Lehmann Type-2 Inverse Lomax Distribution and Applications to Reliability Data

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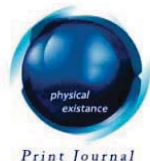*GJSFR-F Classification:* DDC Code: 536.73 LCC Code: QC318.E57

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Lehmann Type-2 Inverse Lomax Distribution and Applications to Reliability Data

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I. Introduction

Over the last century, many significant distributions have been introduced to serve as models in applied sciences. The notable among them, generalized beta distribution developed by McDonald (1995), is at the top of the list in terms of usefulness. The main feature of the generalized beta distribution is that it introduces skewness and kurtosis into the baseline distribution that allows for modeling data of various forms of the shape of the hazard function which may be decreasing, increasing, decreasing-increasing, increasing decreasing, and inverted bathtub shapes. In this paper, we focus our attention on one of the most attractive of these distributions, known as the inverse Lomax (IL) distribution. Mathematically, it can be presented in the distribution form as $Y = X^{-1}$, where X is a random variable following the famous Lomax distribution (see Lomax, 1954). Thus, the cumulative distribution function (CDF) of the IL distribution is given by

$$G(x; \lambda, \rho) = \left(1 + \frac{\rho}{x}\right)^{-\lambda}, \ x > 0 \tag{1}$$

And the corresponding PDF is given as

$$g(x; \lambda, \rho) = \lambda \rho x^{-2} \left(1 + \frac{\rho}{x}\right)^{-\lambda - 1}, x > 0 \tag{2}$$

where $\boldsymbol{\rho}$ is a positive scale parameter and $\boldsymbol{\lambda}$ is a positive shape parameter. The reasons forstudying the IL distribution are not limited to the following: It has proved itself as a statistical model in various applications, including economics and actuarial sciences (see Kleiber and Kotz, (2003)) and geophysics (see McKenzie et al. (2004)). Also, the mathematical and inferential aspects of the IL distribution have been studied. See, for example, Lorenz (2004), for the Lorenz ordering of order statistics, Rahman and Aslam

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(2013) studied the estimation of the parameters in a Bayesian setting, Yadav et al. (2016) examined the estimation of the parameters from hybrid censored samples, Singh et al. (2016) the study of the reliability estimator under type II censoring and Reyad, and Othman (2018) studied the Bayesian estimation of a two-component mixture of the IL distribution type I censoring. Despite an interesting compromise between simplicity and accuracy, the IL model suffers from a certain rigidity in the peak (punctual and roundness) and tail properties. This motivates the development of various parametric extensions, such as the inverse power Lomax distribution introduced by Hassan and Abd-Allah 92018), the Weibull IL distribution studied by Hassan, A.S.; Mohamed (2019) and, the Marshall-Olkin IL distribution developed by Maxwell et al. (2019). In this paper, we introduce and discuss a new extension of the IL distribution called Lehman Type-2 Inverse Lomax distribution which can model all forms of data exhibiting any shape of the hazard function because of its flexibility and tractability.

II.   Lehmann Type-2 Inverse Lomax Distribution

The CDF of the Lehmann type-2 family of distribution is given by

$$F(x) = 1 - \left(1 - G(x)\right)^{b} \tag{3}$$

And the associated PDF is given by

$$f(x) = bg(x)\left(1 - G(x)\right)^{b-1} \tag{4}$$

Putting (3) and (4) in (2), we have an expression for the Lehmann Type-2 inverse Lomax (LT-2IL) distribution given by

$$f(x; b, \lambda, \rho) = b\lambda\rho x^{-2}\left(1 - \frac{\rho}{x}\right)^{-\lambda-1}\left(1 - \left(1 + \frac{\rho}{x}\right)^{-\lambda}\right)^{b-1}, x > 0 \tag{5}$$

And the corresponding CDF is given by

$$F(x; b, \lambda, \rho) = 1 - \left(1 - \left(1 + \frac{\rho}{x}\right)^{-\lambda}\right)^{b}, \tag{6}$$

The survival and the hazard function are respectively given by

$$S(x; b, \lambda, \rho) = \left(1 - \left(1 + \frac{\rho}{x}\right)^{-\lambda}\right)^{b} \tag{7}$$

And

$$h(x; b, \lambda, \rho) = \frac{b\lambda\rho\left(1 - \frac{\rho}{x}\right)^{-\lambda-1}\left(1 - \left(1 + \frac{\rho}{x}\right)^{-\lambda}\right)^{b-1}}{x^{2}\left(1 - \left(1 + \frac{\rho}{x}\right)^{-\lambda}\right)^{b}}. \tag{8}$$

The graph of the density, hazard, and the survival function is given in figures 1, 2, and 3 for various values of the parameters.

