Online ISSN : 2249-4626
Print ISSN : 0975-5896
DOI : 10.17406/GJSFR

Global Journal

of Science Frontier Research: F

Mathematics and Decision Science

Highlights

- Many Fibonacci Primes
- Evolution of COVID-19 Disease
- Conducing Cubic Crystal Material
- Maxwell-Auzhan Boundary Conditions

Discovering Thoughts, Inventing Future

VOLUME 20 ISSUE 5 VERSION 1.0

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Global Journal of Science Frontier Research: F
Mathematics & Dicision Science

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Volume 20 Issue 5 (Ver. 1.0)

Open Association of Research Society

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Contents of the Issue

Global Journal of Science Frontier Research: F
Mathematics & Decision Science
Volume 20 Issue 5 Version 1.0 Year 2020
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

There are Infinitely Many Fibonacci Primes

By Fengsui Liu
Zhe Jiang University

Abstract- We invent a novel algorithm and solve the Fibonacci prime conjecture by an interaction between proof and algorithm. From the entire set of natural numbers successively deleting the residue class 0 mod a prime, we retain this prime and possibly delete another one prime retained, then we invent a recursive sieve method, a modulo algorithm on finite sets of natural numbers, for indices of Fibonacci primes. The sifting process mechanically yields a sequence of sets of natural numbers, which converges to the index set of all Fibonacci primes. The corresponding cardinal sequence is strictly increasing. The algorithm reveals a structure of particular order topology of the index set of all Fibonacci primes, then we readily prove that the index set of all Fibonacci primes is an infinite set based on the existing theory of the structure. Some mysteries of primes are hidden in second order arithmetics.

Keywords and phrases: Fibonacci prime conjecture, recursive sieve method, order topology, limit of a sequence of sets.

GJSFR-F Classification: MSC 2010: 11N35, 11N32, 11U09, 11Y16, 11B37, 11B50

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There are Infinitely Many Fibonacci Primes

Fengsui Liu

Abstract- We invent a novel algorithm and solve the Fibonacci prime conjecture by an interaction between proof and algorithm. From the entire set of natural numbers successively deleting the residue class 0 mod a prime, we retain this prime and possibly delete another one prime retained, then we invent a recursive sieve method, a modulo algorithm on finite sets of natural numbers, for indices of Fibonacci primes. The sifting process mechanically yields a sequence of sets of natural numbers, which converges to the index set of all Fibonacci primes. The corresponding cardinal sequence is strictly increasing. The algorithm reveals a structure of particular order topology of the index set of all Fibonacci primes, then we readily prove that the index set of all Fibonacci primes is an infinite set based on the existing theory of the structure. Some mysteries of primes are hidden in second order arithmetics.

Keywords and phrases: Fibonacci prime conjecture, recursive sieve method, order topology, limit of a sequence of sets.

I. Introduction

Fibonacci numbers F_x are defined by the recursive formula

$$\begin{aligned} F_1 &= 1, \\ F_2 &= 1, \\ F_{x+1} &= F_x + F_{x-1}. \end{aligned}$$

The first few Fibonacci numbers are:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, ... (OEIS A005478)

A Fibonacci prime is a Fibonacci number that is prime.

The first few Fibonacci primes are:

2, 3, 5, 13, 89, 233, 1597, 28657, 514229, 433494437, 2971215073, ... (OEIS A005478).

The first few indices x, which give Fibonacci primes are:

3, 4, 5, 7, 11, 13, 17, 23, 29, 43,47, 83, 131, 137, 359, 431, 433, 449, 509, 569, 571, 2971, 4723, 5387, ... (OEIS A001605).

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The study of Fibonacci primes has a long history. At present, Fibonacci primes with thousands of digits have been found. For example, 33-th Fibonacci prime is F_{81839}, with 17103 digits. We do not repeat those stories.

From the aspect of primality, like the Mersenne numbers [7], we have an open problem and a conjecture.

There are infinitely many Fibonacci composites with prime indices.

There are infinitely many Fibonacci primes[12][1].

In another paper, we had discussed the open problem [7].

In this paper, we prove the Fibonacci prime conjecture by a recursive sieve method or algorithm.

In order to be self-contained we repeat some contents in papers [7].

II. A Sifting Process for Primes

For expressing a recursive sieve method by well-formed formulas, we extend both operations addition and multiplication $+, \times$ into finite sets of natural numbers. We introduce several definitions and notations.

We use small letters a, x, t to denote natural numbers and capital letters A, X, T sets of natural numbers except for F_x.

For arbitrary both finite sets of natural numbers A, B we write

$$A = \langle a_1, a_2, \dots, a_i, \dots, a_n \rangle, \ a_1 < a_2 < \dots < a_i < \dots < a_n,$$

$$B = \langle b_1, b_2, \dots, b_j, \dots, \ b_m \rangle, \ b_1 < b_2 < \dots < b_j < \dots < b_m.$$

We define

$$A + B = \langle a_1 + b_1, a_2 + b_1, \dots, a_i + b_j \dots, a_{n-1} + b_m, a_n + b_m \rangle,$$

$$AB = \langle a_1 b_1, a_2 b_1, \dots, a_i b_j \dots, a_{n-1} b_m, a_n b_m \rangle.$$

For example:

$$\langle 1, 5 \rangle + \langle 0, 6, 12, 18, 24 \rangle = \langle 1, 5, 7, 11, 13, 17, 19, 23, 25, 29 \rangle,$$

$$\langle 6 \rangle \langle 0, 1, 2, 3, 4 \rangle = \langle 0, 6, 12, 18, 24 \rangle.$$

For the empty set $\emptyset$ we define $\emptyset + B = \emptyset$ and $\emptyset B = \emptyset$.

We write $A \setminus B$ for the set difference of A and B.

Let

$$X \equiv A = \langle a_1, a_2, \dots, a_i, \dots, a_n \rangle \bmod a,$$

be several residue classes mod a.

We define the solution of the system of congruences

$$X \equiv A = \langle a_1, a_2, \dots, a_i, \dots, a_n \rangle \bmod a,$$

$$X \equiv B = \langle b_1, b_2, \dots, b_j, \dots, b_m \rangle \bmod b$$

Ref

1. Chris Caldwell, The Top Twenty: Fibonacci Number from the Prime Pages. Retrieved 2017-03-03.

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to be

$$X \equiv D = \langle d_{11}, d_{21}, \ldots, d_{ij}, \ldots, d_{n-1m}, d_{nm} \rangle \bmod ab,$$

where $x \equiv d_{ij} \bmod ab$ is the solution of the system of congruences

$$x \equiv a_i \bmod a,$$

$$x \equiv b_j \bmod b.$$

If $\gcd(a, b) = 1$, the solution $X \equiv D \bmod ab$ is computable and unique by the Chinese remainder theorem.

For example, $X \equiv D = \langle 5, 25 \rangle \bmod 30$ is the solution of the system of congruences

$$X \equiv \langle 1, 5 \rangle \bmod 6,$$

$$X \equiv \langle 0 \rangle \bmod 5.$$

We known that the residue class $a_i \mod a$ is the set of natural numbers $\{x : x \equiv a_i \mod a\}$, several residue classes $A \mod a$ is the union of several sets. Thus we may write the relation $x \in A \mod a$ and set operation $B \cup (A \mod a)$.

Except extending $+, \times$ into finite sets of natural numbers, we continue the traditional interpretation of the formal language $0, 1, +, \times, \in$. The reader who is familiar with model theory may know, we have found a new model or structure of the second order arithmetic by a two-sorted logic

$$\langle P(N), N, +, \times, 0, 1, \in \rangle,$$

and we have found a formal system

$$\langle P(N), N, +, \times, 0, 1, \in \rangle \models PA \cup ZF,$$

where N is the set of all natural numbers, and $P(N)$ is the power set of N; PA is the Peano theory and ZF is the set theory.

We denote this model by $P(N)$.

In the traditional first order arithmetics N, we have computed out 33 Fibonacci primes, but the numerical evidence does not provide theoretically information about infinitude.

In the second order arithmetics $P(N)$, we may construct a recursive function on sets of natural numbers by arithmetical operations $+, \times$ and set-theoretical operations $\cup, \cap, \setminus$, its inputs and outputs are sets of natural numbers, it expresses our intuitable idea: we delete all indices of non-Fibonacci primes and retain all indices of Fibonacci primes. Then we obtain a sequence of sets of natural numbers (T_i'), which converges to the set T_e of all indices of Fibonacci primes. We reveal an exotic structure of the set T_e

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in second order arithmetics. The existing theory of those structures, order topology, allows us to prove the conjecture.

First we repeat the recursively sifting process for primes.

Let p_i be the i-th prime, $p_0 = 2$. Let

$$m_{i+1} = \prod_0^i p_j.$$

From the entire set of natural numbers we successively delete the residue class 0 mod p_i, i.e., the set of all numbers x such that the least prime factor of x is p_i. We leave the residue class T_{i+1} mod m_{i+1}. Then the left residue class T_{i+1} mod m_{i+1} is the set of all numbers x such that x does not contain any prime $p_j \leq p_i$ as a factor $gcd(x, m_{i+1}) = 1$.

Let T_{i+1} be the set of least nonnegative representatives of the residue class T_{i+1} mod m_{i+1}, the reduced residue system mod m_{i+1}. We design a primitive recursive formula for the set T_{i+1} and the prime p_{i+1}. This formula expresses a recursive sieve method for primes.

$$\begin{aligned} T_1 &= \langle 1 \rangle, \\ p_1 &= 3, \\ T_{i+1} &= (T_i + \langle m_i \rangle \langle 0, 1, 2, \ldots, p_i - 1 \rangle) \setminus \langle p_i \rangle T_i, \\ p_{i+1} &= g(T_{i+1}), \end{aligned} \tag{2.1}$$

where $X \equiv \langle p_i \rangle T_i$ mod m_{i+1} is the solution of the system of congruences

$$X \equiv T_i \mod m_i,$$
$$X \equiv \langle 0 \rangle \mod p_i,$$

and $g(T)$ is a projective function

$$g(T) = g(\langle t_1, t_2, \ldots, t_n \rangle) = t_2.$$

The cardinality of the set T_{i+1} is

$$|T_{i+1}| = \prod_0^i (p_j - 1). \tag{2.2}$$

For example:

$$\begin{aligned} T_1 &= \langle 1 \rangle, \\ p_1 &= 3, \\ T_2 &= (\langle 1 \rangle + \langle 2 \rangle \langle 0, 1, 2 \rangle) \setminus \langle 3 \rangle = \langle 1, 5 \rangle, \\ p_2 &= 5, \\ T_3 &= \langle 1, 5 \rangle + \langle 6 \rangle \langle 0, 1, 2, 3, 4 \rangle) \setminus \langle 5, 25 \rangle = \langle 1, 7, 11, 13, 17, 19, 23, 29 \rangle. \\ p_3 &= 7. \end{aligned}$$

Notes

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It is easy to prove this primitive recursive formula by mathematical induction.

Look through a list of primes, the sequence of primes seems chaotic and random. It offers no clues as to how to determine the next number.

Randomness and chaos are anathema to the mathematician [10].

Despite over two thousand years one pursues an order, pattern, structure, and finds a formula that generates all primes exactly and without exception, our endeavour is in vain.

In the second arithmetics the prime formula 2.1 reveals the recursive structure of each primes. Now primes do not appear randomly. They are computed one after the other by $+, \times$. They are governed by a recursive rule. Recursion opens new theoretical windows onto our understanding of the primes. It seems this is a prime conspiracy.

Gauss prime number theorem $\pi(x) \approx x/\ln x$ is an explicitly approximation for the recursive formula $\pi(p_i) = i$. The prime number theorem provides a heuristic model. Due to parity problem, we ought to reflect whether or not the heuristic model describes the sequence of primes enough accurately.

The prime formula is a reformulation of Eratosthenes sieve method.

In contrast with Eratosthenes sieve, which does not automatically yield theoretical information, the recursive sieve method itself mechanically yields a constructive proof that there are infinitely many primes[7].

We can not directly extend this constructive proof to Fibonacci primes.

Below we will refine and modify formula 2.1 to invent a recursive sieve for Fibonacci primes. A recursive formula on the sets of naturel numbers reveals a secret of the patterns of Fibonacci primes.

III. A Recursive Sieve for Fibonacci Primes

Based on the recursive sieve method for primes, formula (2.1), we successively delete all numbers x such that x contains the least prime factor p_i, we delete all composites together with the prime p_i. The sifting condition or 'sieve' is

$$x \equiv 0 \mod p_i \wedge p_i \leq x.$$

We refine the sifting condition to be

$$x \equiv 0 \mod p_i \wedge p_i < x. \tag{3.1}$$

According to this new sifting condition or 'sieve', we successively delete the set C_i of all numbers x such that x is composite with the least prime

Ref

10. M. D. Sautoy, *The Music of the Primes*, HarperCollins Publishers, (2003), p.6.

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factor p_i,

$$C_i = \{x : x \in X \equiv T_i \bmod m_i \wedge x \equiv 0 \bmod p_i \wedge p_i < x\},$$

but save the prime p_i as a survivor.

We delete all sets C_j of composites with $0 \leq j < i$ from the entire set N of natural numbers and leave the sifted set

$$L_i = N \setminus \bigcup_{0}^{i-1} C_j.$$

The end-sifted set, the set of all primes T_e, is

$$T_e = N \setminus \bigcup_{0}^{\infty} C_i.$$

Let A_i be the set of all survivors less than p_i,

$$A_i = \langle 2, 3, 5, 7, \ldots, p_{i-1} \rangle.$$

From the recursive formula (2.1), we deduce that the sifted set L_i is the union of the set A_i of survivors and the residue class T_i mod m_i,

$$L_i = A_i \bigcup (T_i \bmod m_i) \qquad (3.2)$$

We slightly modify the above sifting process for primes to invent a recursive sieve for indices of Fibonacci primes.

We know that the Fibonacci sequence is a divisibility sequence. In fact, the Fibonacci sequence satisfies the stronger divisibility property[13]

$$\gcd(F_m, F_n) = F_{\gcd(m,n)}.$$

In other words, for $n \geq 3$, F_n divides F_m iff n divides m. It follows that except for the case $n = 4, F_4 = 3$, every Fibonacci prime have a prime index, but not every prime is the index of a Fibonacci prime.

We discuss indices of Fibonacci primes and their infinitude in the set of indices of all Fibonacci numbers.

If we only consider Fibonacci numbers as factors of Fibonacci numbers within the Fibonacci divisibility system, then the set of all primes T_e is the set of indices of all Fibonacci primes except for $F_2 = 1, F_4 = 3$. But some Fibonacci numbers with prime index contain normal prime factors that are not Fibonacci numbers. For example $F_{19} = 4181 = 37 \times 113, F_{31} = 1346269 = 557 \times 2417$.

We must remove every prime index q such that F_q is a Fibonacci composite with a normal prime factor.

One uses the rank of a prime p to discuss prime factors of Fibonacci composites with prime index.

Notes

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Definition: The rank of a prime p, denoted $\alpha(p)$, is the least positive integer k such that $p|F_k$.

The first few ranks of primes are:

3, 4, 5, 8, 10, 7, 9, 18, 24, 14, 30, 19, 20, 44, 16, 27, 58, 15, 68, 70, 37, 78, 84, 11, 49, 50, 104,36, 27, 19, 128, 130, 69, 46, 37, 50, 79, 164, 168, 87, 178, 90, 190, 97, 99, 22, 42, 224, 228, 114, 13, 238, 120, 250, 129, 88, 67, 270, 139, 28, 284, 147, 44, 310,. . . . . (sequence A001602 in the OEIS).

We exhibit some simple facts about normal prime factors of Fibonacci composites with prime index.

(1) A prime number divides at most one Fibonacci number with prime index due to the stronger divisibility property.

(2) If $q > 4$ is a prime, then every prime p that divides F_q is congruent to 1 mod 4 [8].

(3) Let $p > 5$ be a prime, if $p \equiv \pm 1 \mod 10$, then $\alpha(p)|p-1$; if $p \equiv \pm 3 \mod 10$, then $\alpha(p)|p+1$ [5]. We have $\alpha(p) \leq \frac{p+1}{2}$.

(4) if $p > 7$ is a prime such that $p \equiv 2, 4 \mod 5$, and $2p-1$ is also prime, then $2p-1|F_p$ [14].

It follows that we only need to modify the set A_i of survivors for each prime p_i to obtain a new set A_{i+1} of survivors, such that if $q \in A_{i+1}$ then F_q contains neither normal prime $p \leq p_i$ nor Fibonacci prime $F_p \leq F_{p_i}$ as a factor except itself.

Given any prime $p_i > 5$, suppose that we have a modified set A_i, then we obtain the next set A_{i+1} by the following rules.

If the prime p_i is congruent to 1 mod 4, and if there is an odd prime $q \leq \frac{p_i+1}{2}$ in the set A_i such that

$$p_i < F_q \wedge \alpha(p_i) = q,$$

then F_q is a Fibonacci composite, which contains the least normal prime factor p_i. We add the prime p_i into the set A_i and remove the prime q from the set $A_i \cup \langle p_i \rangle$ to obtain the set A_{i+1}.

$$A_{i+1} = (A_i \cup \langle p_i \rangle) \setminus \langle q \rangle.$$

If there is no such a prime q, then for every number x in the sifted set L_i the Fibonacci number F_x does not contain the normal prime p_i as a factor. We add the prime p_i into the set A_i

$$A_{i+1} = A_i \cup \langle p_i \rangle.$$

Ref

5. Marc Renault, The Fibonacci Sequence Modulo m. http://webspace.ship.edu/msrenault/fibonacci/fib.htm.

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Now for every number x in the sifted set L_{i+1}, the Fibonacci number F_x does not contain Fibonacci number M_q, $q \leq p_i$ as a factor, and does not contain the normal prime $p_j \leq p_i$ as a factor except itself.

After modification the first few set A_i are

$$\begin{aligned} A_5 &= \langle 3,4,5,7.11\rangle, \\ A_8 &= \langle 3,4,5,7.11,13,17,19\rangle, \\ A_{11} &= \langle 3,4,5,7.11,13,17,23,29,31,37\rangle, \\ A_{12} &= \langle 3,4,5,7.11,13,17,23,29,31,37,41\rangle, \\ A_{13} &= \langle 3,4,5,7.11,13,17,23,29,31,37,41,43\rangle, \\ A_{14} &= \langle 3,4,5,7.11,13,17,23,29,31,37,41,43,47\rangle. \end{aligned}$$

The sifting condition formula (3.1) is converted into

$$(x \equiv 0 \mod p_i \wedge p_i < x) \vee (F_x \equiv 0 \mod p_i \wedge p_i < F_x). \qquad (3.3)$$

According to this sifting condition or 'sieve', we successively delete the set C_i of all numbers x, such that F_x is composite with the least factor F_{p_i}, or F_x is composite with the least normal prime factor $p_i > 5$,

$$C_i = \{x : x \in (A_i \cup (X \equiv T_i \mod m_i)) \wedge ((F_x \equiv 0 \mod p_i \wedge p_i < F_x) \\ \vee (x \equiv 0 \mod p_i \wedge p_i < x))\},$$

but remain the survivor x if $p_i = x$ or $p_i = F_x$.

