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From Majorana Neutrino and Dark Neutrino ν_0 to Dark Spin Particles

By ShaoXu Ren

Institute of Physical Science and Engineering Tongji University

Abstract- In current theory of particle physics, the values of Casimir Operator, that is abbreviated to CO, of spin angular momentum for elementary particles are thought to be greater than zero. Both Majorana Neutrino ν and Majorana Antineutrino $\bar{\nu}$ are *all* with CO $\frac{+3\hbar^2}{4}$. Now the above limited region of CO is enlarged, this paper assumes that for the particles, the values of CO are still positive; for the antiparcles, however, the values of CO are negative. In this point of view, something similar is expected to happen in the case of Majorana Neutrino ν and Majorana Antineutrino $\bar{\nu}$. The Majorana Neutrino ν would be still with CO $\frac{+3\hbar^2}{4}$ and the Majorana Antineutrino $\bar{\nu}$ would be with CO $\frac{-3\hbar^2}{4}$. Further, leading to the possible existence of the so-called *Dark Neutrino*, *DN* or $\nu_0.\nu_0$ is with CO $0\hbar^2$ that is the superposition of Majorana Neutrino ν_L(CO$\frac{+3\hbar^2}{4}$) and Majorana Antineutrino $\bar{\nu}_R$(CO $\frac{-3\hbar^2}{4}$). And ν_0 is the one-half spin fermion with zero-charge $0e$ and zero-CO $0\hbar^2$, which is a more neutral neutrino than Majorana neutrino. ν_0 is one kind of Peculiar Dark Spin Particles.

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I. Introduction

What is the physical certification mark that distinguishes particles and antiparticles ? " When considering an electrically charged particle, say an electron, the difference between this particle and its antiparticle is evident: one has charge $-e$, the other $+e$. What happens when the particle is neutral? There is no general answer,......" then the graphic expression of *two states of a Majorana massive field* is given by G. Fantini, A. Gallo Rosso, V. Zema and F. Vissani [1], that indicates Majorana particle [2] is a neutrial charge massive fermion consist of two opposite charges, charge $-e$ and charge $+e$.

We see Majorana particle possesses a symmetrical picture shown in Table 1 and Figure 1. Here, the lefthand demonstrates the four states of a Dirac massive field, or a Dirac particle and a Dirac antiparticle. The righthand shows the two states of a Majorana massive field. Majorana particle is a neutrial fermion made of negative charge $-e$ and positive charge $+e.$, that called the *duality of charges*. All these particles mentioned are with the same CO $\frac{+3\hbar^2}{4}$ and with spin eigenvalues $+\hbar/2$ and $-\hbar/2$ respectively.

Being inspired, similar to Majorana did, this paper poses an assumption: Except the *duality of charges* of particle charge and antiparticle charge, maybe, there is another so-called *duality of Casimir Opertors* of positive Casimir Operator and negative Casimir Operator shown in Table 2 and Figure 2 .

In contrast with the lefthands of the two Figures, now, the four states of a DIrac massive field are replaced by a Majorana massive field, or replaced by a Majorana particle and a Majorana antiparticle, and at the same time, Majorana particle and a Majorana antiparticle are carry opposite CO each other.

Author: *Institute of Physical Science and Engineering Tongji University, 200092, Shanghai, China. e-mail: shaoxu-ren@hotmail.com*

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The righthands of the two Figures indicate: a *more neutrial* fermion that comprises *not only* positive charge $+e$ and negative charge $-e$, *but also* positive Casimir Operator $\frac{+3\hbar^2}{4}$ and negative Casimir Operator $\frac{-3\hbar^2}{4}$, than the Majorana Neutrino did. There are two dualities now the *more neutrial* fermion is so-called *Dark Neutrino*, *DN* or ν_0 that labelled with two physical quantities 0 e and 0 $\hbar^2$.

Table 1: $0\,e \;=\; (-\,e) \;+\; (+\;e)$

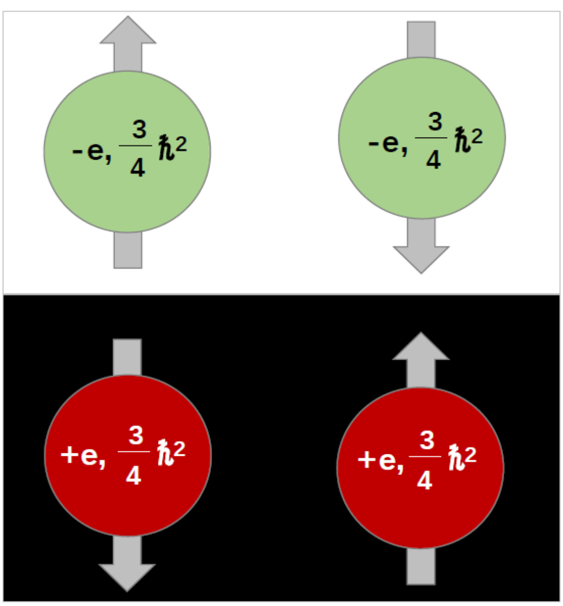
	Charge	‖	Casimir Operator
Dirac particle	$-\,e$	‖	$\frac{+3\hbar^2}{4}$
— *Majorana Neutrino* —	— 0 e —	— — —	— $\frac{+3\hbar^2}{4}$ —
Dirac antiparticle	$+\,e$	‖	$\frac{+3\hbar^2}{4}$

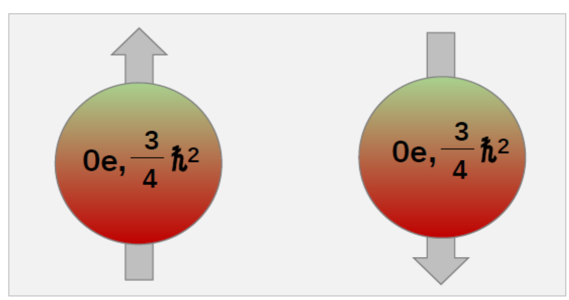
$\Downarrow \qquad \Downarrow \qquad \Downarrow \qquad \Downarrow$

Table 2: $0\,\hbar^2 \;=\; (+\frac{3^{\hbar^2}}{4}) \;+\; (-\frac{3^{\hbar^2}}{4})$

	Casimir Operator	‖	Charge
Majorana Neutrino ν	$+\frac{3\hbar^2}{4}$	‖	0 e
— *Dark Neutrino* ν_0 —	— 0 $\hbar^2$ —	— — — —	— 0 e —
Majorana Antineutrino $\bar{\nu}$	$-\frac{3\hbar^2}{4}$	‖	0 e

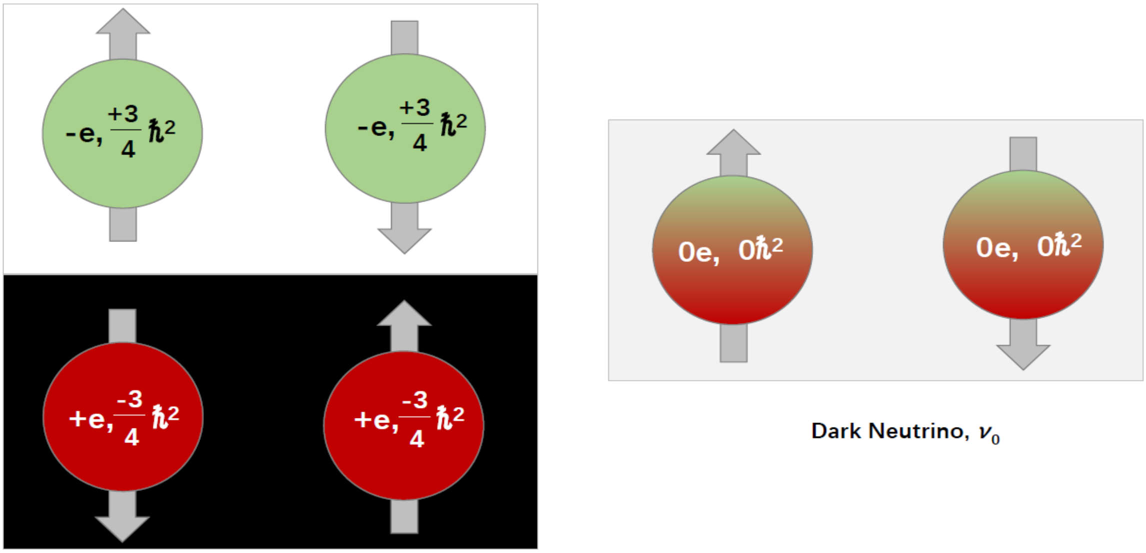
The following: schematics of Table 1 and table 2





