



GLOBAL JOURNAL OF SCIENCE FRONTIER RESEARCH: A
PHYSICS AND SPACE SCIENCE
Volume 21 Issue 6 Version 1.0 Year 2021
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Dark Matter and Real-Particle Field Theory

By Zhong-Cheng Liang

Nanjing University of Posts and Telecommunications

Abstract- Based on real-particle field theory, this research demonstrates that dark matter comprises elastic electrons with a full cosmic background. In the real-particle field theory, real particles are elastic particles with both mass and volume. Real particles have three independent motion modes and two symmetrical interactions. The evolution of the real-particle field follows a set of Poisson equations. The theory shows that the electric and magnetic potentials represent electronic interactions of mass attraction and motion repulsion. The electromagnetic and dark-matter fields are essentially elastic electron fields. The laws of mechanics, gravitation, and electromagnetism of classical physics can be inferred from the real-particle field theory. In the electron field, electronic clusters are photons with the energy proportional to vibration frequency. The mechanic features of photons can be characterized by their volume, mass, and elasticity. Furthermore, the radiation of electronic clusters follows Planck's law, which indicates that dark matter has a uniform density and no photonic current in the cosmic background. It is shown that matter essentially comprises discrete particles, and the matter field is merely a statistical convolution effect of a large number of particles. Consequently, dark matter particles contribute to the structural formation of all matter in the universe.

Keywords: *dark matter, elastic particles, photons, interactions, unified field.*

GJSFR-A Classification: *FOR Code: 020199*



DARKMATTERANDREALPARTICLEFIELDTHEORY

Strictly as per the compliance and regulations of:



Dark Matter and Real-Particle Field Theory

Zhong-Cheng Liang

Abstract- Based on real-particle field theory, this research demonstrates that dark matter comprises elastic electrons with a full cosmic background. In the real-particle field theory, real particles are elastic particles with both mass and volume. Real particles have three independent motion modes and two symmetrical interactions. The evolution of the real-particle field follows a set of Poisson equations. The theory shows that the electric and magnetic potentials represent electronic interactions of mass attraction and motion repulsion. The electromagnetic and dark-matter fields are essentially elastic electron fields. The laws of mechanics, gravitation, and electromagnetism of classical physics can be inferred from the real-particle field theory. In the electron field, electronic clusters are photons with the energy proportional to vibration frequency. The mechanic features of photons can be characterized by their volume, mass, and elasticity. Furthermore, the radiation of electronic clusters follows Planck's law, which indicates that dark matter has a uniform density and no photonic current in the cosmic background. It is shown that matter essentially comprises discrete particles, and the matter field is merely a statistical convolution effect of a large number of particles. Consequently, dark matter particles contribute to the structural formation of all matter in the universe.

Keywords: dark matter, elastic particles, photons, interactions, unified field.

I. INTRODUCTION

Modern astrophysical and cosmological observations have shown that the motion of galaxies deviates significantly from the predictions of the Newtonian and Einsteinian gravitation theories. Therefore, it is necessary to hypothesize the existence of invisible dark matter in the universe, which can compensate for the gravitational effects of galactic motion [1–5]. It has been estimated that only approximately five percent of the universe is visible “bright matter,” while the rest is dark matter and dark energy, which are invisible and intangible (called dark phenomenon) [2]. Since the dark matter hypothesis was suggested, dark matter has been investigated based on various theories [3, 4]. However, because dark matter is considered to interact only gravitationally and not via electromagnetic fields, it is difficult to detect. So far, despite the deployment of expensive equipment to detect dark matter, no reliable observational evidence has been provided. The null result of observations raises questions regarding the existence of dark matter—with gravity but no electrical effect—and the reliability and

completeness of the current gravitation theory, including Newtonian gravitational theory and Einsteinian general relativity. Some researchers insist that dark phenomenon can be explained by modifying the current theory without assuming the existence of dark matter [5, 6]. However, most researchers support the existence of a hypothetical dark matter. In terms of extending the current gravitational theory, the research trend is to combine relativity and quantum theories to develop a unified field theory [1, 2, 5].

In axiomatic set theory, reliability means that a logical system is self-consistent and there is no contradiction; completeness means that all propositions in the logical system can be proved or falsified. The Newtonian gravitational theory is a limit-case approximation of general relativity for a low speed and weak field. The planets in the solar system follow the inverse-square law of gravitation. However, at the galactic scale, galactic rotation deviates from the prediction of Newton's gravitational theory, whereas general relativity can predict the rotation curves under appropriate conditions. At the scale of galaxy clusters, we have to move beyond Newtonian physics and use a relativistic theory and sometimes also a quantum theory. At the cosmological scale, researchers have focused on developing quantum gravity theories, such as the superstring theories. Nevertheless, the gravitational theory remains an inconsistent and incomplete logical system, although the theory has become profound and difficult to understand. This casts doubt on whether gravitation is really that complicated and whether a more reasonable logical framework exists. To address these concerns, we need to scrutinize current physics theories at their foundation.

The core concepts of physics include matter, particles, space, time, motion, energy, and interaction. The differences among the core concepts constitute the basis for the classification of different theoretical systems. Recently, real physics theory, which differs from classical and modern physics, was proposed based on the principle of objectivity. Real physics includes the motion state theory [7–10], statistical thermodynamics [7, 8, 11], and real-particle field theory [7, 8, 12, 13], which is a unified theory of gravitational and electromagnetic interactions. Table I summarizes the core concepts of these three physics theories, where the concepts of real physics are proposed as physical axioms.

Real physics and modern physics theories differ in two aspects. First, unlike modern physics where

Author: College of Electronic and Optical Engineering & College of Microelectronics, Nanjing University of Posts and Telecommunications. Nanjing 210046, China. e-mail: zcliang@njupt.edu.cn



matter is a continuous field, in real physics, matter is composed of discrete particles. Second, in real physics, space and time are independent of each other. The former proposition is of decisive significance for eliminating the opposition between general relativity and

quantum theory [14]. The latter proposition eliminates relativity-induced conceptual confusion, such as between space and time, mass and energy, and matter and spacetime [2]. However, these two propositions are completely consistent with classical mechanics.

Table I: Core concepts of three physics theories

	Classical physics	Modern physics	Real physics
Matter	Discrete particle	Continuous field	Discrete particle
Particle	Point-like geometry, mass conservation, zero volume	Wave-like exciton, mass non-conservation, indefinite volume	Body-like elastomer, mass conservation, finite volume
Space	Absolute-empty, 3-dimensional space	Energy-suffused, 4-dimensional spacetime	Particle-filled, 3-dimensional space
Time	Math reversible	Math reversible	Math irreversible
Motion	Translation mode	Vibration mode	Translation mode, vibration mode, rotation mode
Energy	Motion attribute	Matter attribute	Motion attribute
Interaction	Gravitational force, electromagnetic force	Gravitational force, electromagnetic force, weak force, strong force	Mass attraction, motion repulsion

There are three main differences between real physics and classical physics theories. First, the absolute empty space described by Newton is modified to a real space filled with particles. Second, Newtonian point-like particles are modified to elastic real particles. Third, motion repulsion is added to Newtonian gravitational theory. The first correction considers the existence of dark matter between stars. The second correction helps determine the species of dark matter particles. The third correction unifies gravitational and electromagnetic interactions. This paper introduces real-particle theory and presents analyses of the properties of the real-particle and dark matter fields.

II. REAL-PARTICLE THEORY

a) Theoretical model

Real physics establishes physical axioms and basic principles and involves the conversion of physical concepts into mathematical forms.

i. Real quantity

In real physics, the real quantity, q , refers to the physical quantity defined in terms of real numbers in the following form

$$q = q_s \cdot \tilde{q}; \quad (|q| < \infty, 0 < q_s < \infty) \quad (1)$$

where q_s is a scale factor, and \tilde{q} is a digit factor. Scales are the measures and identifiers of physical quantities and are uniform in space. The significance of introducing real quantities is to provide a new quantitative method for physics theories. In Section II B,

it is demonstrated that the essence of scale is quantum, and their semantics are equivalent.

ii. Real space

Real space is a three-dimensional Euclidean space full of particles. The position in real space is represented by the position vector as

$$r = r_s \cdot \tilde{r} = \overrightarrow{OP} = (x, y, z); \quad \tilde{r} = (\tilde{x}, \tilde{y}, \tilde{z}). \quad (2)$$

The scale r_s is called a space quantum and the digit \tilde{r} is a digital vector. The volume quantum $V_s = r_s^3$ is called a space cell and has a uniform size in space. Space cells must contain particles, i.e., the particle density in real space is non-zero.

