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Bianchi Type-I Anisotropic String Cosmological Models in the Chassis of Normal Gauge for Lyra's Manifold with Variable Deceleration Parameter

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Abstract- Present paper is the study of the anisotropic spatially Bianchi type-I (B-I) homogeneous cosmological models in the chassis of normal gauge for Lyra's manifold. A deterministic solution has been obtained by taking deceleration parameter to be dependent on time which results in average scale factor $a(t) = [\sinh(\alpha t)]^{\frac{1}{n}}$ has been obtained. Modified field equations given by Einstein for the homogeneous Bianchi Type I metric are solved. Our models are in accelerating phase which is consistent to the recent observations. It has been found that the displacement vector β behaves like cosmological term Λ in the normal gauge treatment and the solutions are consistent with recent observations of SNe Ia. It has been found that massive strings dominate in the decelerating universe whereas strings dominate in the accelerating universe. The strings dominate in the early universe and eventually disappear from the universe for sufficiently large times. This is in consistent with the current observations. Some physical and geometric behaviour of these models are also discussed.

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Abstract- Present paper is the study of the anisotropic spatially Bianchi type-I (B-I) homogeneous cosmological models in the chassis of normal gauge for Lyra's manifold. A deterministic solution has been obtained by taking deceleration parameter to be dependent on time which results in average scale factor $a(t) = [\sinh(\alpha t)]^{\frac{1}{n}}$ has been obtained. Modified field equations given by Einstein for the homogeneous Bianchi Type I metric are solved. Our models are in accelerating phase which is consistent to the recent observations. It has been found that the displacement vector β behaves like cosmological term Λ in the normal gauge treatment and the solutions are consistent with recent observations of SNe Ia. It has been found that massive strings dominate in the decelerating universe whereas strings dominate in the accelerating universe. The strings dominate in the early universe and eventually disappear from the universe for sufficiently large times. This is in consistent with the current observations. Some physical and geometric behaviour of these models are also discussed.

I. INTRODUCTION

Bianchi type-I cosmological models are analogous and anisotropic in nature and also which gives physically and geometrically better structure and it helps in describing and understanding the physical properties of early universe. Bianchi-I give rise to an ellipsoidal structure of the universe in spite of inflation. It has been evident by many theories that the universe is accelerating [1-8]. Recent observations of large scale distribution of galaxies and supernovae, a negative-gravity like substance known as dark energy seems to dominate the Universe [8-18], which accelerates the rate at which the universe is expanding. As the universe expands the dark energy clusters more weakly than matter and also dilutes more slowly than matter. Present time rate of expansion is given by Hubble parameter H whereas expansion of current observable universe is speeding up is given by deceleration parameter. These are vital observational parameters for analysing various properties of any cosmological structures. The time dependence of deceleration parameter and its effect on various cosmologi-

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cal models has also been analysed by Pradhan et al. [19]. Time dependence of deceleration parameter q is in concurrence with the present study of escalating universe

II. FIELD EQUATIONS AND METRIC

Considering totally anisotropic B-I metric,

$$ds^2 = A^2 dx^2 + B^2 dy^2 + C^2 dz^2 - dt^2, \quad (1)$$

Here A , B and C are the metric potentials.

The energy-momentum tensor for massive string in perfect fluid is

$$T_i^j = -\lambda x_i x^j + p g_i^j + (\rho + p) v_i v^j, \quad (2)$$

Isotropic pressure is given by p ; Rest energy density for the strings is given by ρ ; λ is tension density of string is given by λ ; x^i represents unit space-like vector and v^i is the particle's four-velocity where $x^2 = 0 = x^3 = x^4$ and $x^1 \neq 0$. Also

$$v_i v^i = -x_i x^i = -1, \quad v^i x_i = 0. \quad (3)$$

let

$$x^i = (A^{-1}, 0, 0, 0). \quad (4)$$

and

$$\rho = \lambda + \rho_p. \quad (5)$$

where represents particle density is being represented by ρ_p . In normal gauge, the field equations has been obtained by Sen [4] and given as

$$R_i^j - \frac{1}{2} g_i^j R + \frac{3}{2} \phi_i \phi^j - \frac{3}{4} g_i^j \phi_k \phi^k = -8\pi T_i^j, \quad (6)$$