We delete all sets C_j with $0 \leq j < i$ from the set N of all indices of Fibonacci numbers and leave a sifted set

$$L_i = N \setminus \bigcup_0^{i-1} C_j.$$

The sifted set L_i is descending

$$L_1 \supset L_2 \supset \cdots \supset L_i \supset \cdots\cdots.$$

The end-sifted set, the set T_e of all indices of Fibonacci primes, is

$$T_e = N \setminus \bigcup_0^{\infty} C_i.$$

We easily verify that the primes 3,4,5,7.11,13,17,23,29,43,47 all are indices of Fibonacci primes by the above algorithm.

The set A_i of survivors is a set of indices of Fibonacci primes or probable primes, the candidates.

Obviously, we have

$$|A_i| \leq |A_{i+1}|.$$

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From the recursive formula (2.1), we deduce that the sifted set L_i is the union of the set A_i of survivors and the residue class T_i mod m_i.

$$L_i = A_i \bigcup (T_i \bmod m_i). \tag{3.4}$$

We intercept the initial segment T_i' from the sifted set L_i, which is the union of the set A_i of survivors and the set T_i of least nonnegative representatives, then we obtain a new recursive formula

$$T_i' = A_i \bigcup T_i. \tag{3.5}$$

Except remaining all survivors x less than p_i in the initial segment T_i', both sets T_i' and T_i are the same.

For example

$$\begin{aligned} A_3 &= \langle 3,4,5 \rangle. \\ T_3' &= \langle 3,4,5 \rangle \cup \langle 1,7,11,13,17,19,23,29 \rangle \\ &= \langle 3,4,5,1,7,11,13,17,19,23,29 \rangle. \end{aligned}$$

Let $|A_i|$ be the number of survivors less than p_i. Then the number of elements of the initial segment T_i' is

$$|T_i'| = |A_i| + |T_i|. \tag{3.6}$$

From formula (2.2) we deduce that the corresponding cardinal sequence $(|T_i'|)$ is strictly increasing

$$|T_i'| < |T_{i+1}'|.$$

Based on cardinal arithmetics we have

$$\lim T_i' = \bigcup T_i' = \aleph_0.$$

Based on order topology we have

$$\lim |T_i'| = \aleph_0. \tag{3.7}$$

Next section we will prove that the set of all indices of Fibonacci primes is an infinite set.

IV. The Infinitude of Fibonacci Primes

We call an index of Fibonacci prime a F-index.

Let A_i' be the subset of F-indices in the initial segment T_i',

$$A_i' = \{x \in T_i' : x \text{ is a F-index}\}. \tag{4.1}$$

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For example:

$$\begin{aligned} A'_2 &= \langle 3,4,5\rangle, \\ A'_3 &= \langle 3,4,5,7.11,13,17,23,29\rangle, \\ A'_4 &= \langle 3,4,5,7,11,13,17,23,29,43,47,83,131,137\rangle. \end{aligned}$$

We consider the properties of both sequences of sets (T'_i) and (A'_i) to prove the conjecture.

Theorem 4.1. The sequence of the initial segments (T'_i) *and the sequence of its subsets* (A'_i) *of F-indices both converge to the set of all F-indices* T_e.

First from set theory [2], next from order topology [6] we prove this theorem.

Proof. For the convenience of the reader, we quote a definition of the set-theoretic limit of a sequence of sets [2].

Let (F_n) be a sequence of sets, we define $\limsup_{n=\infty} F_n$ and $\liminf_{n=\infty} F_n$ as follows

$$\begin{aligned} \limsup_{n=\infty} F_n &= \bigcap_{n=0}^{\infty}\bigcup_{i=0}^{\infty} F_{n+i}, \\ \liminf_{n=\infty} F_n &= \bigcup_{n=0}^{\infty}\bigcap_{i=0}^{\infty} F_{n+i}. \end{aligned}$$

It is easy to check that $\limsup_{n=\infty} F_n$ is the set of those elements x which belongs to F_n for infinitely many n. Analogously, x belongs to $\liminf_{n=\infty} F_n$ if and only if it belongs to F_n for almost all n, that is it belongs to all but a finite number of the F_n.

If

$$\limsup_{n=\infty} F_n = \liminf_{n=\infty} F_n,$$

we say that the sequence of sets (F_n) converges to the limit

$$\lim F_n = \limsup_{n=\infty} F_n = \liminf_{n=\infty} F_n.$$

We know that the sequence of sifted sets (L_i) is descending

$$L_1 \supset L_2 \supset \cdots \supset L_i \supset \cdots\cdots .$$

According to the definition of the set-theoretic limit of a sequence of sets, we obtain that the sequence of sifted sets (L_i) converges to the set T_e

$$\lim L_i = \bigcap L_i = T_e.$$

The sequence of subsets (A'_i) of F-indices is ascending

$$A'_1 \subset A'_2 \subset \cdots \subset A'_i \subset \cdots\cdots ,$$

Ref

6. J. R. Munkres, *Topology*, (2rd ed.), Prentice Hall, Upper Saddle River, (2000), 84.

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Ref

4. Jesper M. Møller, *General Topology* web, math.ku,dk (2005)20.

we obtain that the sequence of subsets (A'_i) converges to the set T_e,

$$\lim A'_i = \bigcup A'_i = T_e.$$

The initial segment T'_i locates between two sets A'_i and L_i

$$A'_i \subset T'_i \subset L_i.$$

It is easy to prove that the sequence of the initial segments (T'_i) converges to the set T_e

$$\lim T'_i = T_e.$$

According to set theory, we have proved that both sequences of sets (T'_i) and (A'_i) converge to the set of all F-indices T_e.

$$\lim T'_i = \lim A'_i = T_e.$$

In general, for any sequence of finite sets (G_i), if G_i locates between two sets A'_i and L_i,

$$A'_i \subset G_i \subset L_i,$$

then we have

$$\limsup G_i \subset \lim L_i,$$

$$\liminf G_i \supset \lim A'_i.$$

Thus

$$\lim G_i = T_e,$$

Even $T_e = \emptyset$ the limit of set theory is valid too.

We can not use analytic techniques for limits of set theory so that we try to endow them with an order topology, and prove that both sequences of sets (T'_i) and (A'_i) converge to the set of all F-indices T_e.

We quote J.R.Munkres's definition of the order topology [4][6].

Let X be a set with a linear order relation; assume X has more one element. Let $\mathbb{B}$ be the collection of all sets of the following types:

(1) All open intervals (a, b) in X.

(2) All intervals of the form $[a_0, b)$, where a_0 is the smallest element (if any) in X.

(3) All intervals of the form $[a, b_0)$, where b_0 is the largest element (if any) in X.

The collection $\mathbb{B}$ is a base of a topology on X, which is called the order topology.

The empty or singleton is not a linear order $<$ set. There is no order topology on the empty set or sets with a single element. For example a

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constant function $f(n) = 0$ has a limit $\lim f(n) = 0$ in real analysis, but it has no limit in order topology, as $n \to \infty$.

The recursively sifting process formula (3.5) produces both sequences of sets together with the set-theoretic limit point T_e.

$$\mathbf{X_1}: \quad T'_1, T'_2, \ldots, T'_i, \ldots \ldots; T_e,$$

$$\mathbf{X_2}: \quad A'_1, A'_2, \ldots, A'_i, \ldots \ldots; T_e.$$

We further consider the structures of sets $\mathbf{X_1}$ and $\mathbf{X_2}$ using the recursively sifting process (3.5) as an order relation

$$i < j \to T'_i < T'_j, \ \forall i(T'_i < T_e),$$

$$i < j \to A'_i < A'_j, \ \forall i(A'_i < T_e).$$

The set $\mathbf{X_1}$ has no repeated term. It is a well ordered set with the order type $\omega + 1$ using the recursively sifting process (3.5) as an order relation. Thus the set $\mathbf{X_1}$ may be endowed an order topology.

In general, the set $\mathbf{X_2}$ may have no repeated term, or may have some repeated terms, or may be a set with a single element $\mathbf{X_2} = \{\emptyset\}$.

We have computed out some patterns of the first few F-indices. The set $\mathbf{X_2}$ contains more than one element, may be endowed with an order topology. using the recursively sifting process (3.5) as an order relation.

Obviously, for every neighborhood $(c, T_e]$ of T_e, there is a natural number i_0, for all $i > i_0$, we have $T'_i \in (c, T_e]$ and $A'_i \in (c, T_e]$, thus both sequences of sets (T'_i) and (A'_i) converge to the set of all F-indices T_e.

$$\lim A'_i = T_e,$$

$$\lim T'_i = T_e.$$

According to the order topology, we have again proved that both sequences of sets (T'_i) and (A'_i) converge to the set of all F-indices T_e. We also have

$$\lim T'_i = \lim A'_i. \tag{4.2}$$

If $T_e = \emptyset$, the set $\mathbf{X_2} = \{\emptyset\}$ only has a single element, which has no order topology. In this case formula (4.2) is not valid and we prove nothing by the order topology.

Theorem (4.1) reveals a particular order topological structure of the set of all F-indices built into the sequences of sets. Now we easily prove that the cardinality of the set of all F-indices is infinite.

Theorem 4.2. The set of all F-indices is an infinite set.

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We give two proofs.

Proof. A. We consider the cardinalities $|T_i'|$ and $|A_i'|$ of sets on two sides of the equality (4.2), and the order topological limits of cardinal sequences $(|T_i'|)$ and $(|A_i'|)$ with the usual order relation $\leq$, as the sets T_i' and A_i' both tend to T_e.

From general topology, we know that if the limits of both cardinal sequences $(|T_i'|)$ and $(|A_i'|)$ on two sides of the equality (4.2) exist, then both limits are equal; if $\lim |A_i'|$ does not exist, then the condition for the existence of the limit $\lim |T_i'|$ is not sufficient [3].

For F-indices, the set T_e is nonempty $T_e \neq \emptyset$, the formula (4.2) is valid, obviously, the order topological limits $\lim |A_i'|$ and $\lim |T_i'|$ on two sides of the equality (4.2) exist, thus both limits are equal

$$\lim |A_i'| = \lim |T_i'|.$$

From formula (3.7) $\lim |T_i'| = \aleph_0$ we have

$$\lim |A_i'| = \aleph_0. \tag{4.3}$$

Usually, let $\pi(n)$ be the counting function, the number of F-indices less than n. Normal sieve theory is unable to provide non-trivial lower bounds of $\pi(n)$ due to the parity problem. Let n be a natural number. Then the number sequence (m_i) is a subsequence of the number sequence (n), we obtain

$$\lim \pi(n) = \lim \pi(m_i).$$

By formula (4.1), the A_i' is the set of all F-indices less than m_i, and the $|A_i'|$ is the number of all F-indices less than m_i, thus $\pi(m_i) = |A_i'|$. We have

$$\lim \pi(m_i) = \lim |A_i'|.$$

From formula (4.3) we prove

$$\lim \pi(n) = \aleph_0. \tag{4.4}$$

Next, we give another proof by the continuity of the cardinal function.

Proof. B. Let $f : \mathbf{X} \to \mathbf{Y}$ be the cardinal function from the order topological space $\mathbf{X}$ to the order topological space $\mathbf{Y}$ such that $f(T) = |T|$.

$$\mathbf{X} \;:\; T_1',\ T_2',\ \ldots,\ T_i', \ldots\ldots;\ T_e,$$

$$\mathbf{Y} \;:\; |T_1'|, |T_2'|, \ldots, |T_i'|, \ldots\ldots; \aleph_0.$$

It is easy to check that for every open set $[|T_1'|, |d|), (|c|, |d|), (|c|, \aleph_0]$ in $\mathbf{Y}$ the preimage $[T_1', d), (c, d),\ (c, T_e]$ is also an open set in $\mathbf{X}$. So that the

Ref

3. L. D. Kudryavtsev, Encyclopaedia of Mathematics, limit, [Online] Available: http://eom.springer.de/l/1058820.htm.

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cardinal function $|T|$ is continuous at T_e with respect to the above particular order topology.

Both order topological spaces are first countable, hence the cardinal function $|T|$ is sequentially continuous. By a usual topological theorem [6] (Theorem 21.3, p130), the cardinal function $|T|$ preserves limits

$$|\lim T_i'| = \lim |T_i'|. \quad (4.5)$$

Order topological spaces are Hausdorff spaces. In Hausdorff spaces the limit point of the sequence of sets (T_i') and the limit point of cardinal sequence $(|T_i'|)$ are unique.

We have proved theorem 4.1, $\lim T_i' = T_e$, and formulas (3.7), $\lim |T_i'| = \aleph_0$. Substitute, we obtain that the set of all F-indices an infinite set,

$$|T_e| = \aleph_0. \quad (4.6)$$

Without any estimation or statistical data, without the Riemann hypothesis, with the recursive sieve, we well understand the recursive structure, set theoretic structure and order topological structure of the set of all F-indices on sequences of sets. The theory of those structures allows us obtain an elementary proof of the conjecture.

We prove that the set of indices of all Fibonacci primes is a infinite set. In other words we have proved the theorem

Theorem 4.3. There are infinitely many Fibonacci primes.

V. Discussion

In general, we can not prove that the cardinal function on sequences of sets is continuous. There is a counterexample, the Ross-Littwood paradox [9] [11].

For example: consider the limit of the sequence of sets, which have no pattern

$$T_i = \langle i+1, i+2, \ldots, 10i \rangle.$$

From set theory we know $\lim T_i = T_e = \emptyset$, thus $|T_e| = 0$. But we also have $\lim |T_i| = \infty$ from real analysis. If the cardinal function is continuous, then there is a contradiction in real analysis, the empty has an infinite cardinality. In this case, one can only get up the continuity and says that there is no relation between the cardinality of the end sifted set $|T_e| = 0$ and the limit $\lim |T_i| = \infty$.

Ref

6. J. R. Munkres, *Topology*, (2rd ed.), Prentice Hall, Upper Saddle River, (2000), 84.

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By the recursive sieve method, we have revealed that the set T_e of all F-indices has the structure of a particular order topology

$$\lim T_i' = T_e.$$

So that we consider the conjecture in the particular order topological space, which is generated naturally by the recursively sifting process (3.5), rather than in real analysis, a metric space.

We consider all sequences of finite sets (G_i), such that $A_i' \subset G_i \subset L_i$, and they converge to the end sifted set T_e from set theory

$$\lim G_i = T_e.$$

We try to endow all set-theoretical convergence $\lim G_i = T_e$ with an order topology using the recursively sifting process (3.5) as an order relation, then construct a particular order topological space $\mathbf{G}$.

Here we must be careful about the existence of order topological limits.

First according to the definition, there exists no order topology on the empty set or sets with a single element[4] [6].

Next, we quote topologist L.D. Kudryavtsev's 'The existence of limits of a function' for $\lim |G_i|$.

If the space $\mathbf{X}$ satisfies the first axiom of countability at the point T_e and the space $\mathbf{Y}$ is Hausdorff, then for the existence of the limit $\lim |G_i|$ of the cardinal function $|G_i|$, it is necessary and sufficient that for any sequence (G_i) , such that $\lim G_i = T_e$, the limit $\lim |G_i|$ exists. If this condition holds, the limit $\lim |G_i|$ does not depend on the choice of the sequence (G_i) , and the common value of these limits is the limit of $(|G_i|)$ at T_e [3].

If the end sifted set is empty $T_e = \emptyset$. Since the sequence of subsets (A_i') is ascending, it converges to the set T_e based on the recursively sifting process (3.5), from set theory we know that the set of sets of patterns $\mathbf{X_2} : A_1', A_2', \ldots, A_i', \ldots\ldots, T_e$. only have a single element $\emptyset$ and the cardinal set $\mathbf{Y_2} : |A_1'|, |A_2'|, \ldots, |A_i'|, \ldots\ldots, |T_e|$. only have a single element 0, both sets have no order topology, both sequences(A_i'), $(|A_i'|)$ have no limit of order topology.

Note, we consider the set sequences, the empty $\emptyset$ is as an element. If we find out at least one prime pattern, then the set sequence (A_i') has more one element.

Only if the end sifted set is empty $T_e = \emptyset$, the limits $\lim A_i'$ and $\lim |A_i'|$ have no existence. The existence of all other limits $\lim |G_i|$ is not sufficient from the above general topology. Thus at the point $T_e = \emptyset$ there is no

Ref

6. J. R. Munkres, *Topology*, (2rd ed.), Prentice Hall, Upper Saddle River, (2000), 84.

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"continuous" or "non-continuous". There is no contradiction. In this case, one needs no order topology.

If the end sifted set is not empty $T_e \neq \emptyset$, since the inclusion relation $G_i \supset A'_i$, the sequence (A'_i) has more one element, every sequence (G_i) has more one element, every limit of set theory $\lim G_i$ may be endowed with an order topology. Every $\lim G_i$ has existence; every $\lim |G_i|$ has existence. The condition of existence of $\lim |T'_i|$, $\lim T'_i$ and $\lim |A'_i|$, $\lim A'_i$ is sufficient. Thus our proof of theorem 3.1 and 3.2 is correct.

In the formal system $P(N)$ we deal with the Ross-Littwood paradox and find out a proof of the Fibonacci prime conjecture in the particular order topological space.

The Ross-Littwood paradox shows that the restricted definition for order topology, assume X has more one element, is necessary.

In fact, we consider the set of various prime patterns T_e, in advance, we have known at least one prime pattern, and in advance, we have known that the end sifted set is not empty $T_e \neq \emptyset$.

By the same paradigm, we may prove another prime conjecture, including the twin prime conjecture.

Acknowledgments

Thank for Qian Liu in Hug Chinese for her support.

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Global Journal of Science Frontier Research: F
Mathematics & Decision Science
Volume 20 Issue 5 Version 1.0 Year 2020
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Understanding the Early Evolution of COVID-19 Disease Spread using Mathematical Model and Machine Learning Approaches

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Abstract- In response to the global COVID-19 pandemic, this work aims to understand the early time evolution and the spread of the disease outbreak with a data driven approach. To this effect, we applied Susceptible- Infective-Recovered/Removed (SIR) epidemiological model on the disease. Additionally, we used the Machine Learning linear regression model on the historical COVID-19 data to predict the earlier stage of the disease. The evolution of the disease spread with the Mathematical SIR model and Machine Learning regression model for time series forecasting of the COVID-19 data without, and with lags and trends, was able to capture the early spread of the disease. Consequently, we suggest that if using a more advanced epidemiological model, and sophisticated machine learning regression models on the COVID-19 data, we can understand, as well as predict the long time evolution of the disease spread.

Index Terms: *COVID-19, mathematical model, SIR model, machine learning, linear regression, time-series, forecast.*

GJSFR-F Classification: *MSC 2010: 93A30*

UNDERSTANDINGTHEEARLYEVOLUTIONOFCOVID19DISEASESPREADUSINGMATHEMATICALMODELANDMACHINELEARNINGAPPROACHES

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Global Journal of Science Frontier Research (F) Volume XX Issue V Version I Year 2020

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Abstract- In response to the global COVID-19 pandemic, this work aims to understand the early time evolution and the spread of the disease outbreak with a data driven approach. To this effect, we applied Susceptible- Infective-Recovered/Removed (SIR) epidemiological model on the disease. Additionally, we used the Machine Learning linear regression model on the historical COVID-19 data to predict the earlier stage of the disease. The evolution of the disease spread with the Mathematical SIR model and Machine Learning regression model for time series forecasting of the COVID-19 data without, and with lags and trends, was able to capture the early spread of the disease. Consequently, we suggest that if using a more advanced epidemiological model, and sophisticated machine learning regression models on the COVID-19 data, we can understand, as well as predict the long time evolution of the disease spread.