A set of particles in a space cell is called a cluster and reflects the entity of a quantum that contains more than one particle. For example, the atom is a cluster containing a nucleus and electrons, and the solar system is a cluster containing the sun and planets.

In a Cartesian coordinate system with reference origin O (space origin) and space coordinates (x, y, z) , the position vector depends on the origin (origin-relevant), whereas the space quantum does not (origin-irrelevant).

iii. Real time

Real time is independent of real space and is defined as

$$t = t_s \cdot \tilde{t}; \quad \tilde{t} = 0, 1, 2, 3, \dots \quad (3)$$

The scale t_s is called a time quantum. The digit \tilde{t} is a sequence of natural numbers, indicating the

uniformity and irreversibility of real time. $\tilde{t} = 0$ is the reference origin of real time (time origin) and is determined by a synchronization protocol. The protocol stipulates that when a signal is generated at a space origin O at t_0 and propagates at a communication speed c , the time at any position $P(r)$ is set to $t = t_0 + r/c$. For example, if Greenwich is taken as the origin and the speed of light as the communication speed, the synchronization of the global time can be established by radio waves. The signal speed used for time synchronization is a system constant, rather than a universal constant. Both the speeds of light and sound can be employed as the signal speed.

iv. Real particle

Real particles are three-dimensional elastic objects. Elastic objects have both mass and volume and thus can spin and undergo elastic deformation. Electrons, protons, and atoms are all elastic particles. The mass of elastic particles is conserved, but their volume, density, and shape are variable. Real physics does not consider particles without mass or volume, and hence, there are only two kinds of primitive real particles in nature: protons and electrons. Their masses are the same as those of protons and electrons in modern physics, but they have no electric charge. The concept of charge is unnecessary in real physics. Apart from protons, the particles that fill the universe can only be electrons. Ubiquitous electrons are the so-called dark matter particles.

v. Matter

Matter in nature is solely composed of discrete elastic particles. Mathematical analysis shows that the field is not an independent form of matter, but rather a statistical convolution effect composed of a large number of particles. The field theory of real particles suggests that the ubiquitous electronic gas forms an ocean of dark matter in the universe, and the space in which protons gather forms islands of "bright matter."

vi. Motion

The spatial state of a real particle includes its position, profile, and posture, which are characterized by the position vector $r_c = (x_c, y_c, z_c)$ of the center of mass, the eigenvalues $I_c = (I_{c1}, I_{c2}, I_{c3})$ of the rotary inertia matrix, and the eigenvector directions $\theta_c = (\theta_{c1}, \theta_{c2}, \theta_{c3})$ of the rotary inertia matrix, respectively. Real particles have three independent motion modes of translation, rotation, and vibration, corresponding to the temporal variations of position, posture, and profile, respectively. Each motion mode has three degrees of freedom, and a real particle has nine degrees of freedom of motion. The rotation mode refers to the particle spin. The vibration mode refers to the elastic oscillation in which the particle returns to its original shape after deformation. The superposition of translation and vibration modes is the cause of wave-particle duality. The motion of real particles encompasses the translation mode of point-

like particles in classical physics and the vibration mode of wave-like particles in modern physics. As an entity of quantum physics, clusters are real particles that comprise three independent motion modes.

vii. Energy

If a particle has a vibration energy, $H_{i\alpha}$, rotation energy, $L_{i\alpha}$, and translation energy, $K_{i\alpha}$, then the total motion energies for an N -particle system are

$$H = \sum_{i=1}^N \sum_{\alpha=1}^3 H_{i\alpha} > 0, \quad (4)$$

$$L = \sum_{i=1}^N \sum_{\alpha=1}^3 L_{i\alpha} > 0, \quad (4)$$

$$K = \sum_{i=1}^N \sum_{\alpha=1}^3 K_{i\alpha} > 0. \quad (4)$$

According to the motion energies, particle systems are classified as vibration (radiative) system ($H > L$ and $H > K$), rotation (magnetic) system ($L > K$ and $L > H$), and translation (thermal) system ($K > H$ and $K > L$). A system with a constant proportion ($H : L : K$) is in a state of equilibrium, and an atomic system in equilibrium is also called a stationary state. In real physics, other forms of energy can be derived from motion energy [11]. In a translation system, for example, the potential energy $J = H - K$, thermal energy $Q = L + K$, chemical energy $G = L - H$, and internal energy $U = K + L - H$ can be derived. Specifically, H equals the mechanical energy, and $-H$ equals the Helmholtz free energy. Energy is essentially attributed to motion, and not directly to matter. The mass of real particles is conserved, and mass and energy cannot be converted into one another.

viii. Interaction

The interaction between real particles includes mass attraction and motion repulsion. Mass attraction represents the aggregation tendency of particles, and motion repulsion indicates the existence of gaps between particles necessary for motion. The mathematical constraint of motion repulsion is that the intervals between different particles are greater than zero, i.e., $r_{ij} = (r_i \cdot r_j)^{1/2} > 0$. The direct inference of motion repulsion is that the mass density of any object is finite, namely, $\rho = M/V < \infty$. In the field theory of real particles, mass potential represents mass attraction and momentum potential represents motion repulsion. Analysis shows that the forms of interaction include the gradient (gravitation and electrostatic), curl (magnetic), and divergence (alternating electromagnetic) forces, which correspond to the translation, rotation, and vibration forces, respectively. The weak and strong



forces originate from the combined effects of the gradient, curl, and divergence forces.

b) Scale systems

In real physics, quantization is the procedure of expressing physical quantities in terms of real quantities and determining their scale relations. There are only three independent scales (basic quanta) in the three dimensional real space. Different physics theories adopt different basic quanta.

i. Scales of classical mechanics

In classical mechanics, the basic quanta are the space scale r_s , time scale t_s , and mass scale M_s . Other scales can be expressed as functions of the basic quanta, e.g.

$$\begin{aligned} u_s &= r_s/t_s, & \omega_s &= 1/t_s, \\ p_s &= M_s(r_s/t_s), & I_s &= M_s r_s^2. \end{aligned} \quad (5)$$

In Eq. (5), u_s is the velocity scale, ω_s is the angular velocity scale, p_s is the momentum scale, and I_s is the rotary inertia scale. The basic quanta $\{r_s, t_s, M_s\}$ stand for the basic physical dimensions of length, time, and mass in classical mechanics.

ii. Scales of motion energy

In real physics, H , L , and K are three independent motion energies. The corresponding energy scales are the vibration quantum H_s , rotation quantum L_s , and translation quantum K_s .

$$\begin{aligned} H_s &= H/N = Y_s V_s = hv, \\ L_s &= L/N = I_s \omega_s^2 = lB, \\ K_s &= K/N = M_s u_s^2 = kT. \end{aligned} \quad (6)$$

In Eq. (6), Y_s is the elastic modulus scale; h , l , and k are the Planck, Bohr magneton, and Boltzmann constants, respectively; and v , B , and T are the vibration frequency, magnetic induction, and absolute temperature, respectively. The energy quanta, H_s , L_s , and K_s , apply to the vibration, rotation, and translation systems, respectively. In equilibrium state, the energy digits $(\tilde{H}, \tilde{L}, \tilde{K})$ take integer values. The stationary states of an atom can be easily determined for predicting the corresponding emission spectrum [10].

iii. Scales of real-particle field

In the real-particle field theory, the basic quanta are the mass scale M_s , time scale t_s , and velocity scale $u_s = c$ (communication speed). Some scales of the field include

$$\begin{aligned} r_s &= ct_s = \lambda, & v &= 1/t_s, & p_s &= M_s c, \\ V_s &= r_s^3 = \lambda^3, & \rho_s &= M_s/\lambda^3, & j_s &= p_s/\lambda^3. \end{aligned} \quad (7)$$

In Eq. (7), λ is the wavelength, v is the frequency, and ρ_s and j_s are the scales of the mass and momentum densities, respectively. In the real-particle field, the vibration quantum $H_s = hv$ corresponds to the Planck energy, and the translation quantum $K_s = M_s c^2$ corresponds to the Einstein energy. The scale relations of quantum mechanics depicted in Eq. (8) are valid under the special condition $H_s = K_s = E_s$.

$$M_s = h/(\lambda c), \quad p_s = h/\lambda, \quad h = r_s p_s = E_s t_s. \quad (8)$$

c) Objectivity principle

The objectivity principle claims that the laws of matter and motion are objective and do not depend on the subjective consciousness of the observer; therefore, human subjective factors must be excluded from the physical formulation.

