.Accounting for the above factors can significantly simplify and improve the accuracy of forecasting calculations. When determining the forecast values, the components of electricity consumption for the main, auxiliary and additional production are directly included in the calculation. In addition, the method allows to investigate and optimize the power consumption modes of both a separate production unit and the enterprise as a whole in conjunction with the indicators of processed products and components of the technological process used. The reliability of the method is justified by calculating the energy indicators of a specific section rolling shop of a steel industry.

Keywords: forecasting method, power consumption, specific energy consumption, products, raw materials, technological factors, production factors, calculation accuracy, energy indicators, components of the technological process.

I. Introduction

The resulting indicators of energy saving are the reduction in the specific consumption of energy resources per unit of industrial production. The complexity of the problem of energy conservation lies in the fact that this indicator is a function of many variables - quantitative and qualitative indicators of production. Considering this problem in terms of power consumption, it can be argued that, in addition to electrical factors, both technological and operational, as well as factors causing deviations from the norms of consumption of raw materials, intermediate products and auxiliary

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components, should be involved in the analysis and calculation of specific energy consumption, as well as in identifying reserves of savings. technological process (compressed air, oxygen, water, etc.). The latter, to one degree or another, are usually taken into account, but are not directly included in the calculation of standards. In many cases, this interferes with an objective assessment of energy performance. The increasing scale of electrification of enterprises requires continuous improvement of methods for analysis, assessment and calculation of energy consumption levels, especially in industry, which accounts for more than half of all electricity produced. Although at present there are a fairly large number of methods for predicting energy consumption, the results of their application do not always reflect the real situation. In this regard, we set the task of scientifically-based calculation, analysis and forecasting of power consumption for each production, taking into account its specific features [1,2].

Ferrous metallurgy has a number of specific production features that distinguish it from other industries. Existing methods for forecasting electricity do not take into account these features that characterize the production process, modes of power consumption of technological equipment, as well as the influence of technological and operational factors on energy performance. In addition, energy services are forced to normalize, evaluate and distribute energy resources based on experience and intuition, taking into account data on the actual and planned performance of individual enterprises, which leads to certain inaccuracies and errors. It should be noted that power consumption depends on the quality of the metal, on the technical perfection of technological equipment and the production process. Sometimes, to obtain 1 ton of finished products, it is necessary to process several tons of metal. And if the organization of production is not perfect, then there may be excess losses and, accordingly, energy overruns. Therefore, today it is relevant to improve the work on forecasting electricity [2,3,4].

II. The Main Part

Assessment of the levels of absolute and specific energy consumption of integrated production, which includes a number of independent units, requires taking into account a large number of indicators.

The complexity of the analysis of power consumption modes and the calculation of energy indicators of ferrous metallurgy enterprises is caused by a number of reasons. Firstly, the range of finished products of the enterprise is very wide, has dozens of items and various energy intensity. At the same time, energy accounting, both for each production and the plant as a whole, is single. Secondly, a number of industries receive part of the intermediate production through internal and external production relations, while at the same time, they sell a certain amount of their products to the side. At the same time, depending on which production unit received the intermediate products from the outside, and what is the number of processing conversions, differences in energy consumption are produced [5,6].

Currently, due to the lack of developed methods for accounting for the above features, forecasting is carried out according to the general indicators of energy consumed from the network and the number of finished products. Naturally, the results of such forecasting calculations in some cases are inaccurate or erroneous and do not reflect the actual level of power consumption.

The task is to find ways to take into account the characteristics of production with a prerequisite for reflection in the calculations for predicting the energy intensity

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of processes. A technique is proposed for predicting energy indicators for a complex of industries as a whole, taking into account the above requirements.

To ensure reliable forecasting, it is necessary to study the patterns of change in the energy intensity of products and determine the conditions under which a change in time of energy consumption indicators can change significantly. In addition, the accuracy of forecasting is influenced by the development of new technologies, the improvement of equipment, the scale of modernization, which are different in each industry. The energy intensity of products and the level of energy consumption also depend on changes in the structure of energy consumption, and the latter depends on the need for such components of the process as: compressed air, oxygen, nitrogen, etc. [7,8,9].

Therefore, it is possible to formulate a set of tasks related to the above factors in order to increase the accuracy of forecasting. According to the proposed method, the projected total electric energy consumption of the enterprise is divided into the following components:

- Energy consumption by enterprises where equipment does not change during the entire forecast period and traditional technologies are applied (W_b);
- Energy consumption by enterprises where it is planned to modernize technological equipment, switch to a new technology (W_m);
- Energy consumption by new enterprises, with new equipment and technology (W_n).

The projected energy consumption will be:

$$W = W_b + W_m + W_n; \qquad (1)$$

or

$$W = \Pi(\beta_b d_b + \beta_m d_m + \beta_n d_n) \; ; \qquad (2)$$

where Π is the number of planned products; $\beta_b, \beta_m, \beta_n$ - the share of planned types of products attributable to the relevant enterprises; d_b, d_m, d_n - respectively, the specific consumption of electricity.

The planned output for the forecast period (Π) and, accordingly, the shares of existing, modernized and new industries ($\beta_b, \beta_m, \beta_n$) are determined in accordance with the plan for the future development of the industry.

The I level of power consumption W_1 of the production unit for the production of intermediate product Π_1 includes the specific consumption e1 of electricity W_1 for the production of products Π_1 and the specific consumption q_1 of product Π_1 per unit of final product F.

The II level of power consumption W_2 of the production unit for the production of process components and secondary energy resources B includes specific costs e_2 of electricity W_2 for product B and specific consumption q_2 of product B per unit of final product F.

The III level of other electricity costs W_3 for the production of final products includes the specific consumption of e_3 electric power W_3 per unit of final product F.

It should be noted that the above levels, in addition to indicators of energy consumption, also include indicators of the consumption of raw materials and technological components.

