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Force-Mediating Particle of Coupling of Spin Angular Momenta

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I. Introduction

In conventional spin theory [1],[2],[3],[4],[5], each spin particle has its own spin space; one particle possesses one spin space; two spin particles, two spin spaces,...... and n particles, n spin spaces. These spaces are independent each other. To couple two or more angular momentums and then obtain the angular momentum of the combined system. The spin space of the combined system can be expressed as the direct product of two or more single spin particles

$$V = V_1 \otimes V_2 \otimes V_3 \otimes \ldots \otimes V_n \quad (0.1)$$

The way the dimensionalities, an example of two spin-1/2 fermions, work out is as follows:

$$V = 1/2 \otimes 1/2 = 1 \oplus 0 \quad (0.2)$$

(V_1=1/2, V_2=1/2) The decomposition of the direct product $V_1 \otimes V_2$ space into a sum of space 1 and space 0 above. The dimensionality of each spin -1/2 is $2\cdot\frac{1}{2}+1=2$, spin-1 is $2\cdot 1+1=3$ and spin-0 is $2\cdot 0+1=1$. The total dimensionality is 2+2=3+1=4.

Now if suppose $V_1=V_2=V$, What will happen to their spin angular momentum couple (0.2)? The analysis shows: an amusing spin angular momentum picture, so-called Spin Topological Space, STS [6],[7],[8],[9] is introduced. The spin space dimensionalities of the two spin-1/2 fermions V_1, V_2 and V, mentioned before, all become to be infinite.

One of the achievements is that STS could invest spin-zero particle with math constructive constituent of angular momentum such as other bosons and fermions in the conventional spin theory. In STS math frame, spin-zero particle is no longer a " point-particle ".

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But on the other hand, STS in itself always encounters with troubles for angular momentum z-component addtion of many single spin particles, some types of z-component addtion, in which the total z-components of the combined system are always less than the those that should be, that is so-called the puzzle of spin angular momentum *losing*, from the point of view of common sense for math and physice world today.

But if the spin-zero particle mentioned previously is supposed to be viewed as an invisible boson, a force-mediating mediator, which actually interacts with the two spin-1/2 fermions, is participanting in the synthetic course of z-component addition of the two fermioms, the problem of *losing* with z-component addition of angular momentums would be solved. It will be lucky, spin-zero boson particle will eliminate the faults of z-component addtion in STS.

Using the above peculiar ideas and new concepts, the matrix representations of C-G Coefficients of two spin-1/2 fermions coupling in STS are worked out, which predict many "excited states" of C-G Coefficients that have not been obverserd so far.

II. Spin Topological Space, STS

We work with systems made up of two angular momenta, $\vec{j_1}$ and $\vec{j_2}$

$$\vec{j_1} \times \vec{j_1} = i\,\vec{j_1}, \quad \vec{j_2} \times \vec{j_2} = i\,\vec{j_2}, \tag{1}$$

If an interaction between $\vec{j_1}$ and $\vec{j_2}$ is such as to have the coupling of two angular momenta

$$\vec{j} = \vec{j_1} + \vec{j_2}$$

Then $$\vec{j} \times \vec{j} = (\vec{j_1} + \vec{j_2}) \times (\vec{j_1} + \vec{j_2}) = i\,(\vec{j_1} + \vec{j_2}) + (\vec{j_1} \times \vec{j_2} + \vec{j_2} \times \vec{j_1}) \tag{2}$$

Suppose $\overleftarrow{j_1} \subset$ Space V_{j_1}, $\overleftarrow{j_2} \subset$ Space V_{j_2}

It will allow the following two cases of (2) to happen:

【1】If *both* $\vec{j_1}$ *and* $\vec{j_2}$ attribute to *different* space

$$V_{j_1} \neq V_{j_2} \tag{3}$$

【2】If $\vec{j_1}$ *and* $\vec{j_2}$ all attribute to one same space

$$V_{j_1} = V_{j_2} = V, \tag{4}$$

Next we give the math construction of physics reality in cases of【1】and【2】

■ 【1】Obviously, $[\, j_{1,\alpha} \,,\, j_{2,\beta} \,]_- = 0, \qquad \alpha, \beta = 1, 2, 3$

we have $$\vec{j_1} \times \vec{j_2} + \vec{j_2} \times \vec{j_1} = 0 \tag{5}$$

And obtain angular momentum commutation rules:

$$(\vec{j_1} + \vec{j_2}) \times (\vec{j_1} + \vec{j_2}) = i(\vec{j_1} + \vec{j_2}) \quad (6)$$

where $$\vec{j} \times \vec{j} = i\vec{j}, \quad \vec{j} = \vec{j_1} + \vec{j_2} \quad (7)$$

As $(j_1, m_1), (j_2, m_2)$ and (j, m) are the eigenvalues of j_1, j_2 and j respectively, by unitary transformation with Wigner or Clebsch-Gordan coefficients, we can have some matrix relationships among (j_1, m_1), (j_2, m_2) and (j, m).

Continue to discuss the example (0.2), let j_1 and j_2 be two spin-1/2 fermions, we can get formula below

	1/2	$\otimes$	1/2	=	1	$\oplus$	0	(8)
Casimir operator	$\frac{3\hbar^2}{4}$		$\frac{3\hbar^2}{4}$		$2\hbar^2$		$0\hbar^2$	

Here, reducible $(2j_1+1)(2j_2+1)=4$ dimensional representation of rotation group is denoted with multiplication 1/2 $\otimes$ 1/2, by unitary transformation, reduce it to its irreducible representation, a triplets, a 3 dimension of spin-1 and a singlet, one dimension, labelled addition 1 $\oplus$ 0. (8) called direct product of two spin-1/2 Hilbert spaces is a direct sum of a spin-1 space and a spin-0 space.

Here: spin-1 space is spanned by a triplet state of two symmetric fermions with Casimir operator eigenvalue $1(1+1)=2\hbar^2$. And spin-0 space, by a singlet state of two antisymmetric fermions with Casimir operator eigenvalue $0(0+1)=0\hbar^2$.

Formula (8), due to spin angular momentum couple, is based on the restriction (5) of case 【1】. We wonder what will happen to (8) in case【2】

if $$\vec{j_1} \times \vec{j_2} + \vec{j_2} \times \vec{j_1} \neq 0 \quad (9)$$

that is: how can we deal with the angular momentum coupling between the two spin-1/2 fermions if they *all attribute to one same spin space V* ?

■ 【2】 Call $(j, k), (r, s)$ Spin Topological Coordinate in real region of STS, and now sign $\vec{\pi}$ is used to represent spin angular momentum.

