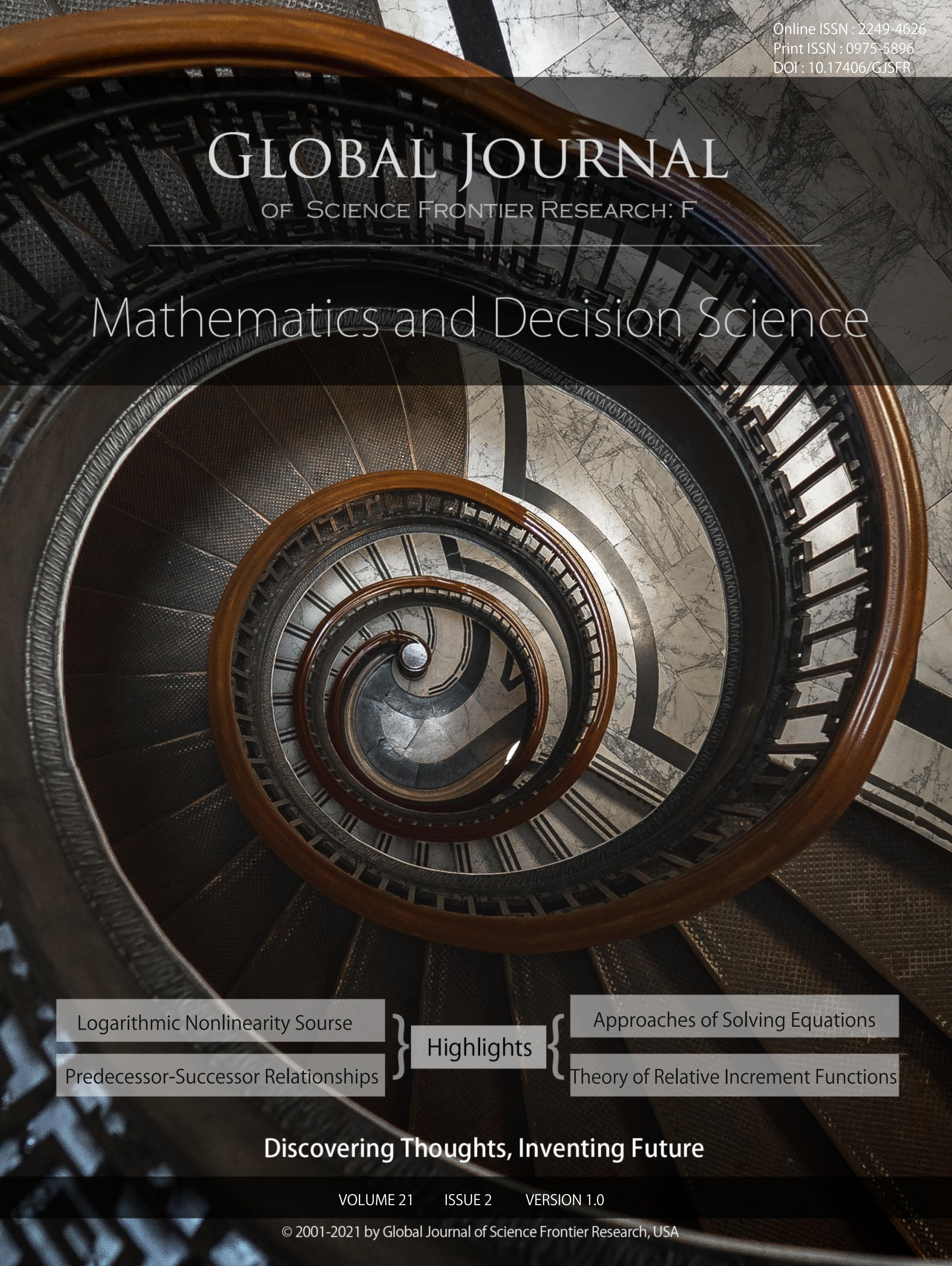

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Predecessor-Successor Relationships

Highlights

Approaches of Solving Equations

Theory of Relative Increment Functions

Discovering Thoughts, Inventing Future

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A Note on Simulating Predecessor-Successor Relationships in Critical Path Models

By Gregory L. Light
Providence College

Abstract- For any *n* entities, we exhaust all possible ordered relationships, from rank (or the highest number of connections in a linear chain, comparable to matrix rank) 0 to (n - 1). As an example, we use spreadsheets with the "RAND" function to simulate the case of *n = 8* with the order-length = *3*, as from a total of *10000* possibilities by the number of combinations of selecting 2 (a pair of predecessor-successor) out of 5 (= card{A, B, C, D} + 1) matching-destinations followed by an exponentiation of 4 (= 8 – card{A, B, C, D}). Since the essence of this paper is about ordered structures of networks, our findings here may serve multi-disciplinary interests, in particular, that of the critical path method (CPM) in operations with management. In this connection, we have also included, toward the end of this exposition, a linear algebraic treatment that renders a deterministic mathematical programming for optimal predecessor-successor network structures.

Keywords: *CPM simulation, directed graphs, ordered networks, circuitry designs, logical flows, transmission patterns, optimal organizational rank.*

GJSFR-F Classification: *MSC 2010: 91G50*

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A Note on Simulating Predecessor-Successor Relationships in Critical Path Models

Gregory L. Light

Abstract- For any *n* entities, we exhaust all possible ordered relationships, from rank (or the highest number of connections in a linear chain, comparable to matrix rank) 0 to (n - 1). As an example, we use spreadsheets with the "RAND" function to simulate the case of $n = 8$ with the order-length $= 3$, as from a total of *10000* possibilities by the number of combinations of selecting 2 (a pair of predecessor-successor) out of 5 (= card{A, B, C, D} + 1) matching-destinations followed by an exponentiation of 4 (= 8 – card{A, B, C, D}). Since the essence of this paper is about ordered structures of networks, our findings here may serve multi-disciplinary interests, in particular, that of the critical path method (CPM) in operations with management. In this connection, we have also included, toward the end of this exposition, a linear algebraic treatment that renders a deterministic mathematical programming for optimal predecessor-successor network structures.

Keywords: *CPM simulation, directed graphs, ordered networks, circuitry designs, logical flows, transmission patterns, optimal organizational rank.*

I. Introduction

In simulating alternative optimal solutions of a critical path model [1], it is a simple matter to vary the needed time durations for the activities of the project in question, but it becomes more challenging to make variations in the involved predecessor-successor relationships as the geometric patterns of the associated network flow charts lend to overwhelming varieties. We thus consider the apparatus of the activity-node incidence matrix composed of entries –*(activity i, incidence time node j)*˝in values-*1, 0* and *+1,* but this is where ambiguities arise: the "start time" is ill-defined since activity *i* can start at an earliest time or else at a later time point [2]. The fact of the matter is that the time interval in any pair of time nodes (points) is totally independent of the needed time interval for activity *i* to be completed. To be sure, this predecessor-successor relationship can be devoid of the connotation of time altogether [3], [4]. For example, of eight relatives (activities) in a family gathering with the longest lineage being from great grandfather (activity A), to grandfather (activity B), to father (activity C), and to son (activity D), how many incidence matrices (modulo permutation of the eight activity-labels) are possible? That is, we are pursuing the general theme of network or organization structure that has a directional orientation.

As such, in this paper we define any time point (that is yet to be solved for in a critical path model, "*CPM*") to be the contact time from some predecessor(s) to its (their) successor(s), which coexist in the *same* (time) column respectively as *1('s)* and -

Author: *Department of Finance, Providence College, Providence, Rhode Island 02918 USA. e-mail: glight@providence.edu*

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1. Conant, D. D., Examining critical and near-critical paths: an Excel-based classroom exercise, *J. Edu. Bus.*, 93(6), 2018, 285-291.

1('s) of the incidence matrix, with the meaning of "contact" being the act of transferring the finished work from a predecessor to any (all) of its successors. Every activity has thus exactly two contact time points, appearing as *(-1)* and *(+1)* in its associated row amid all other *0's*. In this way, by subtracting an earlier time point from a later time point, we obtain a time interval, which we constrain to be greater than or equal to any given values.

As our study here is about network or, in general, graph theory, what we have observed may have multi-disciplinary applications, such as traffic control [5], [6], civil engineering [7], electric/data-logic circuitry [8], [9], and neural network [10].From an extensive literature research, we did not find any similar work that used spreadsheets to simulate predecessor-successor relationships, with the closest being [11].

In the next section, we will first obtain a formula that calculates the total number of activity-node incidence matrices given some n activities and then show by an example how to simulate all the possible activity-node incidence matrices for the case of five contact time points of eight activities. Lastly, we will close with a summary remark in Section 3, where we outline a mathematical schema to solve for an optimal network or organization structure in general.

II. Analysis

Without loss of generality, we consider a project comprising of *8* activities, *A, B, C, D, E, F, G,* and *H*, and pursue the total number of network possibilities as built on predecessor-successor relationships in the context of critical path analysis. Then a familiar spreadsheet treatment is, as an example, to enter − *1*for activity *C* in the row of *C* that intersects a column as associated with a (yet-to-be-solved-for) time point in a tabular array, and + *1* in the same row of another column for a later time point. In this way the "*sum product*" of *Row C* with the header time row gives the time interval between the "contact time" of *C* with its immediate predecessor(s) and that with its immediate successor(s); this time interval is to be constrained to be greater than or equal to the needed time interval for *C* to be completed. The objective is then to minimize the ending time point of the "last activity/activities" (which has/have no successors). A natural question arises: viz., how many network flowcharts of the predecessor-successor relationships are possible? This answer can be found by a consideration of the number of columns as the contacting time points, where the least is *2* - -when all the eight activities have no predecessors or successors with *-1's* all in the first column headed by time *0*, and*1's* all in the second column headed by the longest time needed by the activities, and also where the greatest is *9* - - when the *8* activities form a linear chain of predecessor-successor relationship, from *A* to *B*, to *C*, ..., and to *H*. As such, the task now is to pursue the cases of the number of columns from *3* to *8*.

For the case of exactly *3* columns, as an illustration, we may have the following spreadsheet depiction:

Ref

8. Keller, J., Rauber, T. and Rederlechner, B., Scalability Analysis for Conservative Simulation of Logical Circuits, *VLSI Design*, 9 (3), 1999, 219-235.

Notes

Activity	0		
A	-1	1	0
B	0	-1	1
C	-1	0	1
D	-1	0	1
E	-1	1	0
F	-1	1	0
G	-1	0	1
H	0	-1	1

indicating the existence of precisely two contact time points. As such, there must exist two activities that form a linear chain; find such two activities and label them by *A* and *B*. Since each activity row has one (*-1*) and one (*+1*), by combinations of selecting two cells for these two non-zero numbers from three available cells in a row, we obtain 3 varieties for any of the remaining six rows, from *C* to *H*, i.e., $3^6 = 729$ ways. Incidentally, the above display, as one of the 729 possibilities, translates to

Activity	Immediate predecessors
A	-
B	A, E, F
C	-
D	-
E	-
F	-
G	-
H	A, E, F

Similarly, for the case of exactly *4* columns, we may have, as an example, the following spreadsheet depiction:

Activity	0			
A	-1	1	0	0
B	0	-1	1	0
C	0	0	-1	1
D	0	-1	0	1
E	-1	0	1	0
F	-1	1	0	0
G	0	0	-1	1
H	0	-1	1	0

showing the existence of a combination of *C(4, 2)*that is to be raised to the *5^th^* power, i.e., a total of 7,776 ways. It is now clear that the grand total number of the predecessor-successor relationships of any eight activities is (in *M.S. Excel* notation):

C(2, 2)ˆ7 + C(3, 2)ˆ6 + C(4, 2)ˆ5 + C(5, 2)ˆ4 + C(6, 2)ˆ3 + C(7, 2)ˆ2 + C(8, 2)ˆ1 + C(9, 2)ˆ0

= 1 + 729 + 7776 + 10000 + 3375 + 441 + 28 + 1 = 22351;

thus, for any $n > 1$ activities, we have the total possible number of the activity-node incidence matrices equal to:

$$\sum_{m=2}^{n+1} C(m,2)^{n-m+1} \; .$$

We now present a way to simulate the predecessor-successor relationships for eight activities as above and for the case of five columns, which incidentally has the greatest number of the ordering varieties - - *10,000*, among which being:

Activity	0				
A	-1	1	0	0	0
B	0	-1	1	0	0
C	0	0	-1	1	0
D	0	0	0	-1	1
E	-1	1	0	0	0
F	-1	1	0	0	0
G	0	0	0	-1	1
H	0	-1	0	0	1

where the first four rows are (fixed) values and the remaining four rows carry the same set of five formulas; in matrix notation, we have for activity E

- for *entry (5, 1)* = *-1* (which was in cell "$D44"):=-ROUND(RAND(),0)
- for *entry (5, 2)* = *1*:=IF($D44=-1, ROUND(RAND(),0),-ROUND(RAND(),0))
- for *entry (5, 3)*= *0*:=IF(AND(MIN($D44:E44)=0, MAX($D44:E44)=0), -ROUND(RAND(),0), IF(MAX($D44:E44)=1,0, ROUND(RAND(),0)))
- for *entry (5, 4)* = *0*:=IF(AND(MIN($D44:F44)=0,MAX($D44:F44)=0), -1, IF(MAX($D44:F44)=1,0,ROUND(RAND(),0)))
- for *entry (5, 5)* = *0*:=IF(MAX($D44:G44)=1,0,1)

Note that one can control the probabilities of the distribution of *-1, 0,* and *1* over the available cells in a row by suitably modifying the formula *RAND*. With the needed time durations of the activities also randomly generated, one proceeds to solve for a critical path and, if so desired, uses a spreadsheet macro to generate any collection of examples, such as

Activity	0	40	53	78	106	106
A	-1	1	0	0	0	40
B	0	-1	1	0	0	13
C	0	0	-1	1	0	25
D	0	0	0	-1	1	28
E	0	-1	1	0	0	13
F	-1	0	0	0	1	106
G	-1	1	0	0	0	40
H	0	0	-1	1	0	25

or

Notes

Notes

Activity	0	26	62	106	156	156
A	-1	1	0	0	0	26
B	0	-1	1	0	0	36
C	0	0	-1	1	0	44
D	0	0	0	-1	1	50
E	0	-1	0	0	1	130
F	-1	1	0	0	0	26
G	0	0	-1	1	0	44
H	0	0	0	-1	1	50

III. Summary

In this note, we have obtained a general formula that calculates the total number of predecessor-successor relationships of n entities, or equivalently, the total number of activity-node incidence matrices of n activities, and we have also shown a way to simulate them on spreadsheets. As such, one may gain insight into some "optimal" network structures given the individual activities' required space time lengths. In fact, future studies may pursue such optimization by including the whole activity-node incidence matrix, of n (activities) by $(n + 1)$ (time points, beginning with 0), as decision variables in *CPM* problems with the following added constraints:

(1) The $(1, 1)$*-entry* $= -1$, with the entire (first) column free from 1; this can readily be accomplished by constraining this column to be less than or equal to 0.

