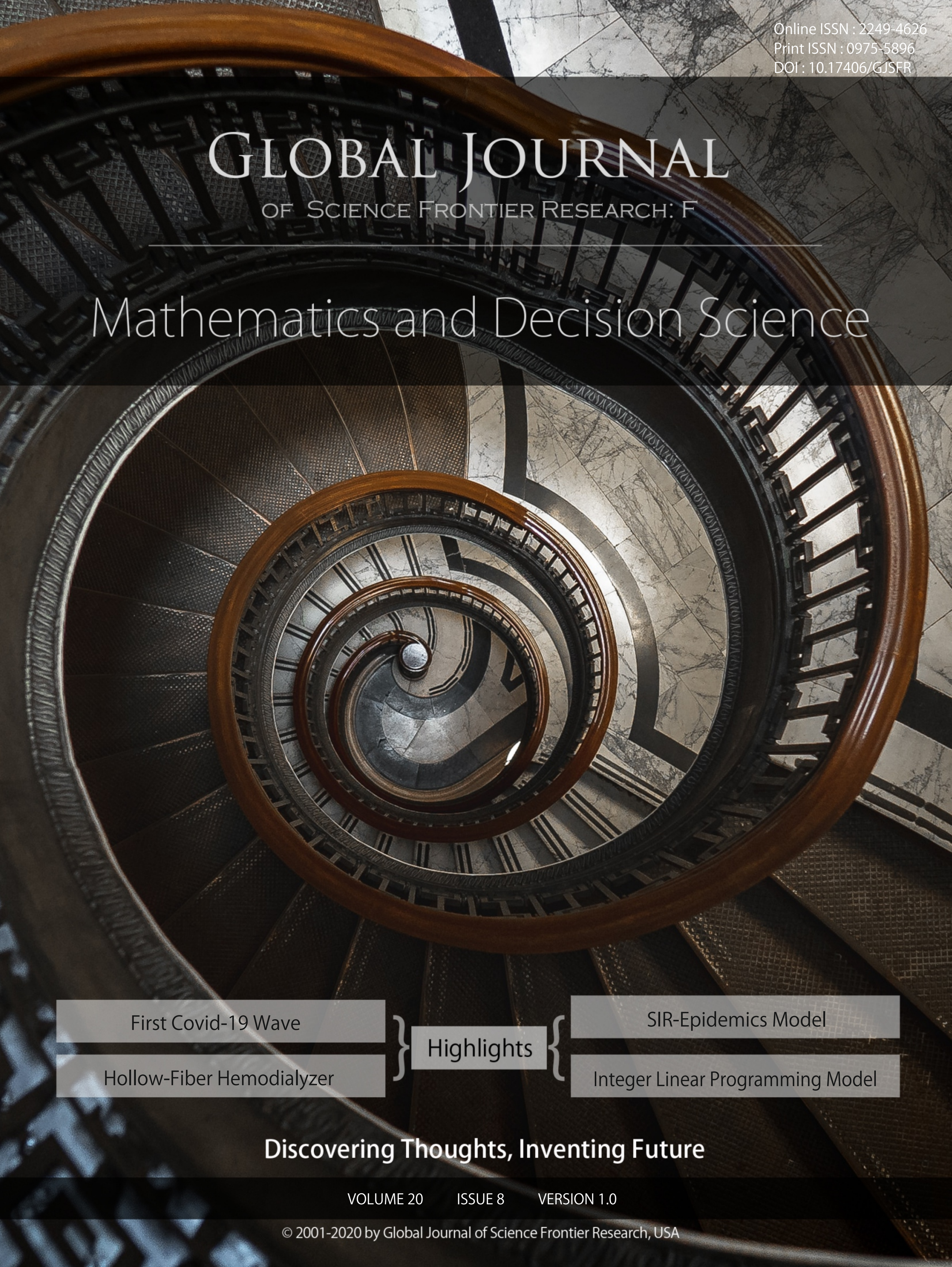
Online ISSN : 2249-4626
Print ISSN : 0975-5896
DOI : 10.17406/GJSFR



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Highlights

SIR-Epidemics Model

Integer Linear Programming Model

Discovering Thoughts, Inventing Future

VOLUME 20 ISSUE 8 VERSION 1.0

Global Journal of Science Frontier Research: F
Mathematics & Decision Sciences

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Volume 20 Issue 8 (Ver. 1.0)

Open Association of Research Society

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License No.: 42125/022010/1186
Registration No.: 430374
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Employer Identification Number (EIN):
USA Tax ID: 98-0673427

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Contents of the Issue

Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 20 Issue 8 Version 1.0 Year 2020
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Modified Integer Linear Programming Model for Bin-Packing Problems

By Niluka P. Rodrigo, W. B. Daundasekera & A. A. I. Perera
University of Peradeniya

Abstract- In this paper, our objective is to compare the solutions obtained from develop mathematical model and general model for Two-Dimensional Bin Packing Problem (*BPP*) with different sizes of boxes. It has become one of the most important mathematical applications throughout the time. In our study, Modified Branch and Bound Algorithm (*MBBA*) is developed to generate all the feasible packing patterns of given boxes to required containers for Two-Dimensional *BPP*. A computer program is developed using Matlab software package to generate feasible packing patterns and optimum packing plan. As a case study, three different sizes of rectangular shape packing items are selected with given demand in order to pack into two different sizes of pallets. Applying the proposed algorithm, demand is satisfied and total unused area significantly minimized. Accordingly, proposed mathematical model is more appropriate for medium size two dimensional *BPP*.

Keywords: bin-packing problem, modified branch and bound algorithm, integer linear programming model, matlab software environment.

GJSFR-F Classification: *MSC 2010: 15A86*

MODIFIEDINTEGERLINEARPROGRAMMINGMODELFORBINPACKINGPROBLEMS

Modified Integer Linear Programming Model for Bin-Packing Problems

Niluka P. Rodrigo [α], W. B. Daundasekera [σ] & A. A. I. Perera [ρ]

Abstract- In this paper, our objective is to compare the solutions obtained from develop mathematical model and general model for Two-Dimensional Bin Packing Problem (*BPP*) with different sizes of boxes. It has become one of the most important mathematical applications throughout the time. In our study, Modified Branch and Bound Algorithm (*MBBA*) is developed to generate all the feasible packing patterns of given boxes to required containers for Two-Dimensional *BPP*. A computer program is developed using Matlab software package to generate feasible packing patterns and optimum packing plan. As a case study, three different sizes of rectangular shape packing items are selected with given demand in order to pack into two different sizes of pallets. Applying the proposed algorithm, demand is satisfied and total unused area significantly minimized. Accordingly, proposed mathematical model is more appropriate for medium size two dimensional *BPP*.

Keywords: bin-packing problem, modified branch and bound algorithm, integer linear programming model, matlab software environment.

I. Introduction

Competence of production system becomes a key factor of the success in today's competitive manufacturing environment and industries. Productivity can be enhanced by minimizing waste, lead time and hence reducing the cost of manufacture. For that reason, Operations Research plays a major role in minimizing manufacture waste. Many people including scientists have contributed and engaged in their research and other activities to overcome above challenges. *BPP* is become an important way of optimization of resources in manufacturing and trading industries. *BPP* could be one-dimensional (1D), two-dimensional (2D) and three-dimensional (3D) packing which gives optimum layout of packing items such that to improve the utilization ratio of bins. In other word; minimizing the bin's slack without overlapping the packages. Output obtains by BPP optimize all the aspects of resources while meeting the given supply-demand by minimizing wastage leading to reduction of cost of production. The *BPP* can be interpreted as a finite collection of items with varying specifications to be packed into one or more box utilizing the maximum volume of the box while satisfying the supply-demand. Each container can hold any subset of the collection of objects without exceeding its capacity (customer requirements). Bin packing is known as container loading, box packing, cargo loading, knapsack, etc. A burning issue faced by the industries is how to find the optimum layout (packing arrangement) of boxes or packing items such that to improve the utilization ratio of bins or minimize the bin slack without overlapping the packages. Researchers have worked on the *BPP* and used different approaches to solve the existing problem.

Among them, Gilmore et al [1] conducted some of the earliest research in this area and one-dimensional cutting stock problem is solved using *Linear Programming Technique*. In this study, unlimited numbers of raw materials with different lengths are assumed available in stock, and a mathematical model has been developed to minimize

Ref

1. P. C. Gilmore and R. E. Gomory, "*A Linear Programming Approach to the Cutting Stock Problem*", Operations Research, Vol. 9, No. 2 (1961), 849 - 859.

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the total cutting cost of the stock length of the feasible cutting patterns and *Column Generation Algorithm* has been developed to generate feasible cutting patterns. Then, Gilmore has claimed that feasible cutting patterns are increased with the required cutting items and *Linear Programming Technique* is not applicable to solved mathematical model with too many variables. Gilmore [2] has made an approach for one-dimensional cutting stock problem as an extended of earliest paper (Gilmore et al (1961)) and cutting stock problem has been described as a NP-hard problem. A new and rapid algorithm for the knapsack problem and changes in the mathematical formulation1 has been evolved and Gilmore has explained the procedure of the *Knapsack Method* using a test problem. Solve the one dimensional *BPP* with island parallel grouping genetic algorithms was the main motivation of Dokeroglu T. et al (2014). Combining state-of-the-art computation tools; parallel processing, GGAs and bin oriented heuristics to efficiently solve the intractable one-dimensional BPP. Different size case studies discussed in the paper using Minimum Bin Slack (MBS) and Best Fit Decrease (BFD) / First Fit Decrease (FFD) [3].

Christian Blum and Verena Schmid (2013) have dealt with two-dimensional *BPP* under free guillotine cutting, a problem in which a set of oriented rectangular items are given which must be packed into minimum number of bins of equal size. An evolutionary algorithm has been discussed and the results of the proposed algorithm are compared with some of the best approaches from the literature [4]. Puchinger has described a combined genetic algorithm/branch & bound approach for solving a real world glass-cutting problem. The Genetic Algorithm uses an order-based representation, which is decoded using a greedy heuristic. The Branch & Bound algorithm was applied with a certain probability enhancing the decoding phase by generating locally optimal sub patterns. Reported results indicate that the approach of occasionally solving sub patterns to optimality may increase the overall solution quality [5]. Rodrigo et al (2012) developed an algorithm, based on Modified Branch and Bound algorithm to determine the feasible cutting patterns for Two-Dimensional cutting stock problem with rectangular shape cutting items. The method was illustrated with the use of a case study, where the data were obtained from a floor tile company known as Mega Marble Company located in London. A computer programme was coded using Matlab inbuilt functions [6]. As an extension of the above study, Rodrigo et al (2012) redesigned the developed algorithm to determine the locations of each cutting item using Cartesian Coordinate Geometry [7]. Other than that, Octarina et al (2019) has used Branch and Bound Algorithm and Gilmore and Gomary model on two dimensional multiple stock size cutting stock problem. A case study has discussed with four stock sizes and five products. Optimum packing schedule illustrated for each stock [8].

Further, Fernandez (2010) has presented a method to find Integer Solutions to One-dimensional CSP to minimize the trim loss or to minimize the number of master rolls needed to satisfy the orders. Integer linear programming models for the One-dimensional CSP with different objective cost functions have been considered. Fernandez has studied an approach based on the classical column-generation procedure by Gilmore and Gomary for solving the linear programming (LP) relaxations. Also to obtain an integer solution, a final integer CSP after using an extra column-generation procedure has been solved [9]. Also, Jatinder N. D. Gupta and Johnny C. (1999) have described a new Heuristic Algorithm to solve the one-dimensional Bin Packing problem. Effectiveness of the proposed algorithm has been compared with the First Fit Decreasing (FFD) and the Best Fit Decreasing (BFD) algorithms using five different data sets [10].

In this paper, packing items with rectangular shapes are selected to nest into pallets with rectangular shape base. An algorithm based on *MBBA* and a computer program using Matlab software package to generate feasible packing patterns and to solve the formulated integer linear programming model is used. Then, the optimum packing schedule obtained from *MILPP* with the results of *BBA* with Gilmore & Gomory for *BPP* are compared.

Ref

2. P. C. Gilmore and R. E. Gomory, "*A Linear Programming Approach To The Cutting Stock Problem – Part II*", Operations Research, Vol. 11, No. 6 (1963), 863 – 888.

II. Materials and Methods

a) *Bin-Packing Problem*

Any firm's main objective is to maximize the annual contribution margin accruing from its production and sales. By reducing wastages and maximizing sales, productivity can be improved. Two-dimensional *BPP* can be defined as packing rectangular shape boxes with known dimensions into the base of the container while minimizing the unused area in order to meet the given demand.

Notes

b) *Gilmore and Gomory Model*

A mathematical model is formulated based on the concept of packing patterns to satisfy the requirements of each item while minimizing the unused packing area or volume:

Mathematical Model (Gilmore and Gomory, 1961)

$$\text{Minimize} \quad z = \sum_{k=1}^{r} \sum_{j=1}^{n} c_{jk} x_{jk} \quad (\text{Total Unused area})$$

$$\text{Subject to} \quad \sum_{j=1}^{n} p_{ijk} x_{jk} \geq d_i \quad \text{for all } i = 1,2,\ldots,m \quad (\text{Demand Constraints})$$

$$x_{jk},\ p_{ijk} \geq 0 \text{ and integer for all } i, j, k.$$

where r = Number of containers,
m = Number of items
n = Number of patterns,
p_{ijk} = The number of occurrences of the i^{th} item in the j^{th} pattern of the k^{th} container,
c_{jk} = Unused area for each j^{th} pattern of the k^{th} container,
d_i = Demand of the i^{th} item,
x_{jk} = The number of containers used to be packed according to the j^{th} pattern.

According to the mathematical model developed by Gilmore and Gomory, total unused area is minimized with satisfying the customer requirements. Sometimes, it is impracticable to obtain the optimal packing schedule only minimizing the unused area. Because, the total number of pieces of each packing item should be satisfy at least the demand of each packing item. That is there can be more pieces than to the demand of packing items in the optimal packing schedule. In this context, allocating extra pieces of each packing item is wastage.

In order to rectify the above disparity, mathematical model has developed to meet the exact demand.

Modified Integer Linear Programming Problem (MILPP) for BPP:

$$\text{Minimize} \quad z = \sum_{k=1}^{r} \sum_{i=1}^{m} \sum_{j=1}^{n} \left[(a_{ijk} x_{jk}) - (d_i' x_{jk}) \right]$$

$$(\text{Total packing items used from each item})$$

$$\text{Subject to} \quad \sum_{j=1}^{n} a_{ijk} x_{jk} \geq d_i \quad \text{for all } i = 1,2,\ldots,m$$

$$(\text{Demand Constraints})$$

$$k = 1,2,\ldots,r$$

$$x_{jk},\ a_{ijk} \geq 0 \text{ and integer for all } i, j, k$$

where r = Number of containers,
m = Number of pieces,
n = Number of patterns,
a_{ijk} = The number of occurrences of the i^{ht} item in the j^{th} pattern of the k^{th} container,
c_{jk} = Unused packing area for each j^{th} pattern of the k^{th} container,
d_i = Demand value of the i^{th} item,

x_{jk} = The number of containers should be used according to the j^{th} pattern from the k^{th} container.

c) Modified Branch and Bound Algorithm in stepwise form:

Step 1: Arrange required lengths, Lk , $k = 1, 2, \ldots, r$ in decreasing order, ie $L_1 > L_2 > \ldots > L_r$, where r = number containers.

Arrange required widths, Wk, $k = 1, 2, \ldots, r$ according to the corresponding length Lk , $k = 1, 2, \ldots, r$.

Step 2: Arrange required lengths, l_i, $i = 1, 2, \ldots, m$ in decreasing order, ie $l_1 > l_2 > \ldots > l_m$, where m = number of packing items.

Arrange required widths, w_i, $i = 1, 2, \ldots, m$ according to the corresponding length l_i, $i = 1, 2, \ldots, m$.

Step 3: For $i = 1, 2,\ldots, m$ and $j = 1$ do Steps 4 to 6.

Step 4: Set $a_{111} = \left[\left[L_1 / l_1 \right]\right]$;

$$a_{ijk} = \left[\left[\left(L_k - \sum_{z=1}^{i-1} a_{zjk} l_z \right) \Big/ l_i \right]\right], \qquad (B.\ 1)$$

where L_k is the length of the base of the k^{th} container.

Here, a_{ijk} is the number of pieces of the i^{th} packing items in the j^{th} pattern of the k^{th} container along the length of the k^{ht} container and $[[y]]$ is the greatest integer les than or equal to y.

Step 5: If $a_{ijk} > 0$, then set

$$b_{ijk} = \left[\left[W_k / w_i \right]\right], \qquad (B.\ 2)$$

else set $b_{ijk} = 0$,
where W_k is the width of the base of the k^{ht} container.

Here, b_{ijk} is the number of pieces of the i^{th} packing item in the j^{th} pattern along the width of the base of the k^{th} container.

Step 6: Set $p_{ijk} = a_{ijk} \times b_{ijk}$,

where p_{ijk} is the number of pieces of the i^{th} item in the j^{th} pattern of the k^{th} container.

Step 7: Unused packing area

(i) Unused packing area along the length of the k^{ht} container:

$$c_{uk} = \left(L_k - \sum_{i=1}^{m} a_{ijk} l_i \right) \times W_k$$

For $i = 1, 2, \ldots, m$

If $\left(L_k - \sum_{i=1}^{m} a_{ijk} l_i \right) \geq w_i$ and $W_k \geq l_i$, then

Notes

(Considering 90° rotation for the given packing items)

$$\text{set } A_{ijk} = \left[\left[\left(L_k - \sum_{i=1}^{m} a_{ijk}\, l_i\right) \Big/ w_i \right]\right];$$

Notes

$$B_{ijk} = \begin{cases} \left[\left[W_k \big/ l_i \right]\right], & \text{if } A_{ijk} > 0 \\ 0, & \text{otherwise.} \end{cases}$$

$$p_{ijk} = p_{ijk} + \left(A_{ijk} \times B_{ijk}\right).$$

$$\text{else set } A_{ijk} = 0;$$

$$B_{ijk} = 0;$$

$$P_{ijk} = P_{ijk}.$$

If $A_{ijk} > 0$, then

$$\text{set } C_{uk} = \left[\left(L_k - \sum_{i=1}^{m} a_{ijk}\, l_i\right) - A_{ijk}\, w_i\right] \times B_{ijk}\, l_i;$$

$$C_{vk} = \left(L_k - \sum_{i=1}^{m} a_{ijk}\, l_i\right) \times \left(W_k - B_{ijk} l_i\right).$$

$$\text{else} \quad C_{uk} = \left(L_k - \sum_{i=1}^{m} a_{ijk}\, l_i\right) \times W_k,$$

where, A_{ijk} and B_{ijk} are the number of pieces of the i^{th} item in the j^{th} pattern of the k^{ht} container along the length and width of the c_{uk} rectangle respectively and C_{uk} and C_{vk} are the total unused area along the length and width of the c_{uk} rectangle respectively.

(ii) Unused area along the width of the container:

$$c_{vk} = \left(a_{ijk} \times l_i\right) \times K_{ijk}.$$

$$\text{Here, } K_{ijk} = W_k - \left(b_{ijk} \times w_i\right);$$

$$\text{If } \left(b_{ijk} \times w_i\right) = 0, \text{ then}$$

$$\text{set } K_{ijk} = 0,$$

where K_{ijk} is the remaining width of each item in each pattern.

For $z \neq i$

If $\left(a_{ijk} \times l_i\right) \geq l_z$ and $K_{ijk} \geq w_z$, then

$$\text{set } A_{zjk} = \left[\left[\left(a_{ijk} \times l_i\right) \Big/ l_z \right]\right];$$

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$$B_{zjk} = \begin{cases} \left[\left[\left(K_{ijk} / w_z\right)\right]\right], & \text{if } A_{zjk} > 0 \\ 0, & \text{otherwise.} \end{cases}$$

$$p_{zjk} = p_{zjk} + \left(A_{zjk} \times B_{zjk}\right)$$

$$\text{else set } A_{ijk} = 0;$$

$$B_{ijk} = 0;$$

$$P_{ijk} = P_{ijk}.$$

If $A_{zjk} > 0$, then

$$\text{set } C_{uk} = \left(\left(a_{ijk} \times l_i\right) - \left(A_{zjk} \times l_z\right)\right) \times B_{zjk}\, w_z;$$

$$C_{vk} = \left(a_{ijk} \times l_i\right) \times \left(K_{ijk} - \left(B_{zjk} \times l_z\right)\right).$$

$$\text{else } C_{vk} = \left(a_{ijk} \times l_i\right) \times K_{ijk},$$

where, A_{zjk} and B_{zjk} are the number of pieces of the i^{th} item in the j^{th} pattern of the k^{ht} container along the length and width of the c_{vk} rectangle respectively. C_u and C_v are the total unused area along the length and width of the c_{vk} rectangle respectively.

Step 8: Set $r = m - 1$.

While $r > 0$, do Step 9.

Step 9: While $a_{rjk} > 0$

set $j = j + 1$ and do Step 10.

Step 10: If $a_{rjk} \geq b_{rjk}$, then generate a new pattern according to the following conditions:

$$\text{For } z = 1, 2, \ldots, r-1$$

$$\text{set } a_{zjk} = a_{z(j-1)k};$$

$$b_{zjk} = b_{z(j-1)k}.$$

$$\text{For } z = r$$

$$\text{set } a_{zjk} = a_{z(j-1)k} - 1;$$

$$\text{if } a_{zjk} > 0, \text{ then set } b_{zjk} = \left[\left[W_k / w_z\right]\right];$$

$$\text{else set } b_{zjk} = 0.$$

For $z = r + 1, \ldots, m$

calculate a_{zjk} and b_{zjk} using Equations (2.3) and (2.4). Go to Step 5.

else generate a new pattern according to the following conditions:

$$\text{For } z = 1, 2, \ldots, r-1$$

$$\text{set } a_{zjk} = a_{z(j-1)k};$$

$$b_{zjk} = b_{z(j-1)k}.$$

$$\text{For } z = r$$

$$\text{set } a_{zjk} = a_{z(j-1)k};$$

$$b_{zjk} = b_{z(j-1)k} - 1.$$

Notes

For $z = r+1, \ldots, m$

calculate a_{zjk} and b_{zjk} using Equations (2.3) and (2.4).
Go to Step 6.

Step 11: Set $r = r - 1$.

Step 12: STOP.