Physical processes are objective, but physical observations are subjective. For physical observations, measurement units and a reference system must be chosen, both of which are subjective factors. The objectivity principle requires that physical formulas have scale covariance and origin irrelevance.

i. Scale covariance

The scale covariance ensures that the digital relation has the same form as the physical relation, i.e.

$$z = f(x, y) = z_s \cdot \tilde{z}; \quad z_s = f_s, \quad \tilde{z} = f(\tilde{x}, \tilde{y}), \quad (9)$$

where f represents the physical relationship among quantities $\{x, y, z\}$. The formula $z_s = f_s$ is called scale covariance, and the formula $\tilde{z} = f(\tilde{x}, \tilde{y})$ is called digit independence. The operation rules of real quantities can be determined according to Eq. (9).

a. Addition and subtraction

$$z = x \pm y = x_s \cdot (\tilde{x} \pm \tilde{y}); \quad x_s = y_s = z_s, \quad \tilde{z} = \tilde{x} \pm \tilde{y}. \quad (10)$$

b. Multiplication

$$z = xy = (x_s y_s) \cdot (\tilde{x} \tilde{y}); \quad z_s = x_s y_s, \quad \tilde{z} = \tilde{x} \tilde{y}. \quad (11)$$

c. Division

$$z = \frac{y}{x} = \frac{y_s}{x_s} \cdot \frac{\tilde{y}}{\tilde{x}}; \quad z_s = \frac{y_s}{x_s}, \quad \tilde{z} = \frac{\tilde{y}}{\tilde{x}}. \quad (12)$$

d. Real differential

$$z = f(x), \quad x_i = x_0 + i \cdot x_s, \quad i = 0, 1, 2, \dots, n;$$

$$dz = f(x_{i+1}) - f(x_i) = f_s \cdot [f(\tilde{x}_{i+1}) - f(\tilde{x}_i)];$$

$$d\tilde{z} = f(\tilde{x}_{i+1}) - f(\tilde{x}_i), \quad z_s = f_s. \quad (13)$$

e. *Real derivative*

$$\frac{dz}{dx} = \frac{z_s}{x_s} \cdot d\tilde{z}; \quad \left(\frac{dz}{dx} \right)_s = \frac{z_s}{x_s}, \quad \frac{d\tilde{z}}{d\tilde{x}} = d\tilde{z}. \quad (14)$$

f. *Real integral* $\int dx$

$$S(x_0, x_n) = \int_{x_0}^{x_n} f(x) dx = (x_s f_s) \cdot \sum_{i=1}^n f(\tilde{x}_i);$$

$$S_s = x_s f_s, \quad \tilde{S} = \sum_{i=1}^n f(\tilde{x}_i). \quad (15)$$

g. *Digit operation*

Digit operations contain exponential, logarithmic, and trigonometric functions, which can be regarded as the case of $x_s = 1$, e.g.

$$e^x = e^{x_s \cdot \tilde{x}} = e^{\tilde{x}},$$

$$\ln x = \ln(x_s \cdot \tilde{x}) = \ln \tilde{x},$$

$$\sin x = \sin(x_s \cdot \tilde{x}) = \sin \tilde{x}. \quad (16)$$

ii. *Origin irrelevance*

Origin irrelevance requires that the definition of physical quantity be independent of the reference origins in time and space. Therefore, the physical quantity must be defined at any time ($\tilde{t} = k$). The position vector of particles $\mathbf{r}_i(k)$ can only appear in the form of intervals, $\mathbf{r}_{ij}(k)$, and displacements, $d\mathbf{r}_i(k)$, to eliminate the influence of the space origin, as proved in Eq. (17).

$$\mathbf{r}_{ij}(k) = \mathbf{r}_j(k) - \mathbf{r}_i(k) = \overrightarrow{OP_j(k)} - \overrightarrow{OP_i(k)}$$

$$= \overrightarrow{P_i(k)P_j(k)}, \quad (i \neq j)$$

$$d\mathbf{r}_i(k) = \mathbf{r}_i(k+1) - \mathbf{r}_i(k) = \overrightarrow{OP_i(k+1)} - \overrightarrow{OP_i(k)}$$

$$= \overrightarrow{P_i(k)P_i(k+1)}. \quad (16)$$

The particle velocity is defined as displacement divided by time quantum

$$\mathbf{u}_i(k) = \frac{d\mathbf{r}_i(k)}{dt} = \frac{r_s}{t_s} \cdot d\tilde{\mathbf{r}}_i(k);$$

$$u_s = r_s/t_s, \quad \tilde{\mathbf{u}}_i(k) = d\tilde{\mathbf{r}}_i(k). \quad (18)$$

The displacement velocity thus defined is independent of the space and time reference origins.

iii. *Principle of unity*

Scale covariance indicates that physical laws apply to any scale range of physical quantities. All

particles follow the same equations of motion in both the classical and quantum worlds, under slow and fast motion, and at low and high energies. Origin irrelevance indicates that the physical laws apply to any time and space, independent of the choice of the reference system. The principle of objectivity, which combines the concepts of relativity and quantum theories, is a principle of unity and universality.

The observation of motion prerequisites the selection of a coordinate system, which is based on two subjective factors: the reference origin and coordinate unit. In Newtonian mechanics, the instantaneous velocity ($d\mathbf{r}_i/dt$) is a classical derivative related to the reference origin. Therefore, the inertia law is a cornerstone of Newtonian mechanics, and the motion law of Newton directly depends on the reference system. The relativity principle demands that the law of motion be independent of the choice of coordinate system, and therefore, its mathematical structure should be invariant (or covariant) under a coordinate (or metric) transform. Similar to the objectivity principle, the relativity principle serves to eliminate the subjectivity of the coordinate system.

According to real physics, the essence of quantum is scale, and the entity of quantum is cluster. The state function of quantum mechanics (ψ) describes the motion of clusters, rather than individual particles. The cluster motion is determined by the momentum (translation), angular momentum (rotation), and Hamiltonian (vibration) operators. The quantum eigenstates belong to the sub-modes of a certain motion operator, and the state superposition results in a hybrid motion mode. The modulus square of the state function ($|\psi|^2$) represents the probability of the motion mode. In real-particle field theory, the motion of clusters also needs statistical description. However, the statistical function is not a probability but the distribution of mass and momentum densities. The translation, rotation, and vibration of clusters are described by the gradient, curl, and divergence, respectively. The starting point of quantum mechanics is the quantization of energy, whereas the basis of real physics is the quantization of all physical quantities. Scale covariance indicates that real physics is a full-scale theory, and objectivity principle is the ultimate criterion of unifying physics theory.

d. *Real-particle field*

The real-particle field describes the motion of elastic particles in real space. Based on the mass and momentum statistics of particles, we derived a complete set of field equations and inferred the classical laws of mechanics, gravitation, and electromagnetism from the real particle field theory.

i. *Density field*

Consider a finite field space of volume V , free boundary S , and particle number N . The field space is

divided by the volume quantum into \tilde{V} space cells; a field with volume $V = V_s$. \tilde{V} has a total of \tilde{V} clusters. The mass distribution of clusters constitutes a field of mass density $\rho(\mathbf{r}, t)$, and the momentum distribution of clusters constitutes a field of momentum density $\mathbf{j}(\mathbf{r}, t)$. ρ and \mathbf{j} are collectively called the density field. The mass and momentum of a cluster are the sums of the mass and momentum, respectively, of particles in the space cell; thus, the density field is essentially the statistical result of discrete particles.