(2) The $(n, n+1)$*-entry* $= 1$, with the entire (last) column free from -1; this can readily be accomplished by constraining this column to be greater than or equal to 0.

(3) Each row contains exactly one -1 and one $+1$. Here, consider taking the exponential (*EXP* in *M.S. Excel*) of every entry for any of the n rows and constrain every row sum equal to $e^{-1} + e^{1} + (n - 2)$.

(4) For any column from the 2^{nd} to the n^{th}, it is either filled entirely with *0's* or with at least one pair of $(+1, -1)$ for the simple reason that these $(n - 1)$ columns represent predecessor-successor contact time points. Here, consider: (i) adding a positive integer $a > 1$ uniformly to each of the n entries in any column for these $(n - 1)$ columns, (ii) taking the product of these n transformed entries for any of these $(n - 1)$ columns, $[(a + 1)^i a^j (a - 1)^k]$, $i + j + k = n$, $i, j, k >= 0$, (iii) dividing the preceding product by $(a^2\text{-}1)$ and denoting the remainder by R, (iv) setting the column-sum-of-squares for each of these $(n - 1)$ columns equal to S, and (v) constraining the product $R \times S = 0$. Then, if $R = 0$, there exists at least one pair of $(1, -1)$ in this column; otherwise, $S = 0$, showing the entire column being of *0's*. The case of $\{1, 0, \ldots\}$, or $\{0, -1, \ldots\}$, without the opposite sign, cannot occur since it would render $R \times S > 0$. The significance of such implementation is that one would be solving for the optimal "rank" (the highest order/length of a linear chain) of any network (organization). We caution, however, that to implement the above approach, one would need to use computing packages more robust than ordinary spreadsheets. In addition, the optimal solution cannot be unique since the trivial column(s) composed of all *0's* can be placed anywhere within these $(n - 1)$ columns. This undesirable situation can be removed by constraining the column-sum-of-squares to be non-increasing from the 2^{nd} to the n^{th} column so that the "*0-column(s)*"

can precede only the $(n+1)^{st}$ column. Note that this interchangeability among the "middle" $(n-1)$ columns is owing to the additive commutativity of vector spaces, $a_2v_2 + a_3v_3 + \ldots + a_nv_n$; i.e., the decision variables $a_2, a_3, \ldots, a_n$ as the $(n-1)$ time points in *CPM* can be re-arranged. To be clear, for any critical activity (row) i, one has $a_j - a_k =$ *the needed time for the completion of i,* with a_j and a_k uniquely solved in a *CPM*; however, for any predecessor-successor pair of non-critical activities, their contact time a_s has slack variability so that the employed algorithm may present different solutions of a_s upon permutations of $a_2, a_3, \ldots, a_n$. In summary, one may thus arrive at a *CPM* with an optimal predecessor-successor structure in network designs for diverse fields.

Notes

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Estimating Distributions using the Theory of Relative Increment Functions

By Sereko Kooepile-Reikeletseng
University of Botswana

Abstract- Bounded growth processes can be modelled, approximately by different mathematical models. The challenge for statisticians and mathematicians is finding suitable models for these processes. In this paper we illustrate a non-parametric method, using the the theory of relative increment functions, of estimating density functions of these processes. For a long time, mathematicians attempted to describe the cumulative prevalence of caries with the assumption that there is a mathematical model that would describe the caries prevalence and may be used for predicting caries incidences. In 1960 Porter and Dudman [12] introduced The relative increment function and called it the relative increment of decay as they designed it to compare dental caries increments among children. Further studies of this led to the motivation that the best suitable model for describing the cumulative prevalence of caries should be chosen from a set of distributions that have relative increment functions with the same monotonic behaviour as the relative increment of decay [1]. We illustrate how relative increment functions may be used to estimate the unknown indefinitely smooth probability density function of unimodal populations.

Keywords: relative increments, distribution, continuous. unimodal.

GJSFR-F Classification: MSC 2010: 12E12

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Sereko Kooepile-Reikeletseng

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Keywords: *relative increments, distribution, continuous. unimodal.*

I. Introduction

Density estimation using non-parametric methods was first proposed by Fix and Hodges in 1951 as a way of moving away from distributional assumptions which at times restrict estimation. The methods proposed by Fix and Hodges were the Histograms, Naive estimator, Kernel estimator , Nearest neighbour estimator,Variable Kernel estimator and many others. [14].

In 1989 [16] proposed the use of relative increment functions for density estimation.

The relative increment function, h, of a distribution function, F, is defined as

$$h_{F(x)}(x) = \frac{F(x+a) - F(x)}{1 - F(x)} \text{ where } a = x_{k+1} - x_k.$$

He defined the function

$$\Psi(x) = \frac{(F(x) - 1).f'(x)}{f^2(x)}$$

and used the fact that if $\Psi < 1(\Psi > 1)$, then the function h strictly increases (strictly decreases) to classify some well known distribution functions according to their monotonic behaviour. Szabo [20] developed an algorithm for finding the distributions of unknown unimodal population by eliminating a large class of continuous distributions whose behaviour of relative increments do not match the behaviour

Author: *Department of Mathematics, University of Botswana. e-mail: reikeletsengsn@mopipi.ub.bw*

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of the empirical relative increment functions of the population being investigated. We illustrate this method by two numerical examples that showed that when the distribution relative increments behaves the same way as the empirical relative increments, the fit is superior to the ones with different monotonic behaviour of relative increments.

In this section we explain how we use relative increment functions to estimate density. Assume we have a large sample of a continuous random variable. We form the empirical cumulative distribution function F_{emp} at equidistant points x_k. Our aim is to find the smooth unimodal distribution which our sample belongs to. We assume that the distribution is a indefinitely smooth unimodal distribution whose probability density function has at most two points of inflection.

Suppose all the intervals $I_k = [x_{k-1}, x_k]$ have the same length a, for $k = 1, 2, ..., n$. Let v_k be the frequency distribution defined as the number of sample values in I_k, then we have relative frequency $r_k = \frac{v_k}{N}$ and the cumulative relative frequency as $y_k = \sum_{j \le k} r_j$, for all k.

The empirical cumulative distribution function F_{emp} whose points of discontinuity are at equidistant points, x_k, is therefore given by y_k, so

$$F_{emp}(x_k) = y_k \text{ for } k = 0, 1, ..., n$$

Define the relative increment function, h, for a distribution with cumulative distribution function $F(x)$ as

$$h_F(x) = \frac{F(x+a) - F(x)}{1 - F(x)} \text{ where } a = x_{k+1} - x_k$$

and define the empirical relative increment function, h_{emp}, for our population as

$$h_{emp}(x_k) = \frac{y_{k+1} - y_k}{1 - y_k} \text{ for } k = 0, 1, ..., n-1$$

Assume we have a large sample of a continuous random variable. We form the empirical cumulative distribution function F_{emp} at equidistant points x_k. Our aim is to find the smooth unimodal distribution which our sample belongs to. We assume that the distribution is a indefinitely smooth unimodal distribution whose probability density function has at most two points of inflection.

From this sample we calculate the empirical relative increment function $h_{emp}(x_k)$. If the monotonic behaviour of the empirical relative increment function $h_{emp}(x_k)$ is different from the monotonic behaviour of the theoretical relative increment function $h(x_k)$ of the cumulative distribution function $F(x)$, then we drop the corresponding smooth distribution $F(x)$. If $h(x_k)$ and $h_{emp}(x_k)$ have the same monotonic behaviour, we keep the corresponding cumulative distribution function $F(x)$ and put them in a class of possibilities, S.

From the set S a best fitting function is found by using the method of least squares. A distribution function $F(x) \in S$ providing the best fit to the cumulative relative frequency y_k such that $\sum_{k=1}^{n} [F(x_k) - y_k]^2$ is minimal

is selected or a distribution whose probability density function $f(x)$ provides the best fit to the relative frequencies r_k such that $\sum_{k=1}^{n} [f(x_k) - r_k]^2$ is minimal is selected.

Notes

This method can also be used to model bounded growth processes. Let g_k be a sequence of values measured at some equidistant points x_k. An upper bound B for g_k has to be determined such that B is greater than any value of g_k). To model the growth process of (x_k, g_k) we consider the transformed data

$$y_k = \frac{g_k}{B}(< 1)$$

as the values of of the empirical cumulative distribution function F_{emp} at the points x_k. The upper bound B is determined by building a parameter,B, into the distribution functions we want to fit, so instead of fitting $F(x)$, we fit $B * F(x)$. The estimated value of B gives the upper bound.

To use this method, we need to know the monotonic behaviour of relative increment functions h of distributions. A great number of classical smooth unimodal distributions has been investigated and classified according to the behaviour of their relative increments. These are listed in the following section.

II. Summary of Investigated Distributions

Here is a summary of distributions grouped according to the monotonic behaviour of their relative increment functions investigated by Szabo Z.[20] and myself.

2.1. The following probability distributions have increasing RIFs:

1. $F(x) = 1 - (-x)^k$ where $I = (-1, 0), k \in \mathbb{N}$
2. $F(x) = \sin x$ where $I = (0, \frac{\pi}{2})$
3. $F(x) = 1 + \tan x$ where $I = (-\frac{\pi}{4}, 0)$
4. $F(x) = 1 + \sinh x$ where $I = (\ln(\sqrt{2} - 1), 0)$
5. $F(x) = 2 - \cosh x$ where $I = (\ln(2 - \sqrt{3}), 0)$
6. $F(x) = 1 - x^2$ where $I = (-1, 0)$
7. $F(x) = \ln x$ where $I = (1, e)$
8. Uniform Distribution
9. $F(x) = (1 - \exp(-\lambda x))^k$ where $I = (0, \infty), \lambda > 0,\ k > 1$
10. $F(x) = 1 - \exp(-\lambda.e^x)$ where $I = (-\infty, \infty),\ \lambda > 0$
11. $F(x) = (1 + e^{-x})^{-k}$ where $I = (-\infty, \infty),\ k > 0$
12. $F(x) = 2^{-k}.(1 - \tanh(x))^k$ where $I = (-\infty, \infty),\ k > 0$
13. Logistic Distribution

$$F(x) = (1 + e^{-\lambda x})^{-1} \text{ where } I = (-\infty, \infty),\ \lambda > 0$$

14. Fisher Distribution (or z-distribution)

$$F(x) = C.\int_{-\infty}^{x} e^{nt}.(1 + k.e^{2t})^{-\alpha} dt, \text{ where } I = (-\infty, \infty),\ n, n' \in \mathbb{N}\ k = \tfrac{n}{n'},$$

$$\alpha = \tfrac{n+n'}{2},\ C = 2.k^{\frac{n}{2}}.\Gamma(\alpha).\left[\Gamma(\tfrac{n}{2}).\Gamma(\tfrac{n'}{2})\right]^{-1}$$

15. Weibull Distribution when $\alpha > 1$

$$F(x) = 1 - \exp(-\lambda x^{\alpha}) \text{ where } I = (0, \infty),\ \lambda > 0$$

16. Extreme value Distribution

$$F(x)=\int_{-\infty}^{x}\exp(-t-e^{-t})dt \text{ where } I=(-\infty,\infty)$$

17. $F(x)=1-2[c.(1+e^{x})^{k}-c+2]^{-1}$ where $I=(-\infty,\infty),\ c>0,k=1,2$

18. Normal Distribution

$$F(x)=K.\int_{-\infty}^{x}\exp(-\tfrac{1}{2}.\sigma^{-2}.(t-m)^2)dt,\ \text{where } I=(-\infty,\infty),\ \sigma>0,\ K=\tfrac{1}{\sigma\sqrt{2\pi}}$$

19. Special Gamma Distribution

$$F(x)=K.\int_{0}^{x}t^{\alpha-1}\exp(-\lambda t)dt \text{ where } I=(0,\infty),\ \alpha>1,\ \lambda>0,\ K=\tfrac{\lambda^{\alpha}}{\Gamma(\alpha)}$$

20. Beta Distribution of the first kind

$$F(x)=C.\int_{0}^{x}t^{\alpha}(1-t)^{\beta}dt \text{ where } I=(0,1),\ \alpha,\beta>-1,\ C=\tfrac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)}$$

21. $F(x)=C.\int_{-s}^{x}(1-\frac{t^2}{s^2})^{n}dt$ where $I=(-s,s),\ s>0,\ C=\left[s.\beta(\frac{1}{2},n+1)\right]^{-1}$

where n is a positive integer.