Giving following two definitions

$$\pi_{1;j,k} = \frac{1}{2}(\pi_j^+ + \pi_k^-) \quad (10.1)$$

$$\pi_{2;j,k} = \frac{1}{2i}(\pi_j^+ - \pi_k^-) \quad (10.2)$$

After calculations obtain

$$\pi_{3;j,k} = \pi_{1;j,k}\pi_{2;j,k} - \pi_{2;j,k}\pi_{1;j,k} = \frac{1}{2}(\pi_j^+\pi_k^- - \pi_k^-\pi_j^+) \quad (10.3)$$

From above three formulas, it can be shown $\pi_{1;j,k}, \pi_{2;j,k}$ and $\pi_{3;j,k}$ satisfy angular momentum rule.

$$\vec{\pi}_{j,k} \times \vec{\pi}_{j,k} = i\vec{\pi}_{j,k} \tag{10}$$

The similar results to

$$\pi_{1;\,r,s} = \frac{1}{2}(\pi_r^+ + \pi_s^-) \tag{11.1}$$

$$\pi_{2;\,r,s} = \frac{1}{2i}(\pi_r^+ - \pi_s^-) \tag{11.2}$$

$$\pi_{3;\,r,s} = \pi_{1;\,r,s}\pi_{2;\,r,s} - \pi_{2;\,r,s}\pi_{1;\,r,s} = \frac{1}{2}(\pi_r^+\pi_s^- - \pi_s^-\pi_r^+) \tag{11.3}$$

$$\vec{\pi}_{r,s} \times \vec{\pi}_{r,s} = i\vec{\pi}_{r,s} \tag{11}$$

Instesd of sign $\vec{j}$, making the substitutions

$$\vec{j}_1 \Rightarrow (\vec{j}_1)_{j,k} = \vec{\pi}_1 \equiv \vec{\pi}_{j,k} = \{\pi_{1;\,j,k}, \pi_{2;\,j,k}, \pi_{3;\,j,k}\} \tag{12}$$

$$\vec{j}_2 \Rightarrow (\vec{j}_2)_{r,s} = \vec{\pi}_2 \equiv \vec{\pi}_{r,s} = \{\pi_{1;\,r,s}, \pi_{2;\,r,s}, \pi_{3;\,r,s}\} \tag{13}$$

Then the lefthand of (9) becomes to

$$\vec{j}_1 \times \vec{j}_2 + \vec{j}_2 \times \vec{j}_1 \Rightarrow (\vec{\pi}_{j,k} \times \vec{\pi}_{r,s}) + (\vec{\pi}_{r,s} \times \vec{\pi}_{j,k}) \tag{14}$$

After the primordial representations introduced above, the raising operator π_j^+ can be defined by $\pi_j(0)$ and I_{+1} as (15). Similarly the lowering operator π_k^- defined by $\pi_k(0)$ and I_{-1} as (16)

$$\pi_j^+ = +\pi_j(0)I_{+1} \tag{15}$$

$$\pi_k^- = -I_{-1}\pi_k(0) \tag{16}$$

Where $$\pi_j(0) = \pi_0(0) + jI_0, \quad \pi_k(0) = \pi_0(0) + kI_0 \tag{17}$$

Here: Subscript, " 0 " of I_0, represents the unit principle diagonal in STS. Subscripts, " $+1$ " and " -1 " the first up unit principle diagonal and the first down unit principle diagonal.

Substitute (15) and (16) into (10.3), and the same means into (11.3), we obtain

$$\pi_{3;\,j,k} = \pi_0(0) + \frac{1}{2}(j + k + 1) \tag{18.1}$$

$$\pi_{3;\,r,s} = \pi_0(0) + \frac{1}{2}(r + s + 1) \tag{18.2}$$

$$\pi_0(0) = \text{diag}\{\ \cdots,\ +5, +4, +3, +2, +1, \underline{0}, -1, -2, -3, -4, -5,\ \cdots\ \}_0 \tag{19}$$

$\pi_0(0)$ is the spin basic state of all bosons, called the vacuum background of spin angular momentum.

Global Journal of Science Frontier Research (A) Volume XXI Issue IV Version I Year 2021

Now back to (14), for convenience, we are referred to the third component of (14)

$$(\vec{\pi}_{j,k} \times \vec{\pi}_{r,s})_3 + (\vec{\pi}_{r,s} \times \vec{\pi}_{j,k})_3 \tag{20}$$

Firstly calculate

$$(\vec{\pi}_{j,k} \times \vec{\pi}_{r,s})_3 = \pi_{1;j,k}\pi_{2;r,s} - \pi_{2;j,k}\pi_{1;r,s} = \frac{i}{2}(\pi_j^+\pi_s^- - \pi_k^-\pi_r^+) \tag{21.1}$$

$$(\vec{\pi}_{r,s} \times \vec{\pi}_{j,k})_3 = \pi_{1;r,s}\pi_{2;j,k} - \pi_{2;r,s}\pi_{1;j,k} = \frac{i}{2}(\pi_r^+\pi_k^- - \pi_s^-\pi_j^+) \tag{21.2}$$

Then combine the two above expressions, yield

$$(\vec{\pi}_{j,k} \times \vec{\pi}_{r,s})_3 + (\vec{\pi}_{r,s} \times \vec{\pi}_{j,k})_3$$

$$= \frac{i}{2}(\pi_j^+\pi_s^- - \pi_k^-\pi_r^+ + \pi_r^+\pi_k^- - \pi_s^-\pi_j^+) = i(\pi_{3;j,s} + \pi_{3;r,k})$$

$$= i(\pi_{3;j,k} + \pi_{3;r,s}) = i(\vec{\pi}_{j,k} + \vec{\pi}_{r,s})_3 \tag{22}$$

it allows the following to happen:

$$\{ (\vec{\pi}_{j,k} + \vec{\pi}_{r,s}) \times (\vec{\pi}_{j,k} + \vec{\pi}_{r,s}) \}_3$$

$$= (\vec{\pi}_{j,k} \times \vec{\pi}_{j,k})_3 + (\vec{\pi}_{r,s} \times \vec{\pi}_{r,s})_3 + (\vec{\pi}_{j,k} \times \vec{\pi}_{r,s})_3 + (\vec{\pi}_{r,s} \times \vec{\pi}_{j,k})_3$$

$$= i(\vec{\pi}_{j,k})_3 + i(\vec{\pi}_{r,s})_3 + i(\vec{\pi}_{j,k} + \vec{\pi}_{r,s})_3 = 2i(\vec{\pi}_{j,k} + \vec{\pi}_{r,s})_3 \tag{23}$$

Finally approaching to commutation rule of the third component of (14)

$$\frac{1}{2}(\vec{\pi}_{j,k} + \vec{\pi}_{r,s})_3 \times \frac{1}{2}(\vec{\pi}_{j,k} + \vec{\pi}_{r,s})_3 = i\frac{1}{2}(\vec{\pi}_{j,k} + \vec{\pi}_{r,s})_3 \tag{24.3}$$