Graph of density function of LT-2IL

Graph of density function of LT-2IL
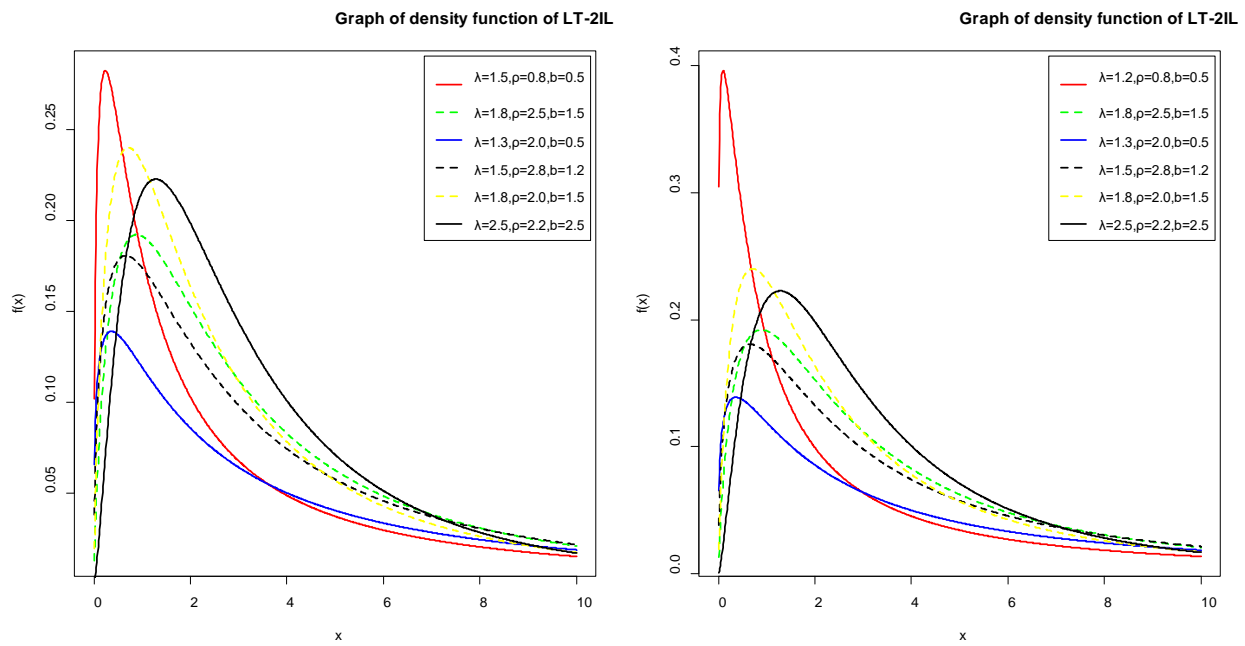
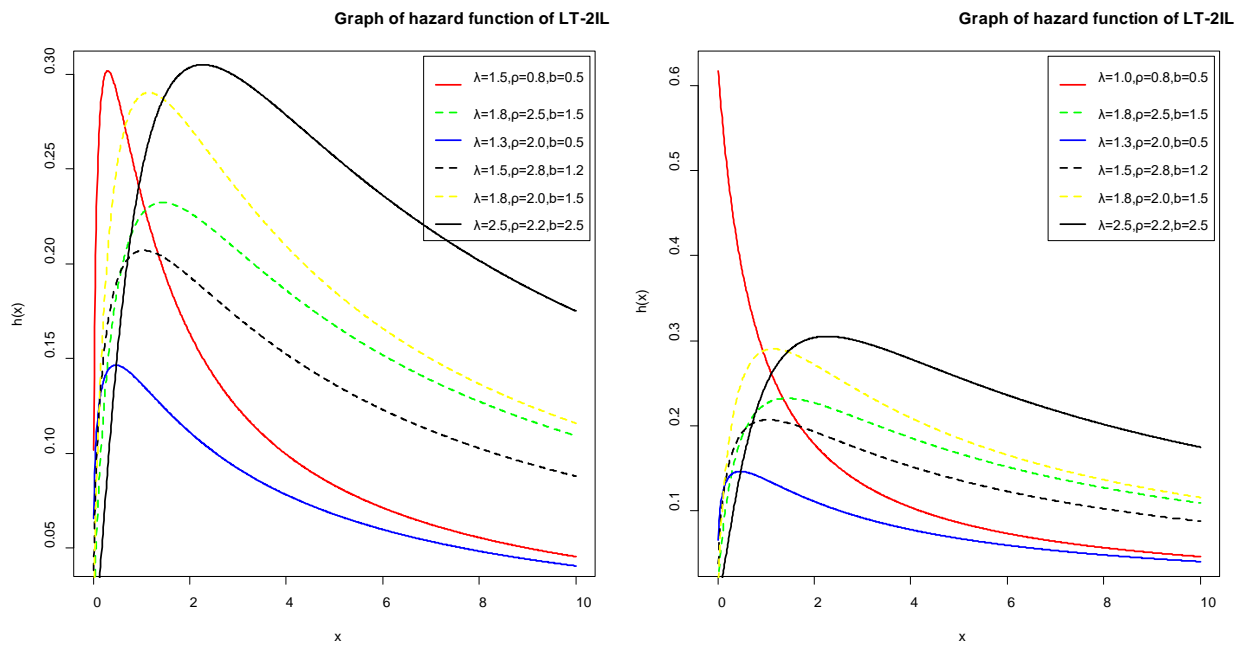*Figure 1:* The graph of the density function of LT-2IL distribution

