Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 22 Issue 4 Version 1.0 Year 2022
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Boosting Human Insight by Cooperative AI: Foundations of Shannon-Neumann Logic

By Edouard Siregar

Abstract- We present the logical foundation of an artificial intelligence (AI) capable of dealing with complex dynamic challenges, that would be very hard to handle using traditional approaches (e.g. predicate logic and deep learning). The AI is based on a cooperative questioning game, to boost insight. Insight gains are measured by information, probability, uncertainty (Shannon), as well as utility (von Neumann).

The framework is a two-person cooperative iterated Q&A game, in which both players (human, AI agent) benefit (positive-sum): the human player gains insight and the AI player learns to improve its suggestions. Generally speaking, valuable insight is typically gained by asking 'good' questions about the 'right' topic, at the 'appropriate' time and place: by posing insightful questions. In this study, we propose a logical and mathematical framework, for the meanings of 'good, right, appropriate', within clearly-defined classes of human intentions.

Keywords: artificial general intelligence, complexity, cooperative learning games, frame drift problem, information entropy, insight problems, predicate logic, renormalization, utility, value-alignment problem.

GJSFR-F Classification: DDC Code: 006.3 LCC Code: Q335

Boosting Human Insight by Cooperative AI: Foundations of Shannon-Neumann Logic

Edouard Siregar

Abstract- We present the logical foundation of an artificial intelligence (AI) capable of dealing with complex dynamic challenges, that would be very hard to handle using traditional approaches (e.g. predicate logic and deep learning). The AI is based on a cooperative questioning game, to boost insight. Insight gains are measured by information, probability, uncertainty (Shannon), as well as utility (von Neumann).

The framework is a two-person cooperative iterated Q&A game, in which both players (human, AI agent) benefit (positive-sum): the human player gains insight and the AI player learns to improve its suggestions. Generally speaking, valuable insight is typically gained by asking 'good' questions about the 'right' topic, at the 'appropriate' time and place: by posing insightful questions. In this study, we propose a logical and mathematical framework, for the meanings of 'good, right, appropriate', within clearly-defined classes of human intentions.

AI based on this Shannon-Neumann Logic, combines symbolic AI with cooperative learning. It is transparent (no hidden layers), explainable (no unjustifiable moves), and remains human-aligned (no AI vs human contradictions) because of continuous cooperation (positive-sum game). In this paper, we focus uniquely on logical validity, and leave the complex topic scientific soundness for future research.

Keywords: *artificial general intelligence, complexity, cooperative learning games, frame drift problem, information entropy, insight problems, predicate logic, renormalization, utility, value-alignment problem.*

I. Introduction & Motivation

Purely algorithmic AI, from Predicate Logic [1] to Deep Learning neural nets [2–4], have proven highly effective for static, well-defined, narrow problems [5]. For *dynamic, complex challenges*, traditional AI becomes too 'brittle' (fails due to inappropriate application), and human *insight* is necessary to guarantee sound, human-aligned solutions. Solutions built on insufficient insight, can have deep long-lasting, human and economic consequences (e.g. conflict avoidance, war on drugs, pandemics or climate ill-preparedness).

Insight is usually gained (besides randomness and serendipity), by knowing *when/where* to pose *which* types of questions, about *what* topic: that is, by posing *'insightful questions'*. This ability thus requires a precise logical and mathematical meaning for the variables $\{when, where, what, which\}$, within well-defined contexts C, of human *cognitive mindsets*.

In this paper, the task of generating insightful questions, uses a framework we call Shannon-Neumann or SN-Logic, to cope with the fundamental concepts in

Author: Sofia Labs, AI systems research, Maryland, USA. e-mail: edsiregar@SofiaLabsLLC.com

Ref

1. Leon Sterling L. and Ehud Shapiro E. (1986) *The Art of Prolog: Advanced Programming Techniques* (MIT Press Series in Logic Programming), MIT Press; First Ed., ISBN-10: 0262192500 ISBN-13: 978-0262192507

insight-gains (see paper I [8]): built by combining information, probability, uncertainty [6] and utility [7]. This paper is structured as follows:

- In section 1, we discussed algorithmic vs human intelligence, and the purpose of SN-Logic.
- In section 2, we present the two-person (human H, AI agent A_{SN}) cooperative Iterated Questioning (IQ) game's role, from both H's and A_{SN}'s perspectives
- In section 2.3, we discuss the dynamic drift problem: coping with the changing human understanding of a given complex challenge, using a dynamic optimization process. It's impossible to clearly define a single problem, in complex challenges (e.g. war on drugs) so that they can last for decades
- In sections 3.1-3.2, we discuss SN-Logic's requirements to cope with insight (which involves causality, information, logic, probability, uncertainty and utility) and the spaces over which SN-Logic operates
- In sections 3.3-3.4, we introduce SN-Logic's grammar: semantics + syntax The syntax is used by *question generators*, to build millions of possible questions
- In section 3.5, we present SN-Logic predicates of two classes: problem difficulty-minimizing, and solution quality-maximizing, used in all inferences
- In section 3.6, we discuss the complexity and scope of SN-Logic, and section 3.7 highlights the distinction between knowledge acquisition (symbolic AI) and cooperative (machine) learning, both present in our AI
- In section 3.8, we introduce the *normal form* for making *SN-inferences*, about a question's insightfulness
- In section 4, we introduce the *Insight Gain Tensor* $\mu(when, where, what, which)$ to select *sound* inferences, from the many *valid* normal-form inferences, and measures of insight gains associated to these questions
- In section 5, we illustrate the use of SN-Logic, and we perform a validation test, to show how SN-Logic/IQ-game helps finding a solution path, to a component of a hard real-world solved case (quantum field theory research topic)

II. Two-Person Cooperative IQ-Game

a) IQ-game: Human player perspective

The Iterated Questioning or IQ game, is described in paper I. During a game session, the AI-agent, A_{SN}, poses the human player H, a question $q \in Q$, it thinks is most insightful, given H's current cognitive mindset $C(t)$. H then explores it, and reports if it was insightful. These are the game's *cooperative policies*, both players agree to adopt for each Q&A episode. The game serves several purposes which benefits both players (positive-sum game) [7, 9]

For the human player, H, the IQ-game has the following main roles:

- The IQ-game is a Q&A process that reduces uncertainty and increases information about a specific problem, via a sequence of Q&As. It provides an *effective tool*, to gain insight on the many aspects of a complex challenge.
- The IQ-game drives a sequential (mostly left-hemispheric) conscious reasoning for solving well-defined (narrow) tasks. This process is mirrored by algorithmic AI. For complex tasks, this process alone fails to deliver full solutions. Conceptual solutions to such problems require the next process: insight-gaining.