Majorana Neutrino

Figure 1: $0\,e \;=\; (-\,e) \;+\; (+\;e)$



Dark Neutrino, ν_0

Figure 2: $0\ \hbar^2 \;=\; (+\frac{3\hbar^2}{4}) \;+\; (-\frac{3\hbar^2}{4})$

II. The Conditional Statements for Casimir Operators in Table. 2 and Figure. 2

The Casimir operator, a quantum operator, is the square sum $j^2 = j_1^2 + j_2^2 + j_3^2$ of three operator components of angular momentum $\vec{j}$. Due to the Hermiticity of angular momentum, the square sum always are $j^2 \geqq 0$, that is, the CO is a *positive* operator.

In particle physics Pauli matrices are positive operators, so the Casimir operators s^2 of spin 1/2 particles, $s_i = \frac{1}{2}\sigma_i$. $s^2 = \vec{s}\cdot\vec{s}$, are positive too. Pauli matrices are the constituents of Dirac equation, further, the solutions of Dirac equation naturally implies *a priori concept* (0) below

- $s^2(\text{particle, antiparticle}) \;=\; \vec{s}\cdot\vec{s} \;=\; \frac{+3\hbar^2}{4} \;=\; (\frac{1}{2})(\frac{1}{2}+1) \;\geq\; 0\hbar^2$ (0)

Formula (0) shows: In Table.1 and Figure.1, the *Spin* Casimir operators $s^2(e^-)$ and $s^2(e^+)$ of both *electron* and *positron* are all *positive* operators. further, (0) is suitable for $s^2(v)$ and $s^2(\overline{v})$ of *neutrinos* v and *antineutrinos* $\overline{v}$ as well in current theory.

This paper bases on the assumption Table.2 and Figure.2, so we see formula (0) is merely the CO of spin 1/2 particles, and however, formula (1) is the CO of spin 1/2 antiparticles

- $s^2(\text{antiparticle}) \;=\; \vec{s}\cdot\vec{s} \;=\; \frac{-3\hbar^2}{4} \;=\; (\frac{i}{2})(\frac{i}{2}+i) \;<\; 0\hbar^2$ (1)

And the formula (2) is the CO of the peculiar *Dark Neutrino* ν_0

- $s^2(DN\ \text{Particle}) \;=\; \vec{s}\cdot\vec{s} \;=\; 0(0+1) \;=\; 0\hbar^2$ (2)

(1) and (2) really are two amusing questions, to find them, let us appeal to the math frame STS.

III. Spin Topological Space, STS (Complex Region)

Go back to Spin Topological Space, STS [3], this time we concern about another important concept: Casimir Operator π^2 of spin 1/2 particles.and spin 1/2 antiparticles.

Remind: the two dimension Hermitan spin matrix operators $\vec{s}=\frac{1}{2}\vec{\sigma}$ that appear in formula (0) are instead by the infinite dimension matrices $\vec{\pi}_{j,k}$, and 1st and 2nd Hermitan components s_1 and s_2 become *non-Hermitan* matrices π_1 and π_2.

Firstly, in order for the assumption (1) to be self-consistent, Spin Topological Coordinate should be extended from real region (j,k) to complex region $(j,b;k,d)$ (3) (4) below

■ Define the transformation

$$\pi_j^+ \Rightarrow \pi_{j,b}^+ = \pi_j^+ + ibI_{+1} \tag{3}$$

$$\pi_k^- \Rightarrow \pi_{k,d}^- = \pi_k^- + idI_{-1} \tag{4}$$

Imaginary numbers b and d now are introduced to raising operator π_j^+ and lowering operator π_k^- respectively.

Using $\pi_{j,b}^+, \pi_{k,d}^-$ to construct the spin angular momentum in complex region

$$\pi_{1\,;j,b\,,\,k,d} = \frac{1}{2}(\pi_{j,b}^+ + \pi_{k,d}^-) \tag{5}$$

$$\pi_{2\,;j,b\,,\,k,d} = \frac{1}{2i}(\pi_{j,b}^+ - \pi_{k,d}^-) \tag{6}$$

$$\pi_{3;j,b\,,\,k,d} = \frac{1}{2}(\pi_{j,b}^+\pi_{k,d}^- - \pi_{k,d}^-\pi_{j,b}^+) \tag{7}$$

$$= \pi_0(0) + \frac{1}{2}(j+k+1) + i\frac{1}{2}(b-d) \tag{8}$$

It can be shown, $\vec{\pi}_{j,b\,,\,k,d}$ still satisfies the commutative algebra rule (9), which is in accord with the Lie algebraic theory of infinite dimension matrix rotation group.

$$\vec{\pi}_{j,b\,,\,k,d} \times \vec{\pi}_{j,b\,,\,k,d} = i\vec{\pi}_{j,b\,,\,k,d} \tag{9}$$

Further, get the representation of invariant, Casimir Operator formula below

$$\pi_{j,b\,,\,k,d}^2 = \pi_{1\,;j,b\,,\,k,d}^2 + \pi_{2\,;j,b\,,\,k,d}^2 + \pi_{3\,;j,b\,,\,k,d}^2$$

$$= \frac{1}{4}\{\,(j-k)^2 - (\,b+d\,)^2 - 1\,\} + i\frac{1}{2}(j-k)(b+d\,) \tag{10}$$

■ The explicit expressions of Casimir Operators, which are in accordance with Table.2 and Figure.2, are given

For neutrino v

$$j - k = \pm\frac{1}{\sqrt{2}}\sqrt{\frac{13}{\sqrt{10}} + 4}\,, \qquad b + d = \pm\frac{1}{\sqrt{2}}\sqrt{\frac{13}{\sqrt{10}} - 4} \tag{11}$$

For antineutrino $\bar{v}$

$$r - s = \pm\frac{1}{\sqrt{2}}\sqrt{\frac{7}{\sqrt{10}} - 2}\,, \qquad a + c = \mp\frac{1}{\sqrt{2}}\sqrt{\frac{7}{\sqrt{10}} + 2} \tag{12}$$

After substituting them into (10) obtain two pairs of conjugative CO between v and $\bar{v}$

$$\pi^2_{j,b\,,\,k,d}(v) = \frac{+3}{4} + i\varphi = \frac{+1}{2}\,(\frac{+1}{2}+1)(1 + i10^{\frac{-1}{2}})\hbar^2 \tag{13.1}$$

$$\pi^2_{r,a\,,\,s,c}(\bar{v}) = \frac{-3}{4} - i\varphi = \frac{+i1}{2}\,(\frac{+i1}{2}+i1)(1 - i10^{\frac{-1}{2}})\hbar^2 \tag{13.2}$$

$$\pi^2_{j,b\,,\,k,d}(v) = \frac{+3}{4} - i\varphi = \frac{+1}{2}\,(\frac{+1}{2}+1)(1 - i10^{\frac{-1}{2}})\hbar^2 \tag{14.1}$$

$$\pi^2_{r,a\,,\,s,c}(\bar{v}) = \frac{-3}{4} + i\varphi = \frac{+i1}{2}\,(\frac{+i1}{2}+i1)(1 + i10^{\frac{-1}{2}})\hbar^2 \tag{14.2}$$

$$\frac{+3}{4} = 0.75, \qquad \varphi = \frac{3}{4\sqrt{10}} \simeq 0.237 \tag{15}$$

We see the real part, $\frac{+3}{4}$ and $\frac{-3}{4}$, of the above formulas are just the previous assumption (0) and (1), the Casimir operators for spin 1/2 particles and for spin 1/2 antiparticles repectively, which are what we except to be originally.

