

# GLOBAL JOURNAL

OF SCIENCE FRONTIER RESEARCH: F

## Mathematics and Decision Science

Bi-Directional Infinity Box

Reduced Symmetrizer Equation

} Highlights {

Quantum Perspective Model

Analysis of Crude Oil Prodction

Discovering Thoughts, Inventing Future

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## Bi-Directional Infinity Box

By Jayant Hooda

**Abstract-** The following article introduces a new concept of Bi-directional Box. The concept is explained by simultaneously calculating a new result: sum of all the natural multiples of each natural number upto infinity. The concept of Bi-directional Box helps to organise multiple infinite series and study different patterns in multiple infinite series. This Bi-directional box can be converted into a triangle by rearranging the already organised terms of the initial box. Similar to Pascal's triangle, this box has many patterns and properties instilled in it too. Along with the initial standard box, infinite such boxes can be made depending upon the sequence/series in which the pattern is to be observed: an example with different sequences is provided at the end of the article.

**Keywords:** *bi-directional infinite box, infinite series, sum of all natural multiples, pascal's triangle.*

**GJSFR-F Classification:** MSC 2010: 26A12



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# Bi-Directional Infinity Box

Jayant Hooda

**Abstract-** The following article introduces a new concept of Bi-directional Box. The concept is explained by simultaneously calculating a new result: sum of all the natural multiples of each natural number upto infinity. The concept of Bi-directional Box helps to organise multiple infinite series and study different patterns in multiple infinite series. This Bi-directional box can be converted into a triangle by rearranging the already organised terms of the initial box. Similar to Pascal's triangle, this box has many patterns and properties instilled in it too. Along with the initial standard box, infinite such boxes can be made depending upon the sequence/series in which the pattern is to be observed: an example with different sequences is provided at the end of the article.

**Keywords:** bi-directional infinite box, infinite series, sum of all natural multiples, pascal's triangle.

## I. RESULT

*The Sum of all multiples combined of every natural number upto infinity is  $1/120$ .*

The series looks this:

$$(1+2+3+4...) + (2+4+6+8...) + (3+6+9+12...) + (4+8+12+16...) \dots$$

In other words, the sum of infinite terms of a series, each of whose terms is further an infinite sum - the first term being the sum of all natural multiples of 1, second term being the sum of all natural multiple of 2, the third term being the sum of all natural multiple of 3, and so on - is  $1/120$ .

To solve such kind of series and observe patterns in them *bi-directional infinity boxes* can be used.

To understand this particular series, first, we need to understand the concept of *bi-directional infinity box*.

(Col 1)



Row 1: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ..., infinity

Row 2: 2, 4, 6, 8, 10, 12, ..., infinity

3, 6, 9, 12, 15, 18, ..., infinity

4, 8, 12, 16, 20, 24, 28, ..., infinity

5, 10, 15, 20, 25, 30, 35, ..., infinity

6, 12, 18, 24, 30, 36, ..., infinity

.

.

.

Infinity

Author: e-mail: [workmail92431@gmail.com](mailto:workmail92431@gmail.com)

Row 1 contains multiples of 1 upto infinity

Row 2 contains multiples of 2 upto infinity

And so on, neither the number of rows ends because we have to take multiple of each natural number upto infinity into consideration, nor the terms in a row ends because we have to take every multiple that exists(infinite) of each natural number into consideration. Thus, if considered as a box, and row 1 and column 1 as its sides, then we can call it a *bi-directional infinity box*(both sides extending to infinity).

In this way, we have organised all the multiples upto infinity of each natural number upto infinity.

This *bi-directional infinity box* can be rearranged into a triangle in 2 ways, one of which will help us to get our result.

(Col 1)



Row 1: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ..., infinity  
 Row 2: 2, 4, 6, 8, 10, 12, ..., infinity  
 3, 6, 9, 12, 15, 18, ..., infinity  
 4, 8, 12, 16, 20, 24, 28, ..., infinity  
 5, 10, 15, 20, 25, 30, 35, ..., infinity  
 6, 12, 18, 24, 30, 36, ..., infinity  
 .  
 .  
 .  
 Infinity



1  
 2 2  
 3 4 3  
 4 6 6 4  
 5 8 9 8 5  
 6 10 12 12 10 6

Form 1.

1  
 2 4 2  
 3 6 9 6 3  
 4 8 12 16 12 8 4  
 5 10 15 20 25 20 15 10 5  
 6 12 18 24 30 36 30 24 18 12 6

Form 2.

Here, the diagonal of the bi - directional box is transitioned into perpendicular height form: where the terms at the diagonal of initial box are kept adjacent while conversion, and the terms to the finite ends from a particular diagonal term are symmetrically arranged on both sides parallel to the base of triangle in straight line. In the other form of conversion, the diagonal terms are not adjacent, and they are



arranged in a way similar to that of in pascal's triangle. Each of these forms contain several unique patterns instilled in them, but for now, we will use the 1st form.

*The sum of each row of this triangle is  $n^3$ .*

	Sum of Numbers In a row	
1	= 1	= $1^3$
2 4 2	= 8	= $2^3$
3 6 9 6 3	= 27	= $3^3$
4 8 12 16 12 8 4	= 64	= $4^3$
5 10 15 20 25 20 15 10 5	= 125	= $5^3$
6 12 18 24 30 36 30 24 18 12 6	= 216	= $6^3$

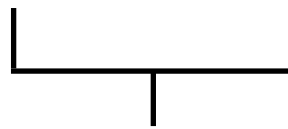
*Proof:*

$S_n$  = Sum of numbers in nth row

$$= n + 2n + 3n + \dots + n^2 + (n-1)n + (n-2)n + \dots + n$$



n numbers



(n - 1) numbers

$$\begin{aligned}
 &= n(1 + 2 + 3 + \dots + n) + n[1 + 2 + 3 + \dots + (n-1)] \\
 &= n \cdot n(n+1)/2 + n(n-1)(n)/2 \quad (\text{sum of first } n \text{ natural numbers} = n(n+1)/2) \\
 &= (n^2)[n+1+n-1]/2 \\
 &= n^3
 \end{aligned}$$

To get the sum of all the terms of this triangle, we will now add the sum of all rows. According to the proof above:

Sum of terms in nth row =  $n^3$

Sum of row 1 =  $1^3$

Sum of row 2 =  $2^3$

And so on.

On adding all the rows, we get:

$$1^3 + 2^3 + 3^3 + \dots = 1/120$$

We have now reduced the initial Bi-directional box into this  $(1^3 + 2^3 + 3^3 + \dots)$

We know that the result of this series is  $1/120$  (initially stated by Ramanujan, and later it was proved in many ways). Some of the proofs can be found here:

<https://math.stackexchange.com/questions/2052233/how-does-one-get-that-13233343-cdots-frac1120/2052282>

From one of the given proofs, we end up with the following form for the Zeta function whenever  $s \in \mathbb{N}$ ,  $s > 0$ :

$$\zeta(-s) = \frac{1}{1 - 2^{1+s}} \lim_{r \rightarrow 1^-} \left( \underbrace{r \cdot \frac{d}{dr} r \cdot \frac{d}{dr} \dots r \cdot \frac{d}{dr} \frac{-1}{1+r}}_s \right)$$

Below are some worked out values; however, for our purpose, we will go with  $s = 3$

$$\zeta(-1) = \frac{1}{1 - 2^{1+1}} \lim_{r \rightarrow 1^-} \left( r \cdot \frac{d}{dr} \frac{-1}{1+r} \right) = -\frac{1}{3} \lim_{r \rightarrow 1^-} \left( \frac{r}{(1+r)^2} \right) = -\frac{1}{12}$$

$$\begin{aligned} \zeta(-2) &= \frac{1}{1 - 2^{1+2}} \lim_{r \rightarrow 1^-} \left( r \cdot \frac{d}{dr} r \cdot \frac{d}{dr} \frac{-1}{1+r} \right) = -\frac{1}{7} \lim_{r \rightarrow 1^-} \left( r \cdot \frac{d}{dr} \frac{r}{(1+r)^2} \right) \\ &= -\frac{1}{7} \lim_{r \rightarrow 1^-} \left( \frac{r - r^2}{(1+r)^3} \right) = 0 \end{aligned}$$

$$\begin{aligned} \zeta(-3) &= \frac{1}{1 - 2^{1+3}} \lim_{r \rightarrow 1^-} \left( r \cdot \frac{d}{dr} r \cdot \frac{d}{dr} r \cdot \frac{d}{dr} \frac{-1}{1+r} \right) = -\frac{1}{15} \lim_{r \rightarrow 1^-} \left( r \cdot \frac{d}{dr} \frac{r - r^2}{(1+r)^3} \right) \\ &= -\frac{1}{15} \left( \frac{r - 4r^2 + r^3}{(1+r)^4} \right) = \frac{1}{120} \end{aligned}$$

For  $s = 3$ , we get  $1/120$

Thus, we can say that:

The sum of all multiples upto infinity of each natural number upto infinity is  $1/120$

OR

Sum of all multiples combined of every natural number upto infinity is  $1/120$ .

The terms in a bi-directional infinity box can be different according to the series which we are dealing with and the patterns we are trying to observe. The one we discussed above is the standard bi-directional infinity box. Another example with of series converted into bi-directional infinity box is

(Col 1)



Row 1:  $1^{**1}, 1^{**2}, 1^{**3}, 1^{**4}, \dots$  going like this upto infinity  
 Row 2:  $2^{**1}, 2^{**2}, 2^{**3}, 2^{**4}, 2^{**5}, 2^{**6} \dots$  infinity  
 $3^{**1}, 3^{**2}, 3^{**3}, 3^{**4}, \dots$  infinity  
 $\dots \dots \dots$  infinity  
 $\dots \dots \dots$  infinity  
 $\dots \dots \dots$  infinity  
 Infinity

Where Row 1 contains a sequence of  $1^n$ , Row 2 contains an individual sequence of  $2^n$ , Row 3 contains an individual sequence of  $3^n$ , and so on ( $n$  belongs to all the natural numbers).

## II. CONCLUSION

Such conversions can be helpful in organising multiple infinite series or for even doing an organised study of patterns and results when multiple series/sequences are combined. Along with it, the article helps us in understanding in a new form of numerical distribution (Bi-directional Infinite Boxes) and its two forms of triangle, on which further development can be done.

## REFERENCES RÉFÉRENCES REFERENCIAS

1. <https://math.stackexchange.com/questions/2052233/how-does-one-get-that-13233343-cdots-frac11202052282>
2. Simply Beautiful Art (<https://math.stackexchange.com/users/272831/simply-beautiful-art>), How does one get that  $1^3+2^3+3^3+4^3+\dots=\frac{1}{120}$ ?, URL (version: 2016-12-10): <https://math.stackexchange.com/q/2052472>

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# With Respect to Quantum Perspective Model, Can Euler Numbers be Related to Biochemistry?

By Tahir Ölmez  
*Selçuk University*

**Abstract-** This article researches whether there is a link between Euler's numbers and genetic codes. At first, the sum of the numbers of the first fifteen "15" digits of Euler's numbers after the comma are converted to bases in genetic codes. Secondly, after the comma, Euler's numbers with eighteen fifteen groups are converted to nucleotide bases. So, the results obtained by this way are expressed as nucleotide bases (A, T, C, G, U). (A)Adenine, (T)Thymine, (C)Cytosine, (G),Guanine, (U)Uracil. Thirdly, the search result is similar to ZEBRAFISH-DANIO RERIO, and even bat coronavirus after the NCBI (National Biotechnology Information Center) searched this sequence "AUGUUGAUAUTAAUCATC". Fourthly, the genetic codes of Zebrafish have been proven to be very similar to human genetic codes. Fifthly, multiple spawning of these fish species also means that Euler's numbers are increasing. In sum, the relationship between the Euler's numbers in mathematical science and the atomic weights of atomic elements in genetic codes also shed lights on Biochemistry.

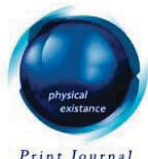
**Keywords:** zebrafish-danio rerio, biochemistry, euler's numbers, bat coronavirus, quantum perspective model, and NCBI (national biotechnology information center).

**GJSFR-F Classification:** MSC 2010: 35Q31



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1. Farrell R E. RNA Methodologies A Laboratory Guide For Isolation and Characterization. 4th Edition, Amsterdam Elsevier Acad. Press, Pennsylvania State University York, 2010; 704-710.

# With Respect to Quantum Perspective Model, Can Euler Numbers be Related to Biochemistry?

Tahir Ölmez

**Abstract-** This article researches whether there is a link between Euler's numbers and genetic codes. At first, the sum of the numbers of the first fifteen "15" digits of Euler's numbers after the comma are converted to bases in genetic codes. Secondly, after the comma, Euler's numbers with eighteen fifteen groups are converted to nucleotide bases. So, the results obtained by this way are expressed as nucleotide bases (A, T, C, G, U). (A)Adenine, (T)Thymine, (C)Cytosine, (G),Guanine, (U)Uracil. Thirdly, the search result is similar to ZEBRAFISH-DANIO RERIO, and even bat coronavirus after the NCBI (National Biotechnology Information Center) searched this sequence "AUGUUGAUATAAUCATC". Fourthly, the genetic codes of Zebrafish have been proven to be very similar to human genetic codes. Fifthly, multiple spawning of these fish species also means that Euler's numbers are increasing. In sum, the relationship between the Euler's numbers in mathematical science and the atomic weights of atomic elements in genetic codes also shed lights on Biochemistry.

**Keywords:** zebrafish-danio rerio, biochemistry, euler's numbers, bat coronavirus, quantum perspective model, and NCBI (national biotechnology information center).

## I. EULER NUMBERS

Euler's numbers are e:

2.7182818284590452353602874713526624977572470936999595749669676277240766303535  
47594571382178525166427427466391932003059921817413596629043572900334295260595  
63073813232862794349076323382988075319525101901157383418793070215408914993488  
4167509244761460668082264800168477411853 [1]

## II. METHODS AND DISCUSSION

The chemical structures of bases include Carbon(C), Nitrogen (N), Oxygen (O), and Hydrogen (H). Calculation of bases with chemical atoms (See also Table-1). (Ölmez T, 2020).

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**Table 1:** Representation of nucleotide bases (A, T, C, G and U) in chemical atoms

ATOMS / NUCLEOTIDE BASES	C=6	H=1	O=8	N=7	SUM
ADENINE: C <sub>5</sub> H <sub>5</sub> N <sub>5</sub>	5	5	-	5	70
THYMINE: C <sub>5</sub> H <sub>6</sub> N <sub>2</sub> O <sub>2</sub>	5	6	2	2	66
CYTOSINE: C <sub>4</sub> H <sub>5</sub> N <sub>3</sub> O <sub>1</sub>	4	5	1	3	64
GUANINE: C <sub>5</sub> H <sub>5</sub> N <sub>5</sub> O <sub>1</sub>	5	5	1	5	78
URACIL : C <sub>4</sub> H <sub>4</sub> N <sub>2</sub> O <sub>2</sub>	4	4	2	2	58

The atomic numbers of them: Carbon(C): 6, Nitrogen (N): 7, Oxygen (O): 8, Hydrogen (H): 1 (Wieser E M et al, 2013).

The chemical structures of bases (A, T, C, G, and U) are shown at below. (Ölmez T, 2020)

- (A) Adenine: C<sub>5</sub>H<sub>5</sub>N<sub>5</sub>:70;  
 (T) Thymine: C<sub>5</sub>H<sub>6</sub>N<sub>2</sub>O<sub>2</sub>:66,  
 (C) Cytosine: C<sub>4</sub>H<sub>5</sub>N<sub>3</sub>O<sub>1</sub>:64,  
 (G) Guanine: C<sub>5</sub>H<sub>5</sub>N<sub>5</sub>O<sub>1</sub>:78, and  
 (U) Uracil: C<sub>4</sub>H<sub>4</sub>N<sub>2</sub>O<sub>2</sub>: 58  
 (Lodish H et al, 2018).

### III. CALCULATION OF EULER NUMBERS AND GENETIC CODES

Euler's numbers are e:

2.7182818284590452353602874713526624977572470936999595749669676277240766303535  
 47594571382178525166427427466391932003059921817413596629043572900334295260595  
 63073813232862794349076323382988075319525101901157383418793070215408914993488  
 4167509244761460668082264800168477411853 [1]

Euler's numbers fifteen groups can be shown as [AUGUUGAUAUTAAUCATC]

The first fifteen groups of Euler's numbers after the comma:

$7+1+8+2+8+1+8+2+8+4+5+9+0+4+5+9+0+4+5=72$ . Just like as in Adenine (A): 70.

The second fifteen groups of Euler's numbers after the comma:

$2+3+5+3+6+0+2+8+7+4+7+1+3+5+2=58$ . Just like as in Uracil (U): 58.

The third fifteen groups of Euler's numbers after the comma:

$6+6+2+4+9+7+7+5+7+2+4+7+0+9+3=78$ . Just like as in Guanine (G): 78

The fourth fifteen groups of Euler's numbers after the comma:

$6+9+9+9+5+9+5+7+4+9+6+6+9+6+7=106^*$ . Just like as in Uracil (U): 58\*

The fifth fifteen groups of Euler's numbers after the comma:

$6+2+7+7+2+4+0+7+6+6+3+0+3+5+3=61$ . Just like as in Uracil (U): 58.

Ref

1. Farrell R E. RNA Methodologies A Laboratory Guide For Isolation and Characterization. 4th Edition, Amsterdam Elsevier Acad. Press, Pennsylvania State University York, 2010; 704-710.

The sixth fifteen groups of Euler's numbers after the comma:

$5+4+7+5+9+4+5+7+1+3+8+2+1+7+8=76$ . Just like as in Guanine (G): 78.

The seventh fifteen groups of Euler's numbers after the comma:

$5+2+5+1+6+6+4+2+7+4+2+7+4+6+6=67$ . Just like as in Adenine (A): 70

The eighth fifteen groups of Euler's numbers after the comma:

$3+9+1+9+3+2+0+0+3+0+5+9+9+2+1=56$ . Just like as in Uracil (U): 58

The ninth fifteen groups of Euler's numbers after the comma:

$8+1+7+4+1+3+5+9+6+6+2+9+0+4+3=68$ . Just like as in Adenine (A): 70

The tenth fifteen groups of Euler's numbers after the comma:

$5+7+2+9+0+0+3+3+4+2+9+5+2+6+0=57$ . Just like as in Uracil (U): 58.

The eleventh fifteen groups of Euler's numbers after the comma:

$5+9+5+6+3+0+7+3+8+1+3+2+3+2+8=65$ . Just like as in Thymine (T): 66

The twelfth fifteen groups of Euler's numbers after the comma:

$6+2+7+9+4+3+4+9+0+7+6+3+2+3+3=68$ . Just like as in Adenine (A): 70

The thirteenth fifteen groups of Euler's numbers after the comma:

$8+2+9+8+8+0+7+5+3+1+9+5+2+5+1=73$ . Just like as in Adenine (A): 70

The fourteenth fifteen groups of Euler's numbers after the comma:

$0+1+9+0+1+1+5+7+3+8+3+4+1+8+7=58$ . Just like as in Adenine (U): 58

The fifteenth fifteen groups of Euler's numbers after the comma:

$9+3+0+7+0+2+1+5+4+0+8+9+1+4+9=62$ . Just like as in Uracil (C): 64

The sixteenth fifteen groups of Euler's numbers after the comma:

$9+3+4+8+8+4+1+6+7+5+0+9+2+4+4=74$ . Just like as in Adenine (A): 70

The seventeenth fifteen groups of Euler's numbers after the comma:

$7+6+1+4+6+0+6+6+8+0+8+2+2+6+4=66$ . Just like as in Thymine (T): 66

The eighteenth fifteen groups of Euler's numbers after the comma:

$8+0+0+1+6+8+4+7+7+4+1+1+8+5+3=63$ . Just like as in Cytosine(C): 64

This sequence is" [AUGUUGAUAUTAAUCATC]. Let me try to explain this sequence with the "Quantum Perspective Model."For example, The first fifteen groups of Euler's numbers after comma equal to Adenine (A):68 with the lack of two"2" Hydrogen bonds(H:1).(Remember, See Table-1; (A)Adenine:70) This result could be the meaning of Euler's numbers sequence in fifteen groups. [AUGUUGAUAUTAAUCATC]. The fourth fifteen groups of Euler's numbers after the comma is regarded as Uracil (U). Because Phosphate group "PO4"(P:15\*1+O:8\*4=47) and Uracil (U): 58 and one



Hydrogen (H). Totally,  $47+58+1:106^*[5]$ . The fifth, sixth and seventh fifteen groups of Euler's numbers after the comma is UGA [Uracil, Guanine and Adenine] which also means STOP codon [13]. So, the deviations in the calculation of Euler's numbers can be derived from the Adenine (A) - Thymine (T) Hydrogen bonds because of Adenine (A) pairs with Thymine (T) by *two* hydrogen bonds. Cytosine(C) - Guanine (G) pairs with by *three* hydrogen bonds [3]. The reason for the lack of hydrogen bonds: Hydrogen bonding is a very versatile attraction. (Ölmez T, 2020) Hydrogen bonds are relatively *weak and easily broken* by increasing hardness (Farrell R E, 2010).

a) *The National Center for Biotechnology Information (NCBI) results for Euler's numbers*

After searching Euler's numbers result [AUGUUGAUAUTAAUCATC] in NCBI databases, some conceptual relationships can be found with bony fish. Types of bony fish are based on Zebra fish(DANIO RERIO) (See FIGURE -4), European star fish, blunt-snouted clingfish, pinecone soldierfish, orbiculate cardinal fish and crown of thorns star fish (See FIGURE -1). Types of other living creatures are human chromosome18, domestic cat, European eel, domestic cat, American alligator, common sunflower, wine grape, golden and black snub-nosed monkey, green monkey, Rhesus monkey, house fly, giant panda, fall armyworm, Porcine epidemic diarrhea virusan, and chinese hamster[4] (See FIGURE -2 ). The most interesting result of this research result is bat coronavirus(spike protein –chaerephon bat coronavirus ADX59458.1) [11] (See FIGURE -3 ).