22. Maxwell Boltzmann distribution

$$f(x)=\sqrt{\tfrac{2}{\pi}}\tfrac{1}{\sigma^3}x^2\exp\left(\tfrac{-x^2}{2\sigma^2}\right) \text{ where } I=(0,\infty),\ \sigma>0$$

23. $f(x)=2\lambda e^{-\lambda x(1-e^{-\lambda x})}$ where $I=(0,\infty)$

24. $f(x)=c(1-\frac{x^2}{a^2})^{n}$ where $I=(-a,a),\ a>0,\ n\in\mathbb{N}$ and $c=\frac{1}{a.B\left(\frac{1}{2},n+1\right)}$

25. Rayleigh Distribution $f(x)=\frac{x}{\sigma^2}\exp\left(\frac{-x^2}{2\sigma^2}\right)$ where $I=(0,\infty),\ \sigma>0$

26. $f(x)=\frac{x}{\sqrt{1-x^2}}$ where $I=(0,1)$

27. $f(x)=kx^{c-1}e^{-\frac{x^2}{2}}$ where $I=(0,\infty),\ c>1,$ and $k>0$

28. Reciprocal distribution

$$f(x)=\tfrac{\ln x-\ln a}{\ln b-\ln a} \text{ where } I=[a,b],\ a,b\in\mathbb{R},\ 0<a<b,\ \text{and } \tfrac{a}{b}<e$$

29. $f(x)=c.\exp\left(\arctan\left(x\right)\right)$ where $c=\frac{1}{\int_0^a\exp\left(\arctan x\right)dx}$, and $I=(0,a),\ a>0$

30. $F(x)=1-x^2$ where $I=(-1,0)$

312. $F(x)=\ln x$ where $I=(1,e)$

32. Nakagami distribution

$$f(x)=\tfrac{2n^n}{\Gamma(n)\Omega^n}x^{2n-1}\exp\left(\tfrac{-n}{\Omega}x^2\right) \text{ where } I=(0,\infty)\ n>\tfrac{1}{2},\ \text{and } \Omega>0$$

Notes

2.2. The following probability distributions have decreasing RIFs:

1. $F(x) = 1 - x^{-\lambda}$, where $I = (1, \infty)$, and $\lambda > 0$
2. $F(x) = 1 - (\ln x)^{-\lambda}$, where $I = (e, \infty)$, and $\lambda > 0$
3. $F(x) = 1 - (\ln(\ln x))^{-\lambda}$, where $I = (e^e, \infty)$, and $\lambda > 0$
4. Weibull Distribution when $0 < \alpha < 1$

$$F(x) = 1 - \exp(-\lambda.x^{\alpha}),\ \text{where } I = (0, \infty),\ \text{and } \lambda > 0$$

5. $F(x) = 1 - a\exp(-b.x) - c.\exp(-d.x)$, where $I = (0, \infty)$, and $a, b, c, d > 0, a + c = 1$
6. $F(x) = 1 - \sum_{j=1}^{N} a_j \exp(-b_j.x)$, where $I = (0, \infty)$, and $a_j, b_j > 0, \sum_{j=1}^{N} a_j = 1$
7. Pareto Distribution of the third kind

$$F(x) = 1 - k.\exp(-b.x).x^{-a},\ \text{where } I = (k, \infty),\ \text{and } a, b, k > 0$$

8. Special Chi-Square Distribution

$$F(x) = K.\int_0^x t^{\frac{-1}{2}}.\exp(\tfrac{-t}{2})dt,\ \text{where } I = (0, \infty),\ \text{and } K = \frac{1}{\sqrt{2}\Gamma(\frac{1}{2})}$$

9. Pareto Distribution of the second kind

$$F(x) = 1 - x^{-k},\ \text{where } I = (1, \infty),\ \text{and } k > 0$$

10. Special Gamma Distribution

$$F(x) = K.\int_0^x t^{\alpha-1}.\exp(-\lambda t)dt,\ \text{where } I = (0, \infty),\ K = \frac{\lambda^{\alpha}}{\Gamma(\alpha)},\ \lambda > 0, \alpha < 1$$

11. $f(x) = \lambda x^{-1}(\ln x)^{-\lambda-1}$ where $I = (\exp, \infty)$, and $\lambda > 0$

2.3. The Exponential Distribution Function

$$F(x) = 1 - \exp[-\lambda(x - a)],\ \text{where } I = (a, \infty),\ \text{and } \lambda > 0$$

has a constant relative increment function.

2.4. The following probability distributions have RIFs that increase first and, having culminated, they decrease:

1. Cauchy Distribution

$$F(x) = \tfrac{1}{2} + \tfrac{1}{\pi}\tan^{-1} x,\ \text{where } I = (-\infty, \infty)$$

2. Inverse Gaussian Distribution

$$F(x) = \int_0^x (\tfrac{2\pi t^3}{\lambda})^{\frac{-1}{2}}.\exp(-\lambda.\tfrac{(t-m)^2}{(2m^2.t)})dt,\ \text{where } I = (0, \infty),\ \lambda > 0, m > 0$$

3. Lognormal Distribution

$$F(x) = \int_0^x \frac{1}{t\sigma\sqrt{2\pi}} \exp\{-\frac{(\ln t - \mu)^2}{2\sigma^2}\}dt,\ \text{where } I = (0, \infty),\ ,\ \sigma > 0, -\infty <$$

$\mu < \infty$

Notes

4. Beta type II distribution

$$f(x) = C\frac{x^p}{(1+x)^{p+q}} \text{ where } I = (0,\infty),\ p,q > 0,\ \text{and } C > 0$$

5. Burr type XII distribution

$$f(x) = ck\frac{x^{c-1}}{(1+x^c)^{k+1}} \text{ where } I = (0,\infty)\ c > 0,\ \text{and } k > 0$$

6. $f(x) = \frac{k}{1+x^4}$ where $I = \mathbb{R}$ and $k > 0$

7. $f(x) = cx^{-n}\exp\left(\frac{-k}{x}\right)$ where $I = (0,\infty)$ and $c,k > 0$

8. Frechet Distribution $F(x) = \exp\left(-(\frac{x-n}{t})^{-a}\right)$ where $I = (n,\infty),\ a,t \in (0,\infty),$ and $n = t\left(\frac{a}{a+1}\right)^{\frac{1}{a}}$.

9. Gumbel Distribution $F(x) = e^{-bx^{-a}}$ where $I = (0,\infty)$ and $a,b \in \mathbb{R}^+$ and $m = \sqrt[a]{\frac{ab}{a+1}}$

2.5. The following probability distributions have RIFs that decrease first and, having reached their minima, they increase:

1. $F(x) = 1 + \frac{2}{\pi}\arcsin x,$ where $I = (-1,0)$
2. $F(x) = \sqrt{x},$ where $I = (0,1)$
3. $F(x) = (1-x^2)^{\frac{1}{2}}$ where $I = (-1,0)$
4. Reciprocal distribution

$$f(x) = \frac{\ln x - \ln a}{\ln b - \ln a} \text{ where } I = [a,b],\ a,b > 0,\ \text{and } \frac{a}{b} > e$$

III. Numerical Examples

In this section, we illustrate the method described in section 2 by two examples.

1. The distribution of the the population of Botswana.
2. The distribution of people living with HIV globally.

Numerical Example 1: Botswana Population Growth. Below is a table of the population of Botswana from 1960 to 2012 in 5 year periods. This data was obtained from the World data bank.

Table 6.1: Botswana Population

$x_k(Time)$	g_k(Population)
5	579729
10	671416
15	793164
20	960807
25	1146205
30	1343440
35	1544865
40	1724924
45	1854739
50	1951715

We wish to find the probability distribution function of this sample.

Notes

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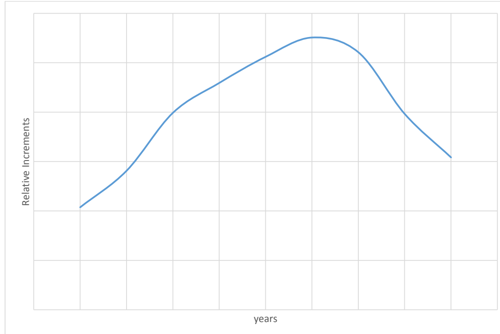


Figure 6.1: Graph of empirical relative increments

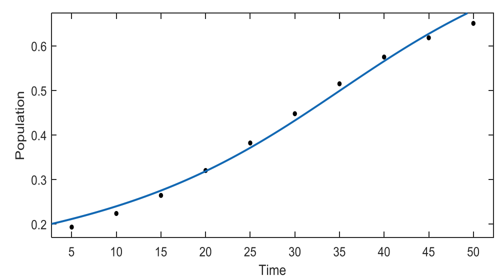
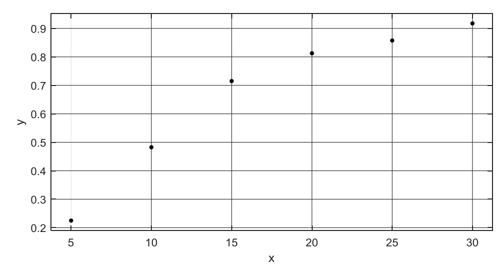
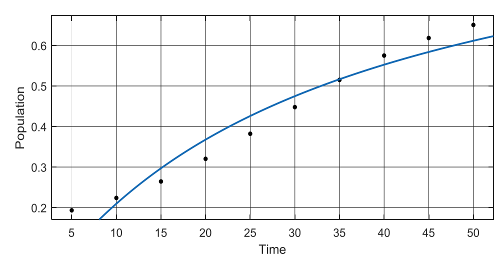
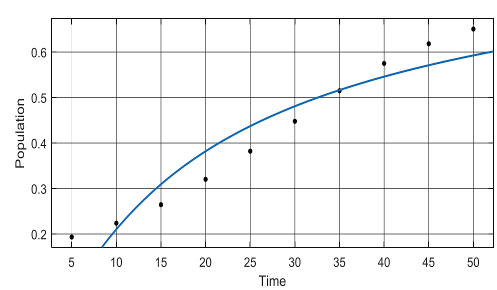
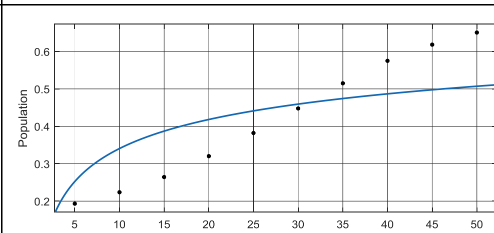
Figure 6.1 shows the empirical relative increment function of the distribution of the population. We see that the relative increments increase and then decrease. The distribution functions of section 2.4 display the same monotonic behaviour of first increasing and then decreasing. For these models, the values of B ranged between $24,900,000$ and $28,600,000$. We therefore picked $3,000,000$ as a reasonable estimate of the upper bound for all the models. Below is the table of time in years and the adjusted values of the population.

Table 6.2: Adjusted Population

$x_k(Time)$	y_k(Adjusted Population)
5	0.193243
10	0.223805
15	0.264388
20	0.320269
25	0.382068
30	0.447813
35	0.514955
40	0.574975
45	0.618246
50	0.650572

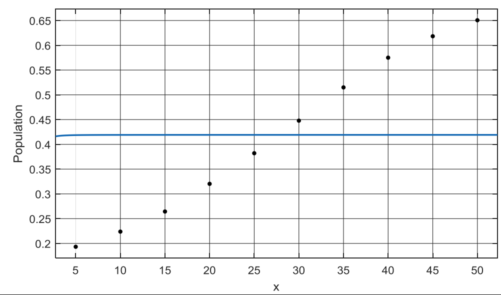
The adjusted population values were fitted to the distributions in section 6.3.4 as they exhibit the same monotonic behaviour of relative increment functions as that of our data. Table 6.2 shows the fitted values of the distributions.

Table 6.3: Fitted Models (Botswana Population)

Distribution function	Fit Results	Fitted Curve
Cauchy Distribution $$\frac{1}{2}+\frac{1}{\pi}*\tan^{-1}\left(\frac{x-\alpha}{\beta}\right)$$	Coefficients (with 95% confidence bounds): $p1 = 35.06$ $p2 = 23.53$ *Goodness of fit*: SSE: 0.002344	
Inverse Gaussian Distribution General model: $$1/2\ \ (1+erf(\sqrt{(\lambda/2x)}\ (x/\mu-1)))+1/2\ \ e\text{^}(2\lambda/\mu)\ (1-erf(\sqrt{(\lambda/2x)}\ (x/\mu+1)))$$	Coefficients (with 95% confidence bounds): a = 0.3371 b = 0.1622 Goodness of fit: SSE: 1.012	
Lognormal Distribution $$\frac{1}{2}*erfc\left(-\frac{\ln(x)-\mu}{\sigma\sqrt{2}}\right)$$ Where $erfc$ is the complementary error function.	Coefficients (with 95% confidence bounds): $\sigma = 2.95$ $\mu = 3.494$ Goodness of fit: SSE: 0.01796	
Beta type II Distrribution $$I_{\frac{x}{1+x}}(\alpha,\beta)$$ Where $I_x(\alpha,\beta)$ is the incomplete beta function.	Coefficients (with 95% confidence bounds): $\alpha = 9.512$ $\beta = 0.5614$ Goodness of fit: SSE: 0.02997	
Burr type XII Distribution $$1-(1+x^c)^{-k}$$ Where $$c,k>0$$	Coefficients (with 95% confidence bounds): $c = 7.823$ $k = 0.02311$ Goodness of fit: SSE: 0.09004	