Index Terms: *COVID-19, mathematical model, SIR model, machine learning, linear regression, time-series, forecast.*

I. Introduction

An outbreak of Severe Acute Respiratory Syndrome Coronavirus 2 (SARS-CoV-2) which was first reported in Wuhan, China in December 2019 [1], [2], has led to 3, 910, 738 confirmed cases, 272, 778 death cases, with 2, 352, 811 active cases, and 1, 306, 204 recovered cases as of May 8, 2020, and has spread to 215 countries of the world, [3], [16], [17], [18]. Coronaviruses are a large family of viruses that may cause respiratory illnesses in humans, ranging from common colds to more severe conditions such as Severe Acute Respiratory Syndrome (SARS) and Middle Eastern Respiratory Syndrome (MERS). People that are infected may be sick with the virus for 1 to 14 days before developing any symptoms, [19], [20]. The most common symptoms of coronavirus disease (COVID-19) are fever, tiredness, dry cough, and in severe cases difficulty in breathing [28]. From the statistics given above, it suggests that about 33% of the people infected with the virus will recover from the disease without going through special treatment.

Thus, “Novel Coronavirus” is a new, previously unidentified strain of Coronavirus. This novel coronavirus that is involved in the current outbreak has been named Severe Acute Respiratory Syndrome Coronavirus-2 (SARS-CoV-2) by the International Committee on Taxonomy of Viruses on 11 February, 2020 [28]. While the

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disease it causes has been named "Coronavirus Disease 2019" (or "COVID-19" in short) by World Health Organization (WHO) [29]. As a world, we now have a critical situation on our hands with the spread of pandemic COVID-19 which has posed a huge threat to global public health. Economic of the world is also suffering greatly as a result of several measures been put in place including total lock-down of cities in order to curb the spread of the disease. It has also had social, environmental and financial consequences, [18], [21], [22], [23], [30].

Understanding the early transmission dynamics of infectious diseases and evaluating the effectiveness of control measures is crucial for assessing the potential for sustained control when occurring in new areas. Preliminary investigation has suggested that the key influencing factor for the rise in cases across the globe is the disease spread by the infectious people. This means that those with the virus can unknowingly infect others before symptoms appear, some as soon as two days after infection. It is assumed that infected individuals are able to spread the infection until they recover, [24], [25], [26]. As a consequence, several measures have been put in place by the governments and health authorities in order to reduce the spread among the populace. There has been a regular campaign to encourage taking personal measures that will help in reducing the spread. These are the practicing of regular hands-washing, observing social distancing, the use of face masks, disinfection, isolation, among others, [27].

As a new infectious disease, the transmission dynamics of COVID-19 is still under investigation. Al- though SARS-CoV-2 is a kind of coronavirus that is similar to Severe Acute Respiratory Syndrome Coron- avirus (SARS-CoV) and the Middle Eastern Respiratory Syndrome Coronavirus (MERS-CoV), many studies are still underway to understand its infectious characteristics.

Since the outbreak of the disease, several study have been made by researchers to understand the COVID-19 disease, to model and to predict the spread with the help of scientific tools such as mathematical models, predictive analytics using machine learning algorithms, and time series forecasting.

In [9], the author presented a comprehensive under- standing of various pathological mechanism of COVID- 19, which can potentially influence the vulnerable development of the disease. In [10], the author build SARS-CoV-2 computed tomography (CT) scan dataset, consisting of 2482 CT scans for both infected and non-infected patients of SARS-CoV-2 (COVID-19). The data which was collected from Sao Paulo in Brazil is publicly available online to encourage research and implementation of artificial intelligent methods, able to identify an individual infected by COVID-19 through analysis of patient's CT scan. In [11], the authors designed a 3- dimensional structure for 2019-nCoV. To elucidate the 3D structure, a computational modeling approach was applied to give insight into the domain architecture of nsp12. In [12] the authors reviewed the characteristics, origin, pathogenicity, genome structure, and replication of SARS-CoV-2, with the goal of explaining how each remedy strategy could act on slowing down or preventing viral infection. Summary of recently investigated treatments (drugs) were presented. In [13], the authors evaluated the safety and efficacy of 2 chloroquine diphosphate (CQ) dosages in patients with critical SARS-CoV-2. The preliminary findings in this study suggest that higher CQ dosage should not be recommended for patients with critical case of the viral infection because of its potential safety hazards. In [14], a set of novel molecular structures of SARS-CoV-2 3C- like protease inhibitors were presented using the Insilico medicine generative chemistry pipeline. 10 representative structures for likely development with 3D representation were presented. [15] in their work, using the data provided by [6], describes the time-line of a live forecasting exercise, and provides objective forecasts for the confirmed

Ref

23. Cennimo, D. J., & Bronze, M. S. (2020). Coronavirus disease 2019 (COVID-19) treatment & management.

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cases of COVID- 19. They discussed also in their work that forecasting of COVID-19 has good potential implications for planning and decision making.

Motivated by [5], [26], [15], the goal of this paper is to study and understand the early-stage evolution of the disease spread with a data-driven approach using Mathematical SIR model and Machine Learning regression model for time series forecasting of the COVID-19 disease, with and without lags and trends.

The rest of the paper is structured as follow: the formulation and application of SIR model to COVID-19 are presented in section II, Data analysis and Time series forecast in section III and section IV give the conclusion to this work.

Ref

15. Petropoulos F, Makridakis S (2020) Forecasting the novel coronavirus COVID-19, PLoS ONE, 15(3): e0231236. https://doi.org/10.1371/journal.pone.0231236.

II. The SIR Model

In this section, we formulate and analyze SIR mathematical model which is the most basic mathematical model for a directly transmitted infectious disease as COVID-19 which is caused by a virus. The transmission of the disease occurs through respiratory droplets, and spread by individual-to-individual contact which happens through a sneeze or cough, through skin-skin contact, or through indirect contact with surfaces in the immediate environment or with objects used on the infected person [4].

a) Mathematical Formulation of SIR Model

The model partitioned the total human population at a particular time into three sub-populations to explains the transmission of the disease in the human population. The total human population which we denoted $N(t)$, is divided into sub-populations of Susceptible class $S(t)$, Infective compartment $I(t)$ and Recovered/Removed class $R(t)$ So that the total population, $N(t)$ is given as:

$$N(t) = S(t) + I(t) + R(t). \tag{1}$$

Susceptible class represent individuals who are at risk of contracting the infection if they had contact with infected individuals. Infective compartment consists of individuals that may not or showing the symptoms of the disease and can infect others. The recovered/removed human compartment are the individuals who have recovered or diseased from the disease. Figure 1 shows flow diagram of COVID-19 disease in a deterministic population. The model variables and their meaning are presented in table I.

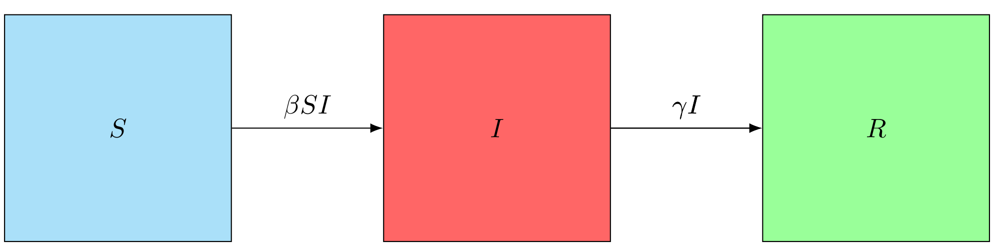

Fig. 1: Flow Diagram of COVID-19 Disease in a Population

Table 1: State variables used and their meanings

Variable	Meaning
$S(t)$	The number of susceptible individuals at a given time, t
$I(t)$	The number of infective individuals at a given time, t
$R(t)$	The number of recovered individuals at a given time, t

The SIR COVID-19 disease transmission model is formulated by assuming the following assumptions as used in [5].

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1) There is a large and homogeneously mixing population.
2) The outbreak of the disease is short lived.
3) The model assumed that there is no natural births or natural deaths that occur.
4) The infection has zero latent period. This means that an individual can spread the disease as soon as they become infected with the disease.
5) Recovering from infection may not necessary confers lifetime immunity since there is no vaccine yet to cure the disease.
6) There is an assumption of the mass-action mixing of individuals. By mass action mixing it is assumes that the rate of an encounter between Susceptible and Infective individuals is directly proportional to the product of the population sizes. By this we mean, doubling the size of either Susceptible or Infective population will result in twice as many new infection cases per unit time. This requires that the individuals of both populations are homogeneously distributed in space and thus do mix either in any smaller subgroups or larger. Understandably, every individual will encounter every other individual per unit time with equal probability. Readers should, however, keep it in mind that the SIR model is a deterministic model and there are no probabilities assumptions involved in the formulation of the model.

Notes

In the formulation of the model, we do not consider the effect of the natural death or birth rate. This is because the model assumes the effective period of the disease is much shorter than the lifetime of the human. The model lets us know the importance of knowing two parameters, namely β and γ. We will consider also that people develop immunity (in the long term, immunity may be lost and the COVID-19 may come back within a certain seasonality such as also find in the case of common flu disease), and there is no transition from recovered to the remaining two classes.

With the model flow diagram and assumptions above, the differential equations that governed the system are now given below:

$$\frac{dS}{dt} = -\beta SI \tag{2}$$

$$\frac{dI}{dt} = \beta SI - \gamma I \tag{3}$$

$$\frac{dR}{dt} = \gamma I \tag{4}$$

Where β is the contagion rate of the disease and γ is the recovery rate. The disease transmission rate β is > 0 and the recovery rate γ is > 0 also (that is, the duration of infection denoted D is equal $\frac{1}{\gamma}$). The incidence term βSI for the number of new infected individuals per unit time corresponds to homogeneous mixing of the infected and susceptible sub-populations.

It can be easily verified that $\frac{dN}{dt} = 0$, and $N = S+I+R$ is therefore constant. That is, the total population is constant. This easily follows from the SIR system above that the sum of the left hand sides of the three equations is the derivative of the total population size and the sum of the right hand sides is zero. Where the total population size is denoted by N. Since $R(t) = N - S(t) - I(t)$, the system can thus be reduced to a system of two ODEs namely (2) and (3).

Now, Suppose that each infective individual has C contacts that will potentially be from the susceptible class and provided that each infective individual is capable of transmitting the disease per unit time, where C is independent of the population size, then, $C * \frac{S}{N}$ of these contacts are with other susceptible individuals. If the fraction, denoted τ, of adequate contacts result in transmission of the disease, then it follows that each infected individual infects $C * \tau * \frac{S}{N}$ susceptible individuals per unit time. Thus,

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we have that $\beta = \frac{b}{N}$, where $b = C * \tau$. The parameter τ is called the transmissibility of the COVID-19 disease.

b) Analysis of the SIR Model

1) The Long Term Limits Existence: We show that the long term limits of the SIR model exist.
   Now, since the right hand side of (2) is negative and the right hand side of (4) is positive, this implies that $\frac{ds}{dt} \leq 0$ and $\frac{dR}{dt} \geq 0$.
   Also, since $0 \leq S(t) \leq S(0) \leq N$ and $0 \leq R(0) \leq R(t) \leq N$, this implies that the limits $S(\infty) = \lim_{t\to\infty} S(t)$, $R(\infty) = lim_{t\to\infty} R(t)$, and thus $I(\infty) = \lim_{t\to\infty} I(t) = N - S(\infty) - R(\infty)$ exist.
2) *The Disease Always Dies Out:* It is also easy to prove that the disease always dies out. Now, $I(\infty) = 0$ for all initial conditions, without formulating a formula for $I(t)$. Suppose now that the disease will not dies out, (4) implies that for sufficiently large t, $\frac{dR}{dt} > \frac{\gamma I(\infty)}{2} > 0$ and this implies that $R(\infty) = \infty$, this is a contradiction. Hence, the disease will dies out.
3) *Theorem of Epidemic Threshold:* We define the effective reproductive number, which we denoted $R_e = \frac{S(0)}{N} * \frac{b}{\gamma}$ and the basic reproduction number denoted, $R_0 = \frac{b}{\gamma}$.

Now, if the entire population is initially susceptible, with one infective case, that is, $S(0) = N-1$, $I(0) = 1$, $R(0) = 0$ and large (see the model assumptions), then $R_e = \frac{N-1}{N} * \frac{b}{\gamma}$ is approximately equal to R_0. Hence, to modify formula involving R_0, we shall assume that the quantity $\frac{N-1}{N}$ is equal to 1. We now show that R_e is the threshold parameter that determines whether the infectious disease will quickly die out or whether it will permeate the population and cause an epidemic.

Theorem 1.

1. *If $R_e \leq 1$, then I(t) decreases monotonically to zero as $t \to \infty$*
2. *If $R_e > 1$, then I(t) starts increasing, reaches its maximum, and then decreases to zero as $t \to \infty$. The scenario of increasing numbers of infected individuals is called an epidemic. It follows that an infectious disease can get into a population and cause an epidemic in an entirely susceptible population if $R_0 > 1$ or $b > \gamma$.*

Proof.

Equation (3) and the discussion in Section 2.2.1 imply that $\frac{dI}{dt} = (\beta S - \gamma) I \leq (\beta S(0) - \gamma) I = \gamma(R_e - 1) I \leq 0$ for $R_e < 1$. This observation together with I(∞)=0 (see Section 2.2.2) proves the first statement.

Equation (3) implies $(\frac{dI}{dt}(0) = \gamma(R_e - 1)\, I(0) > 0$ for $R_e > 1$. Thus $I(t)$ is increasing at $t = 0$. Equation (3) also implies that I(t) has only one non-zero critical point. These observations, together with $I(\infty) = 0$ imply the second statement is true. This end the proof.

c) Existence and Uniqueness of Solution for the SIR Model

The first-order ODE is generally in the form:

$$x' = f(t, x), \qquad x(t_0) = x_0 \tag{5}$$

In order to established the existence and uniqueness of solution of (5), we are interested in answering the following questions:

(I) Under what conditions can we say solution to equation (5) exists?

(II) Under what conditions can we say there is a unique solution to equation (5)?

In order to answers these questions:

Let:

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$$f_1 = -\beta SI,$$

$$f_2 = \beta SI - \gamma I,$$

and

$$f_3 = \gamma I$$

We use the following theorem to established the existence and uniqueness of solution for our SIR model.

Theorem 2 (Uniqueness of Solution)
Let D denotes the domain:

$$|t - t_0| \leq a, ||x - x_0|| \leq b, x = (x_1, x_2, ..., x_n),\ x_0 = (x_{10}, x_{20}, ..., x_{n0}) \tag{6}$$

and suppose that f(t, x) satisfies the Lipschitz condition:

$$||f(t, x_1) - f(t, x_2)|| \leq k||x_1 - x_2||, \tag{7}$$

and whenever the pairs (t, x_1) and (t, x_2) belong to the domain D, where k represents a positive constant.

Then, there exist a constant δ > 0 such that there exists a unique (exactly one) continuous vector solution x(t) of the system (5) in the interval $|t - t_0| \leq \delta$. It is important to note that condition (2.12) is satisfied by requirement that:

$$\left\{ \frac{\partial f_i}{\partial x_j}, \quad {}_{i,j=1,2,\ldots,n} \right.$$

be continuous and bounded in the domain D.

Lemma 1: $f(t,x)$ *has continuous partial derivative* $\frac{\partial f_i}{\partial x_j}$ *on a of real bounded closed convex domen* $\mathbb{R}$ *(i.e, convex set of real numbers) where* $\mathbb{R}$ *is used to denotes real numbers, then it satisfies a Lipschitz condition in* $\mathbb{R}$*. Our interest is in the domain:*

$$1 \leq \epsilon \leq \mathbb{R}. \tag{8}$$

So, we look for a bounded solution of the form

$$0 < \mathcal{R} < \infty.$$

We now prove the following existence theorem.

Theorem 3: (Existence of Solution)
Let D denote the domain defined in (6) such that (7) and (8) hold. Then there exist a solution of model system of equations (2)-(4) which is bounded in the domain D.

Proof. Let:

$$f_1 = -\beta SI, \tag{9}$$

$$f_2 = \beta SI - \gamma I, \tag{10}$$

and

$$f_3 = \gamma I \tag{11}$$

We shows that:
$\frac{\partial f_i}{\partial x_j},\ i,j = 1,2,3$ are continuous and bounded. That is, the partial derivatives are continuous and bounded. We explored the following partial derivatives for all the model equations:
From equation (9);

$$\left|\frac{\partial f_1}{\partial S}\right| = \left| - \beta I\right| < \infty,$$

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$$\left|\frac{\partial f_1}{\partial I}\right| = \left| -\beta S\right| < \infty,$$

$$\left|\frac{\partial f_1}{\partial R}\right| = |0| < \infty.$$

Similarly, from equation (10) we also have that:

$$\left|\frac{\partial f_2}{\partial S}\right| = |\beta I| < \infty,$$

$$\left|\frac{\partial f_2}{\partial I}\right| = \left|\beta S - \gamma\right| < \infty,$$

$$\left|\frac{\partial f_2}{\partial R}\right| = |0| < \infty.$$

Finally we have from equation (11);

$$\left|\frac{\partial f_3}{\partial S}\right| = |0| < \infty,$$

$$\left|\frac{\partial f_3}{\partial I}\right| = |\gamma| < \infty,$$

$$\left|\frac{\partial f_3}{\partial R}\right| = |0| < \infty.$$

We have clearly established that all these partial derivatives are continuous and bounded, hence, by Theorem (1), we can say that there exist a unique solution of (9) to (11) in the region D.

d) Implementing the SIR Model Numerically

The SIR model can be implemented in many ways. It can be implemented from the differential equations governing the system, that is within a mean-field approximation. Also, it can be implemented running the system dynamics in a social network (i.e. graph). We will choose the first option and will run a numerical method (with 4th order Runge-Kutta method) to solve the differential equations system.

We define the functions governing the differential equations (2)-(4) using the python programming language. The functions are given below: "

```
# Susceptible equation
def fa(N, a, b, beta):
fa = -beta*a*b
return fa

# Infected equation
def fb(N, a, b, beta, gamma):
fb = beta*a*b - gamma*b
return fb

# Recovered/Deceased equation
def fc(N, b, gamma):
fc = gamma*b
return fc
```

Where $a = S$, $b = I$ and $c = R$.

In order to solve the differential equations system, we develop a 4th order Runge-Kutta (RK4) numerical method using python programming language. Details of RK4 are discussed in [7].

Ref

7. Sowole, S. O., Sangare, D., Ibrahim, A. A., & Paul, I. A. (2019) On the Existence, Uniqueness, Stability of Solution and Numerical Simulations of a Mathematical Model for Measles Disease, *International Journal of Advances in Mathematics*, 2019(4), Pages 84 – 111, 2019.

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The results obtained for N = world population, with one initial infected case (that is $I(0) = 1$), $\beta = 0.7$, $\gamma = 0.2$ and a time step $\delta t = 0.1$ are shown in the figure 2.

