Global Journals Inc.

Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 21 Issue 3 Version 1.0 Year 2021
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

A Modified Taylor Series Expansion Method for Solving Fredholm Integro-Differential Equations

By Itthithep Navarasuchitr

Srinakharinwirot University

Abstract- In this paper, we use a modified Taylor series expansion method for solving the linear Fredholm integro-differential equations. This method transfroms the equation to linear system equations that can be solved easily with reduced row echelon method. Finally, we show the efficiency of this method with numerical examples by comparing the approximate solutions with exact solutions.

Keywords: fredholm integro-differential equations, numerical method, taylor, series.

GJSFR-F Classification: MSC 2010: 45D05; 45E10; 65M20

AMODIFIEDTAYLORSERIESEXPANSIONMETHODFORSOLVINGFREDHOLMINTEGRODIFFERENTIALEQUATIONS

A Modified Taylor Series Expansion Method for Solving Fredholm Integro-Differential Equations

Itthithep Navarasuchitr

Abstract- In this paper, we use a modified Taylor series expansion method for solving the linear Fredholm integro-differential equations. This method transfroms the equation to linear system equations that can be solved easily with reduced row echelon method. Finally, we show the efficiency of this method with numerical examples by comparing the approximate solutions with exact solutions.

Keywords: fredholm integro-differential equations, numerical method, taylor, series.

I. Introduction

Mathematical modeling of real-life problems usually results in functional equations, e.g. differential equations, integro-differential equations, stochastic equations and others. Integro-differential equation is a hybrid of integral and differential equations which have found extensive applications in sciences and engineering.

In particular, integro-differential equations arise in fluid dynamics, biological models and chemical kinetics. The analytical solutions of some integro-differential equations (IDEs) cannot be found, thus numerical method are required. The numerical methods for linear integro-differential equations have been extensively studied by many authors [4, 6, 9]. There is an alternative method for approximating the solution of IDEs that is a Taylor series expansion. The Taylor series expansion is one of the methods used to calculate the solution of differential equations(DEs) and integral equations (IEs) since it is easy to compute and efficient [1–3, 5, 9]. Those who started to use taylor in solving IEs were Y.Ren et al. [10] for Fredholm integral equation and Pallop et al. [11] have modified Y.ren's method for more accurate results and used for wider class of Fredholm integral equation and Itthithep et al. [8] used this method for the solution of Volterra integro-differential equation.

In this research, we use [8] methods to approximate the solution of Fredholm integro-differential equations (FIDEs), given in the form

$$y'(x) - \int_a^b k(x,t)y(t)\mathbf{d}t = f(x), \quad y(0) = y_0. \tag{1}$$

where the functions $f(x)$ and the kernel $k(x,t)$ are known.

Author: *Department of Mathematics, Srinakharinwirot University, Bangkok 10110, Thailand. e-mail: itthithep@g.swu.ac.th*

Ref

10. Y. Ren, B. Zhang, H. Qiao, A simple Taylor-series expansion for a class of second kind integral equations, J. Comput. Appl. Math., Vol.110(1999), 15-24.

II. Modified Taylor-Series Expansion Method

We consider the FIDE in form (1)

$$y'(x) - \int_a^b k(x,t)y(t)\mathbf{d}t = f(x), \quad y(0) = y_0.$$

The Taylor series approximation can be made for the solution $y(t)$ in the (1):

$$y(t) \approx y(x) + y'(x)(t-x) + \frac{y''(x)}{2!}(t-x)^2 + \frac{y'''(x)}{3!}(t-x)^3 + \cdots + \frac{y^{(n)}}{n!}(x)(t-x)^n. \tag{2}$$

Substituting (2) for $y(t)$ in the integral in (1),

$$\begin{aligned} y'(x) - \int_a^b k(x,t)\Big[y(x) + y'(x)(t-x) + \frac{y''(x)}{2!}(t-x)^2 + \cdots \\ + \frac{y^{(n)}(x)}{n!}(t-x)^n\Big]\,\mathbf{d}t = f(x). \end{aligned} \tag{3}$$

