Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 22 Issue 1 Version 1.0 Year 2022
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Modeling Supply Chains by Critical Paths and Leontief Input-Output Table

By Gregory L. Light
Providence College

Abstract- We formulate a supply-chain problem by:(1)casting it in the model of critical-path analysis as defined by predecessor/successor relations, and (2)allowing for mutual dependency among the activity nodes and applying Leontief's input-output structure.

Keywords: *scheduling bottleneck, network disruption, pandemic disequilibria.*

GJSFR-F Classification: *DDC Code: 658.7, LCC Code: HD38.5*

Strictly as per the compliance and regulations of:

5. Wei, J. and Krajewski, L., A model for comparing supply chain schedule integration approaches, Int. J. Prod. Res., 38 (9), 2000, 2099-2123.

Modeling Supply Chains by Critical Paths and Leontief Input-Output Table

Gregory L. Light

Abstract- We formulate a supply-chain problem by:(1)casting it in the model of critical-path analysis as defined by predecessor/successor relations, and (2)allowing for mutual dependency among the activity nodes and applying Leontief's input-output structure.

Keywords: *scheduling bottleneck, network disruption, pandemic disequilibria.*

I. Introduction

Recent global supply-chain problems have received acute attention from every corner on the planet. While ad hoc ex-post analyses are perhaps timely[1], [2], preventative ex-ante treatments are more fundamental. Standard topics of supply chain management include: the bullwhip effect, vertical integration, and point-by-point statistical control[3];this paper seeks to add to the list the critical paths method(CPM)[4], [5] (for applications of CPM, cf. [6], [7]), which bears the common construct of a partially ordered set. In addition, we extend the scope of a directed tree of uni-directional edges/paths connecting all the vertices/nodes[8], [9], to a network of mutually dependent economic agents, which then readily leads to Leontief's input-output table for the gross domestic product (GDP) [10].As such, Section 2 below will connect CPM to a supply-chain problem, and Section 3 will show how an input-output analysis can address a global disruption over an economy, where we will incorporate the apparatus of elasticities of substitution, proportional changes in the ratio of two factors due to a change in the ratio of their prices. Section 4 will draw a summary.

II. Supply Chain by CPM

Let $A \equiv \{a_i \,|\, i = 1, 2, \cdots, n \in N - \{1\}\}$ be a set of activities with strict partial order relations $\prec$, where "$a_j \prec a_k$" denotes a_j being a predecessor of a_k, and denote the set of all the largest elements of A by L, which have no successors. By Hausdorff maximum principle [11], there exists a maximal simply ordered subset B_m of A that has its largest element $a_m \in L$; i.e., B_m is a critical path.

Next, collect all these critical paths $\{B_m\}$ and conduct the usual CPM analysis for each B_m[6]; then one arrives at a complete set of optimal solutions for a supply-chain problem.

Author: *Department of Finance, Providence College, Providence, Rhode Island 02918 USA. e-mail: glight@providence.edu*

III. Supply Chain by Input-Output Analysis

Let $\mathbf{A}=\left(a_{ij}\right)_{n\times n}$ be a matrix of the trading values of economic agent i sold to agent j in GDP. Obtain the row sums $\sum_{j=1}^{n} a_{ij}, \forall i=1,2,\cdots,n;$ analogously, obtain the column sums $\sum_{i=1}^{n} a_{ij}, \forall j=1,2,\cdots,n.$ Measure the economic value of agent$k$ by $\left(\sum_{j=1}^{n} a_{kj}+\sum_{i=1}^{n} a_{ik}\right)\equiv v_k, k=1,2,\cdots,n,$ which represents the total trade value of k, analogous to the sum of exports and imports of an economy.