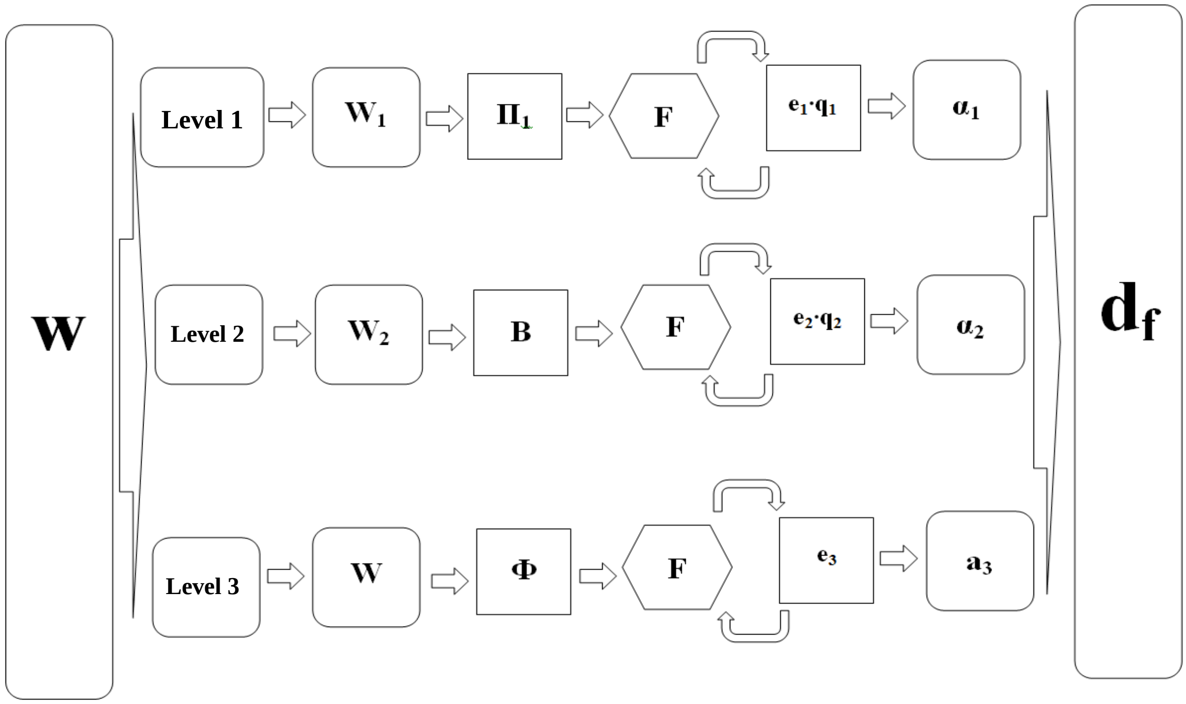

Fig. 1: The scheme for the formation of the predicted specific energy consumption

The predicted values of specific electricity consumption by levels are determined by the following expressions:

a) The specific energy consumption for the production of this workshop is calculated according to the following formulas:

for tier I

$$e_1 = \frac{W_{1.}}{\Pi_1}; \tag{3}$$

for tier II

$$e_2 = \frac{W_{2.}}{B}; \tag{4}$$

for tier III

$$e_3 = \alpha_3 = \frac{W_{3.}}{F} \quad . \tag{5}$$

b) The specific consumption of intermediate production products, technology components per unit of final product can be calculated by the formulas:

for tier I

$$q_1 = \frac{\Pi_1}{F}; \tag{6}$$

Notes

for tier II

$$q_2 = \frac{B_i}{F} \ . \tag{7}$$

The specific energy consumption of the enterprise is determined by the components of each level, using the following expressions:

$$a_1 = \varepsilon_1 \cdot q_1 ; \tag{8}$$

$$a_2 = \varepsilon_2 \cdot q_2 ; \tag{9}$$

$$a_3 = e_3 \ . \tag{10}$$

Therefore, the total specific energy consumption will be:

$$d_f = \sum_1^n \alpha_1 + \sum_1^{n'} \alpha_2 + \alpha_3 , \tag{11}$$

where n, n' are the number of levels of types I and II, respectively, which characterize the total value of the specific power consumption of the main and auxiliary industries.

The developed method for generating the predicted specific energy consumption by types of energy consumption and production processes (Fig. 1) can significantly simplify and improve the accuracy of calculations. This allows you to directly include in the calculation the component values of specific electricity consumption and indicators of individual energy-intensive facilities. The considered parameters are taken from the reporting data. In addition, they are considered, taken into account and controlled separately as in-plant indicators, without a direct link to the general production specific power consumption factors (for example, electric steel furnaces, compressors, etc.) [10,11].

The basis of the proposed method for forecasting energy consumption is, firstly, the division of the total value of this consumption into three categories:

1) Electricity consumption by existing and newly commissioned enterprises, in which existing equipment is stored in the forecast period;
2) Energy consumption by modernized enterprises;
3) Energy consumption by enterprises commissioned in the forecast period with new equipment and new technologies.

Secondly, differentiation is carried out within each category:

1) Electricity consumption by the main production units;
2) Energy consumption by type of energy-intensive auxiliary needs;
3) Other expenses.

As a result, all energy consumption by the industry and its units is differentiated and each calculation can be performed by levels, i.e. make it extremely easy. The calculation allows you to take into account the specifics of technological progress in each of the links in production.

As the basic (initial) data for metallurgical enterprises, the energy consumption indicators of the corresponding production facilities with operating equipment are taken.

As a result, the electricity demand for a given perspective in the industry (W_b) can be determined based on the values of Π_b, B_b, F_b,

Ref

10. I.U.Rakhmonov, K.M.Reymov. Regularities of change of energy indicators of the basic technological equipment of the cotton-cleaning industry. Journal of Physics: Conference Series. APITECH-2019. 1399 (2019) 055038 doi:10.1088/1742-6596/1399/5/055038.

Where

$$W_b = d_b \cdot F_b \ . \qquad (12)$$

The proposed method for determining the predicted value of specific electricity consumption can be applied in industrial enterprises, as well as by type of product of the main divisions of the enterprise for short-term forecasting [12-14].

The reliability of the proposed method is demonstrated as an example by determining the energy indicators of the section rolling shop №1 equipped with traditional technology equipment for the main types of products.

The initial data for the calculation is the products of the section rolling shop №1, shown in table 1.

The value of the specific energy consumption for the production of a given technological cycle by production units is calculated according to formulas 2-4:

for block I:

for ball rolling Ø 100 - $e_{100} = \dfrac{111959{,}4}{3167} = 35{,}3\, kW \cdot h/t$;

Table 1: The main energy indicators of the rolling shop №1

№	Type of product	Output volume	Heat consumption	Power consumption: For technology W_{tech}, kWh	Power consumption: For auxiliary needs W_{aux}, kWh	Power consumption: For other needs W_{oth}, kWh
1	Ball rolling Ø 100	3167	1254	111959,4	66782,8	17677,8
2	Ball rolling Ø 68	8573	3568	351861	209882	55557
3	Ball rolling Ø 40	431	568	36879	21998	5823
	the section rolling shop №1	12171	5390	500699	298663	79057,8

for ball rolling Ø 68 - $e_{68} = \dfrac{351861}{8573} = 41\, kW \cdot h/t$;

for ball rolling Ø 40 - $e_{40} = \dfrac{36879}{431} = 85{,}6\, kW \cdot h/t$.