IV. Casimir Operator of DN, Dark Neutrino v_0

For the implement of the progress of $(+\frac{3\hbar^2}{4}) + (-\frac{3\hbar^2}{4}) = 0\,\hbar^2$, v_0 is written into the superposition of spin angular momentums $\vec{\pi}_{j,b\,,\,k,d}(v)$ and $\vec{\pi}_{r,a\,,\,s,c}(\bar{v})$ in complex region below

$$\pi^+(v_0) = \Pi^+_{j,b\,,\,r,a}(v + \bar{v}) = \frac{1}{2}\{\pi^+_{j,b}(v) + \pi^+_{r,a}(\bar{v})\} \tag{16}$$

$$\pi^-(v_0) = \Pi^-_{k,d\,,\,s,c}(v + \bar{v}) = \frac{1}{2}\{\pi^-_{k,d}(v) + \pi^-_{s,c}(\bar{v})\} \tag{17}$$

and

$$\vec{\Pi} \times \vec{\Pi} = i\vec{\Pi} \tag{18}$$

In much same way as discussed in section III by using (11) and (12), obtain an important formula below

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$$\blacksquare \quad \Pi^2_{j,b\,,\,r,a\,;\,k,d\,,\,s,c}(\nu + \bar{\nu}) = \{\, 0(0{+}1) \pm i\Phi \,\}\hbar^2 \tag{19}$$

Thus we have CO of DN

$$\pi^2_{w,e\,,\,z,f}(\nu_0) = \Pi^2(\nu + \bar{\nu}) = 0\hbar^2 \pm i\Phi\hbar^2 \tag{20}$$

where

$$\Phi = \tfrac{1}{8}(10^{\frac{+1}{2}} - 10^{\frac{-1}{2}}) \simeq 0.356 \tag{21}$$

and

$$w - z = \tfrac{1}{2}(10^{\frac{+1}{4}} + 10^{\frac{-1}{4}}) \tag{22}$$

$$e + f = \tfrac{1}{2}(10^{\frac{+1}{4}} - 10^{\frac{-1}{4}}) \tag{23}$$

We see the real part $0(0{+}1)\hbar^2$ of (20) is just the previous assumption (2), which is the CO of *Dark Neutrino* ν_0, and is in accord with the CO which appears in Table, 2, Figure 2. and formular (2).

V. The Spin Third Components of DN, Dark Neutrino ν_0

For convenience, in the following we take the imaginarys (ref.(8)) of the third components of neutrino ν and antineutrino $\bar{\nu}$ to be zero

$$b = d, \quad a = c \tag{24}$$

$$\blacktriangleright \quad \pi_{3;\,j,b\,,\,k,d}(\nu) = \pi_0(0) + \tfrac{1}{2}(j + k + 1) \tag{25}$$

$$\blacktriangleright \quad \pi_{3;\,r,a\,,\,s,c}(\bar{\nu}) = \pi_0(0) + \tfrac{1}{2}(r + s + 1) \tag{26}$$

Further obtain

$$\blacksquare \quad \Pi_{3;\,j,b\,,\,r,a\,;\,k,d\,,\,s,c}(\nu + \bar{\nu}) = \tfrac{1}{2}\{\pi_{3;\,j,b\,,\,k,d}(\nu) + \pi_{3;\,r,a\,,\,s,c}(\bar{\nu})\} \tag{27}$$

Thus we have CO of DN

$$\pi_{3;\,w,e\,,\,z,f}(\nu_0) = \Pi_3(\nu + \bar{\nu}) = \pi_0(0) + \tfrac{1}{2}(w + z + 1) \tag{28}$$

Where

$$w + z = \tfrac{1}{2}(j + k + r + s) \tag{29}$$

Applying above results to lead to two groups of solutions for (27): solution of integers (30) and solution of half-integers (31), and illustrated in Table. 3

- $\Pi_3\{\nu + \bar{\nu}\} \quad = \ldots, +2, \quad +1, \quad 0, \quad -1, \quad -2, \ldots$ integers (30)

- $\Pi_3\{\nu + \bar{\nu}, \} \quad = \ldots, \frac{+5}{2}, \frac{+3}{2}, \frac{+1}{2}, \frac{-1}{2}, \frac{-3}{2}, \frac{-5}{2}, \ldots$ half-integers (31)

Table 3: $\Pi_{3;\, j,b\, ,\, r,a\, ;\, k,d\, ,\, s,c}(\nu + \bar{\nu})$

$\Pi^2(\nu + \bar{\nu})$ (19)	Antineutrino $\pi_{3;\, r,a\, ,\, s,c}(\bar{\nu})$	$\frac{+5}{2}$	$\frac{+3}{2}$	$\frac{+1}{2}$	$\frac{-1}{2}$	$\frac{-3}{2}$	$\frac{-5}{2}$
Neutrino $\pi_{3;\, j,b\, ,\, k,d}(\nu)$	$\Pi_3(\nu + \bar{\nu})$						
$\frac{+5}{2}$		$\frac{+5}{2}$	$+2$	$\frac{+3}{2}$	$+1$	$\frac{+1}{2}$	0
$\frac{+3}{2}$		$+2$	$\frac{+3}{2}$	$+1$	$\frac{+1}{2}$	0	$\frac{-1}{2}$
$\frac{+1}{2}$		$\frac{+3}{2}$	$+1$	$\frac{+1}{2}$	0	$\frac{-1}{2}$	-1
$\frac{-1}{2}$		$+1$	$\frac{+1}{2}$	0	$\frac{-1}{2}$	-1	$\frac{-3}{2}$
$\frac{-3}{2}$		$\frac{+1}{2}$	0	$\frac{-1}{2}$	-1	$\frac{-3}{2}$	-2
$\frac{-5}{2}$		0	$\frac{-1}{2}$	-1	$\frac{-3}{2}$	-2	$\frac{-5}{2}$

(19) and (30) construct a zero-spin boson (32). with one invariant Co of group representation

$$\Pi^2 = \{\, 0(0{+}1) \pm i\Phi \,\}\hbar^2 \quad \text{and} \quad \Pi_3 = \text{zero}, \pm \text{integers} \tag{32}$$

(19) and (31) construct ν_0 (33) which we have never seen such kind of representation of group before

$$\Pi^2 = \{\, 0(0{+}1) \pm i\Phi \,\}\hbar^2 \quad \text{and} \quad \Pi_3 = \pm \text{half-integers} \tag{33}$$

Where for (33) below

$$\Pi = \Pi(\nu + \bar{\nu}) = \pi(\nu_0) \tag{34}$$

Next following, using (20),(28) and (31) to research Dark Neutrino ν_0 in the case of $\nu = \nu_L$, Left-handed Neutrino and $\bar{\nu} = \bar{\nu}_R$, Right-handed Antineutrino. The more details are shown in Table 4 and section 5.