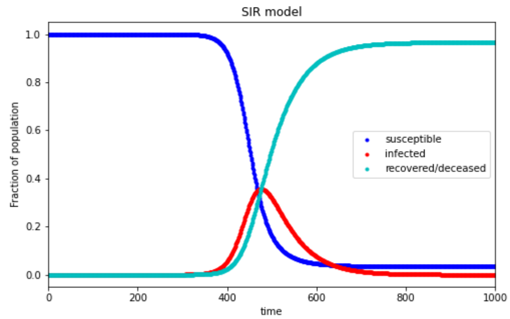

Fig. 2: SIR Model when $\beta = 0.7$ and $\gamma = 0.2$

From the simulation result depicted in the figure 2, the following observations was made:

- The number of infected cases increases for a certain period of time, and then eventually decreases, given that individuals will recovered/deceased from the disease.
- The susceptible compartment of the population de- creases as the virus is transmitted and eventually drop to the absorbent state 0
- However for the recovered/deceased case, the opposite happens.

Readers should however, note that different initial conditions and parameter values will lead to other scenarios from the simulation.

1) *Fitting the SIR model to the Real Data:* Since the beginning of the COVID-19 pandemic, efforts have been put in place to collate live updates of the disease across the globe. For the purpose of our analysis in this paper we are using the data provided by the Center for Systems Science and Engineering (CSSE) at Johns Hopkins University [6] which provide daily updates on the COVID-19 cases that are curated from different official sources.

The SIR model we modeled relied heavily on theoretical assumptions, and we are interested into real approximation of the COVID-19 expansion in order to extract insights and understand the transmission of the virus. Hence, there is need for us to extrapolate the values of β and γ parameters for each case if we hope to be able to predict the evolution of the system.

We considered selected country of interest and fitted the model with the real COVID-19 data of the country and observed how the evolution of the system looks like with the optimal parameters values of β and γ for the country. See figure 3 below.

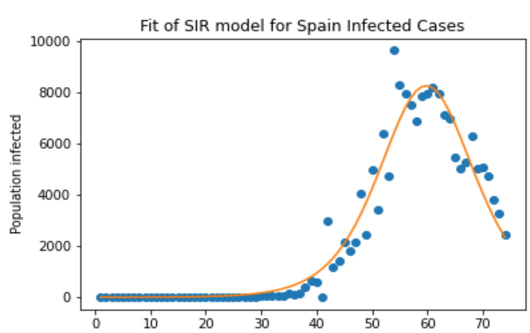

Fig. 3: The Fit of SIR Model for Spain Infected Cases

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The optimal parameters extrapolated for the model fit in the figure 3 above are $\gamma = 9.27$ and $\beta = 9.45$. The yellow line indicates the SIR model while the blue dots represent the data points.

III. Data Analysis and Time Series Forecasting of Covid-19 Data

Notes

Analyzing the SIR model and the simulations given in the section 2 was meant to understand a model that approximately resembles the transmission mechanism of the COVID-19 virus. However, there are alternative methods that prove to be equally useful both to predict and to understand the pandemic evolution, namely ma- chine learning methods. Many of the popular machine learning methods rely on having rich data to extract conclusions and allow algorithms to extrapolate patterns in data. On the COVID-19 data, we will be considering the machine learning regressions approach under this section.

The COVID-19 dataset we used in the analysis includes dates from January 22 to April 14, 2020, totalling 84 days. This dataset covers 184 countries affected by the disease. Countries with Province/State informed are Australia, Canada, China, Denmark, France, Netherlands, the US, and the United Kingdom. Since the data is close to 3 full months from 2020, this is enough data to get some clues about the pandemic. We give the description of the dataset in Table II below:

Table II: Description of COVID-19 Dataset Used

Feature	Description	Type
Id	Unique ID	Integer
Province_State	Province/State in Country/Region	Character
Country_Region	Country/Region with COVID-19 Cases	Character
Date	Date of case reported	string
ConfirmedCases	Number of Confirmed Cases	float
Fatalities	Number of Fatalities Cases	float

We performed some preprocessing to prepare the dataset for training. This consist of filtering of dates to remove Confirmed Cases and Fatalities post-March 12, 2020 for the short term forecasting purpose, we created additional date columns, analyzed and fixed missing values from the data, and also did data transformations.

Understanding the evolution of the disease spread is vital for the early forecast, so we plot some visuals to understand the evolution of the disease.

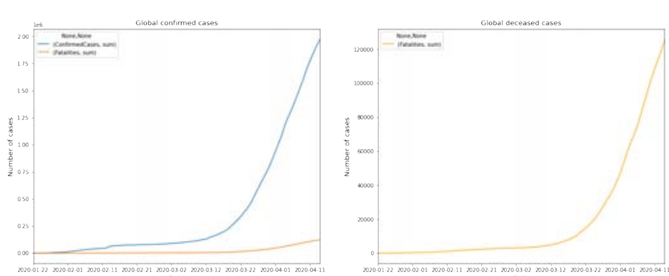

Fig. 4: Evolution of Global COVID-19 Confirmed and Fatalities Cases

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Figure 4 show the global COVID-19 curve. It shows a rich fine structure, but these numbers are strongly affected by the vector zero country-China. We called China vector zero country due to the fact that the COVID-19 disease pandemic started there. Now, during the initial expansion of the virus in China, there was no reliable information about the real infected cases. In fact, the criteria that was used to consider as 'infectious cases' in the COVID-19 dataset was modified around February 11, 2020. This strongly perturbed the curve, this is evidence as can seen in the figure 4.

It was observed in figure 5 that the global COVID-19 general behaviour looks cleaner, and in fact, the curve resembles a typical epidemiology model like SIR. We recalled that the SIR mathematical model presents a large increase in the number of infections cases that, once the infectious cases reaches the maximum of the contagion, will then decrease with a lower slope.

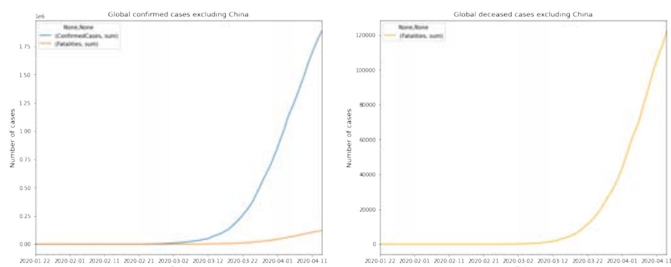

Fig. 5: Evolution of Global COVID-19 Confirmed and Fatalities Cases Excluding China

a) Linear Regression Versus Log-Linear Regression Models

Since we are interested in predicting the future time evolution of the pandemic, our first approach in the paper was to use Machine Learning Linear Regression model for the prediction of the earlier spread of the disease. However, we realized that the evolution is not linear but exponential (this has been discovered from the beginning of the disease spread) so that a preliminary log transformation will be required on the data in order to trained linear model.

The figure 6 below shows a visual comparison of both Confirmed Cases and log Confirmed Cases cases for Spain and with the data information from the last 10 days, beginning from March 1, 2020:

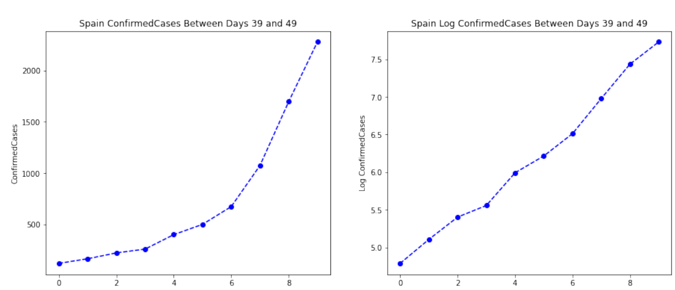

Fig. 6: Spain Confirmed Cases and Log Confirmed Cases

As can be seen in figure 6, the log transformation results in a fancy straight-like line, which is impressive for Linear Regression. However, some important points need to be clarified. This "roughly exponential behaviour" of the curve is only true for the initial disease spread of the COVID-19 pandemic (that is the initial increase of infections on

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the SIR model). But that is exactly the point where most countries are at as at the time of writing this paper. In our analysis, we only extract the last 10 days of data in order to capture exactly the very short term component of the evolution to prevent the effects of certain variables that have been impacting the transmission speed, for instance, the effect of quarantine in comparison to when there is free circulation, etc, and also to prevent differences on criteria when confirming new cases.

We follow the steps below in training the linear regression model:

- Features Selection. We select features for the training.
- Dates filtering. We filtered the train data from March 1 to March 18, 2020.
- Application of Log transformation. We applied log transformation to Confirmed Cases and Fatalities features.
- Handled infinities. We Replace infinities from the logarithm with 0 values. Given the asymptotic behaviour of the logarithm for log(0), this implies that when applying the inverse transformation (exponential) a 1 will be returned instead of a 0. This problem does not impact many countries. In our subsequent work, we intend to address this issue in order to obtain a cleaner solution.
- train/test split. Split the data into train/validation/test sets
- Prediction. Apply Linear Regression, and trained the model country by country after which the results was joined together.

Figures 7 to 10 shows the results for some of the countries after training the model.

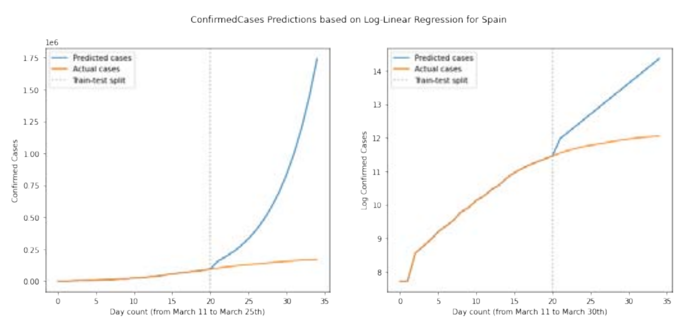

Fig. 7: 1st Predictions for Confirmed Cases based on Log- Linear Regression for Spain

From the regression results, some of which were depicted in the figure 7 to 10, the following general observations were noticed:

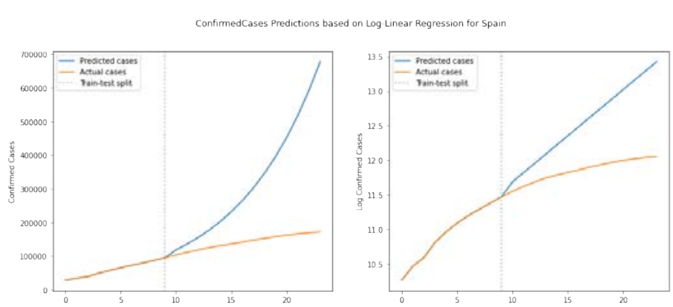

Fig. 8: 2nd Predictions for Confirmed Cases based on Log-Linear Regression for Spain.

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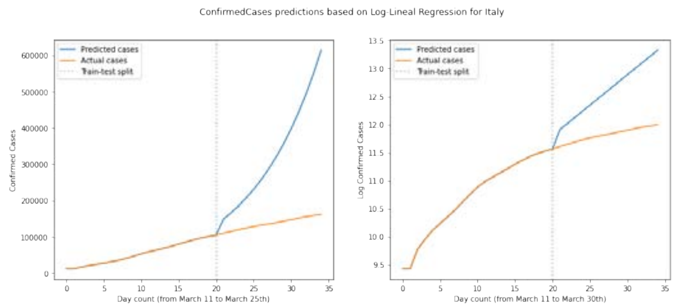

Fig. 9: 1st Predictions for Confirmed Cases based on Log- Linear Regression for Italy

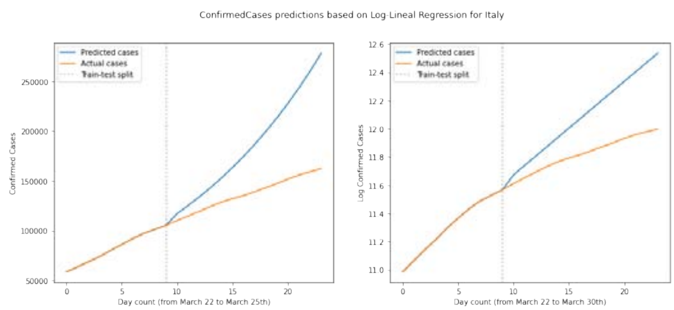

Fig. 10: 2nd Predictions for Confirmed Cases based on Log-Linear Regression for Italy

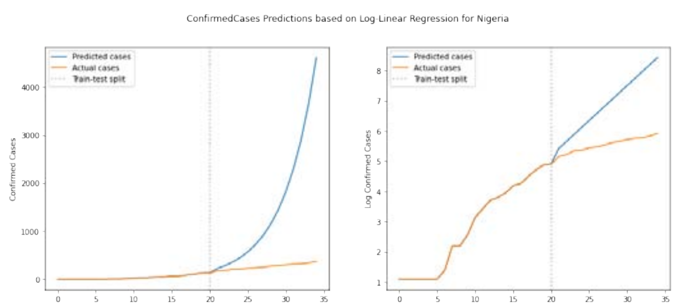

Fig. 11: 1st Predictions for Confirmed Cases based on Log-Linear Regression for Nigeria

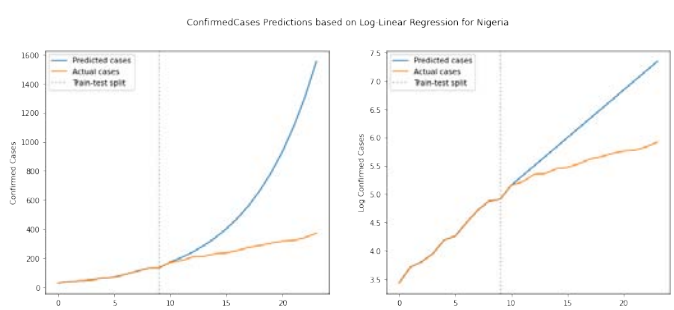

Fig. 12: 2nd Predictions for Confirmed Cases based on Log-Linear Regression for Nigeria

- The general evolution of the disease spread is captured despite the simplicity of the model.

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- The cumulative infected cases have been changing since March, so that using the whole month data for training the model will result in overestimated predictions. When we reduced the training set to only a few days prior to the testing region, results are better. This is capturing the problem of the exponential behaviour that is only true for the early stages of the disease spread. The stage we are now, the disease spread behaviour is more complex, and in order to predict the evolution with large portions of the historic data, alternative and better models will be required (e.g sigmoid, ARIMA, etc.).
- Estimations are increasingly worse as time passes (getting harder to extrapolate).
- Countries that recently confirmed their first contagions are difficult to predict (i.e, countries with fewer data points)
- Countries with 0 cases in the whole training dataset are predicted as non-infected (countries with no data points).

b) Linear regression with Lags and Trends

With the results obtained in the previous subsection, we can deduce that the Linear Regression Model is a good model for predicting the early stages of the COVID-19 spread. That is, Linear regression model can predict the initial outbreak of the disease from the data we are analyzing, and there's no way our model could predict when the number of new infections is going to decrease. But for short-term prediction purposes, everything is fine. We will now try to improve the results obtained previously in this subsection.

Time series data possess specific properties such as trend and structural break, common methods used to analyze other types of data may not be appropriate for the analysis of time series data [8]. So, enriching time series data is key to obtain good results, therefore, we applied two different transformations to the COVID-19 data in this subsection, namely lags and trends. For example, on Confirmed Cases column, lags can be seen as a way to compute the previous value of the column so that the lag_3 for Confirmed Cases would inform this column from the previous day. By definition, the lag_3 of a feature X is given by the formula:

$$X_{lag_3}(t) = X(t-3) \tag{12}$$

Transforming a column into its trend gives the natural tendency of this column, which is different from the raw value. The definition of trend we applied is given by:

$$Trend_X = \frac{X(t) - X(t-1)}{X(t-1)} \tag{13}$$

The backlog of Lags we will apply is 14 days while for Trends is 7 days both for Confirmed Cases and Fatalities.

In order to apply lags in the model, there is a problem to solve: If we use our dataset to predict the next following days of contagions, for the first day, all the lags will be reported (from the previous days), but for the next days, many of the lags will be unknown (and will be flagged as 0), since the number of Confirmed Cases is only known for the training subset. We take the following simple approach to overcome this problem:

(i) We begin with the train dataset, with all cases and lags reported

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8. Min B. Shresthaa, Guna R. Bhatta. (2018) Selecting appropriate methodological framework for time series data analysis, *The Journal of Finance and Data Science,* 2019(4), Pages 71 – 89, 2018.

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(ii) We forecast only for the next day through the Linear Regression
(iii) We set the new prediction as a Confirmed Cases and Fatalities
(iv) We recompute lags
(v) We then repeat from step (ii) to step (iv) for all the remaining days.

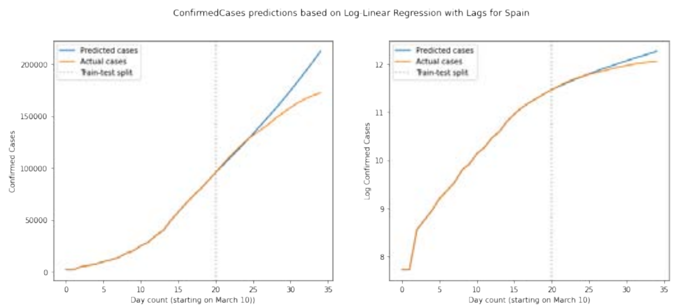

Fig. 13: Predictions for Confirmed Cases of Log-Linear Regression with Lags for Spain

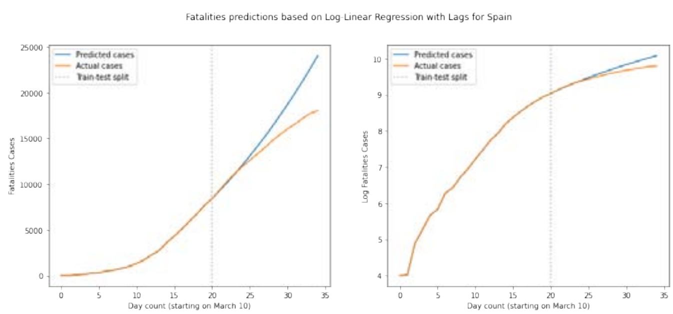

Fig. 14: Predictions of Fatalities of Log-Linear Regression with Lags for Spain

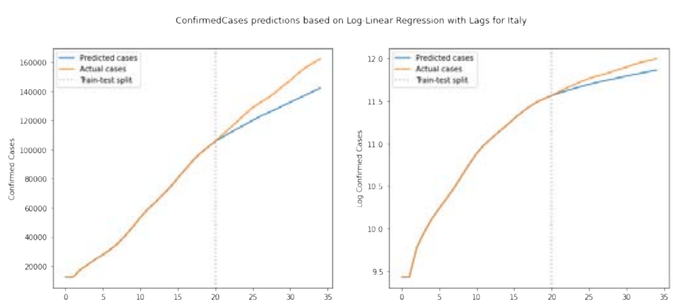

Fig. 15: Predictions for Confirmed Cases of Linear Regression with Lags for Italy

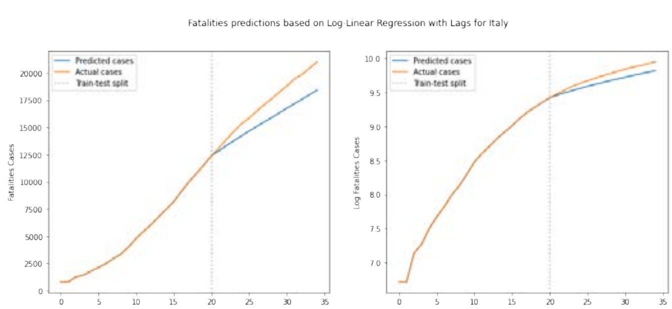

Fig. 16: Predictions of Fatalities of Linear Regression with Lags for Italy

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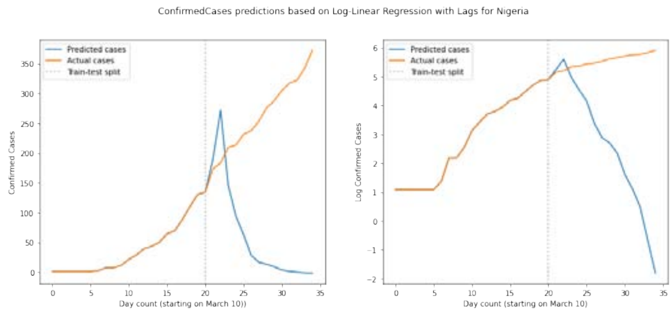

Fig. 17: Predictions for Confirmed Cases of Linear Regression with Lags for Nigeria

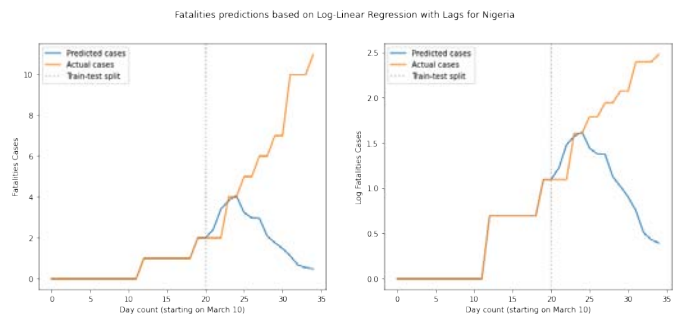

Fig. 18: Predictions of Fatalities of Linear Regression with Lags for Nigeria

Figure 13 to 18 shows some of the regression results with the addition of lags. Two full weeks of the data was used for the training (from February 26th to March 11th), with their previous 14 lags. With the addition of the lags, the following observations were made:

1. *Countries with enough data:* For countries with several Confirmed Cases and Fatalities (For ex- ample, Spain, Italy) in the train dataset (prior to March 11th), predictions are very precise and similar to actual data points.
2. *Countries with poor data:* Countries with a small number of data points (e.g. Nigeria) in the train dataset show a potentially disastrous prediction. Given the small number of cases, the log transformation followed by a Linear Regression is not able to capture the future behavior of the disease spread.
3. *Countries with no data:* When the number of con- firmed cases in the train dataset is 0 or negligible, the model predicts always no infections.