We obtain,

$$\begin{aligned} \Big[-\int_a^b k(x,t)\,\mathbf{d}t\Big]y(x) + \Big[1 - \int_a^b k(x,t)(t-x)\,\mathbf{d}t\Big]y'(x) + \cdots \\ + \Big[-\frac{1}{n!}\int_a^b k(x,t)(t-x)^n\,\mathbf{d}t\Big]y^{(n)}(x) \approx f(x). \end{aligned} \tag{4}$$

Next, we integrate both sides of (1) with respect to t from 0 to x,

$$\int_0^x y'(t)\,\mathbf{d}t - \int_0^x \int_a^b k(s,t)y(t)\,\mathbf{d}t\mathbf{d}s = \int_0^x f(t)\,\mathbf{d}t. \tag{5}$$

Application of integration by parts yields

$$\begin{aligned} \int_0^x P_k(t)y^{(k)}(t)\,\mathbf{d}t = P_k(t)y^{(k-1)}(t)|_0^x - P_k'(t)y^{(k-2)}(t)|_0^x + P_k''(t)y^{(k-3)}(t)|_0^x + \cdots \\ + (-1)^{(k-1)}P_k^{(k-1)}(t)y(t)|_0^x + (-1)^{(k)}\int_0^x P_k^{(k)}(t)y(t)\,\mathbf{d}t. \end{aligned} \tag{6}$$

Where $P_k(t) = 1$ and $k = 1$. So $P_k^{(i)}(t) = 0$ for $i = 1, 2, \ldots, k$.
So that,

$$\begin{aligned} \int_0^x y'(t)\,\mathbf{d}t &= y(t)|_0^x \\ &= y(x) - y(0) \end{aligned} \tag{7}$$

Notes

Substituting (7) for $\int_0^x y'(t)\,\mathbf{d}t$ in the integral in (5), we obtain

$$y(x) - y(0) - \int_0^x \int_a^b k(s,t)y(t)\,\mathbf{d}t\mathbf{d}s = \int_0^x f(t)\,\mathbf{d}t. \tag{8}$$

Similarly, Substitute $y(t)$ is replaced by $y(x)$ in (8) by the right sides of (2) to obtain

Notes

$$\begin{aligned} y(x) - y(0) - \int_0^x \int_a^b k(s,t)\Big[y(x) + y'(x)(t-x) + \frac{y''(x)}{2!}(t-x)^2 + \cdots \\ + \frac{y^{(n)}(x)}{n!}(t-x)^n\Big]\,\mathbf{d}t\mathbf{d}s = \int_0^x f(t)\,\mathbf{d}t. \end{aligned} \tag{9}$$

We obtain,

$$\begin{aligned} &\left[1 - \int_0^x \int_a^b k(s,t)\,\mathbf{d}t\,\mathbf{d}s\right]y(x) - \left[\int_0^x \int_a^b k(s,t)(t-x)\,\mathbf{d}t\,\mathbf{d}s\right]y'(x) \\ &- \left[\frac{1}{2!}\int_0^x \int_a^b k(s,t)(t-x)^2\,\mathbf{d}t\,\mathbf{d}s\right]y''(x) - \cdots \\ &- \left[\frac{1}{n!}\int_0^x \int_a^b k(s,t)(t-x)^n\,\mathbf{d}t\,\mathbf{d}s\right]y^{(n)}(x) = y(0) + \int_0^x f(t)\,\mathbf{d}t. \end{aligned} \tag{10}$$

Next, we differentiate both sides of (1) n times, one obtains

$$y''(x) - \frac{\partial}{\partial x}\left[\int_a^b k(x,t)y(t)\,\mathbf{d}t\right] = f'(x), \tag{11}$$

$$y'''(x) - \frac{\partial^2}{\partial x^2}\left[\int_a^b k(x,t)y(t)\,\mathbf{d}t\right] = f''(x), \tag{12}$$

$$\vdots$$

$$y^{(n)}(x) - \frac{\partial^n}{\partial x^n}\left[\int_a^b k(x,t)y(t)\,\mathbf{d}t\right] = f^{(n-1)}(x). \tag{13}$$