Proceeding similarly as the above discussion, we can get:

$$\frac{1}{2}(\vec{\pi}_{j,k} + \vec{\pi}_{r,s})_1 \times \frac{1}{2}(\vec{\pi}_{j,k} + \vec{\pi}_{r,s})_1 = i\frac{1}{2}(\vec{\pi}_{j,k} + \vec{\pi}_{r,s})_1 \tag{24.1}$$

$$\frac{1}{2}(\vec{\pi}_{j,k} + \vec{\pi}_{r,s})_2 \times \frac{1}{2}(\vec{\pi}_{j,k} + \vec{\pi}_{r,s})_2 = i\frac{1}{2}(\vec{\pi}_{j,k} + \vec{\pi}_{r,s})_2 \tag{24.2}$$

And $$\frac{1}{2}(\vec{\pi}_1 + \vec{\pi}_2) \times \frac{1}{2}(\vec{\pi}_1 + \vec{\pi}_2) = i\frac{1}{2}(\vec{\pi}_1 + \vec{\pi}_2) \tag{24}$$

★ Summary:

a▸ Angular momentum coupling between two spin particles $\vec{\pi}_1$ and $\vec{\pi}_2$ in STS

$$\vec{\Pi} \times \vec{\Pi} = i\vec{\Pi} \tag{25}$$

Global Journal of Science Frontier Research (A) Volume XXI Issue IV Version I Year 2021

$$\vec{\Pi} = \frac{1}{2}(\vec{\pi}_1 + \vec{\pi}_2) \tag{26}$$

$$\Pi_3 = \frac{1}{2}(\pi_{3;\,j,k} + \pi_{3;\,r,s}) \tag{27}$$

$$= \pi_0(0) + \frac{1}{2}\{ \frac{1}{2}(j + k + r + s) + 1 \} \tag{28}$$

b▸ The extension of the spin coupling among three spin particles $\vec{\pi}_1, \vec{\pi}_2$ and $\vec{\pi}_3$ in STS

$$\vec{\Pi} \times \vec{\Pi} = i\vec{\Pi} \tag{28}$$

$$\vec{\Pi} = \frac{1}{3}(\vec{\pi}_1 + \vec{\pi}_2 + \vec{\pi}_3) \tag{29}$$

$$\Pi_3 = \frac{1}{3}(\pi_{3;\,j,k} + \pi_{3;\,r,s} + \pi_{3;\,u,v}) \tag{30}$$

$$= \pi_0(0) + \frac{1}{2}\{ \frac{1}{3}(j + k + r + s + u + v) + 1 \} \tag{31}$$

III. Puzzles of Spin Angular Momentum Addition of the Third Components

First using a▸ (27) (28) to discuss coupling of two spin-1/2 fermions $\vec{\pi}_1$ and $\vec{\pi}_2$ in STS.

(A) Put

$$j + k = r + s = 0 \tag{32}$$

Get

$$\Pi_3(\uparrow,\uparrow) = \frac{1}{2}(m_1 + m_2) = \frac{1}{2}\left(\frac{+\hbar}{2}\uparrow + \frac{+\hbar}{2}\uparrow\right) = \frac{+\hbar}{2}\uparrow \neq \frac{+\hbar}{2} + \frac{+\hbar}{2} = +1\hbar\uparrow \tag{33}$$

(B) Put

$$j + k = 0, \quad r + s = -2 \tag{34}$$

Get

$$\Pi_3(\uparrow,\downarrow) = \frac{1}{2}(m_1 + m_2) = \frac{1}{2}\left(\frac{+\hbar}{2}\uparrow + \frac{-\hbar}{2}\downarrow\right) = 0\hbar \tag{35}$$

(C) Put

$$j + k = -2, \quad r + s = 0 \tag{36}$$

Get

$$\Pi_3(\downarrow,\uparrow) = \frac{1}{2}(m_1 + m_2) = \frac{1}{2}\left(\frac{-\hbar}{2}\downarrow + \frac{+\hbar}{2}\uparrow\right) = 0\hbar \tag{37}$$

(D) Put

$$j + k = r + s = -2 \tag{38}$$

Get

$$\Pi_3(\downarrow,\downarrow) = \frac{1}{2}(m_1 + m_2) = \frac{1}{2}\left(\frac{-\hbar}{2}\downarrow + \frac{-\hbar}{2}\downarrow\right) = \frac{-\hbar}{2}\downarrow \neq \frac{-\hbar}{2} + \frac{-\hbar}{2} = -1\hbar\downarrow \tag{39}$$

Where m_1 and m_2 are the eigenvalues of $\pi_{3;\,j,k}$ and $\pi_{3;\,r,s}$.

We see there are some amusing phenomenons in the above spin addition of the third components of two spin-1/2 particles:

Results (35) and (37) are the those we expected: " positive $\frac{+\hbar}{2}$ plus negative $\frac{-\hbar}{2}$ equal to zero" that agree with current angular theory (8). Unfortunately, for caculation (33): " positive $\frac{+\hbar}{2}$ plus positive $\frac{+\hbar}{2}$ equal to positive $\frac{+\hbar}{2}$ ", this result conflicts with (8) and physics common sense in lab ! actually, (33) shoud be $+1\hbar$, which now loses its half value. Similar puzzles for (39), which should be $-1\hbar$. This type of puzzles also exist in three body coupling b▸ (30) (31). The sum values of the spin third components of coupled-spin particles would always be less than that should be, except when the sum values is zero, such as cases of (35) (37)

If we still want, in STS math world, to obtain the same results, which correspond with current angular momentum theory such as (8) do, some new concepts should be required to be introduced, even if the ideas of physics background of those new phenomenons are truly impossible to understand.

★ The purpose of this paper is to use three-body coupling b▸ (30) (31) to research the spin angular momentum coupling of two spin-1/2 particles $\vec{\pi}_1, \vec{\pi}_2$. We suggest there may exist a seclusive hidden spin particle $\vec{\pi}_3 \equiv \pi_{u,v}$, that actually and stealthily is participating the formation process of spin coupling between $\vec{\pi}_1$ and $\vec{\pi}_2$. $\vec{\pi}_3$ is a spin particle, a spin-force-mediating particle. $\overleftarrow{\pi}_3$ can interact with $\vec{\pi}_1$ and $\vec{\pi}_2$ through spin angular momentum coupling, then to solve the puzzles.

Obviously, zero spin particle, with Casimir operator eigenvalue $\pi_3^2 = 0 = 0(0+1)\hbar^2$, may seems to be the perfect candidate, because spin-0 particle possesses the following advantages of its properties of spin-dual-role: "nothing" and "everything".

In current spin angular momentum couple theory frame, zero spin particle is a trivial spin particle, it is a "point spin", no spin effect with any other spin particeles. Zero spin particle is "nothing", all for naught, superfluous in spin addition.

But on the other hand, in Spin Topological Space STS frame, zero spin particle turns to be "constructive", that is, has spin ability to interact with other spin particles, at this time the zero spin is "everything", a physical reality as an invisible mediator, actually is participating the spin couple between the two spin-1/2 fermions .