VI. CHIRAL ARROW $\Uparrow$ AND MOTION ARROW $\uparrow$ OF ν_0

A particle is called *chiral* if it is distinguishable from its mirror image [4], Mark *chiral arrow* $\Uparrow$ is used to play the role in particle physics. The direction of the chiral arrows for particle and antiparticle are the same. Mark *motion arrow* $\uparrow$ is used to represent the direction of a particle momentum $\vec{p}$ that is aligned along with the z-axis.

Base on the two marks, $\Uparrow$ and $\uparrow$, we constructe Table.4 and Table.5 below

Table 4: Formation of DN with $\pi_3(\nu_0) = \frac{+1}{2}$

Diraction of motion $\vec{p}$ motion arrow	Left-handed Neutrino ν_L chiral arrow	$\pi_3(\nu_L)$ flavour	$\pi_3(\bar{\nu}_R)$ flavour	Right-handed Antineutrino $\bar{\nu}_R$ chiral arrow	Diraction of motion $\vec{p}$ motion arrow	DN	$\pi_3(\nu_0)$
$\uparrow$	$\Downarrow$	$\frac{-5}{2}$	$\frac{+7}{2}$	$\Uparrow$	$\uparrow$	$\uparrow$	$\frac{+1}{2}$
$\uparrow$	$\Downarrow$	$\frac{-3}{2}$	$\frac{+5}{2}$	$\Uparrow$	$\uparrow$	$\uparrow$	$\frac{+1}{2}$
$\uparrow$	$\Downarrow$	$\frac{-1}{2}$	$\frac{+3}{2}$	$\Uparrow$	$\uparrow$	$\uparrow$	$\frac{+1}{2}$
$\downarrow \blacklozenge$	$\Uparrow$	$\frac{+1}{2} \blacklozenge$	$\blacklozenge \frac{+1}{2}$	$\Uparrow$	$\blacklozenge \uparrow$	0	$\frac{+1}{2}$
$\downarrow$	$\Uparrow$	$\frac{+3}{2}$	$\frac{-1}{2}$	$\Downarrow$	$\downarrow$	$\downarrow$	$\frac{+1}{2}$
$\downarrow$	$\Uparrow$	$\frac{+5}{2}$	$\frac{-3}{2}$	$\Downarrow$	$\downarrow$	$\downarrow$	$\frac{+1}{2}$
$\downarrow$	$\Uparrow$	$\frac{+7}{2}$	$\frac{-5}{2}$	$\Downarrow$	$\downarrow$	$\downarrow$	$\frac{+1}{2}$

Table 5: Formation of DN with $\pi_3(\nu_0) = \frac{-1}{2}$

Diraction of motion motion arrow	Left-handed Neutrino ν_L chiral arrow	$\pi_3(\nu_L)$ flavour	$\pi_3(\bar{\nu}_R)$ flavour	Right-handed Antineutrino $\bar{\nu}_R$ chiral arrow	Diraction of motion motion arrow	DN	$\pi_3(\nu_0)$
$\downarrow$	$\Uparrow$	$\frac{+5}{2}$	$\frac{-7}{2}$	$\Downarrow$	$\downarrow$	$\downarrow$	$\frac{-1}{2}$
$\downarrow$	$\Uparrow$	$\frac{+3}{2}$	$\frac{-5}{2}$	$\Downarrow$	$\downarrow$	$\downarrow$	$\frac{-1}{2}$
$\downarrow$	$\Uparrow$	$\frac{+1}{2}$	$\frac{-3}{2}$	$\Downarrow$	$\downarrow$	$\downarrow$	$\frac{-1}{2}$
$\uparrow \blacklozenge$	$\Downarrow$	$\frac{-1}{2} \blacklozenge$	$\blacklozenge \frac{-1}{2}$	$\Downarrow$	$\blacklozenge \downarrow$	0	$\frac{-1}{2}$
$\uparrow$	$\Downarrow$	$\frac{-3}{2}$	$\frac{+1}{2}$	$\Uparrow$	$\uparrow$	$\uparrow$	$\frac{-1}{2}$
$\uparrow$	$\Downarrow$	$\frac{-5}{2}$	$\frac{+3}{2}$	$\Uparrow$	$\uparrow$	$\uparrow$	$\frac{-1}{2}$
$\uparrow$	$\Downarrow$	$\frac{-7}{2}$	$\frac{+5}{2}$	$\Uparrow$	$\uparrow$	$\uparrow$	$\frac{-1}{2}$

In the above two tables, the rotation orientation of these spin particles maybe clockwise or counterclockwise, which is discribed by their third components π_3 of spin angular momentum [5],[6]. The clockwise, corresponding to the postive value of π_3 and the counterclockwise to the negative values of π_3. Further, People usually provide: if $\vec{\pi}_3$ parallel to direction $\uparrow$ of its momentum $\vec{p}$, we speak of a particle with right-handed helicity, RH, and if $\vec{\pi}_3$ antiparallel to direction $\uparrow$ of its momentum $\vec{p}$, with left-handed hecility, LH. Remind $\vec{\pi}_3$ parallel chiral arrows $\Uparrow$.

In nature there are only right-handed antineutrinos and only left-handed neutrinos, so, we have $\nu \Rightarrow \nu_{\mathrm{L}}$ and $\bar{\nu} \Rightarrow \bar{\nu}_{\mathrm{R}}$ in Table.4 and in Table.5.

Cntinuing with the concept of Table 4 and Table 5, then Table.3 turns into Table.6

Table 6: Formation of DN ν_0 and Zero Spin from ν_{L} and $\bar{\nu}_{\mathrm{R}}$ with common CO $\{\,0(0+1) \pm i\Phi\,\}\hbar^2$

Diraction of motion $\vec{p}$ motion arrow	LH Neutrino ν_{L} chiral arrow	$\pi_3(\nu_{\mathrm{L}})$ flavour	|	$\pi_3(\bar{\nu}_{\mathrm{R}})$ flavour	RH Antineutrino $\bar{\nu}_{\mathrm{R}}$ chiral arrow	Diraction of motion $\vec{p}$ motion arrow	|||		DN $\pi_3(\nu_0)$ (33)	zero spin $\pi_3(zs)$ (32)
$\cdots\cdots$	$\cdots\cdots$	$\cdot\cdot$	$\cdot$	$\cdot\cdot$	$\cdots\cdots$	$\cdots\cdots$		$\circ$	$\circ\circ\circ\circ$	$\circ\circ$
$\uparrow$	$\Downarrow$	$\frac{-1}{2}$	|	$\frac{+7}{2}$	$\Uparrow$	$\uparrow$	|||	$\uparrow$	$\frac{+3}{2}$	
$\uparrow$	$\Downarrow$	$\frac{-1}{2}$	|	$\frac{+5}{2}$	$\Uparrow$	$\uparrow$	|||	$\uparrow$		$+1$
$\uparrow$	$\Downarrow$	$\frac{-1}{2}$	|	$\frac{+3}{2}$	$\Uparrow$	$\uparrow$	|||	$\uparrow$	$\frac{+1}{2}$	
$\uparrow$	$\Downarrow$	$\frac{-1}{2}$	|	$\frac{+1}{2}$	$\Uparrow$	$\uparrow$	|||	$\uparrow$		0
—	—	—	–	—	—	—	–	–	—	–
$\downarrow$	$\Uparrow$	$\frac{+1}{2}$	|	$\frac{-1}{2}$	$\Downarrow$	$\downarrow$	|||	$\downarrow$		0
$\downarrow$	$\Uparrow$	$\frac{+1}{2}$	|	$\frac{-3}{2}$	$\Downarrow$	$\downarrow$	|||	$\downarrow$	$\frac{-1}{2}$	
$\downarrow$	$\Uparrow$	$\frac{+1}{2}$	|	$\frac{-5}{2}$	$\Downarrow$	$\downarrow$	|||	$\downarrow$		-1
$\downarrow$	$\Uparrow$	$\frac{+1}{2}$	|	$\frac{-7}{2}$	$\Downarrow$	$\downarrow$	|||	$\downarrow$	$\frac{-3}{2}$	
$\cdots\cdots$	$\cdots\cdots$	$\cdot\cdot$	$\cdot$	$\cdot\cdot$	$\cdots\cdots$	$\cdots\cdots$		$\circ$	$\circ\circ\circ\circ$	$\circ\circ$

VII. Dark Spin Particles, DSP

DSP are spin particles that have yet to be observed in nature up to now, but are existing in math Spin Topological Space, STS. To give a succinct explanation account of DSP, we back to real region, spin topological coordinate (j,k) can help us describe spin particles and write down their group representations. In the following Tables, $\pi^2_{j,k}$ and $\pi_{3;j,k}$ are Casimir Operator and the third components of spin particles.