Using Leibnitz rule, we find that

$$\frac{\partial^n}{\partial x^n}\left[\int_a^b k(x,t)y(t)\,\mathbf{d}t\right] = \int_a^b \frac{\partial^n}{\partial x^n}\Big[k(x,t)\Big]y(t)\,\mathbf{d}t = \int_a^b \Big[k_x^{(n)}(x,t)\Big]y(t)\,\mathbf{d}t$$

Therefore,

$$y''(x) - \int_a^b k'_x(x,t)y(t)\,\mathbf{d}t = f'(x), \tag{14}$$

$$y'''(x) - \int_a^b k''_x(x,t)y(t)\,\mathbf{d}t = f''(x), \tag{15}$$

$$\vdots$$

$$y^{(n)}(x) - \int_a^b k_x^{(n-1)}(x,t)y(t)\,\mathbf{d}t = f^{(n-1)}(x). \tag{16}$$

Notes

Substitute $y(t)$ is replaced by $y(x)$ in (14)-(16) by the right sides of (2) to obtain

$$y''(x) - \int_a^b k'_x(x,t)\Big[y(x) + y'(x)(t-x) + \frac{y''(x)}{2!}(t-x)^2 + \cdots + \frac{y^{(n)}(x)}{n!}(t-x)^n\Big]\,\mathbf{d}t = f'(x), \tag{17}$$

$$y'''(x) - \int_a^b k''_x(x,t)\,ds\Big[y(x) + y'(x)(t-x) + \frac{y''(x)}{2!}(t-x)^2 + \cdots + \frac{y^{(n)}(x)}{n!}(t-x)^n\Big]\,\mathbf{d}t = f''(x), \tag{18}$$

$$\vdots$$

$$y^{(n)}(x) - \int_a^b k_x^{(n)}(x,t)\Big[y(x) + y'(x)(t-x) + \frac{y''(x)}{2!}(t-x)^2 + \cdots + \frac{y^{(n)}(x)}{n!}(t-x)^n\Big]\,\mathbf{d}t = f^{(n-1)}(x). \tag{19}$$

We obtain,

$$\Big[-\int_a^b k'_x(x,t)\,\mathbf{d}t\Big]y(x) + \Big[-\int_a^b k'_x(x,t)(t-x)\,\mathbf{d}t\Big]y'(x) + \Big[1 - \frac{1}{2!}\int_a^b k'_x(x,t)(t-x)^2\,\mathbf{d}t\Big]y''(x) + \cdots + \Big[-\frac{1}{n!}\int_a^b k'_x(x,t)(t-x)^n\,\mathbf{d}t\Big]y^{(n)}(x) = f'(x) \tag{20}$$

$$\Big[-\int_a^b k''_x(x,t)\,\mathbf{d}t\Big]y(x) + \Big[-\int_a^b k''_x(x,t)(t-x)\,\mathbf{d}t\Big]y'(x) + \Big[-\frac{1}{2!}\int_a^b k''_x(x,t)(t-x)^2\,\mathbf{d}t\Big]y''(x) + \cdots + \Big[-\frac{1}{n!}\int_a^b k''_x(x,t)(t-x)^n\,\mathbf{d}t\Big]y^{(n)}(x) = f''(x) \tag{21}$$

$$\ddots$$

$$\left[-\int_a^b k_x^{(n-1)}(x,t)\,\mathbf{d}t\right]y(x)+\left[-\int_a^b k_x^{(n-1)}(x,t)(t-x)\,\mathbf{d}t\right]y'(x)$$

$$+\left[-\frac{1}{2!}\int_a^b k_x^{(n-1)}(x,t)(t-x)^2\,\mathbf{d}t\right]y''(x)+\cdots$$

$$+\left[1-\frac{1}{n!}\int_a^b k_x^{(n-1)}(x,t)(t-x)^n\,\mathbf{d}t\right]y^{(n)}(x)=f^{(n-1)}(x) \tag{22}$$