<input checked="" type="checkbox"/> neogenin-like isoform X11 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2112	<a href="#">XP_022084118.1</a>
<input checked="" type="checkbox"/> neogenin-like isoform X10 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2114	<a href="#">XP_022084109.1</a>
<input checked="" type="checkbox"/> neogenin-like isoform X9 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2117	<a href="#">XP_022084103.1</a>
<input checked="" type="checkbox"/> neogenin-like isoform X8 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2130	<a href="#">XP_022084096.1</a>
<input checked="" type="checkbox"/> neogenin-like isoform X7 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2162	<a href="#">XP_022084087.1</a>
<input checked="" type="checkbox"/> neogenin-like isoform X6 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2166	<a href="#">XP_022084078.1</a>
<input checked="" type="checkbox"/> neogenin-like isoform X5 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2171	<a href="#">XP_022084069.1</a>
<input checked="" type="checkbox"/> neogenin-like isoform X4 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2180	<a href="#">XP_022084061.1</a>
<input checked="" type="checkbox"/> neogenin-like isoform X3 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2182	<a href="#">XP_022084052.1</a>
<input checked="" type="checkbox"/> neogenin-like isoform X2 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2184	<a href="#">XP_022084043.1</a>
<input checked="" type="checkbox"/> neogenin-like isoform X1 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2198	<a href="#">XP_022084033.1</a>
<input checked="" type="checkbox"/> hypothetical protein [Legionella hackeliae]	<a href="#">Legionella hackeliae</a>	32.9	32.9	83%	17	28.57%	45	<a href="#">WP_172480797.1</a>
<input checked="" type="checkbox"/> PREDICTED: uncharacterized protein LOC104816379 [Tarenaya hassleriana]	<a href="#">Tarenaya hassleriana</a>	32.9	32.9	83%	18	30.43%	101	<a href="#">XP_010543517.1</a>
<input checked="" type="checkbox"/> hypothetical protein VOLCADRAFT_58884 [Volvox carteri f. nagariensis]	<a href="#">Volvox carteri f. nagariensis</a>	32.9	32.9	94%	18	29.41%	116	<a href="#">XP_002948916.1</a>
<input checked="" type="checkbox"/> hypothetical protein FM113_14290 [Leucobacter sp. 7(1)]	<a href="#">Leucobacter sp. 7(1)</a>	32.9	32.9	83%	18	29.41%	159	<a href="#">S_JN12217.1</a>
<input checked="" type="checkbox"/> hypothetical protein [Rhodopirellula sp. JC645]	<a href="#">Rhodopirellula sp. JC645</a>	32.9	32.9	88%	18	28.57%	275	<a href="#">WP_150078846.1</a>
<input checked="" type="checkbox"/> spike protein [Porcine epidemic diarrhea virus]	<a href="#">Porcine epidemic diarrhea virus</a>	32.9	60.7	83%	18	26.67%	1386	<a href="#">AEW24858.1</a>
<input checked="" type="checkbox"/> spike protein [Chaerephon bat coronavirus/Kenya/KY41/2006]	<a href="#">Chaerephon bat coronavirus/Kenya/KY4...</a>	32.9	32.9	88%	18	31.25%	1386	<a href="#">ADX59458.1</a>
<input checked="" type="checkbox"/> hypothetical protein RSAG8_03191 [Rhizoctonia solani AG-8 WAC10335]	<a href="#">Rhizoctonia solani AG-8 WAC10335</a>	32.5	32.5	77%	22	26.67%	40	<a href="#">KDN47771.1</a>
<input checked="" type="checkbox"/> hypothetical protein [Salmonella enterica subsp. salamae]	<a href="#">Salmonella enterica subsp. salamae</a>	32.5	92.7	83%	25	21.74%	68	<a href="#">EDU0501388.1</a>
<input checked="" type="checkbox"/> hypothetical protein [Bacteroidales bacterium]	<a href="#">Bacteroidales bacterium</a>	32.5	32.5	100%	25	28.00%	78	<a href="#">MBE6332989.1</a>
<input checked="" type="checkbox"/> keratin-associated protein 17-1 [Cricetulus griseus]	<a href="#">Chinese hamster</a>	32.5	58.5	94%	25	21.05%	112	<a href="#">XP_035317626.1</a>

Figure 1: The NCBI (National Biotechnology Information Center) Result for Nucleotide Sequence "AUGUUGAUAUTAAUCATC" [4]

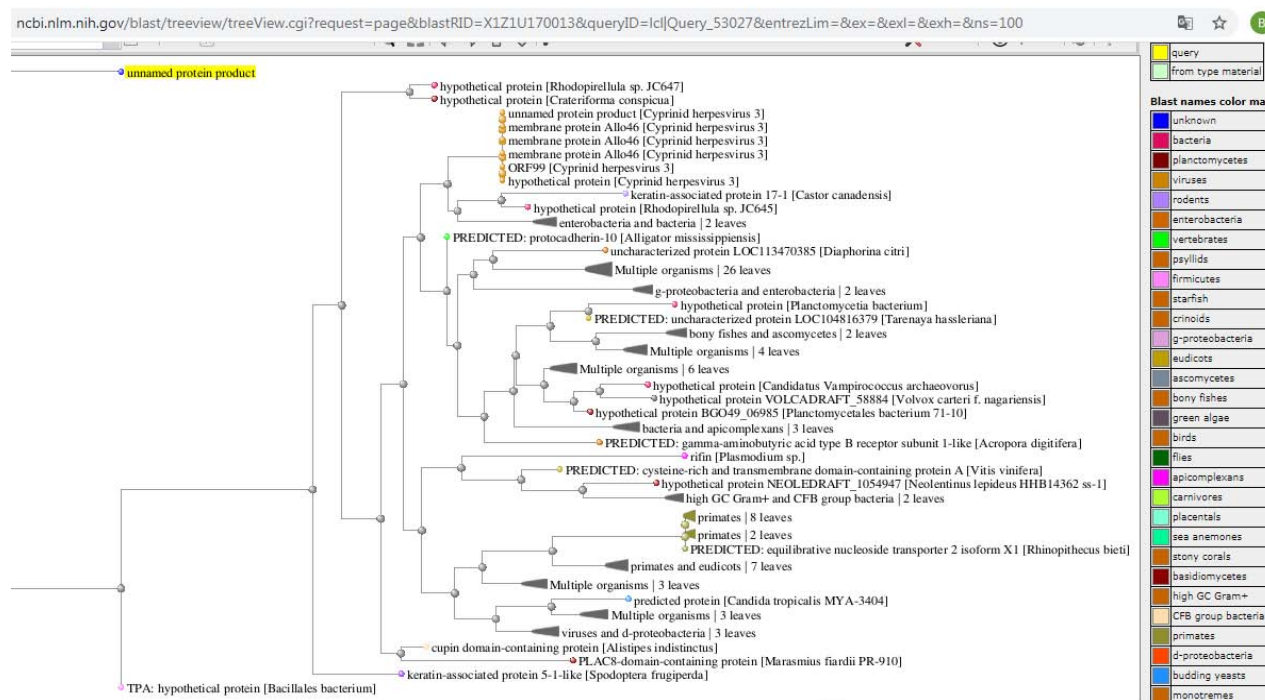


Figure 2: The NCBI (National Biotechnology Information Center) Result Blast Tree View Widget for “AUGUUGAUAUTAAUCATC” Nucleotide Sequence [4]

neogenin-like isoform X10 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2114	<a href="#">XP_022084109.1</a>
neogenin-like isoform X9 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2117	<a href="#">XP_022084103.1</a>
neogenin-like isoform X8 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2130	<a href="#">XP_022084096.1</a>
neogenin-like isoform X7 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2162	<a href="#">XP_022084087.1</a>
neogenin-like isoform X6 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2166	<a href="#">XP_022084078.1</a>
neogenin-like isoform X5 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2171	<a href="#">XP_022084069.1</a>
neogenin-like isoform X4 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2180	<a href="#">XP_022084061.1</a>
neogenin-like isoform X3 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2182	<a href="#">XP_022084052.1</a>
neogenin-like isoform X2 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2184	<a href="#">XP_022084043.1</a>
neogenin-like isoform X1 [Acanthaster planci]	<a href="#">crown-of-thorns starfish</a>	33.3	84.6	100%	13	30.00%	2198	<a href="#">XP_022084033.1</a>
hypothetical protein [Legionella hackeliae]	<a href="#">Legionella hackeliae</a>	32.9	32.9	83%	17	28.57%	45	<a href="#">WP_172480797.1</a>
PREDICTED: uncharacterized protein LOC104816379 [Tarenaya hassleriana]	<a href="#">Tarenaya hassleriana</a>	32.9	32.9	83%	18	30.43%	101	<a href="#">XP_010543517.1</a>
hypothetical protein VOLCADRAFT_58884 [Volvox carteri f. nagariensis]	<a href="#">Volvox carteri f. nagariensis</a>	32.9	32.9	94%	18	29.41%	116	<a href="#">XP_002948916.1</a>
hypothetical protein FM113_14290 [Leucobacter sp. 7(1)]	<a href="#">Leucobacter sp. 7(1)</a>	32.9	32.9	83%	18	29.41%	159	<a href="#">S_JN12217.1</a>
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spike protein [Porcine epidemic diarrhea virus]	<a href="#">Porcine epidemic diarrhea virus</a>	32.9	60.7	83%	18	26.67%	1386	<a href="#">AEW24858.1</a>
spike protein [Chaerophon bat coronavirus/Kenya/KY41/2006]	<a href="#">Chaerophon bat coronavirus/Kenya/KY4...</a>	32.9	32.9	88%	18	31.25%	1386	<a href="#">ADX59458.1</a>
hypothetical protein RSAG8_03191 [Rhizoctonia solani AG-8 WAC10335]	<a href="#">Rhizoctonia solani AG-8 WAC10335</a>	32.5	32.5	77%	23	26.67%	40	<a href="#">KDN47771.1</a>
hypothetical protein [Salmonella enterica subsp. salamae]	<a href="#">Salmonella enterica subsp. salamae</a>	32.5	92.7	83%	25	21.74%	68	<a href="#">EDU0501388.1</a>
hypothetical protein [Bacteroidales bacterium]	<a href="#">Bacteroidales bacterium</a>	32.5	32.5	100%	25	28.00%	78	<a href="#">MBE6332989.1</a>
keratin-associated protein 17-1 [Cricetus griseus]	<a href="#">Chinese hamster</a>	32.5	58.5	94%	25	21.05%	112	<a href="#">XP_035317626.1</a>

Figure 3: The NCBI (National Biotechnology Information Center) Result for bat coronavirus [4]

## IV. RESULTS

The relationships between the numerical value of the numbers of light velocities against genetic codes were researched based on the quantum perspective model (Köklü K, 2019b). Further, after the manual division of Twenty-Two (22) to seven (7) numbers, each of the *fourteen (14) number sequences* obtained after the comma was followed by approximately continuously same nucleotide base codes (Köklü K, 2019a). If you take Pi numbers as fourteen sequences a hundred times, The output of the genetic code is [ATU]. this result is similar to ZEBRAFISH- DANIO RERIO in NCBI Blast. (See FIGURE -4). [4] (NCBI: The National Center for Biotechnology). This fish is a perfect favorite example of many experiments related to biology and gene sequences. [2] The square of the speed of light is written just like those gene sequences: [UUATACCTC] or [UUAUACCTC]. After searching three times this sequence (27), the output of this nucleotide blast is similar to Zebrafish-Danio Rerio. (Köklü K, 2019b). In other words, the output of this nucleotide blast is common to Zebra fish in groups of fourteen pi numbers and Euler numbers in groups of fifteen. In sum, the common feature of the square of the speed of light and pi numbers and Euler numbers is Zebra fish. Insulin receptor (IR) signaling is thought to be important in growth and development. The role of insulin receptor signaling in Zebrafish embryogenesis has vital roles in vertebrate embryogenesis and growth [9]. Also, new genetic models have been produced to study resistance to the thyroid hormone receptor in Zebrafish [10]. Besides, even both the CAAT Box and TATA Box NCBI results in consist of bony fishes (Denticle Herring). In other words, CAAT Box and TATA Box are also related to the golden ratio. Remember, the NCBI result of Euler's numbers also consist of bony fishes (Zebrafish), too. (Ölmez T, 2020). Denticle Herring is the most primitive living clupeiform. [6]. Interestingly, it also consists of "phosphoserine phosphatase" protein - coding [7]. Let alone, the length of it's *15(fifteen)* cm. [8] Remember, this research is taken Euler's numbers as *fifteen* groups, too. Monkeys and humans have a different number of chromosomes, but they probably have the same number of genes. [12] This similarity can be regarded as a Quantum Perspective Model. In summary, this similarity may be the beginning of the mutual relations of the sciences based on the Quantum Perspective Model at minor level systems. Namely, atoms with the smallest base structure can be taken as a small unit of analysis from the same point of view as mathematical numbers. As a result, with this quantum perspective model, the relationships between chemical formulas and numbers have introduced different *paradigms* to obtain new clues.

Ref

4. Lodish H, Berk A, Zipursky S L, Matsudaira P, Baltimore D and Darnell J. Molecular Cell Biology, 6th edition, Translation: Geçkil H, Özmen M, Yeşilada Ö, Palme Publishing, New York, 2018, 294-302.

<a href="#">4_Ts1_b3v08</a>	<a href="#">Timema bart...</a>	34.2	34.2	94%	36	100.00%	353296	<a href="#">QD564569.1</a>
<a href="#">PREDICTED: Patiria miniata uncharacterized LOC119723493 (LOC119723493). transcript variant X1...</a>	<a href="#">bat star</a>	34.2	34.2	94%	36	100.00%	3973	<a href="#">XM_038194174.1</a>
<a href="#">Moraxella nonliquefaciens strain FDAARGOS_869 chromosome. complete genome</a>	<a href="#">Moraxella no...</a>	34.2	34.2	94%	36	100.00%	2272684	<a href="#">CP065728.1</a>
<a href="#">PREDICTED: Glossina fuscipes uncharacterized LOC119642928 (LOC119642928). transcript variant X...</a>	<a href="#">Glossina fusc...</a>	34.2	34.2	94%	36	100.00%	1955	<a href="#">XM_038042244.1</a>
<a href="#">PREDICTED: Glossina fuscipes uncharacterized LOC119642928 (LOC119642928). transcript variant X...</a>	<a href="#">Glossina fusc...</a>	34.2	34.2	94%	36	100.00%	2049	<a href="#">XM_038042243.1</a>
<a href="#">2_Ts1_b3v08</a>	<a href="#">Timema shep...</a>	34.2	34.2	94%	36	100.00%	113746	<a href="#">OC002461.1</a>
<a href="#">2_Ts1_b3v08</a>	<a href="#">Timema shep...</a>	34.2	34.2	94%	36	100.00%	184815	<a href="#">OC001171.1</a>
<a href="#">Darwinula stvensoni</a>	<a href="#">Darwinula ste...</a>	34.2	34.2	94%	36	100.00%	30022	<a href="#">LR903044.1</a>
<a href="#">Corylus avellana genome assembly. chromosome: ca2</a>	<a href="#">Corylus avell...</a>	34.2	136	94%	36	100.00%	50858233	<a href="#">LR899424.1</a>
<a href="#">Homo sapiens DNA. chromosome 18. nearly complete genome</a>	<a href="#">human</a>	34.2	34.2	94%	36	100.00%	77846715	<a href="#">AP023478.1</a>
<a href="#">Vagococcus carniphilus strain ATCC BAA-640 chromosome. complete genome</a>	<a href="#">Vagococcus c...</a>	34.2	34.2	94%	36	100.00%	3020833	<a href="#">CP060720.1</a>
<a href="#">Staphylococcus epidermidis strain LM087. complete genome</a>	<a href="#">Staphylococc...</a>	34.2	34.2	94%	36	100.00%	2494355	<a href="#">CP060528.1</a>
<a href="#">Torulaspora sp. CBS 2947 strain CBS2947 chromosome 1</a>	<a href="#">Torulaspora s...</a>	34.2	34.2	94%	36	100.00%	1700142	<a href="#">CP059267.1</a>
<a href="#">PREDICTED: Anguilla anguilla major histocompatibility complex class I-related gene protein-like (LOC1...</a>	<a href="#">European eel</a>	34.2	34.2	94%	36	100.00%	4118	<a href="#">XM_035425578.1</a>
<a href="#">Danio rerio genome assembly. chromosome: 21</a>	<a href="#">zebrafish</a>	34.2	66.4	94%	36	100.00%	48052324	<a href="#">LR812058.1</a>
<a href="#">Danio rerio genome assembly. chromosome: 21</a>	<a href="#">zebrafish</a>	34.2	66.4	94%	36	100.00%	43492497	<a href="#">LR812614.1</a>
<a href="#">Danio rerio strain Nadia (NA). genome assembly. chromosome: 21</a>	<a href="#">zebrafish</a>	34.2	34.2	94%	36	100.00%	44761126	<a href="#">LR812589.1</a>
<a href="#">Danio rerio genome assembly. chromosome: 8</a>	<a href="#">zebrafish</a>	34.2	34.2	94%	36	100.00%	55327790	<a href="#">LR812070.1</a>
<a href="#">Carpoglyphus lactis mitochondrion. complete genome</a>	<a href="#">prune mite</a>	34.2	34.2	94%	36	100.00%	14060	<a href="#">MN073839.1</a>
<a href="#">Felis catus Senzu DNA. chromosome: D2. American Shorthair breed</a>	<a href="#">domestic cat</a>	34.2	66.4	94%	36	100.00%	90643714	<a href="#">AP023162.1</a>
<a href="#">PREDICTED: Cyclopterus lumpus A-kinase anchoring protein 13 (akap13). transcript variant X6. mRNA</a>	<a href="#">lumpfish</a>	34.2	34.2	94%	36	100.00%	7215	<a href="#">XM_034536038.1</a>
<a href="#">PREDICTED: Cyclopterus lumpus A-kinase anchoring protein 13 (akap13). transcript variant X5. mRNA</a>	<a href="#">lumpfish</a>	34.2	34.2	94%	36	100.00%	7638	<a href="#">XM_034536036.1</a>

Figure 4: The NCBI (National Biotechnology Information Center) Result for Zebra fish [4]

## V. CONCLUSION

First, the results of this research can be summarized by obtaining Euler's numbers through the chemical structure of chemical elements. At a minor level, Euler's numbers can be thought of as an indicator of chemical formulas. One of the results of both Biochemistry and Mathematics common feature is NCBI blast results. Because these are bony fishes especially DANIO RERIO. Even not only the NCBI result of pi and square number of light is DANIO RERIO, but also NCBI result of Euler's numbers is DANIO RERIO. Since fish are one of the vertebrates that make the most eggs, this is similar to Euler's numbers in terms of multifunctional. This may be an indicator of the Euler's numbers for living things, especially fish.