Ref

7. Von Neumann, J. and Morgenstern O. (1944) *Theory of Games and Economic Behavior*, Princeton University Press: Princeton, NJ.

- The IQ-game drives a parallel (mostly right-hemispheric) non-conscious process, for gaining insights leading to an 'aha' moment. Largely non-conscious processing can be used, where the first process proves too slow or impossible (task is too broad, ill-defined and complex).
- The IQ-game is driven by dual goals: minimizing obstacles and maximizing solution qualities. The *minimizing questions* guide H to eliminate or reduce difficulties in the problem, when possible. The *maximizing questions* guide H to boost specific solution qualities, when constraints allow it. It is a dynamic optimization (changes with H's understanding). We discuss this process in section 3.4.
- The IQ-game provides a non-brittle reasoning *framework*, which continuously adapts to the human player H's cognitive *intentions* C. This mindset C evolves as H's understanding of the challenge progresses. The IQ-game copes with the framework drift problem (section 2.3).

b) IQ-game: AI player perspective

For the AI-agent, A_{SN}, the IQ-game has these roles:

- The IQ game produces game session episodes, from which the agent A_{SN} can learn via cooperative learning.
- The IQ game ensures the agent remains *human-aligned* [10], because of the continuous human judgments. What is useful, informative, insightful for a human player H, does not necessarily mean the same for A_{SN}, even if it starts that way. In the learning process, these values can drift apart, due to many factors. In the IQ game, human valuation is the *ultimate arbiter*, for the insight value of a question (since any AI short of a full AGI super-intelligence, will fail miserably at this task), while SN-Logic estimates the insight values, given $C(t)$.
- The IQ game taps into a most valuable human resource: our collective evidence-based knowledge, undeniably our greatest accomplishment (culture, science, technology).

Note that our collective *belief-based* human selections are often poor (e.g. who we put in power as our leader). The forces here are complex and evolutionary: desire for control, cognitive biases and herd mentality from the fear of social isolation (e.g. [11]).

These factors are absent in the IQ procedure, since decisions are individual, and based *directly* on one's own *experience* of a question's insight, within a very specific cognitive context $C(t)$. It uses *direct evidence-based* judgment, where H's main incentive is to make life easier for herself. There are, of course individual variations in the experienced insightfulness of questions, but only stable patterns (across many individuals) are retained in cooperative learning (not presented in this paper).

c) Framework drift problem

A *complex challenge* is typically time-evolving, multi-objective, multi-solution, multi-discipline, multi-level and open-ended, making it hard from the start, to clearly define a single problem, even when it is *urgent* (e.g. a crisis) or *critical* (e.g. sustainability), or both (e.g. a pandemic)

Instead, there is a drift in the framing of problem and its solutions, as we accumulate new insights about a challenge: a framework drift problem. The drift cannot be handled with a static AI/ML system, focused on a given narrow problem.

The IQ-game, copes with the framework drift, by using an *adaptive* reasoning *framework*, and an *adaptive* cognitive intention $C = \{framework, where, when, what\}$

Ref

10. Russell S. (2019) *Human Compatible: Artificial Intelligence and the Problem of Control*, Viking, New York.

(section 3.3-3.4) which tracks the human player H's current understanding of the conceptual framework. It follows H's evolving understanding of the challenge, helping the SN-logic suggest the insightful questions, within each context C. The IQ-game doesn't define a problem from the start, but instead, let's H describe the

III. Predicate SN-Logic

a) SN-Logic requirements

Standard Logic Programming (predicate logic) is very effective when making strict deductions, but it cannot cope with the cooperative 2-person IQ-game. The purpose of SN-Logic is to provide an inference engine with the following requirements: it has to be ...

- *precise* (ambiguity-free) semantics axioms
- *consistent* (contradiction-free) framework within which, all SN-inferences can be made (normal-form inferencing)
- *transparent* (natural language, no hidden layers)
- *explainable* (no unjustifiable moves)
- *human-aligned* (no conflicts of with human cognitive intentions)
- *non-brittle* able to cope with fundamental concepts related to human-insight: causality (causes of insight), time-dependence (evolving understanding), information, probability, uncertainty (Shannon), utility (von Neumann), and insight (paper I). Brittleness is a common cause of AI failures.

To satisfy these requirements, we need a consistent set of SN-Logic definitions, axioms and rules, to which we now turn.

b) SN-Logic Spaces

To reason using a predicate logic (such as SN-Logic), the variables x need *spaces* X, to *scope* the *quantification*: $\forall x \in X$, $\exists x \in X$. SN-Logic's concepts are partitioned in six compact concept spaces, over which we can perform inferences (see appendices A-F):

Five *vector* spaces $\{T, S_D, S_C, S_G, S_S\}$, are used to describe the human player H's changing cognitive mindset $C(t)$, during the IQ-game. The AI agent, A_{SN}, needs to know $C(t)$, because the insightfulness of a question, depends on H's increasing understanding of the challenge and its possible solutions, as insight is accumulated.

The (tensor product) space S_A, of possible *conceptual actions* (operation x object) provide the raw material to build conceptual solutions.