Ref

8. Sisca Octarina, Vinny Ananda, Evi Yuliza, "Gilmore and gomory model on two dimensional multiple stock size cutting stock problem", Sriwijaya International Conference on Basic and Applied Science, J. Phys.: Conf. Ser. 1282 (2019) 012015. doi: 10.1088/1742-6596/1282/1/012015

d) Case Study

Proposed *MBBA* to solve one-dimensional BPP is tested and analyzed to determine feasible and optimal packing patterns. Following examples will illustrate how to generate feasible packing patterns by minimizing the total bin slack. Following data table represents two one-dimensional bin packing problem given in the paper [8] and proposed algorithm in this paper has been applied to solve those problems.

Table 1: The sizes of the stocks

i^{th} stock	Length (inches)	Width (inches)
1	24	14
2	24	13
3	18	10
4	13	10

Table 2: The sizes and demands of the products

i^{th} product	Length (inches)	Width (inches)	Number of Demand
1	8	5	2
2	7	4	3
3	5	3	2
4	4	2	5
5	2	1	5

III. Results and Discussion

MBBA is applied to the above examples to generate feasible packing patterns.

Effectiveness of the proposed *Integer Linear Programming Problem* has been compared with the results given in the paper [8].

Table 1: No of bins for problems given in the paper [8]

No of Containers from different algorithms with LP models / i^{th} stock	*MBBA* with *MILPP*	BBA with Gilmore & Gomory model
1	1	1
2	1	1
3	2	2
4	2	3

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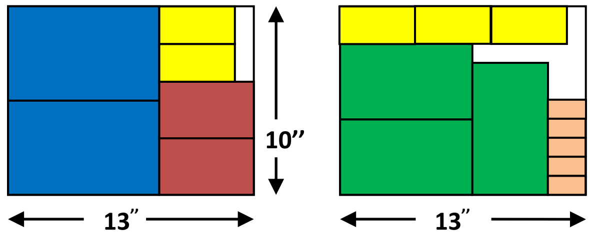

IV. Conclusion

In this study, feasible packing patterns are generated using *MBBA* and the integer linear programming model developed by Gilmore and Gomory and *MILPP* are applied to generate the optimum packing schedule. According to the optimum solution obtain from *MILPP*, only 2 plates of 13 inches 10 inches are used to fulfil the customer demand while 3 plates of 13 inches 10 inches are used with Gilmore and Gomory model. Comparing the solutions obtained from *MBBA* with *MILPP* and *BBA* with Gilmore and Gomory model, it is clear that the less number of plates needed to satisfy the customer demand using the proposed method in this research than to the *BBA* with Gilmore and Gomory model [8].

References Références Referencias

1. P. C. Gilmore and R. E. Gomory, "*A Linear Programming Approach to the Cutting Stock Problem*", Operations Research, Vol. 9, No. 2 (1961), 849 - 859.
2. P. C. Gilmore and R. E. Gomory, "*A Linear Programming Approach To The Cutting Stock Problem – Part II*", Operations Research, Vol. 11, No. 6 (1963), 863 – 888.
3. Dokeroglu T., Bayir M. A and Cosar A. "Integer linear programming solution for the multiple query optimization problem", Information Science and Systems, pp 51-60, Springer, Cham (2014). DOI: 10.1007/978-3-319-09465-6-6.
4. Christian B. and Verena S. "Solving the 2D bin packing problem by means of a Hybrid Evolutionary Algorithm", Elsevier, International Conference on Computational Science, ICCS (2013), 899-908.
5. Jakob Puchinger, Gunther R. Raidl and Gabriele Koller, "*Solving a Real-World Glass Cutting Problem*".
6. W. N. P Rodrigo, W.B. Daundasekera, A.A.I Perera "Pattern Generation for Two-Dimensional Cutting Stock Problem" International Journal of Mathematics Trends and Technology (IJMTT), Vol. 3 Issue2 (2012), 54-62.
7. Rodrigo WNP, Daundasekera WB, Perera AAI, "Pattern Generation for Two-Dimensional Cutting Stock Problem with Location", Indian Journal of Computer Science and Engineering, (IJCSE). 2012;3(2):354-368.
8. Sisca Octarina, Vinny Ananda, Evi Yuliza, "Gilmore and gomory model on two dimensional multiple stock size cutting stock problem", Sriwijaya International Conference on Basic and Applied Science, J. Phys.: Conf. Ser. 1282 (2019) 012015. doi: 10.1088/1742-6596/1282/1/012015
9. L. Fernjandez, L. A. Fernjandez, C. Pola, "*Integer Solutions to Cutting Stock Problems*", 2nd International Conference on Engineering Optimization, Sep 6-9, 2010.
10. Jatinder N. D Gupta and Johnny C. "A new heuristic algorithm for the one-dimensional bin-packing problem", Production planning and Control, ISSN 0953-7287 (1999), Vol. 10, No. 6, 598-603.

Ref

8. Sisca Octarina, Vinny Ananda, Evi Yuliza, "Gilmore and gomory model on two dimensional multiple stock size cutting stock problem", Sriwijaya International Conference on Basic and Applied Science, J. Phys.: Conf. Ser. 1282 (2019) 012015. doi: 10.1088/1742-6596/1282/1/012015

Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 20 Issue 8 Version 1.0 Year 2020
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Shock Propagation in a Hollow-Fiber Hemodialyzer

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Abstract- Hemodialysis (HD) is one type of procedure for eliminating toxic chemicals and infusing bicarbonate in patients with end-stage renal disease (ESRD). We have developed a comprehensive mathematical model to describe the dynamic exchange process of solutes in a prototype hemodialyzer. The model, which is represented by a coupled set of transport equations, delineates the blood and dialyzate compartments of the hemodialyzer, and includes bicarbonate-buffering reaction in the blood channel and bicarbonate replenishment mechanism in the dialyzate. In a paper submitted by the author, we ignored the inherent velocity discontinuity in the blood channel as the radius of the blood channel r approaches the semi-permeable membrane RB, that is, $r \to RB$. In this paper, we will investigate the evolution of bicarbonate and carbon dioxide in the blood compartment as the radius of the blood channel approaches the semi-permeable membrane. That is, we will investigate the solutions to the simplified form of the model in the blood compartment near the velocity shock vector, which manifests a discontinuity when $vz(r)=0$ of the simplified non-steady state model. We will investigate the cases of analytical solutions of the model in the blood channel with negligible diffusion and also shock solutions with diffusion.

Keywords: *hemodialysis; bicarbonate; dialyzate; hemodialyzer.*

GJSFR-F Classification: *MSC 2010: 00A05*

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Strictly as per the compliance and regulations of:

Shock Propagation in a Hollow-Fiber Hemodialyzer

Edward K. Boamah

Abstract- Hemodialysis (HD) is one type of procedure for eliminating toxic chemicals and infusing bicarbonate in patients with end-stage renal disease (ESRD). We have developed a comprehensive mathematical model to describe the dynamic exchange process of solutes in a prototype hemodialyzer. The model, which is represented by a coupled set of transport equations, delineates the blood and dialyzate compartments of the hemodialyzer, and includes bicarbonate-buffering reaction in the blood channel and bicarbonate replenishment mechanism in the dialyzate. In a paper submitted by the author, we ignored the inherent velocity discontinuity in the blood channel as the radius of the blood channel r approaches the semi-permeable membrane RB, that is, $r \to RB$. In this paper, we will investigate the evolution of bicarbonate and carbon dioxide in the blood compartment as the radius of the blood channel approaches the semi-permeable membrane. That is, we will investigate the solutions to the simplified form of the model in the blood compartment near the velocity shock vector, which manifests a discontinuity when $vz(r)=0$ of the simplified non-steady state model. We will investigate the cases of analytical solutions of the model in the blood channel with negligible diffusion and also shock solutions with diffusion.

Keywords: hemodialysis; bicarbonate; dialyzate; hemodialyzer.

I. Introduction

One of the leading goals of HD therapy, aside the elimination of electrolytes, toxic chemicals, and water, is correcting metabolic acidosis by the infusion of bicarbonate as a buffer from the dialyzate into the bloodstream in a prototype hemodialyzer. Normalization of metabolic acidosis sometimes results in, for example, metabolic alkalosis due to over compensation of the acidosis, causing dialysis symptoms such as mental confusion and muscle cramps [1], [2], [3]. Thus, it is of vital importance to obtain the closest if not the exact correction of metabolic acidosis during HD as time progresses.

Research and development in the hemodialyzer technology and HD therapy have depended mostly on empirical evidence. This is costly and often involves numerous clinical trials. In an experiment performed over 12 weeks, Ward et al. [5] were faced with the problem of how to control bicarbonate during HD therapy and that resulted in abnormally high pH value which puts the patient at risk of post-dialytic alkalosis. In an attempt to address the discrepancies in the experimental results, Gotch et al. [4], considered a black box input-output ordinary differential equation model to describe the mass balance of hydrogen ions during HD. Black box model in this case means the internal variables of the hollow fibers were not taken into consideration. Due to variations that were still observed in the experimental results of HD

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Ref

1. Ahmad S, Pagel M, Vizzo J and Scribner BH, Effect of the normalization of acid-base balance on postdialysis plasma bicarbonate, Trans Am Soc Artif Intern Organs 26 (1980) 318-340.

procedures, there have been increasing need to consider mathematical models. Monti et al. [6] also used a black box input-output ordinary differential equation model to describe the dynamic exchange process of bicarbonate during and after HD. This black-box model was oversimplified because the internal structure and variables of the prototype hemodialyzer unit were not taken into account. Thus, the model did not allow for the prediction of internal variables such as the surface area of the dialyzer, dialyzate flow rates, the thickness and permeability of the membrane [7], [8].

In order to go beyond black-box input-output ordinary differential equation models, and efficiently address the dynamic exchange of solutes in the prototype hemodialyzer, a comprehensive partial differential equation model incorporating internal properties, the dimensions and the surface area of the hollow fibers used during hemodialysis, the nature of solute transfer across the hemodialyzer semi permeable membrane, the hemodialysis flow rates and the duration of the administration of hemodialysis were taken into account. The knowledge gained from this study will eventually lead to a means of predicting the underlying mechanisms of solute transfer in a prototype hemodialyzer, thereby minimizing the need to perform costly and time-consuming clinical trials.

II. Model Description

a) Notation

x Species,
$x = 1: CO_2;\ x = 2: HCO_3^-;\ x = 3: H^+$
ϕ_x Concentration of species in the blood,
$\phi_{0,x}$ Initial concentration of species in the blood,
ψ_x Concentration of species in the dialysate,
$\psi_{0,x}$ Initial concentration of species in the dialysate,
v_z^B Velocity of blood in the axial direction,
v_z^D Velocity of dialyzate in the axial direction,
v_r^B Velocity of blood in the radial direction,
v_r^D Velocity of dialyzate in the radial direction,
v_w^B Wall velocity in the membrane-blood channel,
v_w^D Wall velocity in the membrane-dialyzate channel,
r_B Radius of the blood channel,
r_D Radius of the dialyzate channel,
t_m Membrane thickness,
r_m Sum of t_m and r_B,
S_B Blood-membrane Sherwood number,
S_D Dialyzate-membrane Sherwood number,
Pe_z^B Length Pecle′t number (blood side),
Pe_r^B Radial Pecle′t number (blood side),
Pe_z^D Length Pecle′t number (dialyzate side),
Pe_r^D Radial Pecle′t number (dialyzate side),
L Coaxial entrance length,
B_r Radius of the annulus,
P_x^m Membrane permeability of species,
U_z^B Maximum axial velocity of blood flow,

Ref

6. Monti JP, Sarrazin MY, Baz M, Murisasco A, Crevat AD and Elsen R, Kinetic modeling of intradialytic and interdialytic pH shifts during and after acetate and bicarbonate hemodialysis Int. J Artif. Organs 13 (1990) 799 – 802.

Notes

U_z^D Maximum axial velocity of dialyzate flow,
t_0 Dimensionless time scale,
P_e Pressure at the entrance of the annular,
P_L Pressure at the end of the annular,
$D_{x,r}$ Diffusion coefficient of species in the radial direction,
N_z Axial grid size,
N_r Radial grid size,
N_t Number of tubes,
Q_B Blood flow rate,
Q_D Dialyzate flow rate,
Q_u Ultrafiltration flow rate,
$n(z)$ Mass flux in the blood side channel,
$n(L-z)$ Mass flux in the dialyzate side channel,
B_x The nonlinear reactive term which deals with buffering of the blood,
R_x The replenishment term which replenishes bicarbonate concentration in the dialysate.

b) Model assumptions

We will use the following fundamental assumptions to formulate the models.

- We consider dilute, aqueous and Newtonian flow mechanism in the hemodialyzer.
- We consider permeability, diffusivity and density as constants.
- Axial diffusion was considered negligible.
- Flows were considered laminar.
- The flow mechanisms for the blood and the dialyzate are countercurrent.
- There are no angular gradients (axis-symmetry approximation).

The rate of change of species concentration in the blood and dialyzate channels are given as

$$\frac{\partial \phi_1}{\partial t} + v_z \frac{\partial \phi_1}{\partial z} = D_{r,1} \frac{\partial}{r\partial}\left(r \frac{\partial \phi_1}{\partial r}\right) + B_1 \ , \qquad (2.1)$$

and

$$v_z(r) = 2\bar{v}_z \left(1 - \left(\frac{r}{R}\right)^2\right) \qquad (2.2)$$

where $D_{r,1}$, $\bar{v}_z$ and v_z are the diffusion coefficient for partial pressure of carbon dioxide, the average velocity, and the axial velocity respectively. In equation (2.1), the second and third terms represent convection and diffusion effects. The initial and boundary conditions of (2.1) and (2.2) are given by

$$\phi_1 = \phi_{1,0} \text{ at } t = 0,\ z > 0; \qquad (2.3)$$

$$\phi_1 = \phi_{1,0} \text{ at } t > 0,\ z = 0. \qquad (2.4)$$

The condition of no flux at $r = 0$ requires that

$$\frac{\partial \phi_1}{\partial r} = 0. \qquad (2.5)$$

At the blood membrane interface, we write the expression of flux at the boundary $r = R_B$ in the form

$$\frac{-D_{r,1}\partial\phi_1}{\partial r} = P_1^m \phi_1 \ \forall z. \tag{2.6}$$

where T_r called the transmittance coefficient is the fraction of solutes that penetrate the membrane pores. Similarly, the convection-diffusion-reaction equation on the dialyzate side is given as

$$\frac{\partial\phi_2}{\partial t} + v_z\frac{\partial\phi_2}{\partial z} = D_{r,2}\frac{\partial}{r\partial}\left(r\frac{\partial\phi_2}{\partial r}\right) + B_2, \tag{2.7}$$

and

$$v_z(r) = 2\bar{v}_z\left(1 - \left(\frac{r}{R}\right)^2\right) \tag{2.8}$$

where $D_{r,1}, \bar{v}_z$ and v_z are the diffusion coefficient for bicarbonate, the average velocity, and the axial velocity respectively. In equation (2.7), the second and third terms represent convection and diffusion effects. The initial and boundary conditions of (2.7) and (2.8) are given by

$$\phi_2 = \phi_{2,0} \text{ at } t = 0,\ z > 0; \tag{2.9}$$

$$\phi_2 = \phi_{2,0} \text{ at } t > 0,\ z = 0. \tag{2.10}$$

The condition of no flux at $r = 0$ requires that

$$\frac{\partial\phi_2}{\partial r} = 0. \tag{2.11}$$

At the blood membrane interface, we write the expression of flux at the boundary $r = R_B$ in the form

$$\frac{-D_{r,2}\partial\phi_2}{\partial r} = P_2^m \phi_2 \ \forall z. \tag{2.12}$$

For simplicity, we assumed that radial convection is negligible. Since the radius of the blood channel is very small as compared to the length of the tube, we shall assume that the fluid is a fully-formed flow through a semi-infinite hollow fiber where $0 < z < \infty$ and $0 < r < R_B$.

c) Chemical reactions

The most important buffering reaction in the blood is the inter-conversion of CO_2 and HCO_3^-. These undergo the following reversible reaction (catalyzed in the presence of carbonic anhydrase (C.A) or uncatalyzed)

$$CO_2 + H_2O \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} HCO_3^- + H^+ \tag{Reaction 2.13}$$

where k_1 and k_{-1} are the forward and reversible reaction rate constants respectively. Chemically, (Reaction 2.13) forms a major buffer system of the blood. The net rate of reaction of species by chemical (reaction 2.13) per unit volume can be expressed as

Notes

$$B_x = k_1[CO_2] - k_{-1}[HCO_3^-][H^+] \qquad \text{(Reaction 2.14)}$$

The interrelationship between $pH, HCO_3^-, and\ pCO_2$ is then expressed by the Henderson-Hasselbalch equation as

$$pH = pK + log_{10}\left(\frac{[HCO_3^-]}{disolved\ CO_2}\right), \qquad (2.15)$$

where $(disolved\ [CO_2] = \alpha pCO_2)$ is the partial pressure of carbon dioxide and α is the solubility constant in $mmol/l$ [9]. The concentration of H^+ is obtained from the pH of the blood by the equation,

$$[H^+] = 10^{-pH} \qquad (2.16)$$

By (Reaction 2.13),

$$B_1 = -k_1[CO_2] + k_{-1}[HCO_3^-][H^+] \qquad (2.17)$$

and

$$B_2 = k_1[CO_2] - k_{-1}[HCO_3^-][H^+] \qquad (2.18)$$

d) Nondimensionalization

We nondimensionalize the models (2.1) to (2.18) by writing

$$\phi_1^* = \frac{\phi_1}{\phi_{1,0}}; \phi_2^* = \frac{\phi_2}{\phi_{2,0}}\ ;\ r^* = \frac{r}{B_r};\ z^* = \frac{z}{L};\ v_z^* = \frac{v_z}{\bar{v}_z};\ t_1^* = \frac{t}{t_{1,0}}; t_2^* = \frac{t}{t_{2,0}}\ ;\ t_{1,0} = \frac{B_r^2}{D_{1,r}};$$

$$t_{2,0} = \frac{B_r^2}{D_{2,r}};\ \ Pe_1^B = \frac{t_1\bar{v}_z}{L};\ Pe_2^B = \frac{t_2\bar{v}_z}{L}$$

For simplicity, we drop the asterisks to obtain the following nondimensional model and parameters. The dimensionless mass transport model in the blood channel is given as

$$\frac{\partial\phi_1}{\partial t} + Pe_1^B v_z \frac{\partial\phi_1}{\partial z} = D_{r,1}\frac{\partial}{r\partial}\left(r\frac{\partial\phi_1}{\partial r}\right) + t_1(k_{-1}\phi_2\phi_3 - k_1\phi_1), \qquad (2.13)$$

$$\frac{\partial\phi_2}{\partial t} + Pe_2^B v_z \frac{\partial\phi_2}{\partial z} = D_{r,2}\frac{\partial}{r\partial}\left(r\frac{\partial\phi_2}{\partial r}\right) - t_2(k_{-1}\phi_2\phi_3 - k_1\phi_1), \qquad (2.14)$$

$$v_z(r) = 2(1 - r^2)\ , \qquad (2.15)$$

$$B_1 = -t_0 k_1\left(\alpha\phi_1 - \frac{\phi_2}{10^{pH}K}\right) \qquad (2.16)$$

and

$$B_2 = k_1\left(\alpha\phi_1 - \frac{\phi_2}{10^{pH}K}\right) \qquad (2.17)$$

where $K = \frac{k_1}{k_{-1}}\ and\ B_3 = 0$. The concentration of hydrogen ions in the blood in (2.13) and (2.14) may be simplified as

Ref

9. Sargent JA and Gotch FA, Bicarbonate and carbon dioxide transport during hemodialysis, ASAIO Journal 2, (1979) 61 – 72.

$$\phi_3 = \frac{\alpha\phi_1}{10^{pK}\phi_2}. \tag{2.18}$$

Thus, the reaction terms containing ϕ_3 are simplified as

$$\pm t_{1,2}(k_{-1}\phi_2\phi_3 - k_1\phi_1) = \pm\beta_{1,2}\phi_1, \tag{2.19}$$

where $\pm t_{1,2}k_1\left(\frac{\alpha-10^{pK}K}{10^{pK}K}\right)$. The dimensionless, initial, boundary and no-flux conditions and the interfacial membrane relations are given as follows:

Boundary conditions:

$$\phi_1(z,r,0) = \phi_2(z,r,0) = 1,\ z > 0, \tag{2.20}$$

$$\phi_1(0,r,t) = \phi_2(0,r,t) = 1,\ t > 0, \tag{2.21}$$

No-flux conditions:

$$\frac{\partial\phi_1(z,0,t)}{\partial r} = \frac{\partial\phi_2(z,0,t)}{\partial r} = 0, \tag{2.22}$$

Interfacial membrane relations:

$$-\frac{D_{r,1}\partial\phi_1(z,1,t)}{\partial r} = sh_1\phi_1 \tag{2.23}$$

$$-\frac{D_{r,2}\partial\phi_1(z,1,t)}{\partial r} = sh_2\phi_2 \tag{2.24}$$

where $sh_1 = \frac{P_{m,1}R_B}{D_{r,1}}$ and $sh_2 = \frac{P_{m,2}R_B}{D_{r,2}}$. The dimensionless model in (2.13) to (2.15) may be written as.

$$\frac{1}{Pe_1^B}\frac{\partial\phi_1}{\partial t} + v_z\frac{\partial\phi_1}{\partial z} = D_{r,1}\frac{\partial}{Pe_1^B r\partial}\left(r\frac{\partial\phi_1}{\partial r}\right) + \frac{\beta\phi_1}{Pe_1^B}, \tag{2.25}$$

$$\frac{1}{Pe_2^B}\frac{\partial\phi_2}{\partial t} + v_z\frac{\partial\phi_2}{\partial z} = D_{r,2}\frac{\partial}{Pe_2^B r\partial}\left(r\frac{\partial\phi_2}{\partial r}\right) - \frac{\beta\phi_1}{Pe_2^B}, \tag{2.26}$$

with the initial and boundary conditions as given in (2.20) and (2.21).