Finally, at a macro level, the calculation results of Euler's numbers with chemical structures (especially A, T, C, G, and U) are related to both Biochemistry and Mathematics. Briefly, Euler's numbers are not only attributed to numbers in Mathematics but also attributed to chemical formulas of Biochemistry (Carbon(C), Nitrogen (N), Oxygen (O), and Hydrogen (H)). In summary, this similarity may be the beginning of the mutual relations of the sciences on based on Quantum Perspective Model at minor level systems. Namely, atoms with the smallest base structure can be taken as a small *unit of analysis* from the same point of view as mathematical numbers.

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## Reduced Symmetrizer Equation

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**Abstract-** This paper revisits the real symmetrizer equation in the literature to transform it into a reduced symmetrizer equation. This reduction can be accomplished by decomposing the symmetrizer of the equation. The reduced equation has a diagonal matrix as its symmetrizer and can be further decomposed into more such equations. These reduced equations are coexisting and synchronized with the original symmetrizer equation. Associated results concerning the reduced symmetrizer equation are introduced. A numerical algorithm for symmetrizer computation is developed based on these results. Typical symmetrizer problems in the literature are solved using the algorithm and the results are presented.

**Keywords:** *symmetric matrix factorization, symmetrizer equation, symmetrizer, similarity symmetrization, diagonal matrix symmetrizer.*

**GJSFR-F Classification:** MSC 2010: 15A21, 15A23, 65J10



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**Abstract-** This paper revisits the real symmetrizer equation in the literature to transform it into a reduced symmetrizer equation. This reduction can be accomplished by decomposing the symmetrizer of the equation. The reduced equation has a diagonal matrix as its symmetrizer and can be further decomposed into more such equations. These reduced equations are coexisting and synchronized with the original symmetrizer equation. Associated results concerning the reduced symmetrizer equation are introduced. A numerical algorithm for symmetrizer computation is developed based on these results. Typical symmetrizer problems in the literature are solved using the algorithm and the results are presented.

**Keywords:** symmetric matrix factorization, symmetrizer equation, symmetrizer, similarity symmetrization, diagonal matrix symmetrizer.

## I. INTRODUCTION

F.G Frobenius [6] introduced symmetric factorization of matrices in 1910. Frobenius finding was that any given matrix  $A \in R^{n \times n}$  can be factorized as  $A = S_1 S_2$ ;  $S_1 = S_1^T$ ,  $S_2 = S_2^T$ ;  $S_1, S_2 \in R^{n \times n}$ . Frobenius also stated that one among  $S_1, S_2$  or both can be nonsingular. Thus Frobenius hints that there can be left and right symmetrizers for  $A$ . Here for convenience, we choose  $S_1$  as nonsingular. Accordingly there exists a matrix  $B = S_1^{-1}$  so that  $BA = S_2$  is symmetric. The nonsingular symmetric matrix  $B$  is called a symmetrizer of  $A$  [1,3,4,6,21]. In the same year, J. Marty [15] introduced symmetrization of linear integral operators. They were the pioneers who worked in the area of symmetrizing matrices. Their work remained dormant for quite some decades till 1950s. Towards the end of 1950s, matrix symmetrization again drew the attention of many researchers. Notably among them, Professor J.L Howland [8] and his team of researchers, E.J Desautels [3], F.J. Farrel [9] and other mathematicians, Olga Taussky and Hans Zassenhaus [22], Marcus and Khan [14] actively pursued this work of symmetrizing  $A$ . Marcus and Khan [14] studied in a detailed way, a more general equation  $XA = A^T X$  where  $X$  need not be symmetric and discussed its complete solution. These studies contributed to strengthening many theoretical aspects of symmetrizers. It is reported in Dopico

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and Uhlig [4] that several attempts by many researchers to compute numerically a symmetrizer during 1960s to early half of 1970s landed into instability and were abandoned mid-way. An account of those failed attempts is presented in Uhlig [24]. It was Olga Taussky and Hans Zassenhaus [22] who gave a new definition for the Frobenius equation as  $BA = A^T B$ , or the similarity transformation  $BAB^{-1} = A^T$ . Later Taussky [21] independently developed the necessary and sufficient conditions and associated theories for  $A$  to be symmetrized and the role of symmetrizer  $B$  in the study of  $A$ . Due to these contributions, Frobenius equation became known as Taussky's Symmetrizer Equation. Sen and Venkaiah [23] proposed a solution to similarity symmetrization in the context of solving asymmetric control system problems. It is reported in [4] that method proposed in [23] can be used for low dimensional matrices  $A$ .

Adhikari's [1] work in year 2000 states that symmetrizing associated asymmetric matrices of linear systems will be convenient. We can then extend the application of established analytical tools for symmetric systems to such cases also. Illustrative examples solving dynamic linear systems in this manner are presented in [1]. What Adhikari [1] introduced is termed as equivalence symmetrization. In line with Adhikari, C.Q. Liu [11] introduced an alternative equivalence symmetrization method. Liu also worked on Adhikari's same illustrative examples in [1] for highlighting its easiness and effectiveness.

It is reported in [4] that first successful numerical computation of a symmetrizer was in 2013 by Frank Uhlig [24,25] based on an iterative linear equation solver of Huang and Nong[10]. Recently in 2016, Froilan Dopico and Frank Uhlig [4] successfully came out with more methods of computing symmetrizers using eigen data and linear equations solver.

The motivation behind [1] was that equivalence transformations are the most general class of nonsingular linear transformations achieving more generality and convenience. We here attempt to transform  $BA = A^T B$ ,  $A, B \in R^{n \times n}$ ,  $B = B^T$ ,  $\det(B(1:k, 1:k)) \neq 0$ ,  $k = 1:n$ ,  $\det(A) \neq 0$  into a reduced symmetrizer equation as  $DM = M^T D$ ;  $D, M \in R^{n \times n}$ , where  $M$  is similar to  $A$  and  $D$  is a non singular diagonal matrix. Parter and Youngs [17] considered this topic of symmetrizing  $A$  by a diagonal matrix  $D \in R^{n \times n}$ . Their work was to derive the necessary and sufficient conditions on the existence of  $D$  symmetrizing  $A$  based on probabilistic rules called conservation and reversibility laws. It was based on George Pimbley's [19] initial value problem for the multi-group transport equation. Pimbley addressed the condition on a non-negative  $A \in R^{n \times n}$  that leads to the existence of a positive diagonal matrix  $D \in R^{n \times n}$  such that  $DAD^{-1}$  is a nonnegative symmetric matrix.

The advantages and specialties of reduced symmetrizer equation are highlighted. Frobenius equation [6] is extended to this reduced symmetrizer equation to decompose it into more such reduced equations. Based on this, we prove that nonderogatory  $M$  can be expressed as a linear sum of  $n^2$  linearly independent matrices having diagonal matrix symmetrizers. A numerical algorithm is developed based on these results for computing a symmetrizer of  $A$ . This is applied

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4. F. Dopico, F. Uhlig, Computing matrix symmetrizers, part 2: New methods using eigendata and linear means; a comparison, Linear Algebra and Its Applications 504 (2016) 590–622, doi:10.1016/j.laa.2015.06.031.

for symmetrizing those coefficient matrices of undamped and damped dynamical systems from Adhikari [1]. We also discuss how Frobenius and Taussky's equations are linked to each other.

## II. REDUCED SYMMETRIZER EQUATION

Taussky and Zassenhaus [22] and Taussky [21] introduced the conditions of symmetrizability of an asymmetric matrix. According to that, for every  $A \in R^{n \times n}$  there is a nonsingular symmetric matrix  $S \in R^{n \times n}$  transforming it into  $A^T$ . The conditions are :

- i)  $A$  is the product of two symmetric matrices, one of which is positive definite.
- ii)  $A$  is similar to a symmetric matrix.
- iii)  $A^T = S^{-1}AS$  with  $S = S^T \succ 0$ .
- iv)  $A$  has real characteristic roots and a full set of characteristic vectors.

Taussky's similarity symmetrization of  $A$  and conditions are also discussed in [1,11]. Symmetrization of  $A$  can also be achieved by other type of transformations such as equivalent  $[1]$ ,  $(A + A^T)/2$ ,  $AA^T$ ,  $A^T A$ , elementary symmetrization [20] etc. Adhikari [1] classified the similarity transformation based symmetrization as first kind. He introduced the equivalence transformation based symmetrization and termed it as second kind. This second kind of symmetrization is defined as follows:

- v) A matrix  $A$  is symmetrizable of the second kind if and only if there exist two nonsingular matrices  $L, R$  such that  $\tilde{A} = L^T A R$  is symmetric.

This is a more general classification. It includes the first kind as a special case,  $L^T = R^{-1} = R^{-T}$ . Let  $A \in R^{n \times n}$  be nonsingular as well as all its leading principal submatrices,  $A(1:k, 1:k)$ ,  $k = 1:n-1$ . Recalling from [7], consider elementary reduction,  $L^T A R = D$ ,  $L, R = U$ ,  $D \in R^{n \times n}$ ,  $L, U$  are unit upper triangular matrices and  $D$  is a nonsingular diagonal matrix. Thus it is of the second kind. In a reverse way, this second kind symmetrization can be expressed in terms of diagonalization of  $A$  as well. We have  $L^T A U = D$ . Let  $B = LL^T$ . Then  $B$  is symmetric positive definite and nonsingular. Consider the nonsingular matrices  $S_1, S_2 \in R^{n \times n}$  given by  $S_1 = LB$  and  $S_2 = UB$ .

$$\tilde{A} = S_1^T A S_2 \quad (1)$$

Same  $B = LL^T$  applied in (1) may be used for simultaneously symmetrizing other coefficient matrices of a given linear system. Now from the definition of a symmetrizer in Frobenius [6], Taussky [21] and Desautels [3], we shall transform the symmetrizer equation using elementary reduction of its symmetrizer.

Let  $A \in R^{n \times n}$  be nonsingular. Let it be a symmetrizable matrix as discussed in [1,6,20]. Let  $B = B^T \in R^{n \times n}$ ,  $\det(B(1:k, 1:k)) \neq 0$ ,  $k = 1:n$  be a symmetrizer of  $A$ . Recalling from [7], we can decompose  $B$  as  $LBL^T = D$  applying Gauss Elimination (GE) and is unique. Matrices  $D, L \in R^{n \times n}$ . Here  $L$  is a unit lower triangular matrix and  $D$  is a nonsingular diagonal matrix.

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1. S. Adhikari, On symmetrizable systems of second kind, Journal of Applied Mechanics 67 (7) (2000) 797-802, doi:10.1115/1.1322038.

We may note that  $LBL^T = D$  will be stable if  $B$  is positive definite by the Cholesky theorem [7]. Recalling from Desautels [3], if  $A$  is nonderogatory,  $B$  will be diagonalizable as it is congruent to diagonal matrix of eigen values of  $A$ . Also recall from [7] that if  $B$  is column diagonally dominant, we may be able to apply GE without pivoting. If  $B$  is symmetric positive definite or semidefinite and ill-conditioned, referring to [7], we may apply permutation matrices for symmetric pivoting and diagonal dominance so that  $L(PBP^T)L^T = D$ . Here  $P$  is an appropriate permutation matrix and  $D = \text{diag}(d_1, d_2, \dots, d_n)$ ,  $d_i \geq d_{i+1}$ ,  $i = 1 : n - 1$ .

$$BA = A^T B. \quad (2)$$

Consider  $LBL^T = D$ . We shall rewrite (2) as

$$L^{-1}DL^{-T}A = A^TL^{-1}DL^{-T}. \quad (3)$$

Let  $M = L^{-T}AL^T$ . Then equation (3) can be represented as

$$DM = M^TD. \quad (4)$$

**Lemma 1.** Given a symmetrizer  $B \in R^{n \times n}$ ,  $\det(B(1:k, 1:k))$ ,  $k = 1 : n \neq 0$  of  $A \in R^{n \times n}$ , the symmetrizer equation  $BA = A^TB$  can be transformed into  $DM = M^TD$  by decomposing  $B$  as  $LBL^T = D$ ,  $L$  is unit lower triangular,  $D$  is diagonal and  $M = L^{-T}AL^T$ .

**Proof.** The proof follows from (2), (3) and (4). Uniqueness of matrices  $L, D$  as a pair follows as the decomposition  $LBL^T = D$  is unique [7].

We see that  $M$  is similar to  $A$  and  $D$  is congruent to  $B$ . From (4) it becomes clear that the symmetrizer  $B$  should be nonsingular. Frobenius [6] states that in  $A = S_1S_2$ ,  $S_1^T = S_1$ ,  $S_2^T = S_2$ , either  $S_1$  or  $S_2$  can be nonsingular so that  $A$  can have a left or right symmetrizer. In (2), we consider the case  $B = S_1$  as nonsingular for convenience. Equations (3) and (4) prove that symmetrizing  $A$  by  $B$  is synchronized with symmetrizing  $M$  by  $D$ . This leads us to,

**Lemma 2.** Consider  $B = B^T \in R^{n \times n}$ ,  $\det(B(1:k, 1:k))$ ,  $k = 1 : n \neq 0$ . Let  $LBL^T = D$ ,  $L$  is unit lower triangular and  $D$  diagonal. Then  $B$  symmetrizes  $A \in R^{n \times n}$  iff  $D$  symmetrizes  $M = L^{-T}AL^T$ .

**Proof.** We have

$$\begin{aligned} DM = M^TD &\Leftrightarrow DL^{-T}AL^T = LA^TL^{-1}D \\ &\Leftrightarrow L^{-1}DL^{-T}A = A^TL^{-1}DL^{-T} \\ &\Leftrightarrow BA = A^TB \end{aligned} \quad (5)$$

Thus (5) is a necessary and sufficient condition for similarity symmetry.



**Lemma 3.** Let  $BA = A^T B$ ,  $A, B = B^T \in R^{n \times n}$ ,  $\det(B(1 : k, 1 : k))$ ,  $k = 1 : n \neq 0$  so that  $LBL^T = D$ ,  $L$  is unit lower triangular and  $D$  is diagonal. Consider  $M = L^{-T}AL^T = (m_{ij})$ ;  $i, j = 1 : n$  where  $m_{ij}, m_{ji} \neq 0$ ,  $j = 1 : n$ . Let  $D_i = \text{diag}(x_1, x_2, \dots, x_n)$ ,  $x_i \neq 0$ ,  $i = 1 : n$  such that  $x_i m_{ij} = x_j m_{ji}$ ,  $j = 1 : n$ . Then  $D_i$  is a symmetrizer of  $M$  and  $B_i = L^{-1}D_i L^{-T}$  is a symmetrizer of  $A$ .

**Proof.** Let  $D = \text{diag}(d_1, d_2, \dots, d_n)$ . Since column- $i$  and row- $i$  of  $M$  are without zeros and from the hypothesis and by Lemma 2, we have

$$x_i m_{ij} = x_j m_{ji}, d_i m_{ij} = d_j m_{ji}, j = 1 : n. \quad (6)$$

From (6), we get

$$\begin{aligned} x_i/x_j &= d_i/d_j, j = 1 : n \Rightarrow d_j = x_j d_i/x_i, j = 1 : n. \\ &\Rightarrow x_j = x_i m_{ij}/m_{ji}, j = 1 : n. \end{aligned} \quad (7)$$

Consider any other row- $k$ , column- $k$  of  $DM$ . Applying  $d_j, j = 1 : n$  of (7)

$$\begin{aligned} d_k m_{kj} &= d_j m_{jk} \Rightarrow d_k/d_j = m_{jk}/m_{kj}; j = 1 : n \\ &\Rightarrow x_k/x_j = m_{jk}/m_{kj} \Rightarrow x_k m_{kj} = x_j m_{jk}, j = 1 : n, k = 1 : n. \end{aligned} \quad (8)$$

From (8),  $D_i$  symmetrizes  $M$ ,  $\det(D_i) \neq 0$ . So  $D_i$  is a symmetrizer of  $M$ . Hence by Lemma 2,  $B_i = L^{-1}D_i L^{-T}$  is a symmetrizer of  $A$ .

**Corollary 3.1.** If  $M$  in Lemma 3 has at least one row- $i$ , column- $i$  of non-zero entries, then it has a diagonal matrix symmetrizer  $D_i$  as derived in (7). This diagonal matrix symmetrizer  $D_i$  of  $M$  is embedded in  $M$  itself.

**Corollary 3.2.** As diagonal matrix  $D_i$  can be derived from (7) using arbitrary scalar  $x = x_i \neq 0$ , infinitely many symmetrizers of  $A$  exist against  $LBL^T = D$ .

**Remark 3.1.** This result can be used to test the symmetrizability of a given matrix  $M$  by a diagonal matrix  $D_i$  as well as to derive  $D_i$  using (7). Coupled with Lemma 2, it can also be used to test the symmetrizability of  $A$  by  $B$ .

**Lemma 4.** Let  $B = B^T \in R^{n \times n}$ ,  $\det(B(1 : k, 1 : k))$ ,  $k = 1 : n \neq 0$ ,  $LBL^T = D$ , be a symmetrizer of  $A \in R^{n \times n}$ ,  $L$  unit lower triangular and  $D$  diagonal. If  $M = L^{-T}AL^T$  has a column- $i$  and row- $i$  are without zeros, and  $D_1, D_2 \in R^{n \times n}$  are its two diagonal matrix symmetrizers, then they are linearly dependent.

**Proof.** Let  $D_1 = \text{diag}(x_1, x_2, \dots, x_n)$ ,  $D_2 = \text{diag}(y_1, y_2, \dots, y_n)$ . Then we have

$$x_j = x_i m_{ij}/m_{ji}; y_j = y_i m_{ij}/m_{ji}; j = 1, 2, \dots, n \quad (9)$$

From (9), it follows that  $D_1 = (x_i/y_i)D_2$ ,  $(x_i/y_i) \neq 0$ . Hence the result.

**Corollary 4.1.** If  $M$  is without zero entries, then the  $n$  symmetrizers  $D_i$  of  $M$  derived from column- $i$ , row- $i$ ;  $i = 1 : n$  using (7) will be linearly dependent.

**Corollary 4.2.** Let  $A \in R^{n \times n}$  and  $B_i \in R^{n \times n} = L^{-1}D_iL^{-T}, i = 1 : n$  are  $n$  diagonalizable symmetrizers of  $A$  where  $L$  is a unit lower triangular matrix and  $D_i, i = 1 : n$  are diagonal matrices. Then  $B_i, i = 1 : n$  are linearly dependent.

*Proof.* Let  $M = L^{-T}AL^T$ . By Lemma 2,  $D_i$  are symmetrizers of  $M$ . So by Lemma 4, they are linearly dependent and hence  $B_i, i = 1 : n$  are so.

**Corollary 4.3.** Let  $A, B = B^T \in R^{n \times n}$ . Let  $B$  be diagonalizable symmetrizer of  $A$  so that  $B = L^{-1}DL^{-T}$  and  $BA = A^TB$ ,  $L$  a unit lower triangular matrix and  $D$  a nonsingular diagonal matrix. Let  $D = \sum_{i=1}^n \alpha_i D_i$  where  $D_i, i = 1 : n$  constitute a basis of subspace of all diagonal matrices of  $R^{n \times n}$ . Consider  $B = \sum_{i=1}^n \alpha_i B_i$  where  $B_i = L^{-1}D_iL^{-T}, i = 1 : n$ . Then  $B_i, D_i, i = 1 : n$  are not symmetrizers of  $A$  and  $M = L^{-T}AL^T$  respectively.

**Remark 4.1.** From Corollary 4.3, we see that in several occasions of handling  $B_i, i = 1 : n$ , we may miss  $L$  but if we use Lemma 3, this will not happen.

**Remark 4.2.** Reduced equation  $DM = M^TD$  is more convenient to handle than  $BA = A^TB$  and from equation (5), these are inter-dependent. As the decomposition  $LBL^T = D$  is unique in Lemma 4,  $M = L^{-T}AL^T$  has only one linearly independent diagonal matrix symmetrizer  $D$ . If  $M_1, M_2 \in R^{n \times n}$  are linearly dependent, so do their diagonal matrix symmetrizers, say,  $D_1, D_2$ .