Graph of hazard function of LT-2IL

Graph of hazard function of LT-2IL

Figure 2: The graph of the hazard function of LT-2IL distribution

Graph of survival function of LT-2IL

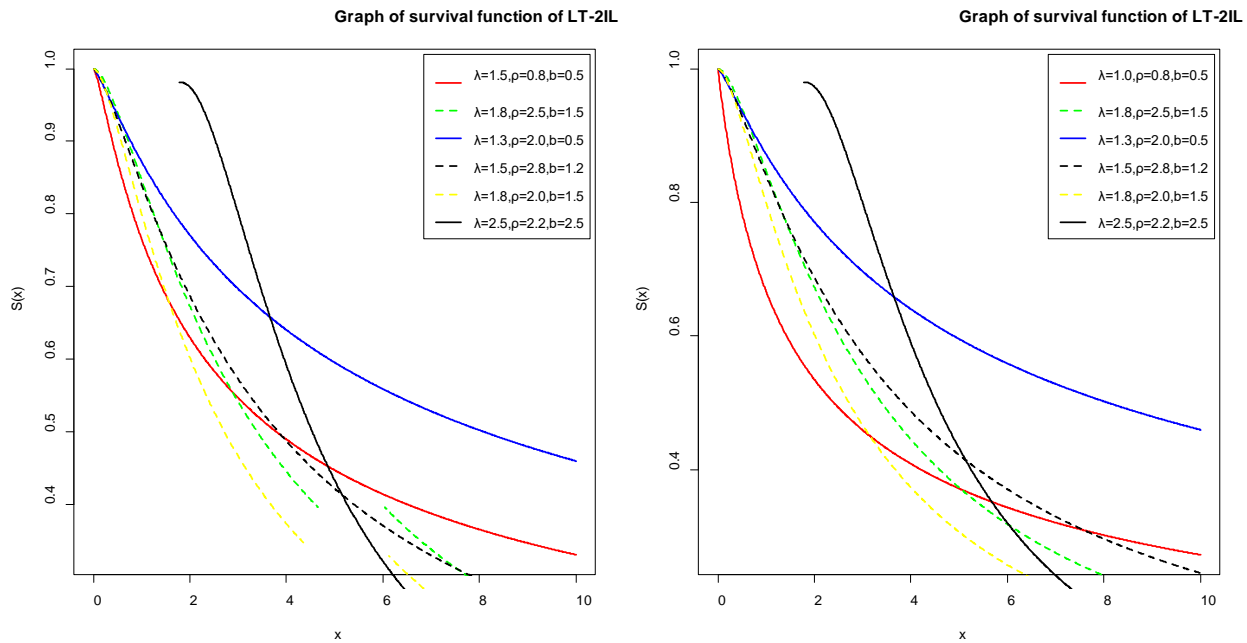**Graph of survival function of LT-2IL**

Figure 3: The graph of the survival function of $LT - 2IL$ distribution

III.   Useful Expansions

The binomial theorem, for $m > 0$ and $|v| < 1$, can be expressed as

$$(1 - v)^m = \sum_{i=0}^{\infty} (-1)^i \binom{m}{i} v^i \tag{9}$$

Then, applying the binomial series expansion given in (9) to (5), we have

$$\left(1 - \left(1 + \frac{\rho}{x}\right)^{-\lambda}\right)^{b-1} = \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} \left(1 + \frac{\rho}{x}\right)^{-\lambda i}$$

Finally, we have

$$f(x) = b\lambda\rho x^{-2} \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} \left(1 + \frac{\rho}{x}\right)^{-[\lambda(i+1)+1]} \tag{10}$$

Equation (10) represents the Exponentiated Inverse Lomax distribution, with shape parameter $[\lambda(i + 1) + 1]$ and scale parameter ρ. It then follows that the properties of Lehmann type-2 inverse Lomax distribution can be obtained from that of the Exponentiated inverse Lomax distribution.