Notes

Notes

<table>
<tr>
<td>Number 6
$$\frac{1}{8}k\sqrt{2}\left(\ln - \frac{x^2 + x\sqrt{2} + 1}{-x^2 + x\sqrt{2} - 1}\right) + 2\tan^{-1}\left(x\sqrt{2}+1\right) + 2\tan^{-1}\left(x\sqrt{2}-1\right))$$</td>
<td>Coefficients (with 95% confidence bounds):
$k = 0.6022$
Goodness of fit:
SSE: 0.3504</td>
<td>
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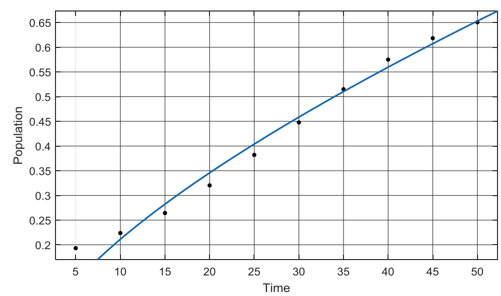
<tr>
<td>Number 7
$$cx^{-n}x\left(-\frac{k\,\mathrm{Ei}\left(1,\frac{k}{x}\right)}{x} + e^{-\frac{k}{x}}\right)$$
Where Ei is the exponential Integral</td>
<td>Coefficients (with 95% confidence bounds):
$c = 0.03116$
$k = -3.3$
$n = 0.1954$
Goodness of fit:
SSE: 0.006749</td>
<td>
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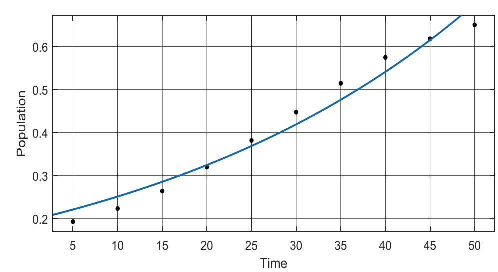
<td>Frechet
$$e^{-\left(\frac{x-m}{s}\right)^{-a}}$$
Where
a is the shape parameter
s is the scale parameter
m is the location parameter</td>
<td>Coefficients (with 95% confidence bounds):
$a = 2.002$
$s = 78.36$
Goodness of fit:
SSE: 0.007987</td>
<td>
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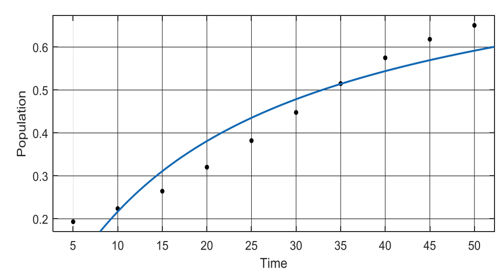
<tr>
<td>Gumbel
$$e^{-bx^{-a}}$$
where a is real and b is the shape parameter</td>
<td>Coefficients (with 95% confidence bounds):
$a = 0.6644$
$b = 7.06$
Goodness of fit:
SSE: 0.02744</td>
<td>
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</table>

Amongst these, the Cauchy distribution function,

$$F(x) = \tfrac{1}{2} + \tfrac{1}{\pi}\tan^{-1}\left(\tfrac{x-35.06959}{23.5291}\right)$$

provided the best least squares fit

$$\sum_{k=1}^{10}[F(x_k - y_k]^2 \approx 0.002341.$$

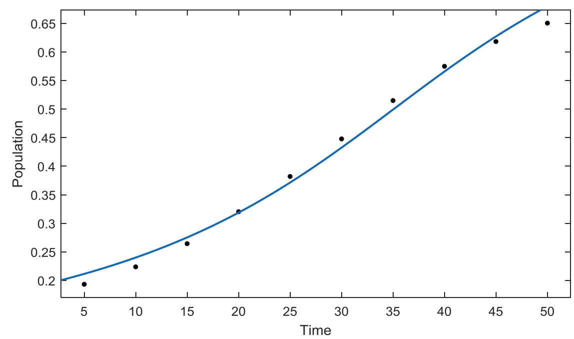


Figure 6.2: Graph of F and (x_k, y_k)

Figure 6.2 shows the fitted curve of the Cauchy model.

Numerical Example 2: Number of people living with HIV/AIDS (1990 to 2015): In the 1990s almost 3.5 million people were diagnosed with HIV every year. In 1997 the number declined and was reduced to 2.1 million in 2015. This table, obtained from UNAIDS data, shows the number of people living with HIV/AIDS from 1990 to 2015 in millions. The estimated value of B, which we adjust g_k by, gives the upper bound. For our models in section 2.4 the estimated values of B ranged between 36.82 and 39.77, we therefore picked 40 as a reasonable estimate of the upper bound for all the models.

Table 6.4

x_k(Time)	g_k(in Millions)(People living with HIV)
5	9
10	19.3
15	28.6
20	32.5
25	34.4
30	36.7

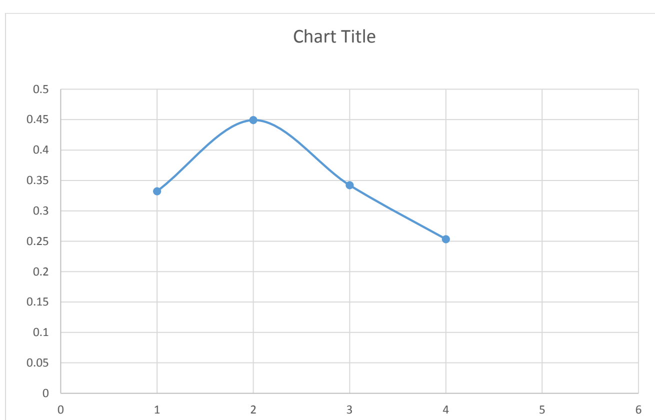


Figure 6.3: Graph of empirical relative increments

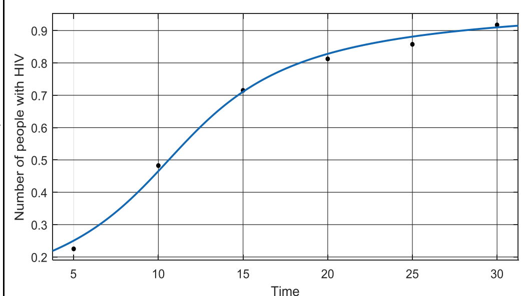
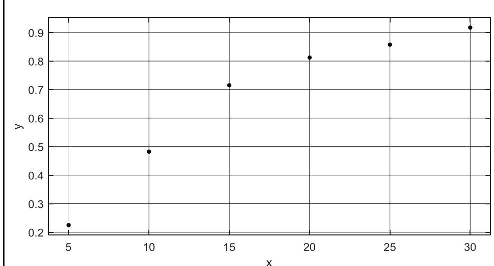
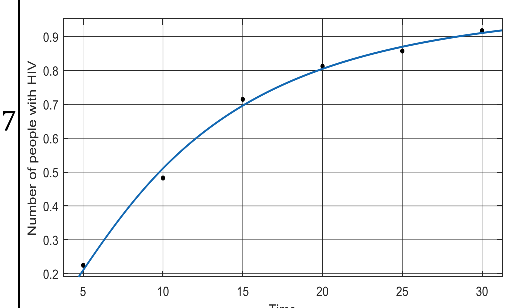
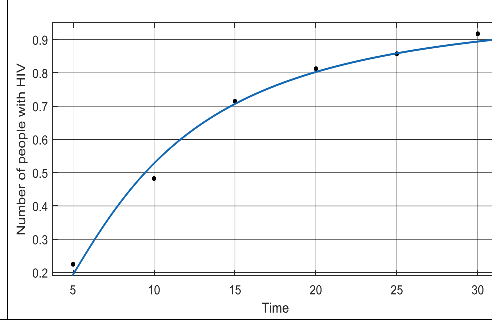
As we can see, the empirical relative increments increase up to a certain point and then decrease. Below is the table of distributions with the same behavior of relative Increments, the distributions in *section*6.3.4. The table shows the fitted values of the distributions.

Notes

Notes

Table 6.5

x_k(Time)	g_k(in Millions)(People living with HIV)	y_k (Adjusted g_k)
5	9	0.225
10	19.3	0.4825
15	28.6	0.715
20	32.5	0.8125
25	34.4	0.86
30	36.7	0.9175

Distribution function	Fit Results	Fitted Curve
Cauchy Distribution $\frac{1}{2}+\frac{1}{\pi}*\tan^{-1}\left(\frac{x-\alpha}{\beta}\right)$	Coefficients (with 95% confidence bounds): $\alpha\ =\ 10.61$ $\beta\ =\ 5.623$ Goodness of fit: SSE: 0.001816	
Inverse Gaussian Distribution General model: $1/2\ \ (1+erf(\sqrt{}(\lambda/2x)\ (x/\mu-1))) + 1/2\ \ e\text{^}(2\lambda/\mu)\ (1 - erf(\sqrt{}(\lambda/2x)\ (x/\mu + 1)))$	Coefficients (with 95% confidence bounds): a = 0.3371 b = 0.1622 Goodness of fit: SSE: 1.012	
Lognormal Distribution $\frac{1}{2}*erfc\left(-\frac{\ln(x)-\mu}{\sigma\sqrt{2}}\right)$ Where $erfc$ is the complementary error function.	Coefficients (with 95% confidence bounds): $\sigma\ =\ 1.667$ $\mu\ =\ 2.28$ Goodness of fit: SSE: 0.001625	
Beta type II Distrribution $I_{\frac{x}{1+x}}(\alpha,\beta)$ Where $I_x(\alpha,\beta)$ is the incomplete beta function.	Coefficients (with 95% confidence bounds): $\alpha\ =\ 16.06$ $\beta\ =\ 1.986$ Goodness of fit: SSE: 0.003733	

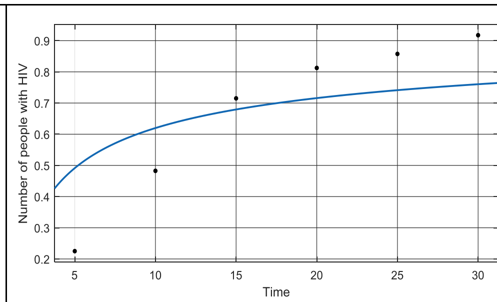
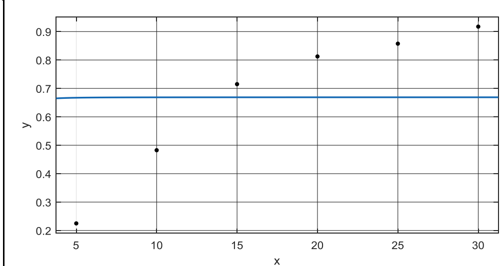
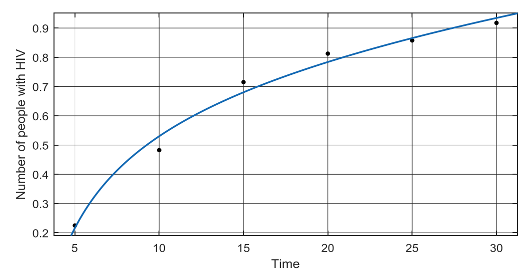
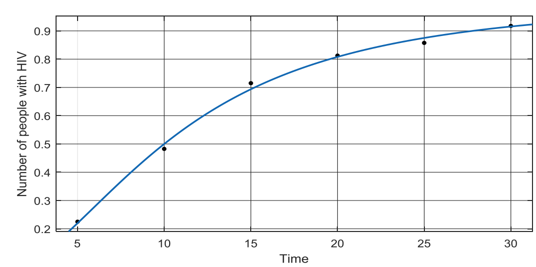
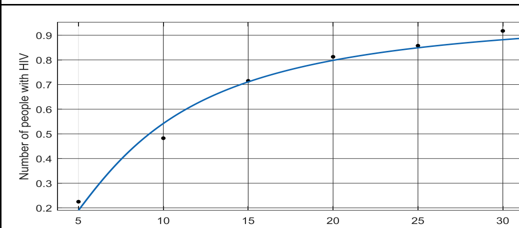
Burr type XII Distribution $$1-(1+x^c)^{-k}$$ Where $$c,k>0$$	Coefficients (with 95% confidence bounds): $c = 7.078$ $k = 0.05934$ Goodness of fit: SSE: 0.1386	
Number 6 $$\frac{1}{8}k\sqrt{2}\left(\ln-\frac{x^2+x\sqrt{2}+1}{-x^2+x\sqrt{2}-1}\right)+2\tan^{-1}(x\sqrt{2}+1)+2\tan^{-1}(x\sqrt{2}-1))$$	Coefficients (with 95% confidence bounds): $k = 0.6022$ Goodness of fit: SSE: 0.3504	
Number 7 $$cx^{-n}x\left(-\frac{k\,\mathrm{Ei}\left(1,\frac{k}{x}\right)}{x}+e^{-\frac{k}{x}}\right)$$ Where Ei is the exponential Integral	Coefficients (with 95% confidence bounds): $a = 0.218$ $k = 2.24$ $n = 0.5929$ Goodness of fit: SSE: 0.00472	
Frechet $$e^{-\left(\frac{x-m}{s}\right)^{-a}}$$ Where a is the shape parameter s is the scale parameter m is the location parameter	Coefficients (with 95% confidence bounds): $a = 3.506$ $m = -15.01$ $s = 22.53$ Goodness of fit: SSE: 0.001123	
Gumbel $$e^{-bx^{-a}}$$ where a is real and b is the shape parameter	Coefficients (with 95% confidence bounds): $a = 1.44$ $b = 16.85$ Goodness of fit: SSE: 0.006365	

Figure 6.4: Graph of F and (x_k, y_k)

Notes

Notes

Amongst these distributions , the Lognormal distribution function,

$$F(x) = K. \int_0^x \exp \frac{(-\frac{1}{2}.\sigma^2.(\ln t)^2)}{t} dt$$

provided the best least squares fit

$$\sum_{k=1}^{6} [F(x_k - y_k]^2 \approx 0.001625$$

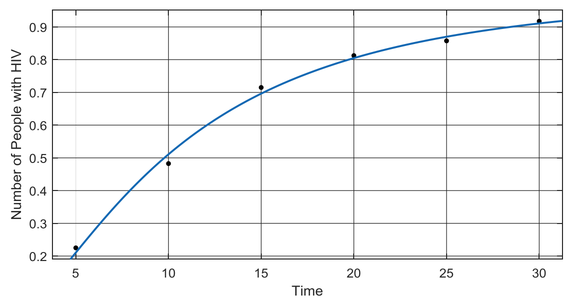


Figure 6.4: Graph of F and (x_k, y_k)

According to our model, the number of people with HIV/AIDS will not reach $40,000,000$.