Notes

Notes

- *Vector space T of exploration stages*: vector variable $[when \in T]$ describe the current stage $when$ of the exploration cycle. The vector $[when]$ rotates in T over time (appendix A).
- *Vector space S_D of mental obstacles*: vector variable $[where \in S_D]$ describes $where$ the human player's H difficulties reside. The vector $[where]$ rotates in S_D over time while exploring the challenge (appendix B).
- *Vector space S_C of difficulty causes*: vector variable $[what \in S_C]$ describes $what$ in the reasoning's framework, is causing H difficulty. The vector $[what]$ rotates in S_C over time while exploring the challenge (appendix C).
- *Vector space S_G of mental goals*: vector variable $[where \in S_G]$ describes the solution quality, H intends to improve. The vector $[where]$ rotates in S_G over time while exploring the challenge (appendix D)
- *Vector space S_S of solution elements*: vector variable $[what \in S_S]$ describe $what$ aspect of the solution, H intends to improve. The vector $[what]$ rotates in S_S over time while exploring the challenge (appendix E)
- *Tensor space of conceptual actions $S_A = O_p \times O_b$*: action variable $[which \equiv action \in S_A]$ is composed of a mental operation (verb $\in O_p$) attached to a target object (noun $\in O_b$)). Space S_A provides the *building-blocks* of conceptual solutions. (appendix F).

c) SN-Grammar: Axioms of Semantics

SN-Logic's role, is to provide guidance for insight-building via a Q&A process: suggesting $when/where$ to pose $which$ questions about $what$ topic. To be used in inferences, the meanings of the parts of speech (variables $\{when, where, what, which\}$), and the sentence structure (questions $which \equiv q \in Q$), have to be both *consistent* and *precise*.

A_{SN} needs a basic grammar (syntax, semantics, vocabulary) to communicate effectively with the human player H, in a consistent and precise manner. SN-Logic is based on four consistent (contradiction-free) axioms, to define its semantics precisely (ambiguity-free).

Let the human-player H's cognitive mindset $C(framework, p)$ be defined by the current reasoning $framework$ (next section), and three (intention) parameters: $p = \{when = p_1, where = p_2, what = p_3\}$, then:

(Sem 1) *Shannon-informative* questions: a question (which) $q(p, action)$, that *reduces uncertainty* (Shannon entropy) for H, who's mindset is $C(framework, p)$

(Sem 2) *Neumann-useful* questions: a question (which) $q(p, action)$, that has a *human-aligned* (via the 2-person IQ-game) *utility*, within a mindset $C(framework, p)$. It helps H make progress towards a solution.

(Sem 3) *SN-insightful* questions: question (which) $q(p, action)$ satisfying (Sem 1, Sem 2) is *SN-insightful*, within a mindset $C(framework, p)$, otherwise it is *SN-insightless*.

(Sem 4) *SN-Valid* inferences: an inference is SN-valid, if and only if it has the *SN normal form* (section 3.6)

These SN axioms of semantics, allow the AI to cope with core concepts of causality (causes of insight), dynamics (changing reasoning frames) information, probability, uncertainty [6], utility [7] and insight (paper I). These are necessary components of an insight-boosting AI. The axioms Sem1, Sem2 restrict the form of allowed questions. This constraint is used by a *Q-generator* of questions $q \in Q$, to which we now turn.

d) SN-Grammar: Syntax for Dual-Optimization

The cooperative IQ-game is driven by *dual-objectives*: to *minimize* the problem's *causes of difficulty*, and to *maximize* the solution's *quality*. The optimization must continuously adapt to H's understanding of the challenge, over an IQ-game session).

The SN-grammar has a simple syntax, specified for each question class Q. All questions $q \in Q$ will fall into two classes $Q = \{Q_{min}, Q_{max}\}$, from two complementary (dual) perspectives: (a) causes of cognitive *difficulty* (to *minimize*), (b) *qualities* of solution (to *maximize*). Each question class *generates* many of specific questions, aimed at making insight-gains.

> The purpose of SN-Logic is to incrementally boost our insight about solutions, by suggesting *when/where* to pose *which* types of questions about *what* topic, while adapting to a moving target: our *current* understanding the *obstacles* in a challenge

The question generator, or *Q-gen*, of *difficulty-minimizing* questions, uses a specific syntax for an evolving cognitive mindset $C_{min}(frame, topic, p_1, p_2, p_3)$. There is a lot of freedom in which questions to pose, even at a specific place and time, within a well-defined framework. We select a set of six commonly useful problem-solving questions, to illustrate the procedure.

Q-Gen Syntax: difficulty-minimizing questions $q(p, action) \in Q_{min}$

> q_{min1}: at what exploration stage are we in now? (specifies $when = p_1 \in T$)
> q_{min2}: what reasoning *frame* are we operating in, now? (specifies [frame])
> q_{min3}: what *topic* in [frame] are we focusing on, now? (specifies [topic])
> q_{min4}: where does the main difficulty reside? (specifies $where = p_2 \in S_D$)
> q_{min5}: what, more specifically, causes this difficulty? (specifies $what = p_3 \in S_C$)
> q_{min6}: can you *reduce* the difficulty (*where*) and *avoid* its causes (*what*),
> by using these actions? (specifies $action \in S_A$ and $which = q_{min6} \in Q_{min}$)

The variable [*action*] ($\in S_A \equiv O_P \times O_b$), is a product [*verb operation*] ($\in O_p$) x [*noun object*] ($\in O_b$) (appendices and section 5).

The [frame] variable, labels the reasoning framework *currently* being used (e.g. a discipline, a subject, a specialty, a model, a system, a theory, a technology etc.). This framework can change from one exploration stage to the next. It is a moving target, which mirrors our current understanding of a complex challenge.

The [topic] variable, labels a set of items we're focusing on, within [frame] (e.g. agents, assumptions, bounds, properties, qualities, relations, statements, strategies, tactics, techniques etc.). Typically, [topic] is a tool we use within [frame], to make progress. For a concrete example, see section 5.

Questions $q \in Q_{min}$ are *SN-insightful*, only if they are *SN-informative* (axiom Sem 1): they attempt to reduce a maximum possible amount of uncertainty (alternatives, ignorance, options, possibilities), within the context C_{min}.