III. THE SOLUTION WITHOUT DIFFUSION

As a first approximation, we neglect the diffusion and the time derivative terms in (2.25) and (2.26) since $\frac{1}{Pe_1^B} = \frac{1}{Pe_2^B} \approx 1$ and so the reduced equations are given as

Notes

Notes

$$v_z \frac{\partial \phi_1}{\partial z} = \frac{\beta \phi_1}{Pe_1^B}, \tag{2.27}$$

$$v_z \frac{\partial \phi_2}{\partial z} = \frac{\beta \phi_1}{Pe_2^B}, \tag{2.28}$$

together with the given boundary conditions. Since v_z is a function of only r, we may use the transformation

$$X = \frac{zt}{v_z(r)}, \tag{2.29}$$

and so equations (2.27) and (2.28) become

$$\frac{\partial \phi_1}{\partial X} = \frac{\beta}{tPe_1^B}\phi_1, \tag{2.30}$$

$$\frac{\partial \phi_2}{\partial X} = \frac{\beta}{Pe_2^B}\phi_1, \tag{2.31}$$

$$\phi_1(0, r, t) = 1, t > 0 \text{ becomes } \phi_1 = 1 \text{ at } X = 0,$$

$$\phi_2(0, r, t) = 1, t > 0 \text{ becomes } \phi_2 = 1 \text{ at } X = 0.$$

The solutions to the problems in (2.30) to (2.31) are given as

$$\phi_1 = exp\left(\frac{\beta}{tPe_1^B}X\right) = exp\left(\frac{\beta z}{v_z Pe_1^B}\right) \tag{2.32}$$

$$\phi_2 = 1 + \frac{Pe_1^B}{Pe_2^B}\left(1 - exp\left(\frac{\beta}{tPe_1^B}X\right)\right) = 1 + \frac{Pe_1^B}{Pe_2^B}\left(1 - exp\left(\frac{\beta z}{v_z Pe_1^B}\right)\right) \tag{2.33}$$

The solutions in (2.30) and (2.31) satisfy the boundary conditions on $X = 0$, but physically, this is not significantly interesting. We are rather interested in the observation away from $z = 0$.

IV. The Solution with Diffusion

Using the approximation, $\frac{1}{Pe_1^B} \approx \frac{1}{Pe_2^B} \approx \frac{\beta}{Pe_1^B} \approx \frac{\beta}{Pe_2^B} = \varepsilon \ll 1$, we retain the diffusion terms in (2.25) and (2.26) and perform the computational analysis to examine the behavior of species concentrations near the membrane boundary. We may write,

$$\frac{1}{Pe_1^B}\frac{\partial \phi_1}{\partial t} + v_z \frac{\partial \phi_1}{\partial z} = D_{r,1}\frac{\partial}{Pe_1^B r \partial}\left(r\frac{\partial \phi_1}{\partial r}\right) + \frac{\beta \phi_1}{Pe_1^B}, \tag{2.34}$$

$$\frac{1}{Pe_2^B}\frac{\partial \phi_2}{\partial t} + v_z \frac{\partial \phi_2}{\partial z} = D_{r,2}\frac{\partial}{Pe_2^B r \partial}\left(r\frac{\partial \phi_2}{\partial r}\right) - \frac{\beta \phi_1}{Pe_2^B}, \tag{2.35}$$

where $\frac{1}{Pe_1^B} \approx \frac{1}{Pe_2^B} \approx \frac{\beta}{Pe_1^B} \approx \frac{\beta}{Pe_2^B} \ll 1$

We require that the solutions for (2.34) and (2.35) satisfy the given dimensionless boundary conditions. Now, we study the behavior of solute transfer near the shock $z = 2(1 - r^2)t$. Thus, we introduce the new variables

$$\xi = z - v_z(r)t, \;\; \eta = r, \;\; \tau = t \tag{2.36}$$

Notes

to transform the differential equations in (2.34) and (2.35) as follows:

$$\frac{\partial \phi_1}{\partial t} = \frac{\partial \xi}{\partial t}\frac{\partial \phi_1}{\partial \xi} + \frac{\partial \eta}{\partial t}\frac{\partial \phi_1}{\partial \eta}, + \frac{\partial \tau}{\partial t}\frac{\partial \phi_1}{\partial \tau} \tag{2.37}$$

$$\frac{\partial \phi_1}{\partial t} = -v_z(\eta)\frac{\partial \phi_1}{\partial \xi} + \frac{\partial \phi_1}{\partial \tau} \tag{2.38}$$

Also,

$$\frac{\partial \phi_1}{\partial x} = \frac{\partial \xi}{\partial x}\frac{\partial \phi_1}{\partial \xi} + \frac{\partial \eta}{\partial x}\frac{\partial \phi_1}{\partial \eta}, + \frac{\partial \tau}{\partial x}\frac{\partial \phi_1}{\partial \tau} \tag{2.39}$$

$$v_z(r)\frac{\partial \phi_1}{\partial x} = v_z(\eta)\frac{\partial \phi_1}{\partial \xi} \tag{2.40}$$

and finally,

$$\frac{\partial \phi_1}{\partial r} = \frac{\partial \xi}{\partial r}\frac{\partial \phi_1}{\partial \xi} + \frac{\partial \eta}{\partial r}\frac{\partial \phi_1}{\partial \eta}, + \frac{\partial \tau}{\partial r}\frac{\partial \phi_1}{\partial \tau} \tag{2.41}$$

$$\frac{\partial \phi_1}{\partial r} = 2\eta\tau\frac{\partial \phi_1}{\partial \xi} + \frac{\partial \phi_1}{\partial \eta}, \tag{2.42}$$

from which it follows that

$$\frac{\partial^2 \phi_1}{\partial r^2} = 4\tau^2\eta^2\frac{\partial^2 \phi_1}{\partial \xi^2} + 4\tau\eta\frac{\partial^2 \phi_1}{\partial \xi \partial \eta} + 2\tau\frac{\partial \phi_1}{\partial \xi} + \frac{\partial^2 \phi_1}{\partial \eta^2}. \tag{2.43}$$

Substituting (2.38), (2.40), (2.42) and (2.43) into (2.34) to (2.35) and simplifying, we obtain

$$\frac{\partial \phi_1}{\partial \tau} = \frac{1}{Pe_1^B}\left(4\tau^2\eta^2\frac{\partial^2 \phi_1}{\partial \xi^2} - 4\tau\eta\frac{\partial^2 \phi_1}{\partial \xi \partial \eta} - 4\tau\frac{\partial \phi_1}{\partial \xi} + \frac{\partial \phi_1}{\eta \partial \xi} + \frac{\partial^2 \phi_1}{\partial \eta^2}\right) + \frac{\beta \phi_1}{Pe_1^B} \tag{2.44}$$

$$\frac{\partial \phi_2}{\partial \tau} = \frac{1}{Pe_2^B}\left(4\tau^2\eta^2\frac{\partial^2 \phi_2}{\partial \xi^2} - 4\tau\eta\frac{\partial^2 \phi_2}{\partial \xi \partial \eta} - 4\tau\frac{\partial \phi_2}{\partial \xi} + \frac{\partial \phi_2}{\eta \partial \xi} + \frac{\partial^2 \phi_2}{\partial \eta^2}\right) + \frac{\beta \phi_2}{Pe_2^B} \tag{2.45}$$

For simplicity, we transform equations (2.44) and (2.45) by using

$$\xi = \frac{X}{\sqrt{Pe^B_{1,2}}} \tag{2.46}$$

and use the outer solutions from the previous section so that

$$\phi_1 = \phi_2 = 1 \tag{2.47}$$

Notes

and

$$\phi_1 = exp\left(\frac{\beta}{\tau Pe_1^B}X\right), \quad \phi_2 = 1 - \frac{Pe_1^B}{Pe_2^B}\left(1 - exp\left(\frac{\beta}{\tau Pe_1^B}X\right)\right), \quad \xi > 0 \tag{2.48}$$

This choice of transformation in (2.46) helps to simplify and remove fractions from the system. We substitute (2.46) into equations (2.44) and (2.45) and simplify to obtain

$$\frac{\partial \phi_1}{\partial \tau} = 4\tau^2\eta^2 \frac{\partial^2 \phi_1}{\partial X^2} + \varepsilon\phi_1 + O\left(\mu, \frac{1}{\sqrt{Pe_1^B}}\right) \tag{2.49}$$

$$\frac{\partial \phi_2}{\partial \tau} = 4\tau^2\eta^2 \frac{\partial^2 \phi_2}{\partial X^2} - \varepsilon\phi_1 + O\left(\mu, \frac{1}{\sqrt{Pe_2^B}}\right) \tag{2.50}$$

The following are the associated boundary conditions:

$$\phi_1 \to 1 \text{ as } X \to -\infty \tag{2.51}$$

$$\phi_1 \to exp\left(\frac{\beta}{\tau Pe_1^B}X\right) \text{ as } X \to +\infty \tag{2.52}$$

$$\phi_1 \to 1 \text{ as } \tau \to 0, \tag{2.53}$$

and

$$\phi_2 \to 1 \text{ as } X \to -\infty \tag{2.54}$$

$$\phi_2 \to 1 - \frac{Pe_1^B}{Pe_2^B}\left(1 - exp\left(\frac{\beta}{\tau Pe_1^B}X\right)\right) \text{ as } X \to +\infty \tag{2.55}$$

$$\phi_2 \to 1 \text{ as } \tau \to 0\,. \tag{2.56}$$

We may now apply the method of multiple scales (Nayfeh, 1973) to solve (2.49) to (2.56) with $\varepsilon \ll 1$ and $\mu \ll 1$. We introduce the fast time $T_0 = t$ and the slow time $T = \varepsilon t$ for (2.49) and (2.50). Expanding the solutions to (2.49) and (2.50) as

$$\phi_{1,2}(X,T)=\phi_{1,2}^{(0)}(X,T_0,T_1)+\ \phi_{1,2}^{(1)}(X,T_0,T_1)+\cdots, \tag{2.57}$$

and substituting $\phi_1^{(0)}$ will yield

$$\frac{\partial\phi_{1,2}^{(0)}}{\partial T_0}=4\eta^2T_0^2\frac{\partial^2\phi_{1,2}^{(0)}}{\partial X^2}, \tag{2.58}$$

$$\phi_{1,2}^{(0)}\to 1 \text{ as } X\to-\infty, \tag{2.59}$$

$$\phi_{1,2}\to 1+exp\left(\frac{\beta}{Pe_1^B}X\right),\ 1-\frac{Pe_1^B}{Pe_2^B}\left(1-exp\left(\frac{\beta}{Pe_1^B}X\right)\right) \text{ as } X\to+\infty, \tag{2.60}$$

$$\phi_{1,2}^{(0)}\to 1 \text{ as } T_0,T_1\to-\infty. \tag{2.61}$$

We define a new variable that combines both X and T_0 in the form

$$\rho=\frac{\sqrt{3}}{4\eta T_0^{\frac{3}{2}}}X, \tag{2.62}$$

and then convert the derivatives of $\frac{\partial}{\partial\tau}$ and $\frac{\partial}{\partial X}$ to $\frac{\partial}{\partial T_0}$ and $\frac{\partial}{\partial\rho}$ respectively to obtain

$$\frac{\partial\phi_{1,2}^{(0)}}{\partial T_0}=-\frac{3\sqrt{3}}{8\eta T_0^{\frac{5}{2}}}X\frac{\partial\phi_{1,2}}{\partial\rho}, \tag{2.63}$$

and

$$\frac{\partial^2\phi_2}{\partial X^2}=-\frac{3}{16\eta^2T_0^3}X\frac{\partial^2\phi_{1,2}}{\partial\rho^2}. \tag{2.64}$$

Substituting, (2.58) will become

$$-2\left(\frac{X\sqrt{3}}{4\eta T_0^{\frac{3}{2}}}\right)\frac{\partial\phi_{1,2}}{\partial\rho}=\frac{\partial^2\phi_{1,2}}{\partial\rho^2}, \tag{2.65}$$

which is simplified to the form

$$\frac{\partial^2\phi_{1,2}}{\partial\rho^2}+2\rho\frac{\partial\phi_{1,2}}{\partial\rho}=0, \tag{2.65}$$

Since was defined such that $\rho=\frac{\sqrt{3}}{4\eta T_0^{\frac{3}{2}}}X$. Here, we note that the original differential equation for $\phi_{1,2}(\tau,X)$ has been transformed to an equation for $\phi_{1,2}(\rho)$ with boundary conditions given as follows:

Notes

$$\phi_{1,2}^{(0)} \to 1 \text{ as } X \to -\infty \text{ will become } \phi_{1,2}^{(0)} \to 1 \text{ as } \rho \to \infty \quad (2.66)$$

$$\phi_{1,2}^{(0)} \to 1 \text{ as } T_0, T_1 \to -\infty \text{ will become } \phi_{1,2}^{(0)} \to 1 \text{ as } \rho \to \infty \quad (2.67)$$

Notes

and equation (2.60) will also be transformed to the corresponding matching condition. We note that the two boundary conditions in τ and X collapse to a single boundary condition on ρ. In summary, we have transformed the partial differential equations and the corresponding boundary conditions to the form

$$\frac{\partial^2 \phi_{1,2}^{(0)}}{\partial \rho^2} + 2\rho \frac{\partial \phi_{1,2}^{(0)}}{\partial \rho} = 0, \quad (2.68)$$

$$\phi_{1,2}^{(0)} \to 1 \text{ as } \rho \to \infty, \quad (2.69)$$

$$\phi_{1,2}^{(0)} \to 1 + exp\left(\frac{\beta}{tPe_1^B} X\right), 1 - \frac{Pe_1^B}{Pe_2^B}\left(1 - exp\left(\frac{\beta}{Pe_1^B} X\right)\right) \text{ as } X \to \infty, \quad (2.70)$$

Since the only variable appearing in the partial differential equation is ρ, we conclude that $\phi_{1,2}^{(0)} = \phi_{1,2}^{(0)}(\rho)$ and the new equation is an ordinary differential equation of the form

$$\frac{d^2 \phi_{1,2}^{(0)}}{\partial \rho^2} + 2\rho \frac{d\phi_{1,2}^{(0)}}{d\rho} = 0 \quad (2.71)$$

with the given boundary conditions. We solve (2.71) by denoting and substituting $\frac{d\phi_{1,2}^{(0)}}{d\rho} = f$ to obtain

$$\frac{df}{d\rho} + 2\rho f = 0. \quad (2.72)$$

Solving (2.72), we obtain

$$f = f_0 e^{-\rho^2} \text{ and so } \frac{d\phi_{1,2}}{d\rho} = f_0 e^{-\rho^2} \quad (2.73)$$

which implies

$$\phi_{1,2} = \frac{f_0}{\sqrt{\pi}} \int_{-\infty}^{\rho} e^{-\lambda^2}\, d\lambda \quad (2.74)$$

$$\phi_{1,2} = \frac{f_{0,1,2}}{\sqrt{\pi}} \int_{-\infty}^{\frac{\sqrt{3}X}{4\eta T_0^{3/2}}} e^{-\lambda^2}\, d\lambda \quad (2.75)$$

We apply the corresponding boundary conditions and rewrite (2.75) in terms of original variables to obtain first order solutions that are uniformly valid in the forms

$$\phi_1 = \frac{1+exp\left(\frac{\beta}{Pe_1^B}X\right)}{\sqrt{\pi}} \int_{-\infty}^{\frac{\sqrt{3Pe_1^B}(z-v_z t)}{4rt^{3/2}}} e^{-\lambda^2} d\lambda + O\left(\varepsilon, 1/\sqrt{Pe_1^B}, \mu\right) \quad (2.76)$$

$$\phi_1 \approx \frac{1+exp\left(\frac{\beta}{Pe_1^B}X\right)}{\sqrt{\pi}} \left(\int_{-\infty}^{0} e^{-\lambda^2} d\lambda + \int_{0}^{\frac{\sqrt{3Pe_1^B}(z-v_z t)}{4rt^{3/2}}} e^{-\lambda^2} d\lambda \right), \quad (2.77)$$

$$= \frac{1+exp\left(\frac{\beta}{Pe_1^B}X\right)}{2\sqrt{\pi}} \left(\sqrt{\pi} + \frac{2}{\sqrt{\pi}} \int_{0}^{\frac{\sqrt{3Pe_1^B}(z-v_z t)}{4rt^{3/2}}} e^{-\lambda^2} d\lambda \right), \quad (2.78)$$

$$\phi_1 = \frac{1+exp\left(\frac{\beta}{Pe_1^B}X\right)}{2\sqrt{\pi}} \left(\sqrt{\pi} + erf\left(\sqrt{3Pe_1^B} \left(\frac{Z-V_z t}{4rt^{3/2}} \right) \right) \right), \quad (2.79)$$

Similarly,

$$\phi_2 = \left(\frac{Pe_1^B + Pe_2^B}{Pe_2^B} \right) \frac{1+exp\left(\frac{\beta}{Pe_1^B}X\right)}{\sqrt{\pi}} \int_{-\infty}^{\frac{\sqrt{3Pe_1^B}(z-v_z t)}{4rt^{3/2}}} e^{-\lambda^2} d\lambda + O\left(\varepsilon, 1/\sqrt{Pe_2^B}, \mu\right) \quad (2.80)$$

which implies

$$\phi_2 \approx \left(\left(\frac{Pe_1^B + Pe_2^B}{Pe_2^B} \right) \left(\frac{1+exp\left(\frac{\beta}{Pe_1^B}X\right)}{2\sqrt{\pi}} \right) \right) \left(\sqrt{\pi} + erf\left(\sqrt{3Pe_1^B} \left(\frac{Z-v_z t}{4rt^{3/2}} \right) \right) \right) \quad (2.81)$$

V. Concluding Remarks

We considered the evolution of solute concentrations in the blood compartment during HD therapy by solving the model when diffusion coefficients were negligible. On the other hand, a small but non zero diffusion coefficients, were introduced and as a result, the solutions were modified by smoothing out the discontinuity present as r approaches the membrane in the blood compartment. Equations 2.78 – 2.81 are explicit solutions of the dimensionless concentrations of bicarbonate and partial pressure of carbon dioxide as $r \to R_B$ in the blood compartment. The results we observed were solute concentrations as functions of time and distances specified along the hollow fiber. The result might help to give more insight into the change of solute concentration as diffusion across the semi-permeable membrane occurs.

Notes

We have proposed and given analytical results of the modified model system for bicarbonate HD which may practically be used. The theoretical model presented provides valuable insight into the system characteristics and serves as a rational basis for interpreting data.

References Références Referencias

Notes

1. Ahmad S, Pagel M, Vizzo J and Scribner BH, Effect of the normalization of acid-base balance on postdialysis plasma bicarbonate, Trans Am Soc Artif Intern Organs 26 (1980) 318-340.
2. Ruch TC and Patton HD, Physiology and Biophysics, Philadelphia-London-Toronto: WB Saunders Co. (1979).
3. Raja RM, Kramer MS, Rosenbaum JL, Fernandes M and Barber K, Hemodialysis with varying dialysate bicarbonate concentration. Trans Am Soc Artif. Intern. Organs 28 (1982) 514 – 521.
4. Gotch FA, Sargent JA and Keen ML, Hydrogen ion balance in dialysis therapy, Artificial organs 6 (1982) 388-95.
5. Ward RA, Wathen RL and Wiliams TE, Effects of long-term bicarbonate hemodialysis on acid-base status, Trans Am Soc. Artificial International organs 28 (1982) 295-300.
6. Monti JP, Sarrazin MY, Baz M, Murisasco A, Crevat AD and Elsen R, Kinetic modeling of intradialytic and interdialytic pH shifts during and after acetate and bicarbonate hemodialysis Int. J Artif. Organs 13 (1990) 799 – 802.
7. Ronco C, Brendolan A, Crepaldi C, Rodighiero M and Scabard M, Blood and dialyzate flow distributions in hollow-fiber hemodialyzers analyzed by computerized helical scanning technique J Am Soc Nephrol 13 (2002) S53-S61.
8. Ronco C, Ballestri M and Brendolan A, New developments in hemodialyzers Blood Purif. 18 (2000) 267-275.
9. Sargent JA and Gotch FA, Bicarbonate and carbon dioxide transport during hemodialysis, ASAIO Journal 2, (1979) 61 – 72.

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Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 20 Issue 8 Version 1.0 Year 2020
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Improved Average Penalty Cost (IAPC) Method to Obtain Initial basic Feasible Solution of Transportation Problem

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Abstract- Operational planning, scheduling and synchronization of all production activities are the key responsibilities of the management of a manufacturing plant. Transport modeling is one such activity which is directly involved in the production cost. Therefore, it is necessary for the management of the plant to design the transportation process in such way so that the total production cost is minimized, subject to the constraint that cannot be compromised. In the solution procedure of these transportation problems, an initial basic feasible solution (IBFS) is always require to reach at the optimal solution. In this study, a new algorithm is developed to find IBFS. The result of the proposed method is compared with more classical method naming Vogel's Approximation Method (VAM) and cost cell based method named Least Cost Method (LCM). Here the number of numerical problems is established and found in 58.3% cases the proposed method provides optimal where the rest of the cases it offers very near to optimal solution. For finding the degree of effectiveness of proposed method a study is carried out and simulation results show that Improved Average Penalty Cost (IAPC) yields better IBFS than VAM and LCM.

Keywords: *transportation table (TT), IBFS, VAM, LCM.*

GJSFR-F Classification: *MSC 2010: 91B32*

IMPROVEDAVERAGEPENALTYCOSTIAPCMETHODTOOBTAININITIALBASICFEASIBLESOLUTIONOFTRANSPORTATIONPROBLEM

Strictly as per the compliance and regulations of:

Improved Average Penalty Cost (IAPC) Method to Obtaininitial basic Feasible Solution of Transportation Problem

Md. Munir Hossain

Abstract- Operational planning, scheduling and synchronization of all production activities are the key responsibilities of the management of a manufacturing plant. Transport modeling is one such activity which is directly involved in the production cost. Therefore, it is necessary for the management of the plant to design the transportation process in such way so that the total production cost is minimized, subject to the constraint that cannot be compromised. In the solution procedure of these transportation problems, an initial basic feasible solution (IBFS) is always require to reach at the optimal solution. In this study, a new algorithm is developed to find IBFS. The result of the proposed method is compared with more classical method naming Vogel's Approximation Method (VAM) and cost cell based method named Least Cost Method (LCM). Here the number of numerical problems is established and found in 58.3% cases the proposed method provides optimal where the rest of the cases it offers very near to optimal solution. For finding the degree of effectiveness of proposed method a study is carried out and simulation results show that Improved Average Penalty Cost (IAPC) yields better IBFS than VAM and LCM.

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I. Introduction

With each passing year, ecommerce business transactions are reaching new heights of success. With the entry of large ecommerce marketplace players into the logistics industry, this supply chain management business has become more competitive than conventional logistics service providers. In this competitive global market, each and every company must have a very good planning to deliver their product to the customer in the right place at right time. This type of planning is known as a transportation networking in which the main objective is to decide how to shift goods from various sending locations (known as origins) to various receiving locations (known as destinations) with minimal costs.

In The Mathematical Method of Production Planning and Organization (1939), Kantorovich [1] showed that all problems of economic allocation can be seen as maximizing a function subject to constraints. F.L. Hitchcock [2] in 1941 formally introduced the transportation problem by presenting a paper entitled 'The Distribution of a Product from Several Sources to Numerous Localities'. This presentation is considered as the origin, and first important contribution to the solution of transportation problems. Continuation of the improvement of transportation problems, Koopmans [3] in 1947 presented his historic paper titled 'Optimum Utilization of the Transportation Systems', which was based on his war time experience.