The specific consumption of intermediate production products, technology components per unit of final product is calculated according to formulas 5-6:

for block I:

$$\text{Ball rolling Ø 100 - } q_{100} = \frac{3167}{12171} = 0{,}26\, t/t;$$

12. I.U.Rakhmonov, N.N.Niyozov._Optimization setting of steel-smelting industry in the issue of alloy steels. E3S Web Conf. Volume 139, 2019. Rudenko International Conference "Methodological problems in reliability study of large energy systems" (RSES 2019) 01077. 1-3 p. https://doi.org/10.1051/e3sconf/201913901077.

$$\text{Ball rolling Ø 68 - } q_{68} = \frac{8573}{12171} = 0{,}7\, t/t;$$

$$\text{Ball rolling Ø 40 - } q_{40} = \frac{431}{12171} = 0{,}03\, t/t.$$

Notes

The total specific energy consumption for the I block is determined by the formula (8):

$$\sum_{1}^{n} e_i = 35{,}3 \cdot 0{,}26 + 41 \cdot 0{,}7 + 85{,}6 \cdot 0{,}03 = 40{,}5\, kW \cdot h/t.$$

According to the formula (4), we calculate the value of the specific energy consumption for the production of a given technological cycle by production units:

for block I:

$$\text{for ball rolling Ø 100 - } \gamma_{100} = \frac{66782{,}8}{1254} = 53{,}2\, kW \cdot h/Gkal;$$

$$\text{for ball rolling Ø 68 - } \gamma_{68} = \frac{209882}{3568} = 58{,}8\, kW \cdot h/Gkal;$$

$$\text{for ball rolling Ø 40 - } \gamma_{40} = \frac{21998}{568} = 38{,}7\, kW \cdot h/Gkal.$$

By the formula (7), we calculate the specific heat energy consumption per unit of final product:

for block II:

$$\text{for ball rolling Ø 100 - } \omega_{100} = \frac{1254}{12171} = 0{,}1\, Gkal/t;$$

$$\text{for ball rolling Ø 68 - } \omega_{68} = \frac{3568}{12171} = 0{,}3\, Gkal/t;$$

$$\text{for ball rolling Ø 40 - } \omega_{40} = \frac{568}{12171} = 0{,}05\, Gkal/t.$$

The total specific energy consumption for block II is determined by the formula (3.9):

$$\sum_{1}^{n} \alpha_i = 53{,}2 \cdot 0{,}1 + 58{,}8 \cdot 0{,}3 + 38{,}7 \cdot 0{,}05 = 25\, kW \cdot h/t$$

According to the formula (5), we calculate the value of the specific energy consumption by production units for block III:

$$\text{for ball rolling Ø 100 - } \mu_{100} = \frac{17677{,}8}{12171} = 1{,}56\, kW \cdot h/t;$$

$$\text{for ball rolling Ø 68 - } \mu_{68} = \frac{55557}{12171} = 4{,}56\; kW \cdot h / t;$$

$$\text{for ball rolling Ø 40 - } \mu_{40} = \frac{5823}{12171} = 0{,}5\; kW \cdot h / t.$$

The total specific energy consumption for block III will be:

$$\alpha_{np} = 1{,}56 + 4{,}56 + 0{,}5 = 6{,}56\; kW \cdot h / t.$$

According to the formula (10), we calculate the total specific energy consumption for the workshop:

$$d_z = 40{,}5 + 25 + 6{,}56 = 72\; kW \cdot h / t.$$

According to formula (11), the need for electricity for a given perspective in the industry (W_b) is determined:

$$W_b = 72 \cdot 12171 = 876312\;\; kW \cdot h.$$

According to the calculation indices db, it is possible to determine the predicted values of the specific and absolute energy consumption of the enterprises where modernization is planned (d_m, W_m), and for each level, in accordance with the scheme (Fig. 1), the following are calculated:

for tier I

$$d_{m_1} = \frac{e_b \cdot \varPi_b \pm \sum_1^m \Delta W_m}{\varPi_b + \Delta \varPi_m}; \tag{13}$$

for tier II

$$d_{m_2} = \frac{\alpha_b \cdot B_b \pm \sum_1^m \Delta W_m}{B_b + \Delta B_m}, \tag{14}$$

where m is the number of units or processes to be modernized; ΔW_m– energy saving due to modernization of technological equipment.

Using the formula (9), the predicted values of the consumed raw materials and the flow rate of the components of the technological process q_2 are determined.

For enterprises with new, more advanced technologies and equipment (d_n and $W_н$) for given $\varPi$, B, F, the calculation of electricity demand is performed by the same method as on the basis of design data.

Values of reliable data for individual indicators - projected products, consumption of process components, etc. can be determined by expert judgment.

Thus, according to the proposed method, for each industry, it is possible to obtain the necessary set of mathematical models that can improve the accuracy of forecasting power consumption, as well as flexibly and quickly carry out their correction for any technical and technological changes in individual links of each production.

Notes

III. Conclusions

1. A method for forecasting power consumption parameters has been developed, which allows one to study and optimize power consumption modes of both a separate production unit and the enterprise as a whole in conjunction with indicators of processed products and used components of the technological process.
2. It has been established that the method for determining the predicted specific energy consumption allows us to more fully investigate the trends and patterns of changes in the specific and total energy consumption, taking into account the complex influence of technological and production factors, and allows us to significantly simplify and improve the accuracy of calculations.

Notes

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Elliptic Blending with One-Equation Model

By M. M. Rahman, Xiaoqing Tian, Huachen Pan & A. K. M. Sadrul Islam
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Abstract- A wall-distance-free modification to Wray-Agarwal (WA) one-equation turbulence model is convoked using an elliptic relaxation approach to accurately accounting for non-local characteristics of near-wall turbulence. Model coefficients/functions are parameterized with the elliptic relaxation function to preserve the combined effects of near-wall turbulence and nonequilibrium. The characteristic length scale associated with the elliptic relaxation equation is formulated in terms of viscous and turbulent length scales in conjunction with the invariant of strain-rate tensor. Consequently, non-local effects are explicitly influenced by the mean flow and turbulent variables. A near-wall damping function is introduced to relax the viscous length-scale coefficient adhering to the elliptic relaxation model. Comparisons indicate that the new model improves the accuracy of flow simulations compared to the widely used Spalart-Allmaras model and remains competitive with the SST k-ω model. A good correlation is obtained between the current model and DNS/experimental data.