We shall extend the Frobenius equation [6] as applicable to diagonal matrix symmetrizers. We have for  $D, Z \in R^{n \times n}$ ,  $DZD$  is always symmetric where  $D$  is a nonsingular diagonal matrix and  $Z$  is any symmetric matrix.

**Lemma 5.** A nonsingular diagonal matrix  $D \in R^{n \times n}$  will be a symmetrizer of  $M \in R^{n \times n}$  iff there exists a matrix  $Z = Z^T \in R^{n \times n}$  such that  $M = ZD$ .

*Proof.* Suppose  $D$  is a symmetrizer of  $M$ . Then we have

$$DM = M^TD \Leftrightarrow MD^{-1} = D^{-1}M^T \Leftrightarrow M = D^{-1}M^TD. \quad (10)$$

In (10), let  $Z = D^{-1}M^T$  and the result follows. Then  $Z$  will be nonsingular if and only if  $M$  is so. In  $M$ , if  $r$  columns are dependent then in  $Z$ , so much rows will be dependent. Conversely, if there is a matrix  $Z; Z = Z^T$  such that

$$M = ZD \Leftrightarrow DM = DZD \quad (11)$$

Thus from (11), it is confirmed that  $D$  is a symmetrizer of  $M$ .

**Corollary 5.1.** As  $DM = M^TD \Leftrightarrow M = ZD$ , let  $M = (m_{ij}), Z = (z_{ij}); i, j = 1 : n$ , then  $m_{ij} = m_{ji} = 0$  iff  $z_{ij} = z_{ji} = 0; i, j = 1 : n$ .

**Corollary 5.2.** If  $A, B = B^T \in R^{n \times n}$ , then  $B$  will be a symmetrizer of  $A$  iff there exists a matrix  $S = S^T \in R^{n \times n}$  such that  $A = SB$ .

**Remark 5.1** By Corollary 5.1, in  $Z, M \in R^{n \times n}$ , zeros can be only in identical, symmetric positions  $(i, j), (j, i), 1 \leq i, j \leq n$ . This result is stochastically termed

Ref

6. F. G. Frobenius, Über die mit einer matrix vertauschbaren matrisen, Gesammelte Abhandlungen, Band 3, Springer (1968) 415–427; and in Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften, zu Berlin (1910), 1–15.

in Parter and Youngs [17] as conservative law. If  $m_{ij} = m_{ji} = 0$ , in every column- $i$ , row- $i$  for some  $1 \leq i, j \leq n$  of  $M$ , we see from (11) that there can exist  $D$  symmetrizing  $M$ . In such situations, the test in Lemma 3 is invalid.

**Remark 5.2** Consider  $Z = D^{-1}M^T = MD^{-1}$  in (10). Accounting both the terms, we may assume  $Z = (D^{-1}M^T + MD^{-1})/2$ .

**Remark 5.3** Adapting the expression of  $Z$  in Remark 5.2 into Frobenius equation [6] and treating  $S_2 = B$  as nonsingular, we have  $S_1 = (B^{-1}A^T + AB^{-1})/2$ . The singularity of  $S_1$  is depending on the singularity of  $A$ . Then we have  $A = S_1B$ ,  $BA = BS_1B = A^TB$ . The symmetrizer  $B$  has an independent existence and  $S_1$  is dependent on  $A, A^T$  and  $B$ . This is how the Frobenius equation [6] is extended and how the Taussky's and Frobenius equations are linked.

**Remark 5.4** We may see how Pimbley's [19] condition is true for non-negative  $M, D > 0, DMD^{-1} = DZ$  is non-negative and symmetric. From (11), Pimbley's equation [19] can be true if  $Z$  is a non-negative diagonal matrix.

**Remark 5.5** From (11), it follows that if  $Z$  is nonsingular, then  $Z^{-1}, Z$  are symmetrizers of  $M, M^T$  respectively. From Desautels [3], the set of symmetrizers of  $M$  is a linear subspace of  $R^{n \times n}$  of dimension  $n$ .

Consider the set of those matrices  $M$ , symmetrized by  $D$  and may be denoted as  $D(M)$ . We see that cardinality of largest possible set of linearly independent symmetric matrices  $Z_i; i = 1, 2, \dots \in R^{n \times n}$  which can be applied in  $M = ZD$  of (11) is  $n(n+1)/2$ . Hence  $D$  will be a symmetrizer of matrices  $M_i = Z_iD, i = 1 : n(n+1)/2 \in R^{n \times n}$ , which are linearly independent. If  $D$  symmetrizes  $M_1, M_2$ , then it also symmetrizes  $\alpha_1M_1 + \alpha_2M_2$  for nonzero scalars  $\alpha_1, \alpha_2$ . So  $\alpha_1M_1 + \alpha_2M_2 \in D(M)$ . Zero matrix also belongs to  $D(M)$ . Hence we see that  $D(M)$  is a linear subspace of  $R^{n \times n}$  of dimension  $n(n+1)/2$ . Additionally  $I_n, M, M^{-1} \in D(M)$ . Suppose  $Z_i; i = 1, 2, \dots, n$  are nonsingular and linearly independent set of  $n$  diagonal matrices. Then  $Z_iD; i = 1, 2, \dots, n$  also will be so. This set of diagonal matrices will span the whole subspace of diagonal matrices of  $R^{n \times n}$  and is also a subspace of  $D(M)$ .

**Lemma 6.** Let  $D(M) \subset R^{n \times n}$  denotes the subspace of matrices symmetrized by a diagonal matrix  $D \in R^{n \times n}$ . If  $N \in D(M)$  is nonderogatory, then it can be expressed as a linear sum of  $n$  nonsingular matrices  $N_i, i = 1 : n \in D(M)$ .

**Proof.** Recall from Desautels [2,3,13,18] that when  $N \in R^{n \times n}$  is nonderogatory, it has  $n$  simple eigen values as well as  $n$  distinct symmetrizers spanning the subspace of symmetrizers of  $N$ . Any symmetrizer  $B$  of  $N$  will be congruent to the diagonal matrix of its eigen values and so will be diagonalizable [3]. By Lemma 5, we have  $N = ZD, Z = \alpha_1Z_1 + \alpha_2Z_2 + \dots + \alpha_nZ_n$ , where  $Z_i, i = 1 : n$  are symmetrizers of  $N^T, \alpha_i, i = 1 : n$  are real scalars, as  $N^T$  as well is nonderogatory and as  $ZN^T = NZ$ . Hence  $N = \sum_{i=1}^n \alpha_iN_i = \sum_{i=1}^n \alpha_iZ_iD \in D(M)$  and are nonsingular.

**Corollary 6.1.** Let  $A \in R^{n \times n}$  be a nonderogatory matrix so that  $B_i \in R^{n \times n} = L_i^{-1}DL_i^{-T}$ ,  $i = 1 : n$  are  $n$  diagonalizable and linearly independent symmetrizers of  $A$  where  $L_i, i = 1 : n$  are distinct unit lower triangular matrices and  $D$  is a nonsingular diagonal matrix. Then  $M_i = L_i^{-T}AL_i^T \in D(M)$  will span  $M$ .

**Remark 6.1** Taking further, the expression  $A = SB$  of Corollary 5.2,  $A$  itself will be symmetric if and only if  $SB = BS$ . Such a situation  $M = ZD = DZ$  is there only when  $Z$  is zero or is a diagonal matrix. Also if  $A$  is nonderogatory, we can find exactly  $n$  symmetric matrices  $S_i; i = 1, 2, \dots, n$  and  $A = S_iB_i; S_i = (B_i^{-1}A^T + AB_i^{-1})/2, i = 1, 2, \dots, n$ . The corresponding scenario with diagonal matrix symmetrizer will be  $M_i = Z_iD_i; i = 1, 2, \dots, n$ . According to Frobenius[4,6], for a given matrix  $A \in R^{n \times n}$ , there is a symmetrizer  $B \in R^{n \times n}$ . But, this is not true with diagonal matrix symmetrization. As in Lemma 5, only to matrices  $M = ZD; Z = Z^T$  have diagonal matrix  $D$  as symmetrizer.

**Lemma 7.** Let  $B = B^T, \det(B(1, k : 1, k), k = 1 : n) \neq 0 \in R^{n \times n}$  be a symmetrizer of  $A \in R^{n \times n}$ . Let  $LBL^T = D$  where  $L$  is unit lower triangular and  $D$  diagonal. Consider  $M = L^{-T}AL^T$  so that  $D$  is a symmetrizer of  $M$ . If  $DM$  is diagonalizable, then  $DM$  and symmetrizer  $BA$  of  $A$  are congruent.

**Proof.** We have  $D, DM^i, i = 1, 2, \dots$  are symmetrizers of  $M$ . Consider

$$DM = L_1^{-1}D_1L_1^{-T}. \quad (12)$$

Now by analogy,  $L_1^{-1}D_1L_1^{-T}$  is a symmetrizer of  $M$ ,  $D_1$  is a symmetrizer of  $M_1 = L_1^{-T}ML_1^T$ ,  $D_1M_1$  is a symmetrizer of  $M_1$ . From (12), we have,

$$L_1^{-1}D_1L_1^{-T}L^{-T}AL^T = LA^TL^{-1}L_1^{-1}D_1L_1^{-T}. \quad (13)$$

Equation (13) can be rearranged as below.

$$L^{-1}L_1^{-1}D_1L_1^{-T}L^{-T}A = A^TL^{-1}L_1^{-1}D_1L_1^{-T}L^{-T} \quad (14)$$

Let this derived symmetrizer of  $A$  from (14) be  $B_1$  so that we have

$$\begin{aligned} B_1 &= L^{-1}L_1^{-1}D_1L_1^{-T}L^{-T} \\ &= L^{-1}DML^{-T} = BA. \end{aligned} \quad (15)$$

**Remark 7.1** We may generalize this result that if  $LBL^T = D$ , the symmetrizers  $D, DM, DM^2, \dots$  of  $M$  are congruent to the symmetrizers  $B, BA, BA^2, \dots$  of  $A$ . The specialty of the congruence is presented in (15) and is that the matrices  $L, L^T, L^{-1}, L^{-T}$  are commonly applied for these congruences. The point is that only diagonalization of  $B$  is in demand for these congruences. According to

Desautels [3], Taussky [21], symmetrizers  $D, DM, DM^2, \dots, DM^{n-1}$  of  $M$  span a subspace with dimension  $n$  if and only if  $M$ , to that effect  $A$  is nonderogatory.

We shall see when  $M$  in Lemma 1 will be symmetric.

**Lemma 8.** Let for a given matrix  $A \in R^{n \times n}$ ,  $B = B^T$ ,  $\det(B(1:k, 1:k), k = 1:n) \neq 0$  be a symmetrizer. Let  $LBL^T = D$ ,  $L, D \in R^{n \times n}$ ,  $L$  unit lower triangular and  $D$  diagonal. Then  $M = L^{-T}AL^T$  will be a symmetric matrix iff the positive definite symmetric matrix  $\Lambda = L^{-1}L^{-T}$  is a symmetrizer of  $A$ .

*Proof.* If  $M = L^{-T}AL^T$  is symmetric, then we get

$$\begin{aligned} L^{-T}AL^T &= LA^TL^{-1} \Leftrightarrow \\ L^{-1}L^{-T}A &= A^TL^{-1}L^{-T}. \end{aligned} \quad (16)$$

From (16), positive definite symmetric matrix  $\Lambda = L^{-1}L^{-T}$  symmetrizes  $A$ .

### III. DECOMPOSITION OF REDUCED SYMMETRIZER EQUATION

We may break equation (11) further.

**Lemma 9.** Let  $M \in R^{n \times n}$  and  $D \in R^{n \times n}$  be a nonsingular diagonal matrix. Consider  $N = (M + D^{-1}M^TD)/2$ . Then  $N = M$  iff  $D$  is a symmetrizer of  $M$ .

*Proof.* We have if  $N = M$ , then  $DM = (DM + M^TD)/2$  and hence  $D$  is a symmetrizer of  $M$ . Conversely, if  $D$  is a symmetrizer of  $M$ , then  $DN = (DM + M^TD)/2 = DM$  and so  $N = M$ .  $\square$

**Remark 9.1** Recall that in (11),  $Z = (D^{-1}M^T + MD^{-1})/2$  and  $N = ZD$ . In this context, we may recall from K.Y. Fan and A.J. Hoffman [5] that for  $A \in R^{n \times n}$ ,  $S = (A + A^T)/2$  is the closest to  $A$  of all the symmetric matrices  $X \in R^{n \times n}$ . Thus

$$\|A - S\| \leq \|A - X\|. \quad (17)$$

Analogues to (17), if  $D$  symmetrizes  $M$ , and  $K$  any other nonsingular diagonal matrix,  $D, M, K \in R^{n \times n}$ , let  $Z = (KM + M^TK)/2$ . Then from Lemma 9,

$$M - (M + M^T)/2 = (M - Z)/2 + (Z - M^T)/2 \Rightarrow \quad (18)$$

$$\|M - (M + DMD^{-1})/2\| \leq \|M - (KM + M^TK)/2\|$$

In (18), because of von Neumann's [16] characterization of all unitary invariant norms in  $R^{n \times n}$ , we have  $\|M\| = \|M^T\|$ ,  $\|(M - Z)/2\| = \|(Z - M^T)/2\|$ . From (18) we have the result that among all nonsingular diagonal matrices  $K \neq I_n$  and as  $K \rightarrow D$ , the symmetric matrix  $(N + N^T)/2 \rightarrow (M + M^T)/2$  will be closest to  $M$  than any other symmetric matrices  $(KM + M^TK)/2$ .

Analogously  $\|A - (A + BAB^{-1})/2\| \leq \|A - (A + SAS^{-1})/2\|$  where  $B$  is a symmetrizer of  $A$  and  $S$  is any other symmetric matrix,  $A, B, S \in R^{n \times n}$ .

**Remark 9.2** Let  $M = (N + D^{-1}N^TD)/2, M \in R^{n \times n}$ . We have  $M = ZD$  and  $Z = (D^{-1}M^T + MD^{-1})/2 = MD^{-1} = (D^{-1}N^T + ND^{-1})/2$ .

So  $M^T - N^T = D(N - M)D^{-1}$ . We have another situation that Pimbley's [19] condition is met when  $M^T - N^T$  is nonnegative and symmetric and it is possible even when  $M, N$  are not symmetric.

**Lemma 10.** Let  $D(M) \subset R^{n \times n}$  denotes the subspace of matrices symmetrized by a diagonal matrix  $D \in R^{n \times n}$ . If  $N \in D(M)$  has  $n$  simple eigen values, then  $N = \sum_{i=1}^{n^2} M_i, M_i = 1 : n^2 \in R^{n \times n}$  all having diagonal matrix symmetrizers.

**Proof.** From Lemma 5, we have  $N = ZD$  where  $Z = Z^T \in R^{n \times n}$  is nonsingular. Now  $D = \sum_{i=1}^n \alpha_i D_i, D_i, i = 1 : n \in R^{n \times n}$  are nonsingular and diagonal. Since  $N = ZD \in D(M)$ , by Lemma 6,  $N = \sum_{i=1}^n \beta_i Z_i D, Z_i D \in D(M); i = 1 : n$ . Here  $\alpha_i, \beta_i, i = 1 : n$  are nonzero scalars. Hence,

$$\begin{aligned} N &= \alpha_1 \beta_1 Z_1 D_1 + \alpha_1 \beta_2 Z_1 D_2 + \dots + \alpha_1 \beta_n Z_1 D_n \\ &+ \alpha_2 \beta_1 Z_2 D_1 + \alpha_2 \beta_2 Z_2 D_2 + \dots + \alpha_2 \beta_n Z_2 D_n + \dots \end{aligned} \quad (19)$$

$$+ \alpha_n \beta_1 Z_n D_1 + \alpha_n \beta_2 Z_n D_2 + \dots + \alpha_n \beta_n Z_n D_n \Rightarrow$$

$$N = M_1 + M_2 + \dots + M_k; k = n^2. \quad (20)$$

Since none of the scalars,  $\alpha_i \beta_j = 0, i, j = 1 : n$  is zero in (19), the  $n^2$  matrices in (20) are linearly independent. Each matrix  $M_i$  is nonsingular as it is a product of nonsingular matrices. Also  $D_i; i = 1 : n$  symmetrizes  $M_{n(j-1)+i}, i, j = 1 : n$ .

Under certain conditions, converse of the Lemma 10 is also true.

**Corollary 10.1.** Let  $N \in R^{n \times n}$  be the sum of  $n^2$  linearly independent nonsingular matrices  $M_i, i = 1 : n^2 \in R^{n \times n}$  such that  $(D_i^{-1}M_j^T + M_j D_i^{-1})/2 = (D_i^{-1}M_{n(i-1)+j}^T + M_{n(i-1)+j} D_i^{-1})/2; i, j = 1 : n$  where  $D_i, i = 1 : n \in R^{n \times n}$  are diagonal matrix symmetrizers of  $M_{n(i-1)+j}; i, j = 1 : n$ . If all the matrices  $M_i, i = 1 : n^2$  are with  $n$  simple eigen values, then  $N$  also will be symmetrized by a diagonal matrix  $D \in R^{n \times n}$  and it has  $n$  simple eigen values.

**Proof.** From the hypothesis and Lemma 6, if  $D_i$  symmetrizes  $M_{n(i-1)+j}$ , let

$$\begin{aligned} Z_j &= (D_i^{-1}M_{n(i-1)+j}^T + M_{n(i-1)+j} D_i^{-1})/2 \Rightarrow \\ Z_j D_i &= M_{n(i-1)+j}; i, j = 1 : n \Rightarrow \end{aligned} \quad (21)$$

$$N = Z_1 D_1 + Z_1 D_2 + \dots + Z_1 D_n + Z_2 D_1 + Z_2 D_2 + \dots + Z_2 D_n + \dots +$$

$$Z_n D_1 + Z_n D_2 + \dots + Z_n D_n \Rightarrow N = ZD$$

Ref

19. H. G. Pimbley, Solution of an initial value problem for the multi-velocity neutron transport equation with a slab geometry, J. Math. and Mech. 8(1959) 837–866.



In (21)  $Z_1 + Z_2 + \dots + Z_n = Z$  and  $D_1 + D_2 + \dots + D_n = D$  are assigned. As  $Z$  is nonsingular,  $Z^{-1}$  is a symmetrizer of  $N$ . Hence  $Z$  will be a symmetrizer of  $N^T$ . From Desautels [3],  $N^T$  is nonderogatory and hence  $N$  is so.

**Remark 10.1** The  $n^2$  symmetrizer equations  $D_i^{-1} M_{n(i-1)+j}^T = M_{n(i-1)+j} D_i^{-1}; i, j = 1 : n$  in (21), where  $D = D_1 + D_2 + \dots + D_n$  and  $Z = Z_1 + Z_2 + \dots + Z_n; Z_i = (D_j^{-1} M_{n(j-1)+i}^T + M_{n(j-1)+i} D_j^{-1})/2; i, j = 1 : n$  are synchronized among themselves and with equation (2) as  $M = ZD$  and  $M$  similar to  $A$ .

### a) Symmetrizer Computation Algorithm

Recall from Lemma 5 that a nonsingular diagonal matrix  $D \in R^{n \times n}$  will be a symmetrizer of  $M \in R^{n \times n}$  if and only if  $M = ZD, Z \in R^{n \times n}, Z = Z^T$ . We see that  $Z$  will be then a symmetrizer of  $M$  if it is nonsingular. In a general way, it may be noted that if we can derive a diagonal matrix  $D$  at each step, then  $Z = (DM + M^T D) * 0.5$  can be computed as an approximate symmetrizer of  $M$  for that step. This  $D$  has to be improved upon, in an iterative manner so that  $DM$  is arbitrarily close to symmetry. Note that it is not essential to decompose  $Z$  or  $M$  for deriving  $D$  as presented in the algorithm below:

**Step-1:** Initialize iteration index  $k=1$ , row index  $r=3$  and current row index  $c=3$ . Initially let  $M_r(k) = (m_{ij}(k)); i, j = 1 : n$  is set as a matrix similar to  $A$ . Set  $\epsilon > 0$  to an arbitrarily chosen small quantity, say  $\epsilon = 1.0e - 15$ . Initialize matrix  $M = A, Z = I_n, D = I_n$ .