The generator of *quality-maximizing* questions, uses a specific syntax for an evolving cognitive mindset $C_{max}(frame, topic, p_1, p_2, p_3)$:

Ref

6. Shannon C. and Weaver W. (1949) *The mathematical theory of communications*, Univ. Illinois Press.

Global Journal of Science Frontier Research (F) Volume XXII Issue IV Version I Year 2022

Q-Gen Syntax: quality-maximizing questions $q(p, action) \in Q_{max}$

q_{max1}: at what exploration stage are we in now? (specifies $when = p_1 \in T$)
q_{max2}: what reasoning *frame* are we operating in, now? (specifies [frame])
q_{max3}: what *topic* in [frame] are we focusing on, now? (specifies [topic])
q_{max4}: where do you need a boost (goal)? (specifies $where = p_2 \in S_G$)
q_{max5}: what solution aspect, do you want to focus on? (specifies $what = p_3 \in S_S$)
q_{max6}: can you *boost* your goal ($where$) and the solution's quality ($what$), by using these *actions*? (specifies $action \in S_A$ and $which = q_{max6} \in Q_{max}$)

Questions in Q_{max} are *SN-insightful*, only if they are *SN-informative* (axiom Sem 1): they attempt to reduce a maximum amount of uncertainty (alternatives, ignorance, options, possibilities), within the context C_{max}. They are *specificity-boosting* questions which reduce uncertainty (Shannon entropy) to increase the solution's quality.

e) SN-Logic predicates q(x)

The SN concept of *insight* involves notions in information, logic, probability, uncertainty and utility (see paper I). To cope with these, we need a logic with quantifiers for scoping the variables x to specific spaces X. In standard predicate logic, a predicate is a *function* p of a variable x, which maps a variable $x \in X$, into the predicate's truth values $\{T, F\}$ [12].

$$X \rightarrow \{T, F\} \text{ and } x \in X \rightarrow p(x) = T \text{ } or \text{ } F$$

In SN-Logic, an SN-predicate is a *function* q of a variable x, which maps a variable $x \in X$, into the predicate's insight values $\{insightful\ I^+, insightless\ I^0\}$.

$$X \rightarrow \{I^+, I^0\} \text{ and } x \in X \rightarrow q(x) = I^+ \text{ } or \text{ } I^0$$

In SN-Logic we define the two classes (minimizing, maximizing) of predicates $q(x)$, the mindset parameter $p \in P \equiv \{when, where, what\}$ and the predicate variable 'cognitive *action*':

- SN-predicate questions $q(p, action) \in Q_{min}$, where $p \in P$, $action \in S_A$
- SN-predicate questions $q(p, action) \in Q_{max}$, where $p \in P$, $action \in S_A$

The parameter $p \in P$ is in the space P of *cognitive mindsets* $C_{min}(framework, p)$: the set of H's *intentions*, during the IQ-game. The AI needs to know this intent, to make useful cooperative suggestions. The mindset parameter p, encodes the type of insight, H wants to boost, at any given time.

f) SN-Logic Complexity & Scope

SN-Logic only requires concept spaces $(\{T, S_D, S_C, S_G, S_Q, O_p, O_b\}$ of very small size $N = Card(Space) \approx 10^2$ (see appendices).

- Number of distinct *cognitive mindsets*: $N_{cogn} = O(Card(P)) = O(Card(T) \times Card(S_D) \times Card(S_C) = 10 \times 10 \times 10 = 10^3$
- Number of possible *conceptual actions*: $N_{acts} = O(Card(S_A)) = O(Card(O_p) \times O(Card(O_b)) = 10^2 \times 10^2 = 10^4$
- Number of possible *distinct questions*: $N_{ques} = Card(Q) = N_{cogn} \times N_{acts} = 10^7$ minimizing questions, posed by the Q_{min}-generator (same for maximizing questions).

Ref

12. Andrews P. B. (2002) *An Introduction to Mathematical Logic and Type Theory: To Truth Through Proof*, 2nd ed., Berlin: Kluwer Academic Pub. and Springer.

These numbers already compare favorably to a typical human problem-solver H, working by herself. But the real power of SN-Logic (its scope of applications), comes from the combinatorial possibilities: the possible *combinations* and *permutations* of insight-boosting questions, needed to solve *each class* of challenges:

- Number of combinations: $N_{comb} = 2^{N_{ques}}$
- Number of permutations: $N_{perm} = N_{ques}!$

Thus, the number of distinct classes of challenges SN-Logic can cope with, is *effectively infinite* ($N = 10^7!$), yet, based on a few small, compact concept spaces (cardinality $\approx 10^2$). In this sense, SN-Logic is *economical* (Occam's razor).

Notes

g) Symbolic AI (knowledge acquisition) vs Learning

The computed complexity of SN-Logic is a theoretical *upper bound*, to determine the scope of SN-Logic. In practice the *computational cost* will be much lower, due to *universal* constraints (common to all challenge classes), because they are imposed by (mostly) challenge-independent forces:

- causality: universal root causes of cognitive difficulties (e.g. confusion due to ambiguity, indecision due to missing information) and solution quality (e.g. accuracy, adaptability)
- logic: *valid* inferences with *sound* semantics
- planning: logically *necessary* chronology of solution steps
- problem-solving: *universal tactics* to minimize obstacles (to avoid/reduce), and maximize solution quality (to target/increase/maximize) (e.g. divide-and-conquer, minimize ambiguity, maximize order, simplify)
- information: a question is only informative, if it *reduces uncertainty* by eliminating alternatives, options, outcomes, possibilities, within a cognitive mindset (intention) C, restricting the *insightful* questions to a manageable subset: $q \in Q^*(C) \subset Q$, with $Card(Q^*(C)) << Card(Q)$
- utility: a question is only useful, if it helps H, overcome obstacles, given a cognitive intention C, restricting the *insightful* questions to a manageable subset: $q \in Q^*(C) \subset Q$, with $Card(Q^*(C)) << Card(Q)$

These rules impose a lot of structure on the SN-agent's *insight grain tensor* $\mu(frame, topic, when, where, what, which)$, which is, in its fully general form, a high-dimensional rank-6 tensor, but is in practice, very *sparse* and *decomposable* into simpler tensors and convolution kernels.

The structure imposed by the universal (challenge class-independent) constraints, is sufficient to construct factored ('vanilla') tensors μ^* of much lower dimensions and lower rank: *knowledge acquisition.* A 'flavor' is then learned to *fine-tune* the tensors to each class of challenge, via *cooperative learning* (not described in this paper). Given the complexity upper-bounds of SN-Logic, the fine-tuning possibilities are vast.

h) SN-Logic Normal Form

A_{SN}'s fundamental problem, is to use the IQ-game, to guide a human player H, in *when* and *where*, to pose *which* types of questions about *what* topic, to gain a maximum amount of insight into a complex challenge.