Ref
1. Kantorovich, L.V. (1939), The Mathematical Method of Production Planning and Organization. Management Science, 6, 363-422.

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G. B. Dantzig [4] in 1951 first introduced the logical solution procedure for the transportation problem. In the solution procedure of the transportation problem it is always required to find an initial basic feasible solution (IBFS) to obtain the optimal solution. It was again developed by Charnes et al. [5] in 1953 and referred as North West Corner Method (NWCM) in which the north-west-corner cost cell is considered at every stage of allocation. And then the next developed method is Least Cost Method (LCM) consists in allocating as much as possible in the lowest cost cell of the Transportation Table in making allocation in every stage. Reinfeld and Vogel [6] in 1958 developed a method known as Vogel's Approximation Method (VAM). Including the above mentioned methods, Row Minima Method (RMM) and Column Minima Method (CMM) are also considered as the well reputed methods for solving transportation problems which are discussed in most of the Operation Research books [7-12]. Among these methods, VAM is considered as the most efficient solution procedures for obtaining an initial basic feasible solution for the transportation problems.

Ulrich A. Wagener [13] in 1965 proposed a new procedure for the computation of a transport problem model which uses each column of the cost matrix. Kirca and Satir [14] in 1990, developed a heuristic to obtain an efficient IBFS to the transportation problems. This method is called Total Opportunity Cost Method (TOCM). The TOCM is formed by adding the row opportunity cost matrix (ROCM) and the column opportunity cost matrix (COCM) where, for each row in the initial transportation cost matrix, the ROCM is generated by subtracting the lowest cost in the row from the other cost elements in that row and, for each column in the initial transportation cost matrix, the COCM is generated by subtracting the lowest cost in the column from the other cost elements in that column. Kirca and Satir then essentially use the LCM with some tie-breaking rules on the TOCM to generate a feasible solution to the transportation problem. Mathirajan and Meenakshi [15] in 2004 analyzed some variants of VAM and extended TOCM using the VAM procedure. They coupled VAM with TOCM and achieved very efficient initial solutions. Kasana and Kumar [16] in 2005 proposed Extremum Difference Method (EDM) where they define the penalty as the differences of the highest and lowest unit transportation cost in each row and column and allocate as like as the VAM procedure. Koruko˘glu and Balli [17] in 2011 proposed an improved version of the well-known VAM by taking the total opportunity cost into account. They claimed through computational experiments that this improved VAM provided more efficient initial feasible solution to a large scale transportation problem. Rashid [18] in 2011 developed an effective approach for solving TPs by defining penalty as the differences of the highest and next to the highest cost in each row and column of a transportation table and allocate to the minimum cost cell corresponding to the highest penalty. This method is named as Highest Cost Difference Method (HCDM) for solving TPs. Khan A.R. [19] in 2011 presented a method by defining pointer cost as the difference of the highest and next to the highest cost in each row and column of a transportation table and allocate to the minimum cost cell corresponding to the highest three pointer cost. Again, Singh et al. [20] in 2012 modified the solution procedure of VAM using total opportunity cost and allocation costs.

Deshmukh [21] in 2012 proposed a new method called an innovative method (NMD) to obtain a better IBFS to the transportation problem. Sudhakar et al. [22] in 2012 proposed a new approach called "Zero Suffix Method (ZSM)" for obtaining a minimal total cost solution to the transportation problem. In this method, they proposed to obtain at least one zero in each row and each column of the transportation

Ref

5. Charnes A., Cooper W.W., Henderson A., 1953, 'An Introduction to Linear Programming', John Wiley & Sons, New York.

table by subtracting the least element of each row and then column from all the elements of the corresponding row and column. Get suffix value of each zero and assign the cell corresponding to the greatest suffix value. Delete the exhausted row / column to get the reduced table. Procedures are repeated for the reduced table until all the demands and supplies are exhausted. Islam M.A. et al. [23] in 2012, applied EDM on TOCM, and allocate to the minimum cost cell corresponding to the highest distribution indicator and again HCDM on TOCM for obtaining an IBFS. Md. AshrafulBabu et al. [24] in 2013, proposed a method for solving transportation problems, where first allocation was made in the lowest cost cell which appears along lowest demand/supply. They named the method "Lowest Allocation Method (LAM)". S. Aramuthakannanet al. [25] in 2013 proposed a new method to solve transportation problems in which the allocations are made basing on the minimum demand and supply. This method is known as Revised Distribution Method. Abdul Sattar Soomro et al. [26] in2014modified the VAM algorithm and proposed Minimum Transportation Cost Method (MTCM)by computing row penalty by the difference of two largest transportation costs and column penalty by the difference of two smallest costs. UtpalKanti Das et al. [27] in 2014 brought out a logical development in the solution procedure of VAM. Basically he proposed a different idea to calculation of penalty cost for improving VAM. Ahmed, M.M. et al. [28] in 2014 developed an algorithm for finding an IBFS for both the balanced and unbalanced TP. In this method the transportation matrix is transformed to a Modified Transportation Cost Matrix (MTCM). The MTCM is formed as the differences of the Row Modified Cost Matrix (RMCM) and the Column Modified Cost Matrix (CMCM) where, for each row in the initial transportation cost matrix, the RMCM is generated by subtracting each of the cost of the row from the largest cost of that row of the transportation table and, for each column in the initial transportation cost matrix, the CMCM is generated by subtracting each of the cost of the column from the largest cost of that column of the transportation table. Finally the penalty costs are defined as the differences of the highest and next to the highest cost in each row and column of the MTCM and allocate to the minimum cost cell corresponding to the highest penalty cost. Dr. Muwafaq Alkubaisi [29] in 2015 used the median cost as penalty instead of the difference of two smallest costs in a row and column and applied VAM algorithm in the rest of the procedure. Aminur Rahman Khan et al. [30] in 2015 used TOCM to define the pointer cost as the sum of all entries in the respective row or column of the TOCM and then allocate to the minimum cost cell corresponding to the highest pointer cost. Hossain Md. M. et al.[31] have used TOCM and defined the penalty cost as the difference between the highest and the lowest cell in the respective row and column of the TOCM. Then allocate to the minimum cost cell corresponding to the highest penalty cost where computing penalty cost for each and every allocation is avoided to ease up the computational complication. Z.A.M.S. Juman, M.A. Hoque [32] in 2015, proposed a better efficient heuristic solution technique by JHM (Juman & Hoque Method) to obtain a better IBFS for the TPs. Moreover, Uddin, M.S. et al. [33] in 2016 proposed a method to solve the transportation problems named as Improved Least Cost Mehtod (iLCM). This improvement is basically done by bringing changes in the existing solution procedure of the classical LCM. Using the TOCM, Azad and Hossain [34] in 2017 presented a method where penalties are the average of the row opportunity cost (row penalty) and the average of the column opportunity cost (column penalty). J. Ravi et al. [35] in 2019 used the result of difference from standard deviation (DFSD) method as the smallest unit cost element in the row/column (cell) from the immediate next smallest unit cost element in the same row/column is determining a

Ref

23. Islam, M.A., Haque, M.H., Uddin, M.S., 2012, '*Extremum Difference Formula on Total Opportunity Cost: A Transportation Cost Minimization Technique*', Prime University Journal of Multidisciplinary Quest, 6(1), 125-130.

penalty measure for the target row/column. Recently, Hossain Md. M. et al. [36] in 2020, a computationally easier solution procedure for TP is proposed which is known as 'Least Cost Mean Method (LCMM)'. They presented this LCMM by defining penalty cost as the average of the lowest cell cost and the next lowest cell cost for each row and column.

II. Formulation for a Transportation Problem

Transportation problem is a distribution type problem; to model this type of problems we use the following notations:

m Total number of sources/origins

n Total number of destinations

S_i Amount of supply at source i

d_j Amount of demand at destination j

c_{ij} Unit transportation cost from source i to destination j

x_{ij} Amount to be shipped from source i to destination j

Network representation of a transportation problem, is represented in Figure-2.1

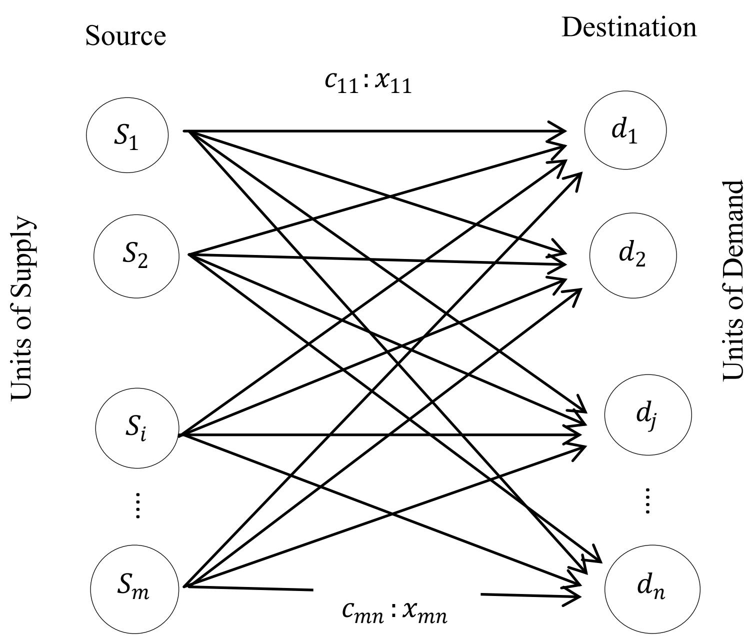


Figure 2.1: Network Diagram for transportation problem

The objective of the model is to determine the unknowns' x_{ij} that will minimize the total transportation cost while satisfying the supply and demand restrictions. Basing on this objective the Hitchcock-Koopman's transportation problem is mathematically formulated as:

Ref

36. Hossain, Md. M; Ahmed, M.M., 2020, 'A *Comparative Study of Initial Basic Feasible Solution by a Least Cost Mean Method (LCMM) of Transportation Problem*', American Journal of Operations Research, 10(4), 122-131. DOI: 10.4236/ajor.2020.104008.

Minimize: $$z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$$ (Objective function)

Subjectto: $$\sum_{j=1}^{n} x_{ij} \le S_i \; ; i = 1,2,\ldots\ldots, m$$ (Capacity constraints)

Notes

$$\sum_{i=1}^{m} x_{ij} \ge d_j \; ; \; j = 1,2,\ldots\ldots., n$$ (Requirement constraints)

$x_{ij} \ge 0$, for all i and j (Non-negative condition)

There are two types of transportation problems, in a case where the supply of goods available for shipping at the origins is equal to the demand for goods at the destinations; the transportation problem is called balanced. In a case where the quantities are different, the problem is unbalanced. When a transportation problem is unbalanced, a dummy variable is used to even out demand and supply. A dummy variable is simply a fictional warehouse or store.

III. Algorithm

Step 1: Formulate the problem mathematically and if the problem is unbalance, balance the given Transportation Problem.

Step 2: Subtract the smallest cost (c_{ik} where $k = 1,2,\ldots\ldots.n$) from each of the cost along the first row ($c_{i1}, c_{i2}, \ldots\ldots. c_{in}$, where $i = 1,2,\ldots\ldots..m$) of the TT and write those on the top right corner of the corresponding cost. Similar operation is applicable for rest of the rows.

Step 3: Applying the same process on each of the column and write the result on the right bottom corner of the corresponding cost.

Step 4: Magnitude of the subtraction of top and bottom element is put at the left bottom corner of the corresponding cost cell.

Step 5: Find the average of the left bottom elements of each row and put it at right side of the corresponding row as a row penalties. Similarly average column penalties place at the below of the corresponding column.

Step 6: Choose the highest average penalty costs and observe the row or column along which it appears. If a tie occurs in the highest average penalty, take the row/column along which lowest-cost appears. In case of tie for lowest cost cells, select the cell where maximum allocation can be put. If maximum allocation is in tie situation then select the cell for which sum of demand and supply is maximum in the TT. When the sum of demand and supply are same then choose any one of them.

Step 7: Place the first allocation then adjust the supply and demand requirements in the respective row and column. If the first allocation equals the demand, cross out the column. Now fulfill the row allocation along the basic cost cell by making the allocation(s) in the successive smallest cost cell/cells. Consider that the row is exhausted at some cell (i, j) with the allocation x_{ij}. Now follow the same procedure to fulfill the allocation along j-th column and continue the process until all the rim condition are satisfied. The process will reverse if first allocation equals the supply.

Step 8: If the allocation equals both the demand and the supply, cross out both the row and column. Find the next smallest cost cell along the crossed out row and column. Assign zero in that cell cost. Consider that the zero allocation x_{pq} is made in the cell (p, q) which is along the exhausted row. Now fulfill the allocation along q-th column following the procedure described in Step 7. In case of exhaust column, process will reverse.

Step 9: Finally calculate the total transportation cost from the cost table. This calculation is the sum of the product of cost and corresponding allocated value of the cost table.

IV. Numerical Examples of Transportation Problem

We consider the following numerical problems to solve by using the proposed and other methods.

Table 4.1: Numerical examples of balanced transportation problems

Problem Number	Type of the Problem	Data of the problems
P-1	5x6	$[c_{ij}]_{5x6}$=[5 3 7 3 8 5; 5 6 12 5 7 11; 2 8 3 4 8 2; 9 6 10 5 10 9; 5 3 7 3 8 5] $[s_i]_{5x1}$=[3, 4, 2, 8, 3] $[d_j]_{1x6}$=[3, 4, 6, 2, 1, 4]
P-2	3x4	$[c_{ij}]_{3x4}$=[9 8 5 7; 4 6 8 7; 5 8 9 5] $[s_i]_{3x1}$=[12, 14, 16] $[d_j]_{1x4}$=[8, 18, 13, 3]
P-3	3x5	$[c_{ij}]_{3x5}$=[5 7 10 5 3; 8 6 9 12 14; 10 9 8 10 15] $[s_i]_{3x1}$=[5, 10, 10] $[d_j]_{1x5}$=[3, 3, 10, 5, 4]
P-4	4x3	$[c_{ij}]_{4x3}$=[2 7 4; 3 3 1; 5 4 7; 1 6 2] $[s_i]_{4x1}$=[5, 8, 7, 14] $[d_j]_{1x3}$=[7, 9, 18]
P-5	3x4	$[c_{ij}]_{3x4}$=[10 2 20 11; 12 7 9 20; 4 14 16 18] $[s_i]_{3x1}$=[15, 25, 10] $[d_j]_{1x4}$=[5, 15, 15, 15]
P-6	3x4	$[c_{ij}]_{3x4}$=[4 6 8 8; 6 8 6 7; 5 7 6 8] $[s_i]_{3x1}$=[40, 60, 50] $[d_j]_{1x4}$=[20, 30, 50, 50]
P-7	3x3	$[c_{ij}]_{3x3}$=[4 3 5; 6 5 4 ; 8 10 7] $[s_i]_{3x1}$=[9, 8, 10] $[d_j]_{1x3}$=[7, 12, 8]
P-8	3x4	$[c_{ij}]_{3x4}$=[19 30 50 12; 70 30 40 60; 40 10 60 20] $[s_i]_{3x1}$=[7, 10, 18] $[d_j]_{1x4}$=[5, 8, 7, 15]
P-9	6x6	$[c_{ij}]_{6x6}$=[12 4 13 18 9 2; 9 16 10 7 15 11; 4 9 10 8 9 7; 9 3 12 6 4 5; 7 11 5 18 2 7; 16 8 4 5 1 10] $[s_i]_{6x1}$=[120, 80, 50, 90, 100, 60] $[d_j]_{1x6}$=[75, 85, 140, 40, 95, 65]
P-10	3x4	$[c_{ij}]_{3x4}$=[50 60 100 50; 80 40 70 50; 90 70 30 50] $[s_i]_{3x1}$=[20, 38, 16] $[d_j]_{1x4}$=[10, 18, 22, 24]
P-11	5x4	$[c_{ij}]_{5x4}$ = [10 20 5 7; 13 9 12 8; 4 15 7 9; 14 7 1 1; 3 12 5 19] $[s_i]_{5x1}$ = [200, 300, 200, 400, 400] $[d_j]_{1x4}$ = [500, 600, 200, 200]
P-12	3x4	$[c_{ij}]_{3x4}$=[13 18 30 8; 55 20 25 40; 30 6 50 10] $[s_i]_{3x1}$=[8, 10, 11] $[d_j]_{1x4}$=[4, 7, 6, 12]

Notes

V. Solution of a Problem with Illustration

Algorithm becomes more clear to the readers if goes through the illustrative solution of the related problems. Consider that the transportation problem is formulated and shown in Table-4.1 as an example 1.

Table 5.1: Transportation Problem of Example 1

Factories	Showrooms						Supply
	D_1	D_2	D_3	D_4	D_5	D_6	(a_i)
W_1	5	3	7	3	8	5	3
W_2	5	6	12	5	7	11	4
W_3	2	8	3	4	8	2	2
W_4	9	6	10	5	10	9	8
W_5	5	3	7	3	8	5	3
Demand (b_j)	3	4	6	2	1	4	

Before selecting the first cost cell as an allocation, we need to form a modified transportation table. In the 1st row of the given transportation cost matrix, 3 is the smallest cost. Subtract it from each of the cost along the first row [i.e.$(5-3)=2,\ (3-3)=0,\ (7-3)=4,\ (3-3)=0,\ (8-3)=5,\ (5-3)=2$]and put those results at the top right on the corresponding cost cell. Similarly, we subtract 5, 2, 5 and 3 from every element of 2nd, 3rd, 4th and 5throw respectively and place all the differences on the top right of the particular elements.

In the same way, we subtract 2, 3, 3, 3, 7 and 2 from each element of the 1st, 2nd, 3rd, 4th, 5th and 6th column respectively and place the result at the bottom left of the corresponding elements.

Forming modified transportation table (Table-5.2) whose elements remain same and place the magnitude of the subtraction of top and bottom element at the left bottom corner of the corresponding cost cell.

Table 5.2: Modified Transportation Table of Example 1

Factories	Showrooms						Supply
	D_1	D_2	D_3	D_4	D_5	D_6	(a_i)
W_1	${}_1 5^2_3$	${}_0 3^0_0$	${}_0 7^4_4$	${}_0 3^0_0$	${}_4 8^5_1$	${}_1 5^2_3$	3
W_2	${}_3 5^0_3$	${}_2 6^1_3$	${}_2 12^7_9$	${}_2 5^0_2$	${}_2 7^2_0$	${}_3 11^6_9$	4
W_3	${}_0 2^0_0$	${}_1 8^6_5$	${}_1 3^1_0$	${}_1 4^2_1$	${}_5 8^6_1$	${}_0 2^0_0$	2
W_4	${}_3 9^4_7$	${}_2 6^1_3$	${}_2 10^5_7$	${}_2 5^0_2$	${}_2 10^5_3$	${}_3 9^4_7$	8
W_5	${}_1 5^2_3$	${}_0 3^0_0$	${}_0 7^4_4$	${}_0 3^0_0$	${}_4 8^5_1$	${}_1 5^2_3$	3
Demand (b_j)	3	4	6	2	1	4	

Now determine the penalty cost for each row of the modified transportation table by taking the average of all left bottom entries in the respective row and place them along the right side of each corresponding rows.[i.e.$\frac{(1+0+0+0+4+1)}{6}=1,\ \frac{(3+2+2+2+2+3)}{6}=2.3,\ \frac{(0+1+1+1+5+0)}{6}=1.3,\ \frac{(3+2+2+2+2+3)}{6}=2.3,\ \frac{(1+0+0+0+4+1)}{6}=1$]

Do the same calculation for each column and place them in the bottom of the modified transportation table below the corresponding columns.

Notes

Table 5.3: Initial Basic Feasible Solution Using IAPC

Factories	Showrooms						Supply (a_i)	Row Penalty
	D_1	D_2	D_3	D_4	D_5	D_6		
W_1	${}_{1}5^{2}_{3}$	[3] ${}_{0}3^{0}_{0}$	${}_{0}7^{4}_{4}$	[0] ${}_{0}3^{0}_{0}$	${}_{4}8^{5}_{1}$	${}_{1}5^{2}_{3}$	3	(1)
W_2	[3] ${}_{3}5^{0}_{3}$	${}_{2}6^{1}_{3}$	${}_{2}12^{7}_{9}$	${}_{2}5^{0}_{2}$	[1] ${}_{2}7^{2}_{0}$	${}_{3}11^{6}_{9}$	4/3	(2.3)
W_3	[0] ${}_{0}2^{0}_{0}$	${}_{1}8^{6}_{5}$	${}_{1}3^{1}_{0}$	${}_{1}4^{2}_{1}$	${}_{5}8^{6}_{1}$	[2] ${}_{0}2^{0}_{0}$	2	(1.3)
W_4	${}_{3}9^{4}_{7}$	${}_{2}6^{1}_{3}$	[6] ${}_{2}10^{5}_{7}$	[2] ${}_{2}5^{0}_{2}$	${}_{2}10^{5}_{3}$	${}_{3}9^{4}_{7}$	8	(2.3)
W_5	${}_{1}5^{2}_{3}$	[1] ${}_{0}3^{0}_{0}$	${}_{0}7^{4}_{4}$	${}_{0}3^{0}_{0}$	${}_{4}8^{5}_{1}$	[2] ${}_{1}5^{2}_{3}$	3	(1)
Demand (b_j)	3	4	6	2	1	4/2		
Column Penalty	(1.6)	(1)	(1)	(1)	(3.4)	(1.6)		

a) Selecting first cost cell as a first allocation

From Table-3.3 it is observed that maximum penalty is (3.4) along with the D_5column/showroom and minimum transportation cost corresponding to this column is 7 in the cell (W_2, D_5). So we allocate 1 unit (min of 1 and 4) to the cell (W_2, D_5). Since the demand of the D_5 column is satisfied, we cross out this column and adjust the supply of the W_2 row/factory.

b) Second Allocation

Now according to Step 7 of our proposed algorithm, we need to fulfill the allocation of W_2 row by selecting the smallest cost cell along it. Here both (W_2, D_1)and (W_2, D_4) having the same cost (5). Since maximum allocation can be put in (W_2, D_1), so we allocate 3 unit in it.

c) Third Allocation

With the second allocation, both W_2 row and D_1 column is satisfied and we delete both of them. Here comes Step 8 to find the next allocation cell. In this case we find the smallest cost (2) along the both crossed out row and column. Assign zero (0) in the cost cell (W_3, D_1).

d) Fourth Allocation

Zero allocation is made in(W_3, D_1) which is along the exhausted columnD_1, therefore we fulfill the fourth allocation along W_3 row by choosing the smallest cost (2) and allocate 2 units (min of 2 and 4)in the cost cell(W_3, D_6). Since the supply of the W_3 row is satisfied, we cross out this row and adjust the demand of the D_6 column.