Combining equations (4), (10), (20)-(22) , we obtain

$$\begin{pmatrix} 1-\int_0^x\int_a^b k(s,t)\,\mathbf{d}t\,\mathbf{d}s & -\int_0^x\int_a^b k(s,t)(t-x)\,\mathbf{d}t\,\mathbf{d}s & \cdots & -\frac{1}{n!}\int_0^x\int_a^b k(s,t)(t-x)^n\,\mathbf{d}t\,\mathbf{d}s \\ -\int_a^b k(x,t)\,\mathbf{d}t & 1-\int_a^b k(x,t)(t-x)\,\mathbf{d}t & \cdots & -\frac{1}{n!}\int_a^b k(x,t)(t-x)^n\,\mathbf{d}t \\ -\int_a^b k'_x(x,t)\,\mathbf{d}t & -\int_a^b k'_x(x,t)(t-x)\,\mathbf{d}t & \cdots & -\frac{1}{n!}\int_a^b k'_x(x,t)(t-x)^n\,\mathbf{d}t \\ -\int_a^b k''_x(x,t)\,\mathbf{d}t & -\int_a^b k''_x(x,t)(t-x)\,\mathbf{d}t & \cdots & -\frac{1}{n!}\int_a^b k''_x(x,t)(t-x)^n\,\mathbf{d}t \\ \vdots & \vdots & \ddots & \vdots \\ -\int_a^b k_x^{(n-1)}(x,t)\,\mathbf{d}t & -\int_a^b k_x^{(n-1)}(x,t)(t-x)\,\mathbf{d}t & \cdots & 1-\frac{1}{n!}\int_a^b k_x^{(n-1)}(x,t)(t-x)^n\,\mathbf{d}t \end{pmatrix}\times$$

$$\begin{pmatrix} y(x) \\ y'(x) \\ y''(x) \\ y'''(x) \\ \vdots \\ y^{(n)}(x) \end{pmatrix}=\begin{pmatrix} y(0)+\int_0^x f(t)\,\mathbf{d}t \\ f(x) \\ f'(x) \\ f''(x) \\ \vdots \\ f^{(n-1)}(x) \end{pmatrix} \tag{23}$$

Equation (23) becomes a linear systems of $n+1$ algebraic equation for $n+1$ unknowns $y(x), y'(x), y''(x), \dots, y^{(n)}(x)$, which can be solved easily use of initial condition.

III. Numerical Examples

We present in this section numerical result for some examples to show efficient and accuracy of the modified Taylor-series expansion method, and the corresponding absolute errors between their values as $e_n(x) = |exact_n(x) - app_n(x)|$.

Example 3.1. Consider

$$y'(x)-\int_0^1 (xt)y(t)\,\mathbf{d}t = 3+6x, \qquad y(0)=0 \tag{24}$$

such that $k(x,t)=xt$, $f(x)=3+6x$, $a=0$, $b=1$ and exact solution is $y(x)=4x^2+3x$.

Let $n = 2$. We apply equation (23) to approach the equation (24) that is,

$$\begin{pmatrix} 1-\int_0^x\int_0^1 (st)\,\mathbf{d}t\,\mathbf{d}s & -\int_0^x\int_0^1 (st)(t-x)\,\mathbf{d}t\,\mathbf{d}s & -\frac{1}{2}\int_0^x\int_0^1 (st)(t-x)^2\,\mathbf{d}t\,\mathbf{d}s \\ -\int_0^1 (xt)\,\mathbf{d}t & 1-\int_0^1 (xt)(t-x)\,\mathbf{d}t & -\frac{1}{2}\int_0^1 (xt)(t-x)\,\mathbf{d}t \\ -\int_0^1 (t)\,\mathbf{d}t & -\int_0^1 (t)(t-x)\,\mathbf{d}t & 1-\frac{1}{2}\int_0^1 (t)(t-x)\,\mathbf{d}t \end{pmatrix} \times$$

$$\begin{pmatrix} y(x) \\ y'(x) \\ y''(x) \end{pmatrix} = \begin{pmatrix} \int_0^x (3+6t)\,\mathbf{d}t \\ 3+6x \\ 6 \end{pmatrix}. \tag{25}$$