ii. Potential field

The potential field is constructed from the density field through an integral transform named the

$$\Phi(\mathbf{r}, t) = \frac{-1}{\varphi} \left[\rho(\mathbf{r}, t) \circledast \left(\frac{1}{|\mathbf{r}|} \right) \right] = \frac{-1}{\varphi} \int_V \frac{\rho(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|} dV' = \frac{-1}{4\pi\varphi_s} \int_V \frac{\rho(\mathbf{r}', t)}{r} dV'; \quad \Phi_s = \frac{\rho_s V_s}{\varphi_s r_s} = c^2. \quad (19)$$

$$\mathbf{A}(\mathbf{r}, t) = \alpha \left[\mathbf{j}(\mathbf{r}, t) \circledast \left(\frac{1}{|\mathbf{r}|} \right) \right] = \alpha \int_V \frac{\mathbf{j}(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|} dV' = \frac{\alpha_s}{4\pi} \int_V \frac{\mathbf{j}(\mathbf{r}', t)}{r} dV'; \quad A_s = \frac{\alpha_s j_s V_s}{r_s} = c. \quad (20)$$

In Eqs. (19) and (20), \circledast is the operator of confined space convolution, $r = |\mathbf{r} - \mathbf{r}'|$ is the distance between clusters, and the minimum distance is r_s (i.e., $\tilde{r} = 1$). The potentials are finite because $\tilde{r} \geq 1$ and $\tilde{V} < \infty$. Φ is a scalar potential with the same form as the gravitational and electric (retarded) potentials, and \mathbf{A} is a vector potential with the same form as the magnetic (retarded) potential [15, 16].

The medium coefficient, $\varphi = 4\pi\varphi_s$, and the dynamics coefficient, $\alpha = \alpha_s/4\pi$, are not constants but satisfy the constraint

$$\alpha\varphi = \alpha_s\varphi_s = c^{-2}. \quad (23)$$

The constraint indicates that a smaller φ corresponds to larger α , that is, the two potentials are directly proportional to each other. In this way, the balance of attraction and repulsion is independent of the values of φ and α .

A convolution operation applies a smoothing effect to an input function [17], which can transform a rough density field into a smooth potential field. Therefore, classical vector calculus can be applied on the potential field to obtain the motion equation of the particle field.

iii. Field constraints

The constraints on the real-particle field include the continuity, conservation, and boundary conditions.

a. Continuity theorem

If $\mathbf{z}(\mathbf{r}, t)$ is the bulk density of any physical quantity $\mathbf{Z}(t)$, then

$$\mathbf{Z}(t) = \int_V \mathbf{z}(\mathbf{r}', t) dV'. \quad (22)$$

confined space convolution. As defined in Eqs. (19) and (20), the mass potential $\Phi(\mathbf{r}, t)$ is the space convolution of mass density, which represents the mass attraction between clusters. The momentum potential $\mathbf{A}(\mathbf{r}, t)$ is the space convolution of momentum density, which represents the motion repulsion between clusters. The opposite signs of Φ and \mathbf{A} reflect the opposite interactions of attraction and repulsion.

The time derivative of $\mathbf{Z}(t)$ can be expressed as

$$\frac{d\mathbf{Z}}{dt} = \frac{d}{dt} \left[\int_V \mathbf{z}(\mathbf{r}', t) dV' \right] \equiv \int_V \frac{D\mathbf{z}}{Dt} dV'. \quad (23)$$

Here, $D\mathbf{z}/Dt$ is the motion derivative of \mathbf{z} , which can be derived as (see Appendix A)

$$\frac{D\mathbf{z}}{Dt} = \frac{\partial \mathbf{z}}{\partial t} + \nabla \cdot (\mathbf{z}\mathbf{u}), \quad (24)$$

where \mathbf{u} is the cluster velocity, and $\mathbf{z}\mathbf{u}$ is the dyad of \mathbf{z} and \mathbf{u} . The term $\partial\mathbf{z}/\partial t$ originates from a density variation, whereas the term $\nabla \cdot (\mathbf{z}\mathbf{u})$ originates from a volume variation. Eq. (24) is known as the continuity theorem.

b. Conservation theorem

If the total amount of \mathbf{Z} is time invariant, i.e., $\mathbf{Z}/dt = 0$, then

$$\frac{D\mathbf{z}}{Dt} = \frac{\partial \mathbf{z}}{\partial t} + \nabla \cdot (\mathbf{z}\mathbf{u}) = 0. \quad (25)$$

This equation is known as the conservation theorem.

c. Boundary condition

The mass of a real-particle field is conserved, and the conservation equation is

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\mathbf{u}) = \frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0. \quad (26)$$

Eq. (26) is equivalent to the free boundary condition of the density field in the form

$$\oint_S \mathbf{j}(\mathbf{r}', t) \cdot d\mathbf{S}' = - \int_V \frac{\partial \rho(\mathbf{r}', t)}{\partial t} dV'. \quad (27)$$

This condition implies the prohibition of matter exchange between the system and its surroundings.

The boundary condition of the potential field imposes another constraint

$$\frac{\alpha_s}{4\pi} \oint_S \frac{\mathbf{j}(\mathbf{r}', t) \cdot d\mathbf{S}'}{r} = D_0(t). \quad (28)$$

This condition implies the permission of exchange of vibration energy between the system and its surroundings.

iv. Action field

The action field is the spatial first derivative of the potential field. The action field includes the gradient field G , curl field C , and divergence field D (see Appendix B).

$$\mathbf{G}(\mathbf{r}, t) = -\nabla\Phi = \frac{-1}{\varphi} \int_V \frac{\rho(\mathbf{r}', t)\mathbf{r}}{r^3} dV'; \quad (30)$$

$$G_s = \frac{r_s}{t_s^2}.$$

$$\mathbf{C}(\mathbf{r}, t) = \nabla \times \mathbf{A} = \alpha \int_V \frac{\mathbf{j}(\mathbf{r}', t) \times \mathbf{r}}{r^3} dV';$$

$$C_s = \frac{1}{t_s}.$$

$$D(\mathbf{r}, t) = \nabla \cdot \mathbf{A} = \frac{1}{c^2} \frac{\partial \Phi}{\partial t} - D_0(t);$$

$$D_s = \frac{1}{t_s}. \quad (31)$$

G has the scale of acceleration and follows the inverse square law with distance, which is the gravitational acceleration of Newton. C also follows an inverse square law with respect to distance, which has the same form as the Biot-Savart law. The form of D is similar to that of the Lorenz gauge, which indicates that the temporal change in mass potential leads to the spatial change in momentum potential. The scales of C and D represent the rotational and vibrational frequencies of the clusters, respectively.

e) Field equations

The field equations are derived from the spatial derivatives of the potential field, which include the equations of the potential and action fields (see Appendix B for details). The two sets of equations completely determine the evolution of the particle field.

i. Equations of potential field

The spatial second derivative of the potential field gives the corresponding equations as follows

$$(\nabla \cdot \nabla) \mathbf{A} = \nabla^2 \mathbf{A} = -\alpha_s \mathbf{j}, \quad (32)$$

$$\nabla \cdot \mathbf{G} = -\nabla^2 \Phi = -\frac{1}{\varphi_s} \rho, \quad (33)$$

$$\nabla \times \mathbf{G} = -\nabla \times \nabla \Phi \equiv 0, \quad (34)$$

$$\nabla \cdot \mathbf{C} = \nabla \cdot (\nabla \times \mathbf{A}) \equiv 0, \quad (35)$$

$$\nabla \times \mathbf{C} = \nabla \times (\nabla \times \mathbf{A}) = \alpha_s \mathbf{j} - \frac{1}{c^2} \frac{\partial \mathbf{G}}{\partial t}, \quad (36)$$

$$\nabla D = \nabla(\nabla \cdot \mathbf{A}) = -\frac{1}{c^2} \frac{\partial \mathbf{G}}{\partial t}. \quad (37)$$

Eq. (33) is the potential equation of Newton, and Eqs. (33){(36) are similar to the Maxwell equations. Eq. (37) expresses a quantity similar to the displacement current.

ii. Equations of action field

The spatial second derivative of the action field (the spatial third derivative of the potential field) gives the corresponding equations as follows

$$\nabla^2 \mathbf{G} = -\frac{1}{\varphi_s} \nabla \rho, \quad (38)$$

$$\nabla^2 \mathbf{C} = -\alpha_s \nabla \times \mathbf{j}, \quad (39)$$

$$\nabla^2 D = \alpha_s \frac{\partial \rho}{\partial t}. \quad (40)$$

There are only three equations of the action field, namely, the Poisson equations of the gradient, curl, and divergence fields. It can be seen that the action field is caused by the spatial or temporal change in the density field. This set of equations has a clear physical meaning; it contains only two free parameters and is of high symmetry and simplicity. The field equations are derived from the potential field, which guarantees the existence of solutions to the equations. The equations of the action field are called the unified field equations.