Keywords: *damping function; wall-distance-free; elliptic blending; non-equilibrium flow.*

GJSFR-F Classification: *MSC 2010: 11G05*

ELLIPTICBLENDINGWITHONEEQUATIONMODEL

Strictly as per the compliance and regulations of:

Elliptic Blending with One-Equation Model

M. M. Rahman [α], Xiaoqing Tian [σ], Huachen Pan [ρ] & A. K. M. Sadrul Islam [ω]

Abstract- A wall-distance-free modification to Wray-Agarwal (WA) one-equation turbulence model is convoked using an elliptic relaxation approach to accurately accounting for non-local characteristics of near-wall turbulence. Model coefficients/functions are parameterized with the elliptic relaxation function to preserve the combined effects of near-wall turbulence and nonequilibrium. The characteristic length scale associated with the elliptic relaxation equation is formulated in terms of viscous and turbulent length scales in conjunction with the invariant of strain-rate tensor. Consequently, non-local effects are explicitly influenced by the mean flow and turbulent variables. A near-wall damping function is introduced to relax the viscous length-scale coefficient adhering to the elliptic relaxation model. Comparisons indicate that the new model improves the accuracy of flow simulations compared to the widely used Spalart-Allmaras model and remains competitive with the SST k-ω model. A good correlation is obtained between the current model and DNS/experimental data.

Keywords: damping function; wall-distance-free; elliptic blending; non-equilibrium flow.

I. Nomenclature

C_1	model closure coefficient
C_b	model closure coefficient
C_w	model closure coefficient
C_f	skin-friction coefficient
$C_{1k\epsilon}$	closure coefficient
$C_{2k\omega}$	closure coefficients
$C_{2k\epsilon}$	closure coefficient
d	distance to the nearest wall
W	mean vorticity invariant
u^+	non-dimensional flow velocity
y^+	non-dimensional wall distance
μ, μ_T	laminar and turbulent eddy viscosities
ν	kinematic viscosity
ρ	density
σ	Schmidt number
f_μ	eddy-viscosity damping function
f_1	blending function
f_R	elliptic blending function
h	channel height
k	turbulent kinetic energy
R	undamped eddy-viscosity; k/ω
Re	Reynolds number
S	mean strain-rate invariant
χ	undamped turbulent-to-laminar viscosity ratio
ω	specific turbulent dissipation rate
Subscript	
T	turbulent condition
i,j	variable numbers
ref	reference condition

II. Introduction

One-equation turbulence models solve directly for the eddy-viscosity (μ_T) without computing the full range of turbulent time and length scales. This form of transport equation is attrac-

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tive due to its simplicity of implementation and less demanding computational requirements, compared with the standard two-equation k-ϵ and k-ω models. However, several one-equation eddy-viscosity (μ_T) turbulence models are unable to accurately predict non-equilibrium flows, e.g., wall-bounded flows in adverse pressure gradient with separation and reattachment. This can be largely attributed to the overestimation of μ_T by the model. Thus, the inclusion of an elliptic blending in the μ_T-transport equation may improve the prediction capability of one-equation model.

In particular, many turbulence models usually include the distance to the wall as an explicit parameter which hinders them from simulating complex flows involving multiple surfaces; the wall distance in this case becomes cumbersome to be defined accurately. The elliptic relaxation method is an excellent way to avoid the use of wall-distance in an eddy-viscosity turbulence model [1-10]. The wall blocking is governed by an elliptic partial differential equation (i.e., the Helmholtz-type equation) which accounts for non-local near-wall effects. Durbin et al. [7], Rahman et al. [8, 9] and Elkhoury [10] have employed a Helmholtz-type relaxation equation along with a one-equation model to account for the wall-blocking effect.

Recently, a new one-equation model, the Wray-Agarwal (WA) model based on $R = k/\omega$ [11, 12] has been proposed which has shown promise for accurately predicting many wall-bounded mildly separated flows. However, the model contains explicitly the distance from the wall. This paper proposes wall-distance-free (wdf) modifications to the WA model with an elliptic blending function. Nevertheless, the damping function is retained to relax the viscous length-scale coefficient embedded with the elliptic relaxation equation. The elliptic WA (EWA) model is applied to compute the fully-developed turbulent flow in a channel, the turbulent flow in an asymmetric planar diffuser and the flow over an Onera-M6 wing. Computations show that the EWA model improves the predictions compared to the original WA (OWA) model.

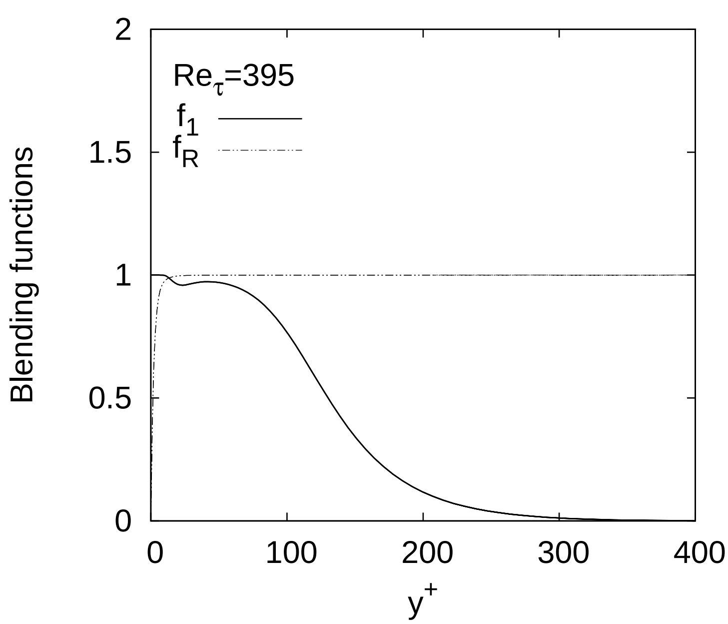

Figure 1: Blending function distributions for turbulent channel flow

III. Original WA(OWA) Model

The OWA model determines the undamped eddy-viscosity $R = k/\omega$ by the following transport equation [11, 12]:

Ref

7. Durbin PA, Mansour NN, Yang Z: Eddy viscosity transport model for turbulent flow. *Physics of Fluids*, 6(2):1007-1015;1994.