In Table 7 the values with underline are the spin eigenvalues of Bosons and Fermions, only those values are what we could explore and see in current theory and experiment now. Any other values without underline are the "spin-excited states " of Bosons and Fermions in STS.

Table 7: Bosons and Fermions

	$\pi^2_{j,k}$	$j-k$							
Boson	0	$+1$	(j,k)	$(+2,+1)$	$(+1,0)$	$(0,-1)$	$(-1,-2)$	$(-2,-3)$	$(-3,-4)$
A1	$0(0+1)$		$\pi_{3;j,k}$	$+2$	$+1$	$\underline{0}$	-1	-2	-3
Boson	2	$+3$	(j,k)	$(+3,0)$	$(+2,-1)$	$(+1,-2)$	$(0,-3)$	$(-1,-4)$	$(-2,-5)$
A2	$1(1+1)$		$\pi_{3;j,k}$	$+2$	$\underline{+1}$	$\underline{0}$	$\underline{-1}$	-2	-3
Boson	6	$+5$	(j,k)	$(+4,-1)$	$(+3,-2)$	$(+2,-3)$	$(+1,-4)$	$(0,-5)$	$(-1,-6)$
A3	$2(2+1)$		$\pi_{3;j,k}$	$\underline{+2}$	$\underline{+1}$	$\underline{0}$	$\underline{-1}$	$\underline{-2}$	-3
Boson	12	$+7$	(j,k)	$(+5,-2)$	$(+4,-3)$	$(+3,-4)$	$(+2,-5)$	$(+1,-6)$	$(0,-7)$
A4	$3(3+1)$		$\pi_{3;j,k}$	$\underline{+2}$	$\underline{+1}$	$\underline{0}$	$\underline{-1}$	$\underline{-2}$	$\underline{-3}$
Fermion	$\frac{3}{4}$	$+2$	(j,k)	$(+3,+1)$	$(+2,0)$	$(+1,-1)$	$(0,-2)$	$(-1,-3)$	$(-2,-4)$
B1	$\frac{1}{2}(\frac{1}{2}+1)$		$\pi_{3;j,k}$	$\frac{+5}{2}$	$\frac{+3}{2}$	$\underline{\frac{+1}{2}}$	$\underline{\frac{-1}{2}}$	$\frac{-3}{2}$	$\frac{-5}{2}$
Fermion	$\frac{15}{4}$	$+4$	(j,k)	$(+4,0)$	$(+3,-1)$	$(+2,-2)$	$(+1,-3)$	$(0,-4)$	$(-1,-5)$
B2	$\frac{3}{2}(\frac{3}{2}+1)$		$\pi_{3;j,k}$	$\frac{+5}{2}$	$\underline{\frac{+3}{2}}$	$\underline{\frac{+1}{2}}$	$\underline{\frac{-1}{2}}$	$\underline{\frac{-3}{2}}$	$\frac{-5}{2}$
Fermion	$\frac{35}{4}$	$+6$	(j,k) $A7$	$(+5,-1)$	$(+4,-2)$	$(+3,-3)$	$(+2,-4)$	$(+1,-5)$	$(0,-6)$
B3	$\frac{5}{2}(\frac{5}{2}+1)$		$\pi_{3;j,k}$	$\underline{\frac{+5}{2}}$	$\underline{\frac{+3}{2}}$	$\underline{\frac{+1}{2}}$	$\underline{\frac{-1}{2}}$	$\underline{\frac{-3}{2}}$	$\underline{\frac{-5}{2}}$

■ Regular Dark Spin Particles

Spin particles in Table 8 are called regular dark spin particles, due to their $\pi^2_{j\,k,}$ is connected to one of the values of their $\pi_{3j,\,k}$. Here the spin topological coordinate (j,k) of spin-1/3, spin-2/3 and spin-1/6, spin-5/6 are listed, the more details about others spin can be referred to author's works. The particles in Table 7 possess the same property.

Table 8: Regular Dark Spin Particles

	$\pi^2_{j,k}$	$j-k$							
	$\frac{7}{36}$	$\frac{+4}{3}$	(j,k)	$(\frac{+7}{3},\frac{+3}{3})$	$(\frac{+4}{3},\frac{0}{3})$	$(\frac{+1}{3},\frac{-3}{3})$	$(\frac{-2}{3},\frac{-6}{3})$	$(\frac{-5}{3},\frac{-9}{3})$	$(\frac{-8}{3},\frac{-12}{3})$
C1	$\frac{1}{6}(\frac{1}{6}+1)$		$\pi_{3;j,k}$	$\frac{+13}{6}$	$\frac{+7}{6}$	$\frac{+1}{6}$	$\frac{-5}{6}$	$\frac{-11}{6}$	$\frac{-17}{6}$
	$\frac{55}{36}$	$\frac{+8}{3}$	(j,k)	$(\frac{+11}{3},\frac{+3}{3})$	$(\frac{+8}{3},\frac{0}{3})$	$(\frac{+5}{3},\frac{-3}{3})$	$(\frac{+2}{3},\frac{-6}{3})$	$(\frac{-1}{3},\frac{-9}{3})$	$(\frac{-4}{3},\frac{-12}{3})$
C2	$\frac{5}{6}(\frac{5}{6}+1)$		$\pi_{3;j,k}$	$\frac{+17}{6}$	$\frac{+11}{6}$	$\frac{+5}{6}$	$\frac{-1}{6}$	$\frac{-7}{6}$	$\frac{-13}{6}$
	$\frac{4}{9}$	$\frac{+5}{3}$	(j,k)	$(\frac{+8}{3},\frac{+3}{3})$	$(\frac{+5}{3},\frac{0}{3})$	$(\frac{+2}{3},\frac{-3}{3})$	$(\frac{-1}{3},\frac{-6}{3})$	$(\frac{-4}{3},\frac{-9}{3})$	$(\frac{-7}{3},\frac{-12}{3})$
D1	$\frac{1}{3}(\frac{1}{3}+1)$		$\pi_{3;j,k}$	$\frac{+7}{3}$	$\frac{+4}{3}$	$\frac{+1}{3}$	$\frac{-2}{3}$	$\frac{-5}{3}$	$\frac{-8}{3}$
	$\frac{10}{9}$	$\frac{+7}{3}$	(j,k)	$(\frac{+10}{3},\frac{+3}{3})$	$(\frac{+7}{3},\frac{0}{3})$	$(\frac{+4}{3},\frac{-3}{3})$	$(\frac{+1}{3},\frac{-6}{3})$	$(\frac{-2}{3},\frac{-9}{3})$	$(\frac{-5}{3},\frac{-12}{3})$
D2	$\frac{2}{3}(\frac{2}{3}+1)$		$\pi_{3;j,k}$	$\frac{+8}{3}$	$\frac{+5}{3}$	$\frac{+2}{3}$	$\frac{-1}{3}$	$\frac{-4}{3}$	$\frac{-7}{3}$

■ Peculiar Dark Spin Particles

Spin particles in the next six tables are referred to peculiar dark spin particles, which are cataloged by different CO. The first three tables are with even number CO, and the last three tables with odd number.