A standard normal form inferencing (analogous to conjunctive and disjunctive normal forms, in digital and predicate logic), is necessary for the AI to cope with the computational complexity of SN-Logic. The AI can efficiently search for predicate variables $action \in S_A$, used as building-blocks for conceptual solutions. Given an *evolving* inferencing framework (*frame, topic*), SN-normal forms are the following:

SN normal-form for minimizing inferences

Given a minimizing mindset $C_{min}(frame, topic, p)$, where
$p \in P = \{when, where, what\}$:

if $\exists$ $action \in S_A$, such that $\mu_{min}(frame, topic, p, action) > \mu_{crit}$,

then

$q(p, action) \in Q^*_{min}(C_{min}) \subset Q_{min}$, and

$q(p, action)$ is *SN-insightful*, within C_{min}

SN normal-form for maximizing inferences

Given a maximizing mindset $C_{max}(frame, topic, p)$, where
$p \in P = \{when, where, what\}$:

if $\exists$ $action \in S_A$, such that $\mu_{max}(frame, topic, p, action) > \mu_{crit}$,

then

$q(p, action) \in Q^*_{max}(C_{max}) \subset Q_{max}$, and

$q(p, action)$ is *SN-insightful*, within C_{max}

The sets $Q^*(C)$, are maximum-insight subsets of Q_{min} or Q_{max}, and $\mu(frame, topic, p, action)$ is an *insight-gain tensor* (discussed shortly) whose insight gains are above a minimum *critical cutoff* μ_{crit}. The purpose of an insight-gain cutoff scale is intuitive, but its mathematical justification is outside the scope of this paper, which focuses only on logical *validity*, and ignores scientific *soundness*. The cutoff is related to a scale-invariance due to a conformal symmetry, under the renormalization of probabilities (unitarity). *Scale-separation* is used in quantum field theories [13], but *justified* by the *conformal symmetry* [14] of a renormalization group [15].

To perform successful inferences autonomously, the AI agent needs to possess the means of deciding whether a predicate variable $action \in S_A$, leads to insight gains above a minimum lower bound (that is, $action \in S^*_A(C) \subset S_A$). The insight-gain tensor provides the SN-agent, the ability to select *sound* inferences, from a vast number of merely, *valid* ones (that is, of SN normal-form).

IV. Insight Gain Tensors μ

a) Need for Insight-Gain Tensors

The AI performs SN normal-form inferences, to suggest insightful questions to explore, given human-targeted insight gains $C(p)$. These 'most insightful' questions, lie in a restricted subspace $Q^*(C) = \{Q^*_{min}(C_{min}),\ Q^*_{max}(C_{max})\}$, within a large space Q, of possible questions ($Card(Q) = 10^7$). Given a current mindset $C(p)$, A_{SN} must find a subspace of questions $Q^*(C)$. This is where an insight-gain measure $\mu(p, action)$ (convolution tensors and their kernels, used to restrict searches to optimal sub-spaces) are essential, to make *sound* inferences (real-world accurate), rather than merely *valid* ones (SN normal-form inferences). This will be presented elsewhere. For now, we simply discuss general constraints imposed by SN-Logic, on the tensor elements.

Ref

13. Dyson F., (1949) The radiation theories of Tomonaga, Schwinger and Feynman, Phys. Rev. 75, 486.

b) Constraints on Insight-Gain Tensors μ

The AI's capacity to generate *SN-insightful* I^+ questions, from a vast possibility of *insightless* I^0 ones (with $actions \in S_A$), resides in the structure a high-dimensional *insight-gain* tensor $\mu(when, where, what, which) \equiv \mu(p, action)$, for each challenge *class* and reasoning *frame*. So the full rank-7 tensor is actually $\mu(class, frame, topic, p_1, p_2, p_3, action)$. This function outputs the value g of insight gain associated to exploring a question $which \equiv q(p, action) \in Q$, where $p \in P$ encodes H's targeted insight gains. To be useful, the tensor μ is required to satisfy the following properties:

- $\mu : Cl \times Fr \times P \ \times \ S_A \rightarrow [0,1]$, where Cl = set of challenge classes, Fr = set of reasoning frameworks (frame+topic), $P = T \times S_1 \times S_2$, $S_A = O_p \ \times \ O_b$, $S_1 = S_D$ *or* S_G, and $S_2 = S_C$ *or* S_Q
- it is a measure of insight gain $\mu(class, frame, topic, p, action) = g \in [0,1]$ (normalized)
- probability of all possible *actions* with a mindset p, must sum to one (unitarity)
- $\mu_{crit} \in]0,1[$ (minimum critical insight-gain value $\mu > \mu_{crit}$)
- $g = 0$ when $q(p, action)$ is *SN-insightless* I^0 , given the mindset p
- $g = 1$ when $q(p, action)$ is maximally *SN-insightful* I^+, given the mindset p
- μ is initialized by satisfying heuristics from causality, information, logic, planning, problem solving and utility. These constraints provide the initial (challenge class-independent) approximation for μ
- μ gets optimized (fine-tuned) for specific classes of challenges, by *cooperative learning*, using the IQ-game's session episodes

V. Validation Test: Post-Doc Researcher's Dilemma

We can now illustrate how SN-Logic is used, on a real challenge. In the IQ-game, both players (human: H, A_{SN}) agree to use simple *cooperative strategies*, given H's current mindset C:

(1) A_{SN} suggests its guess at a most insightful question ($q \in Q^*(C)$)

(2) H reports questions q she actually finds insightful

The game's Q&A session, cycles over each obstacle, encountered within a challenge. Hundreds of such sub-problems may be encountered, to solve a challenge. Usually, the number and nature of these obstacles is unknown ahead of time, in real-world challenges.

For clarity, we use a single, static, not so complex, yet most difficult challenge. The scenario is: a young post-doctoral researcher, H, is trying to find a good quantum field topic, to spend her next ten years on. The first few moves (Q&As) of the two-person IQ-game, could proceed as follows:

Q from A_{SN}: 'Greetings! What *class* of challenge are we exploring today (sample which depends on what SN-Logic is being used for):

Notes

To *compose* something	e.g. music, arts, literature, programming
To *compute* something	e.g. any domain
To *construct* something	e.g. any domain
To *design* something	e.g. engineering, technology
To *discover* something	e.g. science, mathematics
To *govern* something	e.g. crisis mitigation, leadership
To *invent* something	e.g. engineering, science, technology
To *manage* something	e.g. corporate, government
To *optimize* something	e.g. any domain
To *predict* something	e.g. any domain
To *reconstruct* something	e.g. intelligence, inverse problems, sensing
To *solve* something	e.g. any domain

A by H: I want to improve on standard quantum field theory, its a *discover* class of challenge ([class] = *discover*).