IV. DISCUSSIONS

This method is different from the classical methods of density estimation and kernel estimation because some of these methods do not provide indefinitely smooth models and some provide some twice differentiable models with much more than two points of inflection. In addition our model will give us tools to estimate the behaviour of the considered (natural, social and others) phenomena in the future if the environment and other conditions or circumstances do not change.This method cam only be used for relative increment functions with at most two phases. Further analysis of the algorithm to accommodate sample whose relative increment functions are more than two phased may prove useful to our work of finding continuous distributions. Results from this study sheds light on the use of relative increment functions in determining distributions which would be very helpful in forecasting and estimating percentiles of growth processes. By using indefinitely smooth model of growth processes, one can predict how the process in question will behave in the future.

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A Note on Few Interesting Approaches of Solving Equations to Find the Number of Real Zeros

By Prabir Kumar Paul
Jadavpur University

Abstract- Be it in the world of mathematics or real life, it is often rewarding to think out-of-the box while solving a problem. Accordingly, in this paper, our aim is to explore the various alternative approaches for solving algebraic equations and finding the number of real zeros. We will further delve deeper into the conceptual part of mathematics and understand how implementation of simple ideas can lead to an acceptable solution, which otherwise would have been tedious by considering the conventional approaches. In the pursuit of achieving the objective of this paper, we will consider few examples with full solutions coupled with precise explanation. It is also intended to leave something meaningful for the readers to explore further on their own. The fundamental objective of this paper is to emphasize on the importance of application of basic mathematical logic, concept of inequality, concept of domain and range of functions, concept of calculus and last but not the least the graphical approach in solving mathematical equations. As a further clarification on the scope of this paper, it is highly pertinent to bring to the understanding of the readers two important aspects - firstly, we will only deal with equations involving real variables; and secondly, this paper does not include topics related to number theory.

Keywords: greatest integer function, fractional part of integer, calculus, inequality, domain of definition, periodic function.

GJSFR-F Classification: *MSC 2010: 03C05*

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A Note on Few Interesting Approaches of Solving Equations to Find the Number of Real Zeros

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Abstract- Be it in the world of mathematics or real life, it is often rewarding to think out-of-the box while solving a problem. Accordingly, in this paper, our aim is to explore the various alternative approaches for solving algebraic equations and finding the number of real zeros. We will further delve deeper into the conceptual part of mathematics and understand how implementation of simple ideas can lead to an acceptable solution, which otherwise would have been tedious by considering the conventional approaches. In the pursuit of achieving the objective of this paper, we will consider few examples with full solutions coupled with precise explanation. It is also intended to leave something meaningful for the readers to explore further on their own. The fundamental objective of this paper is to emphasize on the importance of application of basic mathematical logic, concept of inequality, concept of domain and range of functions, concept of calculus and last but not the least the graphical approach in solving mathematical equations. As a further clarification on the scope of this paper, it is highly pertinent to bring to the understanding of the readers two important aspects - firstly, we will only deal with equations involving real variables; and secondly, this paper does not include topics related to number theory.

Keywords: *greatest integer function, fractional part of integer, calculus, inequality, domain of definition, periodic function.*

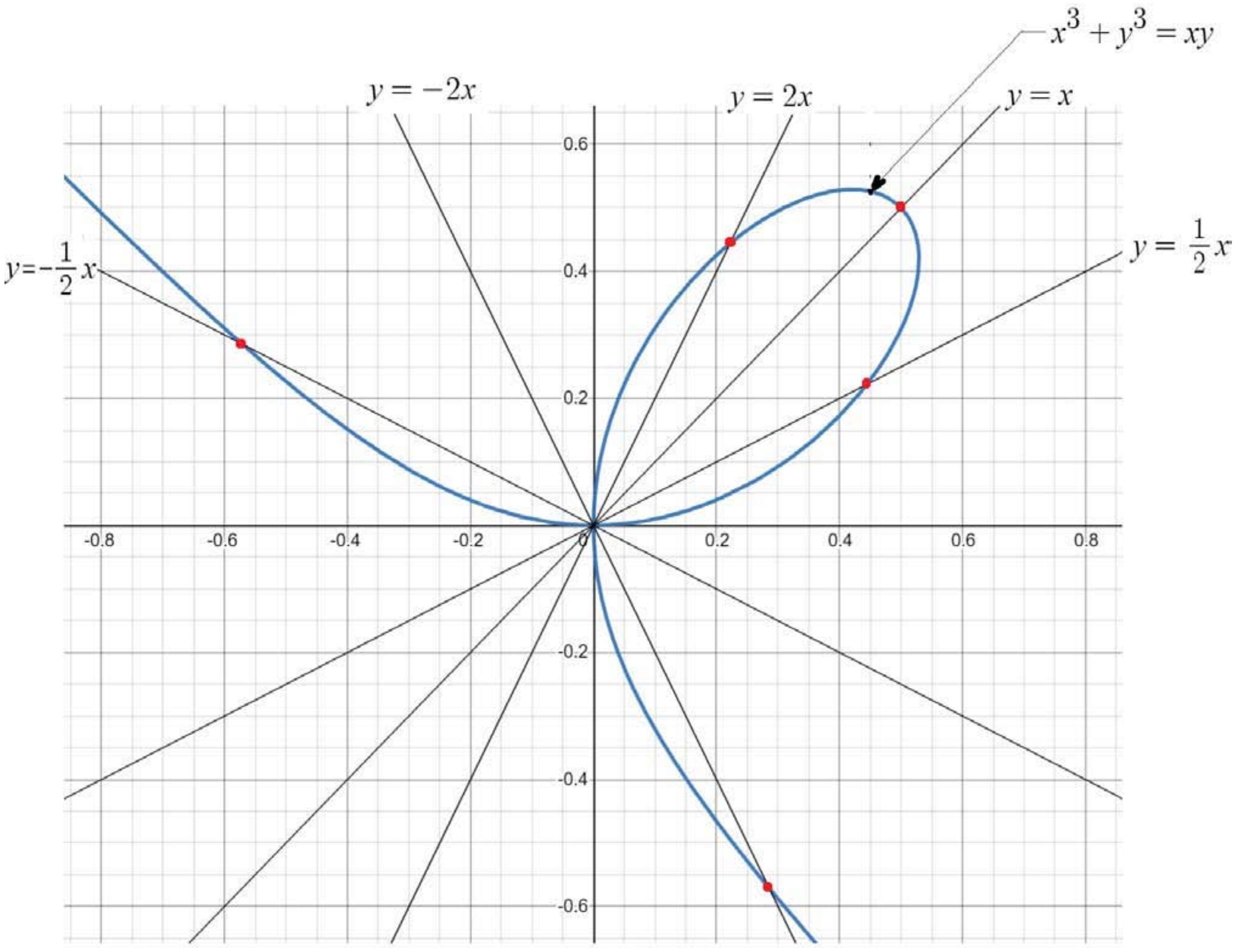
I. Introduction

To put things in perspective, let us explain on what we actually mean by a conventional approach. Let's start with a simple equation such as $\sqrt{x}+\sqrt{1-x^2}=3$. So the conventional, rather the natural way might be trying to remove the square-root and then solve for x without even paying attention to the fact that, the value of left hand side (referred as LHS henceforth) cannot even exceed 2 for any real values of x but why? Because, LHS is defined for $0 \le x \le 1$. Hence, even without any calculation whatsoever, it can be concluded that there is no real x for which the equation is satisfied. It is worth mentioning that any effort towards removing the square-root will only make the problem complex eventually leading to nowhere near conclusion. Similarly let us consider another problem: what are all real non zero solutions of the equation $x^3 + y^3 = xy$? The first thing that may appear in the mind of many readers is how to solve for two unknown real variables with only one equation having no other constraints available. But on the other hand the absence of no additional constraints gives rise to the possibility of existence of multiple solutions. Now what should be the approach? Without any loss of generality, let us think completely out of the box and

Author: B.C.E (Jadavpur University, WB, India), MSc. (OGSE), University of Aberdeen (Scotland, UK). e-mail: prabir555@gmail.com

explore possibility of intersection of the straight line $y = mx$ with the curve $x^3 + y^3 = xy$. Why are we doing this? We shall see the logic as we move along. The equation becomes, $x^3 + m^3 x^3 = m x^2$ that gives value of x in terms of m and the set of all real solutions are $(x, y) = \left(\frac{m}{1+m^3}, \frac{m^2}{1+m^3}\right), x \neq 0, y \neq 0$ since we are only interested in non-zero solutions. So, for all real values of m (except,$m \neq -1$), we can obtain all non zero real solutions of the equation $x^3 + y^3 = xy$.

So a simple substitution was adequate to solve the equation. However, what does it signify geometrically? Why does the substitution make lot of sense, let us explore in the graph below:



Point of Intersection of the curve with the straightline as highlighted in red colour are the soltuions that can be explicitly obtained using all real values of the parameter 'm' which actually denotes the slope of the straight line

Fig. 1

It can be observed that for various values of m, we get different straight line and an intersection point which is nothing but a solution of the equation $x^3 + y^3 = xy$. For illustration, lets' put,m $= -2$ yields a solution of $\left(\frac{m}{1+m^3}, \frac{m^2}{1+m^3}\right) = (0.286, -0.571)$. In fact, $\left(\frac{m}{1+m^3}, \frac{m^2}{1+m^3}\right)$ is the parametric representation of the curve $x^3 + y^3 = xy$.

Prior moving to the next section we shall leave one interesting problem for the readers to explore. How many real solutions does the equation $\{x\}^{[\sin x]} + \{y\}^{[\sin y]} =$

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$[x + y - x^2 - y^2]$ have? Where, { } denotes the fractional part of integer and [ ] denotes the greatest integer function. In the next section, efforts have been made to throw some light on this kind of problem.

In the subsequent section, we shall see few more examples with detailed solutions that essentially focus on various basic yet interesting approaches for equation solving.

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II. Examples

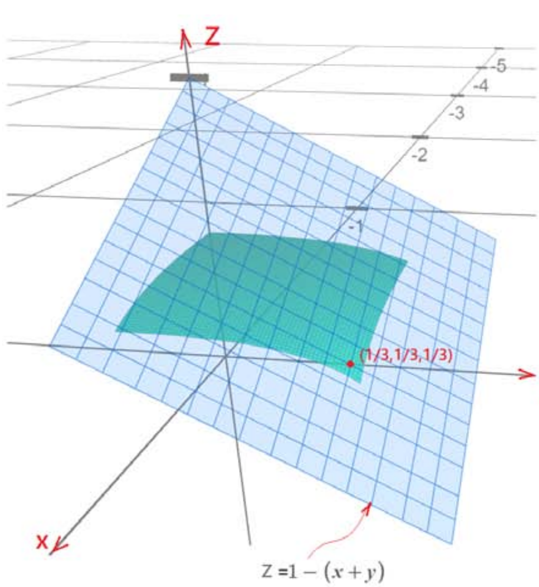
Example-1: Solve: $a + b + c = 1$, $a^3 + b^3 + c^3 = 1/9$ where a, b, c > 0 real numbers.

Solution:

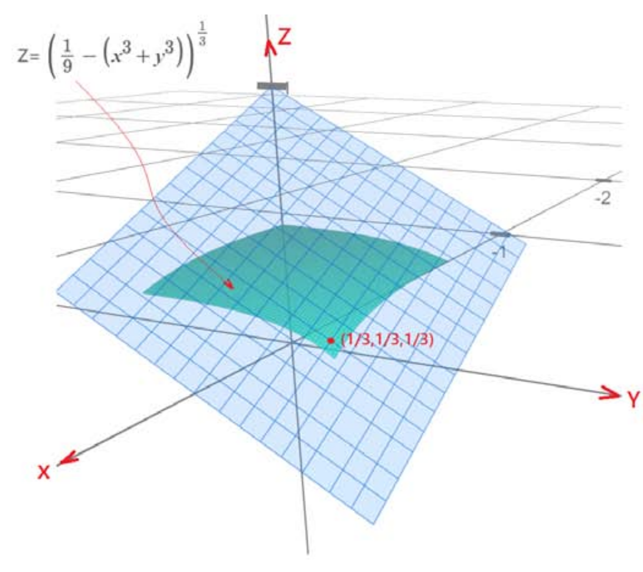
It is to be noted that nos. of variables exceed the nos. of equations provided. However, it is not very difficult to see that $a = b = c = 1/3$ is a solution since it satisfies both equations but, the question still remains what are the other solutions? Or whether there are any other solutions at all?

Using the m-th power Inequality, $\frac{a^3+b^3+c^3}{3} \geq \left(\frac{a+b+c}{3}\right)^3$ implies that $\frac{a^3+b^3+c^3}{3} \geq \left(\frac{1}{3}\right)^3$ So, $a^3 + b^3 + c^3 \geq 1/9$, However given that $a^3 + b^3 + c^3 = 1/9$ hence, it is possible if (and only if) $a = b = c$. Substituting this we get that, $a = b = c = 1/3$ which is the only solution.

Geometrically, if x, y & z represents the variables a, b & c respectively then, the solution indicates the point of tangency (1/3, 1/3, 1/3) of the plane $z = 1 - (x + y)$ and the three-dimensional curve, $z = (1/9 - (x^3 + y^3))^{1/3}$. VIEW-1 and VIEW-2 shows the 3D-plot of the curve and plane on the first quadrant since, $x \geq 0, y \geq 0$.