The above mentioned procedure is repeated for rest of the allocations. Thus we get the Fifth Allocation is 2 in the cell (W_5, D_6), Sixth Allocation is 1 in the cell (W_5, D_2), Seventh Allocation is 3 in the cell (W_1, D_2), Eighth Allocation is 0 in the cell (W_1, D_4), Ninth Allocation is 2 in the cell (W_4, D_4) and the Final Allocation is 6 in the cell (W_4, D_3).

Total transportation cost $z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

$$z = 3\times3 + 0\times3 + 3\times5 + 1\times7 + 0\times2 + 2\times2 + 6\times10 + 2\times5 + 1\times3 + 2\times5$$

$$= 9 + 0 + 15 + 7 + 0 + 4 + 60 + 10 + 3 + 10$$

$$= 118$$

Notes

VI. Performance Evaluation of the Proposed Method

IBFS for the problems obtain by the well reputed methods and proposed method is tabulated below:

Table 6.1: IBFS of the problems

Method	Number of the problems											
	P-1	P-2	P-3	P-4	P-5	P-6	P-7	P-8	P-9	P-10	P-11	P-12
NWCM	129	320	234	102	520	980	150	975	4285	4160	16500	484
RMM	126	248	183	80	505	960	145	1064	2290	4120	9200	589
CMM	132	248	215	111	475	960	150	995	2915	3320	8900	476
LCM	134	248	191	83	475	960	145	894	2455	3500	10200	516
VAM	116	248	187	80	475	930	150	859	2310	3320	9800	476
IAPC	118	240	183	76	460	920	144	799	2290	3320	9200	412
Optimal Solution	116	240	183	76	435	920	139	799	2170	3320	8800	412

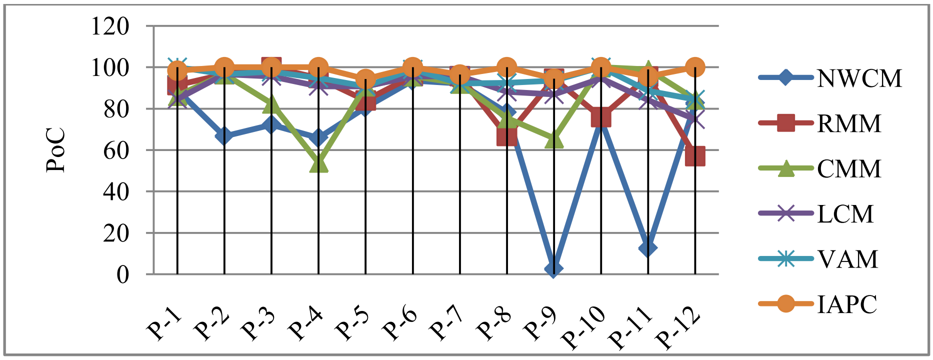
Now we calculate the Percentage of Correctness (PoC) of the IBFS obtained by various methods, just to justify the performance of the proposed method. This PoC do indicate the closeness between the obtained IBFS and the optimal solution (OS).To calculate this percentage, we have used the formula,

PoC= $100 - \{(\text{IBFS} - \text{OS}) \div \text{OS}\} \times 100$. This PoC is shown in Table-6.2

Table 6.2: PoC of IBFS

Method	Number of the problems											
	P-1	P-2	P-3	P-4	P-5	P-6	P-7	P-8	P-9	P-10	P-11	P-12
NWCM	88.79	66.67	72.13	65.79	80.46	93.48	92.09	77.97	02.53	74.70	12.50	82.52
RMM	91.38	96.67	100	94.74	83.91	95.65	95.68	66.83	94.47	75.90	95.45	57.04
CMM	86.21	96.67	82.51	53.95	90.80	95.65	92.09	75.47	65.67	100	98.86	84.47
LCM	84.48	96.67	95.63	90.79	90.80	95.65	95.68	88.11	86.87	94.58	84.09	74.76
VAM	100	96.67	97.81	94.74	90.80	98.91	92.09	92.49	93.55	100	88.64	84.47
IAPC	98.27	100	100	100	94.25	100	96.40	100	94.47	100	95.45	100
Optimal Solution	100	100	100	100	100	100	100	100	100	100	100	100

The calculation of PoC in the Table-6.2gives clear reflection that the IBFS obtained by IAPC are very proximate to optimal solution. The proximity between optimal solution and IBFS obtained by all methods discussed here is shown in the following Graph-1:



Graph 1: Graphical representation of PoC

From the data of Table-6.2, it is also observed that the performances of the proposed method are better than all other methods for solving the balanced transportation problems. In this study we also calculate the average of PoC (APoC) which is shown in the Table-6.3.

Table 6.3: Average of PoC

	NWCM	RMM	CMM	LCM	VAM	IAPC
Limit	02.53 to 93.48	66.83 to 100	53.95 to 100	74.76 to 96.67	84.47 to 100	94.25 to 100
APoC	67.47	87.31	85.19	89.84	94.18	98.24

VII. Degree of Effectiveness

The performance of IAPC is measured using percentage decrease in the total cost associated with an IBFS of a numerical problem obtained by IAPC over the corresponding one obtained by LCM and VAM. Also the performance of IAPC is measured by comparing the percentage increases of each of the total costs associated with the IBFS (obtained by these methods) from the optimal solution. These performance measures are shown in table-6.1. Performance measure of IAPC over LCM and VAM for 12 numerical problems has chosen from the literature.

Table 7.1: Performance Measure of IAPC

Problem Chosen from	Initial Cost with an IBFS by			% decrease in IBFS by IAPC over LCM and VAM		Optimal Solution	% increase from the Optimal Solution		
	LCM	VAM	IAPC	LCM	VAM		LCM	VAM	IAPC
Wagener [13]	134	116	118	11.94	-1.72	116	15.52	0.00	1.72
Azad and Hossain [34]	248	248	240	3.22	3.22	240	3.33	3.33	0.00
Ray and Hossain [12]	191	187	183	4.19	2.14	183	4.37	2.18	0.00
Ahmed et al [28]	83	80	76	8.43	5.00	76	9.21	5.26	0.00
Seethalakshmy et at [37]	475	475	460	3.16	3.16	435	9.19	9.19	5.75
Juman and Hoque [32]	960	930	920	4.17	1.07	920	4.35	1.09	0.00
Azad and Hossain [34]	145	150	144	0.69	4.00	139	4.32	7.91	3.60
Babu et al [24]	894	859	799	10.63	6.98	799	11.89	7.51	0.00
Khan et al [30]	2455	2310	2290	6.72	0.86	2170	13.13	6.45	5.53
Hossain and Ahmed [36]	3500	3320	3320	5.14	0.00	3320	5.42	0.00	0.00
Alkubaisi [29]	10200	9800	9200	9.80	6.12	8800	15.91	11.36	4.54
Sudhakar et al [22]	516	476	412	20.15	13.44	412	25.24	15.53	0.00

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Notes

It can easily be observed from Table 6.1 that the IAPC leads to better IBFS over VAM for 10 out of 12 problems considered. For the remaining 2 problems, in one problem both IAPC and VAM lead to an IBFS with the same total cost and in the other problem VAM leads better IBFS than IAPC. Also IAPC leads to better IBFS over LCM for 12 out of 12 problems considered. Besides, from the results in Table-6.1 it can easily be observed that IAPC led to the optimal solution in 7 out of 12 considered problems, whereas each of LCM and VAM led to the optimal solutions to 0 and 2 out of 12 respectively. In the remaining 5 problems, the percentage increases in the total cost from the optimal solution in case of IAPC is the least.

Comparative efficiencies of IAPC over LCM and VAM in respect of percentage decrease in the total cost for an IBFS and percentage increase of the total cost for an IBFS from the optimal solution, for each category of the considered problems are depicted by bar-charts in Fig. 1-2.

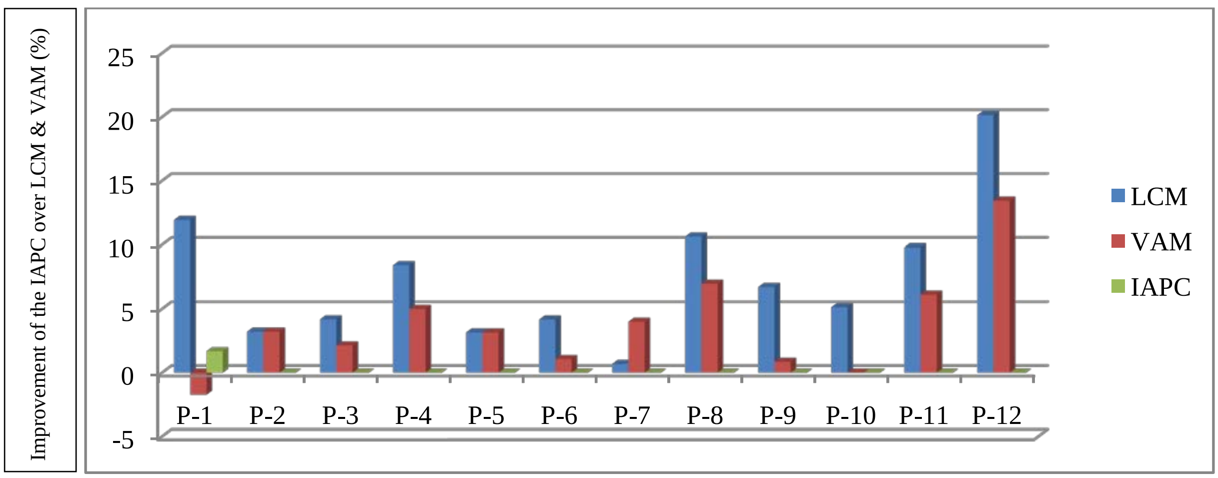


Fig. 1: Improvement of the IAPC over LCM & VAM (%)

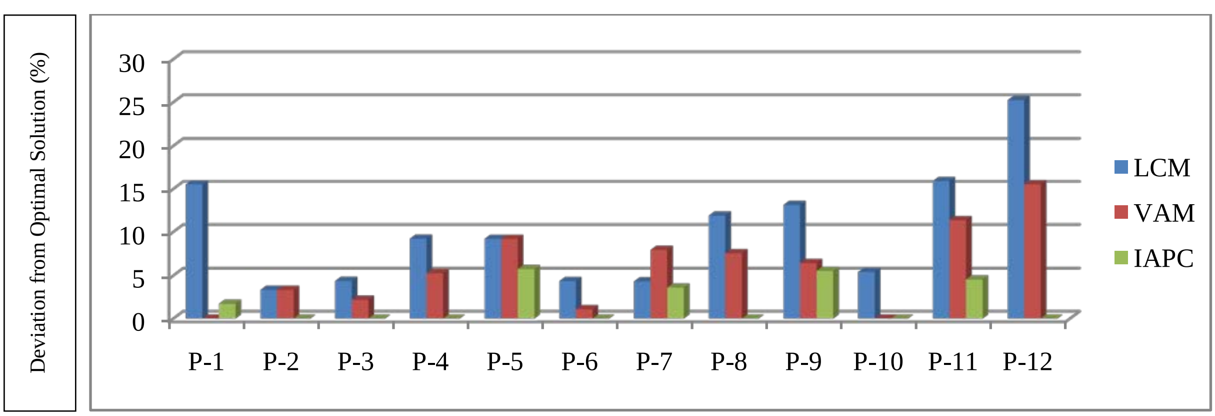
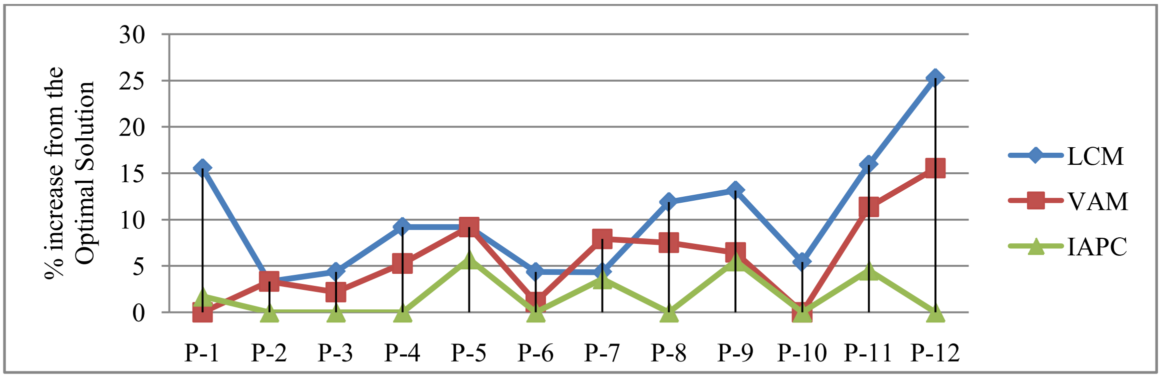


Fig. 2: Deviation from Optimal Solution (%)

According to the simulation results from Table-6.1 it is also perceived that in 41.67% cases IAPC provides increased IBFS than the optimal solution. The percentage increases in the total cost from the optimal solution in case of LCM and VAM is 100% and 83.33% cases respectively. The following Graph-2 represents the percentage increases of IBFS from optimal solution.



Graph-2: Graphical representation of percentage increases of IBFS

VIII. Conclusion

TP is an optimal path selecting procedure to transport goods by spending minimum transportation cost which is sometimes to blame for a company's inability to properly serve customers. To improve the company's position in the market, manager needs to build uptransportation networks in order to save transportation cost and time so that the market prices of daily commodities remain affordable. In this study developed IAPC perform promisingly in finding IBFS to the TP in comparison with other two best available methods VAM and LCM. The performance evaluation of IAPC has been carried out to justify its efficiency by solving twelve numerical examples chosen from the literature.IAPC provides better feasible solutions than existing method which are very close to optimal solution and sometimes it is equal to optimal solution. But it is not grantee that all time IAPC provides least feasible solution but most of the times it gives better approach.

References Références Referencias

1. Kantorovich, L.V. (1939), The Mathematical Method of Production Planning and Organization. Management Science, 6, 363-422.
2. Hitchcock F.L., 1941, 'The distribution of a Product from Several Sources to Numerous Localities', Journal of Mathematics and Physics, 20, 224-230.
3. Koopmans T. C., 1947, 'Optimum Utilization of the Transportation System', Econometrica, 17, 3-4.
4. Dantzig G. B., 1951, 'Application of the Simplex Method to a Transportation Problem, Activity Analysis of Production and Allocation', In: Koopmans, T.C. Ed., John Wiley and Sons, New York 359-373.
5. Charnes A., Cooper W.W., Henderson A., 1953, 'An Introduction to Linear Programming', John Wiley & Sons, New York.
6. Reinfeld N.V., Vogel W.R., 1958, 'Mathematical Programming', Englewood Cliffs, NJ: Prentice-Hall.
7. Hiller F.S., Lieberman G.J., 2001, 'Introduction to Operations Research', Seventh Edition, The McGraw-Hill Companies, Inc., 1221 Avenue of the Americas, New York, NY, 10020.
8. Panneerselvam, R., 2007, 'Operations Research', Second Edition, Prentice-Hall of India Private Limited, New Delhi, pp.71-126.

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9. Hamdy A. Taha, 2004, 'Operations Research: An Introduction', Sixth Edition, Prentice Hall of India, New Delhi, pp. 165,181-183.
10. Swarup K. Gupta P.K., Mohan M., 1995, 'Operation Research' Seventh Edition (Reprint), Sultan Chand & Sons. New Delhi, India, pp. 146-182.
11. Kasana H.S., Kumar K.D., 2005, '*Introductory Operations Research: Theory and Applications*', Springer International Edition, New Delhi.
12. Ray G.C, Hossain M.E., 2007, 'Operation Research', Titas Publications, Banglabazar, Dhaka, Bangladesh, pp. 47-234.
13. U. A. Wagener, 1965, '*A New Method of Solving the Transportation Problem*', Operational Research Society, 16(4), 453–469.
14. Kirca O., Satir A., 1990, '*A Heuristic for Obtaining an Initial Solution for the Transportation Problem*', Journal of the Operational Research Society, 41, 865–871.
15. Mathirajan M., Meenakshi B., 2004, '*Experimental Analysis of Some Variants of Vogel's Approximation Method*', Asia-Pacific Journal of Operational Research, 21(4), 447-462.
16. Kasana H.S., Kumar K.D., 2005, '*Introductory Operations Research: Theory and Applications*', Springer International Edition, New Delhi, 2005.
17. S. Koruko˘glu, S. Balli, 2011, '*An improved Vogel's approximation method for the transportation problem*', Math. Comput. Appl. 16 (2), 370–381.
18. Rashid A., 2011, '*An Effective Approach for Solving Transportation Problems*', Jahangirnagar Journal of Mathematics and Mathematical Sciences, 26, 101–107.
19. Khan A.R., 2011, '*A Re-solution of the Transportation Problem: An Algorithmic Approach*', Jahangirnagar University Journal of Science, 34(2), 49-62.
20. S. Singh, G.C. Dubey, R. Shrivastava, 2012, '*Optimization and Analysis of Some Variants through Vogel's Approximation Method (VAM)*', IOSR Journal of Engineering (IOSRJEN), 2 (9), 20–30.
21. N.M. Deshmukh, 2012, '*An Innovative Method for Solving Transportation Problem*', International Journal of Physics and Mathematical Sciences, 2(3), 86–91.
22. V.J. Sudhakar, N. Arunsankar, T. Karpagam, 2012, '*A New Approach for finding an Optimal Solution for Transportation Problems*', European Journal of Scientific Research, 68(2), 254–257.
23. Islam, M.A., Haque, M.H., Uddin, M.S., 2012, '*Extremum Difference Formula on Total Opportunity Cost: A Transportation Cost Minimization Technique*', Prime University Journal of Multidisciplinary Quest, 6(1), 125-130.
24. Babu M. A., Helal M. A., Hasan M. S. Das U.K., 2013, '*Lowest Allocation Method (LAM): A New Approach to Obtain Feasible Solution of Transportation Model*', International Journal of Scientific and Engineering Research, 4(11), 1344-1348.
25. Aramuthakannan S., Kandasamy P. R., 2013 '*Revised Distribution Method of finding Optimal Solution for Transportation Problems*', IOSR Journal of Mathematics (IOSR-JM), 4(5), 39-42.
26. Soomro A. S., Tularam G. A., Bhayo G.M., 2014, '*A Comparative Study of Initial Basic Feasible Solution Methods for Transportation Problems*', Mathematical Theory and Modeling, 4(1), 11-18.

27. Das U.K., Babu M.A., Khan A.R., Helal M.A., Uddin M.S., 2014, '*Logical Development of Vogel's Approximation Method (LD-VAM): An Approach to Find Basic Feasible Solution of Transportation Problem*', International Journal of Scientific & Technology Research (IJSTR), 3(2), 42-48.
28. Ahmed M.M., Tanvir A.S.M., Sultana S., Mahmud S., Uddin M.S., 2014, '*An Effective Modification to Solve Transportation Problems: A Cost Minimization Approach*', Annals of Pure and Applied Mathematics, 6(2), 199-206.
29. Dr. Muwafaq Alkubaisi, 2015, '*Modified VOGEL Method to Find Initial Basic Feasible Solution (IBFS)- Introducing a New Methodology to Find Best IBFS*', Business and Management Research, 4(2),22–36.
30. Khan A.R., Vilcu A., Sultana N., Ahmed S.S., 2015, '*Determination of Initial Basic Feasible Solution of a Transportation Problem: A TOCM-SUM Approach*', Buletinul Institutului Politehnic Din Iasi, Romania, Secţia Automaticasi Calculatoare, LXI (LXV)(1), 39-49.
31. Hossain, Md. M., Ahmed, M.M., Islam, Md. A., Ukil, S.I., 2020, '*An Effective Approach to Determine an Initial Basic Feasible Solution: A TOCM-MEDM Approach*', Open Journal of Optimization, 9(2), 27-37. https://doi.org/10.4236/ojop.2020.92003
32. Juman Z.A.M.S., Hoque M.A., 2015, '*An Efficient Heuristic to Obtain a Better Initial Feasible Solution to the Transportation Problem*', Applied Soft Computing, 34, 813-826.
33. Uddin M.S., Khan A.R., Kibria C.G., Raeva, I., 2016, '*Improved Least Cost Method to Obtain a Better IBFS to the Transportation Problem*', Journal of Applied Mathematics & Bioinformatics, 6(2), 1-20.
34. Azad, S.M.A.K., Hossain, Md.B., 2017,'*A New Method for Solving Transportation Problems considering Average Penalty*'. IOSR Journal of Mathematics, 13(1), 40-43. https://doi.org/10.9790/ 5728-1301044043
35. Ravi J., Dickson S., Akila R., Sathya K., 2019, '*An Optimal Solution for Transportation Problem-DFSD*', Journal of Computational Mathematica, 3(1), 43-51.
36. Hossain, Md. M; Ahmed, M.M., 2020, '*A Comparative Study of Initial Basic Feasible Solution by a Least Cost Mean Method (LCMM) of Transportation Problem*', American Journal of Operations Research, 10(4), 122-131. DOI: 10.4236/ajor.2020.104008.
37. Seethalakshmy A., Srinivasan N., 2016, '*A Direct Method to Obtain an Optimal Solution in the Transportation Problem*', International Journal of Advance Research (IJAR), 4(10), 473-477. DOI: 10.21474/IJAR01/1817.

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Global Journal of Science Frontier Research (F) Volume XX Issue VIII Version I Year 2020

Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 20 Issue 8 Version 1.0 Year 2020
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

First Consistent Determination of the Basic Reproduction Number for the First Covid-19 Wave in 71 Countries from the SIR-Epidemics Model with a Constant Ratio of Recovery to Infection Rate

By R. Schlickeiser & M. Kröger
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Abstract- The box-shaped serial interval distribution and the analytical solution of the Susceptible Infectious-Recovered (SIR)-epidemics model with a constant time-independent ratio of the recovery (μ_0) to infection rate (a_0) are used to calculate the effective reproduction factor and the basic reproduction number R_0. The latter depends on the positively valued net infection number $x = 13.5(a_0 - \mu_0)$ as $R_0(x) = x(1 - e^{-x})^{-1}$ which for all values of x is greater unity. This dependence differs from the simple relation $R_0 = a_0/\mu_0$. With the earlier determination of the values of k and a_0 of the Covid-19 pandemic waves in 71 countries the net infection rates and the basic reproduction numbers for these countries are calculated.