Next, let $\Sigma_{inputs}^{\ output\ k}\equiv\left(\sigma_{ij}\right)_{n\times n\to k}$ be a matrix of elasticities of substitution between inputs i and j for producer k, $k=1,2,\cdots,n,$ where $\sigma_{ii;k}=1$ and $\sigma_{ij;k}=\sigma_{ji;k}$. Analogously, let $S_{outputs}^{\ consumer\ k}\equiv\left(s_{ij}\right)_{n\times n\to k}$ be a matrix of elasticities of substitution between outputs i and j for consumer k, $k=1,2,\cdots,n,$ where $s_{ii;k}=1$ and $s_{ij;k}=s_{ji;k}$.

Fix k as a producer; measure its inputs substitutability by the product of the elements of the upper triangular sub-matrix of $\Sigma_{inputs}^{\ output\ k}$, or $\Pi_{i<j}\sigma_{ij;k}$; analogously, measure the outputs substitutability for consumer k by $\Pi_{i<j}s_{ij;k}$; now combine these two measures of substitutability for k as a producer and as a consumer by $\left(\Pi_{i<j}\sigma_{ij;k}\cdot\Pi_{i<j}s_{ij;k}\right)\equiv\xi_k$. Finally, measure the economic significance of k by $v_k/\xi_k\equiv\gamma_k$, and establish a decreasing sequence $\left\langle\gamma_{k_j}\right\rangle_{j=1,2,\cdots,n}$, by which one can address an economy-wide supply chain problem more *à propos*, in particular, paying special attention to the case of $\gamma_k=\infty$.

IV. Summary

As has been observed in recent years, there are mainly three kinds of situations that disrupt a general economy: that due to labor shortage for certain specific tasks, that due to the lack of special components in a production process, and that due to disparate outlet distributions when the same commodity does not have a universal availability; all these problems can be alleviated or even prevented by an ex-ante detailed CPM analysis. The Covid-19 pandemic has inflicted losses of GDP's across countries of different degrees; here we contend that by conducting an input-output analysis as outlined above, the public sectors of an economy may act to balance supply and demand strategically over the markets. In summary, this paper has presented two methodologies for dealing with a supply-chain problem. Future studies might pursue Gantt Charts for all the commonly experienced supply chain problems as well as an estimation of the elasticities of substitution in the frame of the above conducted Leontief input-output analyses.

References Références Referencias

1. Bieck, C. and Marshall, A., Redirecting resources to promote post-pandemic growth, Strategy and Leadership, 48 (6), 2020, 45-50.
2. Tsiamas, K. and Rahimifard, S., A simulation-based decision support system to improve the resilience of the food supply chain, Int. J. Comp. Integrated Manufacturing, 34 (9), 2021, 996-1010.
3. Heizer, J., Render, B., and Munson, C., *Operations Management*, Boston, Pearson (2020).

Notes

4. Hanaka, T., Kagawa, S., Ono, H., and Kanemoto, K., Finding environmentally critical transmission sectors, transactions, and paths in global supply chain networks, Ener. Econ., 68 (1), 2017, 44-52.
5. Wei, J. and Krajewski, L., A model for comparing supply chain schedule integration approaches, Int. J. Prod. Res., 38 (9), 2000, 2099-2123.
6. Light, G., A note on identifying critical activities in project scheduling via linear programming on spreadsheets, with incidental pedagogical remarks, Global J. Sci. Frontier Res., 21 (1), 2021.
7. Light, G., A note on simulating predecessor-successor relationships in critical path models, Global J. Sci. Frontier Res., 21 (2), 2021.
8. Gravner, J., Paguyo, J., and Slivken, E., Maximal spanning time for neighborhood growth on the Hamming plane, SIAM J. Discrete Math., 33(2), 2019, 976–993.
9. Sholander, M., Trees, lattices, order, and betweenness, Proc. Amer. Math. Soc. 3, 1952, 369-381.
10. Dynes, S., Johnson, E., Andrijcic, E., and Horowitz, B., Economic costs of firm-level information infrastructure failures: Estimates from field studies in manufacturing supply chains, Int. J. Logis. Manag., 18 (3), 2007, 420-442.
11. Hausdorff, F., Hausdorff on Ordered Sets, Providence, Am. Math. Soc. (2005).