The field equation given by Einstein (6) with (2) for the metric (1) will result in:

$$\frac{\dot{B}\dot{C}}{BC} + \frac{3}{4}\beta^2 + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + p - \lambda = 0, \quad (7)$$

$$\frac{\dot{C}\dot{A}}{CA} + \frac{3}{4}\beta^2 + \frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + p = 0, \quad (8)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{3}{4}\beta^2 + \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + p = 0, \quad (9)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} + \frac{3}{4}\beta^2 = \rho. \quad (10)$$

$T_{i;j}^i = 0$ will give

$$(\rho + p) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + \dot{\rho} = -\lambda \frac{\dot{A}}{A}, \quad (11)$$

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simultaneously Right hand side of Eq. (6) gives

$$\left(R_i^j - \frac{1}{2}g_i^j R\right)_{;j} + \frac{3}{2}(\phi_i \phi^j)_{;j} - \frac{3}{4}(g_i^j \phi_k \phi^k)_{;j} = 0. \quad (12)$$

Equation (12) gives

$$\begin{aligned} \phi_i \left[\frac{\partial \phi^j}{\partial x^j} + \phi^l \Gamma_{lj}^j \right] + \phi^j \left[\frac{\partial \phi_i}{\partial x^j} - \phi_l \Gamma_{ij}^l \right] - \frac{1}{2} g_i^j \phi_k \left[\frac{\partial \phi^k}{\partial x^j} + \phi^l \Gamma_{lj}^k \right] - \\ \frac{1}{2} g_i^j \phi^k \left[\frac{\partial \phi_k}{\partial x^j} - \phi_l \Gamma_{kj}^l \right] = 0. \end{aligned} \quad (13)$$

Eq. (13) reduces to

$$\begin{aligned} \beta \left[\frac{\partial (g^{44} \phi_4)}{\partial x^4} + \phi^4 \Gamma_{44}^4 \right] + g^{44} \phi_4 \left[\frac{\partial \phi_4}{\partial t} - \phi_4 \Gamma_{44}^4 \right] - \frac{1}{2} g_4^4 \phi_4 \left[\frac{\partial \phi^4}{\partial x^4} + \phi^4 \Gamma_{44}^4 \right] - \\ \frac{1}{2} g_4^4 g^{44} \phi^4 \left[\frac{\partial \phi_4}{\partial t} - \phi^4 \Gamma_{44}^4 \right] = 0. \end{aligned} \quad (14)$$

which gives

$$\beta^2 \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = -\beta \dot{\beta}. \quad (15)$$

III. SOLUTIONS OF THE FIELD EQUATIONS

There are five equations from (7)-(10) and (15) where λ , p , ρ , β which are the cosmological parameters and A , B , C metric potentials are the unknowns. For finding its explicit solution Firstly we assumed that the expansion scalar (θ) of cosmological constant is directly proportionate to the integrant σ_1^1 of the shear tensor σ_i^j which will give following relation:

$$A^{\frac{1}{m}} = BC, \quad (16)$$

where m is any positive value which is well explained by Thorne [79]. Also

$$\sigma \leq 0.3H$$

For spatially homogeneous metric, Collins et al. [82] have found out, that the ratio of σ which gives normal congruence of any cosmological model to the θ which gives homogeneous expansion of the model is constant i.e. $\frac{\sigma}{\theta}$ is constant.

The Hubble parameter is

$$3H = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right). \quad (17)$$

Next, we take deceleration parameter to be time dependent

$$q = \frac{-\ddot{a}a}{\dot{a}^2} = b(t), \quad (18)$$

which results in

$$a = [\sinh(\alpha t)]^{1/n} \quad (19)$$

also we know

$$a^3 = V = ABC \quad (20)$$

Subtracting (8) from (9), and taking integral twice, we obtain

$$\frac{B}{C} = k_1 \exp[k_2 \int (ABC)^{-1} dt], \quad (21)$$

where k_1 and k_2 are the constants.