1. Q from A_{SN}: Which *exploration stage* are we in, now: (AI is using q_{min1} in Q-gen)

   to specify a current obstacle
   to minimize the obstacle
   to explore solution ideas
   to question a solution idea
   to verify a solution idea

   A by H: 'I want to identify the current obstacle'

2. Q from A_{SN}: 'What is our current *reasoning framework*? (AI is using q_{min2} in Q-generator)

   The framework is composed of a *topic* and a *frame*

The *topic* can be any useful tool we select, for overcoming the obstacle (select the closest match):

actions	e.g. activities or behaviors
agents	e.g. catalysts or inhibitors
limits	e.g. lower, upper, extremes
computations	e.g. algorithms
equations	e.g. model or representation
laws	e.g. laws of quantum physics
procedures	e.g. protocols or decision
processes	e.g. interactions or communications
properties	e.g. pattern or symmetry
qualities	e.g. strengths or weaknesses
relationships	e.g. hierarchy or priorities
restrictions	e.g. constraints or conditions
rules	e.g. allowed or forbidden
statements	e.g. assumptions, conditions or theorems
states	e.g. equilibrium or criticality
strategies	e.g. divide-and-conquer
structures	e.g. classes, partitions, sets
tactics	e.g. explore special cases
techniques	e.g. calculation or construction
...	

Notes

The reasoning *frame* is the clearly-defined context, within which *topic* is being used (select the closest match):

discipline	e.g. molecular biology
subject	e.g. protein folding
context	e.g. social revolution
environment	e.g. location and time
event	e.g. activity or pandemic crisis
model	e.g. just-in-time supply-chains
principle	e.g. quantum computing
method	e.g. optimization
network	e.g. communication or transport
theory	e.g. general relativity
specialty	e.g. programming
system	e.g. quantum communications
technology	e.g. fresh water extractor
...	

Note SN-logic's non-brittleness: at any given time, the reasoning *frame* can adapt to any required abstraction level and scope. Such frame changes are typically *unpredictable* at the start of a real-world challenge.

A by H: For my research direction, I want to identify a weakness in quantum field theory (QFT) (so here, [frame] ≡ *theory*, and [topic] ≡ *qualities*).

3. Q from A_{SN}: 'What's your main difficulty with the [frame], right now? (select the closest match)
(AI is using q_{min4} from Q-generator + an insight-gain tensor/kernel)

inability to *apply*	[topic] in/of [frame]
inability to *compute*	[topic] in/of [frame]
inability to *construct*	[topic] in/of [frame]
inability to *decide*	[topic] in/of [frame]
inability to *evaluate*	[topic] in/of [frame]
inability to *exploit*	[topic] in/of [frame]
inability to *identify*	[topic] in/of [frame]
inability to *select*	[topic] in/of [frame]
inability to *simplify*	[topic] in/of [frame]
inability to *solve*	[topic] in/of [frame]
inability to *understand*	[topic] in/of [frame]

A by H: I can't *evaluate* the weaknesses [topic = qualities] of quantum field theory [frame = theory]

4. Q from A_{SN}: 'More specifically, why can't you *evaluate* the [topic] in [frame]? (select the closest root cause of the difficulty)
(AI is using q_{min5} from Q-generator + an insight-gain tensor/kernel)

missing *comparison for*	[topic] in/of [frame]
missing *constraint on*	[topic] in/of [frame]
missing *criterion for*	[topic] in/of [frame]
missing *direction in*	[topic] in/of [frame]
missing *information about*	[topic] in/of [frame]
missing *intuition for*	[topic] in/of [frame]
missing *knowledge of*	[topic] in/of [frame]
missing *metric for*	[topic] in/of [frame]
missing *ranking of*	[topic] in/of [frame]
missing *standard for*	[topic] in/of [frame]
missing *value of*	[topic] in/of [frame]

A by H: 'I lack an *intuition for* the weaknesses of QFT'

Notes

Notes

5. Q from A_{SN}: awesome, so our current obstacle is, our *missing intuition* for the *weaknesses* of QFT. Let's try to eliminate this obstacle.

   A by H: 'Okay, I'm all ears!'

6. Q from A_{SN}: can we gain *intuition* to *evaluate* QFT's weakness, by...
   (explore any question you think is promising, or move-on)
   (AI is using q_{min6} in Q-generator + SN normal-form inferences + insight-gain tensors/kernels)

   by exploring:
   idealized cases of the *theory* (QFT)
   solved cases of the *theory* (QFT)
   simple cases of the *theory* (QFT)

   by outlining:
   consequences (causal) of the *theory* (QFT)
   implications (logical) of the *theory* (QFT)
   predictions (temporal) of the *theory* (QFT)
   tests (experimental) of the *theory* (QFT)

   by identifying:
   inconsistent aspects of the *theory* (QFT)
   limitations of the *theory* (QFT)
   problematic aspects of the *theory* (QFT)
   uncertain aspects of the *theory* (QFT)
   unjustified aspects the of *theory* (QFT)
   untested aspects of the *theory* (QFT)
   by looking for:
   ambiguities (imprecision)
   contradictions (logical, evidence)
   counter-examples (exceptions)
   discrepancies (differences)
   dogma (cognitive traps)
   errors (math, procedures)
   falsehoods (logical)
   flaws (procedure, reasoning)
   gaps (missing pieces)
   implicit assumptions (reasoning)
   impossibilities (logical, physical)
   inaccuracies (scientific, technical)
   incompatibilities (between two items)
   inconsistencies (logical)
   limitations (scope of applicability)
   unexplained items (no explanation)
   unjustified items (lack justification)
   unsupported items (lack evidence)
   violations (law-breaking)
   weaknesses (logical)

   A from H: 'I find some questions quite *insightful*, because
   (click on each insightful one, and note the reasons for your record):

   I outlined the *implications* of QFT (e.g. including matrix unitarity), and QFT's *experimental tests* (e.g. including neutron decay experiments). I found reported *incompatibilities* (e.g. known violations of the CKM matrix's unitarity [16], in neutron decay experiments [17]). That seems like an interesting research area of quantum field theory, for me.