a) Quantile function

The quantile function of the $LT - 2IL$ distribution is defined by $Q(u; b, \lambda, \rho) = F^{-1}(u; b, \lambda, \rho)$, $u \in (0,1)$. After some mathematical manipulations, we obtain

$$Q(u; b, \lambda, \rho) = \rho \left\{ \left[1 - (1 - u)^{1/b}\right]^{-1/\lambda} - 1 \right\}, u \in (0,1) \tag{10.1}$$

Notes

From (10.1), we can obtain the lower quartile (q_1), middle quartile (q_2), also known as the median, and the upper quartiles (q_3) of the $LT - 2IL$ distribution by taking the values of u to be 0.25, 0.5, and 0.75 respectively. Then, we obtain an equation for the lower quartile, median, and the upper quartile of $LT - 2IL$ distribution given respectively, by

$$Q(0.25; b, \lambda, \rho) = \rho \left\{ \left[1 - (0.75)^{1/b} \right]^{-1/\lambda} - 1 \right\}, \tag{10.2}$$

$$Q(0.5; b, \lambda, \rho) = \rho \left\{ \left[1 - (0.5)^{1/b} \right]^{-1/\lambda} - 1 \right\} \tag{10.3}$$

and

$$Q(0.75; b, \lambda, \rho) = \rho \left\{ \left[1 - (0.25)^{1/b} \right]^{-1/\lambda} - 1 \right\} \tag{10.4}$$

b) *Moments of $LT - 2IL$ distribution*

The kth moment of the $LT - 2IL$ distribution under certain regularity conditions; the kth moment of TIIHLF distribution is obtained as

$$\mu'_k = \int_{-\infty}^{\infty} x^k f(x) dx = b\lambda\rho x^{-2} \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} W(x) \tag{11}$$

where

$$W(x) = \int_{-\infty}^{\infty} x^{k-2} \left(1 + \frac{\rho}{x} \right)^{-[\lambda(i+1)+1]} dx \tag{12}$$

Taking $z = \frac{\rho}{x}$ and putting it in (12), we have

$$W(x) = -\rho^{k-1} \int_{-\infty}^{\infty} z^{2-k} (1+z)^{-[\lambda(i+1)+1]} dz \tag{13}$$

Also, letting $y = \frac{v}{1-v}, dy = (1-v)^{-2} dv$ and putting it in (13), we have

$$W(x) = \rho^{k-1} \int_0^1 v^{2-r} (1-v)^{k+\lambda(i+1)-3} \tag{14}$$

Then we have

$$W(x) = \rho^{k-1} B[1 - k, k + \lambda(i+1) - 2]$$

Finally, we obtain an expression for the kth moment of the $LT - 2IL$ distribution given as

$$\mu'_k = b\lambda \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} \rho^k B[1 - k, k + \lambda(i+1) - 2] \tag{15}$$

Where $B(.,.)$ is a beta function

An expression for the first, second and third moments can be obtained by respectively taken the value of $k = 1, 2$ and 3 as

$$\mu_1' = b\lambda \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} \rho B[\lambda(i+1) - 1], \tag{16}$$

$$\mu_2' = b\lambda \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} \rho^2 B[-1, \lambda(i+1)] \tag{17}$$

And

$$\mu_3' = b\lambda \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} \rho^3 B[-2, 1 + \lambda(i+1)] \tag{18}$$

c) Incomplete moments of LT-2IL distribution

The incomplete moment of LT-2IL distribution can be obtained using (13) as

$$\mu_k' = \int_{-\infty}^{\infty} x^k f(x) dx = b\lambda\rho x^{-2} \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} W(x) \tag{16}$$

where

$$W(x) = \int_0^t x^{k-2} \left(1 + \frac{\rho}{x}\right)^{-[\lambda(i+1)+1]} dx \tag{17}$$

Taking $z = \frac{\rho}{x}$ and putting it in (17), we have

$$W(x) = -\rho^{k-1} \int_0^t z^{2-k} (1 + z)^{-[\lambda(i+1)+1]} dz \tag{18}$$

Also, letting $y = \frac{v}{1-v}, dy = (1 - v)^{-2} dv$ and putting it in (18), we have

$$W(x) = \rho^{k-1} \int_0^1 v^{2-r} (1 - v)^{k+\lambda(i+1)-3} \tag{19}$$

Then we have

$$W(x) = \rho^{k-1} B\rho_{/t+\rho} [1 - k, k + \lambda(i+1) - 2] \tag{20}$$

Finally, we obtain an expression for the kth incomplete moment of the $LT - 2IL$ distribution given as

$$\mu_k' = b\lambda \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} \rho^k B\rho_{/t+\rho} [1 - k, k + \lambda(i+1) - 2] \tag{21}$$

$$\mu_1' = b\lambda \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} \rho^k B\rho_{/t+\rho} [1 - k, k + \lambda(i+1) - 2] \tag{22}$$

d) Renyi entropy

Renyi entropy was proposed by Renyi (1961). It can be obtained by

$$I_\varphi(x) = \frac{1}{1-\varphi} \log M \tag{28}$$

Where

$$M = \int_{-\infty}^{\infty} f^\varphi(x) dx, \qquad \varphi > 0, \varphi \neq 1 \tag{29}$$