VIEW-1 showing plane z = 1-(x+y) touches the curve at the point (1/3,1/3,1/3)



VIEW-2 showing plane z = 1-(x+y) touches the curve at the point (1/3,1/3,1/3)

Example-2: Solve: $x^2 + y^2 + z^2 + t^2 - x - y - z - t = \log_e\left(\frac{\sin w}{e}\right)$ where x, y, z, t, w > 0 real numbers

Solution:

$\Rightarrow$ $x^2 + y^2 + z^2 + t^2 - x - y - z - t = \log_e\left(\frac{\sin w}{e}\right)$

$$=> x^2 + y^2 + z^2 + t^2 - x - y - z - t = \log_e(\sin w) - \log_e e$$

$$=> x^2 + y^2 + z^2 + t^2 - x - y - z - t + 1 = \log_e(\sin w)$$

$$=> \left(x^2 - x + \tfrac{1}{4}\right) + \left(y^2 - y + \tfrac{1}{4}\right) + \left(z^2 - z + \tfrac{1}{4}\right) + \left(t^2 - t + \tfrac{1}{4}\right) = \log_e(\sin w)$$

$$=> \left(x - \tfrac{1}{2}\right)^2 + \left(y - \tfrac{1}{2}\right)^2 + \left(z - \tfrac{1}{2}\right)^2 + \left(t - \tfrac{1}{2}\right)^2 = \log_e(\sin w)$$

Now, LHS is non-negative for all real x, y, z, and t while the function $\log_e(\sin w)$ in the RHS is always negative since $0 < \sin w \leq 1$, $w \in R^+$ except at $w = (4n+1)\frac{\pi}{2}$, the maximum value of RHS is zero. So, the equation to hold true for all real variables then RHS must be equal to zero.

$\left(x - \tfrac{1}{2}\right)^2 + \left(y - \tfrac{1}{2}\right)^2 + \left(z - \tfrac{1}{2}\right)^2 + \left(t - \tfrac{1}{2}\right)^2 = 0$, which yields the solution $x = y = z = t = 0.5$

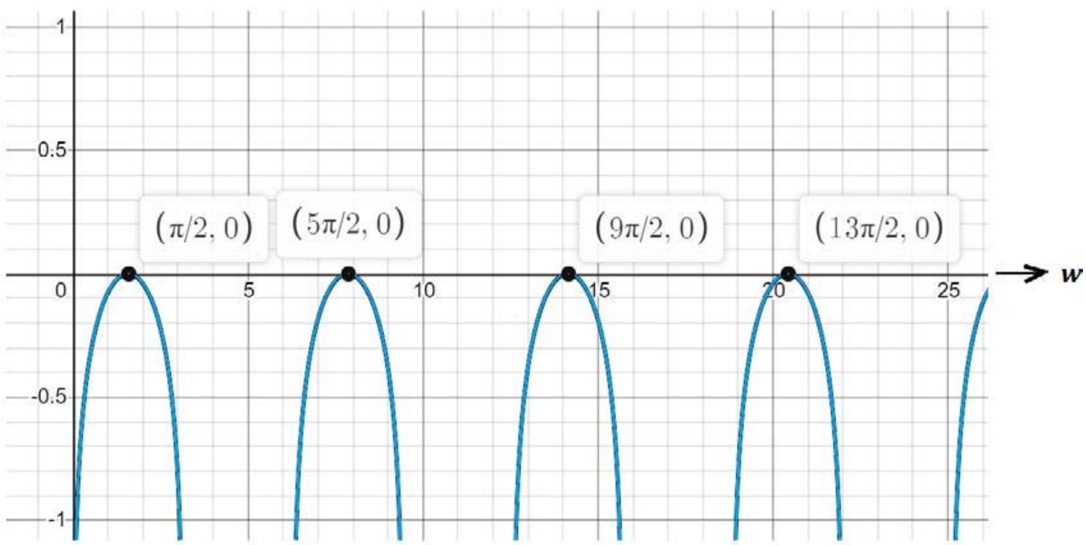


Fig. 2: Graph of $\log(\sin w)$

Example-3: Solve: $y^3 - x^3 = 3x^2 + 1$ where x, y are positive integers

Solution:

By rearranging we can write, $y^3 = x^3 + 3x^2 + 1$

Now we know $(x+1)^3 = x^3 + 3x^2 + 3x + 1 > x^3 + 3x^2 + 1$ since, x is a positive integer.

So, $x^3 + 3x^2 + 1 < (x+1)^3$ but $x^3 + 3x^2 + 1 > x^3$ because $3x^2 + 1 > 0$ for all real x, Hence, combining these two conditions we can write that, $x^3 < (x^3 + 3x^2 + 1) < (x+1)^3$ so, the expression $(x^3 + 3x^2 + 1)$ lies between two consecutive perfect cubes hence, it is clear that, $(x^3 + 3x^2 + 1)$ cannot be a perfect cube. But LHS is a perfect cube i.e y^3 while RHS cannot be a perfect cube hence; the given equation does not yield any solutions in positive integers.

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Example-4: Solve: $(2\sin x-3x)^5+(3\sin x-4x)^5+(4\sin x-5x)^5=0$

Solution:

It is obvious that x=0 satisfies the given equation. Let us investigate whether there are any non-trivial real solutions?

Let us consider $f(x)=(2\sin x-3x)^5+(3\sin x-4x)^5+(4\sin x-5x)^5$

$$f'(x)=5(2\sin x-3x)^4(2\cos x-3)+5(3\sin x-4x)^4(3\cos x-4)+5(4\sin x-5x)^4(4\cos x-5)$$

$$f'(x)=10(2\sin x-3x)^4\left(\cos x-\frac{3}{2}\right)+15(3\sin x-4x)^4\left(\cos x-\frac{4}{3}\right)+20(4\sin x-5x)^4\left(\cos x-\frac{5}{4}\right)$$

Since, $(2\sin x-3x)^4 \geq 0, (3\sin x-4x)^4 \geq 0$ *and* $(4\sin x-5x)^4 \geq 0$

Similarly, $|\cos x| \leq 1, hence, \left(\cos x-\frac{3}{2}\right)<0, \left(\cos x-\frac{4}{3}\right)<0$ *and* $\left(\cos x-\frac{5}{4}\right)<0$

Hence for all real x, $f'(x)\leq 0$

Hence, f(x) is a deceasing function from $-\infty$ to $+\infty$ (in the entire domain of definition) and the function is continuous in its entire domain. So, it will intersect the X-axis only once indicating that there is only one real solution. So, x = 0 is the only (trivial) solution.

Example-5: Solve: $x^2+y^2=[x-x^2+1]$ *and* $2x+y=2$ *for all non-zero real x and y, where* [ ] *denotes the greatest integer function*

Solution:

$(x-x^2+1)=\frac{5}{4}-\left[x^2-2.x\frac{1}{2}+\left(\frac{1}{2}\right)^2\right]=\frac{5}{4}-\left(x-\frac{1}{2}\right)^2<\frac{5}{4}$ Since, $\left(x-\frac{1}{2}\right)^2>0$ for all real x

$\left(x-\frac{1}{2}\right)^2=0$ is omitted as we shall see later that $x=\frac{5}{4}$ does not satisfy the equations.

Negative values of $[x-x^2+1]$ are not admissible since, LHS of the first equation i.e (x^2+y^2) is non-negative. Hence, only two values of the greatest integer function possible as mentioned below:

$0<(x-x^2+1)<1$ Then $[x-x^2+1]=0$ the first equation becomes, $x^2+y^2=0$

$1\leq(x-x^2+1)<\frac{5}{4}$ Then $[x-x^2+1]=1$ the first equation becomes, $x^2+y^2=1$

However, $x^2+y^2=0$ is not possible since only non-trivial solutions are intended.

Hence, the set of equations required to be solved becomes, $x^2+y^2=1$ and $2x+y=2$

These equations can now be solved with usual approach and the solutions are (0.6, 0.8) and (1, 0).

It is an interesting aspect that the graph of $x^2+y^2=1$ should be a full circle however it is shown as half-circle in the Fig-3 below, why? There is a valid reason to this which is left for readers to explore.

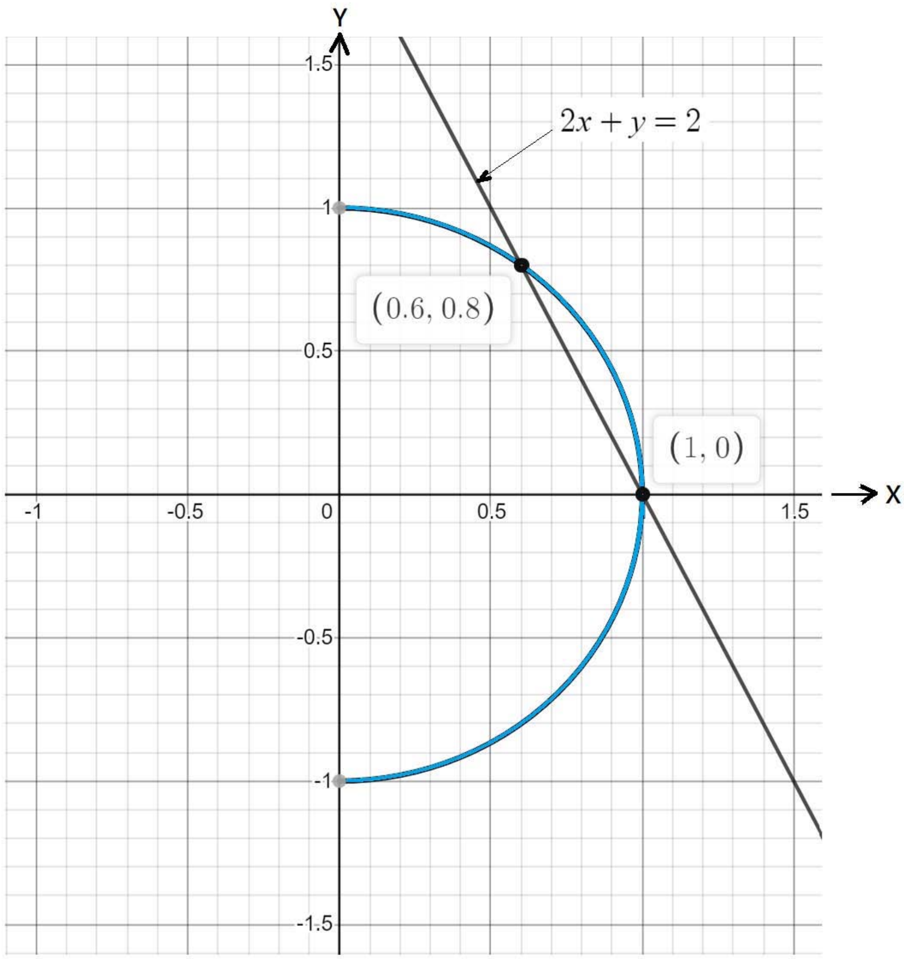

Fig. 3

Example-6: Solve: $a^5 + b^5 + c^5 = a^3 + b^3 + c^3 + 25[\sqrt{\pi}]$ where a, b and c are integers and [ ] denotes the greatest integer function

Solution:

$a^5 + b^5 + c^5 = a^3 + b^3 + c^3 + 25[\sqrt{\pi}]$ is a non-linear algebraic equation which at the first look appears to be complicated to certain extent but with simple rearrangement, it can be shown that there is no such real a, b and c that satisfy the equation. Let us re-arrange the terms and re-write the equation as follows:

$$(a^5 - a^3) + (b^5 - b^3) + (c^5 - c^3) = 25[\sqrt{\pi}]$$

$$a^2.a.(a+1)(a-1) + b^2.b.(b+1)(b-1) + c^2.c.(c+1)(c-1) = 25 \text{ Since, } [\sqrt{\pi}] = 1$$

Since a, b and c are integers hence, a and $(a+1)$are consecutive integers hence, $a(a+1)$ is an even number, hence, $a(a+1)$, $b(b+1)$and $c(c+1)$are all even numbers hence, each of $(a^5 - a^3)$, $(b^5 - b^3)$ and $(c^5 - c^3)$ are even hence, LHS is an even number for all real a, b, c while RHS is an odd number hence, there does not exist any integer solutions.

Example-7: For what real values of parameter 'a' the equation x^4-ax+6=0 will have positive solutions ?

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Solution:

It is clear that, $x \neq 0$ since, $x > 0$ for solution to be positive. The given equation can be written by expressing the variable 'a' as a function of 'x' like $a = x^3 + \frac{6}{x}$ $(x \neq 0)$

Applying the inequality AM ≥ GM, $x^3 + \frac{6}{x} = \left(x^3 + \frac{2}{x} + \frac{2}{x} + \frac{2}{x}\right) \geq 4\left(\sqrt[4]{x^3 . \frac{2}{x} . \frac{2}{x} . \frac{2}{x}}\right) = 4(\sqrt[4]{8})$

So, the minimum value of the parameter 'a' for which real solution exist $= 4(\sqrt[4]{8})$

Hence, only for all real 'a' satisfying $a \geq 4(\sqrt[4]{8}) = 6.727$ (approx) real solutions exist.

This is clearly manifested in the graph below. For different representative values of a, series of graphs are drawn to demonstrate the mathematical logic as stated above.