We obtain,

$$\begin{pmatrix} 1-\frac{1}{4}x^2 & -\frac{1}{6}x^2+\frac{1}{4}x^3 & -\frac{1}{16}x^2+\frac{1}{6}x^3-\frac{1}{8}x^4 \\ -\frac{1}{2}x & 1-\frac{1}{3}x+\frac{1}{2}x^2 & -\frac{1}{8}x+\frac{1}{3}x^2-\frac{1}{4}x^3 \\ -\frac{1}{2} & -\frac{1}{3}+\frac{1}{2}x & \frac{7}{8}+\frac{1}{3}x-\frac{1}{4}x^2 \end{pmatrix} \times$$

$$\begin{pmatrix} y(x) \\ y'(x) \\ y''(x) \end{pmatrix} = \begin{pmatrix} 3x^2+3x \\ 3+6x \\ 6 \end{pmatrix}. \tag{26}$$

And approximation is

$$y(x) = 4x^2 + 3x.$$

Table 1: Numerical approximation for $y(x)$ in Example 3.1 with $n = 2$.

x	$y(x)$		Absolute error	
	Exact	Our approx.	Exact	Our approx.
0.0	0.00000	0.00000	0.00000	0.00000
0.1	0.34000	0.34000	0.00000	0.00000
0.2	0.76000	0.76000	0.00000	0.00000
0.3	1.26000	1.26000	0.00000	0.00000
0.4	1.84000	1.84000	0.00000	0.00000
0.5	2.50000	2.50000	0.00000	0.00000
0.6	3.24000	3.24000	0.00000	0.00000
0.7	4.06000	4.06000	0.00000	0.00000
0.8	4.96000	4.96000	0.00000	0.00000
0.9	5.94000	5.94000	0.00000	0.00000
1.0	7.00000	7.00000	0.00000	0.00000

Notes

Notes

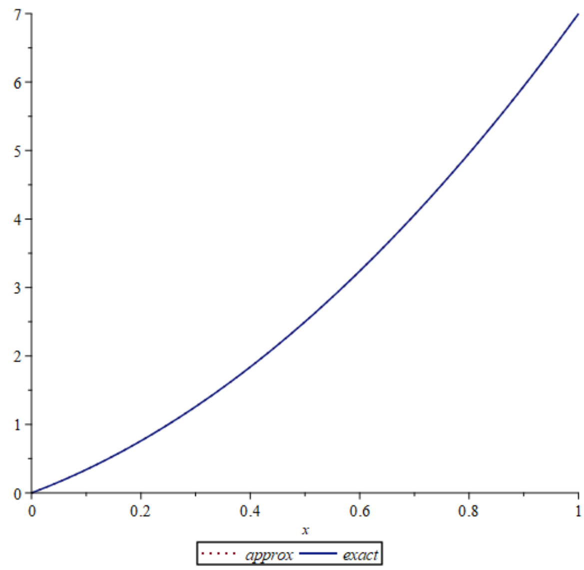

Figure 1: Comparison of approximations and exact solution with $n = 2$.

Example 3.2. As a second example, we solve the following Fredholm integro-differential equation

$$y'(x) - \int_{-1}^{1} (1 - x^2t^2) y(t)\, dt = 4, \qquad y(0) = -2 \tag{27}$$

such that $k(x,t) = 1 - x^2t^2$, $f(x) = 4$, $a = -1$, $b = 1$ and exact solution is $y(x) = -2 + \frac{4}{9}x^3$.