$$\frac{\partial \rho R}{\partial t}+\frac{\partial \rho U_j R}{\partial x_j} = \frac{\partial}{\partial x_j}\left[(\mu+\sigma_R\,\mu_T)\,\frac{\partial R}{\partial x_j}\right]+C_1\rho R S+\rho\, f_1 C_{2k\omega}\frac{R}{S}\frac{\partial R}{\partial x_j}\frac{\partial S}{\partial x_j}$$
$$-\ \rho\,(1-f_1)\left[C_{2k\epsilon}\left(\frac{R}{S}\right)^2\frac{\partial S}{\partial x_j}\frac{\partial S}{\partial x_j}\right] \quad (1)$$

where the eddy viscosity $\mu_T = f_\mu \rho R$ and other variables are evaluated as:

$$\begin{aligned} f_1 &= \tanh\left(arg_1^4\right), & arg_1 &= C_b\frac{\nu+R}{S\kappa^2 d^2} \\ f_\mu &= \frac{\chi^3}{\chi^3+C_w^3}, & \chi &= \frac{R}{\nu} \\ C_1 &= f_1\left(C_{1k\omega}-C_{1k\epsilon}\right)+C_{1k\epsilon}, & \sigma_R &= f_1\left(\sigma_{k\omega}-\sigma_{k\epsilon}\right)+\sigma_{k\epsilon} \end{aligned} \quad (2)$$

where d is the normal distance from the wall and ν represents the kinematic viscosity. The associated constants are [12]: $C_{1k\omega}=0.0833$, $C_{1k\epsilon}=0.16$, $C_{2k\omega}=1.22$, $C_{2k\epsilon}=1.95$, $\sigma_{k\omega}=0.72$, $\sigma_{k\epsilon}=1.0$, $C_b=1.66$, $C_w=8.54$ and $\kappa=0.41$. The in-variants of mean strain–rate and vorticity tensors are given by $S=\sqrt{2S_{ij}S_{ij}}$ and $W=\sqrt{2W_{ij}W_{ij}}$ (required afterwards), respectively. The strain–rate S_{ij} and the vorticity W_{ij} tensors are defined as

$$S_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right),\qquad W_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}-\frac{\partial u_j}{\partial x_i}\right) \quad (3)$$

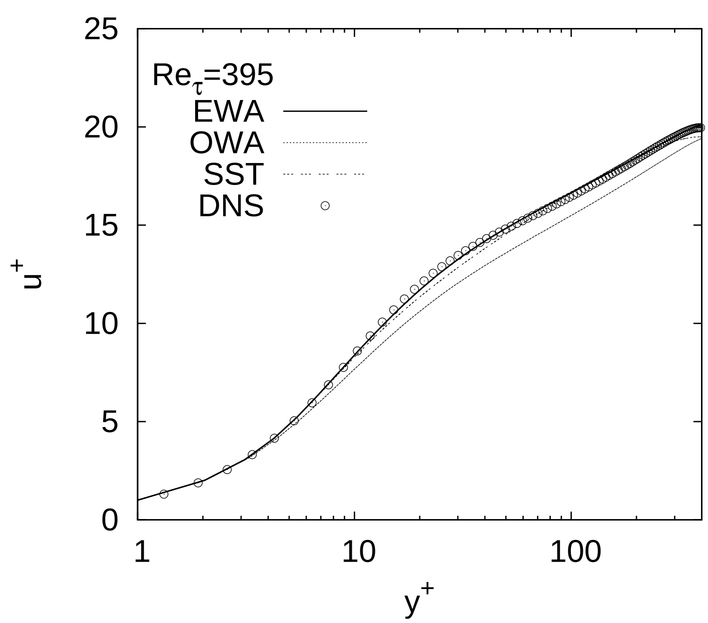

Figure 2: Channel flow predictions compared with DNS results

A plot of blending function related to the OWA model in Eq. (2) for a fully-developed channel flow with $Re_\tau = 395$ is shown in Fig. 1. As can be seen, for the OWA model $f_1 = 1.0$ in the viscous sub-layer and in the logarithmic overlap region; finally approaches 0 far away from the wall.

Ref

12. Wray T: Development of a One-Equation Eddy Viscosity Turbulence Model for Application to Complex Turbulent Flows. Engineering and Applied Science Theses & Dissertations. 214. Washington Uiversity in St. Louis, USA, 2016.

IV. Elliptic WA (EWA) Model

To assess the impact of elliptic relaxation function to account for non-local wall effects and flow in-homogeneity solely through the model coefficients, the OWA model is designed as:

$$\frac{\partial \rho R}{\partial t}+\frac{\partial \rho U_j R}{\partial x_j} = \frac{\partial}{\partial x_j}\left[(\mu+\sigma_R\,\mu_T)\frac{\partial R}{\partial x_j}\right]+C_1\rho RS+\rho\, C_{2k\omega}\frac{R}{S}\frac{\partial R}{\partial x_j}\frac{\partial S}{\partial x_j}$$
$$-\ \rho\, C_{2k\epsilon}\min\left[\left(\frac{R}{S}\right)^2\frac{\partial S}{\partial x_j}\frac{\partial S}{\partial x_j};\frac{C_l S R^2}{\nu}\right] \quad (4)$$

where C_l is defined subsequently and the eddy-viscosity $\mu_T = f_\mu \rho R$.

The physical relevance of elliptic-relaxation blending is to reinforce wall-effect, thereby suppressing the production of eddy-viscosity near a surface. To revive this phenomenon in the current model, the elliptic relaxation equation of Elkhoury [10] is invoked herein with the source term replaced by 1.0:

$$-L_R^2\nabla^2 f_R + f_R = 1.0 \quad (5)$$

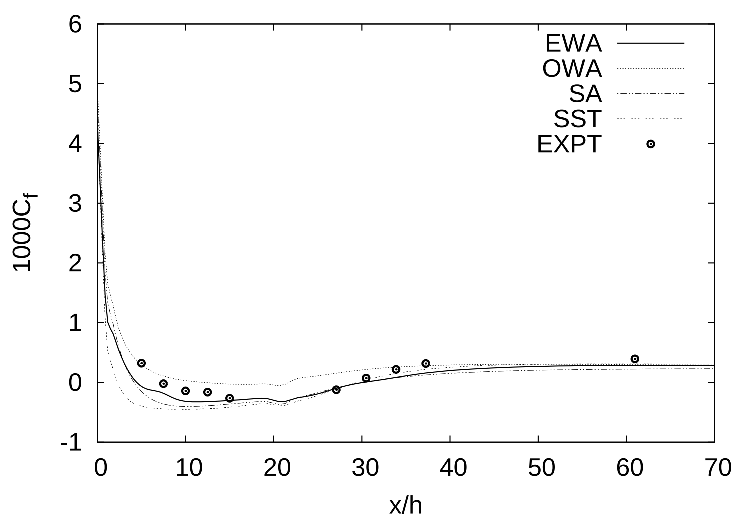

Figure 3: Skin-friction coefficient of diffuser flow along deflected bottom wall

Note-worthily, the near-wall turbulence eddies follow the Kolmogorov scaling (i.e., turbulence fluctuations depend on the laminar viscosity). Therefore, ν/S can be used for the viscous scaling, serving as a lower bound on the turbulent length scale:

$$L_R^2 = \max\left(\frac{C_l\,R}{3};C_l\,\nu\right)\bigg/ S \approx C_l\nu\sqrt{1+\frac{\chi^2}{9}}\bigg/ S \quad (6)$$

where $C_l = 4+\sqrt{\chi}$ and coefficients associated with the length scale L_R are determined from fully-developed turbulent channel and complex (i.e., flow with separation and reattachment) flow simulations. The virtue of Eq. (6) is that unlike the Poisson equation, it requires no special numerical treatment (i.e., the Laplace operator is relatively easy to treat and is particularly easy