Solving the equations (16), (19) and (21), we get

$$A = \sinh(\alpha t)^{\frac{3m}{n(m+1)}}; , \quad (22)$$

$$B = \sqrt{k_1} \sinh(\alpha t)^{\frac{3}{2n(m+1)}} \exp\left[\frac{k_2}{2}\right] F(t), \quad (23)$$

$$C = \frac{1}{\sqrt{k_1}} \sinh(\alpha t)^{\frac{3}{2n(m+1)}} \exp\left[\frac{-k_2}{2}\right] F(t) \quad (24)$$

Hence the model (1) will result in

$$ds^2 = \sinh(\alpha t)^{\frac{6m}{n(m+1)}} dx^2 + k_1 \sinh(\alpha - dt^2 t)^{\frac{6}{2n(m+1)}} \exp[k_2 F(t)] dy^2 \\ + \frac{1}{k_1} \sinh(\alpha t)^{\frac{6}{2n(m+1)}} \exp[-k_2 F(t)] dz^2. \quad (25)$$

Now solving Eq. (15) results $\beta = 0$ or $\dot{\beta} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0$. which results in

$$\dot{\beta} + \beta \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0, \quad (26)$$

which leads to

$$\frac{\dot{\beta}}{\beta} + \frac{3\alpha}{n} \coth(\alpha t) = 0. \quad (27)$$

Integrating Eq. (27), we obtain

$$\beta = [\sinh(\alpha t)]^{-\frac{3}{n}}, \quad (28)$$

where k is an constant.

$$V = (AB^2) = a^3(t) = [\sinh(\alpha t)]^{3/n} \quad (29)$$

The Hubble parameter for our derived model is given by

$$H = \frac{\alpha}{n} \coth(\alpha t) \quad (30)$$

Also,

$$q + 1 = n(1 - \tanh^2 \alpha t) \quad (31)$$

here n and α are positive. The value of q which is a time dependent parameter defines the nature of the universe. From Eq.[31] we can see that $q > 0$ when $\tanh(\alpha t) < (\frac{n-1}{n})^{\frac{1}{2}}$ and $q < 0$ when $\tanh(\alpha t) > (\frac{n-1}{n})^{\frac{1}{2}}$. Current studies, suggests that the present universe is escalating and value of deceleration parameter lies between the interval of $-1 < q < 0$. Currently ($t_0 = 12.36 \text{ Gyr}$) with ($q_0 = -0.52$) (Amirhashchi et al.)(Yu,Ratna and Wang) based on OHD+JLA Data, we have the following equation for α and n .

$$\alpha t_0 = \tanh^{-1} \left[1 - \left(\frac{1+q_0}{n} \right) \right]^{\frac{1}{2}} \quad (32)$$

here the current value of H is denoted by H_0 and the present age of universe by t_0 . We consider the three cases based on different data:

$$\alpha = \frac{1}{12.36} \tanh^{-1} \left[1 - \left(\frac{0.48}{n} \right) \right]^{\frac{1}{2}} \quad (33)$$

For the present Universe, it is quite clear that the model holds good for $n > 0.48$.

It has been observed by Halford [4] that the cosmological constant Λ and displacement field ϕ_i in Lyra's manifold behaves in same manner. From Eq. (28), $\beta(T)$ which is the displacement vector reduces with the rise of cosmic time when k and n are positive and at large times it attains a very small positive value. Recent observations of cosmological bodies Riess et al. [99, 100]; SNe Ia (Garnavich et al. [94, 95]; Schmidt et al. [101]); Perlmutter et al. [96]–[98]; suggest that cosmological constant is positive $\Lambda (G\hbar/c^3) \approx 10^{-123}$. $\beta(T)$ so obtained in our derived cosmological model is in consensus with recent observations.