Keywords: *epidemics; coronavirus; statistical analysis; parameter estimation; basic reproduction number.*

GJSFR-F Classification: *MSC 2010: 30C15*

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R. Schlickeiser [α] & M. Kröger [σ]

Abstract- The box-shaped serial interval distribution and the analytical solution of the Susceptible Infectious-Recovered (SIR)-epidemics model with a constant time-independent ratio of the recovery (μ_0) to infection rate (a_0) are used to calculate the effective reproduction factor and the basic reproduction number R_0. The latter depends on the positively valued net infection number $x = 13.5(a_0 - \mu_0)$ as $R_0(x) = x(1-e^{-x})^{-1}$ which for all values of x is greater unity. This dependence differs from the simple relation $R_0 = a_0/\mu_0$. With the earlier determination of the values of k and a_0 of the Covid-19 pandemic waves in 71 countries the net infection rates and the basic reproduction numbers for these countries are calculated.

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I. Introduction

The effective reproduction factor $R(t)$ and the basic reproduction number R_0 are key quantities to measure the dynamical evolution of epidemics. If the number of daily new cases $c(t)$ (deaths or infections) is known, either from monitoring or from Susceptible-Infectious- Recovered (SIR)-modelling, the effective reproduction factor is defined by[1,2]

$$R(t) = \frac{c(t)}{\int_0^\infty ds\, W(s) c(t-s)}, \quad (1)$$

where $W(t)$ is the properly normalized serial interval distribution[3] and time t taken in units of days. This serial interval distribution describes the probability for the time lag between a person's infection and the subsequent transmission of the virus to a second person. As different choices of the serial interval distribution $W(t)$ are used in the literature this leads to differences in the calculated associated effective reproduction

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factors R(t). Most frequently,[2] for the serial interval distribution *W(t)* either, the gamma distribution

$$W(t) = \frac{b^3}{2} s^2 e^{-bt}, \qquad b = 4/9, \tag{2}$$

with the mean value $\langle t \rangle = 3/b = 27/4 = 6.75$ days, or the box-shaped distribution

$$W_{\rm box}(t) = \frac{\Theta(0, t, t_b)}{t_b}, \qquad t_b = 2\langle t \rangle = 13.5, \tag{3}$$

with the two-sided Heaviside function $\Theta(x, A; B) = 1$ for $A \le x \le B$ and $\Theta(x) = 0$ otherwise, are chosen. The box-shaped serial distribution (3) readily yields for the effective reproduction factor (1) that

$$R_{\rm box}(t) = \frac{c(t)t_b}{\int_0^{t_b} ds\, c(t-s)} = \frac{c(t)t_b}{\int_{t-t_b}^{t} dx\, c(x)} \tag{4}$$

for any given number of daily new cases $c(t)$.

In earlier work[2] we calculated the effective reproduction factor by adopting a Gaussian distribution of daily cases both for the gamma (Eq. (2)) and box-shaped (Eq. (3)) serial distributions, respectively. Fig. 5 of ref.[2] indicates that the two effective reproduction factors are very similar. Here we use the analytical approximations[4] for the daily cases resulting from the SIR-modelling of epidemics to calculate the corresponding effective reproduction factors with the simpler box-shaped serial distribution (4). The SIR model decribes the time evolution of infectious diseases in human populations. The SIR system is the simplest and most fundamental of the compartmental models and its variations (for a recent review see ref.[5]).

The calculated effective reproduction factors $R(t)$ are then used to infer the basic reproduction number R_0 which has been defined[6,7] as the average number of secondary infections produced when one infected individual is introduced into a host population where everyone is susceptible. As such $R_0 = R(t_0)$ is identical to the value of the effective reproduction factor $R(t_0)$ at the starting time of the outbreak.[2] If a specific mathematical model for the time evolution of the pandemic outbreak such as e.g. the SIR-model is used, this model should then obey the initial condition that one infected individual occurs at time t_0. We are particularly interested in the relation of the SIR-basic reproduction number R_0 with the ratio $k = \mu_0/a_0$ of the initial recovery (μ_0) and infection (a_0) rates entering the SIR-model rate equations.

The inverse of this ratio k is often referred to as the inverse microscopic SIR-basic reproduction number.[5] However, the recent application[8] of analytical approximations of the solution of the SIR-model to the monitored death and infection rates in many countries provided values of $k > 0.9$ close to unity for the majority of countries investigated corresponding to microsciopic SIR-basic reproduction numbers $R_0 = 1/k < 1.11$ only slightly greater than unity. These values are significantly smaller than the estimates of $R_0 \in [2.4, 5.6]$ in the mainstream literature on Covid-19 based on the monitored effective reproduction factors.[5,9] It is the purpose of the present manuscript to resolve this discrepancy by calculating the relation between the SIR-basic reproduction number and the value of the ratio k. It is shown that this relationship is different from the simple microscopic $kR_0 = 1$ relationship.

Notes

Notes

II. Analytical Approximations of the SIR-Model

We start by reviewing our main earlier results [4,8] on the analytical solution of the SIR-epidemics model. Here ratio the considered population of $N \gg 1$ persons is assigned to the three compartments s (susceptible), i (infectious) or r (recovered/removed). Persons from the population may progress with time between these compartments with given infection ($a(t)$) and recovery rates ($\mu(t)$) which in general vary with time due to non-pharmaceutical interventions taken during the pandemic evolution.

It is convenient to introduce with $I(t) = i(t)/N$, $S(t) = s(t)/N$ and $R(t) = r(t)/N$ the infected, susceptible and recovered/removed fractions of persons involved in the infection at time t, with the sum requirement $I(t) + S(t) + R(t) = 1$. In terms of the reduced time $\tau(t) = \int_0^t d\xi a(\xi)$, accounting for arbitrary but given time-dependent infection rates, and the medically interesting daily rate of new infections

$$\dot{J}(t) = a(t)j(\tau) = \dot{\tau}j(\tau) = \dot{\tau}\frac{dJ}{d\tau}, \tag{5}$$

where the dot denotes a derivative with respect to t, the SIR-model equations can be written as

$$\frac{dI}{d\tau} = j - KI, \quad \frac{dS}{d\tau} = -j, \quad \frac{dR}{d\tau} = KI, \tag{6}$$

with $K(t) = \mu(t)/a(t)$ and the dimensionless $j(\tau)$.

For the special and important case of a time independent ratio $K(t) = k =$ const., new analytical results of the SIR-model (6) have been recently derived.[4] For a growing epidemics with time the constant ratio $k < 1$ has to be smaller than unity corresponding to the initial infection rate a_0 at time $t = 0$ being larger than the initial recovery rate μ_0, both in units of days^{-1}. The new analytical solutions assume that the SIR equations are valid for all times $t \in [-\infty, \infty]$, and that the time $t = \tau = t_0 = 0$ refers to the 'observing time' when the existence of a pandemic wave in the society is realized and the monitoring of newly infected persons $\dot{J}(t)$ is started. As only initial condition the fraction of initially (at the observing time) infected persons $S(0) = e^{-\varepsilon}$ is adopted which corresponds to

$$I(0) = 1 - k\varepsilon - e^{-\varepsilon} \simeq (1-k)\varepsilon,$$
$$J(0) = 1 - e^{-\varepsilon} \simeq \varepsilon, \tag{7}$$

where the two last approximations hold for small values $\varepsilon \ll 1$, for the initial fraction of infected persons $I(0)$ and the initial cumulative fraction $J(0)$, respectively. In order to meet the above demanded initial condition, that one infected individual is introduced at time $t_0 = 0$ into a host population where everyone is susceptible, demands that $\varepsilon = 1/N$, where N denotes the total number of the host population.

The exact solution for the cumulative number of infected persons $J(\tau)$ as a function of the reduced time is given by[4,8]

$$\tau = \int_{1-e^{-\varepsilon}}^{J} \frac{dy}{(1-y)f(y)}, \quad f(y) = y + k\ln(1-y) \tag{8}$$

Taking the derivative of the exact solution (8) with respect to τ provides the exact relationship

$$j(\tau) = \frac{dJ}{d\tau} = (1-J)[J + k\ln(1-J)] \tag{9}$$

between the daily rate $j(\tau)$ and its corresponding cumulative number $J(\tau)$. Moreover, with $J(\tau)$ known one obtains

$$S(\tau) = 1 - J(\tau),$$
$$I(\tau) = J(\tau) + k\ln[1-J(\tau)], \tag{10}$$
$$R(\tau) = -k\ln[1-J(\tau)].$$

Notes

III. SIR Box-Shaped Effective Reproduction Factor

Identifying in Eq. (4) $c(t) = \dot{J}(t)$ we readily obtain

$$R_{\text{box}}(t) = \frac{\dot{J}(t)t_b}{\int_{t-t_b}^{t} dx\, \dot{J}(x)} = \frac{\dot{J}(t)t_b}{J(t) - J(t-t_b)} \tag{11}$$

Using the invariance of $J(t) = J(\tau)$ and Eq. (5) then provides

$$R_{\text{box}}(t) = \frac{a(t)t_b j(\tau)}{J(\tau) - J(\tau-\tau_b)}$$

$$= \frac{a(t)t_b[1-J(\tau)]\{J(\tau) + k\ln[1-J(\tau)]\}}{J(\tau) - J(\tau-\tau_b)}, \tag{12}$$

where we inserted the exact Eq. (9) in the last step. According to Eq. (12) the SIR box-shaped effective reproduction factor is directly related to the time-dependent infection rate *a(t)* and the cumulative number of new infections depending on the reduced time $\tau = \int_0^t d\xi\, a(\xi)$. For given time-dependencies of the infection rate $a(t)$ we can immediately calculate the reduced time. Then for any value of *k* the cumulative numbers $J(\tau)$ are calculated according to Eq. (8), and as a consequence the SIR box-shaped effective reproduction factor is obtained.

We proceed with two important special limits.

a) *Slowly varying infection rate*

For slowly varying infection rates $a(t)$ we Taylor expand the cumulative number $J(t-t_b)$ on the right hand side of Eq. (11) as

$$J(t-t_b) \simeq J(t) - t_b \left.\frac{dJ}{dt}\right|_t = J(t) - t_b\dot{J}(t) \tag{13}$$

With this approximation the SIR box-shaped effective reproduction factor (11) reduces to $R_{\text{box}}(t) \simeq 1$, correctly agreeing with the limit for stationary case distributions.

b) *Infinite time*

According to the solution (8) the time $\tau = \infty$ corresponds to the cumulative number $J_\infty(k)$ given by the solution of the transcendental equation

$$J_\infty + k\ln(1-J_\infty) = 0 \tag{14}$$

Consequently, according to the relation (9) the daily rate $j(\infty)=0$ vanishes at the infinite time implying with Eq. (12) that the effective reproduction factor vanishes at infinite time $R_{\rm box}(\infty)=0$.

IV. Basic Reproduction Number

Notes

In order to calculate $J(-\tau_b)$ we use its asymptotic behavior for early times by integrating Eq. (68) of ref.[4], i.e.

$$J(\tau)\simeq J(0)e^{(1-k)\tau} \tag{15}$$

With $J(0)=1-e^{-\varepsilon}$ fom Eq. (7) this implies

$$J(0)-J(-\tau_b)\simeq J(0)[1-e^{-(1-k)\tau_b}]$$

$$=[1-e^{-\varepsilon}][1-e^{-(1-k)\tau_b}] \tag{16}$$

With the intial infection rate $a_0 = a(t=0)$ we then calculate from Eq. (12)

$$R_0 = R_{\rm box}(0)$$

$$= \frac{a_0 t_b[1-J(0)]\{J(0)+k\ln[1-J(0)]\}}{J(0)-J(-\tau_b)}$$

$$= \frac{a_0 t_b[1-k\varepsilon-e^{-\varepsilon}]}{[e^{\varepsilon}-1][1-e^{-(1-k)\tau_b}]}$$

$$\simeq \frac{a_0 t_b(1-k)}{1-e^{-(1-k)\tau_b}}, \tag{17}$$

Here, the last approximation assumes small values of $\varepsilon \ll 1$. Note that in this limit the resulting basic reproduction number is independent from the value of ε.

At early negative times no nonpharmaceutical interventions have been taken so that $a(t)\simeq a_0$ is constant and the reduced time $\tau_b\simeq a_0 t_b$. The SIR-basic reproduction number (17) then becomes

$$R_0(x)\simeq \frac{x}{1-e^{-x}}\simeq \begin{cases}1+\frac{x}{2} & \text{for } x\ll 1\\ x & \text{for } x\gg 1\end{cases} \tag{18}$$

in terms of the dimensionless quantity

$$x = a_0 t_b(1-k) = 13.5 a_0(1-k) = 13.5(a_0-\mu_0), \tag{19}$$

to which we refer as net injection number.

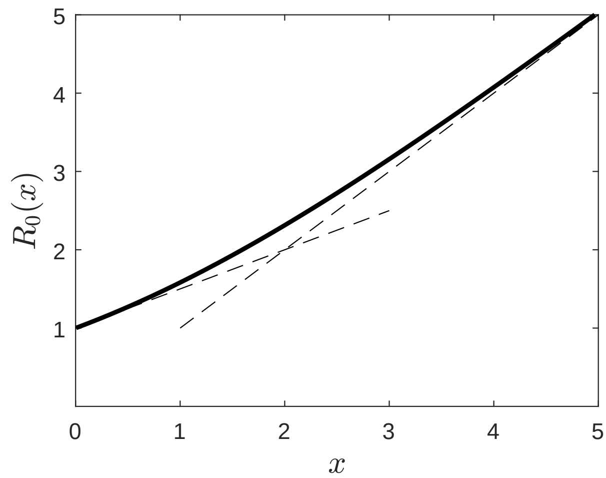

Fig. 1: SIR basic reproduction number (17) and its asymptotics as a function of the net infection number $x = a_0 t_b (1-k)$.

In Fig. 1 we plot the SIR-basic reproduction number $R_0(x)$ as a function of *x*. For small values of $x \ll 1$ we note that $R_0(x \ll 1)$ is only slightly larger than unity, whereas for large values of $x \gg 1$ there is the direct proportionality $R_0(x) \propto x$.

In principle, for values of *k* close to unity as obtained for most countries,[8] large values of $x \gg 1$ are possible provided the infection rate is large enough $a_0 \gg 1/13.5$ days^{-1}. The discrepancy noted in the introduction is therefore resolved even for values of *k* close to unity.

Values of $k > 0.9$ obviously correspond to net infection numbers smaller than $x < 1.3a_0$ which still can be much larger than unity provided the initial infection rate is significantly greater than $a_0 > 1/1.3 = 0.77$ days^{-1}. For $k = 0.9$ reported values of $R_0 \in [2.4, 5.6]$ are possible for initial infection rates greater than $a_0 = [1.8, 4.3]$ days^{-1}. Likewise, for $k = 0.95$ even larger initial infection rates $a_0 \in [3.5, 8.6]$ days^{-1} are needed.

In our earlier modeling[8] of the Covid-19 pandemic wave in 71 countries we determined not only the value of *k* but also the value of the initial infection rate a_0 (see Table 2 and 3 in ref.[8]). It is therefore straightforward to calculate the resulting net infection rates x and the basic reproduction numbers $R_0(x)$ for these 71 countries. The results are summarized in Table 1 and indicate large differences in the values of the basic reproduction numbers.

V. Summary and Conclusions

The box-shaped serial interval distribution and the analytical solution of the SIR-epidemics model with a constant time-independent ratio of the recovery (μ_0) to infection rate (a_0) are used to calculate the effective reproduction factor and the basic reproduction number R_0. The latter depends on the positively valued net infection number $x = 13.5(a_0 - \mu_0)$ as $R_0(x) = x(1-e^{-x})^{-1}$ which for all values of x is greater unity. This dependence differs from the simple relation $R_0 = a_0/\mu_0$. With the more general dependence of $R_0(x)$ values of basic reproduction numbers significantly greater than unity can occur even for values of k close to unity provided the initial infection rate is large enough. As we have determined the values of k and a_0 in our earlier modeling of the Covid-19 pandemic waves in 71 countries we have calculated here the resulting net infection rates and the basic reproduction numbers for these countries.

Notes

Notes

References Références Referencias

1. Fraser, C., Estimating individual and household reproduction numbers in an emerging epidemic, PLoS one 2, e758 (2007).
2. Kröger, M., Schlickeiser, R., Gausian doubling times and reproduction factors of the Covid-19 pandemic disease, Frontiers Phys. 8, 276 (2020).
3. Nishiura, H., Linton, N. M., Akhmetzhanov, A. K., Serial interval of novel coronavirus (COVID-19) infections, Int. J. Infect. Dis. 93, 284 (2020)
4. Kröger M, Schlickeiser R., Analytical solution of the SIR model for the temporal evolution of epidemics. Part A: Time-independent reproduction factor, J. Phys. A: Math. Theor. 53, 505601 (2020)
5. Estrada E., Covid-19 and Sars-Cov-2. Modeling the present, looking at the future, Phys. Rep. 869, 1 (2020).
6. Anderson, R. M., May, R. M., Infectious Diseases of Humans, Oxford University Press, Oxford (1992).
7. Delamater, P. L., Street, E. J., Leslie, T. F., Yang, Y. T., Jacobsen, K. H., Complexity of the basic reproduction number (R_0), Emerg. Inf. Dis. 25, 1 (2019).
8. Schlickeiser, R., Kröger, M., Epidemics forecast from SIR modeling, verification and calculated effects of lockdown and lifting of interventions, Frontiers Phys., in press (2020) Doi 10.3389/fphy.2020.593421. Preprint online medXriv Doi 10.1101/2020.08.12.20173294
9. Flaxman, S., Mishra, S., Gandy, A., Unwin, H. J. T., Mellan, T. A., Coupland, H., et al., Estimating the effects of non-pharmaceutical interventions on Covid-19 in Europe, Nature 584, 257 (2020).

Table 1: x and R_0 for all those countries (alpha-3 country code) for which k and a_0 have been determined in Ref.[8]

country	k	a_0	x	R_0	country	k	a_0	x	R_0	country	k	a_0	x	R_0
AFG	0.995	68.9	4.7	4.7	ALB	0.988	24.4	4.0	4.0	AND	0.931	7.6	7.1	7.1
ARG	0.987	21.8	3.8	3.9	ARM	0.897	1.1	1.5	2.0	AUT	0.992	149.8	16.2	16.2
BEL	0.911	6.1	7.3	7.3	BFA	0.999	1754.5	23.7	23.7	BGR	0.974	10.1	3.5	3.7
BLR	0.975	40.9	13.8	13.8	BOL	0.968	7.9	3.4	3.5	BRA	0.919	12.1	13.2	13.2
CAF	0.999	123.9	1.7	2.1	CHE	0.976	22.0	7.1	7.1	CHL	0.910	2.0	2.4	2.7
CHN	0.999	1544.6	20.9	20.9	COL	0.899	4.0	5.5	5.5	CUB	0.999	631.5	8.5	8.5
CZE	0.997	235.1	9.5	9.5	DEU	0.989	57.7	8.6	8.6	DNK	0.989	52.6	7.8	7.8
DOM	0.747	2.6	8.9	8.9	DZA	0.977	18.4	5.7	5.7	ECU	0.935	15.5	13.6	13.6
EGY	0.994	52.5	4.3	4.3	ESP	0.937	12.4	10.5	10.5	ETH	0.999	215	2.9	3.1
FRA	0.954	10.0	6.2	6.2	GAB	0.997	64.6	2.6	2.8	GBR	0.925	8.9	9.0	9.0
GHA	0.999	464.9	6.3	6.3	GRC	0.998	191.9	5.2	5.2	GTM	0.985	9.7	2.0	2.3
HND	0.978	26.0	7.7	7.7	HRV	0.996	72.3	3.9	4.0	IND	0.997	189.9	7.7	7.7
IRL	0.962	21.2	10.9	10.9	IRN	0.819	3.1	7.6	7.6	ITA	0.940	10.2	8.3	8.3
KOR	0.999	989.6	13.4	13.4	KWT	0.986	21.8	4.1	4.2	LBN	0.999	95.5	1.3	1.8
LUX	0.981	29.5	7.6	7.6	MAR	0.999	628.8	8.5	8.5	MDA	0.973	25.9	9.4	9.4
MEX	0.954	16.2	10.1	10.1	MKD	0.927	5.4	5.3	5.3	MRT	0.996	66.4	3.6	3.7
MYS	0.999	2064.2	27.9	27.9	NGA	0.999	405.4	5.5	5.5	NLD	0.963	15.4	7.7	7.7
NPL	0.999	953.4	12.9	12.9	PAK	0.997	101.1	4.1	4.2	PAN	0.848	3.5	7.2	7.2
PER	0.703	3.2	12.8	12.8	PHL	0.996	142.4	7.7	7.7	POL	0.993	63.1	6.0	6.0
PRT	0.982	86.4	21	21.0	ROU	0.909	13.8	17	17.0	RUS	0.984	60.7	13.1	13.1
SEN	0.998	79.9	2.2	2.4	SMR	0.867	2.9	5.2	5.2	SOM	0.999	591.7	8.0	8.0
SRB	0.931	10.1	9.4	9.4	SWE	0.935	10.1	8.9	8.9	TCD	0.999	1387.0	18.7	18.7
THA	0.999	3463.0	46.8	46.8	TUN	0.999	659.7	8.9	8.9	TUR	0.990	144.0	19.4	19.4
USA	0.938	10.2	8.5	8.5	ZAF	0.968	6.5	2.8	3.0					

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Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 20 Issue 8 Version 1.0 Year 2020
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Availability and Profit Analysis of a Series-Parallel System

By Abdullahi Sanusi
Bayero University

Abstract- This paper presents the study of availability and profit analysis of a series-parallel system consisting of four units A, B, C and D arranged in series in which subsystem D consist of three units arranged in parallel. The system works if any of A, B, C or D work. The Markovian approach was adopted to model the system behaviour with the assumption that the failure and repair rates of each subsystem follow exponential distribution. The differential equations that described the system were formulated to analyse the probability for each state. These equations were solved recursively to obtain explicit expressions for steady-state availability, busy period of repairman and profit function. The computed results are demonstrated by graphs. The results show that increase in repair rate increases the availability and associated profit. The results of the designed model are beneficial to reliability and maintenance managers.

Keywords: *availability, profit, reliability, series-parallel.*

GJSFR-F Classification: *MSC 2010: 33E20*

AVAILABILITYANDPROFITANALYSISOFASERIESPARALLELSYSTEM

Strictly as per the compliance and regulations of:

Ref

7. Sanusi, A., Yusuf, I., Mamuda. B.Y. (2020). Performance evaluation of an industrial configured as series-parallel system, Journal of Mathematical and Computational Science. 10, pp. 692-712.