**Step-2:** Consider the principal submatrix  $M_r(k)(1 : 1, 1 : 1)$ . It is the scalar  $m_{11}(k)$  and is symmetric. Correspondingly to compose a diagonal matrix  $D_k = \text{diag}(d_1(k), d_2(k), \dots, d_n(k))$  for this step- $k$ , we choose  $d_1(k) = 1$ . For the principal submatrix  $M_r(k)(1 : 2, 1 : 2)$  we choose  $d_2(k) = m_{12}(k)/m_{21}(k)$  so that  $M_r(k)(1 : 2, 1 : 2)$  is symmetric in  $D_r(k)M_r(k)$ .

**Step-3:** Now consider the principal submatrix  $M_r(k)(1 : r, 1 : r)$ . There are  $r - 1$  pair of entries  $m_{rj}(k), m_{jr}(k), j = 1 : r - 1$  to be addressed in row- $r$  and column  $r$ . By Corollary 5.1, if any of the row entries  $m_{rj}(k)$  is zero, then corresponding column entry  $m_{jr}(k)$  also should be zero and in this case we may consider  $d_r(k) = 1$ . If non of these pairs are zeros, As a trail, taking into account the first pair, consider  $d_r(k) = m_{1r}(k)/m_{r1}(k)$ . Thus  $(r,1)$  and  $(1,r)$  entries in  $D_r(k)M_r(k)$  are identical. Now the effect of this is computed as  $e_{r1}(k) = \sum_{i=1}^r \sum_{j=1}^r (m_{ij}(k) - m_{ji}(k))^2$ . This computation of error in symmetry is repeated out of row- $r$  and column- $r$  of  $D_r(k)^{-1} M_r(k)^T$  and added together to store as  $e_{r1}(k)$ . Repeat this error computations  $e_{ri}(k), i = 1 : r - 1$  in a similar way with respect to other pairs by applying  $d_r(k) = m_{ir}(k)/m_{ri}(k), i = 1 : r - 1$ . Among these  $(r-1)$  errors, let  $e_{rl}(k)$  be the minimum against row- $r$  for iteration- $k$  and correspondingly we choose  $d_r(k) = m_{lr}(k)/m_{rl}(k)$ .

**Step-4:** Remaining entries of  $D_r(k)$ ,  $d_{r+1}(k), d_{r+2}(k), \dots, d_n(k)$  we set  $d_i(k) = 1, i = r+1 : n$ . Compute the approximate symmetrizer for submatrix  $M_r(k)(1 : r, 1 : r)$  as  $Z_r(k) = 0.5 * (D_r(k)M_r(k) + M_r(k)^T D_r(k))$ . Set column- $i$  and row- $i$  in  $Z_r(k)$  as that of  $I_n$  for  $i = r+1 : n$ .

**Step-5:** If iteration index  $k = 1$ , set  $e_{rmin} = e_{rl}(k)$ , otherwise if  $e_{rl}(k) < e_{rmin}$  set  $e_{rmin} = e_{rl}(k)$ . In this case the iteration- $k$  is treated as a success. If  $e_{rl}(k) > e_{rmin}$  it is a failure. If iteration- $k$  is a success, store  $M = M_r(k)$ ,  $D = D_r(k)$ ,  $Z = Z_r(k)$ . Compute  $B = ZD^{-1}Z$  as an approximate symmetrizer of  $A$  at iteration- $k$  as  $A = Z^{-1}M^T Z$ .

**Step-6:** Test whether  $e_{rmin}(k) < \epsilon$ . If it is so, check whether current row  $c = r$ . If it is true, set  $c = c + 1$ . Exchange row- $c$  with row- $r$  and column- $c$  with column- $r$ . Loop back for carrying out step-2 through step-6. If  $r < c$ , set  $r = c$ . If  $c \geq n$  and  $e_{rmin} < \epsilon$  stop the process, otherwise set  $c = n$ , loop back to step-2 and repeat the steps 2 through 6.

#### b) Assumptions, Theory and Convergence of the Algorithm

The target is to compute a symmetrizer  $B \in R^{n \times n}$  of a given nonsingular matrix  $A \in R^{n \times n}$ .  $A$  is assumed to be as in [3,6,20].

1. So far, we were using the decomposition  $LBL^T = D$  for developing the theoretical background, deriving  $D$  from  $B$  and similar matrix  $M$  from  $A$ . Against this, now both  $B, D$  are derived from  $M$  itself at each iteration. Reduction of assumed symmetrizers  $Z_r(k)$  is not essential for the procedure. It is thus more practical by avoiding the usual difficulties of round off or truncation errors, pivoting, permutations, division by zero or singularities etc. faced in such a decomposition. The basic assumption about symmetrizer  $Z_r(k)$  is that because of the KY Fan and A.J. Hoffman [5] result, it is sufficiently close to the symmetrizer of  $M_r(k)$  at iteration  $k$ . The fact is that when  $n = 3$ , for row-3 and column-3, only one of the pairs of entries,  $(m_{31}(k), m_{13}(k))$  and  $(m_{32}(k), m_{23}(k))$  remains non-identical at iteration- $k$ . In order to carry forward this situation to problems with  $n \geq 4$ , we address first it as a problem with  $n = 3$  and once the error is minimized to the preset and specified tolerance value  $\epsilon > 0$ , row-3 and column-3 are exchanged with row-4 and column-4 and again treat the problem as if it is of dimension  $n=3$ . Once these two levels of minimization with  $n = 3$  are accomplished, stage is set ready for  $n = 4$  as implemented in step-6. Now the procedure treats the  $4 \times 4$  submatrix of the problem and so on. This way, the  $n$ -dimensional symmetrization is conducted in a conducive environment so that the computed symmetrizers remain close to any actual existing symmetrizer if any, obeying the result we derived here from [5].

2. In step-3, we individually compute errors for symmetrizing  $M_r(k), M_r(k)^T$  by  $D_r(k), D_r(k)^{-1}$  respectively and add both them as a single error value as  $e_{ri}(k), i = 1 : r-1$ . This for the stability of the process. If we depend only one of them, soon  $D$  may converge to either zero or infinity. Also note that if  $D$  is a symmetrizer of  $M$  then  $D^{-1}$  symmetrizes  $M^T$ . Thus it is also a theoretical requirement.

Ref

5. K. Y. Fan, A. J. Hoffman, Some metric inequalities in the space of matrices, Proceedings of the American Mathematical Society 6 (1955) 111-116.

3. When  $e_{rmin} < \epsilon$ , the matrix is assumed to be a symmetrizer of  $M_r k$ . Corresponding to  $r = n$ ,  $DM$  will be symmetric,  $BA$  will be symmetric. The process is terminated as it marks the successful convergence of the procedure.

4. We know from Desautels [3] that if  $D$  is a symmetrizer of  $M$  so do  $DM, DM^2, \dots etc.$ . Hence  $Z_r(k) = 0.5 * (D_r(k)M_r(k) + M_r(k)^T D_r(k))$  will be closer to  $D_r(k)M_r(k)$  as and when the successive computations approach  $DM$  at successful iteration- $k = 1, 2, \dots$  and converges to it when  $e_{rmin} < \epsilon$ .

#### IV. NUMERICAL EXAMPLES

The numerical strategy explained in Section 3 is implemented in MATLAB. The coefficient matrices cited in Adhikari [1] of dynamic systems associated with equations of motion of undamped linear systems and that of equations of motion describing free vibration of a viscously damped linear system are considered here to test the practical use of the procedure. There are real coefficient matrices with complex eigen system. Dopico and Uhlig [4] prove that if  $A$  is diagonalizable then the two eigen vectors corresponding to a pair of complex conjugate eigen values of  $A$  can be chosen of complex conjugates of each other. Then the symmetrizer built from such an eigen vector basis will be real. We may note that all these examples from Adhikari [1] are diagonalizable. (Results are rounded to 4 decimals). For all the problems  $\epsilon = 1.0e - 15$  is fixed.

**Example 1:** This is the first real asymmetric matrix cited by Adhikari which is reported as not satisfying Taussky's condition of symmetrizability.

The matrix is  $A = \begin{pmatrix} 1.0 & -2.0 & 1.5 \\ 12.0 & 6.0 & 7.0 \\ -2.0 & 4.0 & 9.0 \end{pmatrix}$ . Its eigen values are

$(1.5543+5.0507i, 1.5543-5.0507i, 12.8914)$ . After 32 iterations, the error reduced to  $6.107564e-16$ , the matrix  $M$  similar to  $A$  is computed as

$$M = \begin{pmatrix} 1.1262 & -2.1628 & 1.1215 \\ 11.5416 & 6.4407 & 7.9459 \\ -2.9727 & 3.9469 & 8.4331 \end{pmatrix}.$$

Its symmetrizer is  $D = \text{diag}(1.0000, -0.1874, -0.3773)$ . The symmetrization of

$M$  is  $DM = \begin{pmatrix} 1.1262 & -2.1628 & 1.1215 \\ -2.1628 & -1.2069 & -1.4890 \\ 1.1215 & -1.4890 & -3.1814 \end{pmatrix}$ . Now the symmetrizer of  $A$  is

computed as  $B = \begin{pmatrix} -18.1025 & -6.6674 & -1.7944 \\ -6.6674 & -1.9765 & -9.7041 \\ -1.7944 & -9.7041 & -14.1343 \end{pmatrix}$ . Symmetrization of  $A$  is

carried out as  $BA = \begin{pmatrix} -94.5221 & -10.9766 & -89.9748 \\ -10.9766 & -37.3404 & -111.1730 \\ -89.9748 & -111.1730 & -197.8291 \end{pmatrix}$ .

**Example 2:** Adhikari [1] considers only real square matrices as these are the coefficient matrices in the equations of motion of linear vibrations. In this case of undamped system  $X\ddot{x}(t) + Yx(t) = 0, t > 0$ , the coefficient matrices  $X$  and  $Y$  are real and symmetrizable of first kind. These matrices are taken from Ma and Caughey [12]. We may consider the matrix  $X =$

$$\begin{pmatrix} 0.5740 & 1.3858 & 1.3858 \\ 0.7070 & 0.7070 & -0.7070 \\ 0.4620 & -0.1914 & -0.1914 \end{pmatrix}. \text{ Results are presented below:}$$

Eigen Values :-1.1360, 1.6648, 0.5608 ; No. of iterations : 72

Error Value : 8.988429e-16 : Symmetrizer  $D = \text{diag}(1.0000, 1.5931, 3.7701)$

$$\text{Similar Matrix } M = \begin{pmatrix} 0.5069 & 1.2911 & 1.3981 \\ 0.8104 & 0.7382 & -0.7057 \\ 0.3708 & -0.2982 & -0.1555 \end{pmatrix}.$$

$$\text{Symmetrization of } M: DM = \begin{pmatrix} 0.5069 & 1.2911 & 1.3981 \\ 1.2911 & 1.1761 & -1.1242 \\ 1.3981 & -1.1242 & -0.5864 \end{pmatrix}.$$

$$\text{Symmetrizer } B = \begin{pmatrix} 3.2023 & 3.7397 & 0.0369 \\ 3.7397 & 6.1330 & 1.2816 \\ 0.0369 & 1.2816 & 1.8605 \end{pmatrix}.$$

$$\text{Symmetrization of } X : BX = \begin{pmatrix} 4.4991 & 7.0747 & 1.7868 \\ 7.0747 & 9.2732 & 0.6011 \\ 1.7868 & 0.6011 & -1.2111 \end{pmatrix}.$$

Now we are considering coefficient matrices  $Y1, Y2$  of an undamped system with

$$\text{complex eigen values where } X \text{ is same as in example-1. } Y1 = \begin{pmatrix} 1.2044 & -5.4424 & 2.7013 \\ 2.1007 & 0.893 & 0.1894 \\ -1.8393 & 0.8953 & 2.5087 \end{pmatrix}.$$

Eigen values are  $1.05192517450474 + 4.01396879901174i$ ,

$1.05192517450474 - 4.01396879901174i$ , and  $2.44854965099053$ .

No.of Iterations: 21 Error Value : 3.245450e-16.

$$\text{Similar Matrix } M = \begin{pmatrix} 1.1739 & -5.4028 & 2.8105 \\ 2.0130 & 0.8309 & 0.3480 \\ -2.0081 & 0.6674 & 2.5477 \end{pmatrix}.$$

$$\text{Symmetrization of } M, DM = \begin{pmatrix} 1.1739 & -5.4028 & 2.8105 \\ -5.4028 & -2.2300 & -0.9341 \\ 2.8105 & -0.9341 & -3.5657 \end{pmatrix}.$$

$$\text{Symmetrizer } B = \begin{pmatrix} 0.9933 & -0.0510 & -0.1018 \\ -0.0510 & -2.6624 & -0.0623 \\ -0.1018 & -0.0623 & -1.4524 \end{pmatrix}.$$

$$\text{Symmetrization of } Y1 \text{ as } BY1 = \begin{pmatrix} 1.2764 & -5.5399 & 2.4180 \\ -5.5399 & -2.0126 & -0.7984 \\ 2.4180 & -0.7984 & -3.9306 \end{pmatrix}.$$

$$\text{Next exmple is Matirx } Y2 = \begin{pmatrix} 0.1995 & 1.8857 & -3.2199 \\ -2.1312 & -0.2236 & 0.3609 \\ 1.0378 & 1.9501 & -1.5606 \end{pmatrix} \text{ with coeffi-}$$

cient matrix  $X$  same as in example-1. The solution is as follows.

Eigen values:  $-1.20087894725344 + 2.67720413881095i$ ,

$-1.20087894725344 - 2.67720413881095i$  and  $0.813057894506878$ .

No.of iterations : 14. Error Value : 2.582360e-16.

Diagonal Matrix Symmetrizer  $D := \text{diag}(1.0000, 1.7457, -1.9129)$ .

$$\text{Similar Matrix } M = \begin{pmatrix} 1.6391 & 1.7133 & -5.6672 \\ 0.9814 & 1.1319 & -1.3944 \\ 2.9626 & 1.2725 & -4.3597 \end{pmatrix}.$$

$$\text{Symmetrizing } M \text{ is } DM = \begin{pmatrix} 1.6391 & 1.7133 & -5.6672 \\ 1.7133 & 1.9760 & -2.4341 \\ -5.6672 & -2.4341 & 8.3398 \end{pmatrix}.$$

$$\text{Symmetrizer } B = \begin{pmatrix} -0.6605 & -6.2810 & 0.2562 \\ -6.2810 & 1.0501 & 3.9742 \\ 0.2562 & 3.9742 & 7.5928 \end{pmatrix}.$$

$$\text{Symmetrizing } Y2 \text{ is } BY2 = \begin{pmatrix} 13.5229 & 0.6586 & -0.5400 \\ 0.6586 & -4.3288 & 14.4011 \\ -0.5400 & 14.4011 & -11.2399 \end{pmatrix}.$$

**Example 3:** Equations of motion describing free vibration of a viscously damped linear system can be expressed as  $X\ddot{x}(t) + Y\dot{x}(t) + Wx(t) = 0, t > 0$ . The matrices  $X, Y, W$  are from  $R^{n \times n}$ . When  $X, Y, W$  are symmetric and positive definite, they represent the mass, stiffness, and viscous damping matrices. The scope of this work is limited to demonstrate the individual symmetrization of these coefficient matrices. In this respect, matrix cited in Adhikari [1] in place of  $W$  is considered. The matrix  $W$  from a damped system claimed to be not symmetrizable of first kind has complex eigen values.

$$\text{The matrix is } W = \begin{pmatrix} 4.3160 & -2.5771 & -1.4626 \\ 2.7122 & 1.8365 & -0.1999 \\ 1.3827 & -2.5631 & 4.3419 \end{pmatrix}.$$

But it is successfully solved using the decomposition method presented here and the results are presented below.

Eigen Values:  $2.7938 + 2.8137i, 2.7938 - 2.8137i, 4.9068$ ; No. of iterations: 44  
Error :  $4.286294e-16$  ; Symmetrizer  $D = \text{diag}(1.0000, -0.8916, -1.1189)$

$$\text{Similar Matrix } M = \begin{pmatrix} 4.3537 & -2.4003 & -2.1766 \\ 2.6922 & 2.0645 & -1.8430 \\ 1.9452 & -1.4686 & 4.0761 \end{pmatrix}.$$

$$\text{Symmetrization of } M: DM = \begin{pmatrix} 4.3537 & -2.4003 & -2.1766 \\ -2.4003 & -1.8407 & 1.6432 \\ -2.1766 & 1.6432 & -4.5609 \end{pmatrix}.$$

$$\text{Symmetrizer } B = \begin{pmatrix} 5.9204 & -16.2577 & -0.7220 \\ -16.2577 & 7.8400 & 4.0794 \\ -0.7220 & 4.0794 & -11.9274 \end{pmatrix}.$$

$$\text{Symmetrization of } W : BW = \begin{pmatrix} -19.5403 & -43.2641 & -8.5442 \\ -43.2641 & 45.8402 & 39.9236 \\ -8.5442 & 39.9236 & -51.5470 \end{pmatrix}.$$

## V. CONCLUSIONS

Frobenius [6] or Taussky's real symmetrizer equation [21] can be transformed into a reduced symmetrizer equation (Lemma 1) when the symmetrizer of the given equation is diagonalizable in elementary terms. This reduced equation has a diagonal matrix as its symmetrizer. It is congruent to the symmetrizer of Taussky's equation. It symmetrizes a matrix similar to that of the original equation. It is proved that diagonal matrix symmetrizer is embedded in the concerned matrix itself. This property can be used as a test for diagonal matrix symmetrization (Lemma 3). The reduced symmetrizer equation can be further decomposed into more such reduced equations as proved in Corollary 10.1.

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1. S. Adhikari, On symmetrizable systems of second kind, Journal of Applied Mechanics 67 (7) (2000) 797-802, doi:10.1115/1.1322038.



We conclude that symmetrization in  $R^{n \times n}$  is mutually synchronized with symmetrization by diagonal matrices whenever the symmetrizer is diagonalizable. This provides necessary and sufficient conditions for a matrix to be symmetrized using Taussky's equation (Lemma 2) as well as new approaches in solving it. The second symmetric matrix factor of the Frobenius equation, which need not be a symmetrizer, is expressed here in terms of the asymmetric matrix, its transpose and the symmetrizer (Lemma 5). This is the matrix that bridges the Frobenius and Taussky's equations. It is attempted to explain or adapt certain results in [5], [16], [17] and [19] using reduced symmetrizer equation. A symmetrizer computing algorithm is introduced. It is applied on some practical problems in the literature and results are presented.

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[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25].



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## Is there a Similiarity between Fibonacci Sequence and Euler's Number with Respect to Quantum Perspective Model?

By Tahir Ölmez

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**Abstract-** According to Quantum Perspective Model, this article studies whether there is a link between the Euler's numbers and the Fibonacci series. When the digits of the Euler's number after the comma are converted from decimal(10) number base system to binary(2) number base system, it corresponds to the number in the Fibonacci series.(0,1,1,2,3,5,8,13,21,34,55...) [7].From this point of view, when the first hundred digits of the Euler's numbers after the comma were calculated, the number "55" (ten times) in the Fibonacci series was found, in particular. Besides, the eleventh number in the Fibonacci series is also "55". In other words, the approximate unchanged numbers of the golden ratio numbers after the comma can be reached for the first time after dividing them from "55" to "34" (1,618). In sum, Euler's numbers are not only attributed to the Fibonacci series in mathematics, but also attributed to the golden ratio in nature.

**Keywords:** *quantum perspective model, euler's numbers, fibonacci series, binary number base system, the golden ratio and the pascal triangle.*

**GJSFR-F Classification:** MSC 2010: 35Q31



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# Is there a Similiarity between Fibonacci Sequence and Euler's Number with Respect to Quantum Perspective Model?

Tahir Ölmez

**Abstract-** According to Quantum Perspective Model, this article studies whether there is a link between the Euler's numbers and the Fibonacci series. When the digits of the Euler's number after the comma are converted from decimal(10) number base system to binary(2) number base system, it corresponds to the number in the Fibonacci series. (0,1,1,2,3,5,8,13,21,34,55...) [7]. From this point of view, when the first hundred digits of the Euler's numbers after the comma were calculated, the number "55" (ten times) in the Fibonacci series was found, in particular. Besides, the eleventh number in the Fibonacci series is also "55". In other words, the approximate unchanged numbers of the golden ratio numbers after the comma can be reached for the first time after dividing them from "55" to "34" (1,618). In sum, Euler's numbers are not only attributed to the Fibonacci series in mathematics, but also attributed to the golden ratio in nature.