IV. Conclusion

Understanding the evolution of the spread of a disease like COVID-19 will ultimately play a major role in prevention measure that can be taken by relevant authorities to flatten the curve. The aim of this paper was to explore and understand the early time evolution of the spread of COVID-19 disease with a data-driven approach. Therefore, we modelled and applied COVID- 19 disease to the SIR epidemiological model. Also, we use the machine learning linear regression model on the COVID-19 data that is publicly available for early-stage time series forecasting of the COVID-19 cases, considering when there is no lags and trends, and with the application of lags and trends. With these approaches, we are able to capture and understand the early spread of the disease. Hence, we suggest that if using a more advanced deterministic epidemiological model, and sophisticated machine learning regression

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models on the COVID-19 data, it is possible to understand, as well as predict the long time evolution of the disease spread.

Declaration of Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Funding Source

The authors declare that there is no funding received for the writing of this paper.

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Global Journal of Science Frontier Research: F
Mathematics & Decision Science
Volume 20 Issue 5 Version 1.0 Year 2020
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Mixed Value Problem for a One-Dimensional Nonlinear Nonstationary Twelve Moment Boltzmann's System Equations with the Maxwell-Auzhan Boundary Conditions

By Sh. Akimzhanova
Satbayev University

Abstract- It is proved existence and uniqueness of solution of the problem with initial and boundary conditions of Maxwell-Auzhan (we consider pure specular reflection from the boundary) for the nonstationary nonlinear one-dimensional Boltzmann's twelve-moment system equations in space of functions continuous in time and summable in square by spatial variable.

Keywords: Boltzmann's homogeneous one-dimensional equation, Boltzmann's moment system equations, Microscopic Maxwell boundary condition, Macroscopic, boundary conditions.

GJSFR-F Classification: *MSC 2010: 35Q20*

MIXEDVALUEPROBLEMFORAONEDIMENSIONALNONLINEARNONSTATIONARYTWELVEMOMENTBOLTZMANNSSYSTEMEQUATIONSWITHTHEMAXWELLAUZHANBOUNDARYCONDITIONS

Strictly as per the compliance and regulations of:

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Mixed Value Problem for a One-Dimensional Nonlinear Nonstationary Twelve Moment Boltzmann's System Equations with the Maxwell-Auzhan Boundary Conditions

Sh. Akimzhanova

Abstract- **В работе доказано существование и единственность решения начально-краевой задачи для нестационарной нелинейной одномерной двенадцати моментной системы уравнений Больцмана при граничных условиях Максвелла-Аужана в пространстве функций, непрерывных по времени и суммируемых в квадрате по пространственной переменной (рассмотрен случай чисто зеркального отражения от границы).**

Ключевые слова: однородное одномерное уравнение больцмана, система моментных уравнений больцмана, микроскопическое граничное условие максвелла, макроскопическое граничное условие.

Abstract- It is proved existence and uniqueness of solution of the problem with initial and boundary conditions of Maxwell-Auzhan (we consider pure specular reflection from the boundary) for the nonstationary nonlinear one-dimensional Boltzmann's twelve-moment system equations in space of functions continuous in time and summable in square by spatial variable.

Keywords: Boltzmann's homogeneous one-dimensional equation, Boltzmann's moment system equations, Microscopic Maxwell boundary condition, Macroscopic, boundary conditions.

I. Introduction

Many problems of the rarefied gas dynamics require the solution of one or another problem for the Boltzmann equation.The prediction of aerodynamic characteristics of aircraft at very high speeds and at high altitudes is an important problem in aerospace engineering. It is impossible to determine the aerodynamic characteristics of aircraft at high altitudes. The interaction of gas molecules with the surfaces of real bodies has been little studied. The aerodynamic characteristics of aircraft at very high speeds and at high altitudes can be determined by the methods of the theory of a rarefied gas [1]. To analyze the aerodynamic characteristics of aircraft in the transient regime, the complete integro-differential Boltzmann equation is used with appropriate boundary conditions. The determination of the boundary conditions on the surfaces that are streamlined with a rarefied gas is one of the most important questions in the kinetic theory of gases. In high-altitude aerodynamics, the interaction of gas with the surface of a streamlined body plays an important role [2]. The aerothermodynamic characteristics of bodies to the gas flow are determined by the transfer of momentum and energy to the surface of the body, that is, the connection between the velocities and the energies of the molecules incident on the surface and the molecules reflected from it,

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which is the essence of the kinetic boundary conditions on the surface. Maxwell's boundary condition for solving specific problems more accurately describes the interaction of gas molecules with the surface. One of the approximate methods for solving the initial-boundary value problem for the Boltzmann equation is the moment method. With the help of the moment method, it is possible to determine the aerodynamic characteristics of aircraft, such as atmospheric parameters, flight speed, geometric parameters, and the like. We note that in work [3] two new models of boundary conditions were proposed: diffusive-moment and mirror-moment, generalizing the known boundary conditions of Cherchinyani, and in [4] aerodynamic characteristics of space vehicles were studied by the method of direct static modeling (Monte Carlo method) and various model of the interaction of gas molecules with the surface and their effect on aerodynamic characteristic. Moment methods are different from each other as sets of various systems of basic functions. For example, Grad in works [5] and [6] received moment system through decomposition of particles distribution function by Hermitte polynomials near the local Maxwell's distributions. Grad used Cartesian coordinates of velocities and Grad's moment system contained unknown hydrodynamic characteristics as density, temperature, average speed, etc. In [7] obtained the moment system which differs from the system of equations of Grad. In this case was used the spherical coordinates of velocity and distribution function is decomposed into a series by the eigenfunctions of the linearized collision operator [1], [8], which is the product of the Sonin polynomials and spherical functions. The expansion coefficients, the moments of the distribution function are defined differently from Grad. The resulting system of equations corresponding to the partial sum of the series, which was call the Boltzmann's moment system equations, is nonlinear hyperbolic system relative to the moments of the particles distribution function. Differential part of the resulting system is linear and quadratic nonlinearity is shaped as moments of the distribution function. Quadratic forms – the moments of the nonlinear collision integrals – are calculated in [9] and are expressed in terms of coefficients Talmi [10] and Klebsh-Gordon [11].

In [12] - [13], moment systems for the spatially homogeneous Boltzmann equation and the conditions for the representability of the solution of the spatially homogeneous Boltzmann equation in the form of the Poincare series were obtained. The method proposed in [12] (application of the Fourier transform with respect to the velocity variable in the isotropic case) greatly simplified the collision integral and, hence, the calculation of the moments from the collision integral. In [13] the result of [12] is generalized for the case of anisotropic scattering.

In work [14] presented a systematic nonperturbative derivation of a hierarchy of closed systems of moment equations corresponding to any classical theory. This paper is fundamental work where closed systems of moment equations describe a transition regime.

The Boltzmann equation is equivalent to an infinite system of differential equations relative to the moments of the particle distribution function in the complete system of eigenfunctions of linearized operator. As a rule, limited study to finite moment system equations as solving the infinite system of equations does not seem to be possible.

Finite system of moment equations for a specific task with a certain degree of accuracy replaces the Boltzmann equation. It's necessary, also roughly, to replace the boundary conditions for the particle distribution function by a number of macroscopic conditions for the moments, i.e. there arises the problem of boundary conditions for a

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finite system of equations that approximate the microscopic boundary conditions for the Boltzmann equation. The question of boundary conditions for a finite system of moment equations can be divided into two parts: how many conditions must be imposed and how they should be prepared. From microscopic boundary conditions for the Boltzmann equation there can be obtained an infinite set of boundary conditions for each type of decomposition. However, the number of boundary conditions is determined not by the number of moment equations, i.e. it is impossible, for example, take as much boundary conditions as equations, although the number of moment equations affects the number of boundary conditions. In addition, the boundary conditions must be consistent with the moment equations and the resulting problem must be correct.

Grad in [5] described the construction of an infinite sequence of boundary conditions without the consent of the order of approximation for the decomposition of the boundary conditions and the expansion of the Boltzmann equation. Boundary conditions, even a one-dimensional Grad's moment system equations is a very difficult task, because Grad's moment system of equations is a hyperbolic system and this system of equations contains as coefficients unknown parameters like density, temperature, average speed, etc. In such case the characteristic equation is also dependent on the unknown parameters and thus appears to be very difficult to formulate the boundary conditions for the moment system. In work [15] had been discussed the boundary conditions for the 13-moment Grad system.

In work [7] had been shown approximation of the homogeneous boundary condition for the particle distribution function and proved the correctness of the initial-boundary value problem for nonstationary nonlinear Boltzmann's moment system equations in a three-dimensional space. More precisely, was proved the existence of a unique generalized solution of the initial-boundary value problem for the Boltzmann's moment system equations in the space of functions that are continuous in time and square summablein the space of variables. In addition, the same study shows the approximation of the microscopic boundary conditions for the Boltzmann equation. The boundary condition can be formulated as follows: determine the mirrored half of the distribution function from the known half, corresponding to the falling particles. The boundary condition is specified as an integral relation between particles falling to the boundary and particles reflected from the boundary (assuming that we know the probability of an event that a particle falling to the boundary with velocity v_i reflects with velocity v_r).

In this article it is proved existence and uniqueness of solution of the problem with initial and boundary conditions of Maxwell-Auzhan (we consider pure specular reflection from the boundary) for the nonstationary nonlinear one-dimensional Boltzmann's twelve-moment system equations in space of functions continuous in time and summable in square by spatial variable.

Existence and uniqueness of the solutions of initial and boundary value problem for twelve-moment one-dimensional Boltzmann's system of equations with boundary conditions of Maxwell-Auzhan

In this section we prove existence and uniqueness of solutions of the initial and boundary value problem for twelve-moment one-dimensional Boltzmann's system of equations with boundary conditions of Maxwell-Auzhan in space of functionscontinuous in time and summable in square by spatial variable. Theorem of existence of global in time solution of the initial and boundary value problem for 3-dimensional nonlinear Boltzmann equation with boundary conditions of Maxwell proved in work [16].

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Statement of the problem: Find the solution of initial-boundary value problem for a homogeneous one-dimensional Boltzmann equation

$$\frac{\partial f}{\partial t}+|v|\cos\theta\frac{\partial f}{\partial x}=J(f,\ f),\ t\in(0,\ \mathrm{T}],\ x\in(-a,\ a),\ v\in R_3^v, \tag{1}$$

$$f|_{t=0}=f^0(x,v),\ (x,v)\in[-a,a]\times R_3^v, \tag{2}$$

$$f^+(t,x,v_1,v_2,v_3)=\beta f^-(t,x,v_1,v_2,-v_3)+(1-\beta)\eta\exp\left(-\frac{|v|^2}{2RT_0}\right),$$

$$v_3=|v|\cos\theta,(n,\ v)=(n,\ |v|\cos\theta)>0,\ \ x=-a\ \text{ or }\ x=a, \tag{3}$$

where $f\equiv f(t,x,v)$ is a particle distribution function in space of velocity and time; $f^0(x,v)$ is distribution of the particles at the initial time (fixed function); $J(f,\ f)\equiv\int[f(v')f(w')-f(v)f(w)]\sigma(\cos x)dwdv$ nonlinear collision operator, recorded for Maxwell molecules, n is external unit normal vector of the boundary. Condition (3) is a natural boundary condition for the Boltzmann equation, which makes it possible to determine the reflected half of the distribution function f, if we know the half corresponding to the falling particles. According to (3), some part of falling particles reflected specularly, and other particles are absorbed into the wall and emitted with the Maxwell distribution with corresponding wall temperature T_0.

Formula (3) refers to the case of a wall at rest; otherwise v must be replaced by $v-u_0$, u_0 being the velocity of the wall. β, T_0, u_0 may vary from point to point and with time [8].

For one-dimensional problems of the Eigen functions of linearized operator are [1], [8]:

$$g_{nl}(\alpha v)=\left(\frac{\sqrt{\pi}n!(2l+1)}{2\Gamma(n+l+3/2)}\right)^{1/2}\left(\frac{\alpha|v|}{\sqrt{2}}\right)^l S_n^{l+\frac{1}{2}}\left(\frac{\alpha^2|v|^2}{2}\right)P_l(\cos\theta),$$

$$2n+l=0,1,2,\ldots$$

where $S_n^{l+\frac{1}{2}}\left(\frac{\alpha^2|v|^2}{2}\right)$ is Sonin polynomials, $P_l(\cos\theta)$ is Legendre polynomials, Γ is Gamma function.

To find an approximate solution of the problem (1) -(3) we apply the Galerkin method. We define the approximate solution of one-dimensional problem (1)-(3) as follows:

$$f_5(t,x,v)=\sum_{2n+l=0}^{5}f_{nl}(t,x)g_{nl}(\alpha v), \tag{4}$$

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16. S. Mischler, Kinetic equations with Maxwell boundary conditions. Annalesscientifique de l ENS 43, fascicule 5, 719-760 (2010).

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$$\int_{R_3^v}\left(\frac{\partial f_5}{\partial t}+|v|\cos\theta\frac{\partial f_5}{\partial x}-J(f_5,\ f_5)\right)f_0(\alpha|v|)g_{nl}(\alpha v)dv=0,$$

$$2n+l=0,1,...,5,\qquad (t,x)\in(0,\mathrm{T}]\times(-a,a),\tag{5}$$

$$\int_{R_3^v}\left[f_5(0,x,v)-f_5^0(x,v)\right]f_0(\alpha|v|)g_{nl}(\alpha v)dv=0,$$

$$2n+l=0,1,...,5,\qquad x\in(-a,a),\tag{6}$$

$$\int_{(n,v)>0}(n,v)f_0(\alpha|v|)f_5^+(t,x,v)g_{n,2l}(\alpha v)dv-\beta\int_{(n,v)<0}(n,v)f_0(\alpha|v|)\,f_5^-(t,x,v)g_{n,2l}(\alpha v)dv-$$

$$-(1-\beta)\eta\int_{(n,v)<0}(n,v)f_0(\alpha|v|)\exp\left(-\frac{|v|^2}{2RT_0}\right)g_{n,2l}(\alpha v)dv=0$$

$$2(n+l)=0,2,4,\qquad x=-a\ \text{ or } x=-a\,,\tag{7}$$

where $n=(0,0,1)$ with $x=a$ and $n=(0,0-1)$ with $x=-a$;

$f_0(\alpha|v|)$ is the global Maxwell distribution;

$$f_{nl}(t,x)=\int_{R_3^v}f_5(t,x,v)f_0(\alpha|v|)g_{nl}(\alpha v)dv,$$

$$f_5^0(x,v)=\sum_{2n+l=0}^{5}f_{nl}^0(x)g_{nl}(\alpha v)dv,$$

$$f_{nl}^0(x)=\int_{R_3^v}f_5^0(x,v)f_0(\alpha|v|)g_{nl}(\alpha v)dv.\tag{8}$$

In the general case, the approximation of the boundary condition (3) depends on the parity or oddness of the approximation of the Boltzmann's moment system of equations [17]. When approximating a microscopic boundary condition, we took into account the approximation of the Boltzmann equation by moment equations corresponding to the fifth approximation (the twelve moment system of equations).

Thus, the approximation orders for the expansion of the boundary condition and the expansion of the Boltzmann equation are consistent. Macroscopic conditions (7) were called Maxwell – Auzhan boundary conditions [17].

Boltzmann's system of moment equations (5), corresponding to the decomposition (4) can be written in expanded form

$$\frac{\partial f_{nl}}{\partial t}+\frac{1}{\alpha}\frac{\partial}{\partial x}\left[l\left(\sqrt{\frac{2\left(n+l+\frac{1}{2}\right)}{(2l-1)(2l+1)}}f_{n,l-1}-\sqrt{\frac{2(n+1)}{(2l-1)(2l+1)}}f_{n+1,l-1}\right)+\right.$$

$$\left.+(l+1)\left(\sqrt{\frac{2\left(n+l+\frac{3}{2}\right)}{(2l+1)(2l+3)}}f_{n,l+1}-\sqrt{\frac{2n}{(2l+1)(2l+3)}}f_{n-1,l+1}\right)\right]=I_{nl},$$

$$2n+l=0,\ 1,\ ...,\ 5,\tag{9}$$

Ref

17. A. Sakabekov, Auzhani Y. Boundary conditions for the one dimensional nonlinear nonstationary Boltzmann's moment system equations//Journal of mathematical physics,55,123507,(2014).

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where the moments of the collision integral can be expressed in terms of coefficients Talmi and Klebsh-Gordon as follows [6]

$$I_{nl} = \sum \langle N_3 L_3 n_3 l_3 : l \mid nl00 : l \rangle \langle N_3 L_3 n_3 l_3 : l \mid n_1 l_1 n_2 l_2 : l \rangle (l_1 0 l_2 0 / l0) (\sigma_{l_3} - \sigma_0) f_{n_1 l_1} f_{n_2 l_2},$$

$\langle N_3 L_3 n_3 l_3 : l \mid n_1 l_1 n_2 l_2 : l \rangle$ is generalized Talmi coefficients, $(l_1 0 l_2 0 / l0)$ is Klebsh-Gordon coefficients.