The equations for various parameters (p), (ρ), (λ) and (ρ_p) for our model (??) are given by

$$p_1 = \frac{-3 \sinh(\alpha t)^{-2(3+n)/n} (-(1+m)^2 n^2 - (-3+4n+4m^2(-3+2n)+6m(-1+2n)) \alpha^2 / \sinh(\alpha t)^{6/n})}{8(1+m)^2 n^2} \quad (34)$$

$$p_2 = \frac{-3 \sinh(\alpha t)^{-2(3+n)/n} \cosh(2\alpha t) ((1+m)^2 n^2 + 3(1+2m+4m^2) \alpha^2 \sinh(\alpha t)^{6/n})}{8(1+m)^2 n^2} \quad (35)$$

$$p = p_1 + p_2$$

$$\rho_{p1} = \frac{3 \sinh(\alpha t)^{-2(3+n)/n} (-(1+m)^2 n^2 - (3-4n+2m(-9+2n)+4m^2(-3+2n)) \alpha^2 \sinh(\alpha t))}{8(1+m)^2 n^2} \quad (36)$$

$$\rho_{p2} = \frac{3 \sinh(\alpha t)^{-2(3+n)/n} \cosh(2\alpha t) ((1+m)^2 n^2 + 3(-1+6m+4m^2) \alpha^2 \sinh(\alpha t))}{8(1+m)^2 n^2} \quad (37)$$

$$\rho_p = \rho_{p1} + \rho_{p2}$$

$$\lambda = -\frac{3(2m-1)\alpha^2(3-2n+3\cosh(2\alpha t)\cosh^2(\alpha t))}{4(1+m)n^2} \quad (38)$$

$$\rho = \frac{3}{4} \left(\frac{3(1+4m)\alpha^2 \coth^2(\alpha t)}{(1+m)^2 n^2} + \sinh(\alpha t)^{6/n} \right) \quad (39)$$

Under the appropriate choice of constants the energy density and particle density satisfies the energy conditions. We can see that at $T = 0$ all the cosmological parameters diverge which implies that the derived model has a uniqueness at initial time. Uniqueness of this type is a Point Type and is explained by (MacCallum [102]). The cosmological parameters λ, ρ, p, ρ_p and starts with very large values. For $m < 1$ these parameters decreases with the extension of the current universe. In the starting of universe the values of ρ_p and λ were large implying that strings were dominating the beginning of the universe i.e. at initial times. At extremely large values of times, the cosmological parameters ρ_p and λ approaches zero which implies that for extremely large values of times the strings vanishes and because of this the strings are not being detectable in the present time.

IV. RESULT AND DISCUSSION

We can see from Fig.1 that for $n \leq 1$, our derived model is progressing in escalating phase whereas for $n > 1$, the model is progressing from early de-escalating phase to present escalated phase. It can be seen that our model is evolving only in an escalated phase ($q < 0$) for assuming $n = 0.5$ and $\alpha = 0.0164$ (case I) and in (case II) $n = 0.75$ and $\alpha = .0560$. this is the value of joint OHD+JLA dataset used (Amirhashchi et al. [56, 57]).