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## I. EULER'S NUMBERS AND GOLDEN RATIO

Euler's numbers are  $e: 2,718281828459045...$  [1]

The starting point of this study was found as follows. When the first fifteen digits of the number "e" after the comma are subtracted from a quadrillion, the first three digits of the numbers obtained at the result are "618". Also, the golden ratio numbers include "618", too. (Remember, it is approximately 1,618) (For more information about "618" and biochemistry [6]) In fact, in the digits after "618" in the result, Euler's numbers are the same as the digits after the first three digits after the comma (281828459045).

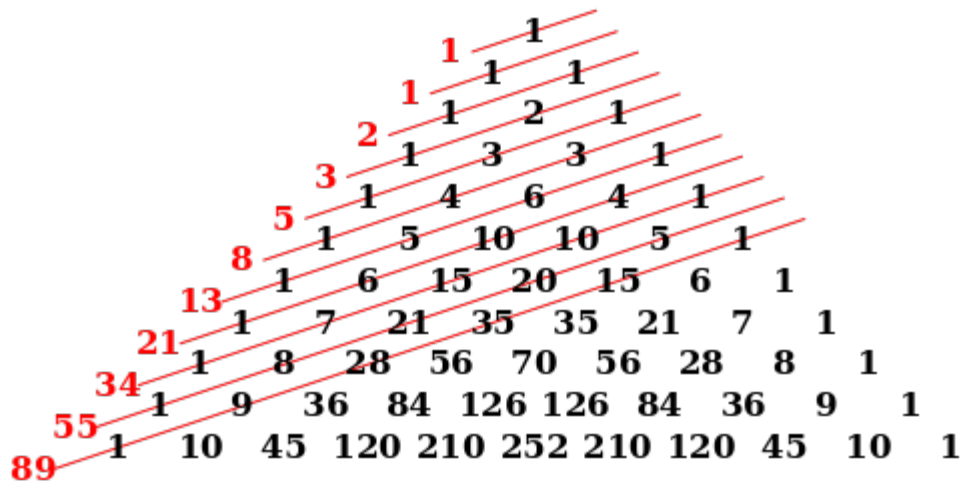
Fibonacci series : .0,1,1,2,3,5,8,13,21,34,55... [7]

The golden ratio has the continued fractions ( $1/1, 2/1, 3/2, 5/3, 8/5, 13/8, 21/13, 34/21, 55/34...$ etc) are ratios of successive Fibonacci numbers. [7]

The starting point of the numbers in the Fibonacci series is the Pascal triangle, which is also formed by the exponents of the eleven "11" digit. Namely, From Fibonacci series, the number of "55" is the eleventh(11) number. Another mysterious point is that if you calculate the diagonals of this triangle. the sum of the numbers in the diagonals will give you the Fibonacci sequence [1]. (1,1,2,3,5,8,13,21,34,55) [7]. Namely, the value of a row is a power of 11. [3]

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Picture 1: The Pascal triangle and Fibonacci series [2]

## II. CALCULATION OF EULER'S NUMBERS FROM DECIMAL BASE SYSTEM (10) TO BINARY BASE SYSTEM (2) AND VICE VERSA

Table 1: The representation of decimal numbers in the binary base and vice versa

DECIMAL BINARY	1 01	2 10	3 11	4 100	5 101	6 110	7 111	8 1000	9 1001	10 1010
DECIMAL BINARY	11 1011	12 1100	13 1101	14 1110	15 1111	16 10000	17 10001	18	19	20
DECIMAL BINARY	21 10101	22	23	24 11000	25	26	27 11011	28	29	30 11110
DECIMAL BINARY	31	32	33	34	35 100011	36 100100	37	38 100110	39	40
DECIMAL BINARY	41	42 101010	43	44	45 101101	46 101110	47 101111	48	49	50
DECIMAL BINARY	51 110011	52 110100	53	54	55	56	57	58	59 111011	60
DECIMAL BINARY	61	62 111110	63	64	65	66 1000010	67 100011	68	69 1000101	70
DECIMAL BINARY	71 1000111	72 1001000	73	74 1001010	75 1001011	76	77 1001101	78 1001110	79	80
DECIMAL BINARY	81 1010001	82 1010010	83	84 1010100	85	86	87 1010111	88	89	90
DECIMAL BINARY	91	92	93	94	95 1011111	96 1100000	97 1100001	98	99 1100011	100

## III. CALCULATION OF EULER'S NUMBERS FROM DECIMAL BASE SYSTEM (10) TO BINARY BASE SYSTEM (2) AND VICE VERSA

The first hundred of Euler's numbers are here:

e:2,71828182845904523536028747135266249775724709369995957496696762772407663035354759457138217852516642742746

At first, Euler's numbers of both digits after the comma was taken each time. For example, 71,82,81,82,84...and so on. Then these numbers are found in the binary number system in Table-1. (For instance, "71", 1000111 and so on).Secondly, convert these binary numbers to decimal number base (For instance, "71" 1000111; 1000=8 and

111=7). Finally, all decimal numbers are subjected to the addition process, respectively.  
(8+7+2+4+2+2+17+2+4+2+5=55). The result of the addition is "55".

Euler's numbers: 71                      82                      81                      82                      84  
Euler's numbers: 1000 111 10 100 10                      10 10001 10 100 10                      101 0  
Euler's numbers: 8 +                      7                      +2+4+2                      + 2 +17                      +2+4+2                      +5  
*EMPTY=55*

Euler's numbers: 84(more)                      59                      04 52                      35                      36                      02 87  
Euler's numbers: 100 111 01 11                      1001 10 100 1000 11 100 100 10                      10 101  
Euler's numbers: 4                      +3                      +2+3 +                      9 +                      2 + 4 +8                      +3 +4                      +4                      + 2                      +2  
+5=55

Euler's numbers: 87(more) 47                      13                      52                      66                      24  
Euler's numbers : 11                      1011 11 11 01 1 10 100 10000 10 1 1000  
Euler's numbers : 3                      +11 +3 +3+1+1+2+4                      +16                      +2 +1+8 =55

Euler's numbers: 97                      75                      72 47                      09                      36  
Euler's numbers: 1 10000 1 100 10 11                      100 1000 10 1 111 10 01 10 01 00  
Euler's numbers: 1+16+                      1 +4                      +2                      +3                      +4                      +8                      +2 +1+7                      +2+1                      +2+1  
*EMPTY=55*

Euler's numbers: 99                      95                      95                      74                      96  
Euler's numbers: 1 1000 11 1 01 1111 1 01 1111 100 10 10 1  
Euler's numbers: 1 + 8 +3 +1+1+15                      +1+1+15 +4 +2 +2                      +1=55

Euler's numbers: 96(more)                      69                      67  
Euler's numbers: 100000 100 00                      101 10000  
Euler's numbers :                      32 + 2                      +*EMPTY*+5                      +16 =55

Euler's numbers: 67(more) 62                      77                      24                      07                      66  
Euler's numbers: 11                      111 110 10 01 101 1 1000 1 11 10000  
Euler's numbers: 3                      +7 +6                      +2+1 +7 +1 +8                      +1+3+16=55

Euler's numbers: 66(more) 30                      35                      35                      47                      59  
Euler's numbers: 10                      111 10 1000 11 1000 11 1011 11 111 01  
Euler's numbers: 2                      +7                      +2                      +8                      +3                      +8                      +3                      +11                      +3                      +7 +1 =55

Euler's numbers : 59(more) 45                      71                      38                      21                      78                      52  
Euler's numbers: 1                      101 101 1000 111 100 110 101 01 100 11 10 11 01  
Euler's numbers: 1                      + 5                      +5                      +8                      +7                      +4                      +6                      +5 +1                      +4                      +3 +2                      +3  
+1 =55

Euler's numbers: 52(more) 51 66 42 74 27 46  
 Euler's numbers :00 1 100 11 10000 10 10 10 10 100 10 10 110 11 10 11 10  
 Euler's numbers :EMPTY +1+4+3 +16 +2 +2 +2+2 +4 +2 +2 +6 +3 +1+3  
 +2=55

#### IV. CONCLUSION

The most widely used number digit system today is decimal. But in this work, Euler's numbers have been converted from decimal base system to binary number base system. Interestingly, the first number of Euler's numbers is "2". Binary numbers have only two digits (0 or 1) too [5].

According to Quantum Perspective Model[4], after calculating the first hundred digits of Euler numbers after the comma, the number "55" (ten times) was found, especially in the Fibonacci series (0,1,1,2,3,5,8,13,21,34,55...)[7]. The 11th digit in the Fibonacci series is also "55". The numbers of the this series can be reached through The Pascal Triangle with the exponents of this number 11. As a result, after calculating the first hundred of Euler's numbers after the comma, the number "55" has been obtained (ten times). It is the sign of the relationship between Euler's numbers and Fibonacci series. During the calculation, the "EMPTY" numbers "00" are disregarded. According to the number-based system, the number "00" has no value, neither in the decimal nor in the binary-based system. According to binary encoding base system, on the case of current not passing, this means 0 (zero) . [8] That's why, it can be the reason of disregardence of "EMPTY" "00" numbers.

As described in the reviews by Mäkelä, and Annila, the Fibonacci sequence is for other mathematical model functions which have useful results. (Mäkelä and Annila, 2010): If Fibonacci numbers are found in Nature, Why not include them in Euler's numbers? Or is it the difference how it discovers parameters in science in terms of the quantum perspective model, especially when the relevant unit of analysis is invariant numbers?

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# Analysis of Crude Oil Production in Nigeria by Servicing Companies

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**Abstract-** The petroleum industry in Nigeria has brought unprecedented changes to the Nigerian economy, particularly in the past five decades when it replaced agriculture as the cornerstone of the Nigeria economy. The oil industry has risen to the commanding heights of the Nigerian economy, contributing the lion share to gross domestic product and accounting for the bulk of federal government revenue and foreign exchange earnings since early 1970. However, Nigeria's considerable endowment in fossil fuel has not translated into an enviable economic performance; rather, the nation's mono-cultural has assumed a precarious dimension in the past decades susceptible to the vagaries of the international oil markets. Empirical analysis was conducted by applying the Multiple Linear Regression of the Ordinary least square techniques, the joint distribution of independent variable contribute to the success of the total production prob(F. Statistic) = 0.00122 which is less than 0.05 thereby establishing the significance of the independent variable. Conclusively, the Servicing Company relationship is not the same, also from estimated regression line only  $x_2$ (Joint Ventures AF/CARRY and  $x_5$ (Sole Risk Independent Companies) has the highest coefficient which implies that they have greater contribution to the total production.

**Keywords:** crude oil, foreign exchange earnings, multiple linear regression, servicing, company, total production.

**GJSFR-F Classification:** MSC 2010: 62J02



*Strictly as per the compliance and regulations of:*





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## I. INTRODUCTION

Oil is a major source of energy in Nigeria and the world in general. Oil being the mainstay of the Nigerian economy plays a vital role in shaping the economic and political destiny of the country. Although Nigeria's oil industry was founded at the beginning of the century, it was not until the end of the Nigeria civil war (1967 - 1970) that the oil industry began to play a prominent role in the economic life of the country. Nigeria can be categorized as a country that is primarily rural, which depends on primary product exports (especially oil products). Since the attainment of independence in 1960 it has experienced ethnic, regional and religious tensions, magnified by the significant disparities in economic, educational and environmental development in the south and the north. These could be partly attributed to the major discovery of oil in the country which affects and is affected by economic and social components. Crude oil discovery has had certain impacts on the Nigeria economy both positively and adversely. On the negative side, this can be considered with respect to the surrounding communities within which the oil wells are exploited. Some of these communities still suffer environmental degradation, which leads to deprivation of means of livelihood and other economic and social factors. Although large proceeds are obtained from the domestic sales and export of petroleum products, its effect on the growth of the

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Nigerian economy as regards returns and productivity is still questionable, hence, the need to evaluate the relative impacts of crude oil on the economy. In the light of the study, the main objective is to assess the impact of crude oil on the Nigerian economy. Given the fact that the oil sector is a very crucial sector in the Nigeria economy, there is the dire need for an appropriate and desirable production and export policy for the sector. In Nigeria, though crude oil has contributed largely to the economy, the revenue has not been properly used. Considering the fact that there are other sectors in the economy, the excess revenue made from the oil sector can be invested in them to diversify and also increase the total GDP of the economy. This study comprises of five sections. Section two presents the background of the study, while the third section focuses on the research methodology. Section four includes data analysis and interpretation of results, and the final section presents and policy proposal and study conclusions.

Therefore, the purpose of this research work is to study the effect of the crude oil production in Nigeria by Servicing Companies for a period of ten years. And to deduce which of this crude oil produced in Nigeria by Servicing Companies between 2002 to 2011 has the highest production or consumption.

#### a) *Historical Background of Oil Industry in Nigeria*

Oil was discovered in Nigeria in 1956 at Oloibiri in the Niger Delta after half a century of exploration. The discovery was made by Shell-BP, at the time the sole concessionaire. Nigeria joined the ranks of oil producers in 1958 when its first oil field came on stream producing 5,100 bpd. After 1960, exploration rights in onshore and offshore areas adjoining the Niger Delta were extended to other foreign companies. In 1965 the EA field was discovered by Shell in shallow water southeast of Warri. In 1970, the end of the Biafran war coincided with the rise in the world oil price, and Nigeria was able to reap instant riches from its oil production. Nigeria joined the Organization of Petroleum Exporting Countries (OPEC) in 1971 and established the Nigerian National Petroleum Company (NNPC) in 1977; a state owned and controlled company which is a major player in both the upstream and downstream sectors [Blair 1976, pp. 98-120]. Following the discovery of crude oil by Shell D'Arcy Petroleum, pioneer production began in 1958 from the company's oil field in Oloibiri in the Eastern Niger Delta. By the late sixties and early seventies, Nigeria had attained a production level of over 2 million barrels of crude oil a day. Although production figures dropped in the eighties due to economic slump, 2004 saw a total rejuvenation of oil production to a record level of 2.5 million barrels per day. Current development strategies are aimed at increasing production to 4million barrels per day by the year 2010. Petroleum production and export play a dominant role in Nigeria's economy and account for about 90 % of her gross earnings. This dominant role has pushed agriculture, the traditional mainstay of the economy, from the early fifties and sixties, to the background.

While the discovery of oil in the eastern and mid-western regions of the Niger Delta pleased hopeful Nigerians, giving them an early indication soon after independent economic development was within reach, at the same time it signaled a danger of grave consequence: oil revenues fueled already existing ethnic and political tension and actually "burned" the country. This tension reached its peak with the civil war that lasted from 1967 to 1970. As the war commenced, the literature reflected the hostility, the impact, and fate of the oil industry. Nigeria survived the war, and was able to recover mainly of the huge revenues from oil in the 1970s. For some three years an oil boom followed, and the country was awash with money. Indeed, there was money for

virtually all the items in its developmental plan. The literature of the postwar years shifted to the analysis of the world oil boom and bust, collectively known as the "oil shock." Starting in 1973 the world experienced an oil shock that rippled through Nigeria until the mid - 1980s. This oil shock was initially positive for the country, but with mismanagement and military rule, it became an economic disaster. The larger middle class produced by the oil boom of the 1970s gradually became disenchanted in the 1980s, and rebellious in the 1990s. The enormous impact of the oil shock could not escape scholarly attention. For almost twenty years (1970s - 1990s), the virtual obsession was to analyze the consequences of oil on Nigeria, using different models and theories. A set of radical-oriented writers was concerned with the nationalization that took place during the oil shock as well as the linkages between oil and an activist foreign policy. Regarding the latter, the emphasis was on OPEC, Nigeria's strategic alliance formation within Africa, the vigorous efforts to establish the Economic Community of West African States (ECOWAS), and the country's attempts to use oil as a political weapon, especially in the liberation of South Africa from apartheid. If many had hoped that oil would turn Nigeria into an industrial power and a prosperous country based on a large middle class, they were to be disappointed when a formally rich country became a debtor nation by the 1980s. The suddenness of the economic difficulties of the 1980s "bust years" had an adverse effect on class relations and the oil workers who understood the dynamics of the industry. As if to capture the labor crisis, writings on oil workers during this period covered many interrelated issues, notably working conditions, strikes, and state labor relations. To be sure, labor issues were not new in the 1980s, since the left-oriented scholars had made a point of exposing labor relations in the colonial era. What was new after 1980 was the focus on oil workers, unions, and class conflict [OPEC annual report 1983].

## II. LITERATURE REVIEW

Etiebet (1999) observed that price of oil products is derived from crude oil prices and it therefore follows that prices of petroleum products should trail crude oil prices. According to the author, it is not always the case for a number of reasons. In the first place, there is always a time lag between crude oil processing and product distribution through network. Secondly, for socio-political reasons, government of both oil producing and consuming countries should invariably intervene in the market to influence products price determination. But in the actual fact, the extent of intervention depends on the specific needs of the country and the level of endowment of the products in question. The author noted that trailing oil products prices down crude oil prices has revealed that, crude oil cost is not the only cost incurred in supply and distribution of petroleum products. Other costs include refining, storing, transporting and distributing, the author asserted. Siddy (1999) asserted that the causes of price instability is attributed to scarcity caused by refinery maintenance and rehabilitation problem, low capacity utilization, supply, and demand inequality. The political change that Nigeria went through, which turned over the administration and endured a lingering economic down turn is enough reason to cause price instability of oil products in Nigeria. The author opined that trailing oil products prices down to crude oil prices has revealed that the instability in the prices of oil products was due to cost of refining, storing, transporting distributing and inefficiencies in the process. Dan (1999) asserted that Nigeria has four refineries, one of which is at Kaduna, Warri and two at Port-Harcourt with a total nominal refining capacity of 445,000 barrels per day.

The author noted that although the refineries find it very difficult to reach that (445,000) due to frequent breakdown and operating problems such as vandalism, which has been reduced and that more products are being pumped throughout the pipelines. Mbendi (2000) argued that in theory, Nigeria's refineries capacity is sufficient to meet its domestic consumption requirement. In practice, however, according to the author, the country has experienced frequent shortage of refined products since its refineries have poor configuration and operation inefficiency. The author stated that it has been estimated that smuggling amounts to over 320,000 barrels per day largely to Benin Republic, Niger, Chad, and Cameroon. The author noted that Nigeria has become a large importer of light petroleum products, importing thousands of tons of refined products. Runl (2010) asserted that people say Nigeria is dominated by oil and they are right because Nigeria seems to be exporting nothing but oil. The government revenues are so dependent on oil, which has been managed quite protectively. But it's still extremely undesirable that internally generated revenue are such a small part of Nigeria's revenue because essentially, it means that all the revenues of the government is just coming down from heaven. It's like a gift and it is easy to waste a gift. The author noted that Nigeria is poor because of oil. Ewa and Agu (2003) shared their view that the dominance of petroleum in Nigerian economy has led to instability in the economy, which as a result makes price instability of oil products to be more prevalent in Nigeria than other countries. The author observed that smuggling is attractive and profitable due to price differential. This act of smuggling oil products from Nigeria to her neighbouring countries is one of the factors which made price instability of oil products to be prevalent in Nigeria. In summary, the works reviewed are the work of many individuals who have shown concern in the area of this study. The most reoccurring term in the works reviewed were that price instability of oil products are prevalent due to ill-refinery maintenance, and rehabilitation problems, low capacity utilization, supply and demand inequality reduction in crude oil allocation, and smuggling of petroleum products.

Nigeria is an oil producing country which depend on its oil income for most of its federal revenue. The share reached 80% in 2008 (Central Bank of Nigeria 2011).

Christos Trisimokos (2011) attempts to estimate the short – run and long- run price and income elasticities of crude oil demand in ten IEA member- countries. Specifically, the price and income elasticities for Sweden, Denmark, Spain, Portugal, Turkey, Finland, Italy, Germany, USA, and Japan are estimated. Crude oil consumption is a function of four explanatory variables real oil price, real GDP per capital, oil consumption lagged one year and a time trend represent Technological improvements.