Putting (5) in (28) followed by binomial expansion, we have

$$M = (b\lambda\rho)^\varphi \sum_{i=0}^{\infty} (-1)^i \binom{\varphi(b-1)}{i} \int_{-\infty}^{\infty} x^2 \left(1 + \frac{\rho}{x}\right)^{[\lambda(\varphi-1)+\varphi]} dx, \tag{30}$$

Taking $y = \frac{\rho}{x}, dx = -\rho y^{-2} dy$, putting it in (30), we have

$$M = (b\lambda)^\varphi \rho^{2+\varphi} \sum_{i=0}^{\infty} (-1)^i \binom{\varphi(b-1)}{i} \int_{0}^{\infty} y^{-2}(1+y)^{[\lambda(\varphi-1)+\varphi]} dy, \tag{31}$$

Furthermore, by letting $y = \frac{u}{1-u}, dw = (1-u)^{-2} du$ and putting it in (31) gives

$$M = (b\lambda)^\varphi \rho^{2+\varphi} \sum_{i=0}^{\infty} (-1)^i \binom{\varphi(b-1)}{i} \int_{0}^{\infty} u^{-2}(1-u)^{-[\lambda(\varphi-1)+\varphi]} dy, \tag{32}$$

$$M = (b\lambda)^\varphi \rho^{2+\varphi} \sum_{i=0}^{\infty} (-1)^i \binom{\varphi(b-1)}{i} B[-1, 1-[\lambda(\varphi-1)+\varphi]]$$

Finally, we have an expression for the Renyi entropy of LT-2IL distribution as

$$I_\varphi(x) = \frac{1}{1-\varphi} \log\left[(b\lambda)^\varphi \rho^{2+\varphi} \sum_{i=0}^{\infty} (-1)^i \binom{\varphi(b-1)}{i} B[-1, 1-[\lambda(\varphi-1)+\varphi]] \right]$$

e) Tsallis Entropy

The Tsallis entropy, also known as β-entropy, was first discovered by Havrada and Charvat (1967) and later developed by Tsallis (1988). The Tsallis entropy of the LT-2IL distribution can be defined as

$$I_T^{(\varphi)} = \frac{1}{\varphi - 1} \left[1 - \int_{-\infty}^{\infty} f(x; b, \rho, \lambda)^\varphi \right], \quad \varphi > 0, \varphi \neq 1 \tag{33}$$

Invariably, it may be written as

$$I_T^{(\varphi)} = \frac{1}{\varphi - 1} [1 - M^\varphi], \quad \varphi > 0, \varphi \neq 1 \tag{34}$$

Putting (29) in (32), we obtain an expression for the Tsallis entropy given as

$$I_T^{(\varphi)} = \frac{1}{\varphi - 1}\left[1 - (b\lambda)^\varphi \rho^{2+\varphi} \sum_{i=0}^{\infty}(-1)^i \binom{\varphi(b-1)}{i} B[-1, 1 - [\lambda(\varphi - 1) + \varphi]]\right] \quad (35)$$

f) Order statistics

Let X be a random variable from the $KGIL$ distribution and, given a random sample size n from X, say say $X_1, \ldots, X_n$, let $x_{i:n}$ be the ith order statistic such that $x_{1:n} \leq x_{2:n} \leq x_{3:n} \leq \cdots \leq x_{n:n}$, given that $x_{h:n} \in \{X_1, \ldots, X_n\}$ for $h = 1, \ldots, n$. In particular, the study of order statistics is very important since naturally it appears in many applications, majorly those involving systems comprises of several components parts that can fail independently of each other. The density of $x_{i:n}$ is given by

$$f_{i:n}(x;) = \frac{n!}{(i-1)(n-i)} F(x; b, \rho, \lambda)^{i-1} R(x; b, \rho, \lambda)^{n-1} f(x; b, \rho, \lambda), \quad x > 0 \quad (36)$$