Next, we enter zero spin territory where no one has gone before. Apply the third component $\pi_{3;\,u,v}$ or m_3 of zero spin particle $\pi_{u,v}$ to explore the puzzles of (33),(39) and (35),(37). Using $\pi_{3;\,u,v}$ to offset the defects, to throw away what we dislike and obtain what we appreciate. Further the coupling of two-body of two spin-1/2 particles in fact turns into those of three-body spin particles.

From (30), write down (40)

$$\Pi_3 = \frac{1}{3}(\pi_{3;\,j,k} + \pi_{3;\,r,s} + \pi_{3;\,u,v}) = \frac{1}{3}\{ m_1 + m_2 + m_3 \} \tag{40}$$

Hence, obtain next four expressions which are the extension of spin angular momentum coupling from the current math frame (8) to math STS frame. The realm of latter is much beyond that of the former.

$$\text{(a)}\ \Pi_3 = \mathbf{m} = \tfrac{1}{3}\{\ m_1 + m_2 + m_3\ \} = \tfrac{1}{3}\{\ \tfrac{+\hbar}{2} + \tfrac{+\hbar}{2} + 2\hbar\ \} = +1\hbar \tag{40.1}$$

$$\text{(b)}\ \Pi_3 = \mathbf{m} = \tfrac{1}{3}\{\ m_1 + m_2 + m_3\ \} = \tfrac{1}{3}\{\ \tfrac{+\hbar}{2} + \tfrac{-\hbar}{2} + 0\hbar\ \} = 0\hbar \tag{40.2}$$

$$\text{(c)}\ \Pi_3 = \mathbf{m} = \tfrac{1}{3}\{\ m_1 + m_2 + m_3\ \} = \tfrac{1}{3}\{\ \tfrac{-\hbar}{2} + \tfrac{+\hbar}{2} + 0\hbar\ \} = 0\hbar \tag{40.3}$$

$$\text{(d)}\ \Pi_3 = \mathbf{m} = \tfrac{1}{3}\{\ m_1 + m_2 + m_3\ \} = \tfrac{1}{3}\{\ \tfrac{-\hbar}{2} + \tfrac{-\hbar}{2} - 2\hbar\ \} = -1\hbar \tag{40.4}$$

Here $m_3 = 0\hbar$ in (40.2) and (40.3) are the eigenvalues of the ground state of zero spin particle, and $m_3 = +2\hbar$ (40.1), $-2\hbar$ (40.4) are the those of the positive-second excited state, the negative-second exicted state in STS.

IV. Fundamental Formulas in Spin Complex Region of STS

The following formulas are the essential in spin complex region of STS, we will use them and zero spin particle to explore the puzzle addition of the third components of two spin-1/2 particles mentioned previously. With this aim in mind.

1) Single-body spin particle

$$\pi_{3\,;\,j,b\,,\,k,d} = \pi_0(0) + \tfrac{1}{2}(j + k + 1) + \tfrac{1}{2}i(\,b - d\,) \tag{41}$$

$$\pi^2_{j,b\,,\,k,d} = \tfrac{1}{4}\{\ (j - k)^2 - (\,b + d\,)^2 - 1\ \} + i\tfrac{1}{2}(j - k)(b + d\,) \tag{42}$$

2) Two-body spin couple

$$\Pi_{3\,;\,j,b\,,\,r,a\,;\,k,d\,,\,s,c} = \pi_0(0) + \tfrac{1}{2}\{\tfrac{1}{2}(j + k + r + s) + 1\} + \tfrac{1}{4}i(\,b - d + a - c\,) \tag{43}$$

$$\Pi^2_{j,b\,,\,r,a\,;\,k,d\,,\,s,c} = \tfrac{1}{16}\{(j - k + r - s\,)^2 - (b + d + a + c)^2 - 4\} + i\tfrac{1}{8}(j - k + r - s)(b + d + a + c) \tag{44}$$

3) Three-body spin couple

$$\Pi_{3\,;\,j,b\,,\,r,a\,;\,k,d\,,\,s,c\,;\,u,e\,,\,v,f} = \pi_0(0) + \tfrac{1}{2}\{\tfrac{1}{3}(j + k + r + s + u + v) + 1\} + \tfrac{1}{6}i(\,b - d + a - c + e - f\,) \tag{45}$$

$$\Pi^2_{j,b\,,\,r,a\,;\,k,d\,,\,s,c\,;\,u,e\,,\,v,f} = \tfrac{1}{36}\{(j - k + r - s + u - v\,)^2 - (b + d + a + c + e + f)^2 - 9\} + i\tfrac{1}{18}(j - k + r - s + u - v)(b + d + a + c + e + f) \tag{46}$$

Global Journal of Science Frontier Research (A) Volume XXI Issue IV Version I Year 2021

Using Spin Topological Coordinates (α, β) (γ, δ) of a spin particle in complex region in STS, two arrays are given

$$(\mathrm{S}, \mathrm{T}) = (\alpha - \beta,\ \gamma + \delta) \tag{47}$$

$$(\mathrm{A}, \mathrm{B}) = (\alpha + \beta,\ \gamma - \delta) \tag{48}$$

array (S, T) is related to Casimir operator of spin particle, and array (A, B) to the spin third component respectively. apply (47) and (48) to discuss the three-body spin couple among two spin-1/2 fermions $\vec{\pi}_1(j, k\ ; b, d)$, $\vec{\pi}_2(r, s\ ; a, c)$ and spin-0 boson $\vec{\pi}_3(u, v\ ; e, f)$ below

spin-1/2 π_1 $\quad (\mathrm{S}_1, \mathrm{T}_1) = (j - k,\ b + d),$ (47.1)

spin-1/2 π_2 $\quad (\mathrm{S}_2, \mathrm{T}_2) = (r - s,\ a + c),$ (47.2)

spin-0 π_3 $\quad (\mathrm{S}_3, \mathrm{T}_3) = (u - v,\ e + f),$ (47.3)

spin-1/2 π_1 $\quad (\mathrm{A}_1, \mathrm{B}_1) = (j + k,\ b - d),$ (48.1)

spin-1/2 π_2 $\quad (\mathrm{A}_2, \mathrm{B}_2) = (r + s,\ a - c),$ (48.2)

spin-0 π_3 $\quad (\mathrm{A}_3, \mathrm{B}_3) = (u + v,\ e - f),$ (48.3)

Spin Topological Coordinates, STC

spin-1/2 π_1 $\quad (j,\ k) = \left(\frac{+1}{2}(\mathrm{A}_1+\mathrm{S}_1),\ \frac{+1}{2}(\mathrm{A}_1-\mathrm{S}_1) \right)$ (49.1)

$(b,\ d) = \left(\frac{+1}{2}(\mathrm{B}_1+\mathrm{T}_1),\ \frac{-1}{2}(\mathrm{B}_1-\mathrm{T}_1) \right)$ (50.1)

spin-1/2 π_2 $\quad (r,\ s) = \left(\frac{+1}{2}(\mathrm{A}_2+\mathrm{S}_2),\ \frac{+1}{2}(\mathrm{A}_2-\mathrm{S}_2) \right)$ (49.2)

