Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 21 Issue 5 Version 1.0 Year 2021
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Parametric Solutions of Fermat's Equation

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Abstract- The values of the variables of the equation, $a^n + b^n = c^n$, are first obtained in terms of two parameters g and h. By substituting these values in the equation it is verified that these parametric solutions as obtained indeed satisfy the Fermat's Equation.

GJSFR-F Classification: MSC 2010: 11D41

1. Ribet, K. A., Hayes, B., Fermat's Last Theorem and Modern Arithmetic, Amer. Sci. 82,1994,144-156.

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I. Introduction

Ever since the French mathematician Pierre de Fermat[1] stated the conjecture in 1630 that the equation, $a^n + b^n = c^n$, cannot have any solutions if a, b, c are non-zero integers each > 0 and n is an integer > 2, the equation has been a subject of intense and often heated discussions amongst mathematicians and aspirants alike. This conjecture is known as Fermat's Last Theorem. The fact that Fermat claimed to have a proof but never wrote it down has put the researchers in a quandary since nobody has yet been able to duplicate the proof the way Fermat originally claimed. Perhaps, the strong appeal of the problem is the simplicity and elegance of its statement contrasted with apparent hopelessness [2] of an elementary way to establish it. Finally, in around 1995 Wiles [3, 4] offered a proof of the Theorem. His paper incorporates by reference [5 ,6] a vastly larger body of mathematical work developed over the last several decades. And it requires an extraordinary arsenal [7] of mathematical tools to understand Wales's complex and very lengthy proof. Consequently, the quest for a simple and short proof continues. In this paper, based on elementary principles, simple parametric solutions of the equation are presented.

II. The Theorem

Fermat asserted [6,7,8] that if a, b, c are nonzero positive integers and n is an integer > 2 then there is no solution of the equation (1).

$$a^n + b^n = c^n \tag{1}$$

This paper offers a simple proof of the theorem based on elementary principles.

III. Simplification of the Theorem

It is enough to prove [1,7] the theorem when the variables are relatively prime integers designated as (a, b, c) = 1, 2|b and the exponent is a prime k > 3.

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IV. Strategy of the Proof

Lemma-1 2

To be candidates for integer solutions of (1) the variables a, b, c must form the three sides of an acute angled triangle ABC under the given conditions.

$$a^k + b^k = c^k \tag{1}$$

Notes

Conditions: (a, b, c) = 1, c > a > b, 2|b, prime k > 3.

Proof:

Case-1: $a + b \le c$ then $a^k + b^k + M < c^k$, $M > 0$. So $a^k + b^k = c^k$ is impossible.
Case-2: a + b > c. Therefore, a, b, c must form the three sides of the triangle ABC.

If ABC is a nonacute triangle then $a^2 + b^2 <= c^2$ then $c^{k-2} a^2 + c^{k-2} b^2 <= c^k$
Since c > a > b . Obviously, $a^k + b^k < c^k$. Hence $a^k + b^k c^k$ is impossible.

Therefore, ABC must be an acute angled triangle.

This proves Lemma-1

Proof of the Theorem.

To prove the theorem an auxiliary equation (1.1) is introduced:

$$x^{2k} + y^{2k} = z^{2k} \tag{1.1}$$

By comparing (1) and (1.1) it is seen that $a = x^2$, $b = y^2$ and $c = z^2$
Therefore, by obtaining solutions of (1.1) solutions of (1) can be obtained.

The variables are first assumed to be integers each > 0.
Under these assumptions parametric solutions of (1.1) are obtained.
To prove the theorem the variables are first to be nonzero integers and a parametric solution of the equation is obtained. And then it is argued that these parametric solutions cannot lead to integer solutions of the equation.

Parametric Solutions of the Equation:

V. Hypothesis-1

There exist integers g and h such that (2) and (3) are satisfied [10,p-536] for integers such that $X = x^2, Y= y^2$, (x, y) = 1 (g, h) = 1, 2|h.

$$X + iY = (g + ih)^k \qquad 2) \qquad X - iY = (g - ih)^k \tag{3}$$

By multiplying the corresponding sides of (2) and (3) one gets (4) where tan H = h/g, $0 < H < \pi/2$.

$$X^2 + Y^2 = (g^2 + h^2)^k [(\cos kH)^2 + (\sin kH)^2] \tag{4}$$

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Since $(\cos kH)^2 + (\sin kH)^2 = 1$ one gets (5).

$$X^2 + Y^2 = (g^2 + h^2)^k \tag{5}$$

By comparing (4) and (5) (6) is obtained.

$$g^2 + h^2 = z^2 \tag{6}$$

Therefore, the variables g, h, z are the three sides of a right triangle ZGH where z is the hypotenuse. Since all the sides and the area of the triangle are rational it is a rational[11] right triangle. Therefore, one gets (7) and (8).

$$x = z(\cos kH)^{1/k} \tag{7}$$

$$y = z(\sin kH)^{1/k} \tag{8}$$

It is verified that (7) and (8) are indeed the parametric solutions of (1.1)

Now by noting that $a = x^2$, $b = y^2$, $c = z^2$ (7.1) and (8.1) are obtained.

$$a = c(\cos kH)^{2/k} \quad (7.1) \qquad b = c(\sin kH)^{2/k} \tag{8.1}$$

By substituting these values of a, b in (1) it is verified that a, b, c as given by (7.1) and (8.1) are indeed the parametric solutions of (1).

Next challenge is to prove that given c is an integer > 0, (a, b) = 1 is impossible in (7.1) and (8.1)

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Notes