Ref
10. Elkhoury M: On the eddy viscosity transport models with elliptic relaxation. Journal of Turbulence, 18(3):240-259;2017.

Notes

with the modification $-L_R^2\nabla^2+1$). It can be easily solved in parallel with the R-equation with an initial guess $0 \le f_R \le 1$ everywhere except on the wall boundaries where $f_R = 0$. It should be noted that the damping function f_μ is introduced to relax the viscous length-scale coefficient C_l. In other words, f_μ avoids the use of a larger value of C_l in the viscous sub-layer and the wall-blocking effect is partially shared by the damping function.

It is worth mentioning that the inverse viscous scaling S/ν is used in Eq. (4) with the $C_{2k\epsilon}$-destruction term to avoid singularity when $\frac{1}{S^2}\frac{\partial S}{\partial x_j}\frac{\partial S}{\partial x_j} \sim \infty$ (i.e., strain-rate S reaches very small values). However, the limiter used in this work is:

$$\frac{C_l S R^2}{\nu} >> \frac{R^2}{S^2}\frac{\partial S}{\partial x_j}\frac{\partial S}{\partial x_j}$$

and the original formulation is eventually recovered. In addition, several numerical computations dictate that the constant coefficients given by the OWA model are too cumbersome to be implemented due to the introduction of elliptic blending. Therefore, the following parametric relations are set to the EWA model:

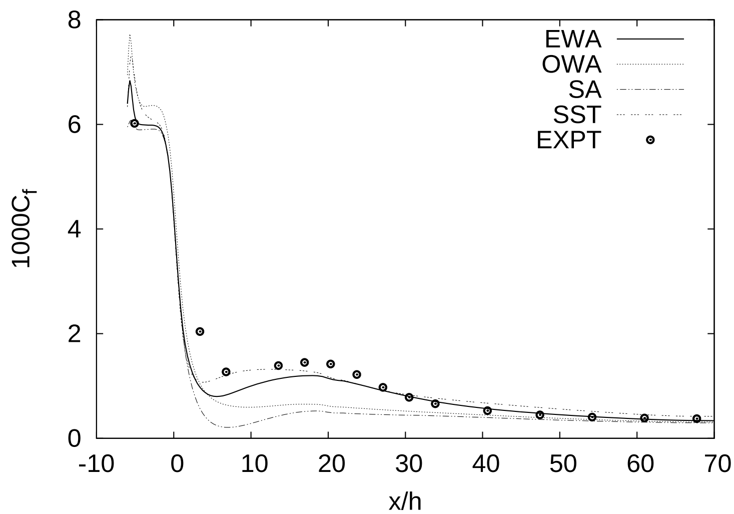

Figure 4: Skin-friction coefficient of diffuser flow along straight top wall

$$\begin{aligned} f_\mu &= \frac{\chi^3}{\chi^3+C_w^3}, & C_1 &= f_R - 1.0 + C_{1k\epsilon} \\ C_{2k\omega} &= 10C_{1k\epsilon}\left(1.0-f_R\right), & C_{2k\epsilon} &= 2.0 - f_R + \min\left(A_{k\omega};C_{1k\epsilon}\right) \\ A_{k\omega} &= \sqrt{\frac{|S-W|}{\max\left(S;W\right)}}, & C_w &= C_l \end{aligned} \quad (7)$$

The revised constants are: $C_{1k\epsilon} = 0.12$ and $\sigma_R = 0.769$. To this end, it can be stressed that the parameter $A_{k\omega}$ is activated when $S \neq W$ (i.e., non-equilibrium flows). A plot of f_R based on Eq. (5) for a developed channel flow is shown in Fig. 1. As can be seen, the relaxation function $f_R \sim 1.0$ after the viscous sub-layer. Note that referring to the OWA model, the $C_{2k\omega}$-cross-diffusion term in the EWA model is limited to be employed near solid walls to minimize the free-stream sensitivity of turbulence model.

Remarkably, with the elliptic blending strategy, the net production $(f_R - 1 + C_{1k\epsilon})\,RS$ has been negative at near-wall (e.g., viscous) regions; however, quickly becomes positive away from the solid wall as is evidenced from Fig.1. This tendency is due to an appropriate choice of length scale associated with the relaxation equation. As mentioned earlier, the wall-blocking effect is partially shared by f_μ signifies that a further reduction in the production term by the elliptic relaxation is suppressed. This is an advantage of integrating both viscous (low-Reynolds number) and blocking (i.e., non-local) effects in a turbulence model, having positive implications. In all computations, R has been non-negative and the proposal for elliptic blending can be thought of as an interpolation method, existing inside the wall-vicinity with the interpolation coefficient provided by a non-local model.