III. REAL-PARTICLE FIELD PROPERTIES

a) Unity of fields

From the constants listed in Table II, the relationship among the gravitational, electromagnetic, and particle fields can be derived. By setting the medium coefficient as the reciprocal of the gravitational constant ($\varphi = \gamma^{-1}$), the gravitational field equations can be obtained from the real-particle field equations. By using the scale conversion factor ($\theta = \epsilon/\varphi$) and the mass-to-charge ratio of an electron ($\sigma = M_e/Q_e$), the relationship between the

Table II: Quantities and relations among the gravitational, electromagnetic, and particle fields

Field quantity	Relation	Constant (N: newton, C: coulomb)
Light speed	$u_s = c = 1/\sqrt{\alpha\varphi}$	$c = 2.9979246 \times 10^8 \text{ m s}^{-1}$
Gravitational constant	$\gamma = \varphi^{-1}$	$\gamma = 6.6742867 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Medium coefficient	$\varphi = 4\pi\varphi_s = \gamma^{-1}$	$\varphi = 1.4982874 \times 10^{10} \text{ N}^{-1} \text{ m}^{-2} \text{ kg}^{-2}$
Dynamics coefficient	$\alpha = \alpha_s/4\pi = \gamma c^{-2}$	$\alpha = 7.4261454 \times 10^{-28} \text{ N s}^2 \text{ kg}^{-2}$
Mass of electron	M_e	$M_e = 9.1093821 \times 10^{-31} \text{ kg}$
Charge of electron	Q_e	$Q_e = 1.6021765 \times 10^{-19} \text{ C}$
Mass-to-charge ratio	$\sigma = M_e/Q_e$	$\sigma = 5.6856296 \times 10^{-12} \text{ kg C}^{-1}$
Scale conversion factor	$\theta = \epsilon/\varphi = 4\pi\epsilon_s\gamma$	$\theta = 7.4261454 \times 10^{-21} \text{ C}^2 \text{ kg}^{-2}$
Vacuum permittivity	$\epsilon_s = \theta\varphi_s$	$\epsilon_s = 8.8541877 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$
Vacuum permeability	$\mu_s = \alpha_s/\theta$	$\mu_s = 4\pi \times 10^{-7} \text{ N C}^{-2} \text{ s}^2$
Charge density	$\rho_e = \sigma\theta\rho$	$\sigma\theta = 4.2222312 \times 10^{-32} \text{ C kg}^{-1}$
Current density	$j_e = \sigma\theta j$	$\sigma\theta = 4.2222312 \times 10^{-32} \text{ C kg}^{-1}$
Electric potential	$\Phi_e = \sigma\Phi$	$\sigma = 5.6856296 \times 10^{-12} \text{ kg C}^{-1}$
Magnetic potential	$\mathbf{A}_e = \sigma\mathbf{A}$	$\sigma = 5.6856296 \times 10^{-12} \text{ kg C}^{-1}$
Electric field	$\mathbf{E}_e = \sigma\mathbf{G}$	$\sigma = 5.6856296 \times 10^{-12} \text{ kg C}^{-1}$
Magnetic induction	$\mathbf{B}_e = \sigma\mathbf{C}$	$\sigma = 5.6856296 \times 10^{-12} \text{ kg C}^{-1}$

electromagnetic and real-particle fields can be found. If the symmetry of charge is “broken”, i.e., there exists no negative charge, the difference between charge and mass is merely a constant factor ($\sigma\theta$). In essence, the gravitational and electromagnetic fields are both real-particle fields described by unified field equations.

b) Equation of motion

i. Motion theorem

The force on clusters can be expressed by the motion derivative of the momentum density as

$$\begin{aligned} \mathbf{f} &= \frac{D\mathbf{j}}{Dt} = \frac{\partial \mathbf{j}}{\partial t} + \nabla \cdot (\mathbf{j}\mathbf{u}) \\ &= \rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right]; \quad f_s = \frac{\rho_s r_s}{t_s^2}. \end{aligned} \quad (41)$$

This equation is called the motion theorem. It can be written in the form of Newton's second law

$$\begin{aligned} \mathbf{f} &= \rho\mathbf{a}, \quad \mathbf{a} = \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u}; \\ a_s &= \frac{f_s}{\rho_s} = \frac{r_s}{t_s^2}, \end{aligned} \quad (42)$$

where \mathbf{a} is the acceleration, $\partial \mathbf{u} / \partial t$ is the linear acceleration, and $(\mathbf{u} \cdot \nabla) \mathbf{u}$ is the curve acceleration.

ii. Force field

The coupling of the density and action fields produces the force field. The force field includes the gradient force \mathbf{f}_G , curl force \mathbf{f}_C , and divergence force \mathbf{f}_D , which represent the translation, rotation, and vibration forces on the cluster, respectively.

$$\mathbf{f}_G = \rho\mathbf{G} = -\rho\nabla\Phi,$$

$$\mathbf{f}_C = \mathbf{j} \times \mathbf{C} = \rho\mathbf{u} \times \mathbf{C},$$

$$\mathbf{f}_D = \mathbf{j}D = \rho D\mathbf{u}. \quad (43)$$

The gradient, curl, and divergence forces cause linear acceleration, curve acceleration, and motion resistance, respectively. The motion resistance is proportional to the velocity \mathbf{u} with the resistance coefficient ρD .

iii. Motion equation

According to $\mathbf{f} = \mathbf{f}_G + \mathbf{f}_C + \mathbf{f}_D$ the motion equation can be obtained as

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \mathbf{G} + \mathbf{u} \times \mathbf{C} + D\mathbf{u}. \quad (44)$$

This equation gives the motion law of clusters in the realparticle field, which is similar to the Navier-Stokes equation in fluid mechanics [15]. The left side of the equation is derived from the density field, and the right side is derived from the density and potential fields. A solution to this equation definitely exists. According to Eq. (25), $\mathbf{f} = 0$ is the case of momentum conservation.

c) Properties of dark matter

i. Waves of dark matter

The solutions to the equations of divergence [Eqs. (31), (37), and (40)] are traveling waves in the form

$$\mathbf{A}(\xi) = \frac{\mathbf{k}}{\omega} W(\xi),$$

$$\Phi(\xi, t) = -W(\xi) + W_0(t),$$

$$D_0(t) = \frac{1}{c^2} \frac{dW_0}{dt}.$$

In Eq. (45), $\xi = \mathbf{k} \cdot \mathbf{r} - \omega t$ is the wave parameter, \mathbf{k} is the wave vector, ω is the angular frequency, and $W(\xi)$

is any positive function. Real physics claims that dark matter particles are electrons and that the dark matter in the universe is electronic gas. Therefore, such traveling waves in an electronic gas are equivalent to electromagnetic and gravitational waves. This implies that electrons not only participate in light emission (such as atomic and X-ray radiations) but also transmit electromagnetic and gravitational waves. In Eq. (45), $c = \omega/|\mathbf{k}|$ is the speed of electromagnetic waves and also the communication speed used for time synchronization. The undulation of the electronic gas is neither a simple transverse wave nor a longitudinal wave; it is a cluster vibration relative to its center of mass, called a spherical wavelet. The electronic clusters are three-dimensional vibrators, and the local vibration of clusters is the physical basis of the Huygens-Fresnel principle [18]. As electronic gas is the medium of undulation, these waves propagate at the speed of light, so there is no instantaneous action.