Applying binomial theorem given in (36) to the expression above, it follows immediately that

$$f_{i:n}(x) = \frac{n!}{(i-1)(n-i)} \sum_{j=0}^{i-1} \binom{i-1}{j}(-1)^j R(x; b, \rho, \lambda)^{j+n-1} f(x; b, \rho, \lambda), x > 0 \quad (37)$$

Putting (5) and (7) in (37) gives us the following series expansion for the ith order statistics for $LT - 2IL$ distribution as

$$f_{i:n}(x) = \frac{n!\, b\lambda\rho x^{-2}}{(i-1)(n-i)} \sum_{j=0}^{i-1} \binom{i-1}{j}(-1)^j \left[\left(1 - \left(1 + \frac{\rho}{x}\right)^{-\lambda}\right)^b\right]^{b+j+n-2} \left(1 + \frac{\rho}{x}\right)^{-\lambda-1} \quad (38)$$

It should be noted that from (38) an expression for the smallest and the largest order statistics can be obtained.

IV. MAXIMUM LIKELIHOOD ESTIMATION (MLE) METHOD

Taking an observed sample $x_1, \ldots, x_n$ from the $LT - 2IL$ distribution, the corresponding likelihood function can be represented as

$$L(b, \lambda, \rho) = \prod_{i=1}^{n} f(x_i: b, \lambda, \rho) \quad (39)$$

$$= \prod_{i=1}^{n} b\lambda\rho x^{-2} \left(1 - \frac{\rho}{x}\right)^{-\lambda-1} \left(1 - \left(1 + \frac{\rho}{x}\right)^{-\lambda}\right)^{b-1}$$

The MLEs of $p, q, \beta,$ and λ are denoted by $\hat{b}, \hat{\rho},$ and $\hat{\lambda}$, respectively. The log-likelihood function is given by

Notes

$$l = n\log(b\lambda\rho) - (\lambda + 1)\sum_{i=1}^{n} \log\left(1 + \frac{\rho}{x_i}\right) + (b-1)\sum_{i=1}^{n} \log\left(1 - \left(1 + \frac{\rho}{x_i}\right)^{-\lambda}\right) \quad (40)$$

And the element of the score vector is given by

$$\frac{\partial l}{\partial b} = \frac{n}{b} + \sum_{i=1}^{n} \log\left(1 - \left(1 + \frac{\rho}{x_i}\right)^{-\lambda}\right) \quad (41)$$

$$\frac{\partial l}{\partial \rho} = \frac{n}{\rho} - (\lambda + 1)\sum_{i=1}^{n}\left[\frac{\left(1 - \frac{1}{x_i}\right)}{\left(1 - \frac{\rho}{x_i}\right)}\right] + \lambda(b-1)\sum_{i=1}^{n}\left[\frac{\left(1 + \frac{1}{x_i}\right)^{-\lambda-1}}{\left(1 - \left(1 + \frac{\rho}{x_i}\right)^{-\lambda}\right)}\right] \quad (42)$$

$$\frac{\partial l}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} \log\left(1 + \frac{\rho}{x_i}\right) + (b-1)\sum_{i=1}^{n}\left[\frac{\left(1 + \frac{\rho}{x_i}\right)^{-\lambda} \log\left(\left(1 + \frac{\rho}{x_i}\right)\right)}{\left(1 - \left(1 + \frac{\rho}{x_i}\right)^{-\lambda}\right)}\right] \quad (43)$$

V. Applications

In this section, two life-time data sets are provided to illustrate the importance of the $LT - 2IL$ distribution in modeling life-time data. We compare the $LT - 2IL$ model with other competitive models such as Kumaraswamy inverse Lomax (KIL), Kumaraswamy Frechet (KF), Exponentiated Lomax (EL), and the inverse Lomax (IL) distribution.

First, we consider the number of failures for the air conditioning system of jet airplanes. These data were reported by Cordeiro and Lemonte (2011) and Huang and Oluyede (2014): 194, 413, 90, 74, 55, 23, 97, 50, 359, 50, 130, 487, 57, 102, 15, 14, 10, 57, 320, 261, 51, 44, 9, 254, 493, 33, 18,209, 41, 58, 60, 48, 56, 87, 11, 102, 12, 5, 14, 14, 29, 37, 186, 29, 104, 7, 4, 72, 270, 283, 7, 61, 100, 61, 502, 220,120, 141, 22, 603, 35, 98, 54, 100, 11, 181, 65, 49, 12, 239, 14, 18, 39, 3, 12, 5, 32, 9, 438, 43, 134, 184, 20, 386, 182,71, 80, 188, 230, 152, 5, 36, 79, 59, 33, 246, 1, 79, 3, 27, 201, 84, 27, 156, 21, 16, 88, 130, 14, 118, 44, 15, 42, 106, 46,230, 26, 59, 153, 104, 20, 206, 5, 66, 34, 29, 26, 35, 5, 82, 31, 118, 326, 12, 54, 36, 34, 18, 25, 120, 31, 22, 18, 216, 139, 67, 310, 3, 46, 210, 57, 76, 14, 111, 97, 62, 39, 30, 7, 44, 11, 63, 23, 22, 23, 14, 18, 13, 34, 16, 18, 130, 90, 163, 208, 1,24, 70, 16, 101, 52, 208, 95, 62, 11, 191, 14, 71 .Some descriptive statistics for these data shows that the smallest and the largest values are 1 and 603, respectively. Further, the mean, median and variance are 92.07, 54.00 and 11645.93, respectively. The total time on test plot for the windshield data set is given in figure 4. The parameter estimates and the values for the goodness of fit test for the model is given is Table 1 and 2 respectively.