### III. METHODOLOGY

Econometric is the branch of economics discipline that brings to together economic theory, mathematics, statistic and computer science with economic phenomena with a view for making economic decision. The model and definition of variable is based on the production of crude oil and how it has been produced by regime and how it contributes to the economic growth of the country spanning between 2005 and 2015. The total production of crude oil [by Regime] can be expressed as a linear function as follow;

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + U$$

Where Y= total production of crude oil by Regime

$x_1$  = the quantity of crude oil produced by joint ventures  
 $x_2$  = the quantity of crude oil produced by venture AF/CARRY  
 $x_3$  = the quantity of crude oil produced by Production Sharing Companies.  
 $x_4$  = the quantity of crude oil produced by Service Contract Companies.  
 $x_5$  = the quantity of crude oil produced by Sole Risk Independent Companies.

*Test of significance:* We use the test statistic

$$t = \frac{\beta - \beta}{S.E(\beta)}$$

Therefore, assuming normality homoscedasticity occur, the test statistic above has t-distribution with  $n - k$  degree of freedom.

*Goodness of fit:* The square of the correlation coefficient,  $R^2$  is called the coefficient of multiple determination or goodness of fit.

$$R^2 = \frac{\sum \hat{Y}^2}{\sum Y^2} = \frac{SSR}{SST} = \frac{ESS}{TSS} = 1 - \frac{e^2}{Y^2}$$

*Hypothesis is to be tested*

$H_0$ : there is no significant difference between  $R^2$  and zero      Versus       $H_1$ :  $H_0$  is not true

Symbolically  $H_0$ :  $R^2 = 0$       Versus  $H_1$ :  $R^2 \neq 0$

*Test static:*  $F = \frac{R^2}{K-1} / \frac{1-R^2}{n-k} = \frac{R^2(n-k)}{(1-R^2)(k-1)} \sim F_{k-1, n-k, d.f}$

where  $k-1$  is the degree of freedom for estimated sum of squares and  $\alpha$  = level of significance

*Decision:* if  $F_{cal} > F_{tab}$  rejecting  $H_0$  otherwise accept  $H_0$

*Interpretation of  $R^2$ :* The higher the value of  $R^2$ , the greater the goodness of fit of the regression and If the null hypothesis,  $H_0$  is rejected at a particular level of significance then the value of  $R^2$  is significantly difference from zero.

*The adjusted coefficient of determination ( $\bar{R}^2$ ):* In determination the adjusted, the adjusted ( $\bar{R}^2$ ), the coefficient of determine  $R^2$  which measures the proportion of the variation in the explanatory variables.

$$\bar{R}^2 = \frac{ESS}{TSS} = R^2 - \frac{(n-1)}{n-K} (1-R^2)$$

*To test the overall significance of the parameter estimate  $\beta_i$ ,* we have an hypothesis which indicates.  $H_0$ :  $\beta_i$ 's = 0  $\forall i$  Versus  $H_1$ :  $\beta_i \neq 0 \forall i$

Therefore the test statistic

$$F = \frac{R^2/(n-k)}{(1-R^2)/(k-1)} \sim f_{\alpha, k-1, n-k}$$

with  $(K-1)$ ,  $(n-K)$  degree of freedom.

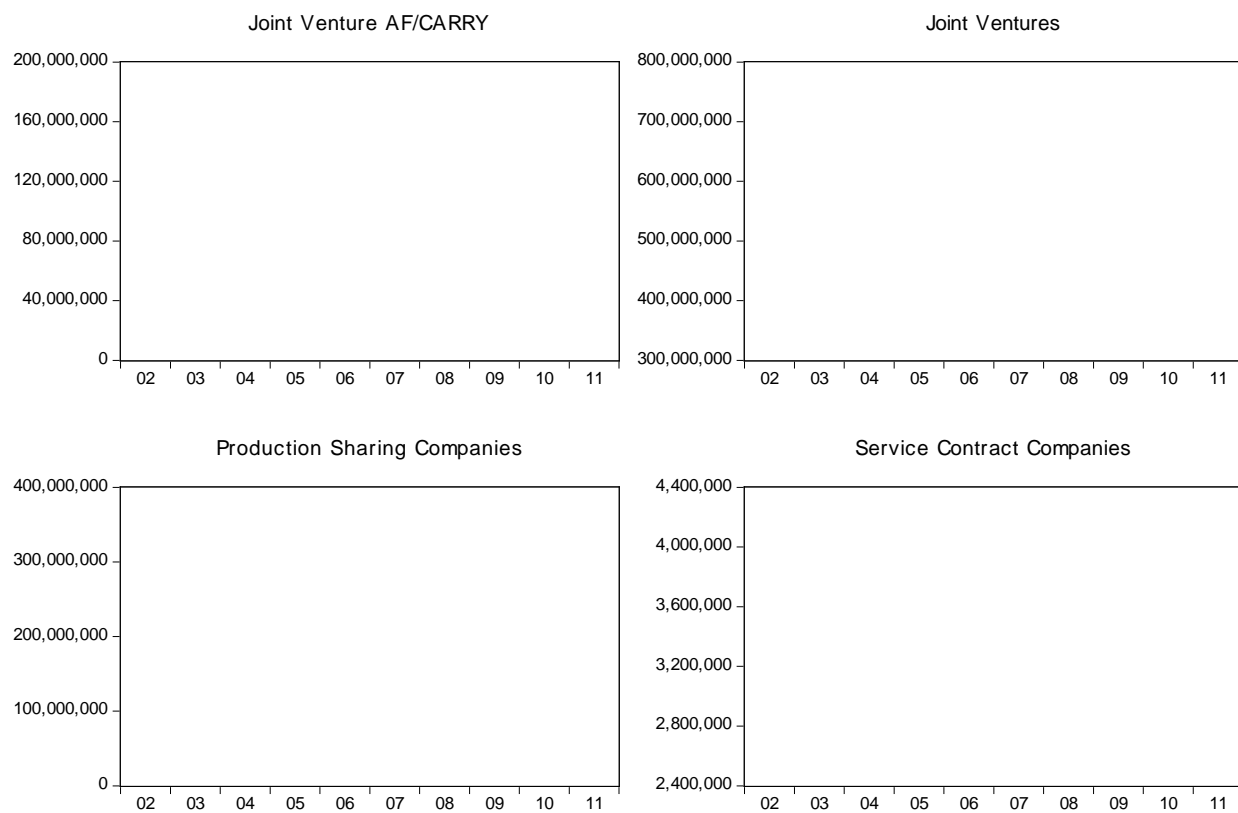
*Autocorrelation:* One of the assumptions of the linear regression model is that errors are independent, that is, error terms are pair wise uncorrelated. This claim was tested in the study as well as multicollinearity and heteroscedasticity as the case may be.

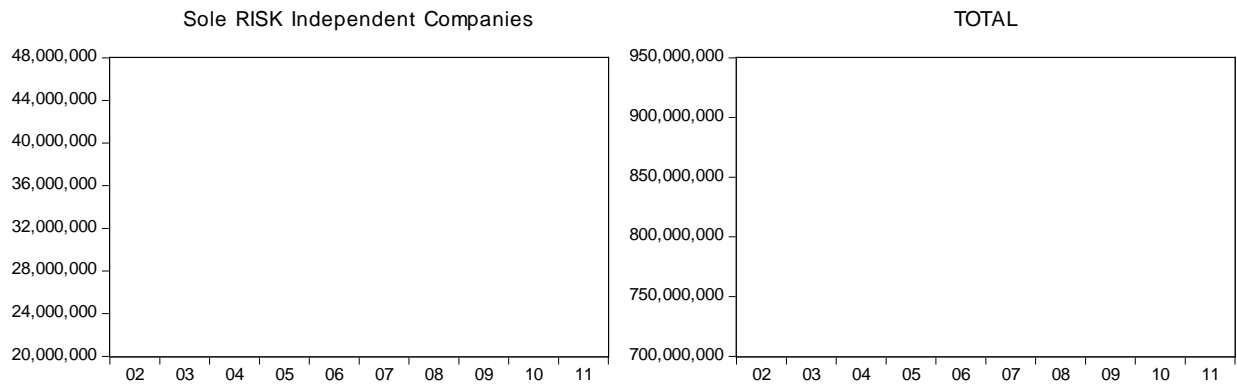
*The examination of residuals:* A residual  $\epsilon_i$  is defined as the difference between the observed value and the fitted value,  $\epsilon_i = \hat{Y}_i - Y_i$ ,  $i = 1, 2, \dots, n$  where  $\hat{Y} = E(Y_i)$ . The analysis of the residuals is an important technique for examining type of departure of the model from what is considered adequate.

#### IV. DATA ANALYSIS

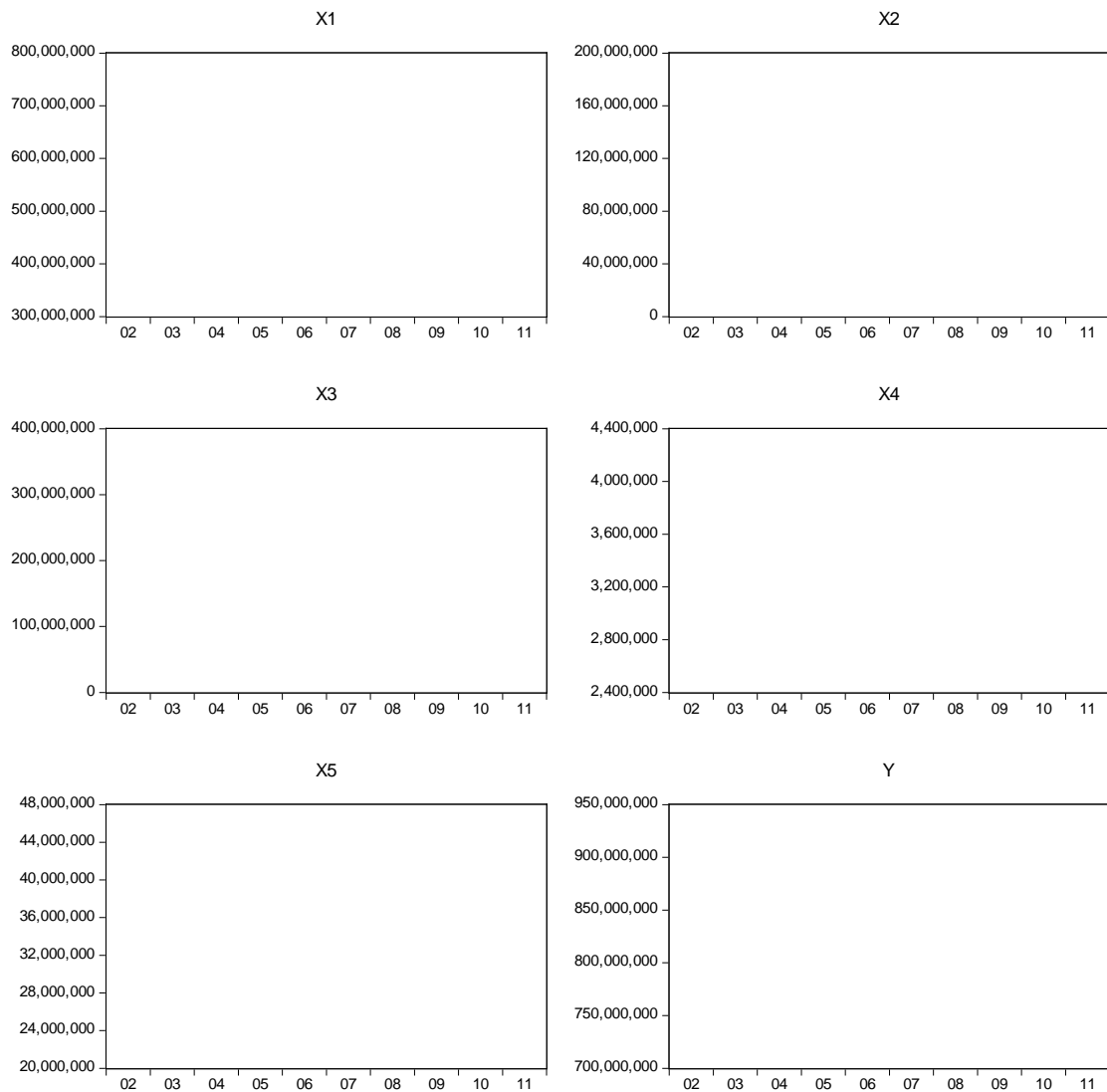
The time plot of all the variables that are of interest in the study is as depicted in figure 1. It shows that the  $x_2$ ,  $x_3$  and  $x_5$  have upward trend over period of interest in the study. This connotes that the three variables increases over the period.  $x_2$  has an erratic movement. It increases from the initial period (2005) up to 2015 and dropped from 2008 till 2010. In addition, it later maintained a steady increase from same 2008 throughout the period of study.  $x_4$  has downward trend between 2005 and 2006. It has an erratic movement between 2003 and 2005. It has steady downward trend i.e. dropped in 2005 till 2015 and.  $x_1$  has an increasing trend between 2005 and 2004 and fell sharply in 2005. It rose between 2005 and 2011, but before that, it maintained downward trend from 2008 throughout the period of study.  $Y$  behaviour is undulating with no particular pattern. However, it should be noted that it maintained its straight trend from its initial period till 2015.

Notes





*Figure 1:* Time plot of variables of interest in the study



*Figure 2:* Bar chart of variables of interest in the study

**Table 1:** Establishment of relationship between the total productions of crude oil in Nigeria and Servicing Companies

Servicing companies	Parameter Estimates		Diagnostics	
	Intercept	Total Production	R-square	F-statistic
Joint Ventures	807518998.6***	0.056	0.018	0.145
Joint Ventures AF/CARRY	732411684.3***	0.921**	0.555	9.973**
Production Sharing Companies	837111357.9***	0.002	0.000	0.000
Service Contract Companies	856942733.2***	-5.454	0.002	0.018
Sole Risk Independent Companies	686269405.2***	4.353**	0.427	5.971**

Y = Total Production,  $x_1$  = Joint Ventures,  $x_2$  = Joint Ventures AF/CARRY,  $x_3$  = Production Sharing Companies,  $x_4$  = Service Contract Companies,  $x_5$  = Sole Risk Independent Companies. From the empirical statistical point of view in the table1 and as F-statistics

(P-value) is < significant level we say it is significant and conclude that servicing companies ( $x$ 's i.e joint ventures, AF/CARRY, Production Sharing Companies, Service Contract Companies and Sole Risk Independent Companies) jointly can influence the Total Production (Y).

*Estimation of the parameters of the econometric model.*

$$E(Y) = B_0 + B_1X_1 + B_2X_2 + B_3X_3 + B_4X_4 + B_5X_5$$

$$E(Y)=19157697+0.950334X_1+0.993823X_2+0.946222X_3+7.035322X_4+0.798971X_5 \dots\dots(1)$$

*Testing the significance of regressors:* From the establishment of the regression analysis table above it was discovered that among the servicing companies; it is only joint ventures AF/CARRY and sole risk independent companies the significant variables to explain the total production.

From appendix (vii)

Testing the significance of the complete regression anova table

$H_0$ : servicing company relationship are the same Versus

$H_1$ : not  $H_0$  at  $\alpha = 0.05$  since the overall Prob (F-statistics) is lesser than 0.05, we say the result is significant and reject  $H_0$  and conclude that servicing company relationship are not the same.

From appendix (viii)

*Testing residuals and the autocorrelation with decision*

*Hypothesis testing*

$H_0$ : residuals are not auto correlated versus

$H_1$ : residuals are auto correlated at  $\alpha = 0.05$

Decision rule: reject  $H_0$  if p-value is greater than 0.05, otherwise accept  $H_0$

From appendix (ix) based Autocorrelation result



The Breusch-Godfrey Serial Correlation LM Test result above indicates that residuals in the model are autocorrelated as the null hypothesis will be rejected since the test is significant. And the Chi square value  $> 0.05$  and meaning that the result is significant.

From appendix (x)

*Test of heteroscedasticity, hypothesis testing, test statistics and decision.*

*Hypothesis testing*

$H_0$ : residuals are not heteroscedastic (homoscedastic) versus

$H_1$ : not  $H_0$  at  $\alpha = 0.05$

*Decision rule*: reject  $H_0$  if p-value is greater than 0.05, otherwise accept  $H_0$

Choosing the obs. R-squared (test stat. = 6.6667) and its corresponding prob. Chi-squared (5) = 0.2466 from the Heteroskedasticity Test: Breusch-Pagan-Godfrey above. The result shows that residuals in the model are not heteroscedastic i.e homoscedastic since its p-value (0.2466)  $> 0.05$ . Meaning that the model is insignificant and null hypothesis will not be rejected.

## V. SUMMARY AND CONCLUSION

This project critically examined the strength of relationship between the total production of crude oil in Nigeria and Servicing Companies (that are producing crude oil per regime). The table in appendix (I) indicates that both  $x_2$  and  $x_5$  have significant relationship on the production of crude oil while others do not. The coefficient interpretation goes thus that joint ventures have a very less significantly relationship with Total Production and positive relationship exist between them which mean that a unit increase of joint ventures will increase total production by 5% provided others independent variables are kept constant. Also, total production will increase by 92% given a 100% increase in joint ventures AF/CARRY while other factors are fixed. More so, a unit increase in Production Sharing Company positively increases total production by 0.2% provided that all other variable are kept constant. However, total production will fall or decrease by 54% for an additional 100% increase of service contract companies and lastly, one unit increase in sole risk independent companies is an increase in total production by 43.5% unit holding other independent variable fixed.

Model R squared (0.983853). 98% variation of total production can be explained the five independent variable. i.e joint ventures, joint venture AF/CARRY, production sharing companies, service contract companies and sole risk independently companies can influence only 98.39% on total company. More so, the  $R^2$  indicates that the model is of good fit or nicely fitted or validity and reliably.

## VI. CONCLUSION

It is apparent from the empirical analysis in chapter four that fitting econometric model is appropriate in establishing the functional relationship that exists between the total production of crude oil and the explanatory variables.

The estimated regression model is given below:

$$E(Y) = 19157697 + 0.950334X_1 + 0.993823X_2 + 0.946222X_3 + 7.035322X_4 + 0.798971X_5$$

However, it is on this basis that the following conclusions were made

- It shows from estimated regression line that the variable  $x_2$  and  $x_5$  has the highest coefficient which implies that they have greater contribution to the response variable Y (Total production of crude oil).
- The coefficient of determination ( $R^2=0.983853$ ) is found to be statistical significant.
- The nature of the data almost follows the ordinary least squares assumptions because there is not heteroscedasticity.

## VII. RECOMMENDATION

The joint distribution of independent variable contribute to the success of the total production prob (F. Statistic) = 0.00122 which is less than 0.05, so we now recommend that the oil production companies should proceed in their production and recruit more expert in order to enhance their production and create necessary facilities that necessitate successful production of oil in Nigeria. Federal government should assist the oil company by funding the major body of oil production at the right time as well as reducing tax rate of the company that are also working under NNPC.