$(a,\ c) = \left(\frac{+1}{2}(\mathrm{B}_2+\mathrm{T}_2),\ \frac{-1}{2}(\mathrm{B}_2-\mathrm{T}_2) \right)$ (50.2)

spin-0 π_3 $\quad (u,\ v) = \left(\frac{+1}{2}(\mathrm{A}_3+\mathrm{S}_3),\ \frac{+1}{2}(\mathrm{A}_3-\mathrm{S}_3) \right)$ (49.3)

$(e,\ f) = \left(\frac{+1}{2}(\mathrm{B}_3+\mathrm{T}_3),\ \frac{-1}{2}(\mathrm{B}_3-\mathrm{T}_3) \right)$ (50.3)

V. Puzzles Solving

Now, for the implement of {(a),(b),(c),(d)} of (40), the above formulas mentioned in section **4.** need to be simplified, so we confine ourself to condition (51), for which the calculations are straightforward.

For z components, we take:

$$\mathrm{B}_1 = \mathrm{B}_2 = \mathrm{B}_3 = 0 \tag{51}$$

Global Journal of Science Frontier Research (A) Volume XXI Issue IV Version I Year 2021

then the imaginaries of $\pi_{3;j,b,k,d}$, $\pi_{3;r,a,s,c}$ and $\pi_{3;u,e,v,f}$ all vanish, (41) become

$$\pi_{3;j,b,k,d} = \pi_0(0) + \tfrac{1}{2}(A_1 + 1) \tag{52.1}$$

$$\pi_{3;r,a,s,c} = \pi_0(0) + \tfrac{1}{2}(A_2 + 1) \tag{52.2}$$

$$\pi_{3;u,e,v,f} = \pi_0(0) + \tfrac{1}{2}(A_3 + 1) \tag{52.3}$$

further (45) turns into

$$\Pi_3 = \pi_0(0) + \tfrac{1}{2}\{ \tfrac{1}{3}(A_1 + A_2 + A_3) + 1 \} \tag{53}$$

For deepper understanding of the role of zero spin particle in the spin angular momentum coupling of two identical spin-1/2 fermions, more detailed processes of calculations (a),(b),(c),(d) of the third components π_3 and Π_3 are demostrated below.

Pay attention to the following correspondence

$$\{ (A),(B),(C),(D) \mid \subset \text{section } 5 \} \Rightarrow \{ (a),(b),(c),(d) \mid \subset \text{section } 3\}$$

(A)

$$(A_1, B_1) = (0, 0) \tag{54.1}$$

$$(A_2, B_2) = (0, 0) \tag{54.2}$$

$$(A_3, B_3) = (+3, 0) \tag{54.3}$$

$$\pi_3(A_1, B_1) = \pi_0(0) + \tfrac{1}{2}(0 + 1) + \tfrac{1}{2}i(0) = \pi_0(0) + \tfrac{1}{2} \tag{55.1}$$

$$\pi_3(A_1, B_1) = \pi_0(0) + \tfrac{1}{2}(0 + 1) + \tfrac{1}{2}i(0) = \pi_0(0) + \tfrac{1}{2} \tag{55.2}$$

$$\pi_3(A_3, B_3) = \pi_0(0) + \tfrac{1}{2}(+3 + 1) + \tfrac{1}{2}i(0) = \pi_0(0) + 2 \tag{55.3}$$

$$\Pi_3 = \pi_0(0) + \tfrac{1}{2}\{ \tfrac{1}{3}(0 + 0 + 3) + 1 \} + \tfrac{1}{6}i(0 + 0 + 0) = \pi_0(0) + 1 \tag{56}$$

Formular (55.3) is the second positive excited eigenvalue of zero spin particle, which is like an invisible force-mediating particle, to make (56) to be in accord with (40.1).

Similar to (A), we get:

(B)

$$(A_1, B_1) = (0, 0) \tag{57.1}$$

$$(A_2, B_2) = (-2, 0) \tag{55.2}$$

$$(A_3, B_3) = (-1, 0) \tag{55.3}$$

$$\pi_3(A_1, B_1) = \pi_0(0) + \tfrac{1}{2}(0 + 1) + \tfrac{1}{2}i(0) = \pi_0(0) + \tfrac{1}{2} \quad (58.1)$$

$$\pi_3(A_1, B_1) = \pi_0(0) + \tfrac{1}{2}(-2 + 1) + \tfrac{1}{2}i(0) = \pi_0(0) - \tfrac{1}{2} \quad (58.2)$$

$$\pi_3(A_3, B_3) = \pi_0(0) + \tfrac{1}{2}(-1 + 1) + \tfrac{1}{2}i(0) = \pi_0(0) + 0 \quad (58.3)$$

$$\Pi_3 = \pi_0(0) + \tfrac{1}{2}\{ \tfrac{1}{3}(0 - 2 - 1) + 1 \} + \tfrac{1}{6}i(0 + 0 + 0) = \pi_0(0) + 0 \quad (59)$$

Formular (58.3) is the lowest eigenvalue, based state, of zero spin particle, to make (59) to be in accord with (40.2).

The same one as (B), we get:

(C)

$$(A_1, B_1) = (-2, 0) \quad (60.1)$$

$$(A_2, B_2) = (0, 0) \quad (60.2)$$

$$(A_3, B_3) = (-1, 0) \quad (60.3)$$

$$\pi_3(A_1, B_1) = \pi_0(0) + \tfrac{1}{2}(-2 + 1) + \tfrac{1}{2}i(0) = \pi_0(0) - \tfrac{1}{2} \quad (61.1)$$

$$\pi_3(A_2, B_2) = \pi_0(0) + \tfrac{1}{2}(0 + 1) + \tfrac{1}{2}i(0) = \pi_0(0) + \tfrac{1}{2} \quad (61.2)$$

$$\pi_3(A_3, B_3) = \pi_0(0) + \tfrac{1}{2}(-1 + 1) + \tfrac{1}{2}i(0) = \pi_0(0) + 0 \quad (61.3)$$

$$\Pi_3 = \pi_0(0) + \tfrac{1}{2}\{ \tfrac{1}{3}(-2 + 0 - 1) + 1 \} + \tfrac{1}{6}i(0_1 + 0 + 0) = \pi_0(0) + 0 \quad (62)$$

Formular (61.3) is the lowest eigenvalue, based state, of zero spin particle, to make (62) to be in accord with (40.3).