ii. Density of dark matter

The characteristic wavelength between an ultraviolet ray and an X-ray is $\lambda_c = 0.01 \mu\text{m}$, and the corresponding characteristic volume is $\lambda_c^3 = 10^{-24} \text{m}^3$. We believe that ultraviolet ray originates from cluster vibration and X-ray from electron vibration, so the characteristic volume contains only a single electron, which corresponds to a characteristic density $n_c = \lambda_c^{-3} = 10^{24} \text{m}^{-3}$. Since the sunlight reaching the earth does not contain X-rays, n_c is a feature density of dark matter between the sun and the earth. The corresponding mass density is $\rho_c = n_c M_e \approx 9.1 \times 10^{-7} \text{kg m}^{-3}$. This value is very small compared with the mass density of air ($\sim 1.2 \text{kg m}^{-3}$) in the standard state, so the existence of electrons is not perceivable by humans. Compared with the mass density of planets in the solar system, the electron mass density is negligible, which explains why the hypothesis of absolute space is valid within the solar system. However, the mass of dark matter cannot be ignored in galactic scales. Newtonian gravitational theory must be replaced by the universal theory of real-particle field.

iii. Structure of matter

Real physics does not consider particles without mass or volume and only recognizes electrons and protons with both mass and volume. Electrons alone constitute dark matter, and electrons and protons together constitute "bright matter". According to the nested structure model, matter is composed of clusters of different levels [9]. For example, protons and electrons form neutrons, protons and neutrons form nuclei, nuclei and electrons form atoms, atoms form molecules, and so on. At the celestial level, planets and satellites form planetary systems, stars and planets form stellar systems, stellar systems form galaxies, galaxies form galaxy clusters, and so on. This case is similar to a computer using a string of $\{1, 0\}$ to express arbitrary

information. From this perspective, dark matter is also "bright matter."

iv. Modulus of elasticity

According to the scale relation in Eq. (6), the vibration energy of electronic clusters is $H_s = Y_s V_s = h\nu$. Then, the elastic modulus of the electronic clusters is

$$Y_s = \frac{H_s}{V_s} = \frac{h\nu}{\lambda^3} = \frac{h\nu^4}{c^3}. \quad (46)$$

According to Eq. (46), the elastic modulus represents the density of the vibration energy, which is proportional to the fourth power of the wave frequency. Because the energy density scale equals the pressure scale, the elastic modulus also represents the wave pressure of dark matter.

v. Nature of photons

Real physics proves theoretically that the dark matter field equals the electromagnetic field. There is no doubt about its existence, because we feel it through light waves. The light propagation depends on the vibration of electronic clusters, and electronic gas is the medium of light waves. As electronic clusters serve as the unit of vibration, they are the entities corresponding to photons. Photons are real particles with volume V_s , mass $\rho_c V_s$, vibration energy $H_s = h\nu$, and elastic modulus $Y_s = (h\nu^4)/c^3$. The number of photons $\tilde{V} = V/V_s = V/\lambda^3$ is not conserved but depends on the wavelength.

vi. Radiation of dark matter

Assuming a dark matter field of volume, V , and photon number, $\tilde{V} = V(v/c)^3$, the vibration energy in the frequency range from v to $v + dv$ is

$$dH(v) = h\nu \cdot d\tilde{V} = \frac{3V}{c^3} h\nu^3 dv. \quad (47)$$

If the thermal equilibrium temperature is T , let $\beta = (kT)^{-1}$, and $H_n = nh\nu$; then, the probability of finding H_n is $P_n(\beta, v) \propto e^{-\beta H_n}$. Subsequently, the probability of finding v at β is

$$P(\beta, v) = a \sum_{n=1}^{\infty} P_n(\beta, v) = \frac{a}{e^{\beta h\nu} - 1}, \quad (48)$$

where a is a normalization factor. Consequently, the vibration energy of dark matter in thermal equilibrium can be derived within the frequency interval dv as

$$dH(\beta, v) = P(\beta, v) dH(v) = \frac{3aV}{c^3} \frac{h\nu^3}{e^{\beta h\nu} - 1} dv. \quad (49)$$

Eq. (49) is the same as the law of black-body radiation first proposed by Planck [19], except for the normalization factor. Cosmological observations prove that the cosmic microwave background (CMB) corresponds to a black-body of temperature

$T = 2.72548K$ [20]. We explain that CMB originates from the thermal equilibrium radiation of dark matter, which has a uniform density; therefore, there is no photonic current in the universe.

vii. Dark matter and ether

To explain the origin of universal gravitation, some scientists hypothesized the existence of gravitation ether. For example, Le Sage proposed an ether model based on the streams of tiny unseen particles, called ultramundane corpuscles [21]. According to this model, any material object is impacted by corpuscles from all directions, and any two material bodies partially shield each other from the impinging corpuscles. The shielding effect results in a net imbalance in the pressure exerted on the bodies and tends to drive the bodies together. Similarly, luminiferous ether was once considered to be the medium of light waves, like air is the medium of sound waves. Since Maxwell established electromagnetic field theory, luminiferous ether was also called electromagnetic ether [22]. These ethers are media filled with space and cannot be seen or touched. They must be stiff enough to transmit gravitation and light waves, yet light enough not to interfere with stellar motions. Because people could not well understand the concept of ether, it was abandoned after the introduction of the special theory of relativity by Einstein [23].

Dark phenomenon is a hypothesis proposed by modern cosmology based on observations of the movement of galaxies. In reality, dark phenomenon is a modern version of the dismissed ether. Real physics confirms that dark matter particles are electrons and dark matter is electronic gas. The properties of the proposed ether are very similar to those of electronic gas. It is invisible, intangible, and very dense in space but extremely light in weight. It is ubiquitous with a uniform density throughout the universe. The motion of "bright matter" hardly disturbs the electronic gas, while the vibration of electronic clusters easily causes ripples of dark matter. Elastic electrons have mass and gravity but no charge or electromagnetic effects. We cannot perceive the existence of electronic gas because the mass of electrons is too small and question its lack of electric charge because of current knowledge limitations. The interactions between real particles solely include mass attraction and motion repulsion and appear in the form of gradient, curl, and divergence forces. In fact, real physics does not require the concept of electric charge.

IV. CONCLUSIONS

This research is based on the real-particle field theory and proves that dark matter comprises elastic electrons with a full cosmic background. The real-particle field theory modifies the core concepts of matter, particle, space, and time, and makes

discoveries regarding the motion forms and interaction laws of elastic particles. Elastic particles have three independent modes of motion-translation, rotation, and vibration. Particle interaction includes mass attraction and motion repulsion. The distribution of particle mass and momentum constitutes a density field; the spatial convolution of densities forms a potential field; and the spatial derivative of potentials determines an action field. The action field includes the gradient, curl, and divergence fields, and its evolution follows a set of Poisson equations. The real particle field theory combines the quantum and relativity principles, and its inferences encompass the classical laws of mechanics and electromagnetism. The theory and associated research demonstrate that the electric and magnetic potentials correspond to the interactions of mass attraction and motion repulsion. The nature of electromagnetic and gravitational waves is the vibration of electronic gas, and the essence of photons is the electronic cluster. The mechanical features of photons can be characterized by their volume, mass, and elasticity.

The essence of matter is discrete particles, and the field is merely a mathematical description of particle motion in a continuous form. Physical quantities are quantized by scales, while volume scale connects discrete particles to continuous fields. Real physics is a full-scale theory based on the principle of objectivity, which applies to the whole space range from atoms to the universe. Once the initial and boundary conditions are given, the evolution of the real-particle system is determined by the unified field equation. It is an exciting and challenging task to determine the structure of atoms and the motion of celestial bodies by using the real-particle field theory.

ACKNOWLEDGMENTS

The author would like to thank Professor L.H. Xie for the helpful discussions. The research leading to these results received funding from the NSFC under Grant No[61775102].