Let $n = 3$. We apply equation (23) to approach the equation (27) that is,

$$\begin{pmatrix} 1 - \int_0^x \int_{-1}^1 (1 - s^2t^2)\, dt\, ds & \cdots & -\frac{1}{3!} \int_0^x \int_{-1}^1 (1 - s^2t^2)(t - x)^3\, dt\, ds \\ -\int_{-1}^1 (1 - x^2t^2)\, dt & \cdots & -\frac{1}{3!} \int_{-1}^1 (1 - x^2t^2)(t - x)^3\, dt \\ -\int_{-1}^1 (-2t^2x)\, dt & \cdots & -\frac{1}{3!} \int_{-1}^1 (-2t^2x)(t - x)^3\, dt \\ -\int_{-1}^1 (-2t^2)\, dt & \cdots & 1 - \frac{1}{3!} \int_{-1}^1 (-st^2)(t - x)^3\, dt \end{pmatrix} \times$$

$$\begin{pmatrix} y(x) \\ y'(x) \\ y''(x) \\ y'''(x) \end{pmatrix} = \begin{pmatrix} -2 + \int_0^x (4)\, dt \\ 4 \\ 0 \\ 0 \end{pmatrix}. \tag{28}$$

We obtain,

$$\begin{pmatrix} 1 + \frac{2}{9}x^3 - 2x & -\frac{2}{9}x^4 + 2x^2 & \frac{1}{9}x^5 - \frac{14}{15}x^3 - \frac{1}{3}x & -\frac{1}{27}x^6 + \frac{4}{15}x^4 + \frac{1}{3}x^2 \\ \frac{2}{3}x^2 - 2 & 1 - \frac{2}{3}x^3 + 2x & \frac{1}{3}x^4 - \frac{4}{5}x^2 - \frac{1}{3} & -\frac{1}{9}x^5 + \frac{2}{15}x^3 + \frac{1}{3}x \\ \frac{4}{3}x & -\frac{4}{3}x^2 & \frac{2}{3}x^3 + \frac{2}{5}x + 1 & -\frac{2}{9}x^4 - \frac{2}{5}x^2 \\ \frac{4}{3} & -\frac{4}{3}x & \frac{2}{3}x^2 + \frac{2}{5} & 1 - \frac{2}{9}x^3 - \frac{2}{5}x \end{pmatrix} \times$$

Global Journal of Science Frontier Research (F) Volume XXI Issue III Version I Year 2021

$$\begin{pmatrix} y(x) \\ y'(x) \\ y''(x) \\ y^{(3)}(x) \end{pmatrix} = \begin{pmatrix} -2+4x \\ 4 \\ 0 \\ 0 \end{pmatrix}. \tag{29}$$

And approximation is

$$y(x) = -2 + \frac{4}{9}x^3.$$

Example 3.3. Next, we discuss the Fredholm integro-differential equation

$$y'(x) - \int_0^{\frac{\pi}{2}} (t)y(t)\,dt = -1 + \cos(x), \qquad y(0) = 0 \tag{30}$$

such that $k(x,t) = t$, $f(x) = -1 + \cos(x)$, $a = 0$, $b = \frac{\pi}{2}$ and exact solution is $y(x) = \sin(x)$.

Table 2: Numerical approximation for $y(x)$ in Example 3.2 with $n = 3$.

x	$y(x)$		Absolute error	
	Exact	Our approx.	Exact	Our approx.
-1.0	-2.44444	-2.44444	0.00000	0.00000
-0.8	-2.22755	-2.22755	0.00000	0.00000
-0.6	-2.09600	-2.09600	0.00000	0.00000
-0.4	-2.02844	-2.02844	0.00000	0.00000
-0.2	-2.00356	-2.00356	0.00000	0.00000
0.0	-2.00000	-2.00000	0.00000	0.00000
0.2	-1.99644	-1.99644	0.00000	0.00000
0.4	-1.97156	-1.97156	0.00000	0.00000
0.6	-1.90400	-1.90400	0.00000	0.00000
0.8	-1.77244	-1.77244	0.00000	0.00000
1.0	-1.55556	-1.55556	0.00000	0.00000