To check the adequacy of the fitted model in fitting the data considered, Akaike information criterion (AIC), consistent Akaike information criterion (CAIC),Bayesian information criterion (BIC), Kolmogorov-Smirnov(KS), Crammer-Von Misses (CM), Anderson Darling(AD) goodness of fit test and its p-value (PV) are obtained. In general, it is considered that the smaller the values of AIC, BIC, CAIC, HQIC and, K statistics and the larger the p-value, the better the fit of the model.
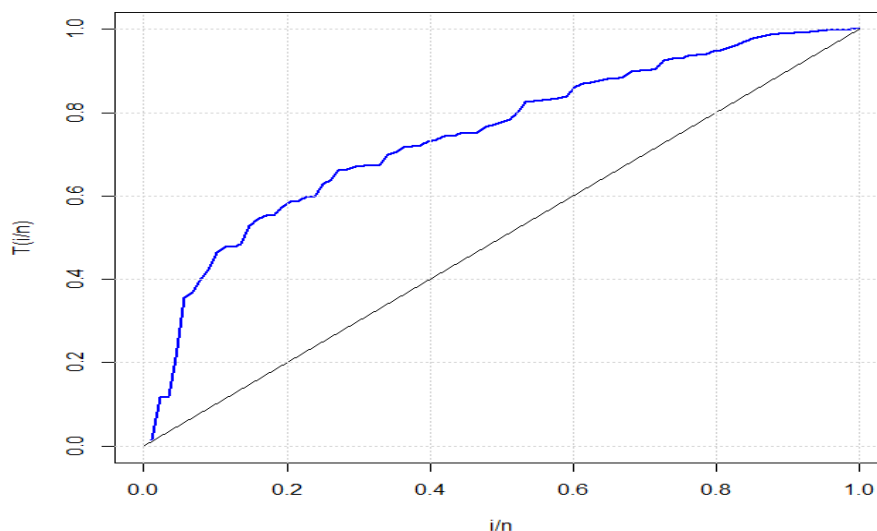
Figure 4: Graph of the total time on test plot for Jet air conditioning data

Table 1: MLEs and standard errors in braces for the first data set

Model	λ	ρ	b	a
$LT - 2IL$	1.247(0.199)	125.111(77.185)	2.7769(0.961)	−(−)
EL	0.6547(0.161)	1.2489(0.368)	15.558(4.407)	−(−)
KIL	0.7724(18.669)	144.4729(101.187)	1.565(37.835)	3.0006(1.204)
KF	0.6137(0.110)	0.6451(0.1625)	8.8399(1.675)	5.001(1.353)
IL	2.0790(0.395)	18.822(4.938)	−(−)	−(−)

Table 2: AIC, BIC, CAIC, CM, AD KS statistic, and P- value (PV) for first data set

Model	−l	AIC	BIC	CAIC	CM	AD	KS	PV
$LT - 2IL$	1034.77	2075.54	2085.25	2075.68	0.057	0.415	0.050	0.736
EL	1054.88	2115.77	2125.48	2115.90	0.357	2.393	0.089	0.099
KIL	1034.74	2077.69	2090.42	2077.69	0.061	0.433	0.049	0.741
KF	1039.52	2087.03	2099.98	2087.25	0.115	0.842	0.052	0.687
IL	1044.10	2092.20	2098.67	2092.26	0.173	1.175	0.068	0.343