Almost same as (A), we get:

(D)

$$(A_1, B_1) = (-2, 0) \quad (63.1)$$

$$(A_2, B_2) = (-2, 0) \quad (63.2)$$

$$(A_3, B_3) = (-5, 0) \quad (63.3)$$

$$\pi_3(A_1, B_1) = \pi_0(0) + \tfrac{1}{2}(-2 + 1) + \tfrac{1}{2}i(0) = \pi_0(0) - \tfrac{1}{2} \quad (64.1)$$

$$\pi_3(A_1, B_1) = \pi_0(0) + \tfrac{1}{2}(-2 + 1) + \tfrac{1}{2}i(0) = \pi_0(0) - \tfrac{1}{2} \quad (64.2)$$

$$\pi_3(A_3, B_3) = \pi_0(0) + \tfrac{1}{2}(-5 + 1) + \tfrac{1}{2}i(0) = \pi_0(0) - 2 \quad (64.3)$$

$$\Pi_3 = \pi_0(0) + \tfrac{1}{2}\{ \tfrac{1}{3}(-2 - 2 - 5) + 1 \} + \tfrac{1}{6}i(0_1 + 0 + 0) = \pi_0(0) - 1 \quad (65)$$

Formular (64.3) is the second negative excited eigenvalue of zero spin particle to make (65) to be in accord with (40.4).

VI. Casimir Operators of Spin Particles

For Casimir operators, we take:

$$T_1 + T_2 + T_3 = 0 \qquad (66)$$

further (46) turns into

$$\Pi^2 = \frac{1}{36}\{(S_1 + S_2 + S_3)^2 - 9\} \qquad (67)$$

Considering two cases of (67)

IF $$S_1 + S_2 + S_3 = +9 \qquad (68)$$

get $$\Pi^2 = \frac{1}{36}\{(+9)^2 - 9\} = \frac{72}{36} = 2 = 1(1+1)\hbar^2 \qquad (69)$$

IF $$S_1 + S_2 + S_3 = +3 \qquad (70)$$

get $$\Pi^2 = \frac{1}{36}\{(+3)^2 - 9\} = \frac{0}{36} = 0 = 0(0+1)\hbar^2 \qquad (71)$$

Now, we present the explicit datas of two arrays (S, T), (72)♦1 and (77)♦2, which satisfy the math requirements of the above two cases of (67)

♦1 Array (72) for case (69), which construct the irreducible **j = 1** representation of (8)

$$(S_1, T_1) = (+2\sqrt{6}, -2\sqrt{5}) \qquad (72.1)$$

$$(S_2, T_2) = (-2\sqrt{6}, -2\sqrt{5}) \qquad (72.2)$$

$$(S_3, T_3) = (+9, +4\sqrt{5}) \qquad (72.3)$$

Then apply (72) to (42), we get:

$$\pi^2_{S_1,T_1}(\text{fermion1}) = \frac{+3}{4} - i2\sqrt{30} \Leftrightarrow \mathbf{j}_1 = \frac{1}{2} \qquad (73.1)$$

$$\pi^2_{S_2,T_2}(\text{fermion2}) = \frac{+3}{4} + i2\sqrt{30} \Leftrightarrow \mathbf{j}_2 = \frac{1}{2} \qquad (73.2)$$

$$\pi^2_{S_3,T_3}(\text{spin-zero3}) = 0 + i18\sqrt{5} \Leftrightarrow \mathbf{j}_3 = 0 \qquad (73.3)$$

Because

$$S_1 + S_2 + S_3 = +2\sqrt{6} - 2\sqrt{6} + 9 = +9 \qquad (74)$$

$$T_1 + T_2 + T_3 = -2\sqrt{5} - 2\sqrt{5} + 4\sqrt{5} = 0 \qquad (75)$$

Obtain Casimir operator

$$\Pi^2 = \frac{1}{36}\{81 - 0 - 9\} = 1(1+1)\hbar^2 \Leftrightarrow \mathbf{j = 1} \qquad (76)$$

♦2 Array (77) for case (71), which construct the irreducible $\mathbf{j}=\mathbf{0}$ representation of (8)

$$(S_1, T_1) = (+\sqrt{6}\,,\ -\sqrt{2}\,) \qquad (77.1)$$

$$(S_2, T_2) = (-\sqrt{6}\,,\ -\sqrt{2}\,) \qquad (77.2)$$

$$(S_3, T_3) = (+3,\ +2\sqrt{2}\,) \qquad (77.3)$$

Then apply (77) to (42), we get:

$$\pi^2_{S_1,T_1}(\text{fermion1}) = \tfrac{+3}{4} - i\sqrt{3} \quad \Leftrightarrow \quad \mathbf{j}_1 = \tfrac{1}{2} \qquad (78.1)$$

$$\pi^2_{S_2,T_2}(\text{fermion2}) = \tfrac{+3}{4} + i\sqrt{3} \quad \Leftrightarrow \quad \mathbf{j}_2 = \tfrac{1}{2} \qquad (78.2)$$

$$\pi^2_{S_3,T_3}(\text{spin-zero3}) = 0 + i3\sqrt{2} \quad \Leftrightarrow \quad \mathbf{j}_3 = 0 \qquad (78.3)$$

Because

$$S_1 + S_2 + S_3 = +\sqrt{6} - \sqrt{6} + 3 = +3 \qquad (79)$$

$$T_1 + T_2 + T_3 = -\sqrt{2} - \sqrt{2} + 2\sqrt{2} = 0 \qquad (80)$$

Obtain Casimir operator

$$\Pi^2 = \tfrac{1}{36}\{\ 9 - 0 - 9\ \} = 0(0+1)\hbar^2 \quad \Leftrightarrow \quad \mathbf{j}=\mathbf{0} \qquad (81)$$

VII. Matrix Represention of $\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$ in STS

Instead of using the matrices of C-G Coefficients in conventional spin theory to depict the spin couple, the results of section 4, 5 and section 6 could be used to build some other new matrix represention pictures of spin angular momentum addtion in STS. Some matrix tables are given below. Among those Table1 is so-called "based state represention", which is just the incarnation of matrices represention of $\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$ in Spin Topological Space. Table2 and Table3 are " excited state represention " of C-G Coefficients, which are the extensions of Table1.

Block ★1, Block ★2 and Block ★3 in Table2, which are the based states of C-G Coefficients with quantum number $\mathbf{j}=1$ in Table1, carry the based state z-components quantum numbers **m** whose values equal to $+\mathbf{1}$, $\mathbf{0}$, $+\mathbf{1}$ respectively. The rest blocks are excited states of $\mathbf{j}=1$

Block ★4 in Table3, is the based state of C-G Coefficients with quantum number $\mathbf{j}=0$ in Table1, carries the based state z-components quantum number **m** whose value equals to **0**. The rest blocks are excited states of $\mathbf{j}=0$

Arrays (A_i, B_i) and (S_i, T_i) or Spin Topological Coordinates (j, k) (b, d), (r, s) (a, c), and (u, v) (e, f) are the characteristic quantum numbers of schematics of spin angular momentum couple.