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Abdullahi Sanusi

Abstract- This paper presents the study of availability and profit analysis of a series-parallel system consisting of four units A, B, C and D arranged in series in which subsystem D consist of three units arranged in parallel. The system works if any of A, B, C or D work. The Markovian approach was adopted to model the system behaviour with the assumption that the failure and repair rates of each subsystem follow exponential distribution. The differential equations that described the system were formulated to analyse the probability for each state. These equations were solved recursively to obtain explicit expressions for steady-state availability, busy period of repairman and profit function. The computed results are demonstrated by graphs. The results show that increase in repair rate increases the availability and associated profit. The results of the designed model are beneficial to reliability and maintenance managers.

Keywords: availability, profit, reliability, series-parallel.

I. Introduction

The series-parallel systems consist of subsystems connected in series where each subsystem consists of units arranged in parallel. The failure of any of the subsystems will lead to the failure of the whole system. These systems can be seen in industries, power stations, manufacturing, production and telecommunications. Owing to their importance in industries and economy, reliability measures of such systems have become an area of interest. System are normally studied in terms their reliability measures such as steady-state availability, busy period, profit function and mean time to system failure (MTSF). Availability and profit of redundant systems can be enhanced using highly reliable structural system design. Improving the system reliability and availability will also increase the production and associated profit. The study of reliability of repairable systems is an important topic in Engineering and Operation Research. System reliability is a very meaningful measure and achieving required level of reliability and availability is an essential requisite. System reliability and availability depends on the system structure.

Due to their importance in promoting and sustaining industries and economy, reliability and economic analysis of such systems have received attention from different researchers, see for instance. Sanusi et al [7] have recently presented the performance evaluation of an industrial configured as series-parallel system. Gahlot et al.[1]have analyzed the performance assessment of serial system with different types of failure and repair policy. M. S. Rao et al. [6] have analyzed a system dynamics model for transient availability modeling of repairable redundant systems.

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Singh and Ayagi [8] presented the study of availability of standby complex system under waiting repair and human failure using Gumbel–Hougaard family copula distribution. Yang et al [10]have discussed the reliability assessment of system with inconsistent priors and multi-level data. Malik *et al*[4] have presented the study of performance modeling and maintenance priorities decision for water flow system of a coal based thermal power plant. Yusuf [12] has used Chapman Kolmogorov differential equation to present the availability and other relevant measures of a two-unit active parallel system connected with external supporting device for operation with online and offline preventive maintenance. Singh et al. [9] discussed the performance of the complex system in the series configuration under different failure and repair discipline. Kumar *et al*[3] have studied the availability and cost analysis of an engineering system involving subsystems in series configuration. Ram Niwas and Harish Garg [5] studied the availability, reliability and profit of an industrial system based on cost free warranty policy. Yusuf, I [11] presented the availability modelling and evaluation of repairable system subject to minor deterioration under imperfect repairs. M. S. Kadyan and Ramesh Kumar [2] have presented the study of the availability and profit analysis of feeding system in the sugar industry.

In this paper, the study of availability and profit analysis of a series-parallel system is presented. The primary focus of this study is to analyse the effects of system parameters on availability and profit and make comparison to know the most critical component in the system. The rest of this paper is organised as follows: Section 2 presents the notations and assumptions used throughout the study. System description is given in section 3. Section 4 deals with the formulation and solution of the model. The results of our numerical simulations and discussion are given in section 5. The paper is concluded in section 6.

II. Notations and Assumptions

a) Notations

$\alpha_1, \alpha_2, \alpha_3, \alpha_4$: Failure rates of subsystem A, B, C and D respectively.

$\beta_1, \beta_2, \beta_3, \beta_4$: Repair rates of subsystem A, B, C and D respectively.

A_V: Steady-state availability.

PF: Profit function.

C_0: Revenue generated when the system is in working state and has no income when in failed state.

C_1: Cost of each repair for supporting device.

C_2: Cost of each repair for failed unit.

B_T: Busy period of repairman.

$\frac{d}{dt}$: Derivative with respect to time t.

b) Assumptions

1. The system may be in operating state or in a failed state.
2. Failure and repair rates are constant overtime and statistically independent.
3. A repair unit is as good as new performance wise for a specified duration.
4. Sufficient repair facilities are provided, i.e. no waiting time to start the repairs.
5. Standby units (if any) are of same nature and capacity as active units.
6. System failure/repair follow exponential distribution.

Ref

12. Yusuf I (2016) Reliability modeling of a parallel system with a supporting device and two types preventive maintenance. Int J Oper Res 25(3):269–287

III. Description of the Model

In this paper, a series-parallel system is considered. Figure 1 and figure 2 depict the block and transition diagrams of the system respectively. The system consists of four subsystems in which at least two units must be in operation in subsystem D for the system to work. The system failed when two units failed. The Markovian approach was adopted to model the system behaviour with the assumption that the failure and repair rates of each subsystem follow exponential distribution. The differential equations that described the system were formulated to analyse the probability for each state. These equations were solved recursively to obtain steady-state availability of the system and profit function. The model consists of four subsystems with the following description:

i) *Subsystem (A):* is a single unit arranged in series. Failure of this unit causes the complete failure of the system.

ii) *Subsystem (B):* a single unit arranged in series. Failure of this unit causes the complete failure of the system.

iii) *Subsystem (C):* is a single unit arranged in series. Failure of this unit causes the complete failure of the system.

iv) *Subsystem (D):* consists of four units arranged in parallel, two are in operative mode and one in cold standby mode. Complete failure of the system will occur only when two units failed at the same time.

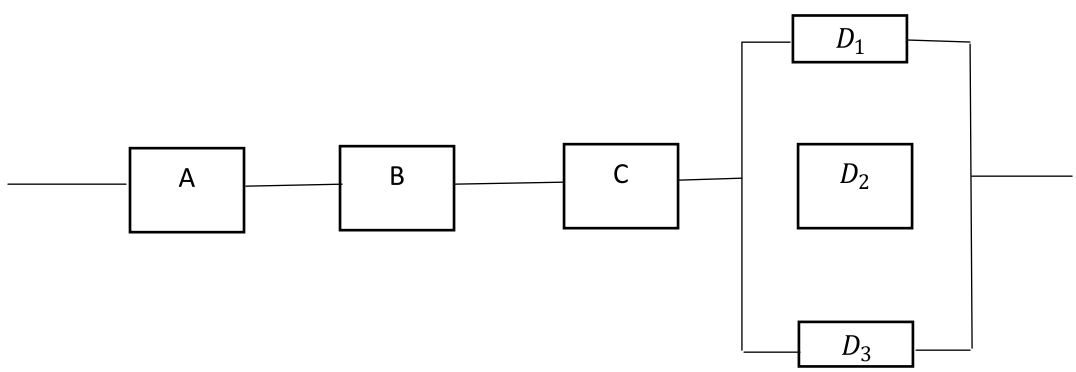

Figure 3.1: Reliability Block Diagram

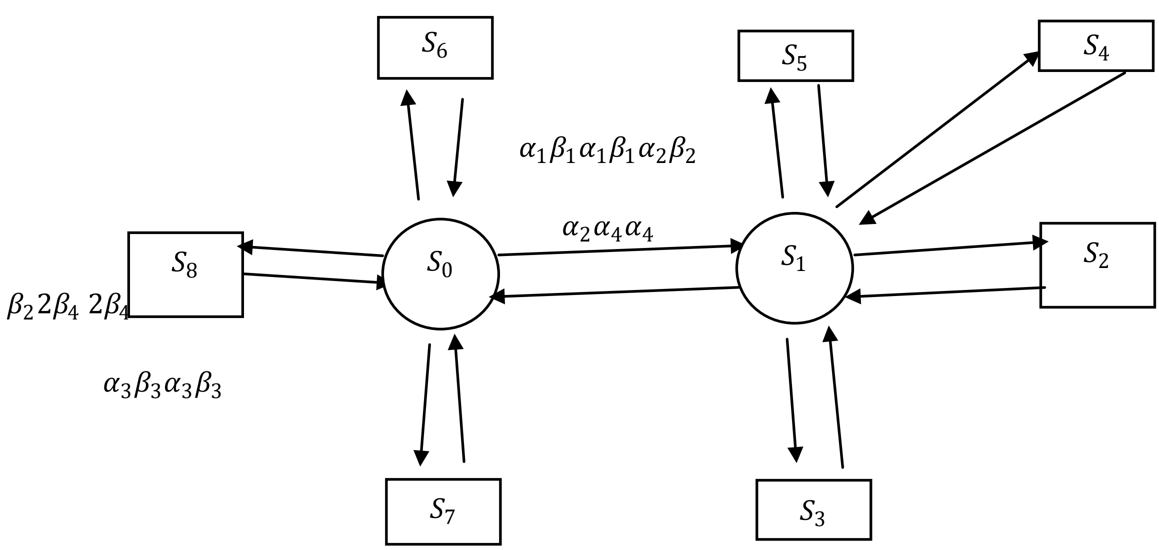

Figure 2: Transition Diagram of the System

IV. Formulation and Solution of Mathematical Model

The differential equations associated with the transition diagram (Figure 2), are derived on the basis of Markov birth-death process. Various probability considerations generate the following sets of differential equations:

$$\left[\frac{d}{dt} + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4\right] P_0 = 2\beta_4 P_1 + \beta_1 P_6 + \beta_3 P_7 + \beta_2 P_8 \quad (1)$$

$$\left[\frac{d}{dt} + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + 2\beta_4\right] P_1 = \alpha_4 P_0 + 2\beta_4 P_2 + \beta_3 P_3 + \beta_2 P_4 + \beta_1 P_5 \quad (2)$$

$$\left[\frac{d}{dt} + 2\beta_1\right] P_2 = \alpha_4 P_1 \quad (3)$$

$$\left[\frac{d}{dt} + \beta_3\right] P_3 = \alpha_3 P_1 \quad (4)$$

$$\left[\frac{d}{dt} + \beta_2\right] P_4 = \alpha_2 P_1 \quad (5)$$

$$\left[\frac{d}{dt} + \beta_1\right] P_5 = \alpha_1 P_1 \quad (6)$$

$$\left[\frac{d}{dt} + \beta_1\right] P_6 = \alpha_1 P_0 \quad (7)$$

$$\left[\frac{d}{dt} + \beta_3\right] P_7 = \alpha_3 P_0 \quad (8)$$

$$\left[\frac{d}{dt} + \beta_2\right] P_8 = \alpha_2 P_0 \quad (9)$$

Where, the initial conditions at time t = 0 are:

$$P(t)_i = \begin{cases} 0, if\ i = 0 \\ 0, if\ i \neq 0 \end{cases} \quad (10)$$

a) Availability equations

To get the steady state availability of the system (ASS), (i.e., time independent performance behaviour) which mean d/dt =0 and t $\rightarrow \infty$, the above equations (eq.1 to 10) become:

$$[\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4] P_0 = 2\beta_4 P_1 + \beta_1 P_6 + \beta_3 P_7 + \beta_2 P_8 \quad (11)$$

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$$[\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + 2\beta_4]P_1 = \alpha_4 P_0 + 2\beta_4 P_2 + \beta_3 P_3 + \beta_2 P_4 + \beta_1 P_5 \quad (12)$$

$$2\beta_1 P_2 = \alpha_4 P_1 \quad (13)$$

$$\beta_3 P_3 = \alpha_3 P_1 \quad (14)$$

$$\beta_2 P_4 = \alpha_2 P_1 \quad (15)$$

$$\beta_1 P_5 = \alpha_1 P_1 \quad (16)$$

$$\beta_1 P_6 = \alpha_1 P_0 \quad (17)$$

$$\beta_3 P_7 = \alpha_3 P_0 \quad (18)$$

$$\beta_2 P_8 = \alpha_2 P_0 \quad (19)$$

These equations were solved recursively, the values of steady state probabilities are as follows:

$$P_1 = \frac{1}{4}\left(\frac{\alpha_4}{\beta_4}\right)P_0, P_2 = \frac{1}{8}\left(\frac{\alpha_4}{\beta_4}\right)^2 P_0, P_3 = \frac{1}{4}\left(\frac{\alpha_3\alpha_4}{\beta_3\beta_4}\right)P_0, P_4 = \frac{1}{4}\left(\frac{\alpha_2\alpha_4}{\beta_2\beta_4}\right)P_0, P_5 = \frac{1}{4}\left(\frac{\alpha_1\alpha_4}{\beta_1\beta_4}\right)P_0,$$

$$P_6 = \frac{\alpha_1}{\beta_1}P_0, P_7 = \frac{\alpha_3}{\beta_3}P_0, P_8 = \frac{\alpha_2}{\beta_2}P_0 \quad (20)$$

The probability of full working capacity, namely, P0 determined by using normalizing condition: (i.e. sum of the probabilities of all working states, reduced capacity and failed states is equal to 1).
$\sum_{I=0}^{8} P_i = 1$, hence,

$$P_0 = \frac{1}{\left(1+\frac{1\alpha_4}{4\beta_4}\right)\left(1+\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}\right)+\frac{1}{8}\left(\frac{\alpha_4}{\beta_4}\right)^2} \quad (21)$$

Now, the steady state availability of the system may be obtained as summation of all working state probabilities as
Hence $Av. = \sum_{i=0}^{1} P_i$ =P0 + P1

$$Av. = \frac{\alpha_4+4\beta_4}{4\beta_4\left[\left(1+\frac{1\alpha_4}{4\beta_4}\right)\left(1+\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}\right)+\frac{1}{8}\left(\frac{\alpha_4}{\beta_4}\right)^2\right]} \quad (22)$$

b) Profit Analysis

Let C_0, $C_1$and C_2 be the revenue generated when the system is in working state and has no income when in failed state, cost of each repair for supporting units and accumulated cost of each repair for failed units (corrective maintenance) respectively. Then the expected total profit per unit time incurred to the system in the steady-state is:

$$P_F = C_0 A_V - C_1 B_{T1} + C_2 B_{T2} \tag{23}$$

Where $B_{T1} = P_1$

Thus,

$$B_{T1} = \frac{\alpha_4}{4\beta_4\left[\left(1+\frac{1\alpha_4}{4\beta_4}\right)\left(1+\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}\right)+\frac{1}{8}\left(\frac{\alpha_4}{\beta_4}\right)^2\right]} \tag{24}$$

Also

$$B_{T2} = P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8$$

$$B_{T2} = \frac{\left(1+\frac{1\alpha_4}{4\beta_4}\right)\left(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}\right)+\frac{1}{8}\left(\frac{\alpha_4}{\beta_4}\right)^2}{\left(1+\frac{1\alpha_4}{4\beta_4}\right)\left(1+\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}\right)+\frac{1}{8}\left(\frac{\alpha_4}{\beta_4}\right)^2} \tag{25}$$

Combining eq.22, 23, 24 and 25 we obtain:

$$P_F = \frac{1}{\left(1+\frac{1}{4}\frac{\alpha_4}{\beta_4}\right)\left(1+\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}\right)+\frac{1}{8}\left(\frac{\alpha_4}{\beta_4}\right)^2}\left[\frac{C_0(\alpha_4+4\beta_4)}{4\beta_4} - \frac{C_1\alpha_4}{4\beta_4} + \left[\left(1+\frac{1}{4}\frac{\alpha_4}{\beta_4}\right)\left(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}\right)+\frac{1}{8}\left(\frac{\alpha_4}{\beta_4}\right)^2\right]\right] \tag{26}$$

V. Discussion of Results

In this section, we numerically obtained and compare the results for system availability and profit function for the developed models. The objectives here are to analyze graphically the effects of system parameters on availability and profit and make comparison. The following set of parameters values are fixed throughout the simulations for consistency.

Figure 4.1: $\alpha_2 = 0.2, \alpha_3 = 0.3, \alpha_4 = 0.4, \beta_2 = 0.01, \beta_3 = 0.02, \beta_4 = 0.04$.

Figure 4.2: $\alpha_1 = 0.15, \alpha_3 = 0.2, \alpha_4 = 0.3, \beta_1 = 0.02, \beta_3 = 0.03, \beta_4 = 0.04$.

Figure 4.3: $\alpha_1 = 0.4, \alpha_2 = 0.5, \alpha_4 = 0.6, \beta_1 = 0.05, \beta_2 = 0.06, \beta_4 = 0.07$.

Figure 4.4: $\alpha_1 = 0.25, \alpha_2 = 0.3, \alpha_3 = 0.35, \beta_1 = 0.03, \beta_2 = 0.04, \beta_3 = 0.05$.

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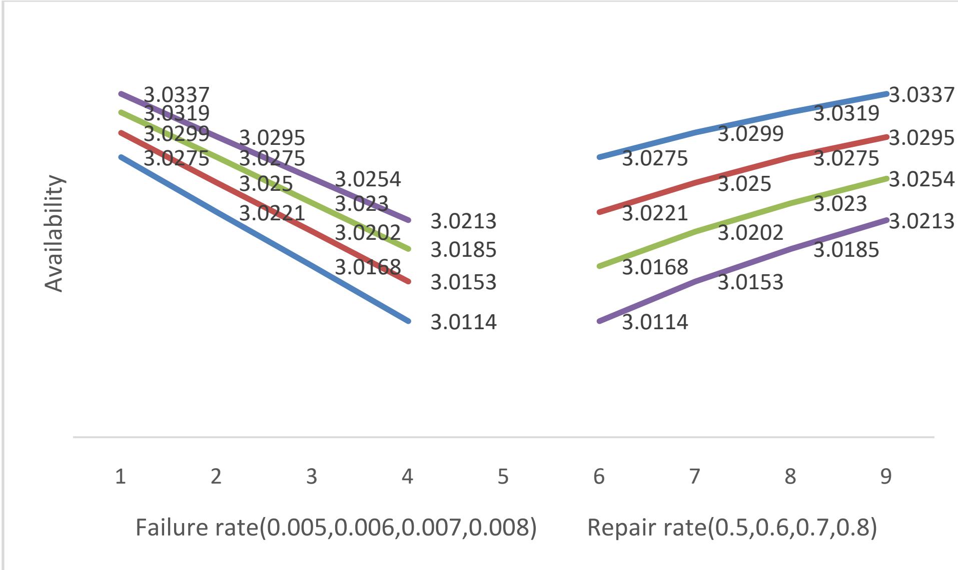

Figure 3: Effect of Failure rate α_1 and Repair rate β_1 on availability

Figure 3 shows the effect of failure rate α_1 and repair rate β_1 on the long run availability of the system. It is seen from this figure that as the failure rate α_1 increases, the system availability decreases considerably. Similarly, as the repair rate $\beta_1$increases, the system availability increases appreciably. This simulation suggests that regular repair will improve the system availability.Figure 4 presents the impact of failure rate α_2 and repair rate β_2 on system availability. We observe from this figure that as the failure rate α_2 grows, the system availability tends to decreases. It is also observed from this figure that the system availability increases with increase in the values of repair rate β_2.

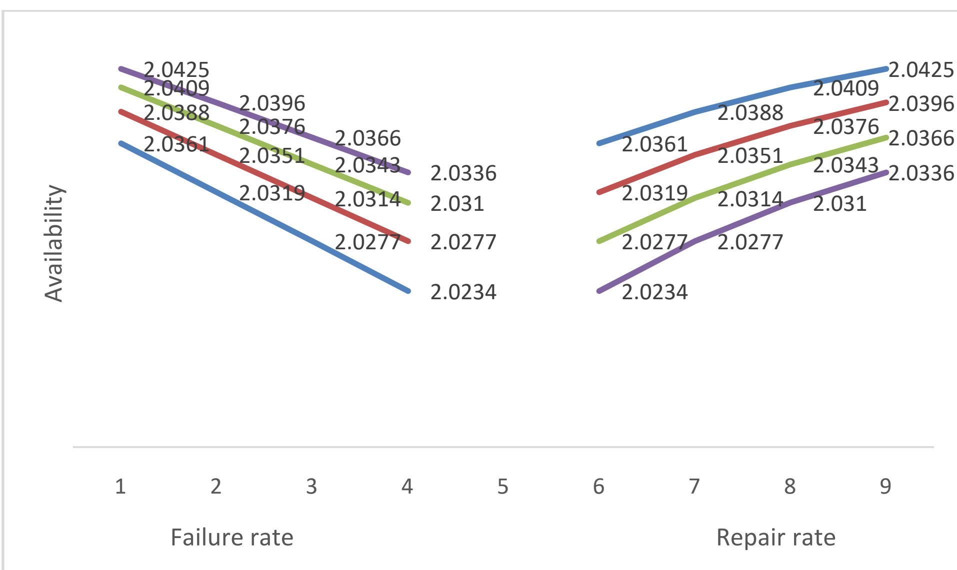

Figure 4: Effect of Failure rate α_2 and Repair rate β_2 on availability

The availability of system with different values of parameters $\alpha_3$and β_3 is correspondingly represented in Figure 5. From this figure, one can observe that these two parameters have significant effect on the system availability. Increasing parameter

$\alpha_3$leads to lower values for system availability, but increasing parameter β_3 leads to higher values for system availability.

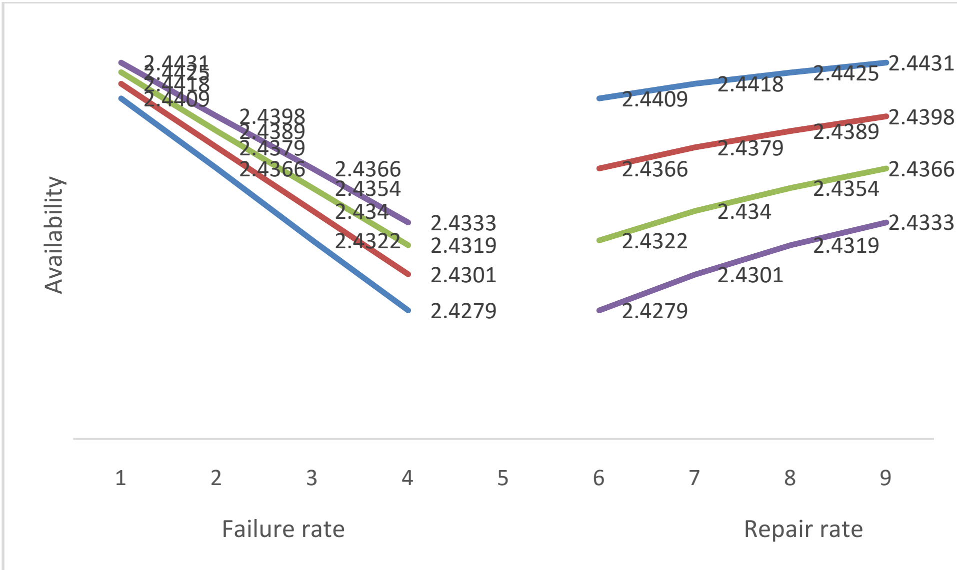

Figure 5: Effect of Failure rate α_3 and Repair rate β_3 on availability

In Figure 6, the availability of System is plotted with respect to parameters $\alpha_4$and β_4 respectively. From this figure, the behavior of availability with increasing parameter α_4 is descending. In the case of increasing parameter β_4 the system availability is ascending.