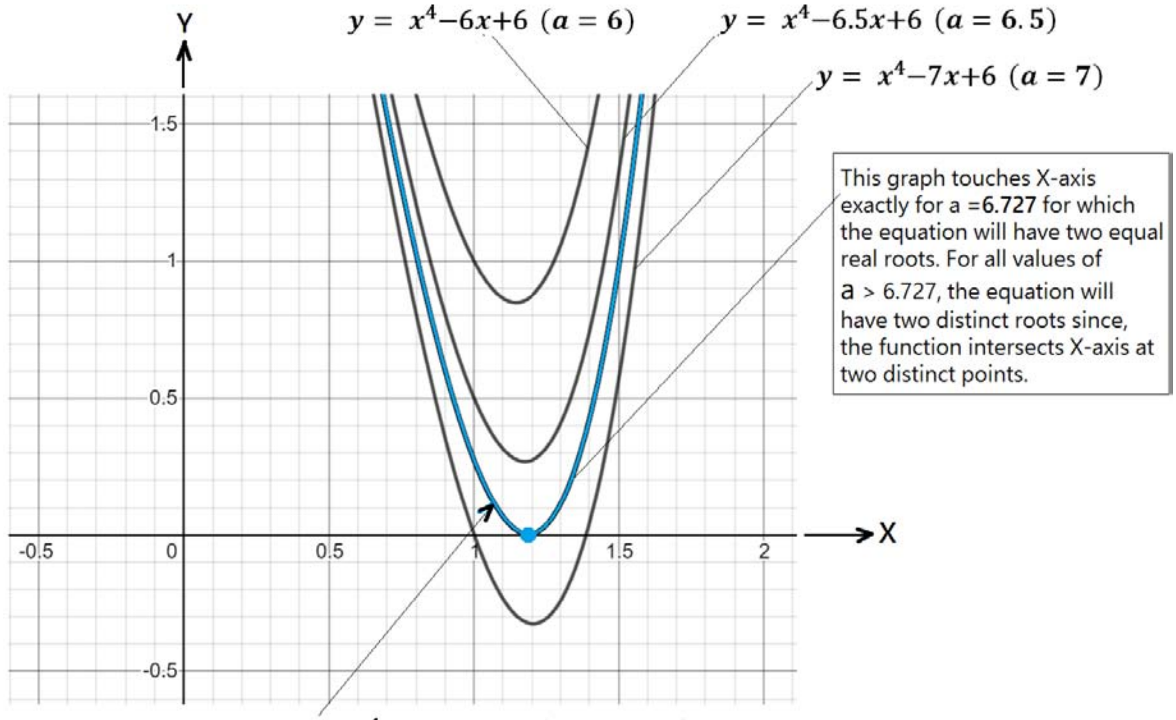

Fig. 4

Example-8: How many real solutions of $x^2 + \{|\sin x| + |\cos x|\}^2 = x + \{|\sin x| + |\cos x|\}$, Where { } denotes the fractional part of integer.

Solution:

Let us say, $y = \{f(x)\}$ where $f(x) = |\sin x| + |\cos x|$ hence, the equation gets transformed to $x^2 + y^2 = x + y$ which can be re-written as $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2$ which represents a circle.

Now the number of point of intersections between the function, $y = \{|\sin x| + |\cos x|\}$ and the circle will provide the nos. of solutions.

Hence, to find exact nos. of real solutions, it is important to draw graph of $y = \{f(x)\}$. This can be done several ways but we chose to do it using the concept of calculus. The function $\{f(x)\}$ is periodic with period of $\frac{\pi}{2}$ so, it repeats itself at every $\frac{\pi}{2}$ interval.

Differentiating $f(x)$ both sides with respect to x and simplifying we get,

$$f'(x) = \left(\frac{\sin x.\cos x}{|\sin x|} - \frac{\sin x.\cos x}{|\cos x|}\right) = \frac{\sin 2x}{2|\sin x|.|\cos x|}(|\cos x| - |\sin x|)$$

It can be shown that the function $f(x)$ has local minima at $x = \frac{n\pi}{2}$ and local maxima at $x = n\pi \pm \frac{\pi}{4}$. By substituting these values of x, the maximum and minimum for $y = \{f(x)\}$ can be evaluated as $0 \leq \{|\sin x| + |\cos x|\} \leq (\sqrt{2} - 1)$

For $0 \leq x \leq \frac{\pi}{4}$ the function $f(x)$ is increasing while for $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$, $f(x)$ is decreasing and this repeats itself for all $n = \pm 1, \pm 2, \pm 3, \ldots$ since the function is periodic. So, the graph of $\{|\sin x| + |\cos x|\}$ should look as indicated below in fig-5. So, $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2$ and $y = \{|\sin x| + |\cos x|\}$ has two real zeros and only one of them is non-trivial as indicated in the fig-5 below.

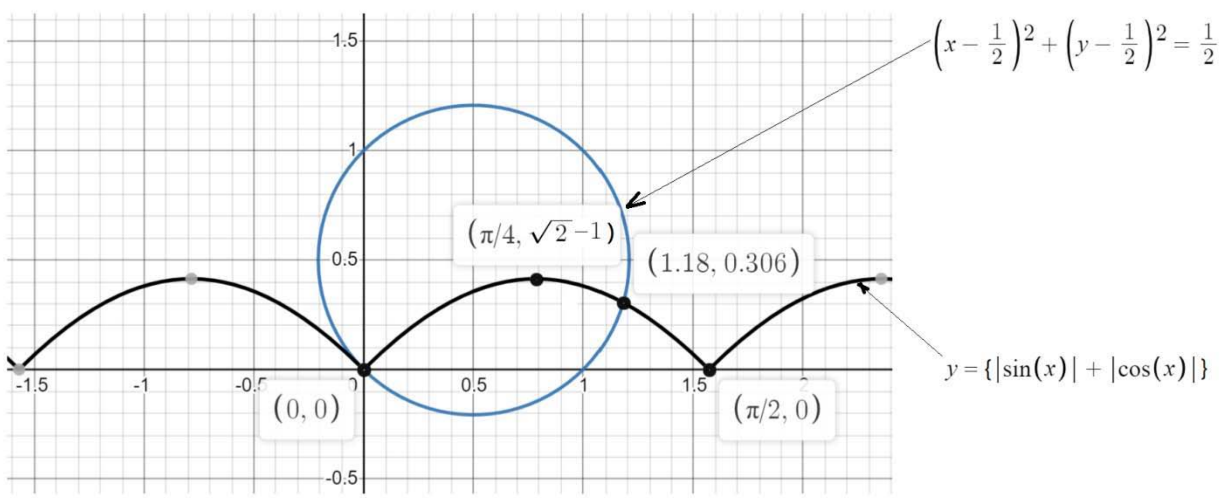

Fig. 5

Two important things to note in the above example:

a) As intended in the question, the above method shows how to find the nos. of real zeros (trivial and non-trivial) but it does not specify how to solve the equation to get exact roots. But if graphs are drawn properly then it will provide an approximate idea on the range values of x close to the actual root.

b) While it is easy to conclude that the function $y = \{|\sin x| + |\cos x|\}$ has local maxima at $x = n\pi \pm \frac{\pi}{4}$ but it is relatively difficult to show that $y = \{|\sin x| + |\cos x|\}$ has local minima at $x = \frac{n\pi}{2}$, even though it is quite clear from the graph. Readers are requested to explore this analytically.

Notes

III. Conclusion

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On the backdrop of all examples and explanations, a pertinent question that may however arise that how one will anticipate the best possible approach to be adopted? It is indeed a tough question to answer since there is no definitive rule. The objective of this paper is to highlight about various approaches to solve an equation but exploring the most relevant one depends on individual intuitiveness coupled with relentless effort. The intention of this paper is to provide the readers with few basic tools and conceptions which can effectively be used and rigorously inculcated to extend the domain of mathematical understanding.

Even though the application of number theory and modular arithmetic was not addressed in this paper, it is worth mentioning that number theory plays a vital role in determining solutions various types of algebraic equations.

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Breakpoint Unit Root Tests on Select Macroeconomic Variables in Nigeria

By Alabi Nurudeen Olawale & Bada Olatunbosun

Auchi Polytechnic

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Keywords: extended augmented dickey fuller, innovational outlier, additive outlier, unit-root test.

GJSFR-F Classification: MSC 2010: 97K80

BREAKPOINTUNITROOTTESTSONSELECTMACROECONOMICVARIABLESINNIGERIA

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Keywords: extended augmented dickey fuller, innovational outlier, additive outlier, unit-root test.

I. Introduction

Unit root test in econometrics is crucial to determining the nature or order of integration of macroeconomic variables. It is the foundation for most statistical techniques such as error correction models, granger causality, cointegration tests to mention but few applied in economics, business and finance.

A large number of literature exists on this subject. However, most researchers determine macroeconomic variables' order of integration based on regular unit-root tests such as Augmented Dickey-Fuller test, Phillips-Perron (PP) test and other regular tests. This present work involves unit-root tests with allowance for a shift in the intercept of the trend function and slope since most macroeconomic time series are interpreted as stationary around a deterministic trend function. Specifically, the extended Augmented Dickey-Fuller test through innovational outlier and additive outlier models were employed as proposed by Perron (1989, 1997). The significance of these methods is that we are able to establish important breakpoints usually associated with shocks on macroeconomic variables.

II. Methods and Results

Three monetary policy variables comprising *interest rate* (*ir*) (proxy by Treasury bill rate), *inflation rate (inf)* and *exchange rate (ex)*. Also, real gross domestic product (*rgdp*) was used as the measure of the Domestic Output and lastly unemployment rate

Author α: Department of Mathematics and Statistics, Federal Polytechnic Ilaro, Ogun State.
e-mail: nurudeen.alabi@federalpolyilaro.edu.ng
Author σ: Department of Statistics, Auchi Polytechnic, Auchi, Edo State. e-mail: badaolatunbosun@yahoo.com

(*um*) were involved in the vector autoregressive (VAR) model. Cubic low to high-frequency conversion method was employed on quarterly *real gdp* and unemployment rate data, without loss of statistical properties (Alabi and Bada, 2018). Furthermore, we transformed the original data into the natural log to ensure that the normality assumptions in the error term in the VAR model can be sustained.

The stationarity of each of the series was analyzed by conducting individual unit-root tests. An extended Augmented Dickey-Fuller test with innovational outlier and additive outlier breakpoints as proposed by Perron (1989, 1997) were employed (Models 1, 2 & 3). According to Zivot & Andrews (1992), Lumsdaine & Papell (1997) & Perron (1989, 1997), if allowance is made for a shift in the intercept of the trend function and slope, most macroeconomic time series are interpreted as stationary around a deterministic trend function. We considered the following three models at levels with dummy variables for different intercepts and slopes. The Model 1 involves including the dummy variable for a change in the intercept of the trend function steadily in a way that relies on the correlation function and the innovation (i.e., noise) function (Perron, 1997). The dummy variables are presumed to be unknown rather than known ex-ante.

$$\Delta y_t = \mu + \theta DU_t + \beta t + \phi D(T_b)_t + \alpha y_{t-1} + \sum_{i=1}^{k} z_i \Delta y_{t-i} + e_t \qquad \textit{Model 1}$$

$$\Delta y_t = \mu + \theta DU_t + \beta t + \omega DT_t + \phi D(T_b)_t + \alpha y_{t-1} + \sum_{i=1}^{k} z_i \Delta y_{t-i} + e_t \qquad \textit{Model 2}$$

$$\Delta y_t = \mu + \theta DU_t + \beta t + \omega DT_t^* + \tilde{y} \qquad \textit{Model 3}$$

where

$$\tilde{y} = \alpha \tilde{y}_{t-1} + \sum_{i=1}^{k} z_i \Delta \tilde{y}_{t-i} + e_t$$

These models were estimated using the Ordinary Least Squares (OLS). The indicator functions $1(\cdot)$ are expressed as $DU_t = 1(t > T_b)$, $D(T_b)_t = 1(t = T_b + 1)$, $DT_t = 1(t > T_b + 1)t$ and $DT_t^* = 1(t > T_b)(t - T_b)$. We test the null hypothesis that $\alpha = 1$ using the *t*-statistic) $t_{\hat{\alpha}}(T_b, k)$, where *k* is the truncation lag parameter which is also unknown. Model 1 and 2 are referred to as the *innovational outlier models* (i.e., IO1 and IO2) respectively. Model 3 is called the *Additive model* (i.e., AO) because a rapidly change in the slope is allowed but the two fragments of the trend function are joined at the breakpoint. According to Perron (1989 & 1997), the breakpoint T_b may be chosen such that $t_{\hat{\alpha}}(T_b, k)$ is minimized. The minimized *t*-statistic is expressed as:
$t_{\hat{\alpha}}^* = \min_{T_b \in (k+1,T)} t_{\hat{\alpha}}(T_b, k)$. The break point was selected in a manner to maximize the *t*-statistic $t_{\hat{\omega}}$ on the shift in slope. Here, the test statistic for testing the null hypothesis $\alpha = 1$ are $t^*{}_{\alpha,|\hat{\theta}|} = (T_b^*, k)$ for model 1 and $t^*{}_{\alpha,|\hat{\omega}|} = (T_b^*, k)$ for models 2 and 3, where T_b^* is such that $t_{\hat{\omega}}^*(T_b^*) = \max_{T_b \in (k+1,T)} |t_{\hat{\omega}}(T_b, k)|$ and $t_{\hat{\theta}}^*(T_b^*) = \max_{T_b \in (k+1,T)} |t_{\hat{\theta}}(T_b, k)|$. T_b was selected by allowing this point to correlate with the data as much as possible although with some loss in power. This was done by imposing no restrictions on the sign of the change. The truncation parameter *k** was selected using the *t*-sig and *F*-sig general to specific recursive procedures as proposed by Perron (1989). These procedures are particularly better than information criteria such as Akaike Information Criterion and Bayesian Information Criterion due to their size stability and better power (Perron, 1989).

Notes

Table 1: Summary of Unit-Root Test results using Models 1, 2 and 3

Variable	Model	ADF test@	*k* max	*k**	Breakpoint	*t*-statistic	*t*-sig *p*-value	*F*-sig *p*-value	Remark
um	IO1	1st Difference	12	12	2016:12	-4.7523	………	0.0637	I(1)
rgdp	AO	1st Difference	13	12	2007:07	-4.6286	………	0.0132	I(1)
inf	AO	Level	13	13	2011:10	-5.2002	0.0177	………	I(0)
ir	IO2	Level	5	1	2015:09	-4.8494	0.011	………	I(0)
ex	IO1	Level	5	1	2015:12	-5.0324	0.0286	………	I(0)

Source: Authors personal computation

Notes

The results of unit-root test reveal that *unemployment rate,* and *real gdp* are stationary of order one under the Innovational Outlier Model 1 and Additive Outlier Model respectively. The truncation lag lengths of k* = 12 were selected using the *F*-sig approach. The *p*-value for the *real gdp* unit-root test is lower than that of the *unemployment rate* unit-root test. This is an indication that the Additive Outlier Model has more power than the Innovational Outlier Model 1 on these series. The remaining series, i.e. *inflation rate*, *interest rate,* and *exchange rate* are stationary at level under Additive Outlier model, Innovational Outlier Models 2, and 3 respectively. The k* = 13 for *inflation rate* and *k** = 1 for *interest rate* and *exchange rate* were chosen using the *t*-sig recursive technique. The *k* max was chosen arbitrarily avoiding the problems of multi collinearity amongst the variables and loss of power usually associated with high values of *k* max. This quantity was 13 lags (for *real gdp* and *inflation rate*) and 5 lags (for both interest rate and exchange rate). Only the *unemployment rate* has a binding *k* max at 12 lags. The breakpoint dates correspond to significant periods of global economic and Nigerian government policy change shocks. The logarithms of the macroeconomic variables are as shown in Fig. 1 below. The breakpoints are selected to maximize the *t*-statistics (Table 1).