Peculiar dark spin particles are *neither* Bosons, Fermions *nor* regular dark spin particles,

Compaire $\pi^2_{w,e\,,\,z,f}(v_0)$ (20), $\pi_{3;\,w,e\,,\,z,f}(v_0)$ (28) and $\Pi_3\{v+\bar{v},\}$ (31) with the Series $\frac{\pm 1}{2}$ in Table 9, we see the former (complex region) is the extension of the latter (real region), the latter is the special case of the former. both of them are all with CO $0\hbar^2$ and $\pi_3=\frac{\pm 1}{2},\frac{\pm 3}{2},\frac{\pm 5}{2},\ldots$ So DN v_0 is one kind of peculiar dark spin particles.

Table 9: $\pi^2_{j,k} = 0(0+1) = 0\hbar^2$, with $j - k = +1$

Series $\frac{+1}{2}$						
(j, k)	$(\frac{+5}{2}, \frac{+3}{2})$	$(\frac{+3}{2}, \frac{+1}{2})$	$(\frac{+1}{2}, \frac{-1}{2})$	$(\frac{-1}{2}, \frac{-3}{2})$	$(\frac{-3}{2}, \frac{-5}{2})$	$(\frac{-5}{2}, \frac{-7}{2})$
$\pi_{3;j,k}$	$\frac{+5}{2}$	$\frac{+3}{2}$	$\frac{+1}{2}$	$\frac{-1}{2}$	$\frac{-3}{2}$	$\frac{-5}{2}$
Series $\frac{+1}{3}$						
(j, k)	$(\frac{+7}{3}, \frac{+4}{3})$	$(\frac{+3}{3}, \frac{+1}{3})$	$(\frac{+1}{3}, \frac{-2}{3})$	$(\frac{-2}{3}, \frac{-5}{3})$	$(\frac{-5}{3}, \frac{-8}{3})$	$(\frac{-8}{3}, \frac{-11}{3})$
$\pi_{3;j,k}$	$\frac{+7}{3}$	$\frac{+4}{3}$	$\frac{+1}{3}$	$\frac{-2}{3}$	$\frac{-5}{3}$	$\frac{-8}{3}$
Series $\frac{+2}{3}$						
(j, k)	$(\frac{+8}{3}, \frac{+5}{3})$	$(\frac{+5}{3}, \frac{+2}{3})$	$(\frac{+2}{3}, \frac{-1}{3})$	$(\frac{-1}{3}, \frac{-4}{3})$	$(\frac{-4}{3}, \frac{-7}{3})$	$(\frac{-7}{3}, \frac{-10}{3})$
$\pi_{3;j,k}$	$\frac{+8}{3}$	$\frac{+5}{3}$	$\frac{+2}{3}$	$\frac{-1}{3}$	$\frac{-4}{3}$	$\frac{-7}{3}$
Series $\frac{+1}{6}$						
(j, k)	$(\frac{+13}{6}, \frac{+7}{6})$	$(\frac{+7}{6}, \frac{+1}{6})$	$(\frac{+1}{6}, \frac{-5}{6})$	$(\frac{-5}{6}, \frac{-11}{6})$	$(\frac{-11}{6}, \frac{-17}{6})$	$(\frac{-17}{6}, \frac{-23}{6})$
$\pi_{3;j,k}$	$\frac{+13}{6}$	$\frac{+7}{6}$	$\frac{+1}{6}$	$\frac{-5}{6}$	$\frac{-11}{6}$	$\frac{-17}{6}$
Series $\frac{+5}{6}$						
(j, k)	$(\frac{+17}{6}, \frac{+11}{6})$	$(\frac{+11}{6}, \frac{+5}{6})$	$(\frac{+5}{6}, \frac{-1}{6})$	$(\frac{-1}{6}, \frac{-7}{6})$	$(\frac{-7}{6}, \frac{-13}{6})$	$(\frac{-13}{6}, \frac{-29}{6})$
$\pi_{3;j,k}$	$\frac{+17}{6}$	$\frac{+11}{6}$	$\frac{+5}{6}$	$\frac{-1}{6}$	$\frac{-7}{6}$	$\frac{-13}{6}$

Table 10: $\pi^2_{j,k} = 1(1+1) = 2\hbar^2$, with $j - k = +3$

Series $\frac{+1}{2}$						
(j, k)	$(\frac{+7}{2}, \frac{+1}{2})$	$(\frac{+5}{2}, \frac{-1}{2})$	$(\frac{+3}{2}, \frac{-3}{2})$	$(\frac{+1}{2}, \frac{-5}{2})$	$(\frac{-1}{2}, \frac{-7}{2})$	$(\frac{-3}{2}, \frac{-9}{2})$
$\pi_{3;j,k}$	$\frac{+5}{2}$	$\frac{+3}{2}$	$\frac{+1}{2}$	$\frac{-1}{2}$	$\frac{-3}{2}$	$\frac{-5}{2}$
Series $\frac{+1}{3}$						
(j, k)	$(\frac{+10}{3}, \frac{+1}{3})$	$(\frac{+7}{3}, \frac{-2}{3})$	$(\frac{+4}{3}, \frac{-5}{3})$	$(\frac{+1}{3}, \frac{-8}{3})$	$(\frac{-2}{3}, \frac{-11}{3})$	$(\frac{-5}{3}, \frac{-14}{3})$
$\pi_{3;j,k}$	$\frac{+7}{3}$	$\frac{+4}{3}$	$\frac{+1}{3}$	$\frac{-2}{3}$	$\frac{-5}{3}$	$\frac{-8}{3}$
Series $\frac{+2}{3}$						
(j, k)	$(\frac{+11}{3}, \frac{+2}{3})$	$(\frac{+8}{3}, \frac{-1}{3})$	$(\frac{+5}{3}, \frac{-4}{3})$	$(\frac{+2}{3}, \frac{-7}{3})$	$(\frac{-1}{3}, \frac{-10}{3})$	$(\frac{-4}{3}, \frac{-13}{3})$
$\pi_{3;j,k}$	$\frac{+8}{3}$	$\frac{+5}{3}$	$\frac{+2}{3}$	$\frac{-1}{3}$	$\frac{-4}{3}$	$\frac{-7}{3}$
Series $\frac{+1}{6}$						
(j, k)	$(\frac{+19}{6}, \frac{+1}{6})$	$(\frac{+13}{6}, \frac{-5}{6})$	$(\frac{+7}{6}, \frac{-11}{6})$	$(\frac{+1}{6}, \frac{-17}{6})$	$(\frac{-5}{6}, \frac{-23}{6})$	$(\frac{-11}{6}, \frac{-29}{6})$
$\pi_{3;j,k}$	$\frac{+13}{6}$	$\frac{+7}{6}$	$\frac{+1}{6}$	$\frac{-5}{6}$	$\frac{-11}{6}$	$\frac{-17}{6}$
Series $\frac{+5}{6}$						
(j, k)	$(\frac{+23}{6}, \frac{+5}{6})$	$(\frac{+17}{6}, \frac{-1}{6})$	$(\frac{+11}{6}, \frac{-7}{6})$	$(\frac{+5}{6}, \frac{-13}{6})$	$(\frac{-1}{6}, \frac{-19}{6})$	$(\frac{-7}{6}, \frac{-25}{6})$
$\pi_{3;j,k}$	$\frac{+17}{6}$	$\frac{+11}{6}$	$\frac{+5}{6}$	$\frac{-1}{6}$	$\frac{-7}{6}$	$\frac{-13}{6}$