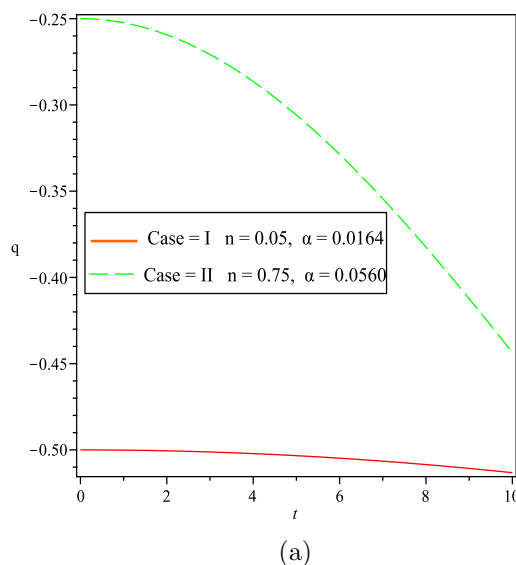


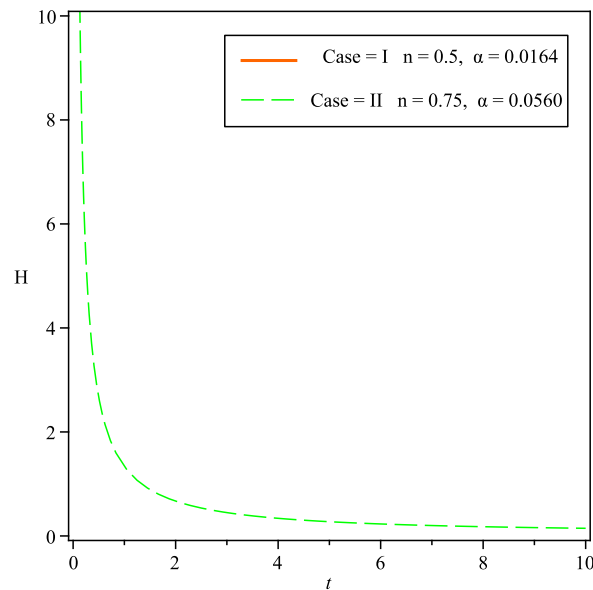
Figure 1: (a) Plot of q to t . Here (a) Case I $n = 0.5, \alpha = 0.0164$ (b) Case II $n = 0.75, \alpha = 0.0560$

Figure.2(a) corresponding to the Eq.30, Plot of Hubble parameter (H) to t . H decreases with increase of t . Figure.2(b) depicts the behaviour of spatial volume V with respect to t . Spatial V increases as cosmic time tends to infinity and becomes zero at $t = 0$. From Eq. (36) and **fig.3**, we can see that isotropic pressure p increases with the increase of time and p approaches to zero for $t > 0$, $n > .48$.

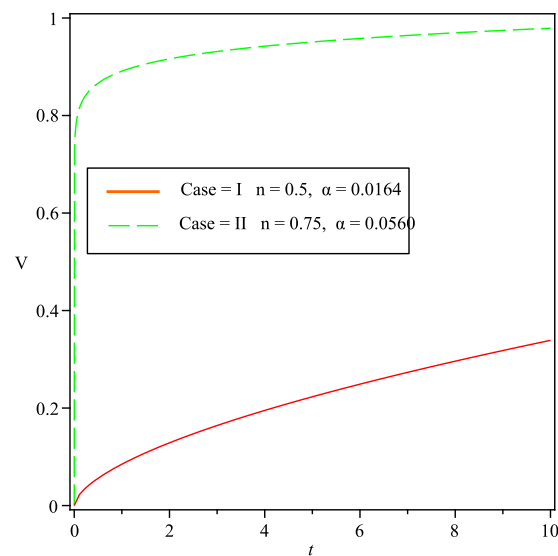
From Eq.(38), We can see that the particle density denoted by ρ_p decreases with the increase of cosmic time and remains positive i.e. $\rho_p > 0$ for all time. **Fig.4** is the plot of particle density with respect to time. Here it is to be noted that ρ_p approaches to zero at large value of times in both cases. It is worth mentioning that ρ_p is decreasing fastly in case 1 in comparison to case 2.

From Eq.(39), we can also see that the tension density λ increases with the increase of time and it is always $\lambda < 0$. **fig.5** is the plot of string tension density with respect to time. It can be seen that the λ remains negative in both cases.

However, it tends towards infinity for $t > 0$. **fig.6** is the plot of ρ with respect to cosmic time. We can see that energy density decreases more sharply in case 1 than 2 with increase of time t and tends to zero as time increases.



(a)



(b)

Figure 2: Plot of Hubble Parameter and Volume to t Here (a) Case I $n = 0.5, \alpha = 0.0164$ (b) Case II $n = 0.75, \alpha = 0.0560$.