If in (9) $2n + l$ is from 0 to 5, we get the system of equations, corresponding fifth approximation of Boltzmann's moment system equations or twelve-moment Boltzmann's system equations.

The initial and boundary value problem for twelve-moment Boltzmann's system equations with boundary conditions of Maxwell-Auzhan (7) can be written in vector-matrix form [18] (we consider pure specular reflection from the boundary $\beta = 1$):

$$\frac{\partial u}{\partial t} + A \frac{\partial w}{\partial x} = J_1(u,\ w)$$

$$\frac{\partial u}{\partial t} + A' \frac{\partial u}{\partial x} = J_2(u,\ w),\ t \in (0,\ \mathrm{T}],\ x \in (-a,\ a), \tag{10}$$

$$u\big|_{t=0} = u_0(x), \quad w\big|_{t=0} = w_0(x), \quad x \in (-a,\ a), \tag{11}$$

$$\left(Aw^+ - Bu^+\right)\big|_{x=-a} = \left(Aw^- + Bu^-\right)\big|_{x=-a},\ t \in [0,\ \mathrm{T}], \tag{12}$$

$$\left(Aw^+ + Bu^+\right)\big|_{x=a} = \left(Aw^- - Bu^-\right)\big|_{x=a},\ t \in [0,\ \mathrm{T}], \tag{13}$$

where A' is transpose matrix; $u_0(x) = \left(f_{00}^0(x),\ f_{02}^0(x),\ f_{10}^0(x),\ f_{04}^0(x),\ f_{12}^0(x),\ f_{20}^0(x)\right)'$, $w_0(x) = \left(f_{01}^0(x),\ f_{03}^0(x),\ f_{05}^0(x),\ f_{11}^0(x),\ f_{13}^0(x),\ f_{21}^0(x)\right)'$ are given initial vector functions,

$$u = \left(f_{00},\ f_{02},\ f_{04},\ f_{10},\ f_{12},\ f_{20}\right),\ w = \left(f_{01},\ f_{03},\ f_{05},\ f_{11},\ f_{13},\ f_{21}\right),$$

$$J_1(u,v) = \left(0,\ I_{04},\ 0,\ I_{12},\ I_{20}\right),\ J_2(u,v) = \left(0,\ I_{03},\ I_{05},\ I_{11},\ I_{13},\ I_{21}\right),$$

$$A = \frac{1}{\alpha}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{\sqrt{3}} & \frac{3}{\sqrt{5}} & 0 & -\frac{2\sqrt{2}}{\sqrt{15}} & 0 & 0 \\ 0 & \frac{4}{\sqrt{7}} & \frac{5}{3} & 0 & -\frac{4\sqrt{2}}{3\sqrt{7}} & 0 \\ -\sqrt{\frac{2}{3}} & 0 & 0 & \sqrt{\frac{5}{3}} & 0 & 0 \\ 0 & -\frac{3\sqrt{2}}{\sqrt{35}} & 0 & \frac{2\sqrt{7}}{\sqrt{15}} & \frac{9}{\sqrt{35}} & -\frac{4}{\sqrt{15}} \\ 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & \sqrt{\frac{7}{3}} \end{bmatrix}$$

Ref

6. G. Grad, Principle of the kinetic theory of gases. Handuch der Physik, Volume 12, Springer, Berlin, p.p. 205-294.

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$$B=\frac{1}{\alpha\sqrt{\pi}}\begin{bmatrix} \sqrt{2} & \sqrt{\frac{2}{3}} & -\sqrt{\frac{2}{105}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{21}} & -\frac{1}{2\sqrt{15}} \\ \sqrt{\frac{2}{3}} & 2\sqrt{2} & \frac{26}{3\sqrt{70}} & -1 & -\frac{2}{\sqrt{7}} & \frac{1}{2\sqrt{5}} \\ -\sqrt{\frac{2}{105}} & \frac{26}{3\sqrt{70}} & -\frac{4}{7\sqrt{2}} & \frac{5}{3\sqrt{35}} & -\frac{13}{\sqrt{245}} & \frac{1}{2\sqrt{7}} \\ -\frac{1}{\sqrt{3}} & -1 & \frac{5}{3\sqrt{35}} & \frac{6}{\sqrt{2}} & \frac{3}{\sqrt{14}} & -\frac{5}{2\sqrt{10}} \\ \frac{1}{\sqrt{21}} & -\frac{2}{\sqrt{7}} & -\frac{13}{\sqrt{245}} & \frac{3}{\sqrt{14}} & \frac{34}{7\sqrt{2}} & \frac{23\sqrt{2}}{4\sqrt{35}} \\ -\frac{1}{2\sqrt{15}} & \frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{7}} & -\frac{5}{2\sqrt{10}} & \frac{23\sqrt{2}}{4\sqrt{35}} & \frac{15}{\sqrt{2}} \end{bmatrix}$$

Notes

The matrices A and B are degenerated. Required to find a solution to the system of equations (10) satisfying the initial condition (11) and boundary conditions (12) and (13).Before we prove the existence and uniqueness of the solution of the problem (10) - (13).

For the problem (10) -(13) following theorem takes place.

Theorem: If $U_0=(u_0(x),w_0(x))\in L^2[-a,a]$, then problem (10)-(13) has unique solution in domain $[-a,a]\times[0,T]$, belonging to the space $C([0,T];L^2[-a,a])$, moreover

$$\|U\|_{C([0,T];L^2[-a,a])}\leq C_1\|U_0\|_{L^2[-a,a]} \tag{14}$$

where C_1 is constant independent from $U=(u,w)'$ and $T\sim 0\left(\|U_0\|^{-1}_{L^2[-a,a]}\right)$.

Proof: Let $U_0\in L^2[-a,a]$. Let's prove estimation (14). We multiple first equation of system (10) by u and second equation by w, and integrate from $-a$ to a:

$$\frac{1}{2}\frac{d}{dt}\int_{-a}^{a}[(u,u)+(w,w)]dx+\int_{-a}^{a}\left[\left(A\frac{\partial w}{\partial x},u\right)+\left(A'\frac{\partial u}{\partial x},w\right)\right]dx=\int_{-a}^{a}[(J_1,u)+(J_2,w)]dx.$$

After integration by parts we receive

$$\frac{1}{2}\frac{d}{dt}\int_{-a}^{a}[(u,u)+(w,w)]dx+\left(u^-,Aw^-\right)\Big|_{x=a}-\left(u^-,Aw^-\right)\Big|_{x=-a}=\int_{-a}^{a}[(J_1,u)+(J_2,w)]dx. \tag{15}$$

Taking into account boundary conditions (12)-(13) we rewrite equality (15) in following form

$$\frac{1}{2}\frac{d}{dt}\int_{-a}^{a}[(u,u)+(w,w)]dx+\left(Bu^-,u^-\right)\Big|_{x=a}+\left(Bu^-,u^-\right)\Big|_{x=-a}-\left(\left(Aw^+-Bu^+\right),u^-\right)\Big|_{x=-a}+$$

$$+\left(\left(Aw^++Bu^+\right),u^-\right)_{x=a}=\int_{-a}^{a}[(J_1(u,w),u)+(J_2(u,w),w)]dx. \tag{16}$$

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Let's use spherical representation [19] of vector $U(t,x)=r(t)\omega(t,x)$, where $\omega(t,x)=(\omega_1(t,x),\omega_2(t,x))', r(t)=\|U(t,.)\|_{L^2[-a,a]}, \|\omega\|_{L^2[-a,a]}=1.$

Substituting the values $u=r(t)\omega_1(t,x)$, $w=r(t)\omega_2(t,x)$ into (17) we have that

$$\frac{dr}{dt}+rP(t)=r^2Q(t) \tag{17}$$

where

$$P(t)=\left(B\omega_1^-,\omega_1^-\right)\Big|_{x=a}+\left(B\omega_1^-,\omega_1^-\right)\Big|_{x=-a}+\Big[\left(A\omega_2^+,\omega_1^-\right)\Big|_{x=a}+\left(B\omega_1^+,\omega_1^-\right)\Big|_{x=a}+$$

$$+\left(B\omega_1^+,\omega_1^-\right)\Big|_{x=-a}-\left(A\omega_2^+,\omega_1^-\right)\Big|_{x=-a}\Big],$$

$$Q(t)=\int_{-a}^{a}\left[\left(J_1(\omega_1,\omega_2),\omega_1\right)+\left(J_2(\omega_1,\omega_2),\omega_2\right)\right]dx.$$

Let's study equation (17) with initial condition

$$r(0)=\|U_0\|=\|U_0\|_{L^2[-a,a]}. \tag{18}$$

Solution of the problem (17) -(18) has following form

$$r(t)=\left\{\exp\left(\int_0^t P(\tau)d\tau\right)\left[\frac{1}{\|U_0\|}-\int_0^t Q(\tau)\exp\left(-\int_0^\tau P(\xi)d\xi\right)d\tau\right]\right\}^{-1}.$$

If $R(t)\equiv\int_0^t Q(\tau)\exp\left(-\int_0^\tau P(\xi)d\xi\right)d\tau\le 0,\forall t$ then $r(t)$ is bounded for $\forall t\in[0,+\infty)$.

Let $R(t)>0$. We denote by T_1 the moment of time at which

$$\frac{1}{\|U_0\|}-\int_0^{T_1} Q(\tau)\exp\left(-\int_0^\tau P(\xi)d\xi\right)d\tau=0.$$

Then $r(t)$ is bounded for $\forall t\in[0,T]$, where $T<T_1$, moreover $T_1\sim 0\left(\|U_0\|^{-1}\right)$, since integrand $Q(\tau)\exp\left(-\int_0^t P(\xi)d\xi\right)$ is bounded. Hence $\forall t\in[0,T]$ takes place a priori estimation (14).

Now we prove the existence of a solution for (10) -(13) with help of Galerkin method. Let us $\{v_l(x)\}_{l=1}^{\infty}$ be a basis in space $L_2[-a,a]$, where dimension of vector $v_l(x)$ is

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19. Pokhozhaev S.I. On an approach to nonlinear equation, Dokl.Akad. Nauk USSR 247, 1327-1331 (1979).

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equal to dimension of vector U. For each m we define an approximate solution U_m of (10)-(13) as follows:

$$U_m = \sum_{j=1}^{m} c_{jm}(t) v_j(x), \tag{19}$$

$$\int_{-a}^{a} \left(\left(\frac{\partial U_m}{\partial t} + A_1 \frac{\partial U_m}{\partial x} \right), v_1(x) \right) dx = \int_{-a}^{a} (J(U_m), v_i(x)) dx, \; i = \overline{1,m}, t \in (0,T] \tag{20}$$

$$U_m \Big|_{t=0} = U_{0m}(x), x \in R, \tag{21}$$

$$\left(Aw_m^- \pm Bu_m^- \right)\Big|_{x=\pm a} = \left(Aw_m^+ \pm Bu_m^+ \right)\Big|_{x=\pm a} \tag{22}$$

where U_{0m} is the orthogonal projection in L^2 of function U_0 on the subspace, spanned by $v_1,...v_m$.

$$J(u_m) = (J_1(u_m, w_m), J_2(u_m, w_m))'$$

We represent $v_j(x)$ in the form $v_j^{(1)} = \left(v_j^1, v_j^2 \right)'$, where

$$v_j^{(1)} = \left(v_{j1}, v_{j2}, v_{j3} \right)', v_j^{(2)} = \left(v_{j4}, v_{j5}, v_{j6} \right)'.$$

The coefficients $c_{jm}(t)$ are determined from the equations

$$\sum_{j=1}^{m} \Bigg\{ \frac{dc_{jm}}{dt} \int_{-a}^{a} (v_j, v_i) dx + c_{jm} \Big[\left(Bv_j^{-(1)}, v_i^{-(1)} \right)\Big|_{x=a} + \left(Bv_j^{-(1)}, v_i^{-(1)} \right)\Big|_{x=-a} +$$

$$+ \left(Bv_i^{-(1)}, v_j^{-(1)} \right)\Big|_{x=a} + \left(Bv_i^{-(1)} + v_j^{-(1)} \right)\Big|_{x=-a} + \left(\left(Av_j^{+(2)} + Bv_i^{+(1)} \right), v_j^{-(1)} \right)\Big|_{x=a} -$$

$$- \left(\left(Av_i^{+(2)} - Bv_i^{+(1)} \right), v_j^{-(1)} \right)\Big|_{x=-a} + \left(\left(Av_j^{+(2)} + Bv_j^{+(1)} \right), v_i^{-(1)} \right)\Big|_{x=a} -$$

$$- \left(\left(Av_j^{+(2)} - Bv_j^{+(1)} \right), v_j^{-(1)} \right)\Big|_{x=-a} \Big] - \int_{-a}^{a} \left(\left(A \frac{\partial v_i^{(2)}}{\partial x}, v_j^1 \right) + \left(A' \frac{\partial v_i^{(1)}}{\partial x}, v_j^{(2)} \right) \right) dx \Bigg\} =$$

$$= \int_{-a}^{a} \left(J\left(\sum_{j=1}^{m} c_{jm} v_j \right), v_i \right) dx, \; i = \overline{1,m}, t \in (0,T] \tag{23}$$

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$$c_{im}(0) = d_{im}, i = \overline{1, m} \tag{24}$$

where d_{im} is i-th component of U_{0m}.

We multiply (21) by $c_{im}(t)$ and sum over i from 1 to m:

$$\int_{-a}^{a}\left(\left(\frac{\partial U_m}{\partial t} + A_1 \frac{\partial U_m}{\partial x}\right), U_m\right)dx = \int_{-a}^{a}\left(J(U_m), U_m\right)dx.$$

With help of above shown arguments now we prove that $r_m(t)$ is bounded in some time interval $[0, T_m]$, where

$$U_m(t,x) = r_m(t) w_m(t,x), T_m \approx 0\left(\|U_{0m}\|^{-1}\right), T_m \geq T, \forall m \text{ and}$$

$$\|U_m\|_{C([0,T];L^2[-a,a])} \leq C_2 \|U_0\|_{L^2[-a,a]}, \tag{25}$$

where C_2 is constant and independent from m. Then solvability of system equations (19) - (22) or (23) -(24) follows from estimation (25).

Thus, the sequence $\{U_m\}$ of approximate solutions of problem (10)-(13) is uniformly bounded in function space $C([0,T]; L^2[-a,a])$. Moreover, homogeneous system of equations $rE + \frac{1}{\alpha}A\xi$ with respect to τ, ξ has only trivial solution. Then it follows from results in [20] that $U_m \to U$ is week in $C([0,T]; L^2[-a,a])$ and $J(U_m) \to J(U)$ is week in $C([0,T]; L^2[-a,a])$ as $m \to \infty$. Further, it can be shown by standard methods that limit element is a weak solution of the problem (10) -(13).

The theorem is proved.

II. Conclusion

We prove the theorem of existence and uniqueness of the local solution of the initial and boundary value problem for twelve-moment one-dimensional Boltzmann's system of equations with boundary conditions of Maxwell-Auzhan in space of functionscontinuous in time and summable in square by spatial variable, because the solution existence time depends on the norm of the initial vector function at the power minus one. Therefore, the smaller the norm of the initial vector function, the longer the solution existence time of the initial and boundary value problem for six-moment one-dimensional Boltzmann's system of equationsand vice versa.

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Global Journal of Science Frontier Research: F
Mathematics & Decision Science
Volume 20 Issue 5 Version 1.0 Year 2020
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Deformation Due to Various Sources in a Thermally Conducing Cubic Crystal Material with Reference Temperature Dependent Properties

By Leena Rani
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Abstract- A homogeneous, thermally conducting cubic crystal, elastic half-plane subjected to normal, tangential force and thermal source under the effect of dependence of reference temperature on all elastic and thermal parameters is investigated. The interaction due to two types of loading: instantaneous and continuous has been considered. The Laplace and Fourier transforms technique has been used to obtain the components of displacement, stresses and temperature distribution for Lord and Shulman (L-S), Green and Lindsay (G-L), Green and Naghdi(G-N) and Chandrasekharaiah and Tzou (CTU) theories of generalized thermoelasticity. The concentrated and distributed loads have been taken to illustrate the utility of the approach. particular case is also deduced. The numerical inversion technique has been used to invert the integral transforms. The comparison of Linear case, quadratic case and exponential case, respectively, are depicted graphically for thermal source for L-S theory.

Keywords: *generalized thermoelasticity, cubic crystal, relaxation time, laplace and fourier transforms.*

GJSFR-F Classification: *MSC 2010: 14D15*

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Abstract- A homogeneous, thermally conducting cubic crystal, elastic half-plane subjected to normal, tangential force and thermal source under the effect of dependence of reference temperature on all elastic and thermal parameters is investigated. The interaction due to two types of loading: instantaneous and continuous has been considered. The Laplace and Fourier transforms technique has been used to obtain the components of displacement, stresses and temperature distribution for Lord and Shulman (L-S), Green and Lindsay (G-L), Green and Naghdi(G-N) and Chandrasekharaiah and Tzou (CTU) theories of generalized thermoelasticity. The concentrated and distributed loads have been taken to illustrate the utility of the approach. particular case is also deduced. The numerical inversion technique has been used to invert the integral transforms. The comparison of Linear case, quadratic case and exponential case, respectively, are depicted graphically for thermal source for L-S theory.

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I. Introduction

In anisotropic bodies, it is necessary to study the response of thermally induced disturbances, which may happen during the manufacturing stages. For example, during the curing stages of filament bound bodies, thermal disturbances may be induced by the heat buildup and cooling processes. The level of these disturbances may exceed the ultimate strength of the material. In the last century, a considerable interest is developed in the theory of thermoelasticity that includes such thermal disturbances. After studying the second sound effect in materials as solid helium, bismuth, and sodium fluoride, a systematic research get started.

The classical dynamical coupled theory of thermoelasticity has been extended to generalized thermoelasticity theories by Lord and Shulman (1967) and Green and Lindsay (1972). Dhaliwal and Sherief (1980) extended the generalized theory of thermoelasticity (1967) to anisotropic media. Green and Naghdi (1993) proposed a new theory of thermoelasticity without energy dissipation and presented the derivation of a complete set of governing equations of the linearized version of the theory for homogenous and isotropic materials in terms of displacement and temperature fields.

Chandrasekharaiah (1998) and Tzou (1995) proposed another generalization to coupled theory is known as dual-phase-lag thermoelasticity, in which Fourier law is

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replaced by an approximation to a modification of the Fourier law with two different translations for the heat flux and temperature gradient. Lin (2004) studied thermoelastic problems in anisotropic half-plane. Kumar and Rani (2007) discussed disturbances due to thermomechanical sources in orthorhombic thermoelastic material.Beom (2013) considered thermoelastic in-plane problems in linear anisotropic solid. Valès et al. (2016) studied determination of heat source dissipation from infrared thermographic measurements. Bockstal and Marin(2017) discussed recovery of a space dependent vector source in anisotropic thermoelastic system. Rani and Singh (2018) studied thermal disturbances in twinned orthotropic thermoelastic material. Zhang et al. (2019) discussed thermo-mechanical coupling analysis of the orthotropic structures by using element-free Galerkin method. Zhou et al. (2020) solved transient heat conduction problems in general anisotropic media and derived three-dimensional Green's functions in bimaterial.