Notes

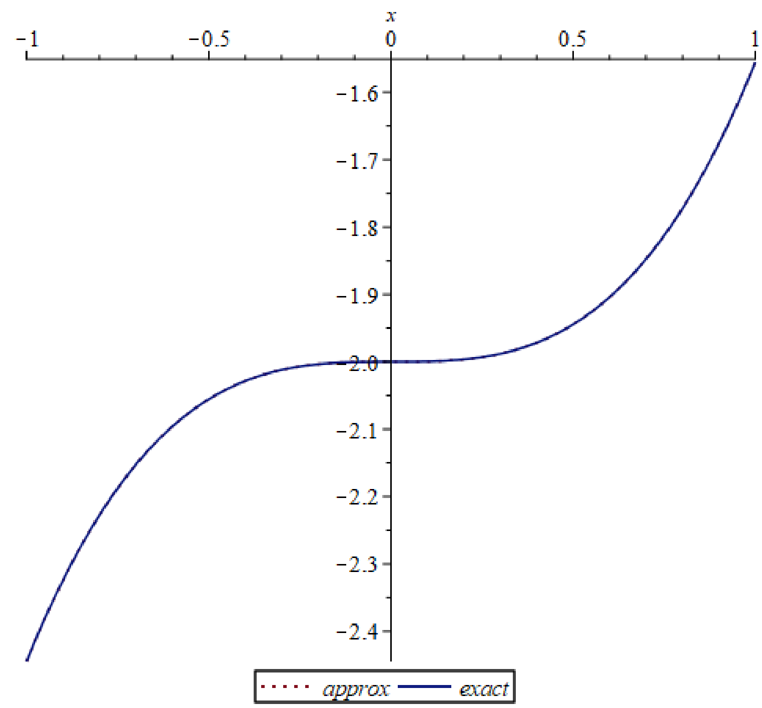

Figure 2: Comparison of approximations and exact solution with $n = 3$.

Let $n = 8$. We apply equation (23) to approach the equation (30) that is,

$$\begin{pmatrix} 1 - \int_0^x \int_0^{\frac{\pi}{2}} (t)\, dt\, ds & \cdots & -\frac{1}{8!} \int_0^x \int_0^{\frac{\pi}{2}} (t)(t-x)^8\, dt\, ds \\ -\int_0^{\frac{\pi}{2}} (t)\, dt & \cdots & -\frac{1}{8!} \int_0^{\frac{\pi}{2}} (t)(t-x)^8\, dt \\ 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix} \times$$

$$\begin{pmatrix} y(x) \\ y'(x) \\ y''(x) \\ \vdots \\ y^{(8)}(x) \end{pmatrix} = \begin{pmatrix} \int_0^x (-1+\cos(t))\, dt \\ -1+\cos(x) \\ -\sin(x) \\ \vdots \\ \sin(x) \end{pmatrix} \tag{31}$$

Table 3: Numerical approximation for $y(x)$ in Example 3.3 with $n = 8$.

x	$y(x)$		Absolute error	
	Exact	Our approx.	Exact	Our approx.
0	0.00000	0.00000	0.00000	0.00000
$\frac{\pi}{20}$	0.15643	0.15644	0.00000	0.00000
$\frac{2\pi}{20}$	0.30901	0.30902	0.00000	0.00000
$\frac{3\pi}{20}$	0.45400	0.45400	0.00000	0.00000
$\frac{4\pi}{20}$	0.58778	0.58778	0.00000	0.00000
$\frac{5\pi}{20}$	0.70711	0.70711	0.00000	0.00000
$\frac{6\pi}{20}$	0.80902	0.80902	0.00000	0.00000
$\frac{7\pi}{20}$	0.89101	0.89101	0.00000	0.00000
$\frac{8\pi}{20}$	0.95106	0.95106	0.00000	0.00000
$\frac{9\pi}{20}$	0.98769	0.98769	0.00000	0.00000
$\frac{10\pi}{20}$	1.00000	1.00000	0.00000	0.00000