The second data sets have been obtained from Murthy et al. (2004) is about the failure times of windshields and is given by 0.04, 0.3, 0.31, 0.557, 0.943, 1.07, 1.124, 1.248, 1.281, 1.281, 1.303, 1.432, 1.48,1.51, 1.51, 1.568, 1.615, 1.619, 1.652, 1.652, 1.757, 1.795, 1.866, 1.876, 1.899, 1.911, 1.912, 1.9141, 0.981, 2.010, 2.038, 2.085, 2.089, 2.097, 2.135, 2.154, 2.190, 2.194, 2.223, 2.224, 2.23, 2.3, 2.324, 2.349, 2.385, 2.481, 2.610, 2.625, 2.632, 2.646, 2.661, 2.688, 2.823, 2.89, 2.9, 2.934, 2.962, 2.964, 3, 3.1, 3.114, 3.117, 3.166, 3.344, 3.376, 3.385, 3.443, 3.467, 3.478, 3.578, 3.595, 3.699, 3.779, 3.924, 4.035, 4.121, 4.167, 4.240, 4.255, 4.278, 4.305, 4.376, 4.449, 4.485, 4.570, 4.602, 4.663, 4.694. Some descriptive statistics for these data shows that the smallest and the largest values are 0.04 and 4.694, respectively. Further, the mean, median and variance are 2.569, 2.367 and 1.286, respectively. The Total time on test plot for the windshield data set is given in figure 4. The parameter estimates and the values for the goodness of fit test for the model is given is Table 3 and 4 respectively.
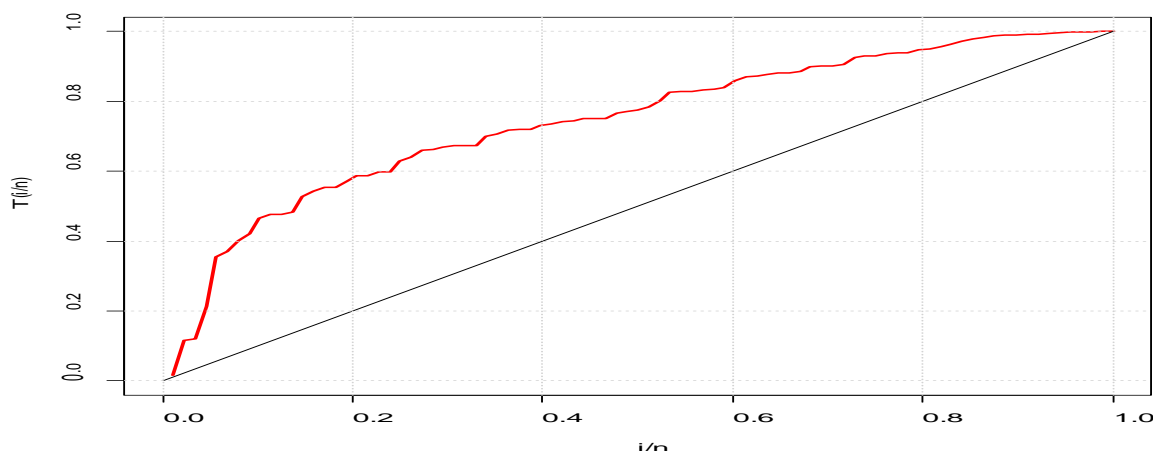
Notes

Figure 5: Graph of the total time on test plot for the second data

Table 3: MLEs and standard errors in braces for the second data set

Model	λ	ρ	b	a
$LT-2IL$	2.591(0.319)	16.442(8.468)	142.560(111.761)	−(−)
EL	1.584(0.351)	1.1683(0.348)	3.428(1.143)	−(−)
KIL	1.549(0.939)	4.3495(1.485)	1.939(1.176)	16.1458(5.434)
KF	0.7820(0.305)	1.0205(0.588)	7.163(3.946)	13.3993(8.685)
IL	4.123(1.321)	0.490(0.184)	−(−)	−(−)

Table 4: AIC, BIC, CAIC, CM, AD, KS statistic and P- value (PV) for the second data set

Model	−l	AIC	BIC	CAIC	CM	AD	KS	PV
$LT-2IL$		283.93	291.07	283.94	0.064	0.668	0.057	0.929
EL	166.92	339.84	347.27	340.13	0.705	4.571	0.169	0.013
KIL	143.95	295.89	305.80	296.38	0.137	1.203	0.094	0.408
KF	150.68	309.83	319.26	309.83	0.272	2.078	0.121	0.152
IL	187.02	378.05	383.0	378.19	0.825	5.233	0.336	$4.6e-09$

It could be observed from the results obtained from the two data sets considered that the Lehmann Type-2 Inverse Lomax model possesses the smallest AIC, BIC, CAIC, CM, AD, KS statistic and, the largest of a P-value. It could therefore be regarded as the best model in the class of the models considered based on the data used.

VI. Concluding Remarks

In this paper, we developed a study a novel three-parameter distribution called Lehmann-type-2 Inverse distribution. Some statistical properties of the new distribution are studied. The maximum likelihood estimation method is used to obtain the parameters. Two real data sets are presented to illustrate the applicability of the new model.

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