V. CONCLUSION

In this paper, Anisotropic spatially homogeneous Bianchi-I cosmological models with perfect fluid within the chassis of normal gauge in Lyra's manifold consid-

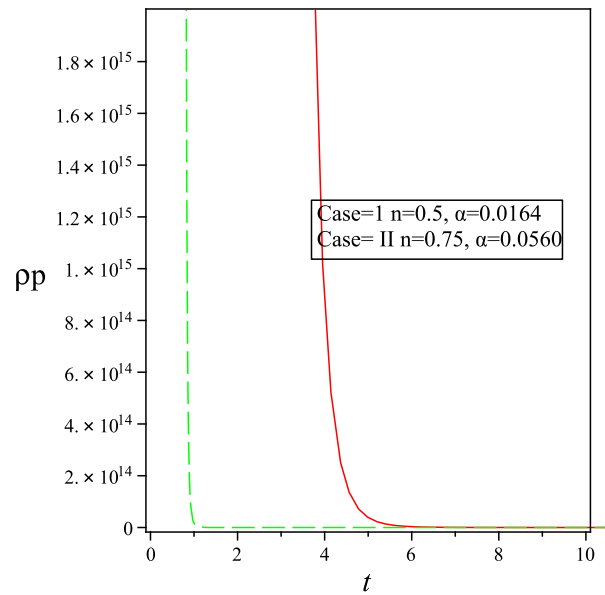


Figure 3: Plot of particle density ρ_p to time t

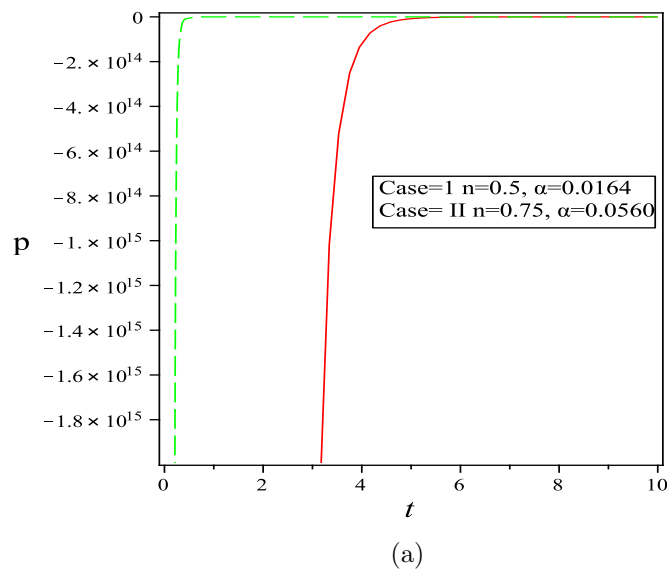


Figure 4: Plot of Isotropic Pressure p to t Here (a) CaseI $n = 0.5, \alpha = 0.0164$
(b) CaseII $n = 0.75, \alpha = 0.0560$.

ering deceleration parameter to be time dependent has been studied.

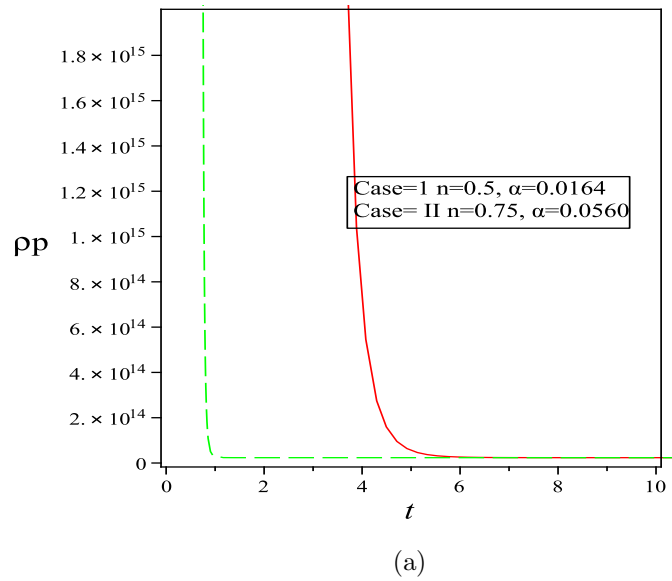


Figure 5: Plot of Particle Density ρ_p to t Here (a) CaseI $n = 0.5, \alpha = 0.0164$
(b) CaseII $n = 0.75, \alpha = 0.0560$.