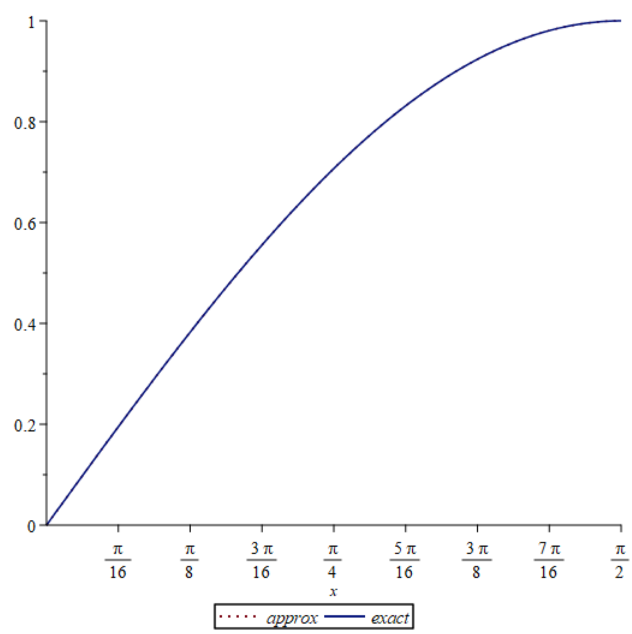

Figure 3: Comparison of approximations and exact solution with $n = 8$.

IV. Conclusion

As illustrated in the examples of this paper, the modified Taylor-series method is a powerful procedure for solving FIDEs. Using the proposed method in solving integral equation shows the high capability of this method compared to other methods.

Acknowledgement

The authors would like to thank referees for their valuable comments and suggestions. This work is supported by Srinakharinwirot University, Thailand (grant number 017/2563).

Notes

Notes

References Références Referencias

1. Y. Huang, X.-F.Li, Approximate solution of a class of linear integro differential equation by Taylor expansion method, International Journal of Computer Mathematics Vol.87(6)(2010), 1277-1288.
2. X.-F.Li and M.Fang, Modified method for determining an approximate solution of the Fredholm-Volterra integral equations by Taylor's expansion, Int. J. Comp. Math. Vol.83 (2006),637-64.
3. S.Nas, S.Yalcinbas and M.Sezer, A Taylor polynomial approach for solving high-order linear Fredholm integro-differential equations, Int. J. Math. Educ. Sci. Technol.Vol. 31 (2000), 213-225.
4. Gökçe Yıldız, Gültekin Tınaztepe and Mehmet Sezer, Bell Polynomial Approach for the Solutions of Fredholm Integro Differential Equations with Variable Coefficients, CMES, vol.123(3)(2020), 973-993.
5. S.Yalcinbas and M.Sezer, The approximate solution of high-order linear Volterra-Fredholm integro-differential equations in terms of Taylor polynomials, Appl. Math. Comp. Vol.112 (2000), 291-308.
6. Z. Pashazadeh Atabakan, A. Kazemi Nasab, and A. KJlJçman, On Solution of Fredholm Integrodifferential Equations Using Composite Chebyshev Finite Difference Method, Abstract and Applied Analysis, Vol.2013, Article ID 694043.
7. Abdul-Majid Wazwaz, Linear and Nonlinear Integral Equations techniques and Applications, Higher Education Press, Beijing and Springer-Verlag Berlin Hei- delberg, 2011.
8. I. Navarasuchitr, P. Huabsomboon, H. Kaneko, Efficient Numerical Technique For Solving Integral Equations, Thai Journal of Mathematics.,Vol.19(1)(2021),261-270.
9. S.M. Hosseini, S. Shahmorad, Numerical solution of a class of integrodifferential equations by the Tau method with an error, Appl. Math. Comput., Vol. 136(2003), 559-570.
10. Y. Ren, B. Zhang, H. Qiao, A simple Taylor-series expansion for a class of second kind integral equations, J. Comput. Appl. Math., Vol.110(1999), 15-24.
11. P. Huabsomboon, B. Novaprateep, H. Kaneko, On Taylor-series expansion techniques for the second kind integral equations, J. Comput. Appl. Math., Vol. 234(2010), 1446-1472.