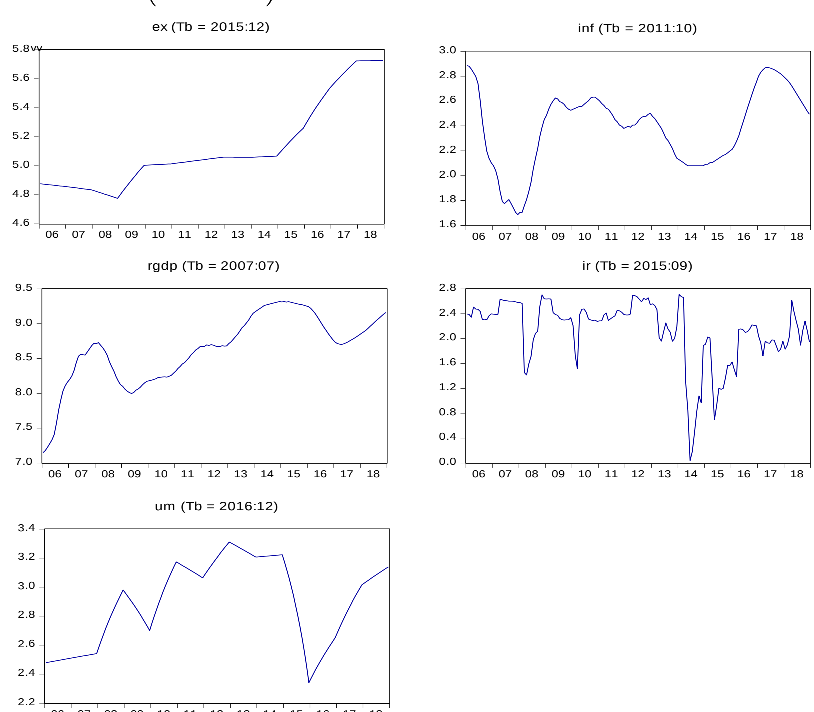

Fig. 1: Log exchange rate, log inflation rate, log real gdp, log interest rate and log unemployment rate for Nigeria

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Firstly, there was a global financial crisis in 2007 when major financial institutions in the United States collapsed. The effect of the global financial crash was observed in Nigeria's *real gdp* in July of 2007. Secondly, Nigeria is known for its inflation targeting monetary policy. Under this policy, the Central Bank of Nigeria (CBN) uses the monetary policy rate (MPR) and cash reserve ratio (CRR) to control rate of inflation in the economy. Hence, the breakpoint of October 2011 in inflation rate series is a consequence of the upward review of CBN's Minimum Rediscount Rate (MRR) from 9.25 percent to 12 percent in October 2011. Furthermore, in 2015, the Central Bank of Nigeria reduced the Monetary Policy Rate (MPR) from 13 percent to 11 per cent culminating into the September 2015 breakpoint date in the *interest rate* series. Thirdly, in October 2015, JP Morgan expelled Nigeria from its Global Bond Index-Emerging Market (GBI-EM). GBI-EM is an index which tracks local currency bonds by emerging market governments. This decision led to the efflux of foreign investors holdings in Nigeria bonds. The effect was revealed in a breakpoint of December 2015 in the *exchange rate* series. Finally, there is a strong connection between economic growth and unemployment rate. According to the United Nations Development Programme 2016 annual report on Nigeria, the country's economy witnessed contraction (recession) for the first time in several decades. This resulted in an escalation of *unemployment rate,* especially amongst the youth, which led to the introduction of several government youth empowerment programmes to reverse the trend. The contraction was captured by the December 2016 breakpoint observed in the *unemployment rate* series. Thus, by introducing trend break functions in the unit-root tests without a priori information, we have been able to establish a good connection between the various breakpoints and the macroeconomic series. This is in line with previous works by Perron (1997), Zivot & Andrews (1992), Banerjee et *al.* (1992), Lumsdaine & Papell (1997), Ling et *al.* (2013), Arestis & Mariscal (2000), Basher & Westerlund (2008), Chiang & Ping (2008), Narayan & Smyth (2005), Ewing & Wunnava (2001) and many other studies.

Notes

III. Conclusion

By introducing trend break functions in the unit-root tests without a priori information, we have been able to establish a good connection between the various breakpoints and the macroeconomic series. These dates represent critical periods of policy changes by the government and external shocks. The unit-root tests with trend functions suggest that structural breaks in the macroeconomic variable series are very important and significant when formulating economic policies. The breakpoints can be included in a VAR model as deterministic terms to further improve the forecast/prediction power without affecting the asymptotic properties of the test statistics involved in the analysis.

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Authors should carefully consider the preparation of papers to ensure that they communicate effectively. Papers are much more likely to be accepted if they are carefully designed and laid out, contain few or no errors, are summarizing, and follow instructions. They will also be published with much fewer delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and suggestions to improve brevity.

Format Structure

It is necessary that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

All manuscripts submitted to Global Journals should include:

Title

The title page must carry an informative title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) where the work was carried out.

Author details

The full postal address of any related author(s) must be specified.

Abstract

The abstract is the foundation of the research paper. It should be clear and concise and must contain the objective of the paper and inferences drawn. It is advised to not include big mathematical equations or complicated jargon.

Many researchers searching for information online will use search engines such as Google, Yahoo or others. By optimizing your paper for search engines, you will amplify the chance of someone finding it. In turn, this will make it more likely to be viewed and cited in further works. Global Journals has compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Keywords

A major lynchpin of research work for the writing of research papers is the keyword search, which one will employ to find both library and internet resources. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining, and indexing.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy: planning of a list of possible keywords and phrases to try.

Choice of the main keywords is the first tool of writing a research paper. Research paper writing is an art. Keyword search should be as strategic as possible.

One should start brainstorming lists of potential keywords before even beginning searching. Think about the most important concepts related to research work. Ask, “What words would a source have to include to be truly valuable in a research paper?” Then consider synonyms for the important words.

It may take the discovery of only one important paper to steer in the right keyword direction because, in most databases, the keywords under which a research paper is abstracted are listed with the paper.

Numerical Methods

Numerical methods used should be transparent and, where appropriate, supported by references.

Abbreviations

Authors must list all the abbreviations used in the paper at the end of the paper or in a separate table before using them.

Formulas and equations

Authors are advised to submit any mathematical equation using either MathJax, KaTeX, or LaTeX, or in a very high-quality image.

Tables, Figures, and Figure Legends

Tables: Tables should be cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g., Table 4, a self-explanatory caption, and be on a separate sheet. Authors must submit tables in an editable format and not as images. References to these tables (if any) must be mentioned accurately.

Figures

Figures are supposed to be submitted as separate files. Always include a citation in the text for each figure using Arabic numbers, e.g., Fig. 4. Artwork must be submitted online in vector electronic form or by emailing it.

Preparation of Eletronic Figures for Publication

Although low-quality images are sufficient for review purposes, print publication requires high-quality images to prevent the final product being blurred or fuzzy. Submit (possibly by e-mail) EPS (line art) or TIFF (halftone/ photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Avoid using pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings). Please give the data for figures in black and white or submit a Color Work Agreement form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution at final image size ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs): >350 dpi; figures containing both halftone and line images: >650 dpi.

Color charges: Authors are advised to pay the full cost for the reproduction of their color artwork. Hence, please note that if there is color artwork in your manuscript when it is accepted for publication, we would require you to complete and return a Color Work Agreement form before your paper can be published. Also, you can email your editor to remove the color fee after acceptance of the paper.

Tips for Writing a Good Quality Science Frontier Research Paper

Techniques for writing a good quality Science Frontier Research paper:

1. Choosing the topic: In most cases, the topic is selected by the interests of the author, but it can also be suggested by the guides. You can have several topics, and then judge which you are most comfortable with. This may be done by asking several questions of yourself, like "Will I be able to carry out a search in this area? Will I find all necessary resources to accomplish the search? Will I be able to find all information in this field area?" If the answer to this type of question is "yes," then you ought to choose that topic. In most cases, you may have to conduct surveys and visit several places. Also, you might have to do a lot of work to find all the rises and falls of the various data on that subject. Sometimes, detailed information plays a vital role, instead of short information. Evaluators are human: The first thing to remember is that evaluators are also human beings. They are not only meant for rejecting a paper. They are here to evaluate your paper. So present your best aspect.

2. Think like evaluators: If you are in confusion or getting demotivated because your paper may not be accepted by the evaluators, then think, and try to evaluate your paper like an evaluator. Try to understand what an evaluator wants in your research paper, and you will automatically have your answer. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

3. Ask your guides: If you are having any difficulty with your research, then do not hesitate to share your difficulty with your guide (if you have one). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work, then ask your supervisor to help you with an alternative. He or she might also provide you with a list of essential readings.

4. Use of computer is recommended: As you are doing research in the field of science frontier then this point is quite obvious. Use right software: Always use good quality software packages. If you are not capable of judging good software, then you can lose the quality of your paper unknowingly. There are various programs available to help you which you can get through the internet.

5. Use the internet for help: An excellent start for your paper is using Google. It is a wondrous search engine, where you can have your doubts resolved. You may also read some answers for the frequent question of how to write your research paper or find a model research paper. You can download books from the internet. If you have all the required books, place importance on reading, selecting, and analyzing the specified information. Then sketch out your research paper. Use big pictures: You may use encyclopedias like Wikipedia to get pictures with the best resolution. At Global Journals, you should strictly follow here.

6. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right? It is a good habit which helps to not lose your continuity. You should always use bookmarks while searching on the internet also, which will make your search easier.

7. Revise what you wrote: When you write anything, always read it, summarize it, and then finalize it.

8. Make every effort: Make every effort to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in the introduction—what is the need for a particular research paper. Polish your work with good writing skills and always give an evaluator what he wants. Make backups: When you are going to do any important thing like making a research paper, you should always have backup copies of it either on your computer or on paper. This protects you from losing any portion of your important data.

9. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several unnecessary diagrams will degrade the quality of your paper by creating a hodgepodge. So always try to include diagrams which were made by you to improve the readability of your paper. Use of direct quotes: When you do research relevant to literature, history, or current affairs, then use of quotes becomes essential, but if the study is relevant to science, use of quotes is not preferable.

10. Use proper verb tense: Use proper verb tenses in your paper. Use past tense to present those events that have happened. Use present tense to indicate events that are going on. Use future tense to indicate events that will happen in the future. Use of wrong tenses will confuse the evaluator. Avoid sentences that are incomplete.

11. Pick a good study spot: Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

12. Know what you know: Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

13. Use good grammar: Always use good grammar and words that will have a positive impact on the evaluator; use of good vocabulary does not mean using tough words which the evaluator has to find in a dictionary. Do not fragment sentences. Eliminate one-word sentences. Do not ever use a big word when a smaller one would suffice.

Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

14. Arrangement of information: Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

15. Never start at the last minute: Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

16. Multitasking in research is not good: Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

17. Never copy others' work: Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

18. Go to seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

19. Refresh your mind after intervals: Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.

20. Think technically: Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

21. Adding unnecessary information: Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

22. Report concluded results: Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

23. Upon conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

The introduction: This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear: Adhere to recommended page limits.

Mistakes to avoid:

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

Title page:

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

Abstract: This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

Approach:

- o Single section and succinct.
- o An outline of the job done is always written in past tense.
- o Concentrate on shortening results—limit background information to a verdict or two.
- o Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

Introduction:

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.

The following approach can create a valuable beginning:

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

Approach:

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

Procedures (methods and materials):

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

Materials may be reported in part of a section or else they may be recognized along with your measures.

Methods:

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

What to stay away from:

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

Figures and tables:

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

Discussion:

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- Give details of all of your remarks as much as possible, focusing on mechanisms.
- Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

Describe generally acknowledged facts and main beliefs in present tense.

The Administration Rules

Administration Rules to Be Strictly Followed before Submitting Your Research Paper to Global Journals Inc.

Please read the following rules and regulations carefully before submitting your research paper to Global Journals Inc. to avoid rejection.

Segment draft and final research paper: You have to strictly follow the template of a research paper, failing which your paper may get rejected. You are expected to write each part of the paper wholly on your own. The peer reviewers need to identify your own perspective of the concepts in your own terms. Please do not extract straight from any other source, and do not rephrase someone else's analysis. Do not allow anyone else to proofread your manuscript.

Written material: You may discuss this with your guides and key sources. Do not copy anyone else's paper, even if this is only imitation, otherwise it will be rejected on the grounds of plagiarism, which is illegal. Various methods to avoid plagiarism are strictly applied by us to every paper, and, if found guilty, you may be blacklisted, which could affect your career adversely. To guard yourself and others from possible illegal use, please do not permit anyone to use or even read your paper and file.

CRITERION FOR GRADING A RESEARCH PAPER (COMPILATION) BY GLOBAL JOURNALS

Please note that following table is only a Grading of "Paper Compilation" and not on "Performed/Stated Research" whose grading solely depends on Individual Assigned Peer Reviewer and Editorial Board Member. These can be available only on request and after decision of Paper. This report will be the property of Global Journals.

Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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