Nowinski (1959,1960,1962) developed thermoelasticity of bodies with temperature dependent properties. Noda (1991) considered thermal stresses in materials with temperature dependent properties. Ezzat et al. (2001) solved a problem of generalized thermoelasticity with two relaxation times in an isotropic elastic medium with temperature-dependent mechanical properties. Othman and Kumar (2009) studied the reflection of magneto-thermoelastic waves with temperature dependent properties in generalized thermoelasticity. Kalkal and Deswal (2014) adopted normal mode technique to investigate the effect of phase lags on three-dimensional wave propagation with temperature-dependent properties. Matysiak et al. (2017) studied temperature and stresses in a thermoelastic half-space with temperature dependent properties. Zhang et al. (2019) studied the effect of temperature dependant material properties on thermoelastic damping in thin beams.

To the best of my knowledge the problem of homogeneous, thermally conducting, cubic crystal material under the effect of dependence of reference temperature on all elastic and thermal parameters has not yet been investigated. In the present problem the component of displacements, stresses and temperature distribution are determined due to mechanical and thermal sources. The solutions are obtained by using Laplace and Fourier technique. The comparison of Linear case, quadratic case and exponential case, respectively, are shown graphically for thermal source for L-S theory.

II. Formulation of the Problem

We consider a homogenous, thermally conducting cubic crystal, elastic half-space in the undeformed state at uniform temperature T_o. The rectangular Cartesian co-ordinate system (x,y,z) having origin on the plane surface z=0 with z-axis pointing vertically into medium is introduced. A concentrated and uniformly distributed mechanical or thermal source is assumed to be acting at the origin of the rectangular Cartesian co-ordinates. Here we consider plane strain problem parallel to xz-plane with displacement vector $\vec{u} = (u,0,w)$ and temperature T(x, z, t), then the field equations and constitutive relations for such a medium in the absence of body forces and heat sources can be written, by following the equations given by Lord-Shulman (1967), Green and Lindsay (1972) and Dhaliwal and Sherief (1980) as

$$c_{11}\frac{\partial^2 u}{\partial x^2}+c_{44}\frac{\partial^2 u}{\partial z^2}+(c_{12}+c_{44})\frac{\partial^2 w}{\partial x\partial z}-\beta\frac{\partial}{\partial x}(T+\delta_{2k}\tau_1\frac{\partial T}{\partial t})=\rho\frac{\partial^2 u}{\partial t^2}, \quad (1)$$

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$$c_{44}\frac{\partial^2 w}{\partial x^2}+c_{11}\frac{\partial^2 w}{\partial z^2}+(c_{12}+c_{44})\frac{\partial^2 u}{\partial x\partial z}-\beta\frac{\partial}{\partial z}\ (T+\delta_{2k}\tau_1\frac{\partial T}{\partial t})=\rho\frac{\partial^2 w}{\partial t^2}, \quad (2)$$

$$K\left(n^*+t_1\frac{\partial}{\partial t}\right)(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2})=\rho c_e\ (n_1\frac{\partial T}{\partial t}+\tau_o\frac{\partial^2 T}{\partial t^2})+T_o\beta(n_1\frac{\partial}{\partial t}+n_0\tau_o\ \delta_{1k}\frac{\partial^2}{\partial t^2})\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right), \quad (3)$$

Notes

and

$$t_{zz}=c_{12}\frac{\partial u}{\partial x}+c_{11}\frac{\partial w}{\partial z}-\beta\,(T+\delta_{2k}\tau_1\frac{\partial T}{\partial t})\ ,\ t_{zx}=c_{44}(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}), \quad (4)$$

where

$$\beta=(c_{11}+2\,c_{12})\alpha\,,$$

and c_{ij} are isothermal elastic parameters, ρ , c_e and τ_0,τ_1 are, respectively, the density, specific heat at constant strain and thermal relaxation times. α is the coefficient of thermal expansion, δ_{1k},δ_{2k} are Kroneck er deltas, K is the coefficient of thermal conductivity. u and w are displacement components along x and z directions respectively, t is time, T is temperature change, t_{zx} and t_{zz} are stresses. Lord-Shulman (L-S) theory, $t_1=\tau_1=0$, $\tau_0>0$,k =1, $n^*=n_o=n_1=1$, and for Green and Lindsay (G-L) theory, $t_1=0$, $\tau_1\geq\tau_0>0$, $n_o=0$, $n^*=n_1=1$. For Green and Naghdi theory(G-N) (type II) $n^*>0, n_1=0$, $n_o=1$, $t_1=\tau_1=0$, $\tau_0=1$. where n^*=constant has the dimension of [1/s], and $n^*K=K^*=c_ec_{11}/4$ is a characteristic constant this theory.

For Chandrasekharaiah and Tzou (CTU) theory is such a modification of classical thermoelasticity model in which Fourier law is replaced by an approximation of the equation

$$q_i(x,t+\tau_q)=-KT_{,i}(x,t+\tau_\theta), \quad (4a)$$

where q_i is the heat flux vector. The model transmits thermoelastic disturbances in a wave like manner (1986) if Eq. (4a) is approximated by

$$(1+\tau_q\frac{\partial}{\partial t})q_i=-K(1+\tau_\theta\frac{\partial}{\partial t})T_{,i},$$

where $0\leq\tau_\theta<\tau_q$.and $t_1=\tau_\theta>0$ and $\tau_\theta=\tau_q>0$, $n^*=n_o=n_1=1$, $0\leq\tau_\theta<\tau_q$, $\tau_1=0$. c_{ijkl} satisfies the (Green) symmetry conditions:

$$c_{ijkl}=c_{klij}=c_{ijlk}=c_{jikl}.$$

The initial and regularity conditions are given by

$$u(x,z,0)=0=\dot{u}(x,z,0),$$

$$w(x,z,0)=0=\dot{w}(x,z,0),$$

$$T(x,z,0)=0=\dot{T}(x,z,0) \quad for\ z\geq 0, \quad -\infty < x < \infty, \tag{5}$$

$$and\ u(x\ z\ t)=w(x\ z\ t)=T(x,z,t)=0 \quad for\, t>0 \quad \text{when} \quad z\to\infty. \tag{6}$$

For dependency of all elastic and thermal parameters on reference temperature we have taken three cases (i) Linear case (ii) quadratic case (iii) Exponential case The material constants are given as (2016, 2001)

For Linear case

$$\mathrm{c_{ij}}=\mathrm{c_{ij0}}(1-\alpha^*T_0),\quad \beta=\beta_0(1-\alpha^*T_0),\ K=K_0(1-\alpha^*T_0),\ v_1=v_{10}(1-\alpha^*T_0),$$
$$c_e=c_{e0}(1-\alpha^*T_0). \tag{7}$$

For quadratic case

$$\mathrm{c_{ij}}=\mathrm{c_{ij0}}(1-\alpha^*T_0)^2,\quad \beta=\beta_0(1-\alpha^*T_0)^2,\ K=K_0(1-\alpha^*T_0)^2,\ v_1=v_{10}(1-\alpha^*T_0)^2,$$
$$c_e=c_{e0}(1-\alpha^*T_0)^2. \tag{8}$$

For exponential case

$$\mathrm{c_{ij}}=\mathrm{c_{ij0}}e^{\alpha^*T_0},\quad \beta=\beta_0 e^{\alpha^*T_0},\ K=K_0 e^{\alpha^*T_0},\ v_1=v_{10}e^{\alpha^*T_0},\ c_e=c_{e0}e^{\alpha^*T_0}. \tag{9}$$

where $\mathrm{c_{ij0}},\beta_0,K_0,v_0,c_{e0}$ are considered as constants, α^* is called empirical material constant. In case of the system independent of reference temperature, $\alpha^*=0$.

III. Solution of the Problem

We introduce dimensionless quantities as

$$x'=\frac{\omega_1^* x}{v_1},\quad z'=\frac{\omega_1^* z}{v_1},\quad t'=\omega_1^* t,\quad \mathrm{u}'=\frac{\rho v_1\omega_1^*}{\beta_0 T_0}\mathrm{u},\quad \mathrm{w}'=\frac{\rho v_1\omega_1^*}{\beta_0 T_0}\mathrm{w},$$

$$\tau_0'=\omega_1^*\tau_0,\quad \mathrm{c}_1=\frac{c_{440}}{c_{110}},\quad \mathrm{c}_2=\frac{c_{120}+c_{440}}{c_{110}},\quad \omega'=\frac{\omega}{\omega_1^*},\quad \in_1=\frac{\beta_0^2 T_o}{\rho^2 c_{e0} v_1^2},$$

$$\tau_1=\omega_1^*\tau_1,\quad \mathrm{a}'=\frac{\omega_1^* a}{v_1},\quad \mathrm{T}'=\frac{T}{T_0}\quad \mathrm{P}'=\frac{\mathrm{P}}{\beta_0\mathrm{T}_0}\,. \tag{10}$$

$$\mathrm{t}'_{zz}=\frac{\mathrm{t}_{zz}}{\beta_0\mathrm{T}_0},\quad \mathrm{t}'_{zx}=\frac{\mathrm{t}_{zx}}{\beta_0\mathrm{T}_0},\quad \mathrm{h}'=\frac{\mathrm{h}v_1}{\omega_1^*}, \tag{11}$$

where $v_1=\left(\frac{\mathrm{c}_{110}}{\rho}\right)^{\frac{1}{2}}$ and $\omega_1^*=\frac{\mathrm{c}_{e0}\,\mathrm{c}_{110}}{\mathrm{K}_0}$ are, respectively, the velocity of compressional waves in x-direction and characteristic frequency of the medium.

Equations (1)-(3) with the help of equations (7)-(9), can be written in non-dimensional form as (dropping the dashes for convenience)

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Global Journal of Science Frontier Research (F) Volume XX Issue V Version I Year 2020

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$$\frac{\partial^2 u}{\partial x^2}+c_1\frac{\partial^2 u}{\partial z^2}+c_2\frac{\partial^2 w}{\partial x\partial z}-\frac{\partial}{\partial x}(T+\delta_{2k}\tau_1\frac{\partial T}{\partial t})=A^*\frac{\partial^2 u}{\partial t^2}\ , \tag{12}$$

$$c_1\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}+c_2\frac{\partial^2 u}{\partial x\partial z}-\frac{\partial}{\partial z}(T+\delta_{2k}\tau_1\frac{\partial T}{\partial t})=A^*\frac{\partial^2 w}{\partial t^2}\ , \tag{13}$$

Notes

$$\left(n^*+t_1\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)=(n_1\frac{\partial T}{\partial t}+\tau_o\frac{\partial^2 T}{\partial t^2})+\in_1(n_1\frac{\partial}{\partial t}+n_0\tau_o\delta_{1k}\frac{\partial^2}{\partial t^2})\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right), \tag{14}$$

For linear case $A^*=\dfrac{1}{(1-\alpha^*T_0)}$, for quadratic case $A^*=\dfrac{1}{(1-\alpha^*T_0)^2}$

and for exponential case $A^*=\dfrac{1}{e^{\alpha^* T_0}}$

Applying the Laplace and Fourier transforms

$$\hat f(x,z,p)=\int_0^\infty f(x,z,t)e^{-pt}dt \quad \text{and}$$

$$\tilde f(\xi,\ z,\ p)=\int_{-\infty}^{\infty}\hat f(x\ \ z\ \ p)e^{i\xi x}dx\ . \tag{15}$$

on equations (12) – (14) and eliminating $\tilde u,\tilde w,\tilde T$ from the resulting expressions, we obtain

$$(\nabla^6+Q\nabla^4+N\nabla^2+I)\quad(\tilde u,\tilde w,\tilde T)=0 \tag{16}$$

where

$$Q=-\frac{1}{c_1}\ [\ c_1\{(1+p\tau_1\delta_{2k})(p+\tau_o\delta_{1k}p^2)\in_1+(p+\tau_o p^2)+\xi^2+(p^2+\xi^2c_1)\ \}$$
$$+(p^2+\xi^2)+\xi^2c_2^2\ ]\ ,$$

$$N=\frac{1}{c_1}\Big[(p^2+\xi^2c_1)(p^2+\xi^2)+(p+\tau_0p^2)(p^2+\xi^2)+\xi^2(p^2+\xi^2)$$
$$+(p^2+\xi^2c_1)\{(p+\tau_0p^2)+\xi^2\}c_1+(1+p\tau_1\delta_{2k})(p+\tau_0\delta_{1k}p^2)\in_1$$
$$\{(p^2+\xi^2)-2\xi^2c_1+\xi^2\}\Big]\ ,$$

$$I=\frac{1}{c_1}\Big[\{(p+\tau_0p^2)+\xi^2\}+(p^2+\xi^2c_1)(p^2+\xi^2)-\xi^2(1+p\tau_1\delta_{2k})(p+\tau_0\delta_{1k}p^2)$$
$$\times\in_1(p^2+\xi^2c_1)\Big].$$

The roots of Eq. (16) are $\pm\lambda_i\ (i=1,2,3)$. Using regularity condition (6),the solutions of Eq. (16) may be written as

$$\tilde u=A_1e^{-\lambda_1 z}+A_2e^{-\lambda_2 z}+A_3e^{-\lambda_3 z}, \tag{17}$$

$$\tilde w=-(a_1A_1e^{-\lambda_1 z}+a_2A_4e^{-\lambda_2 z}+a_3A_6e^{-\lambda_3 z}), \tag{18}$$

$$\tilde T=b_1A_1e^{-\lambda_1 z}+b_2A_2e^{-\lambda_2 z}+b_3A_3e^{-\lambda_3 z}, \tag{19}$$

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where

$$a_i = \frac{\lambda_i(-Q^*\lambda_i^2 + R^*)}{P^*\lambda_i^2 + N^*} \quad , \qquad b_i = \frac{V^* + \lambda_i^2 W^*}{\lambda_i^2 - S^*}; \quad (i = 1, 2, 3),$$

$$P^* = (1 - c_2)\ , \quad N^* = (-p^2 - \xi^2 c_1), \quad R^* = (i\xi c_2 - M^*),$$

$$S^* = [\{\xi^2 + (p + \tau_0 p^2)\} + \frac{(1 + p\tau_1\delta_{2k})(p + \tau_0\delta_{1k}p^2)\in_1}{c_2}] \ ,Q^* = \frac{c_1}{i\xi}\ ,$$

$$W^* = \frac{c_1 \in_1 (p + \tau_0\delta_{1k}p^2)}{(i\xi)c_2}, \quad V^* = -(p + \tau_0\delta_{1k}p^2)\left[\frac{(p^2 + \xi^2)\in_1}{i\xi c_2} - i\xi\in_1\right],$$

and $M^* = -\left(\frac{p^2 + \xi^2}{i\xi}\right)$.

with $A_\ell (\ell = 1,2,3)$ being arbitrary constants.

IV. Application

a) *Instantaneous Load*

i. *Mechanical boundary conditions*

$$t_{zz}(x,z,t) = -P\psi(x)\delta(t),\ t_{zx}(x,z,t) = -P\zeta(x)\delta(t), \quad \frac{\partial T}{\partial z} + hT = 0 \text{ at } z = 0, \tag{20}$$

where $\delta(t)$ is the Dirac's delta function and $\psi(x)$, $\zeta(x)$ specify the vertical and horizontal source distribution functions, respectively, along x-axis. h is heat transfer coefficient.

Using equations (4),(10)-(11),(15), in the boundary conditions given by Eq. (20) and with the help of Eqs. (17) - (19), we obtain the expressions for displacement components, stresses and temperature distribution as

$$\tilde{u}(\xi, z, t) = -P_1\left[\tilde{\psi}(\xi)\left\{\Delta_1 e^{-\lambda_1 z} - \Delta_2 \bar{e}^{\lambda_2 z} + \Delta_3 e^{-\lambda_3 z}\right\} - \tilde{\zeta}(\xi)\left\{\Delta_4 e^{-\lambda_1 z} - \Delta_5 \bar{e}^{\lambda_2 z} + \Delta_6 e^{-\lambda_3 z}\right\}\right],$$

$$\tilde{w} = P_1\left[\tilde{\psi}(\xi)\left\{a_1\Delta_1 e^{-\lambda_1 z} - a_2\Delta_2 \bar{e}^{\lambda_2 z} + a_3\Delta_3 e^{-\lambda_3 z}\right\} + \tilde{\zeta}(\xi)\left\{a_1\Delta_4 e^{-\lambda_1 z} - a_2\Delta_5 \bar{e}^{\lambda_2 z} + a_3\Delta_6 e^{-\lambda_3 z}\right\}\right],$$

$$\tilde{t}_{zx} = -P_1 X^*\left[\tilde{\psi}(\xi)\left\{(i\xi a_1 - \lambda_1)\Delta_1 e^{-\lambda_1 z} - (i\xi a_2 - \lambda_2)\Delta_2 \bar{e}^{\lambda_2 z} + (i\xi a_3 - \lambda_3)\Delta_3 e^{-\lambda_3 z}\right\}\right.$$

$$\left. + \tilde{\zeta}(\xi)\left\{(i\xi a_1 - \lambda_1)\Delta_4 e^{-\lambda_1 z} - (i\xi a_2 - \lambda_2)\Delta_5 \bar{e}^{\lambda_2 z} + (i\xi a_3 - \lambda_3)\Delta_6 e^{-\lambda_3 z}\right\}\right],$$

$$\tilde{t}_{zz} = -P_1\left[\tilde{\psi}(\xi)\left\{p_1\Delta_1 e^{-\lambda_1 z} - p_2\Delta_2 \bar{e}^{\lambda_2 z} + p_3\Delta_3 e^{-\lambda_3 z}\right\} - \tilde{\zeta}(\xi)\left\{p_1\Delta_4 e^{-\lambda_1 z} - p_2\Delta_5 \bar{e}^{\lambda_2 z} + p_3\Delta_6 e^{-\lambda_3 z}\right\}\right],$$

$$\tilde{T} = P_1\left[\tilde{\psi}(\xi)\left\{b_1\Delta_1 e^{-\lambda_1 z} - b_2\Delta_2 \bar{e}^{\lambda_2 z} + b_3\Delta_3 e^{-\lambda_3 z}\right\} + \tilde{\zeta}(\xi)\left\{b_1\Delta_4 e^{-\lambda_1 z} - b_2\Delta_5 \bar{e}^{\lambda_2 z} + b_3\Delta_6 e^{-\lambda_3 z}\right\}\right], \tag{21}$$

where

$$\Delta = \Delta_1^* + h\Delta_2^*$$

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$$\Delta_1^* = X^*[-p_1\{b_2\lambda_2(i\xi a_3-\lambda_3)-b_3\lambda_3(i\xi a_2-\lambda_2)\}+p_2\{b_1\lambda_1(i\xi a_3-\lambda_3)-$$
$$b_3\lambda_3+(i\xi a_1-\lambda_1)\}+p_3\{b_1\lambda_1(i\xi a_2-\lambda_2)-b_2\lambda_2(i\xi a_1-\lambda_1)\}],$$

$$\Delta_2^* = X^*\ [-p_1\{b_3\ (i\xi a_2-\lambda_2)-b_2\ (i\xi a_3-\lambda_3)\}+p_2\{b_3\ (i\xi a_1-\lambda_1)-$$
$$b_1\ (i\xi a_3-\lambda_3)\}\ -p_3\{b_2\ (i\xi a_1-\lambda_1)-b_1\ (i\xi a_2-\lambda_2)\}],$$

Notes

$$\Delta_1 = X^*\left[\left\{b_2\lambda_2\left(i\xi a_3-\lambda_3\right)-b_3\lambda_3\left(i\xi a_2-\lambda_2\right)\right\}\right.$$
$$\left.+h\ \left\{b_3\left(i\xi a_2-\lambda_2\right)-b_2\left(i\xi a_3-\lambda_3\right)\right\}\ \right],$$

$$\Delta_2 = X^*\left[\left\{b_1\lambda_1\left(i\xi a_3-\lambda_3\right)-b_3\lambda_3\left(i\xi a_1-\lambda_1\right)\right\}\right.$$
$$\left.+h\left\{b_3\left(i\xi a_1-\lambda_1\right)-b_1\left(i\xi a_3-\lambda_3\right)\right\}\ \right],$$

$$\Delta_3 = X^*\left[\left\{b_1\lambda_1\left(i\xi a_2-\lambda_2\right)-b_2\lambda_2\left(i\xi a_1-\lambda_1\right)\right\}\right.$$
$$\left.+h\left\{b_2\left(i\xi a_1-\lambda_1\right)-b_1\left(i\xi a_2-\lambda_2\right)\right\}\ \right].$$

$$\Delta_4 = X^*[(p_3b_2\lambda_2-p_2b_3\lambda_3)+h\ (p_2b_3-p_3b_2)],$$

$$\Delta_5 = X^*\ [(p_3b_1\lambda_1-p_1b_3\lambda_3)+h\ (p_1b_3-p_3b_1)]\ ,$$

$$\Delta_6 = X^*\ [(p_2b_1\lambda_1-p_1b_2\lambda_2)+h\ (p_1b_2-p_2b_1)]\ ,$$

$$X^* = \frac{c_{440}}{\rho v_1^2}, \qquad P_1 = \frac{P}{\Delta},$$

$$p_n = \frac{1}{\rho v_1^2}\left(-i\xi c_{12}+c_{110}a_nb_n\right)-\beta b_n(1+p\tau_1\delta_{2k}),(n=1,2,3)\ .$$

Case I: Concentrated Force

In this case, we take

$$\psi(x)=\delta(x), \qquad \zeta(x)=\delta(x), \tag{22}$$

in equation (20).