Global Journal of Science Frontier Research (A) Volume XXI Issue IV Version I Year 2021

Table 1: Based State Representation of the C-G Coefficients of $\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$

j	=	**1**	**1**	**0**	**1**
m	=	+**1**	**0**	**0**	−**1**
j_1=1/2, (S_1, T_1)	=	$(+2\sqrt{6}, -2\sqrt{5})$	$(+2\sqrt{6}, -2\sqrt{5})$	$(+\sqrt{6}, -\sqrt{2})$	$(+2\sqrt{6}, -2\sqrt{5})$
j_2=1/2, (S_2, T_2)	=	$(+2\sqrt{6}, -2\sqrt{5})$	$(+2\sqrt{6}, -2\sqrt{5})$	$(+\sqrt{6}, -\sqrt{2})$	$(+2\sqrt{6}, -2\sqrt{5})$
j_3= 0, (S_3, T_3)	=	$(+9, +4\sqrt{5})$	$(+9, +4\sqrt{5})$	$(+3, +2\sqrt{2})$	$(+9, +4\sqrt{5})$
m_i, (A_i, B_i) / i = 1, 2, 3		$(j, k), (r, s), (u, v)$	$(j, k), (r, s), (u, v)$	$(j, k), (r, s), (u, v)$	$(j, k), (r, s), (u, v)$
‖					
$\frac{+1}{2}$, (0, 0)		$(0+\sqrt{6}, 0-\sqrt{6})$			
$\frac{+1}{2}$, (0, 0)		$(0-\sqrt{6}, 0+\sqrt{6})$	0	0	0
+2, (+3, 0)		(+6, −3)			
$\frac{+1}{2}$, (0, 0)			$(0+\sqrt{6}, 0-\sqrt{6})$	$\frac{0+\sqrt{6}}{2}, \frac{0-\sqrt{6}}{2}$	
$\frac{-1}{2}$, (−2, 0)		0	$(-1-\sqrt{6}, -1+\sqrt{6})$	$\frac{-2-\sqrt{6}}{2}, \frac{-2+\sqrt{6}}{2}$	0
0, (−1, 0)			(+4, −5)	(+1, −2)	
$\frac{-1}{2}$, (−2, 0)			$(-1+\sqrt{6}, -1-\sqrt{6})$	$\frac{-2+\sqrt{6}}{2}, \frac{-2-\sqrt{6}}{2}$	
$\frac{+1}{2}$, (0, 0)		0	$(0-\sqrt{6}, 0+\sqrt{6})$	$\frac{0-\sqrt{6}}{2}, \frac{0+\sqrt{6}}{2}$	0
0, (−1, 0)			(+4, −5)	(+1, −2)	
$\frac{-1}{2}$, (−2, 0)					$(-1+\sqrt{6}, -1-\sqrt{6})$
$\frac{-1}{2}$, (−2, 0)		0	0	0	$(-1-\sqrt{6}, -1+\sqrt{6})$
−2, (−5, 0)					(+2, −7)
m = $\frac{m_1+m_2+m_3}{3}$	=	+**1**	**0**	**0**	−**1**
(b, d)	=	$(-\sqrt{5}, -\sqrt{5})$	$(-\sqrt{5}, -\sqrt{5})$	$(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2})$	$(-\sqrt{5}, -\sqrt{5})$
(a, c)	=	$(-\sqrt{5}, -\sqrt{5})$	$(-\sqrt{5}, -\sqrt{5})$	$(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2})$	$(-\sqrt{5}, -\sqrt{5})$
(e, f)	=	$(+2\sqrt{5}, +2\sqrt{5})$	$(+2\sqrt{5}, +2\sqrt{5})$	$(+\sqrt{2}, +\sqrt{2})$	$(+2\sqrt{5}, +2\sqrt{5})$

Table 2: **j** = 1, Based States & Excited States of $\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$

j	**1**	**1**	**1**	**1**	**1**
	excited state	excited state	based state	excited state	excited state
m	+**3**	+**2**	+**1**	**0**	−**1**
$m_1, (A_1, B_1)$	$\frac{+1}{2}$, (0, 0)	$\frac{+1}{2}$, (0, 0)	$\frac{+1}{2}$, (0, 0)	$\frac{+1}{2}$, (0, 0)	$\frac{+1}{2}$, (0, 0)
$m_2, (A_2, B_2)$	$\frac{+1}{2}$, (0, 0)	$\frac{+1}{2}$, (0, 0)	$\frac{+1}{2}$, (0, 0)	$\frac{+1}{2}$, (0, 0)	$\frac{+1}{2}$, (0, 0)
$m_3, (A_3, B_3)$	+8, (+15, 0)	+5, (+9, 0)	+2, (+3, 0)	−1, (−3, 0)	−4, (−9, 0)
			★1		
$(j\ ,\ k)_1$	$0+\sqrt{6}, 0-\sqrt{6}$	$0+\sqrt{6}, 0-\sqrt{6}$	$0+\sqrt{6}, 0-\sqrt{6}$	$0+\sqrt{6}, 0-\sqrt{6}$	$0+\sqrt{6}, 0-\sqrt{6}$
$(r\ ,\ s)_2$	$0-\sqrt{6}, 0+\sqrt{6}$	$0-\sqrt{6}, 0+\sqrt{6}$	$0-\sqrt{6}, 0+\sqrt{6}$	$0-\sqrt{6}, 0+\sqrt{6}$	$0-\sqrt{6}, 0+\sqrt{6}$
$(u\ ,\ v)_3$	+12, +3	+9, 0	+6, −3	+3, −6	0, −9
– – –	– – – – – –	– – – – – –	– – – – – –	– – – – – –	– – – – – –
m	+**2**	+**1**	**0**	−**1**	−**2**
$m_1, (A_1, B_1)$	$\frac{+1}{2}$, (0, 0)	$\frac{+1}{2}$, (0, 0)	$\frac{+1}{2}$, (0, 0)	$\frac{+1}{2}$, (0, 0)	$\frac{+1}{2}$, (0, 0)
$m_2, (A_2, B_2)$	$\frac{-1}{2}$, (−2, 0)	$\frac{-1}{2}$, (−2, 0)	$\frac{-1}{2}$, (−2, 0)	$\frac{-1}{2}$, (−2, 0)	$\frac{-1}{2}$, (−2, 0)
$m_3, (A_3, B_3)$	+6, (+11, 0)	+3, (+5, 0)	0, (−1, 0)	−3, (−7, 0)	−6, (−13, 0)
			★2		
$(j\ ,\ k)_1$	$0+\sqrt{6}, 0-\sqrt{6}$	$0+\sqrt{6}, 0-\sqrt{6}$	$0+\sqrt{6}, 0-\sqrt{6}$	$0+\sqrt{6}, 0-\sqrt{6}$	$0+\sqrt{6}, 0-\sqrt{6}$
$(r\ ,\ s)_2$	$-1-\sqrt{6}, -1+\sqrt{6}$	$-1-\sqrt{6}, -1+\sqrt{6}$	$-1-\sqrt{6}, -1+\sqrt{6}$	$-1-\sqrt{6}, -1+\sqrt{6}$	$-1-\sqrt{6}, -1+\sqrt{6}$
$(u\ ,\ v)_3$	(+10, +1)	(+7, −2)	(+4, −5)	(+1, −8)	(−2, −11)
– – –	– – – – – –	– – – – – –	– – – – – –	– – – – – –	– – – – – –
m	+**1**	**0**	−**1**	−**2**	−**3**
$m_1, (A_1, B_1)$	$\frac{-1}{2}$, (−2, 0)	$\frac{-1}{2}$, (−2, 0)	$\frac{-1}{2}$, (−2, 0)	$\frac{-1}{2}$, (−2, 0)	$\frac{-1}{2}$, (−2, 0)
$m_2, (A_2, B_2)$	$\frac{-1}{2}$, (−2, 0)	$\frac{-1}{2}$, (−2, 0)	$\frac{-1}{2}$, (−2, 0)	$\frac{-1}{2}$, (−2, 0)	$\frac{-1}{2}$, (−2, 0)
$m_3, (A_3, B_3)$	+4, (+7, 0)	+1, (+1, 0)	−2, (−5, 0)	−5, (−11, 0)	−8, (−17, 0)
			★3		
$(j\ ,\ k)_1$	$-1+\sqrt{6}, -1-\sqrt{6}$	$-1+\sqrt{6}, -1-\sqrt{6}$	$-1+\sqrt{6}, -1-\sqrt{6}$	$-1+\sqrt{6}, -1-\sqrt{6}$	$-1+\sqrt{6}, -1-\sqrt{6}$
$(r\ ,\ s)_2$	$-1-\sqrt{6}, -1+\sqrt{6}$	$-1-\sqrt{6}, -1+\sqrt{6}$	$-1-\sqrt{6}, -1+\sqrt{6}$	$-1-\sqrt{6}, -1+\sqrt{6}$	$-1-\sqrt{6}, -1+\sqrt{6}$
$(u\ ,\ v)_3$	(+8, −1)	(+5, −4)	(+2, −7)	(−1, −10)	(−4, −13)
– – –	– – –	– – –	– – –	– – –	– – –
– – –	– – –	– – –	– – –	– – –	– – –
$(S_1, T_1)=$	$+2\sqrt{6}, -2\sqrt{5}$			$(b,\ d)\ =$	$-\sqrt{5}, -\sqrt{5}$
$(S_2, T_2)=$	$-2\sqrt{6}, -2\sqrt{5}$			$(a,\ c)\ =$	$-\sqrt{5}, -\sqrt{5}$
$(S_3, T_3)=$	$+9, +4\sqrt{5}$			$(e,\ f)\ =$	$+2\sqrt{5}, +2\sqrt{5}$