Table 11: $\pi^2_{j,k} = 2(2+1) = 6\hbar^2,$ with $j - k = +5$

Series $\frac{+1}{2}$						
(j, k)	$(\frac{+9}{2}, \frac{-1}{2})$	$(\frac{+7}{2}, \frac{-3}{2})$	$(\frac{+5}{2}, \frac{-5}{2})$	$(\frac{+3}{2}, \frac{-7}{2})$	$(\frac{+1}{2}, \frac{-9}{2})$	$(\frac{-1}{2}, \frac{-11}{2})$
$\pi_{3;j,k}$	$\frac{+5}{2}$	$\frac{+3}{2}$	$\frac{+1}{2}$	$\frac{-1}{2}$	$\frac{-3}{2}$	$\frac{-5}{2}$
Series $\frac{+1}{3}$						
(j, k)	$(\frac{+13}{3}, \frac{-2}{3})$	$(\frac{+10}{3}, \frac{-5}{3})$	$(\frac{+7}{3}, \frac{-8}{3})$	$(\frac{+4}{3}, \frac{-11}{3})$	$(\frac{+1}{3}, \frac{-14}{3})$	$(\frac{-2}{3}, \frac{-17}{3})$
$\pi_{3;j,k}$	$\frac{+7}{3}$	$\frac{+4}{3}$	$\frac{+1}{3}$	$\frac{-2}{3}$	$\frac{-5}{3}$	$\frac{-8}{3}$
Series $\frac{+2}{3}$						
(j, k)	$(\frac{+14}{3}, \frac{-1}{3})$	$(\frac{+11}{3}, \frac{-4}{3})$	$(\frac{+8}{3}, \frac{-7}{3})$	$(\frac{+5}{3}, \frac{-10}{3})$	$(\frac{+2}{3}, \frac{-13}{3})$	$(\frac{-1}{3}, \frac{-16}{3})$
$\pi_{3;j,k}$	$\frac{+8}{3}$	$\frac{+5}{3}$	$\frac{+2}{3}$	$\frac{-1}{3}$	$\frac{-4}{3}$	$\frac{-7}{3}$
Series $\frac{+1}{6}$						
(j, k)	$(\frac{+25}{6}, \frac{-5}{6})$	$(\frac{+19}{6}, \frac{-11}{6})$	$(\frac{+13}{6}, \frac{-17}{6})$	$(\frac{+7}{6}, \frac{-23}{6})$	$(\frac{+1}{6}, \frac{-29}{6})$	$(\frac{-5}{6}, \frac{-35}{6})$
$\pi_{3;j,k}$	$\frac{+13}{6}$	$\frac{+7}{6}$	$\frac{+1}{6}$	$\frac{-5}{6}$	$\frac{-11}{6}$	$\frac{-17}{6}$
Series $\frac{+5}{6}$						
(j, k)	$(\frac{+29}{6}, \frac{-1}{6})$	$(\frac{+23}{6}, \frac{-7}{6})$	$(\frac{+17}{6}, \frac{-13}{6})$	$(\frac{+11}{6}, \frac{-19}{6})$	$(\frac{+5}{6}, \frac{-25}{6})$	$(\frac{-1}{6}, \frac{-31}{6})$
$\pi_{3;j,k}$	$\frac{+17}{6}$	$\frac{+11}{6}$	$\frac{+5}{6}$	$\frac{-1}{6}$	$\frac{-7}{6}$	$\frac{-13}{6}$

Table 12: $\pi^2_{j,k} = \frac{3}{4} = \frac{+1}{2}(\frac{+1}{2}+1)\hbar^2,$ with $j - k = +2$

Series0						
(j, k)	$(\frac{+5}{2}, \frac{+1}{2})$	$(\frac{+3}{2}, \frac{-1}{2})$	$(\frac{+1}{2}, \frac{-3}{2})$	$(\frac{-1}{2}, \frac{-5}{2})$	$(\frac{-3}{2}, \frac{-7}{2})$	$(\frac{-5}{2}, \frac{-9}{2})$
$\pi_{3;j,k}$	$+2$	$+1$	0	-1	-2	-3
Series $\frac{+1}{3}$						
(j, k)	$(\frac{+17}{6}, \frac{+5}{6})$	$(\frac{+11}{6}, \frac{-1}{6})$	$(\frac{+5}{6}, \frac{-7}{6})$	$(\frac{-1}{6}, \frac{-13}{6})$	$(\frac{-7}{6}, \frac{-19}{6})$	$(\frac{-13}{6}, \frac{-25}{6})$
$\pi_{3;j,k}$	$\frac{+7}{3}$	$\frac{+4}{3}$	$\frac{+1}{3}$	$\frac{-2}{3}$	$\frac{-5}{3}$	$\frac{-8}{3}$
Series $\frac{+2}{3}$						
(j, k)	$(\frac{+19}{6}, \frac{+7}{6})$	$(\frac{+13}{6}, \frac{+1}{6})$	$(\frac{+7}{6}, \frac{-5}{6})$	$(\frac{+1}{6}, \frac{-11}{6})$	$(\frac{-5}{6}, \frac{-17}{6})$	$(\frac{-11}{6}, \frac{-23}{6})$
$\pi_{3;j,k}$	$\frac{+8}{3}$	$\frac{+5}{3}$	$\frac{+2}{3}$	$\frac{-1}{3}$	$\frac{-4}{3}$	$\frac{-7}{3}$
Series $\frac{+1}{6}$						
(j, k)	$(\frac{+8}{3}, \frac{+2}{3})$	$(\frac{+5}{3}, \frac{-1}{3})$	$(\frac{+2}{3}, \frac{-4}{3})$	$(\frac{-1}{3}, \frac{-7}{3})$	$(\frac{-4}{3}, \frac{-10}{3})$	$(\frac{-7}{3}, \frac{-13}{3})$
$\pi_{3;j,k}$	$\frac{+13}{6}$	$\frac{+7}{6}$	$\frac{+1}{6}$	$\frac{-5}{6}$	$\frac{-11}{6}$	$\frac{-17}{6}$
Series $\frac{+5}{6}$						
(j, k)	$(\frac{+10}{3}, \frac{+4}{3})$	$(\frac{+7}{3}, \frac{+1}{3})$	$(\frac{+4}{3}, \frac{-2}{3})$	$(\frac{+1}{3}, \frac{-5}{3})$	$(\frac{-2}{3}, \frac{-8}{3})$	$(\frac{-5}{3}, \frac{-11}{3})$
$\pi_{3;j,k}$	$\frac{+17}{6}$	$\frac{+11}{6}$	$\frac{+5}{6}$	$\frac{-1}{6}$	$\frac{-7}{6}$	$\frac{-13}{6}$

Table 13: $\pi^2_{j,k} = \frac{15}{4} = \frac{+3}{2}(\frac{+3}{2}+1)\hbar^2$, with $j-k=+4$