Using the Laplace and Fourier transforms defined by equations (15) in equation (22), we get

$$\tilde{\psi}(\xi)=1, \qquad \tilde{\zeta}(\xi)=1, \tag{23}$$

where $\delta(x)$ is the Dirac delta function having the property

$$\int_{-\infty}^{\infty}\delta(x)dx=1 \tag{24}$$

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Case II: Uniformly Distributed Force

The solution due to uniformly distributed force applied on the half-space surface is obtained by setting

$$\{\psi(x),\zeta(x)\}=\begin{cases}1 & \text{if } |x|\le a,\\ 0 & \text{if } |x|>a,\end{cases}$$

in equation (20). Taking Laplace and Fourier transforms with respect to the pair (x,ξ), we obtain

$$\{\tilde{\psi}(\xi),\tilde{\zeta}(\xi)\}=\left[2\sin\left(\frac{\xi c_2 a}{\omega_1}\right)\Big/\xi\right],\ \xi\neq 0. \tag{25}$$

The expressions for displacement components, stresses and temperature distribution are obtained for concentrated force, and uniformly distributed force by replacing $\tilde{\psi}(\xi),\tilde{\zeta}(\xi)$ from equations (23),(25) respectively, in equation (20).

b) Thermal boundary conditions

$$t_{zz}=0,\qquad t_{zx}=0,\qquad \text{at } z=0$$

$$\frac{\partial T}{\partial z}=\eta(x)\delta(t)\quad \text{at } z=0,\ \text{for the temperature gradient boundary,}$$

or

$$T=\eta(x)\delta(t)\quad \text{at } z=0,\ \text{for the temperature input boundary,.} \tag{26}$$

Using equations (4), (10)-(11),(15), in the boundary conditions given by Eq. (26) and with the help of Eqs. (17) - (19), we obtain the expressions for displacement components, stresses and temperature distribution as

$$\tilde{u}=\tilde{\eta}(\xi)P_1\left(\Delta_1'\bar{e}^{\lambda_1 z}-\Delta_2'\bar{e}^{\lambda_2 z}+\Delta_3'\bar{e}^{\lambda_3 z}\right),$$

$$\tilde{w}=\tilde{\eta}(\xi)P_1\left(a_1\Delta_1'\bar{e}^{\lambda_1 z}-a_2\Delta_2'\bar{e}^{\lambda_2 z}+a_3\Delta_3'\bar{e}^{\lambda_3 z}\right),$$

$$\tilde{t}_{zx}(\xi,z,t)=\tilde{\eta}(\xi)P_1X^*\left[(i\xi a_1-\lambda_1)\Delta_1'\bar{e}^{\lambda_1 z}-(i\xi a_2-\lambda_2)\Delta_2'\bar{e}^{\lambda_2 z}\right.$$
$$\left.+(i\xi a_3-\lambda_3)\Delta_3'\bar{e}^{\lambda_3 z}\right],$$

$$\tilde{t}_{zz}(\xi,z,t)=\tilde{\eta}(\xi)P_1\left(p_1\Delta_1'\bar{e}^{\lambda_1 z}-p_2\Delta_2'\bar{e}^{\lambda_2 z}+p_3\Delta_3'\bar{e}^{\lambda_3 z}\right),$$

$$\tilde{T}(\xi,z,t)=\tilde{\eta}(\xi)P_1\left(b_1A_2\bar{e}^{\lambda_1 z}+b_2A_4\bar{e}^{\lambda_2 z}+b_3A_6\bar{e}^{\lambda_3 z}\right). \tag{27}$$

where

$$\Delta_1'=X^*\left[p_3(i\xi a_2-\lambda_2)-p_2(i\xi a_3-\lambda_3)\right],$$

$$\Delta_2'=X^*\left[p_3(i\xi a_1-\lambda_1)-p_1(i\xi a_3-\lambda_3)\right],$$

$$\Delta_3'=X^*\left[p_2(i\xi a_1-\lambda_1)-p_1(i\xi a_2-\lambda_2)\right].$$

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Notes

On replacing Δ by $(T_0\omega_1/v_1)\Delta_1^*$ and $T_0\Delta_2^*$ in Eq. (27), we obtain the expressions for temperature gradient boundary and temperature input boundary, respectively.

For temperature gradient boundary we replace and for temperature input boundary we take in Eq. (27).

Case I: Thermal Point Source

In this case

$$\eta(x) = \delta(x),$$

with

$$\tilde{\eta}(\xi) = 1 \tag{28}$$

Case II: Uniformly Distributed Thermal Source

Here

$$\eta(x) = \begin{cases} 1 & \text{if } |x| \le a, \\ 0 & \text{if } |x| > a, \end{cases}$$

with

$$\tilde{\eta}(\xi) = \left[2\sin\left(\frac{\xi c_2 a}{\omega_1}\right) \Big/ \xi \right], \; \xi \neq 0. \tag{29}$$

Replacing $\tilde{\eta}(\xi)$ from equations (28)-(29) in equation (27), we obtain the corresponding expressions for thermal point source and uniformly distributed thermal source, respectively.

c) Continuous Load

i. *Mechanical sources on the surface of half-space*

The boundary conditions in this case are

$$t_{zz}(x,z,t) = -P\psi(x)H(t), \; t_{zx}(x,z,t) = -P\zeta(x)H(t), \; \frac{\partial T}{\partial z} + hT = 0 \text{ at } z = 0, \tag{30}$$

where $H(t)$ is the Heaviside unit step function, P is the magnitude of the force, $\psi(x)$, $\zeta(x)$ specify the vertical and horizontal source distribution functions, respectively, along x-axis. h is heat transfer coefficient.

Adopting the same procedure of previous section (4.1a), using the boundary conditions (30), replacing $\Delta_\ell\,(\ell = 1,2,3......,8)$ *with* $\frac{\Delta_\ell}{p}(\ell = 1,2,3......,8)$, respectively, in equation (21), we obtain the corresponding expressions for the components of displacement, stresses and temperature distribution.

The corresponding expressions for concentrated force and uniformly distributed force are obtained by replacing $\tilde{\psi}(\xi), \tilde{\zeta}(\xi)$ and $\Delta_\ell\,(\ell = 1,2,3......,8)$ *with* $\frac{\Delta_\ell}{p}(\ell = 1,2,3......,8)$, from equations (23), (25) in equation (21), respectively.

ii. *Thermoelastic Interactions due to Thermal Source*

The boundary conditions in this case are

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$$t_{zz} = 0, \quad t_{zx} = 0, \qquad \text{at } z = 0$$

$\frac{\partial T}{\partial z}(x, z = 0) = \eta(x)H(t)$, for the temperature gradient boundary,

or

$$T(x, z = 0) = \eta(x)H(t), \text{ for the temperature input boundary,} \qquad (31)$$

Notes

Adopting the same procedure of previous section (4.1b), using the boundary conditions (31) and replacing $\Delta'(\ell = 1,2,3......,8)$ with $\frac{\Delta'}{p}(\ell = 1,2,3......,8)$, respectively, in equation (27), we obtain the corresponding expressions for the components of displacement, stresses and temperature distribution.

Replacing $\tilde{\eta}(\xi)$ from equations (28)-(29), in equation (27) we obtain the corresponding expressions for thermal point source and uniformly distributed thermal source respectively.

Sub-case 1: If h $\rightarrow$ 0, Eq. (21) yield the considered variables for the insulated boundary.

Sub-case 2: If h $\rightarrow\infty$, Eq. (21) yield considered variables for the isothermal boundary.

Particular Case

Taking

$$c_{11} = \lambda + 2\mu, \qquad c_{12} = \lambda, \qquad c_{44} = \mu$$

we obtain the corresponding expressions for the isotropic thermoelasic material.

V. Inversion of the Transforms

To obtain the solution of the problem in the physical domain, we must invert the transformed equations (21) and (27), for the four theories, i.e., L-S, G-L, G-N and CHT by using the method of inversion described by Kumar and Rani(2007).

VI. Numerical Result and Discussion

Following Dhaliwal and Singh (1980), we take the case of magnesium crystal-like material for numerical calculations. The physical constants used are:

ε = 0.0202, c_{11} = 5.974 x 10^{10} Nm^{-2}, c_{12}= 2.624 x 10^{10} Nm^{-2}, ρ = 1.74 x 10^3 kgm^{-3}, c_{44} = 3.278 x 10^{10} Nm^{-2}, c_e= 1.04 x 10^3 J $kg^{-1}degree^{-1}$ $\omega_1^* = 3.58x10^{11} s^{-1}$, K = 1.7 x 10^2 $Wm^{-1}degree^{-1}$, β= 2.68 x 10^6 $Nm^{-2}degree^{-1}$, P=1, P_1=1 T_0 = 298 ^{0} K.

The variations of normal boundary displacement *w* and boundary temperature field *T* with distance *x* at non-dimensional time t = 1.0 are shown graphically in figures 1-4, for L-S, for non-dimensional relaxation times $\tau_0 = 0.02$. The computations were carried out for time t=1.0 and α^* =0.00051 at z=1.0 in the range $0 \le x \le 10$. The solid lines (——), the small dashed lines (--------) and the long dashed lines (– – – –), in graphs represent the variations for Linear case, quadratic case and exponential case,

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respectively for L-S theory. The results for distributed thermal source are presented for dimensionless width a=1. The figures (1)-(4) are depicted for thermal source.

a) Instantaneous Load

i. *Thermal source on the surface of half-space (Temperature gradient boundary)*

a. *Thermal point source*

Notes

Figure 1. shows the variation of normal displacement 'w' with distance *x*. The values of normal displacement starts with sharp increase and then become oscillatory in the whole range for Linear case, quadratic case and exponential case. The values of normal displacement for linear and quadratic case are more than the exponential case in the whole range $0 \leq x \leq 10$.

Figure 2. depicts the variation of temperature distribution *T* with distance *x*. Initially the values of T start with sharp decrease and then become oscillatory about zero in the whole range for Linear case, quadratic case and exponential case. The values of 'T' shows appreciable effect for all the three cases.

b. *Uniformly Distributed Thermal Source*

Figure 3. depicts the variation of normal displacement w with distance *x* .The values of normal displacement in all the three cases start with sharp increase, the values show very small variation about zero in the whole range for linear and quadratic case. The values of normal displacement for exponential case are more than linear and quadratic case in the range $0 \leq x \leq 10$, which shows the appreciable effect of exponential case.

Figure 4. depicts the variation of temperature distribution *T* with distance *x*. At the point of application of source, the values of *T* decrease sharply for all the three cases. The values of T for linear and quadratic case are more than exponential case in the range $0.5 \leq x \leq 6$. In range $6 \leq x \leq 10$, the values of 'T' for quadratic case shows opposite oscillatory pattern in comparison to linear and exponential case.

VII. Conclusion

1. The comparison of linear and quadratic and exponential case has been depicted for L-S theory for temperature gradient boundary.
2. As 'x' diverse from the point of application of source the components of normal displacement and temperature are observed to follow small variations about zero in the range $1 \leq x \leq 10$ for instantaneous load.
3. The variations of normal displacement and temperature distribution for uniformly distributed thermal source are same as those of Thermal point source with difference in their magnitude for all the three cases.

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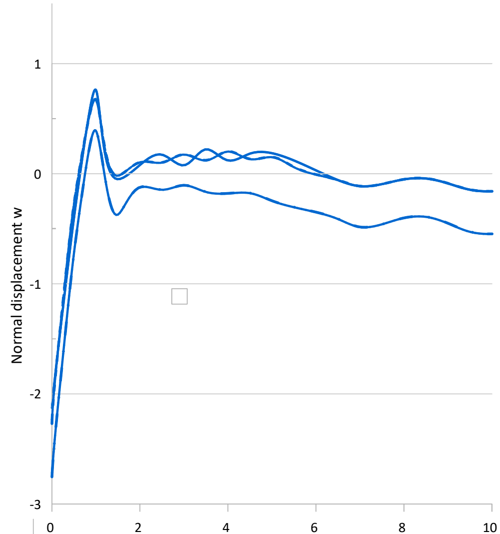

Figure 1: Variation of normal displacement w with distance x.

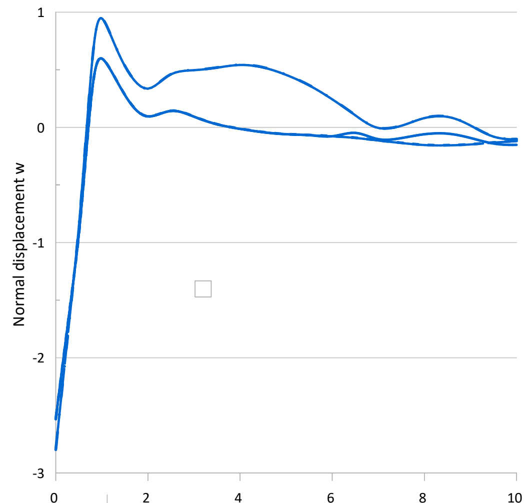

Figure 3: Variation of Normal displacement w with distance x.

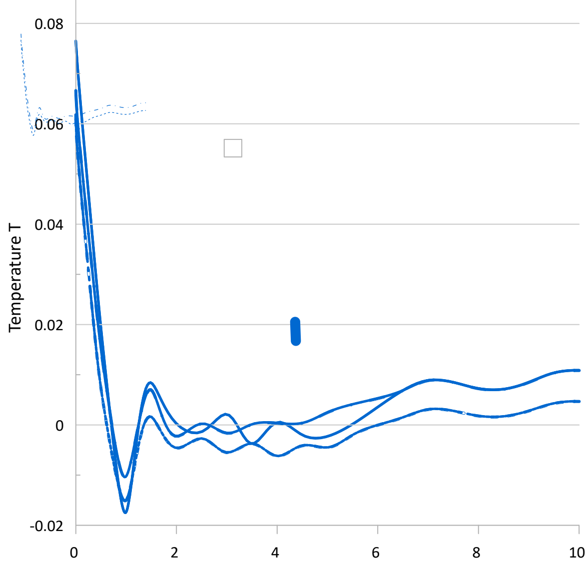

Figure 2: Variation of Temperature T with distance x.

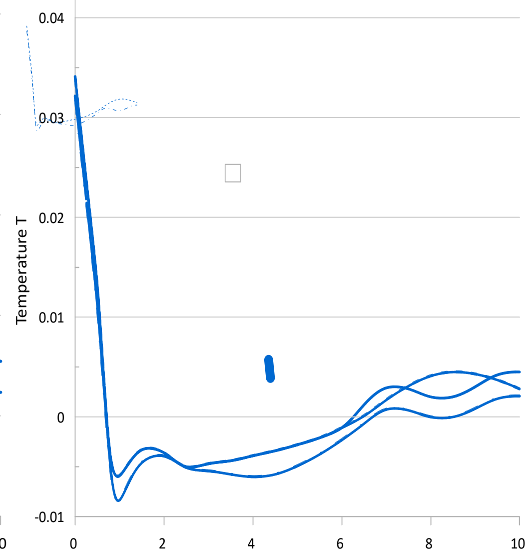

Figure 4: Variation of Temperature T with distance x.

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11. Pick a good study spot: Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

12. Know what you know: Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

13. Use good grammar: Always use good grammar and words that will have a positive impact on the evaluator; use of good vocabulary does not mean using tough words which the evaluator has to find in a dictionary. Do not fragment sentences. Eliminate one-word sentences. Do not ever use a big word when a smaller one would suffice.

Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

14. Arrangement of information: Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

15. Never start at the last minute: Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

16. Multitasking in research is not good: Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

17. Never copy others' work: Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

18. Go to seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

19. Refresh your mind after intervals: Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.

20. Think technically: Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

21. Adding unnecessary information: Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

22. Report concluded results: Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

23. Upon conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

The introduction: This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear: Adhere to recommended page limits.

Mistakes to avoid:

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

Title page:

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

Abstract: This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

Approach:

- o Single section and succinct.
- o An outline of the job done is always written in past tense.
- o Concentrate on shortening results—limit background information to a verdict or two.
- o Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

Introduction:

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.

The following approach can create a valuable beginning:

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

Approach:

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

Procedures (methods and materials):

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

Materials may be reported in part of a section or else they may be recognized along with your measures.

Methods:

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

What to stay away from:

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

Figures and tables:

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

Discussion:

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- Give details of all of your remarks as much as possible, focusing on mechanisms.
- Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

Describe generally acknowledged facts and main beliefs in present tense.

The Administration Rules

Administration Rules to Be Strictly Followed before Submitting Your Research Paper to Global Journals Inc.

Please read the following rules and regulations carefully before submitting your research paper to Global Journals Inc. to avoid rejection.

Segment draft and final research paper: You have to strictly follow the template of a research paper, failing which your paper may get rejected. You are expected to write each part of the paper wholly on your own. The peer reviewers need to identify your own perspective of the concepts in your own terms. Please do not extract straight from any other source, and do not rephrase someone else's analysis. Do not allow anyone else to proofread your manuscript.

Written material: You may discuss this with your guides and key sources. Do not copy anyone else's paper, even if this is only imitation, otherwise it will be rejected on the grounds of plagiarism, which is illegal. Various methods to avoid plagiarism are strictly applied by us to every paper, and, if found guilty, you may be blacklisted, which could affect your career adversely. To guard yourself and others from possible illegal use, please do not permit anyone to use or even read your paper and file.

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Please note that following table is only a Grading of "Paper Compilation" and not on "Performed/Stated Research" whose grading solely depends on Individual Assigned Peer Reviewer and Editorial Board Member. These can be available only on request and after decision of Paper. This report will be the property of Global Journals.

Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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ISSN 9755896

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