APPENDIX A: DERIVATION OF CONTINUITY THEOREM

Let $\mathbf{z}(\mathbf{r}, t)$ be the bulk density of any physical quantity $\mathbf{Z}(t)$, then, the time derivative of $\mathbf{Z}(t)$ can be expressed as

$$\frac{d\mathbf{Z}}{dt} = \frac{d}{dt} \left[\int_V \mathbf{z}(\mathbf{r}', t) dV' \right] \equiv \int_V \frac{D\mathbf{z}}{Dt} dV', \quad (A1)$$

where $D\mathbf{z}/Dt$ is the motion derivative of \mathbf{z} .

As the density \mathbf{z} and volume V are both changeable, we have

$$\begin{aligned}\frac{d\mathbf{Z}}{dt} &\equiv \int_V \frac{D\mathbf{z}}{Dt} dV' = \frac{1}{t_s} \left[\int_{V+\Delta V} \mathbf{z}(\mathbf{r}', t+t_s) dV' - \int_V \mathbf{z}(\mathbf{r}', t) dV' \right] \\ &= \frac{1}{t_s} \left[\int_V \mathbf{z}(\mathbf{r}', t+t_s) dV' - \int_V \mathbf{z}(\mathbf{r}', t) dV' \right] + \frac{1}{t_s} \int_{\Delta V} \mathbf{z}(\mathbf{r}', t) dV' = \mathbf{R}(\Delta\mathbf{z}) + \mathbf{R}(\Delta V), \quad (A2)\end{aligned}$$

where $\mathbf{R}(\Delta\mathbf{z})$ is the variation rate of \mathbf{Z} caused by the change in density and constant volume, and $\mathbf{R}(\Delta V)$ is the variation rate caused by the change in volume and constant density.

$$\begin{aligned}\mathbf{R}(\Delta\mathbf{z}) &= \int_V \frac{\partial \mathbf{z}}{\partial t} dV' \\ &= \frac{1}{t_s} \int_V [\mathbf{z}(\mathbf{r}', t+t_s) - \mathbf{z}(\mathbf{r}', t)] dV',\end{aligned}\quad (A3)$$

$$\mathbf{R}(\Delta V) = \frac{1}{t_s} \int_{\Delta V} \mathbf{z}(\mathbf{r}', t) dV'. \quad (A4)$$

As the volume element on the boundary can be represented by the area element, $dV' = (\mathbf{u}t_s) \cdot d\mathbf{S}'$, $\mathbf{R}(\Delta V)$ can be calculated with the help of a surface integral as

$$\begin{aligned}\mathbf{R}(\Delta V) &= \frac{1}{t_s} \int_{\Delta V} \mathbf{z}(\mathbf{r}', t) dV' \\ &= \frac{1}{t_s} \oint_S \mathbf{z}(\mathbf{u}t_s) \cdot d\mathbf{S}' = \oint_S \mathbf{z}\mathbf{u} \cdot d\mathbf{S}' \\ &= \int_V \nabla \cdot (\mathbf{z}\mathbf{u}) dV',\end{aligned}\quad (A5)$$

where $\mathbf{z}\mathbf{u}$ is the dyad of \mathbf{z} and \mathbf{u} . In the last step of the above derivation, the Gaussian formula of the vector integral is applied to convert a surface integral to a volume integral.

Substituting Eqs. (A3) and (A5) into Eq. (A2), we have

$$\int_V \frac{D\mathbf{z}}{Dt} dV' = \int_V \left[\frac{\partial \mathbf{z}}{\partial t} + \nabla \cdot (\mathbf{z}\mathbf{u}) \right] dV'. \quad (A6)$$

Because the volume V is arbitrary, we can obtain the expression of motion derivative as

$$\frac{D\mathbf{z}}{Dt} = \frac{\partial \mathbf{z}}{\partial t} + \nabla \cdot (\mathbf{z}\mathbf{u}). \quad (A7)$$

This is the continuity theorem given by Eq. (24).

APPENDIX B: DERIVATION OF FIELD EQUATIONS

a) Gradient field

The mass potential given by Eq. (19) is expressed as

$$\Phi(\mathbf{r}, t) = \frac{-1}{4\pi\varphi_s} \int_V \frac{\rho(\mathbf{r}', t)}{r} dV'; \quad \Phi_s = c^2. \quad (B1)$$

The gradient field, \mathbf{G} , is defined as the negative gradient of mass potential, i.e.

$$\begin{aligned}\mathbf{G} &= -\nabla\Phi = \frac{1}{\varphi} \int_V \rho(\mathbf{r}', t) \nabla \left(\frac{1}{r} \right) dV' \\ &= \frac{-1}{4\pi\varphi_s} \int_V \frac{\rho(\mathbf{r}', t) \mathbf{r}}{r^3} dV'; \quad G_s = \frac{r_s}{t_s^2}.\end{aligned}$$

The divergence of the gradient field is calculated as follows

$$\begin{aligned}\nabla \cdot \mathbf{G} &= -\nabla^2\Phi = \frac{1}{4\pi\varphi_s} \int_V \rho(\mathbf{r}', t) \nabla^2 \left(\frac{1}{r} \right) dV' \\ &= -\frac{1}{\varphi_s} \int_V \rho(\mathbf{r}', t) \delta(\mathbf{r} - \mathbf{r}') dV' = -\frac{\rho}{\varphi_s}, \quad (B3)\end{aligned}$$

where $\delta(\mathbf{r})$ is the Dirac delta. $\nabla \cdot \mathbf{G} = -\rho/\varphi_s$ is known as the Gaussian theorem of gradient field, and $\nabla^2\Phi = \rho/\varphi_s$ is the Poisson equation of the mass potential.

According to vector calculus, the curl of a gradient field is always equal to zero, i.e.

$$\nabla \times \mathbf{G} = \nabla \times \nabla\Phi \equiv 0, \quad (B4)$$

which indicates that the gradient field is vortex-free.

Applying the curl operator on both sides of Eq. (B4) gives

$$\nabla \times (\nabla \times \mathbf{G}) \equiv \nabla(\nabla \cdot \mathbf{G}) - \nabla^2 \mathbf{G} = -\frac{\nabla\rho}{\varphi_s} - \nabla^2 \mathbf{G} \equiv 0, \quad (B5)$$

then, we obtain

$$\nabla^2 \mathbf{G} = -\frac{1}{\varphi_s} \nabla\rho. \quad (B6)$$

This is the Poisson equation of the gradient field.

b) Divergence field

The momentum potential given by Eq. (20) is expressed as

$$\mathbf{A}(\mathbf{r}, t) = \frac{\alpha_s}{4\pi} \int_V \frac{\mathbf{j}(\mathbf{r}', t)}{r} dV'; \quad A_s = c. \quad (B7)$$

The divergence field, D , is defined as the divergence of the momentum potential, which can be calculated as follows

$$\begin{aligned}
D &= \nabla \cdot \mathbf{A} = \alpha \int_V \mathbf{j}(\mathbf{r}', t) \cdot \nabla \left(\frac{1}{r} \right) dV' = -\alpha \int_V \mathbf{j} \cdot \nabla' \left(\frac{1}{r} \right) dV' \\
&= -\alpha \int_V \left[\nabla' \cdot \left(\frac{\mathbf{j}}{r} \right) - \frac{\nabla' \cdot \mathbf{j}}{r} \right] dV' = -\alpha \int_V \nabla' \cdot \left(\frac{\mathbf{j}}{r} \right) dV' - \alpha \int_V \frac{1}{r} \frac{\partial \rho}{\partial t} dV' \\
&= -\alpha \oint_S \frac{\mathbf{j} \cdot d\mathbf{S}'}{r} - \alpha \frac{\partial}{\partial t} \left(\int_V \frac{\rho}{r} dV' \right) = -\frac{\alpha_s}{4\pi} \oint_S \frac{\mathbf{j} \cdot d\mathbf{S}'}{r} + \alpha_s \varphi_s \frac{\partial \Phi}{\partial t}.
\end{aligned} \tag{B7}$$

Let the boundary satisfy the constraint

$$D_0(t) = \frac{\alpha_s}{4\pi} \oint_S \frac{\mathbf{j} \cdot d\mathbf{S}'}{r}, \tag{B9}$$

then, the divergence can be written as

$$D = \nabla \cdot \mathbf{A} = \frac{1}{c^2} \frac{\partial \Phi}{\partial t} - D_0(t); \quad D_s = \frac{1}{t_s}. \tag{B10}$$