Series0						
(j,k)	$(\frac{+7}{2},\frac{-1}{2})$	$(\frac{+5}{2},\frac{-3}{2})$	$(\frac{+3}{2},\frac{-5}{2})$	$(\frac{+1}{2},\frac{-7}{2})$	$(\frac{-1}{2},\frac{-9}{2})$	$(\frac{-3}{2},\frac{-11}{2})$
$\pi_{3;j,k}$	$+2$	$+1$	0	-1	-2	-3
Series $\frac{+1}{3}$						
(j,k)	$(\frac{+23}{6},\frac{-1}{6})$	$(\frac{+17}{6},\frac{-7}{6})$	$(\frac{+11}{6},\frac{-13}{6})$	$(\frac{+5}{6},\frac{-19}{6})$	$(\frac{-1}{6},\frac{-25}{6})$	$(\frac{-7}{6},\frac{-31}{6})$
$\pi_{3;j,k}$	$\frac{+7}{3}$	$\frac{+4}{3}$	$\frac{+1}{3}$	$\frac{-2}{3}$	$\frac{-5}{3}$	$\frac{-8}{3}$
Series $\frac{+2}{3}$						
(j,k)	$(\frac{+25}{6},\frac{+1}{6})$	$(\frac{+19}{6},\frac{-5}{6})$	$(\frac{+13}{6},\frac{-11}{6})$	$(\frac{+7}{6},\frac{-17}{6})$	$(\frac{+1}{6},\frac{-23}{6})$	$(\frac{-5}{6},\frac{-29}{6})$
$\pi_{3;j,k}$	$\frac{+8}{3}$	$\frac{+5}{3}$	$\frac{+2}{3}$	$\frac{-1}{3}$	$\frac{-4}{3}$	$\frac{-7}{3}$
Series $\frac{+1}{6}$						
(j,k)	$(\frac{+11}{3},\frac{-1}{3})$	$(\frac{+8}{3},\frac{-4}{3})$	$(\frac{+5}{3},\frac{-7}{3})$	$(\frac{+2}{3},\frac{-10}{3})$	$(\frac{-1}{3},\frac{-13}{3})$	$(\frac{-4}{3},\frac{-16}{3})$
$\pi_{3;j,k}$	$\frac{+13}{6}$	$\frac{+7}{6}$	$\frac{+1}{6}$	$\frac{-5}{6}$	$\frac{-11}{6}$	$\frac{-17}{6}$
Series $\frac{+5}{6}$						
(j,k)	$(\frac{+13}{3},\frac{+1}{3})$	$(\frac{+10}{3},\frac{-2}{3})$	$(\frac{+7}{3},\frac{-5}{3})$	$(\frac{+4}{3},\frac{-8}{3})$	$(\frac{+1}{3},\frac{-11}{3})$	$(\frac{-2}{3},\frac{-14}{3})$
$\pi_{3;j,k}$	$\frac{+17}{6}$	$\frac{+11}{6}$	$\frac{+5}{6}$	$\frac{-1}{6}$	$\frac{-7}{6}$	$\frac{-13}{6}$

Table 14: $\pi^2_{j,k} = \frac{35}{4} = \frac{+5}{2}(\frac{+5}{2}+1)\hbar^2$, with $j-k=+6$

Series0						
(j,k)	$(\frac{+9}{2},\frac{-3}{2})$	$(\frac{+7}{2},\frac{-5}{2})$	$(\frac{+5}{2},\frac{-7}{2})$	$(\frac{+3}{2},\frac{-9}{2})$	$(\frac{+1}{2},\frac{-11}{2})$	$(\frac{-1}{2},\frac{-13}{2})$
$\pi_{3;j,k}$	$+2$	$+1$	0	-1	-2	-3
Series $\frac{+1}{3}$						
(j,k)	$(\frac{+29}{6},\frac{-7}{6})$	$(\frac{+23}{6},\frac{-13}{6})$	$(\frac{+17}{6},\frac{-19}{6})$	$(\frac{+11}{6},\frac{-25}{6})$	$(\frac{+5}{6},\frac{-31}{6})$	$(\frac{-1}{6},\frac{-37}{6})$
$\pi_{3;j,k}$	$\frac{+7}{3}$	$\frac{+4}{3}$	$\frac{+1}{3}$	$\frac{-2}{3}$	$\frac{-5}{3}$	$\frac{-8}{3}$
Series $\frac{+2}{3}$						
(j,k)	$(\frac{+31}{6},\frac{-5}{6})$	$(\frac{+25}{6},\frac{-11}{6})$	$(\frac{+19}{6},\frac{-17}{6})$	$(\frac{+13}{6},\frac{-23}{6})$	$(\frac{+7}{6},\frac{-29}{6})$	$(\frac{+1}{6},\frac{-35}{6})$
$\pi_{3;j,k}$	$\frac{+8}{3}$	$\frac{+5}{3}$	$\frac{+2}{3}$	$\frac{-1}{3}$	$\frac{-4}{3}$	$\frac{-7}{3}$
Series $\frac{+1}{6}$						
(j,k)	$(\frac{+14}{3},\frac{-4}{3})$	$(\frac{+11}{3},\frac{-7}{3})$	$(\frac{+8}{3},\frac{-10}{3})$	$(\frac{+5}{3},\frac{-13}{3})$	$(\frac{+1}{3},\frac{-16}{3})$	$(\frac{-2}{3},\frac{-19}{3})$
$\pi_{3;j,k}$	$\frac{+13}{6}$	$\frac{+7}{6}$	$\frac{+1}{6}$	$\frac{-5}{6}$	$\frac{-11}{6}$	$\frac{-17}{6}$
Series $\frac{+5}{6}$						
(j,k)	$(\frac{+16}{3},\frac{-2}{3})$	$(\frac{+13}{3},\frac{-5}{3})$	$(\frac{+10}{3},\frac{-8}{3})$	$(\frac{+7}{3},\frac{-11}{3})$	$(\frac{+4}{3},\frac{-14}{3})$	$(\frac{+1}{3},\frac{-17}{3})$
$\pi_{3;j,k}$	$\frac{+17}{6}$	$\frac{+11}{6}$	$\frac{+5}{6}$	$\frac{-1}{6}$	$\frac{-7}{6}$	$\frac{-13}{6}$

Remind: For all the above tables mentioned, when transformation

$$(j, k) \;\Rightarrow\; (k, j)$$

lead to

$$j - k = +\Delta \;\Rightarrow\; j - k = -\Delta \quad \text{and} \quad \pi^2_{j,k} = \pi^2_{k,j}, \quad \pi_{3,j,k} = \pi_{3,k,j}$$

VIII. Conclusions

This paper suggests a possibility of the existence of DN, Dark Neutrino ν_0 which is the superposition of Majorana Neutrino ν_{L} (CO $\frac{+3\hbar^2}{4}$) and Majorana Antineutrino $\bar{\nu}_{\mathrm{R}}$ (CO $\frac{-3\hbar^2}{4}$). ν_0 is a charge neutrial particle that could possess CO $0\,\hbar^2$ and half-integer eigenvalues $\frac{\pm1\hbar}{2}\ \frac{\pm3\hbar}{2},\ \frac{\pm5\hbar}{2},\ldots$ of the third component of its own, further we are in a dilemma as to judge the physical certification of spin particle ν_0, to be a Boson or to be a Fermion ?

The only most plsusible explanation is that ν_0 is a kind of dark spin particles.

The tables of section VII are the fundamental representations of spin particles in STS, which are heuristic and useful, the examples are given below:

The spin topological coordinates (j, k) of Fermion B1 of Table 7 are just the flavour quantum numbers of quarks in STS (isospin I=$\hbar/2$) which is the last column of Table 2 in [3]. The (j, k) of regular dark spin particles of Table 8 lead to the colour spectral line array of u quark, $u_{\mathrm{RGB}} \equiv (u_{\mathrm{R}}, u_{\mathrm{G}}, u_{\mathrm{B}})$=($\frac{+2}{3}$, $\frac{+5}{3}$, $\frac{+11}{3}$) [3], then the definition of CSDF, Colour Spectrum Diagram of Flavour is ascertained.

The goal of this paper is mainly to explore the math properties of CO, Casimir Operator of angular momentum of spin particles in STS, and to show the roles of CO in distinguishing the identities between particles and antiparticles of particle physics. The spin-coupling of the third components of two spin particles, multi-body spin particles are rather complex, the difficulty can be seen from the discussions in section V and VI, here we see to ensure the harmony between the consistency of math discipline and reality of physical spin, is not an easy job. There are some critical concepts left are requested to be introduced, the relevant topics will be presented later.

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