7. Q from A_{SN}: 'Do you want to identify a new obstacle, now? ...
   Note: for a complex challenge, limitless combinations of obstacles can be explored in this manner.

This scenario shows how suggested questions from A_{SN}, can replicate real-world solutions to obstacles, via a *cooperative Q&A* dialog. The researchers do something similar between themselves, early-on, to decide what to work on. But AI's *complementary* strength, is to cover many exploration paths, which are very often overlooked, yet may be key to quality solutions. This dynamic 'human-AI' interaction would be even more fruitful, in a *group brainstorming session*, where each member of the team, can select directions to explore and possible answers.

VI. Discussion

a) Tensor Construction & Cooperative learning

We mentioned (section 3.7), that insight-gain convolution tensors and kernels, form the bridge between the SN normal form inferencing (SN-validity), and measures of insight (SN-soundness); the bridge between logic (validity) and science (soundness). Initially, the tensors μ are the AI's 'vanilla' core, then, learned flavors are added to it, via machine learning to optimize the core AI, to distinct challenge classes.

The AI's core will be initialized by heuristics from causality, information, logic, planning, problem-solving, and utility. These apply to all types of challenges. The tensors' added flavor, needs to be learned using *cooperative learning* via a renormalization procedure, from the IQ-game's episodes. The construction of the insight-gain tensors and cooperative learning will be described in future work.

b) Conclusion

We presented the foundations of SN-Logic, designed to boost human insight, to help overcome challenges that are hard to deal with, using traditional AI (mainly, predicate logic and deep learning neural nets). This required a logic, capable of coping with the concepts necessary to measure insight-gains: causality (causes of insight gains), dynamics (adaptive reasoning frameworks), information, probability, uncertainty (Shannon) and utility (von Neumann).

In this paper, we presented the following:

- The two-person (H, A_{SN}) cooperative IQ-game's role from both H's and A_{SN}'s perspectives
- The *frame drift problem*: coping with the *changing understanding* of a challenge, using a (non-brittle) logic and optimization process, which continuously adapt to the current human understanding and intention
- SN-Logic's requirements to compute insightfulness (which involves causality, information, logic, probability, uncertainty and utility) and the concept spaces over which SN-Logic operates (to scope the quantifiers)
- SN-Logic's *grammar: semantics + syntax* for posing questions $q \in Q$ from a vast space of potential questions. The syntax is used by a dual *question generator* ($q \in Q_{min}, q \in Q_{max}$), from which all questions are built ($N_{ques} = O(10^7)$)
- SN-Logic *predicates* of two question classes: problem difficulty-minimizing, and solution quality-maximizing, used in all inferences
- The *complexity* of SN-Logic, and show it's broad scope and capability of coping with a large number of distinct challenge classes.
- The SN *normal-form* for making *valid* inferences, about a question's insightfulness, efficiently within a vast space of possibilities

Ref

17. Zyla, P.A et al (2020) Review of Particle Physics: CKM quark-mixing matrix, *Progress of Theoretical and Experimental Physics*. 2020 (8): 083C01. doi: 10.1093/ptep/ptaa104.

- *Insight Gain Tensors* $\mu(when, where, what, which)$ are necessary to select *sound* inferences (real-world accurate), from a vast (effectively infinite) number of *valid* ones (those with SN normal-form). μ measures the human insight gains, associated to questions posed, within their cognitive mindsets (C_{min}, C_{max})
- A validation test, to show that SN-Logic can replicate the solution steps, to a real-world solved case (discovery in quantum field theory)

Notes

This paper focused solely on logic and validity of SN-inferences. It has not dealt with the equally important issue of scientific soundness and accuracy. We will present the construction of the insight-gain convolution tensors and kernels, and the learned structure (cooperative learning), in future papers.

VII. Appendices

A: Vector Space of Exploration Steps T (sample)

Time basis vector: $when \equiv p_1 \in T$)
to identify an obstacle
to minimize the obstacle
to explore solution ideas
to question a solution idea
to verify a solution idea

B: Vector Space of Cognitive Difficulties S_D (sample)

Basis vectors of cognitive obstacles: $where \equiv p_2 \in S_D$	
inability to *classify*	[frame]
inability to *compute*	[frame]
inability to *connect*	[frame]
inability to *construct*	[frame]
inability to *count*	[frame]
inability to *decide*	[frame]
inability to *design*	[frame]
inability to *eliminate*	[frame]
inability to *evaluate*	[frame]
inability to *exploit*	[frame]
inability to *extract*	[frame]
inability to *identify*	[frame]
inability to *interpret*	[frame]
inability to *organize*	[frame]
inability to *perform*	[frame]
inability to *plan*	[frame]
inability to *predict*	[frame]
inability to *rank*	[frame]
inability to *relate*	[frame]
inability to *select*	[frame]
inability to *simplify*	[frame]
inability to *solve*	[frame]
inability to *transform*	[frame]
inability to *verify*	[frame]
etc.	

C: Vector Space of Difficulty Causes S_C (sample)

Basis vectors of causes: *what* $\equiv p_3 \in S_C$	
level of *abstraction of*	[item]
level of *ambiguity of*	[item]
level of *complexity of*	[item]
level of *dependencies in*	[item]
level of *flaws in*	[item]
level of *fragmentation of*	[item]
level of *implicitness in*	[item]
level of *impracticality of*	[item]
level of *imprecision of*	[item]
level of *incompleteness of*	[item]
level of *inconsistency in*	[item]
level of *indecision about*	[item]
level of *indetermination in*	[item]
level of *inefficiency of*	[item]
level of *insufficiency of*	[item]
level of *uncertainty in*	[item]
level of *unpredictability of*	[item]
level of *weakness of*	[item]
etc.	
missing *assumption about*	[item]
missing *bounds on*	[item]
missing *capacity for*	[item]
missing *classification of*	[item]
missing *confidence in*	[item]
missing *connections in*	[item]
missing *constraints on*	[item]
missing *evidence for*	[item]
missing *explanation for*	[item]
missing *freedom to*	[item]
missing *information about*	[item]
missing *interpretation of*	[item]
missing *intuition for*	[item]
missing *justification for*	[item]
missing *motivation for*	[item]
missing *organization of*	[item]
missing *representation of*	[item]
missing *restriction on*	[item]
missing *scales in*	[item]
missing *statements in*	[item]
missing *tools for*	[item]
missing *verification of*	[item]
etc.	