The gradient of a divergence field can be calculated as follows

$$\begin{aligned}
\nabla D &= \nabla(\nabla \cdot \mathbf{A}) = \nabla \left[\frac{1}{c^2} \frac{\partial \Phi}{\partial t} - D_0(t) \right] \\
&= \frac{1}{c^2} \nabla \left(\frac{\partial \Phi}{\partial t} \right) = -\frac{1}{c^2} \frac{\partial \mathbf{G}}{\partial t}.
\end{aligned} \tag{B11}$$

The Poisson equation of the divergence field can be obtained by applying the divergence operation to Eq. (B11).

$$\begin{aligned}
\nabla \cdot (\nabla D) &= \nabla^2 D = -\frac{1}{c^2} \nabla \cdot \left(\frac{\partial \mathbf{G}}{\partial t} \right) \\
&= -\frac{1}{c^2} \frac{\partial}{\partial t} (\nabla \cdot \mathbf{G}) = \alpha_s \frac{\partial \rho}{\partial t}.
\end{aligned} \tag{B12}$$

c) Curl field

The curl field \mathbf{C} is the curl of momentum potential,

$$\begin{aligned}
\mathbf{C} &= \nabla \times \mathbf{A} \\
&= \alpha \int_V \nabla \times \left[\frac{\mathbf{j}(\mathbf{r}', t)}{r} \right] dV' \\
&= \alpha \int_V \nabla \left(\frac{1}{r} \right) \times \mathbf{j} dV' \\
&= \frac{\alpha_s}{4\pi} \int_V \frac{\mathbf{j} \times \mathbf{r}}{r^3} dV'; \quad C_s = \frac{1}{t_s}.
\end{aligned} \tag{B13}$$

The divergence of a curl field is always equal to zero, i.e.

$$\nabla \cdot \mathbf{C} = \nabla \cdot (\nabla \times \mathbf{A}) \equiv 0. \tag{B14}$$

Applying the Laplace operator to A gives the Poisson equation of momentum potential as

$$\begin{aligned}
\nabla^2 \mathbf{A} &= \frac{\alpha_s}{4\pi} \int_V \mathbf{j}(\mathbf{r}', t) \nabla^2 \left(\frac{1}{r} \right) dV' \\
&= -\alpha_s \int_V \mathbf{j}(\mathbf{r}, t) \delta(\mathbf{r} - \mathbf{r}') dV' = -\alpha_s \mathbf{j}.
\end{aligned} \tag{B15}$$

The curl of a curl field can be obtained based on Eqs. (B10) and (B15) as

$$\nabla \times \mathbf{C} = \nabla \times (\nabla \times \mathbf{A})$$

$$\equiv \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} = \nabla D + \alpha_s \mathbf{j}. \tag{B16}$$

Substituting Eq. (B11) into Eq. (B16), we obtain

$$\nabla \times \mathbf{C} = \alpha_s \mathbf{j} - \frac{1}{c^2} \frac{\partial \mathbf{G}}{\partial t}. \tag{B17}$$

Applying the curl operator on the leftmost side of Eq. (B16) and using the identity $\nabla \cdot \mathbf{C} \equiv 0$, we have

$$\nabla \times (\nabla \times \mathbf{C}) \equiv \nabla(\nabla \cdot \mathbf{C}) - \nabla^2 \mathbf{C} = -\nabla^2 \mathbf{C}. \tag{B18}$$

Applying the curl operator on the rightmost side of Eq. (B16) and using the identity $\nabla \times (\nabla D) \equiv 0$, we have

$$\nabla \times (\nabla D + \alpha_s \mathbf{j}) = \alpha_s \nabla \times \mathbf{j}. \tag{B19}$$

Because Eq. (B18) equals Eq. (B19), we can obtain the Poisson equation of curl field as

$$\nabla^2 \mathbf{C} = -\alpha_s \nabla \times \mathbf{j}. \tag{B20}$$

REFERENCES RÉFÉRENCES REFERENCIAS

1. J. M. Overduin and P. S. Wesson, *Dark Sky, Dark Matter* (Institute of Physics Pub, Bristol, 2003).
2. N. C. M. Martens and D. Lehmkuhl, *Dark matter = modified gravity? Scrutinising the spacetime-matter distinction through the modified gravity/dark matter lens*, Studies in History and Philosophy of Modern Physics 72 237 (2020). <https://doi.org/10.1016/j.shpsb.2020.08.003>
3. G. Bertone, D. Hooper and J. Silk, *Particle dark matter: evidence, candidates and constraints*, Phys. Rep. 405 279 (2005). <https://doi.org/10.1016/j.physrep.2004.08.031>
4. V. Trimble, *Existence and nature of dark matter in the universe*, Ann. Rev. Astro. Astrophys. 25 425

(1987). <https://doi.org/10.1146/annurev.aa.25.090187.002233>

5. T. Clifton, P. G. Ferreira, A. Padilla and C. Skordis, *Modified gravity and cosmology*, Phys. Rep. 513(1) 1 (2012). <https://doi.org/10.1016/j.physrep.2012.01.001>
6. M. Milgrom, *A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis*, Astrophys. J. 270 365 (1983). <https://doi.org/10.1068/161130>
7. Z. C. Liang, *Modeling of real particles*, J. Phys. Conf. Ser. 1391 012026 (2019). <https://doi.org/10.1088/1742-6596/1391/1/012026>
8. Z. C. Liang, *Outline of real physics*, Global Journal of Science Frontier Research: A 20(3) 9 (2020). <https://doi.org/10.34257/gjsfrvol20is3pg9>
9. Z. C. Liang, *Motion, energy, and state of body particle system*, Theoretical Physics 4 66 (2019). <https://doi.org/10.22606/tp.2019.42003>
10. Z. C. Liang, *Motion of elastic particles and spectrum of hydrogen atoms*, Global Journal of Science Frontier Research: F 19(4) 19 (2019). <https://doi.org/10.34257/gjsfrvol19is4pg19>
11. Z. C. Liang, *Cluster ensemble statistics of body particle system*, New Horizons in Mathematical Physics 3 53 (2019). <https://doi.org/10.22606/nhmp.2019.32002>
12. Z. C. Liang, *The origin of gravitation and electromagnetism*, Theoretical Physics 4 85 (2019). <https://doi.org/10.22606/tp.2019.42004>
13. Z. C. Liang, *Essence of light: particle, field and interaction*, Proc. SPIE 10755 107550I (2018). <https://doi.org/10.1117/12.2316422>
14. Einstein himself was deeply concerned regarding the open position between the continuity of gravitational theory and the discreteness of quantum theory. In his last letter to Besso (1954), Einstein stated that “It is quite possible that physics cannot be founded on the concept of field—that is to say—on continuous elements. However, then, out of my whole castle in the air—including the theory of gravitation, but also most of the current physics—there would remain almost nothing.”
15. S. L. Jin and Y. L. Ma, *Theoretical Mechanics* (Higher Education Press, Beijing, 1985).
16. S. H. Guo, *Electrodynamics* (Higher Education Press, Beijing, 2008).
17. S. P. Wang, *Theory and Application of Information Optics* (Beijing University of Posts and Telecommunications Press, Beijing, 2004).
18. M. Born and E. Wolf, *Principle of Optics* (Pergamon Press, Exeter, 1980) 6th ed.
19. M. Planck, *Zur Theorie des Gesetzes der Energieverteilung im Normalspectrum*, Verhandl. Dtsch. phys. Ges. 2 237 (1900).
20. D. J. Fixsen, *The temperature of the cosmic microwave background*, Astrophys. J. 707(2) 916 (2009). <https://doi.org/10.1088/0004-637X/707/2/916>
21. W. Thomson, *On the ultramundane corpuscles of Le Sage, also on the motion of rigid solids in a liquid circulating irrotationally through perforations in them or in a fixed solid*, The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 45 321 (1873). <https://doi.org/10.1080/14786447308640865>
22. J. C. Maxwell, *A Treatise on Electricity and Magnetism* (Cambridge University Press, Cambridge, 1873). <https://doi.org/10.1017/CBO9780511709340.028>
23. A. Einstein, *Sidelights on Relativity* (Methuen, London, 1922).