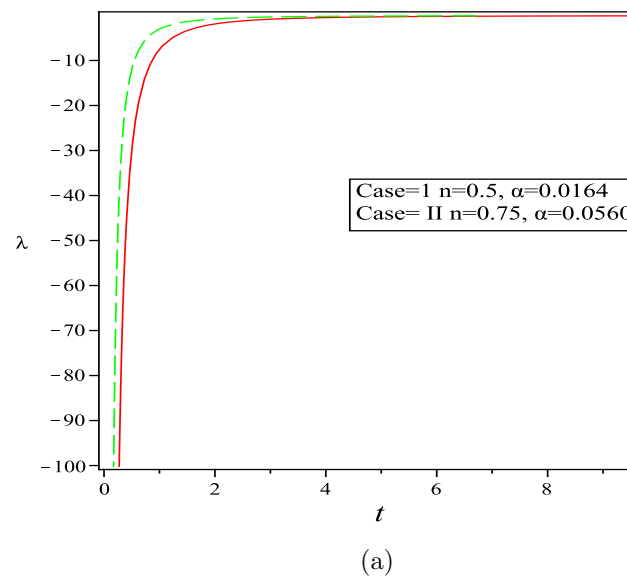


Figure 6: Plot of String Tension λ to t Here (a) CaseI $n = 0.5, \alpha = 0.0164$ (b)
CaseII $n = 0.75, \alpha = 0.0560$.

We have taken that the normal congruence of the model to the homogeneous expansion to be constant. i.e. $\frac{\sigma}{\theta} = \text{constant}$. All the physical quantities are

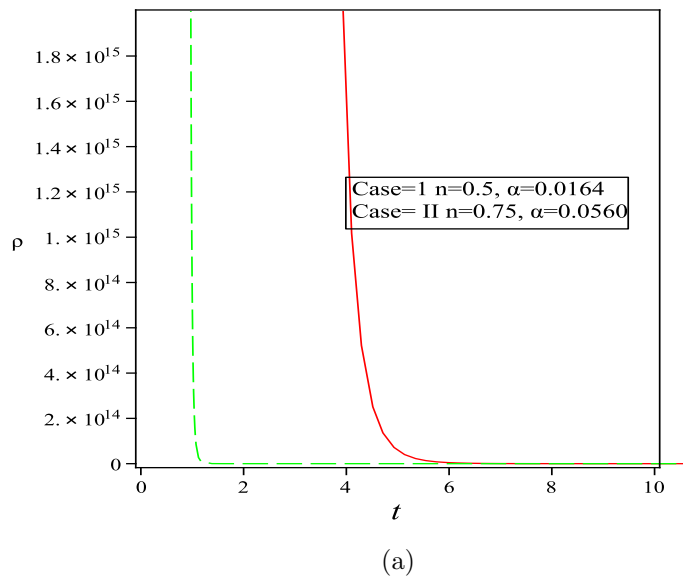


Figure 7: Plot of energy density ρ to t Here (a) CaseI $n = 0.5, \alpha = 0.0164$ (b) CaseII $n = 0.75, \alpha = 0.0560$.

extremely large at initial time and becomes zero as $T \rightarrow \infty$ which is a Point Type uniqueness (MacCallum 1971) at initial time in the derived model.

The derived cosmological model presents the dynamics of strings for various values of $(n, \alpha) = (0.5, 0.0164)$ and $(0.75, 0.0560)$ for various modes of advancement of universe. In the beginning of the universe the strings dominates and at extremely large values of times vanishes which is in concurrence with the current observations.

Further parameters Isotropic Pressure p , Particle Density ρ_p , energy density ρ has been analysed to study their impact with increase in time. The particle density reduces with the increase of cosmic time and becomes negligible at extremely large value of times whereas isotropic pressure and is always negative and at late times it also follows the same pattern and becomes negligible. This negative sign for the pressure (repulsive force) can be explained as a source of the escalation at initial time.

The investigation of such cosmological models in the chassis of Lyra's manifold gives rise to new mode for theoretical formulation for relativistic gravitation and a new prospect for further analysis in astrophysics and cosmology.

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