Notes

D: Vector Space of Mental Goals S_G (sample)

Basis vectors of cognitive goals: $where \equiv p_2 \in S_G$	
clarity about the	[solution item]
confidence in the	[solution item]
construction of the	[solution item]
criticism of the	[solution item]
exploitation of the	[solution item]
imagination for the	[solution item]
intuition for the	[solution item]
understanding of the	[solution item]
etc.	

Note: mental goals [*where*] are intentions one tries to maximize, under constraints. The vector $where \in S_G$ rotates in S_G, with the mindset C about the challenge.

E: Vector Space of Solution Elements S_S (sample)

Basis vectors of solution elements: $what \equiv p_3 \in S_S$
solution's *agents*
solution's *cases*
solution's *components*
solution's *consequences*
solution's *constraints*
solution's *dimensions*
solution's *economy*
solution's *efficiency*
solution's *effectiveness*
solution's *ethics*
solution's *form*
solution's *framework*
solution's *information*
solution's *justification*
solution's *methods*
solution's *plan*
solution's *properties*
solution's *qualities*
solution's *relationships*
solution's *requirements*
solution's *resources*
solution's *restrictions*
solution's *space*
solution's *statements*
solution's *sustainability*
solution's *utility*
solution's *value*
etc.

F: Space of Actions $S_A = O_p \times O_b$ (tiny sample)

Conceptual Action Space: $operation \in O_p \times object \in O_b$
Actions to minimize indecision: avoiding, comparing, demanding, imposing, evaluating, excluding, justifying, maximizing, minimizing, optimizing, prioritizing, ranking, requiring, selecting, weighing items etc. Actions to minimize incomprehension: classifying, collecting, defining, explaining, exploring, exploiting, decomposing, grouping, imposing, interpreting, isolating, reconstructing, relating, removing, separating items etc. Actions to minimize inexperience: exploring cases, exploring examples, exploring idealisations, exploring simplifications etc. Actions to minimize skepticism: comparing, demanding, excluding, explaining, gathering, imposing, justifying, reasoning, refuting, rejecting, requiring, searching for, testing, verifying items etc. Actions to minimize unfamiliarity: building an analogy, building a model, defining concepts, looking for items, outlining facts Actions to maximize ability: training to abstract, training to eliminate, training to exploit, training to organize, training to perform, training to relate, training to select, training to simplify, training to solve, training to transform etc. Actions to maximize clarity: classifying, connecting, defining, idealizing, ordering, organizing, outlining, reducing, relating, removing, separating, simplifying, summarizing items etc. Actions to maximize criticism: questioning an assumption, questioning a premise, questioning the framework, questioning a representation, questioning the necessity, questioning the sufficiency, questioning a method, questioning a path, questioning a solution, questioning the value etc. Actions to maximize exploitation: using an assumption, using a fact, using a given, using a constraint, using a property, using a relationship, using a restriction, using a statement, using a theorem etc. Actions to maximize imagination: weakening an assumption, weakening a bound, weakening a condition, weakening a constraint, weakening a requirement, weakening a restriction, weakening a rule, weakening a statement etc. Actions to maximize intuition: exploring an analogy, exploring a case, exploring an example, exploring a diagram, exploring a metaphor, exploring a model, exploring a story, exploring a simplification etc.

Notes

Notes

References Références Referencias

1. Leon Sterling L. and Ehud Shapiro E. (1986) *The Art of Prolog: Advanced Programming Techniques* (MIT Press Series in Logic Programming), MIT Press; First Ed., ISBN-10: 0262192500 ISBN-13: 978-0262192507
2. Mohri M. (2018) *Foundations of Machine Learning*, The MIT Press, 2nd Ed., Cambridge Massachusetts.
3. Hornik, k. et al. (1989) Multilayer feed forward networks are universal approximators, Neural Networks, Vol. 2, Issue 5, Pages 359-366.
4. Guilhoto L. F. (2018) *An Overview of Artificial Neural Networks for Mathematicians*, Univ. Chicago.
5. Humphreys I. R. et al, (2021) Computed structures of core eukaryotic protein complexes, Science. DOI: 10.1126/science.abm 4805
6. Shannon C. and Weaver W. (1949) *The mathematical theory of communications*, Univ. Illinois Press.
7. Von Neumann, J. and Morgenstern O. (1944) *Theory of Games and Economic Behavior,* Princeton University Press: Princeton, NJ.
8. Siregar E., (2021) Learning human insight by cooperative AI: Shannon-Neumann measure, IOP Publishing Ltd., SciNotes, Volume 2, Number 2
9. Nash J. (1953) Two-Person Cooperative Games, Econometrica, Vol. 21, No. 1 (Jan., 1953), pp. 128-140.
10. Russell S. (2019) *Human Compatible: Artificial Intelligence and the Problem of Control,* Viking, New York.
11. Fromm E. (1994) *Escape from freedom,* Holt Paperbacks; 1st Edition.
12. Andrews P. B. (2002) *An Introduction to Mathematical Logic and Type Theory: To Truth Through Proof,* 2nd ed., Berlin: Kluwer Academic Pub. and Springer.
13. Dyson F., (1949) The radiation theories of Tomonaga, Schwinger and Feynman, Phys. Rev. 75, 486.
14. Guillarmou C., (2020) Conformal bootstrap in Liouville theory, arXiv:2005.11530v2 [math.PR] 11 Nov 2020.
15. Wilson K. G., (1971) Renormalization Group and Critical Phenomena. I. Renormalization Group and the Kadanoff Scaling Picture, Phys. Rev. B 4, 3174.
16. Wikipedia: Cabibbo–Kobayashi–Maskawa matrix, *m, en.wikipedia. org/wiki/Cabibbo–Kobayashi–Maskawa matrix.*
17. Zyla, P.A et al (2020) Review of Particle Physics: CKM quark-mixing matrix, *Progress of Theoretical and Experimental Physics.* 2020 (8): 083C01. doi: 10.1093/ptep/ptaa104.