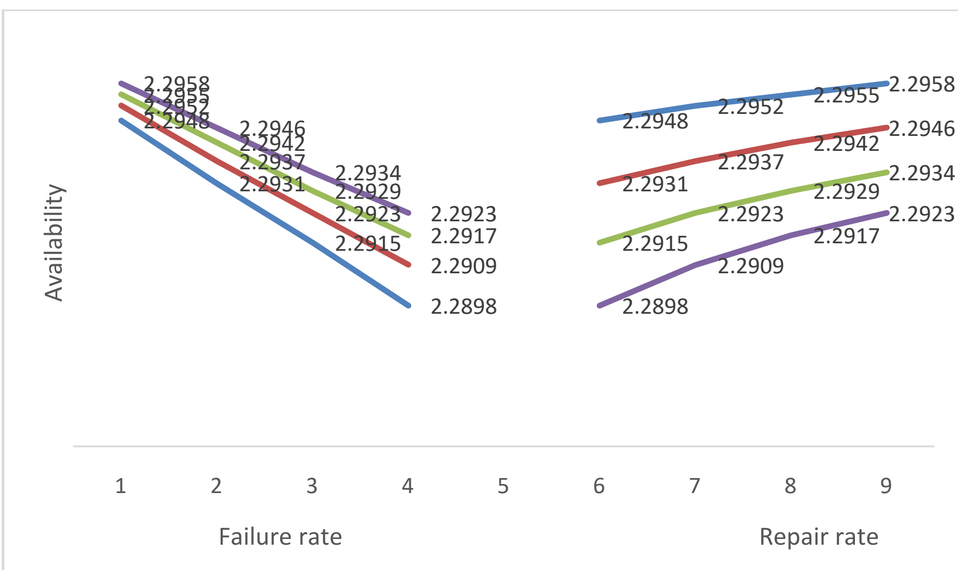

Figure 6: Effect of Failure rate α_4 and Repair rate β_4 on availability

The expected profit function of system versus parameters $\alpha_1$and β_1 respectively, is graphically illustrated in Figure 7. As shown, the behavior of profit function of system is descending with parameter α_1 and is ascending with parameter β_1.

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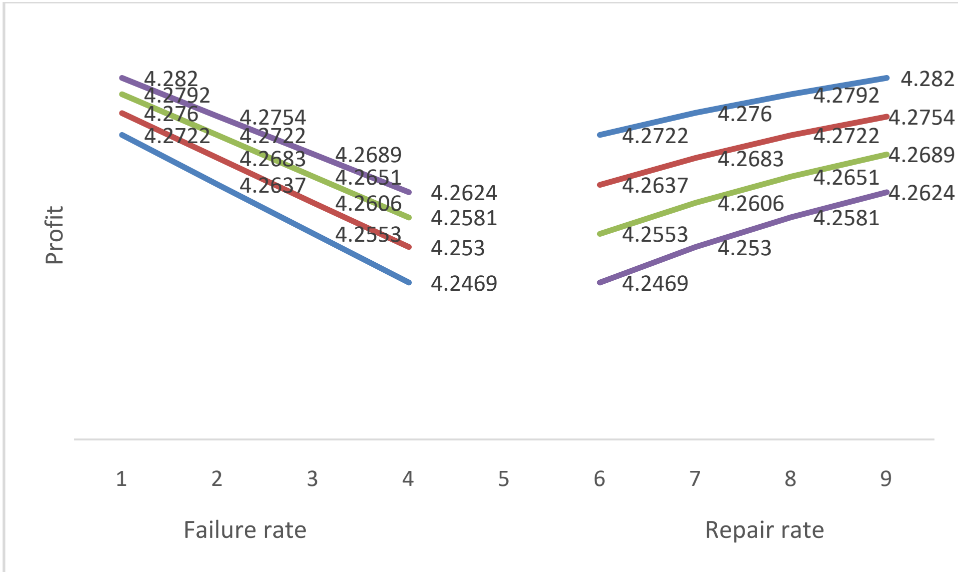

Figure 7: Effect of Failure rate α_1 and Repair rate β_1 on profit

In Figure 8, the profit functions of system are plotted with respect to parameter α_2 and β_2 respectively. From Figure 8, the behavior of profit function with parameter β_2 is rising while with respect to parameter α_2 the profit function is falling.

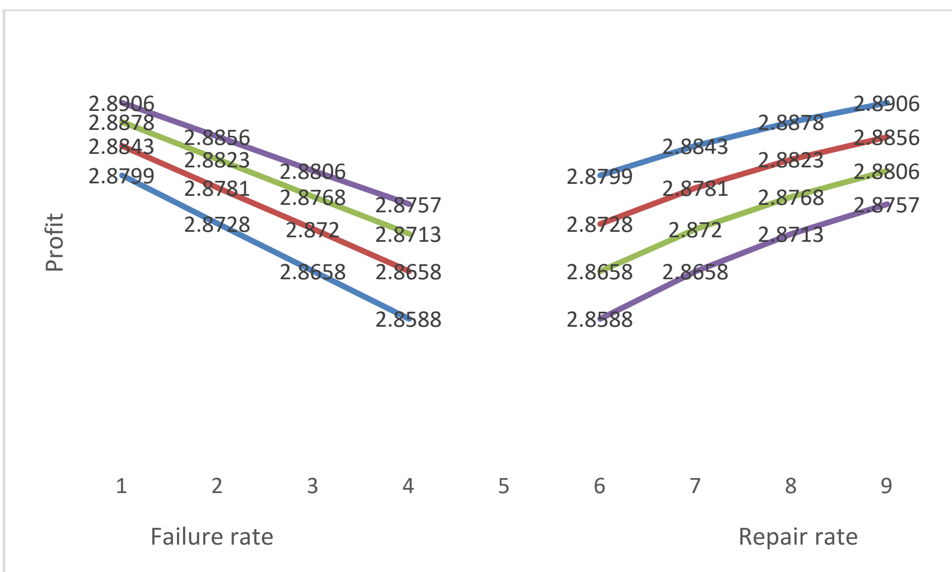

Figure 8: Effect of Failure rate α_2 and Repair rate β_2 on profit

The effect of failure rate α_3 and repair rate β_3 on profit function is depicted in figure 9. This figure indicates that as the values of failure rate increases, the profit function also decreases while the profit function increases with increase in the values of repair rate.

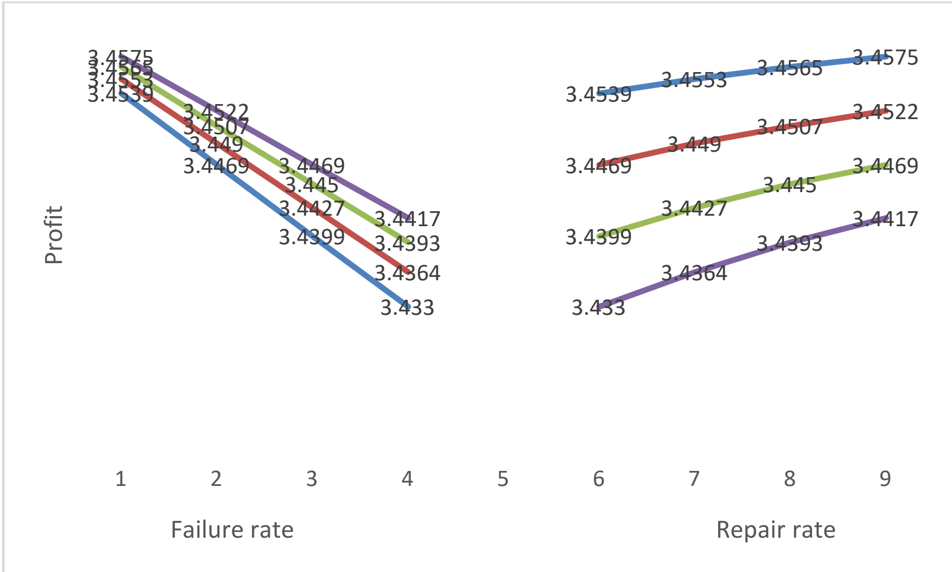

Figure 9: Effect of Failure rate α_3 and Repair rate β_3 on profit

Finally, in order to compare the system availability and profit function of the system, we only compare the availability and profit functions of subsystems A and B respectively. The profit function of subsystem A is drastically more than the profit function of subsystem B with increase in the values of repair rates as evident from figure 7 and figure 8. This simulation suggests that subsystem A gives the highest expected profit function. Thus, subsystem A is the most critical component in the system which needs special attention and careful observation.

VI. Conclusion

In this work, we constructed four different series-parallel systems consisting of subsystems *A*, *B, C* and *D.* Subsystems *A, B* and *C* consist of single unit each in series while subsystem *D* consists of four units arranged in parallel. We developed the explicit expressions for the availability, busy period and profit function for the four systems and performed a comparative analysis. Parametric investigation of various system parameters on system availability and profit function has been captured. It is interesting to see that as the repair rates of each subsystem increase; the availability and profit also increase. This implies that the major repair should be invoked to reduce the failure rate and maximize the system profit.

References Références Referencias

1. Gahlot M, Singh VV, Ayagi HI, Goel CK (2018). Performance assessmentof repairable system in series configuration under differenttypes of failure and repair policies using copula linguistics. Int J Reliab Saf 12(4):348–363. https://doi.org/10.1504/IJRS.2018.09605 9.
2. Kadyan, M. S. and Kumar, R. (2015). Availability and profit analysis of a feeding system in the sugar industry, International Journal of system assurance engineering andmanagement, 8, 301–316.
3. Kumar N., Pant, S., and Singh, S.B. (2017). Availability and cost analysis of an engineering system involving subsystems in series configuration. International of Quality and Reliability Management. 34(6), pp.879-894.

Notes

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4. Malik, S., and Tewari, P.C. (2018). Performance modeling and maintenance priorities decision for water flow system of a coal based thermal power plant. International Journal of Quality and Reliability Management. 35(4), pp. 996-1010.
5. Niwas, R., and Garg, H. (2018). An approach for analysing the reliability and profit of an industrial system based on the cost-free warranty policy. Journal of the Brazilian Society of Mechanical Sciences and Engineering, Vol. 40, pp. 1-9.
6. Rao, M. S. and Naikan, V. N. A. (2015). A system dynamics model for transientavailability modeling of repairable redundant systems, International Journal of PerformabilityEngineering, 11, 203-2011.
7. Sanusi, A., Yusuf, I., Mamuda. B.Y. (2020). Performance evaluation of an industrial configured as series-parallel system, Journal of Mathematical and Computational Science. 10, pp. 692-712.
8. Singh VV, Ayagi HI (2018) Stochastic analysis of a complex system under preemptive resume repair policy using Gumbel-Hougaard family of copula. Int J Math Oper Res 12(2):273–292.
9. Singh VV, Gulati J, Rawal DK, Goel CK (2016) Performance analysis of complex system in series configuration under different failure and repair discipline using copula. Int J Reliab Qual Saf Eng 23(2):812–832.
10. Yang L, Guo Y, Wang Q (2019) Reliability assessment of a hierarchical system subjected to inconsistent priors and multilevel data. IEEE Trans Reliab 68(4):277–292.
11. Yusuf, I. (2015). Availability modeling and evaluation of repairable system subject to minor deterioration under imperfect repairs.International Journal of Mathematics in Operational Research. Vol. 7, pp. 42-51.
12. Yusuf I (2016) Reliability modeling of a parallel system with a supporting device and two types preventive maintenance. Int J Oper Res 25(3):269–287

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Authors can submit papers and articles in an acceptable file format: MS Word (doc, docx), LaTeX (.tex, .zip or .rar including all of your files), Adobe PDF (.pdf), rich text format (.rtf), simple text document (.txt), Open Document Text (.odt), and Apple Pages (.pages). Our professional layout editors will format the entire paper according to our official guidelines. This is one of the highlights of publishing with Global Journals—authors should not be concerned about the formatting of their paper. Global Journals accepts articles and manuscripts in every major language, be it Spanish, Chinese, Japanese, Portuguese, Russian, French, German, Dutch, Italian, Greek, or any other national language, but the title, subtitle, and abstract should be in English. This will facilitate indexing and the pre-peer review process.

The following is the official style and template developed for publication of a research paper. Authors are not required to follow this style during the submission of the paper. It is just for reference purposes.

Manuscript Style Instruction (Optional)

- Microsoft Word Document Setting Instructions.
- Font type of all text should be Swis721 Lt BT.
- Page size: 8.27" x 11'", left margin: 0.65, right margin: 0.65, bottom margin: 0.75.
- Paper title should be in one column of font size 24.
- Author name in font size of 11 in one column.
- Abstract: font size 9 with the word "Abstract" in bold italics.
- Main text: font size 10 with two justified columns.
- Two columns with equal column width of 3.38 and spacing of 0.2.
- First character must be three lines drop-capped.
- The paragraph before spacing of 1 pt and after of 0 pt.
- Line spacing of 1 pt.
- Large images must be in one column.
- The names of first main headings (Heading 1) must be in Roman font, capital letters, and font size of 10.
- The names of second main headings (Heading 2) must not include numbers and must be in italics with a font size of 10.

Structure and Format of Manuscript

The recommended size of an original research paper is under 15,000 words and review papers under 7,000 words. Research articles should be less than 10,000 words. Research papers are usually longer than review papers. Review papers are reports of significant research (typically less than 7,000 words, including tables, figures, and references)

A research paper must include:

a) A title which should be relevant to the theme of the paper.
b) A summary, known as an abstract (less than 150 words), containing the major results and conclusions.
c) Up to 10 keywords that precisely identify the paper's subject, purpose, and focus.
d) An introduction, giving fundamental background objectives.
e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition, sources of information must be given, and numerical methods must be specified by reference.
f) Results which should be presented concisely by well-designed tables and figures.
g) Suitable statistical data should also be given.
h) All data must have been gathered with attention to numerical detail in the planning stage.

Design has been recognized to be essential to experiments for a considerable time, and the editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned unrefereed.

i) Discussion should cover implications and consequences and not just recapitulate the results; conclusions should also be summarized.
j) There should be brief acknowledgments.
k) There ought to be references in the conventional format. Global Journals recommends APA format.

Authors should carefully consider the preparation of papers to ensure that they communicate effectively. Papers are much more likely to be accepted if they are carefully designed and laid out, contain few or no errors, are summarizing, and follow instructions. They will also be published with much fewer delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and suggestions to improve brevity.

Format Structure

It is necessary that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

All manuscripts submitted to Global Journals should include:

Title

The title page must carry an informative title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) where the work was carried out.

Author details

The full postal address of any related author(s) must be specified.

Abstract

The abstract is the foundation of the research paper. It should be clear and concise and must contain the objective of the paper and inferences drawn. It is advised to not include big mathematical equations or complicated jargon.

Many researchers searching for information online will use search engines such as Google, Yahoo or others. By optimizing your paper for search engines, you will amplify the chance of someone finding it. In turn, this will make it more likely to be viewed and cited in further works. Global Journals has compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Keywords

A major lynchpin of research work for the writing of research papers is the keyword search, which one will employ to find both library and internet resources. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining, and indexing.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy: planning of a list of possible keywords and phrases to try.

Choice of the main keywords is the first tool of writing a research paper. Research paper writing is an art. Keyword search should be as strategic as possible.

One should start brainstorming lists of potential keywords before even beginning searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in a research paper?" Then consider synonyms for the important words.

It may take the discovery of only one important paper to steer in the right keyword direction because, in most databases, the keywords under which a research paper is abstracted are listed with the paper.

Numerical Methods

Numerical methods used should be transparent and, where appropriate, supported by references.

Abbreviations

Authors must list all the abbreviations used in the paper at the end of the paper or in a separate table before using them.

Formulas and equations

Authors are advised to submit any mathematical equation using either MathJax, KaTeX, or LaTeX, or in a very high-quality image.

Tables, Figures, and Figure Legends

Tables: Tables should be cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g., Table 4, a self-explanatory caption, and be on a separate sheet. Authors must submit tables in an editable format and not as images. References to these tables (if any) must be mentioned accurately.

Figures

Figures are supposed to be submitted as separate files. Always include a citation in the text for each figure using Arabic numbers, e.g., Fig. 4. Artwork must be submitted online in vector electronic form or by emailing it.

Preparation of Eletronic Figures for Publication

Although low-quality images are sufficient for review purposes, print publication requires high-quality images to prevent the final product being blurred or fuzzy. Submit (possibly by e-mail) EPS (line art) or TIFF (halftone/ photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Avoid using pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings). Please give the data for figures in black and white or submit a Color Work Agreement form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution at final image size ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs): >350 dpi; figures containing both halftone and line images: >650 dpi.

Color charges: Authors are advised to pay the full cost for the reproduction of their color artwork. Hence, please note that if there is color artwork in your manuscript when it is accepted for publication, we would require you to complete and return a Color Work Agreement form before your paper can be published. Also, you can email your editor to remove the color fee after acceptance of the paper.

Tips for Writing a Good Quality Science Frontier Research Paper

Techniques for writing a good quality Science Frontier Research paper:

1. Choosing the topic: In most cases, the topic is selected by the interests of the author, but it can also be suggested by the guides. You can have several topics, and then judge which you are most comfortable with. This may be done by asking several questions of yourself, like "Will I be able to carry out a search in this area? Will I find all necessary resources to accomplish the search? Will I be able to find all information in this field area?" If the answer to this type of question is "yes," then you ought to choose that topic. In most cases, you may have to conduct surveys and visit several places. Also, you might have to do a lot of work to find all the rises and falls of the various data on that subject. Sometimes, detailed information plays a vital role, instead of short information. Evaluators are human: The first thing to remember is that evaluators are also human beings. They are not only meant for rejecting a paper. They are here to evaluate your paper. So present your best aspect.

2. Think like evaluators: If you are in confusion or getting demotivated because your paper may not be accepted by the evaluators, then think, and try to evaluate your paper like an evaluator. Try to understand what an evaluator wants in your research paper, and you will automatically have your answer. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

3. Ask your guides: If you are having any difficulty with your research, then do not hesitate to share your difficulty with your guide (if you have one). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work, then ask your supervisor to help you with an alternative. He or she might also provide you with a list of essential readings.

4. Use of computer is recommended: As you are doing research in the field of science frontier then this point is quite obvious. Use right software: Always use good quality software packages. If you are not capable of judging good software, then you can lose the quality of your paper unknowingly. There are various programs available to help you which you can get through the internet.

5. Use the internet for help: An excellent start for your paper is using Google. It is a wondrous search engine, where you can have your doubts resolved. You may also read some answers for the frequent question of how to write your research paper or find a model research paper. You can download books from the internet. If you have all the required books, place importance on reading, selecting, and analyzing the specified information. Then sketch out your research paper. Use big pictures: You may use encyclopedias like Wikipedia to get pictures with the best resolution. At Global Journals, you should strictly follow here.

6. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right? It is a good habit which helps to not lose your continuity. You should always use bookmarks while searching on the internet also, which will make your search easier.

7. Revise what you wrote: When you write anything, always read it, summarize it, and then finalize it.

8. Make every effort: Make every effort to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in the introduction—what is the need for a particular research paper. Polish your work with good writing skills and always give an evaluator what he wants. Make backups: When you are going to do any important thing like making a research paper, you should always have backup copies of it either on your computer or on paper. This protects you from losing any portion of your important data.

9. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several unnecessary diagrams will degrade the quality of your paper by creating a hodgepodge. So always try to include diagrams which were made by you to improve the readability of your paper. Use of direct quotes: When you do research relevant to literature, history, or current affairs, then use of quotes becomes essential, but if the study is relevant to science, use of quotes is not preferable.

10. Use proper verb tense: Use proper verb tenses in your paper. Use past tense to present those events that have happened. Use present tense to indicate events that are going on. Use future tense to indicate events that will happen in the future. Use of wrong tenses will confuse the evaluator. Avoid sentences that are incomplete.

11. Pick a good study spot: Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

12. Know what you know: Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

13. Use good grammar: Always use good grammar and words that will have a positive impact on the evaluator; use of good vocabulary does not mean using tough words which the evaluator has to find in a dictionary. Do not fragment sentences. Eliminate one-word sentences. Do not ever use a big word when a smaller one would suffice.

Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

14. Arrangement of information: Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

15. Never start at the last minute: Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

16. Multitasking in research is not good: Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

17. Never copy others' work: Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

18. Go to seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

19. Refresh your mind after intervals: Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.

20. Think technically: Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

21. Adding unnecessary information: Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

22. Report concluded results: Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

23. Upon conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

The introduction: This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear: Adhere to recommended page limits.

Mistakes to avoid:

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

Title page:

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

Abstract: This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

Approach:

- o Single section and succinct.
- o An outline of the job done is always written in past tense.
- o Concentrate on shortening results—limit background information to a verdict or two.
- o Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

Introduction:

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.

The following approach can create a valuable beginning:

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

Approach:

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

Procedures (methods and materials):

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

Materials may be reported in part of a section or else they may be recognized along with your measures.

Methods:

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

What to stay away from:

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

Figures and tables:

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

Discussion:

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- o You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- o Give details of all of your remarks as much as possible, focusing on mechanisms.
- o Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- o One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- o Recommendations for detailed papers will offer supplementary suggestions.

Approach:

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

Describe generally acknowledged facts and main beliefs in present tense.

The Administration Rules

Administration Rules to Be Strictly Followed before Submitting Your Research Paper to Global Journals Inc.

Please read the following rules and regulations carefully before submitting your research paper to Global Journals Inc. to avoid rejection.

Segment draft and final research paper: You have to strictly follow the template of a research paper, failing which your paper may get rejected. You are expected to write each part of the paper wholly on your own. The peer reviewers need to identify your own perspective of the concepts in your own terms. Please do not extract straight from any other source, and do not rephrase someone else's analysis. Do not allow anyone else to proofread your manuscript.

Written material: You may discuss this with your guides and key sources. Do not copy anyone else's paper, even if this is only imitation, otherwise it will be rejected on the grounds of plagiarism, which is illegal. Various methods to avoid plagiarism are strictly applied by us to every paper, and, if found guilty, you may be blacklisted, which could affect your career adversely. To guard yourself and others from possible illegal use, please do not permit anyone to use or even read your paper and file.

Criterion for Grading a Research Paper (Compilation) by Global Journals

Please note that following table is only a Grading of "Paper Compilation" and not on "Performed/Stated Research" whose grading solely depends on Individual Assigned Peer Reviewer and Editorial Board Member. These can be available only on request and after decision of Paper. This report will be the property of Global Journals.

Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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ISSN 9755896