Table 3: **j** = 0, Based States & Excited States of $\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$

j	**0**	**0**	**0**	**0**	**0**
	excited state	excited state	based state	excited state	excited state
m	**+2**	**+1**	**0**	**–1**	**–2**
$\mathbf{m}_1,(A_1,B_1)$	$\frac{+1}{2}$, (0, 0)	$\frac{+1}{2}$, (0, 0)	$\frac{+1}{2}$, (0, 0)	$\frac{+1}{2}$, (0, 0)	$\frac{+1}{2}$, (0, 0)
$\mathbf{m}_2,(A_2,B_2)$	$\frac{-1}{2}$, (−2, 0)	$\frac{-1}{2}$, (−2, 0)	$\frac{-1}{2}$, (−2, 0)	$\frac{-1}{2}$, (−2, 0)	$\frac{-1}{2}$, (−2, 0)
$\mathbf{m}_3,(A_3,B_3)$	+6, (+11, 0)	+3, (+5, 0)	0, (−1, 0)	−3, (−7, 0)	−6, (−13, 0)
			★4		
$(j\ ,\ k)_1$	$\frac{0+\sqrt{6}}{2}, \frac{0-\sqrt{6}}{2}$	$\frac{0+\sqrt{6}}{2}, \frac{0-\sqrt{6}}{2}$	$\frac{0+\sqrt{6}}{2}, \frac{0-\sqrt{6}}{2}$	$\frac{0+\sqrt{6}}{2}, \frac{0-\sqrt{6}}{2}$	$\frac{0+\sqrt{6}}{2}, \frac{0-\sqrt{6}}{2}$
$(r\ ,\ s)_2$	$\frac{-2-\sqrt{6}}{2}, \frac{-2+\sqrt{6}}{2}$	$\frac{-2-\sqrt{6}}{2}, \frac{-2+\sqrt{6}}{2}$	$\frac{-2-\sqrt{6}}{2}, \frac{-2+\sqrt{6}}{2}$	$\frac{-2-\sqrt{6}}{2}, \frac{-2+\sqrt{6}}{2}$	$\frac{-2-\sqrt{6}}{2}, \frac{-2+\sqrt{6}}{2}$
$(u\ ,\ v)_3$	(+7, +4)	(+4, +1)	(+1, −2)	(−2, −5)	(−5, −8)

$(S_1\ ,T_1)$	= $(+\sqrt{6}, -\sqrt{2})$	$(b\ ,\ d)$	= $(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2})$
$(S_2\ ,T_2)$	= $(-\sqrt{6}, -\sqrt{2})$	$(a\ ,\ c)$	= $(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2})$
$(S_3\ ,T_3)$	= $(+3, +2\sqrt{2})$	$(e\ ,\ f)$	= $(+\sqrt{2}, +\sqrt{2})$

VIII. Conclusions

In STS, Spin-zero particle possesses non-trivial angular momentum property, with which it could be thought as a force-mediating boson that holding the two spin-1/2 fermions to be coupled together each other, then to form a spin system as a whole. Subsequently, the matrix representions of $\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$ in STS are given.

The works that are the continuation of this paper about the matrix representions of $1 \otimes \frac{1}{2} = \frac{3}{2} \oplus \frac{1}{2}$ and $1 \otimes 1 = 2 \oplus 1 \oplus 0$ in STS are accomplished.

The existence of so-called " based states " may be believed to be math reasonable, after all from Table1, we can obtain what the same informations of the C-G Coefficients just what from the conventional spin theory do. Are there any so-called "excited states" in nature, which appear in Table2 and Table3 ?, we are well not aware of as yet.

As an example of possible " excited states ": If the two spin-1/2 fermions all keep to be stay in based states, that is $m_{1,}(A_1, B_1)$ and $m_{2,}(A_2, B_2)$ stay in their own based states, when spin-zero particle is excited to jump out of its based state $m_{3,}(A_3, B_3) = 0, (-1, 0)$, then the based state quantum number $\mathbf{m} = 0$, of the spin combined system (for both **j** =**1** and **j** = **0**), would turn to be the excited states $\mathbf{m} = \pm 1, \pm 2, \pm 3, \ldots$

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