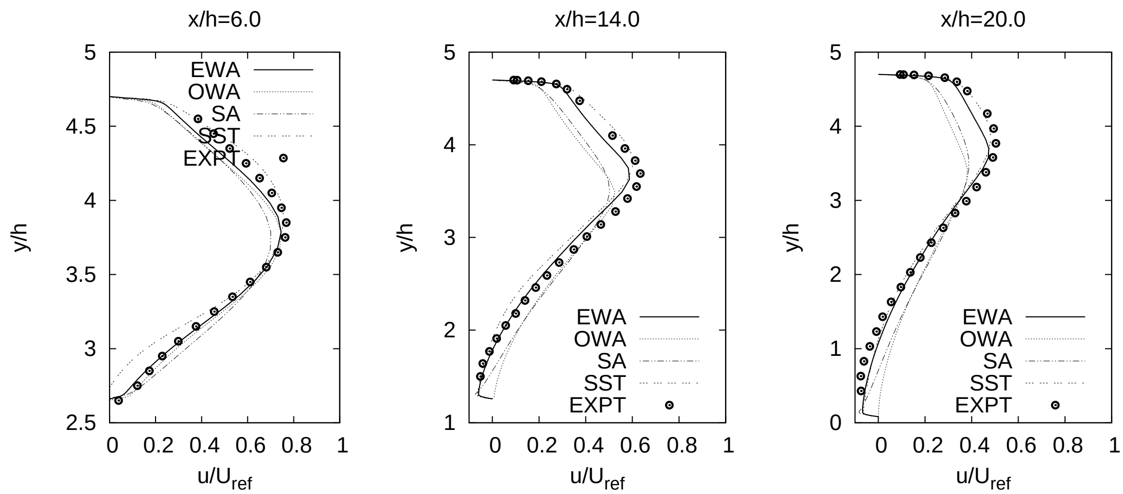

Figure 5: Mean velocity profiles at selected locations for diffuser flow

V. Numerical Computations

To evaluate the accuracy of proposed modifications to the OWA model, a fully developed turbulent channel flow, an asymmetric plane diffuser flow and the flow over an Onera–M6 wing are computed. To evaluate the model reliability, the current model predictions are compared with those from the widely-used SA model [13] and Menter SST (shear–stress transport) k-ω [14] model. A cell-centered finite-volume scheme combined with an artificial compressibility approach [15-18] is employed to solve the governing equations.

a) *Fully-Developed Turbulent Channel Flow*

The computation is carried out for a fully developed turbulent channel flow at $Re_\tau = 395$ for which turbulence quantities are available from the DNS data [19]. The computations are conducted in the half-width h of a channel, using the one-dimensional RANS solver. To ensure the accurate resolution of viscous sub-layer, the first grid node near the wall is placed at $y^+ \approx 0.3$. A 1×64 nonuniform grid is used across the channel half-width which is sufficient to ensure a grid-independent numerical solution. Comparisons are made by plotting the results in wall units. The results for the velocity profile shown in Fig. 2 indicate that the EWA model prediction is qualitatively good relative to the OWA model.

Ref

13. Spalart PR, Allmaras SR: A One-Equation Turbulence Model for Aerodynamic Flows. AIAA Paper No. 92-0439, 1992.

b) *Turbulent Flow in an Asymmetric Plane Diffuser Flow*

To further evaluate the performance, the models are applied to simulate the flow in a plane asymmetric diffuser with an opening angle of 10^o, for which measurements are available [20]. The expansion ratio of 4.7 is sufficient to produce a separation bubble on the deflected wall. This configuration provides a test case for adverse pressure driven wall-bounded separated flow. The

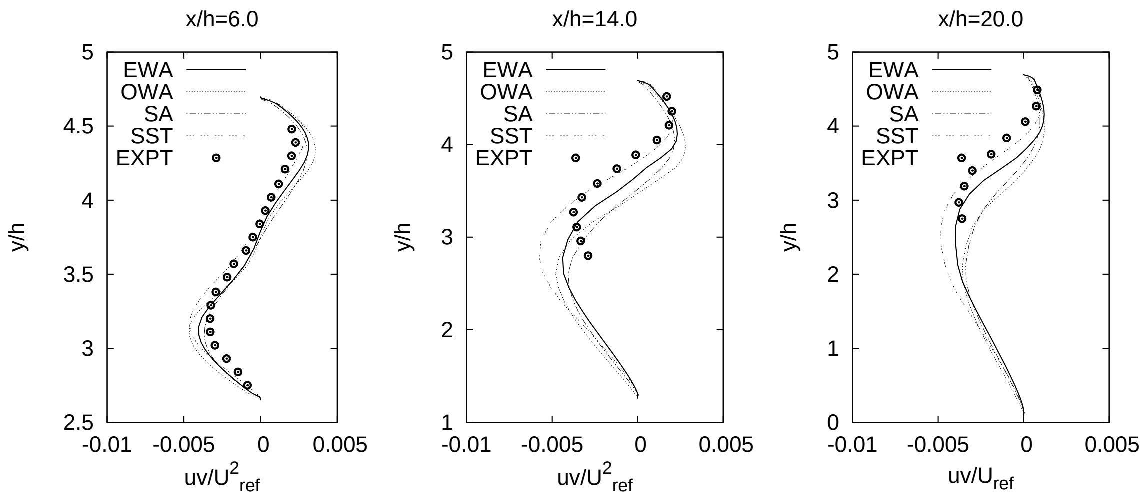

Figure 6: Shear stress profiles at selected locations for diffuser flow

entrance to the diffuser consists of a plane channel to invoke fully developed turbulent flow with $Re = 20000$ based on the center-line velocity and the channel height.

Computations with a 120×72 nonuniform grid resolution are found to be accurate to obtain the grid-independent solution. The length of the computational domain is $76h$, where h is the inlet channel height. The thickness of the first cell remains below one in y^+ unit on both deflected and flat walls. Figures 3 and 4 show the predicted skin-friction coefficients C_f for all models. The proposed elliptic modification to OWA model shows improved predictions. The relatively poor prediction from the OWA model can be attributable to its wall-distance dependent blending function. Significant improvements are also observed in the mean velocity and shear profiles as shown in Figs. 5 and 6, respectively.

c) *Onera-M6Wing*

The Onera-M6 wing is a widely used three-dimensional test case to validate numerical methods and turbulence models for transonic flows. The flow-field is computed at a free-stream Mach number of 0.8395, an angle of attack of 3.06^o and the free-stream Reynolds number of 11.71×10^6. A structured grid used in the simulation consists of four blocks with $1,572,864$ cells and the minimum normalized grid spacing to the wall is 2×10^{-5}. The main feature of this test case is recognized as the interactions of shock-wave and boundary-layer, and the separation induced by the strong shock (i.e., shock induced boundary-layer separation). However, the current study focuses on the validation of turbulence models based on only available experimental data for pressure coefficients [21].

The pressure coefficient results are compared over the wing sections located at $y/b = 20$, 44, 65, 80, 90 and 95% half-span in Fig. 7. It is observed that all models match the experiments very well. Slight over-predictions appear near the leading edge on the upper wing surface, but they are very minor. In addition, the pressures on the lower side of the wing as well as those at

Ref

21. Schmitt V, Charpin F: Pressure distributions on the ONERA M6-Wing at Transonic Mach Number. in AGARD-AR-138, B1-1-B1-44, May 1979.

the trailing edge are well predicted and the overall profiles are captured very well by all models. The difference in the pressure distributions between the SA and other models is not distinct. Nevertheless, other computations differ slightly from those of the SA model, especially on the upper wing surface.