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**APPENDIX (I)**

Dependent Variable: Y

Method: Least Squares

Date: 10/11/14 Time: 11:16

Sample: 2002 2011

Included observations: 10

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	19157697	2.06E+08	0.092982	0.9304
X1	0.950334	0.225717	4.210293	0.0136
X2	0.993823	0.180784	5.497290	0.0053
X3	0.946222	0.352672	2.683011	0.0551
X4	7.035322	15.47833	0.454527	0.6730
X5	0.798971	0.696225	1.147576	0.3151
R-squared	0.983853	Mean dependent var	8.37E+08	
Adjusted R-squared	0.963668	S.D. dependent var	65082177	
S.E. of regression	12405255	Akaike info criterion	35.78885	
Sum squared resid	6.16E+14	Schwarz criterion	35.97040	
Log likelihood	-172.9442	Hannan-Quinn criter.	35.58969	
F-statistic	48.74334	Durbin-Watson stat	1.933953	
Prob(F-statistic)	0.001122			

	Parameter Estimates		Diagnostics	
<i>Servicing companies</i>	Intercept	Total Production	R-square	F-statistic
Joint Ventures	807518998.6***	0.056	0.018	0.145
Joint Ventures AF/CARRY	732411684.3***	0.921**	0.555	9.973**
Production Sharing Companies	837111357.9***	0.002	0.000	0.000
Service Contract Companies	856942733.2***	-5.454	0.002	0.018
Sole Risk Independent Companies	686269405.2***	4.353**	0.427	5.971**

**Regression****Appendix (ii)****Model Summary**

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.134 <sup>a</sup>	.018	-.105	68411246.816

a. Predictors: (Constant), JOINT VENTURES

ANOVA<sup>a</sup>

Model	Sum of Squares	Df	Mean Square	F	Sig.
1 Regression	680418440550422.500	1	680418440550422.500	.145	.713 <sup>b</sup>
Residual	37440789527505352.000	8	4680098690938169.000		
Total	38121207968055776.000	9			

a. Dependent Variable: TOTAL

b. Predictors: (Constant), JOINT VENTURES

Coefficients<sup>a</sup>

Model	Unstandardized Coefficients		Standardized Coefficients	T	Sig.
	B	Std. Error	Beta		
1 (Constant)	807518998.647	81214860.664		9.943	.000
JOINT VENTURES	.056	.146	.134	.381	.713

a. Dependent Variable: TOTAL

Regression  
Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.745 <sup>a</sup>	.555	.499	46054677.105

a. Predictors: (Constant), AF/CARRY

ANOVA<sup>a</sup>

Model	Sum of Squares	df	Mean Square	F	Sig.
1 Regression	21152941702056528.000	1	21152941702056528.000	9.973	.013 <sup>b</sup>
Residual	16968266265999246.000	8	2121033283249905.800		
Total	38121207968055776.000	9			

a. Dependent Variable: TOTAL

b. Predictors: (Constant), AF/CARRY

Coefficients<sup>a</sup>

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
1 (Constant)	732411684.329	36285707.701		20.185	.000
AF/CARRY	.921	.292	.745	3.158	.013

a. Dependent Variable: TOTAL

### Regression Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.003 <sup>a</sup>	.000	-.125	69029735.606

a. Predictors: (Constant), PROD. SHARING COMPANIES

### ANOVA<sup>a</sup>

	Model	Sum of Squares	Df	Mean Square	F	Sig.
1	Regression	372785275734.125	1	372785275734.125	.000	.993 <sup>b</sup>
	Residual	3812083518278004	8	4765104397847505		
	Total	3812120796805577	9	.000		

a. Dependent Variable: TOTAL

b. Predictors: (Constant), PROD. SHARING COMPANIES

### Coefficients<sup>a</sup>

	Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	837111357.869	36214404.862		23.115	.000
	PROD. SHARING COMPANIES	.002	.191	.003	.009	.993

a. Dependent Variable: TOTAL

### Regression Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.047 <sup>a</sup>	.002	-.123	68954551.606

a. Predictors: (Constant), SERVICE CONTRACT COMPANIES

### ANOVA<sup>a</sup>

	Model	Sum of Squares	df	Mean Square	F	Sig.
1	Regression	83366470713763.9	1	83366470713763.9	.018	.898 <sup>b</sup>
	Residual	3803784149734201	8	4754730187167752		
	Total	3812120796805577	9	.000		

a. Dependent Variable: TOTAL

b. Predictors: (Constant), SERVICE CONTRACT COMPANIES

Coefficients<sup>a</sup>

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	856942733.196	149437616.411		5.734	.000
	SERVICE CONTRACT COMPANIES	-5.454	41.191	-.047	-.132	.898

a. Dependent Variable: TOTAL

## Regression

## Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.654 <sup>a</sup>	.427	.356	52235717.324

a. Predictors: (Constant), SOLE RISK IND. COMPANIES

ANOVA<sup>a</sup>

	Model	Sum of Squares	df	Mean Square	F	Sig.
1	Regression	16292646653564018.000	1	16292646653564018.000	5.971	.040 <sup>b</sup>
	Residual	21828561314491756.000	8	2728570164311469.500		
	Total	38121207968055776.000	9			

a. Dependent Variable: TOTAL

b. Predictors: (Constant), SOLE RISK IND. COMPANIES

Coefficients<sup>a</sup>

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	686269405.245	64002592.829		10.723	.000
	SOLE RISK IND. COMPANIES	4.353	1.781	.654	2.444	.040

a. Dependent Variable: TOTAL

## Appendix (vii)

Dependent Variable: Y

Method: Least Squares

Date: 10/11/14 Time: 11:16

Sample: 2002 2011

Included observations: 10

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	19157697	2.06E+08	0.092982	0.9304
X1	0.950334	0.225717	4.210293	0.0136
X2	0.993823	0.180784	5.497290	0.0053
X3	0.946222	0.352672	2.683011	0.0551
X4	7.035322	15.47833	0.454527	0.6730
X5	0.798971	0.696225	1.147576	0.3151
R-squared	0.983853	Mean dependent var		8.37E+08
Adjusted R-squared	0.963668	S.D. dependent var		65082177
S.E. of regression	12405255	Akaike info criterion		35.78885
Sum squared resid	6.16E+14	Schwarz criterion		35.97040
Log likelihood	-172.9442	Hannan-Quinn criter.		35.58969
F-statistic	48.74334	Durbin-Watson stat		1.933953
Prob(F-statistic)	0.001122			

## Appendix (viii)Residuals table

Obs	Actual	Fitted	Residual	Residual Plot
2002	7.3E+08	7.3E+08	-4165665	— . * — . —
2003	8.3E+08	8.3E+08	967072.	— . —* . —
2004	9.1E+08	9.1E+08	2808862	— . —* . —
2005	9.2E+08	9.2E+08	1484941	— . —* . —
2006	8.7E+08	8.7E+08	684523.	— . * . —
2007	8.0E+08	8.1E+08	-3445646	— . * — . —
2008	7.7E+08	7.7E+08	2850474	— . —* . —
2009	7.8E+08	7.8E+08	5237186	— . —* . —
2010	9.0E+08	8.8E+08	1.3E+07	— . —* —
2011	8.7E+08	8.9E+08	-1.9E+07	—* . — . —



## Appendix (ix) Auto correlation result

Breusch-Godfrey Serial Correlation LM Test:

<b>F-statistic</b>	<b>4.861121</b>	<b>Prob. F(2,2)</b>	<b>0.1706</b>
<b>Obs*R-squared</b>	<b>8.293842</b>	<b>Prob. Chi-Square(2)</b>	<b>0.0158</b>

Test Equation:

Dependent Variable: RESID

Method: Least Squares

Date: 10/21/14 Time: 13:47

Sample: 2002 2011

Included observations: 10

Presample missing value lagged residuals set to zero.

<b>Variable</b>	<b>Coefficient</b>	<b>Std. Error</b>	<b>t-Statistic</b>	<b>Prob.</b>
C	-1.57E+08	1.55E+08	-1.014308	0.4172
X1	0.166684	0.170197	0.979361	0.4307
X2	-0.065646	0.110131	-0.596071	0.6116
X3	0.281708	0.271260	1.038514	0.4081
X4	3.183448	10.88803	0.292381	0.7975
X5	0.759717	0.540765	1.404894	0.2952
RESID(-1)	-2.330861	1.047272	-2.225649	0.1560
RESID(-2)	-1.095442	1.409639	-0.777108	0.5184
R-squared	0.829384	Mean dependent var	2.41E-07	
Adjusted R-squared	0.232229	S.D. dependent var	8270170.	
S.E. of regression	7246534.	Akaike info criterion	34.42051	
Sum squared resid	1.05E+14	Schwarz criterion	34.66258	
Log likelihood	-164.1025	Hannan-Quinn criter.	34.15496	
F-statistic	1.388892	Durbin-Watson stat	2.538704	
Prob(F-statistic)	0.480429			

## Appendix (x)

Heteroskedasticity Test: Breusch-Pagan-Godfrey

<b>F-statistic</b>	<b>1.600067</b>	<b>Prob. F(5,4)</b>	<b>0.3347</b>
<b>Obs*R-squared</b>	<b>6.666760</b>	<b>Prob. Chi-Square(5)</b>	<b>0.2466</b>
<b>Scaled explained SS</b>	<b>1.833184</b>	<b>Prob. Chi-Square(5)</b>	<b>0.8717</b>

Test Equation:

Dependent Variable: RESID^2

Method: Least Squares

Date: 10/21/14 Time: 14:09

Sample: 2002 2011

Included observations: 10

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-8.73E+14	1.73E+15	-0.504592	0.6404
X1	1522066.	1895543.	0.802971	0.4670
X2	-719661.0	1518205.	-0.474021	0.6602
X3	2272979.	2961696.	0.767459	0.4856
X4	-86175750	1.30E+08	-0.662966	0.5436
X5	4808340.	5846812.	0.822387	0.4571
R-squared	0.666676	Mean dependent var	6.16E+13	
Adjusted R-squared	0.250021	S.D. dependent var	1.20E+14	
S.E. of regression	1.04E+14	Akaike info criterion	67.67583	
Sum squared resid	4.34E+28	Schwarz criterion	67.85738	
Log likelihood	-332.3791	Hannan-Quinn criter.	67.47667	
F-statistic	1.600067	Durbin-Watson stat	1.887617	
Prob(F-statistic)	0.334692			



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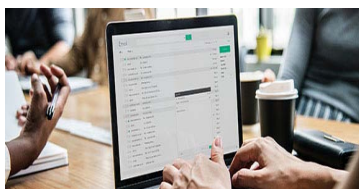
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- Main text: font size 10 with two justified columns.
- Two columns with equal column width of 3.38 and spacing of 0.2.
- First character must be three lines drop-capped.
- The paragraph before spacing of 1 pt and after of 0 pt.
- Line spacing of 1 pt.
- Large images must be in one column.
- The names of first main headings (Heading 1) must be in Roman font, capital letters, and font size of 10.
- The names of second main headings (Heading 2) must not include numbers and must be in italics with a font size of 10.

### ***Structure and Format of Manuscript***

The recommended size of an original research paper is under 15,000 words and review papers under 7,000 words. Research articles should be less than 10,000 words. Research papers are usually longer than review papers. Review papers are reports of significant research (typically less than 7,000 words, including tables, figures, and references)

A research paper must include:

- a) A title which should be relevant to the theme of the paper.
- b) A summary, known as an abstract (less than 150 words), containing the major results and conclusions.
- c) Up to 10 keywords that precisely identify the paper's subject, purpose, and focus.
- d) An introduction, giving fundamental background objectives.
- e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition, sources of information must be given, and numerical methods must be specified by reference.
- f) Results which should be presented concisely by well-designed tables and figures.
- g) Suitable statistical data should also be given.
- h) All data must have been gathered with attention to numerical detail in the planning stage.

Design has been recognized to be essential to experiments for a considerable time, and the editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned unrefereed.

- i) Discussion should cover implications and consequences and not just recapitulate the results; conclusions should also be summarized.
- j) There should be brief acknowledgments.
- k) There ought to be references in the conventional format. Global Journals recommends APA format.

Authors should carefully consider the preparation of papers to ensure that they communicate effectively. Papers are much more likely to be accepted if they are carefully designed and laid out, contain few or no errors, are summarizing, and follow instructions. They will also be published with much fewer delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and suggestions to improve brevity.



## FORMAT STRUCTURE

***It is necessary that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.***

All manuscripts submitted to Global Journals should include:

### **Title**

The title page must carry an informative title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) where the work was carried out.

### **Author details**

The full postal address of any related author(s) must be specified.

### **Abstract**

The abstract is the foundation of the research paper. It should be clear and concise and must contain the objective of the paper and inferences drawn. It is advised to not include big mathematical equations or complicated jargon.

Many researchers searching for information online will use search engines such as Google, Yahoo or others. By optimizing your paper for search engines, you will amplify the chance of someone finding it. In turn, this will make it more likely to be viewed and cited in further works. Global Journals has compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

### **Keywords**

A major lynchpin of research work for the writing of research papers is the keyword search, which one will employ to find both library and internet resources. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining, and indexing.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy: planning of a list of possible keywords and phrases to try.

Choice of the main keywords is the first tool of writing a research paper. Research paper writing is an art. Keyword search should be as strategic as possible.

One should start brainstorming lists of potential keywords before even beginning searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in a research paper?" Then consider synonyms for the important words.

It may take the discovery of only one important paper to steer in the right keyword direction because, in most databases, the keywords under which a research paper is abstracted are listed with the paper.

### **Numerical Methods**

Numerical methods used should be transparent and, where appropriate, supported by references.

### **Abbreviations**

Authors must list all the abbreviations used in the paper at the end of the paper or in a separate table before using them.

### **Formulas and equations**

Authors are advised to submit any mathematical equation using either MathJax, KaTeX, or LaTeX, or in a very high-quality image.

### **Tables, Figures, and Figure Legends**

Tables: Tables should be cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g., Table 4, a self-explanatory caption, and be on a separate sheet. Authors must submit tables in an editable format and not as images. References to these tables (if any) must be mentioned accurately.



## Figures

Figures are supposed to be submitted as separate files. Always include a citation in the text for each figure using Arabic numbers, e.g., Fig. 4. Artwork must be submitted online in vector electronic form or by emailing it.

## PREPARATION OF ELETRONIC FIGURES FOR PUBLICATION

Although low-quality images are sufficient for review purposes, print publication requires high-quality images to prevent the final product being blurred or fuzzy. Submit (possibly by e-mail) EPS (line art) or TIFF (halftone/ photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Avoid using pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings). Please give the data for figures in black and white or submit a Color Work Agreement form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution at final image size ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs): >350 dpi; figures containing both halftone and line images: >650 dpi.

Color charges: Authors are advised to pay the full cost for the reproduction of their color artwork. Hence, please note that if there is color artwork in your manuscript when it is accepted for publication, we would require you to complete and return a Color Work Agreement form before your paper can be published. Also, you can email your editor to remove the color fee after acceptance of the paper.

## TIPS FOR WRITING A GOOD QUALITY SCIENCE FRONTIER RESEARCH PAPER

Techniques for writing a good quality Science Frontier Research paper:

**1. Choosing the topic:** In most cases, the topic is selected by the interests of the author, but it can also be suggested by the guides. You can have several topics, and then judge which you are most comfortable with. This may be done by asking several questions of yourself, like "Will I be able to carry out a search in this area? Will I find all necessary resources to accomplish the search? Will I be able to find all information in this field area?" If the answer to this type of question is "yes," then you ought to choose that topic. In most cases, you may have to conduct surveys and visit several places. Also, you might have to do a lot of work to find all the rises and falls of the various data on that subject. Sometimes, detailed information plays a vital role, instead of short information. Evaluators are human: The first thing to remember is that evaluators are also human beings. They are not only meant for rejecting a paper. They are here to evaluate your paper. So present your best aspect.

**2. Think like evaluators:** If you are in confusion or getting demotivated because your paper may not be accepted by the evaluators, then think, and try to evaluate your paper like an evaluator. Try to understand what an evaluator wants in your research paper, and you will automatically have your answer. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

**3. Ask your guides:** If you are having any difficulty with your research, then do not hesitate to share your difficulty with your guide (if you have one). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work, then ask your supervisor to help you with an alternative. He or she might also provide you with a list of essential readings.

**4. Use of computer is recommended:** As you are doing research in the field of science frontier then this point is quite obvious. Use right software: Always use good quality software packages. If you are not capable of judging good software, then you can lose the quality of your paper unknowingly. There are various programs available to help you which you can get through the internet.

**5. Use the internet for help:** An excellent start for your paper is using Google. It is a wondrous search engine, where you can have your doubts resolved. You may also read some answers for the frequent question of how to write your research paper or find a model research paper. You can download books from the internet. If you have all the required books, place importance on reading, selecting, and analyzing the specified information. Then sketch out your research paper. Use big pictures: You may use encyclopedias like Wikipedia to get pictures with the best resolution. At Global Journals, you should strictly follow here.



**6. Bookmarks are useful:** When you read any book or magazine, you generally use bookmarks, right? It is a good habit which helps to not lose your continuity. You should always use bookmarks while searching on the internet also, which will make your search easier.

**7. Revise what you wrote:** When you write anything, always read it, summarize it, and then finalize it.

**8. Make every effort:** Make every effort to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in the introduction—what is the need for a particular research paper. Polish your work with good writing skills and always give an evaluator what he wants. Make backups: When you are going to do any important thing like making a research paper, you should always have backup copies of it either on your computer or on paper. This protects you from losing any portion of your important data.

**9. Produce good diagrams of your own:** Always try to include good charts or diagrams in your paper to improve quality. Using several unnecessary diagrams will degrade the quality of your paper by creating a hodgepodge. So always try to include diagrams which were made by you to improve the readability of your paper. Use of direct quotes: When you do research relevant to literature, history, or current affairs, then use of quotes becomes essential, but if the study is relevant to science, use of quotes is not preferable.

**10. Use proper verb tense:** Use proper verb tenses in your paper. Use past tense to present those events that have happened. Use present tense to indicate events that are going on. Use future tense to indicate events that will happen in the future. Use of wrong tenses will confuse the evaluator. Avoid sentences that are incomplete.

**11. Pick a good study spot:** Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

**12. Know what you know:** Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

**13. Use good grammar:** Always use good grammar and words that will have a positive impact on the evaluator; use of good vocabulary does not mean using tough words which the evaluator has to find in a dictionary. Do not fragment sentences. Eliminate one-word sentences. Do not ever use a big word when a smaller one would suffice.

Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

**14. Arrangement of information:** Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

**15. Never start at the last minute:** Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

**16. Multitasking in research is not good:** Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

**17. Never copy others' work:** Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

**18. Go to seminars:** Attend seminars if the topic is relevant to your research area. Utilize all your resources.

**19. Refresh your mind after intervals:** Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.



**20. Think technically:** Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

**21. Adding unnecessary information:** Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

**22. Report concluded results:** Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

**23. Upon conclusion:** Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium through which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

## INFORMAL GUIDELINES OF RESEARCH PAPER WRITING

### Key points to remember:

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

### Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

*The introduction:* This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

### The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

### General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

**To make a paper clear:** Adhere to recommended page limits.



### *Mistakes to avoid:*

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

### **Title page:**

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

**Abstract:** This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

*Reason for writing the article—theory, overall issue, purpose.*

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

### **Approach:**

- Single section and succinct.
- An outline of the job done is always written in past tense.
- Concentrate on shortening results—limit background information to a verdict or two.
- Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

### **Introduction:**

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.



*The following approach can create a valuable beginning:*

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

#### **Approach:**

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

#### **Procedures (methods and materials):**

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

#### **Materials:**

*Materials may be reported in part of a section or else they may be recognized along with your measures.*

#### **Methods:**

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

#### **Approach:**

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

#### **What to keep away from:**

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.





**Results:**

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

**Content:**

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

**What to stay away from:**

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

**Approach:**

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

**Figures and tables:**

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

**Discussion:**

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."



Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- Give details of all of your remarks as much as possible, focusing on mechanisms.
- Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

#### **Approach:**

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

Describe generally acknowledged facts and main beliefs in present tense.

### THE ADMINISTRATION RULES

Administration Rules to Be Strictly Followed before Submitting Your Research Paper to Global Journals Inc.

*Please read the following rules and regulations carefully before submitting your research paper to Global Journals Inc. to avoid rejection.*

*Segment draft and final research paper:* You have to strictly follow the template of a research paper, failing which your paper may get rejected. You are expected to write each part of the paper wholly on your own. The peer reviewers need to identify your own perspective of the concepts in your own terms. Please do not extract straight from any other source, and do not rephrase someone else's analysis. Do not allow anyone else to proofread your manuscript.

*Written material:* You may discuss this with your guides and key sources. Do not copy anyone else's paper, even if this is only imitation, otherwise it will be rejected on the grounds of plagiarism, which is illegal. Various methods to avoid plagiarism are strictly applied by us to every paper, and, if found guilty, you may be blacklisted, which could affect your career adversely. To guard yourself and others from possible illegal use, please do not permit anyone to use or even read your paper and file.



CRITERION FOR GRADING A RESEARCH PAPER (COMPILATION)  
BY GLOBAL JOURNALS

Please note that following table is only a Grading of "Paper Compilation" and not on "Performed/Stated Research" whose grading solely depends on Individual Assigned Peer Reviewer and Editorial Board Member. These can be available only on request and after decision of Paper. This report will be the property of Global Journals.

Topics	Grades		
	A-B	C-D	E-F
<b>Abstract</b>	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
<b>Introduction</b>	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
<b>Methods and Procedures</b>	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
<b>Result</b>	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
<b>Discussion</b>	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
<b>References</b>	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring



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save our planet



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