VI. Conclusion

The one-equation Wray-Agarwal (WA) model is modified to replace the blending function f_1 (containing the wall-distance) by an elliptic relaxation method that accounts for the non-local characteristics of near-wall turbulence. The parameterized coefficients with the elliptic function are optimized for several simple and complex flows, three of which are presented herein. It is shown that the elliptic blending function f_R improves the predictions of wall-bounded flows with small regions of separation. Comparing the predicted results with DNS and experimental data demonstrates that the EWA model offers improvements over the OWA and SA models and remains competitive with the SST k-ω model. In addition, the EWA model is wall-distance-free (wdf) and can be easily applied to arbitrary complex computational domains with structured/unstructured grids; the wdf-feature makes it advantageous over OWA, SA and SST models.

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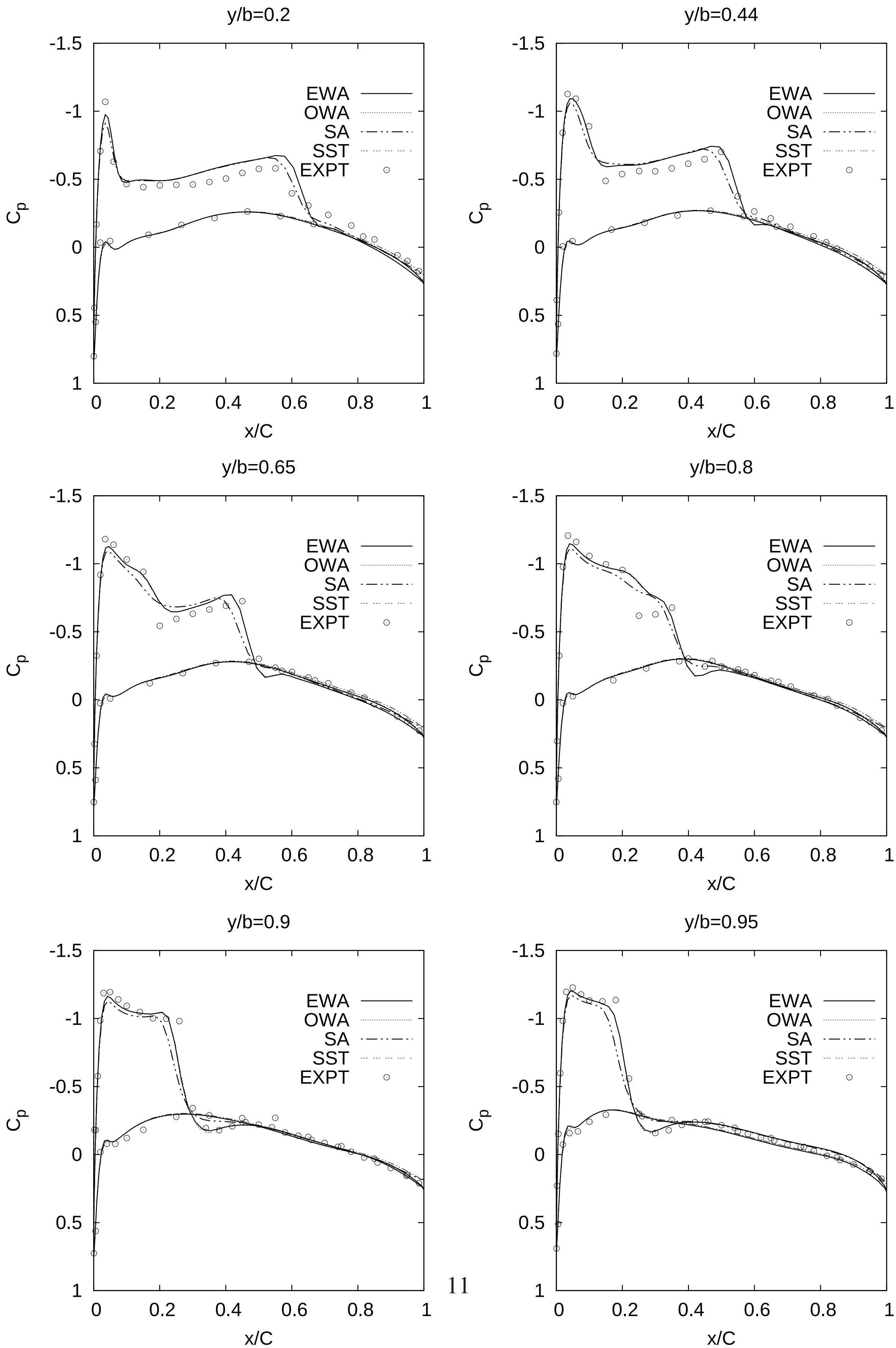

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Figure 7: Wall–pressure coefficients at selected cross sections of Onera–M6 wing.

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15. Never start at the last minute: Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

16. Multitasking in research is not good: Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

17. Never copy others' work: Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

18. Go to seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

19. Refresh your mind after intervals: Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.

20. Think technically: Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

21. Adding unnecessary information: Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

22. Report concluded results: Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

23. Upon conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

The introduction: This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear: Adhere to recommended page limits.

Mistakes to avoid:

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

Title page:

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

Abstract: This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

Approach:

- Single section and succinct.
- An outline of the job done is always written in past tense.
- Concentrate on shortening results—limit background information to a verdict or two.
- Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

Introduction:

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.

The following approach can create a valuable beginning:

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

Approach:

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

Procedures (methods and materials):

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

Materials may be reported in part of a section or else they may be recognized along with your measures.

Methods:

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

What to stay away from:

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

Figures and tables:

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

Discussion:

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- Give details of all of your remarks as much as possible, focusing on mechanisms.
- Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

Describe generally acknowledged facts and main beliefs in present tense.

The Administration Rules

Administration Rules to Be Strictly Followed before Submitting Your Research Paper to Global Journals Inc.

Please read the following rules and regulations carefully before submitting your research paper to Global Journals Inc. to avoid rejection.

Segment draft and final research paper: You have to strictly follow the template of a research paper, failing which your paper may get rejected. You are expected to write each part of the paper wholly on your own. The peer reviewers need to identify your own perspective of the concepts in your own terms. Please do not extract straight from any other source, and do not rephrase someone else's analysis. Do not allow anyone else to proofread your manuscript.

Written material: You may discuss this with your guides and key sources. Do not copy anyone else's paper, even if this is only imitation, otherwise it will be rejected on the grounds of plagiarism, which is illegal. Various methods to avoid plagiarism are strictly applied by us to every paper, and, if found guilty, you may be blacklisted, which could affect your career adversely. To guard yourself and others from possible illegal use, please do not permit anyone to use or even read your paper and file.

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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